Tensor Network applications to quantum computing

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Esa ESRIN, October 12th

T-NiSQ
Tensor Networks in Simulation of Quantum Matter

Dipartimento di Fisica e Astronomia Galileo Galilei

QUANTUM COMPUTING AND SIMULATION CENTER

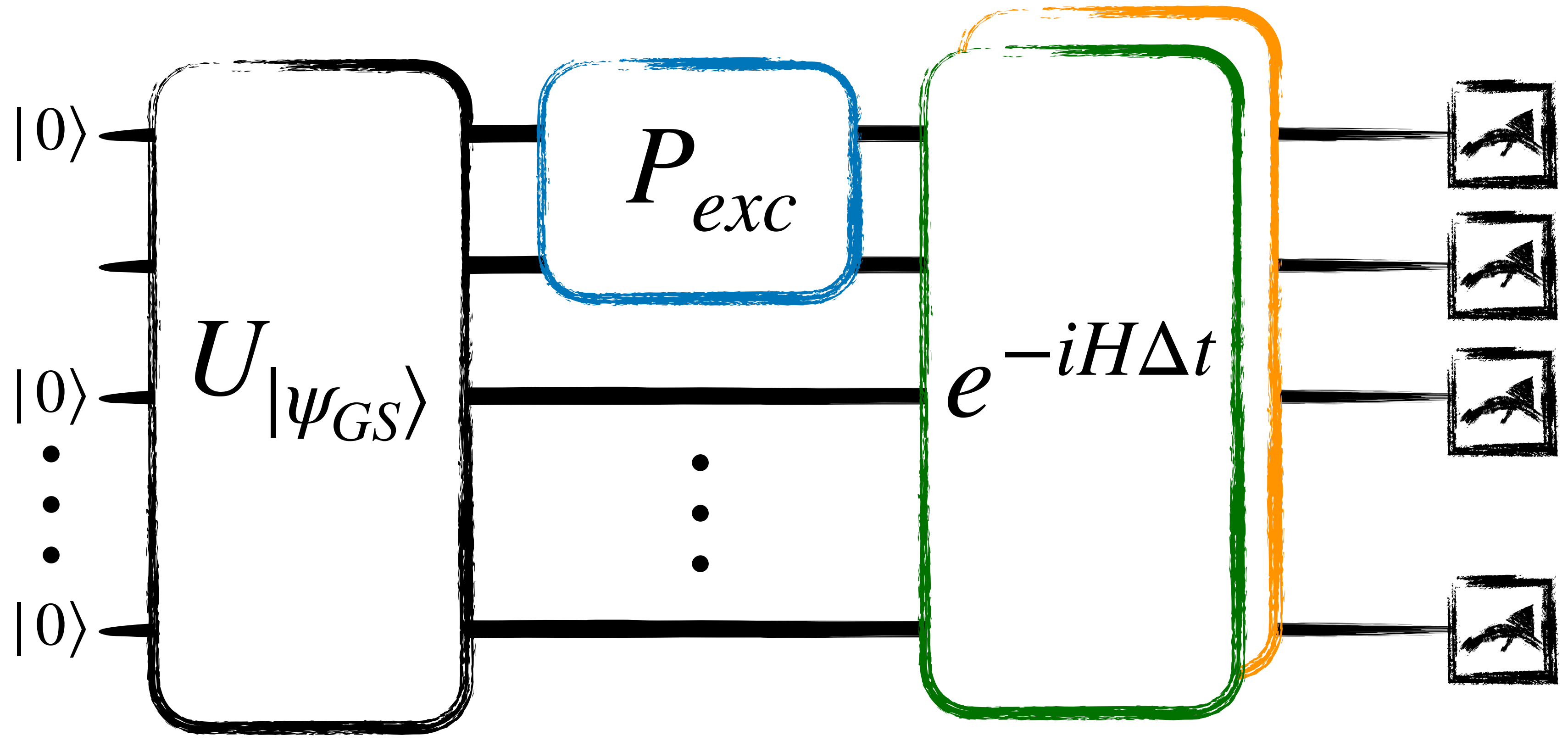
Quantum Circuit Emulator

https://baltig.infn.it/quantum_tea

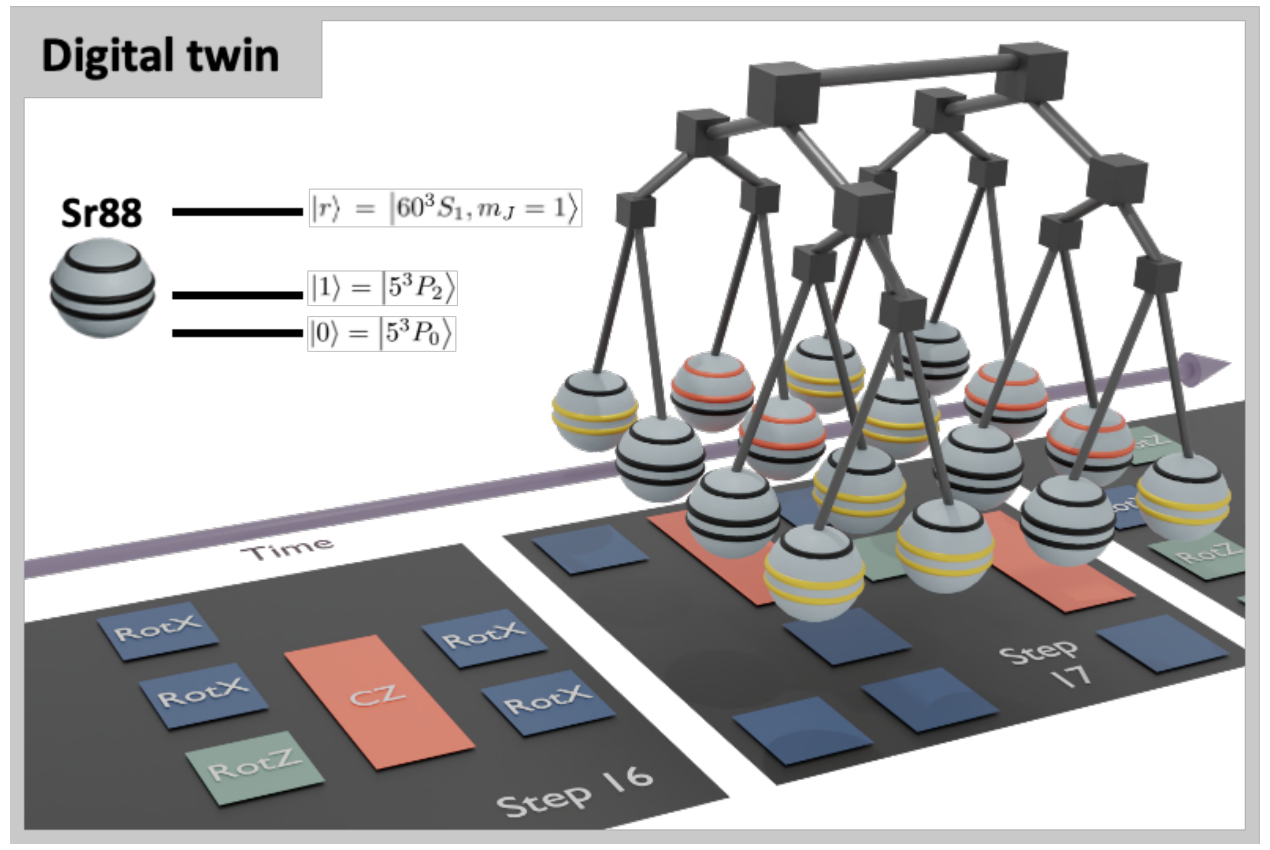
Ground state search & time evolution Many Body systems

Digital quantum simulation

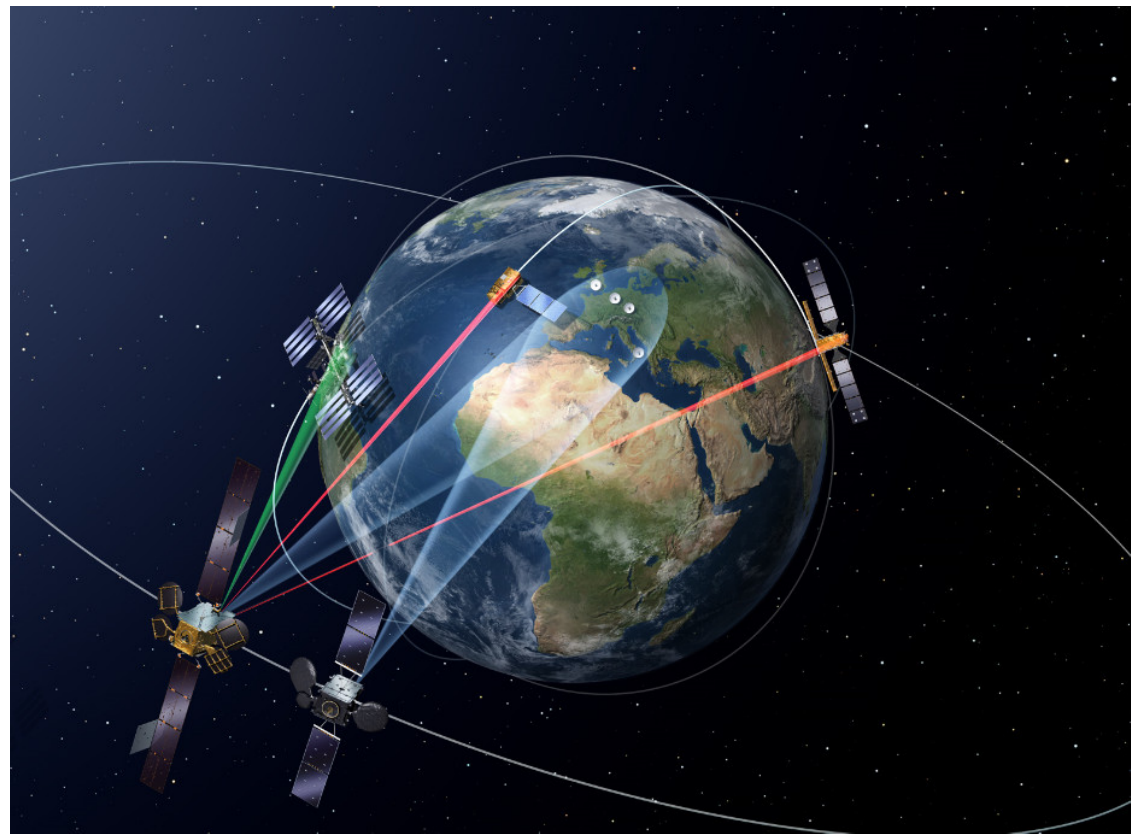
Variational Circuits



Digital Twin



Hard-Optimization problems



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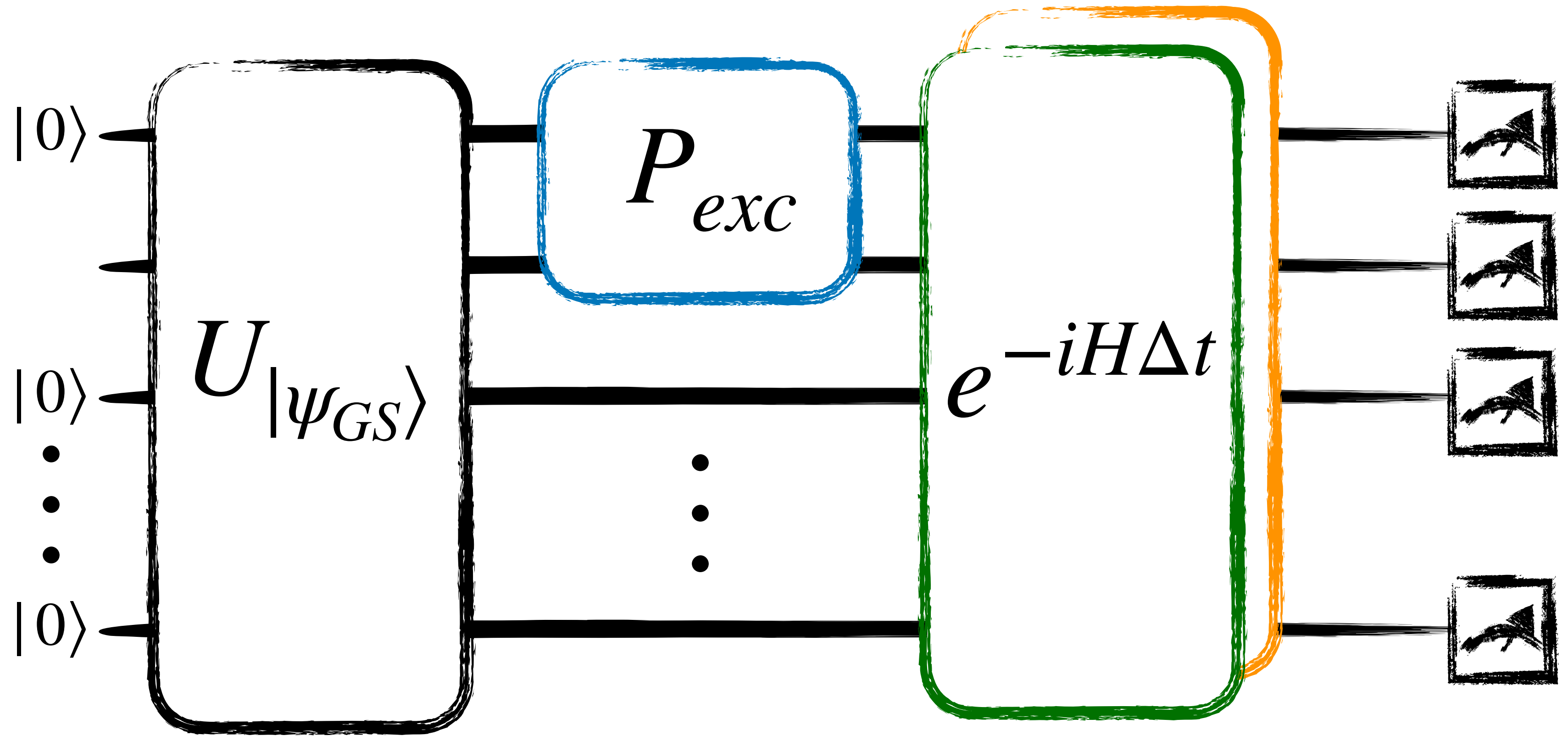
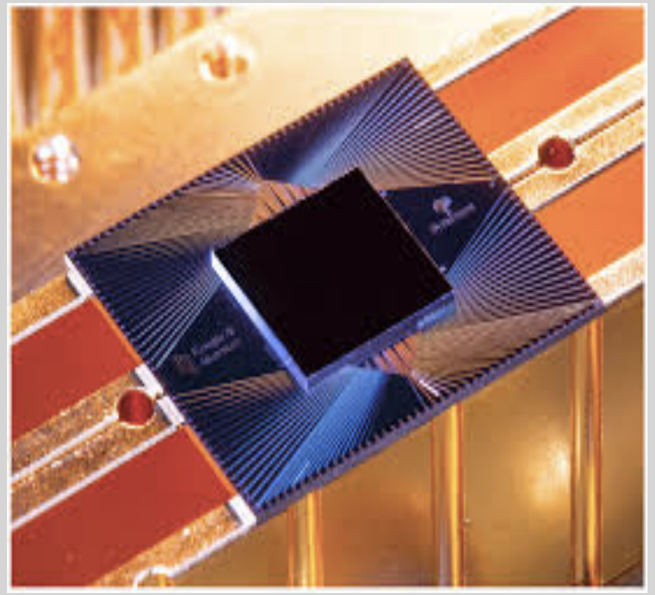


Digital Twin

Hard-Optimization problems

Running quantum algorithms



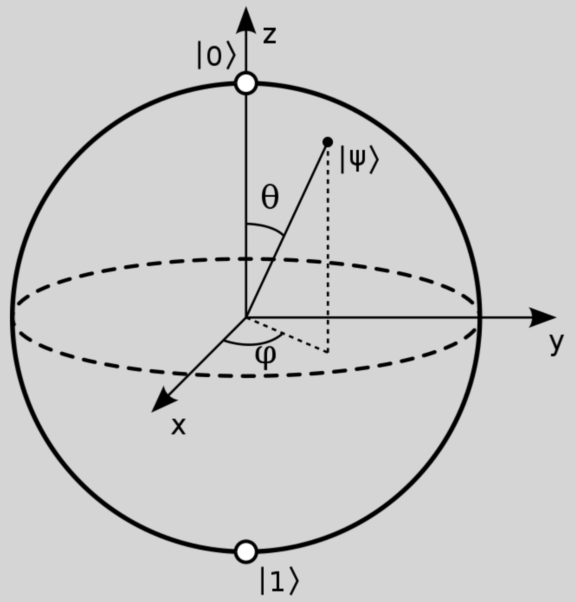
+ Real hardware
- Noisy
- Limited number of qubits

Quantum hardware

High # of qubits +
Flexibility (observables) -
Depth of the circuit -

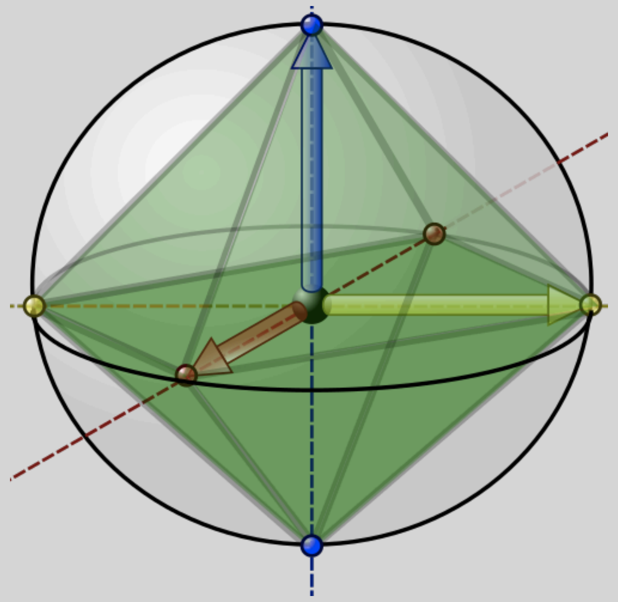
cuQuantum

Quantum algorithm



+ Access to exact state
- Limited number of qubits

Exact simulator



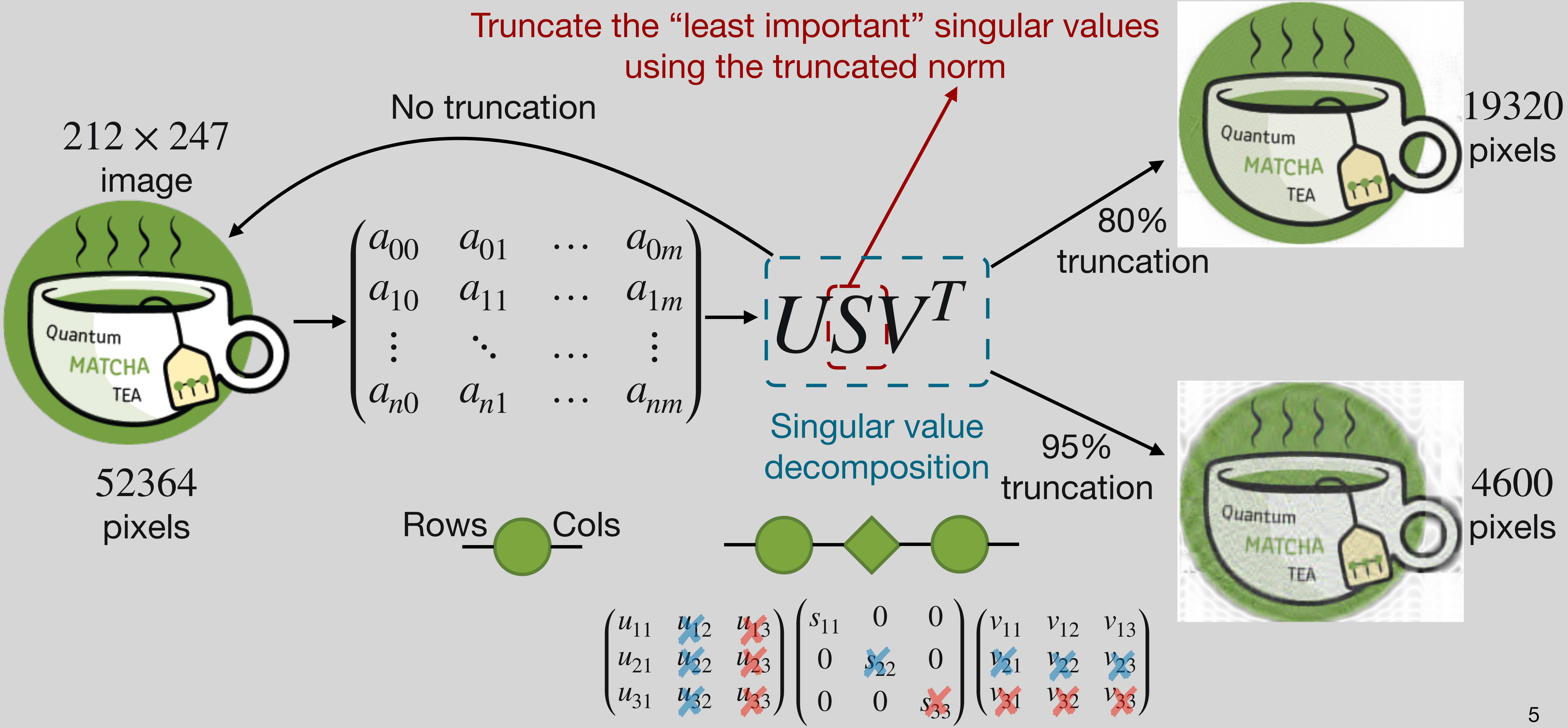
High # of qubits +
Flexibility (# of T gates) -

Clifford simulator

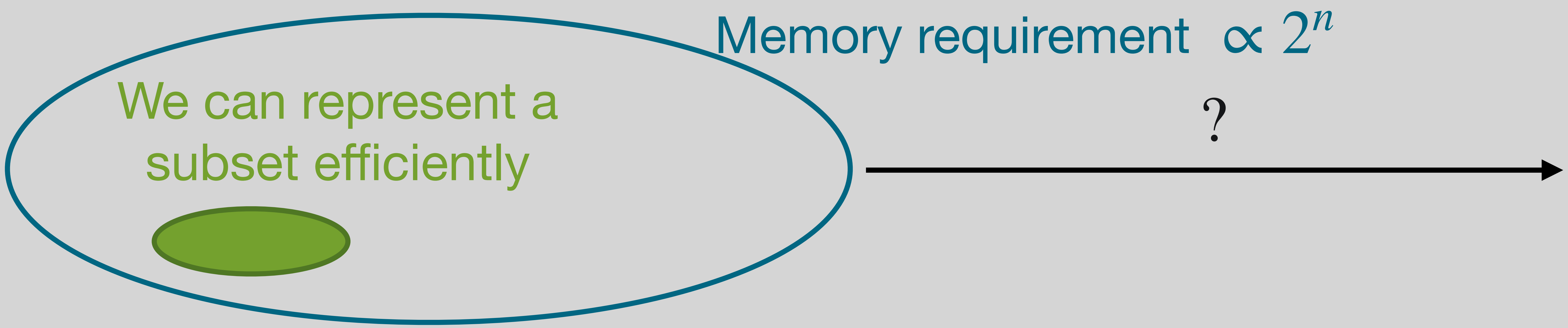
Tensor Network simulator

+ High # of qubits
- Flexibility (entanglement)

Image compression through SVD

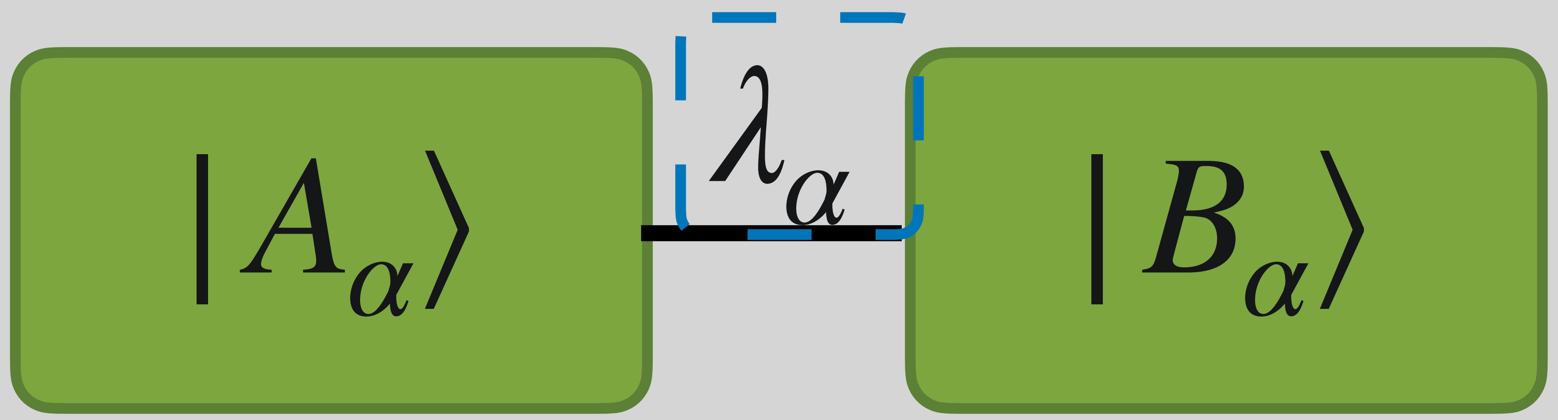


Entanglement and compression



Tensor networks compress the quantum correlations between subsystems ⇒ **compress entanglement**

$$|\psi\rangle = \sum_{\alpha=1}^{\chi}$$



Only keep highest χ Schmidt

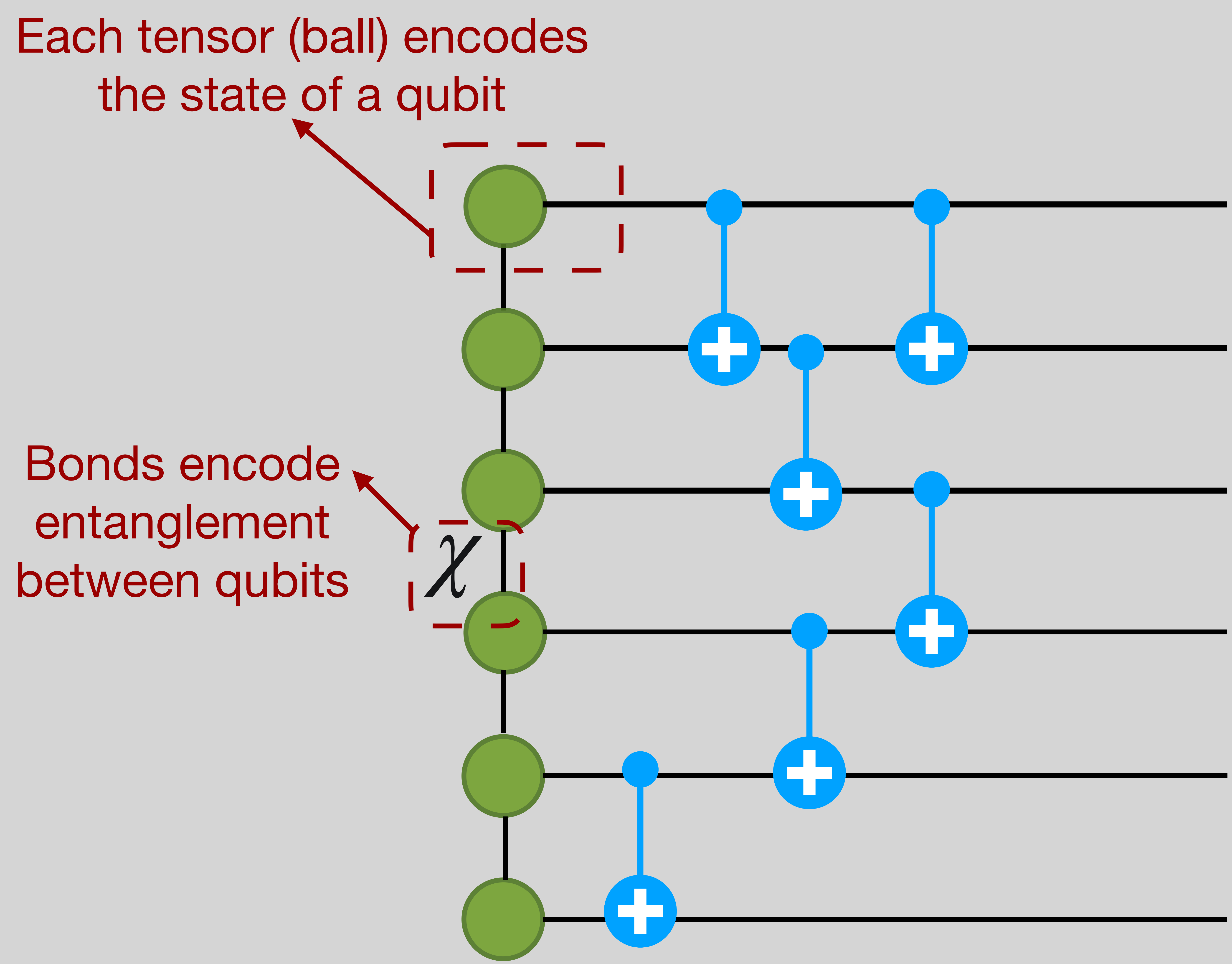
By considering only the highest χ Schmidt values we perform the BEST approximation in terms of entanglement

$$S_V = -Tr(\rho_A \ln \rho_A) = -Tr(\rho_B \ln \rho_B) \longrightarrow S_V = -\sum_{\alpha=1}^{N} \lambda_\alpha^2 \ln \lambda_\alpha^2$$

$$\rho_A = Tr_B(\rho) = Tr_B(|\psi\rangle\langle\psi|) = \sum_{\alpha=1}^{N} \lambda_i^2 |A_\alpha\rangle\langle A_\alpha|$$

Quantum circuit emulator

Memory requirements $O(2^n) \rightarrow O(2n\chi^2)$



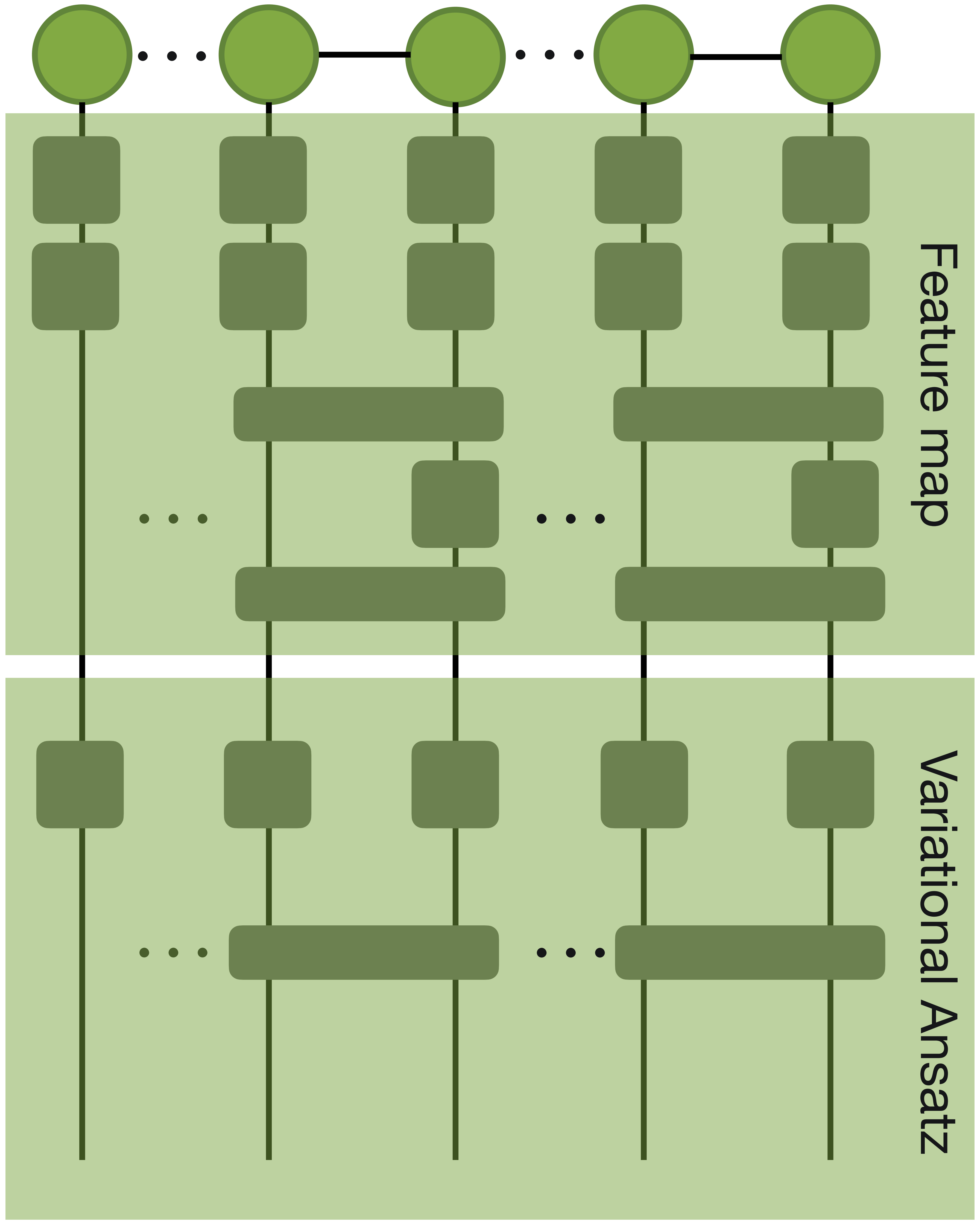
MPS SIMULATIONS ARE NOT LIMITED BY THE NUMBER OF QUBITS BUT BY THE ENTANGLEMENT

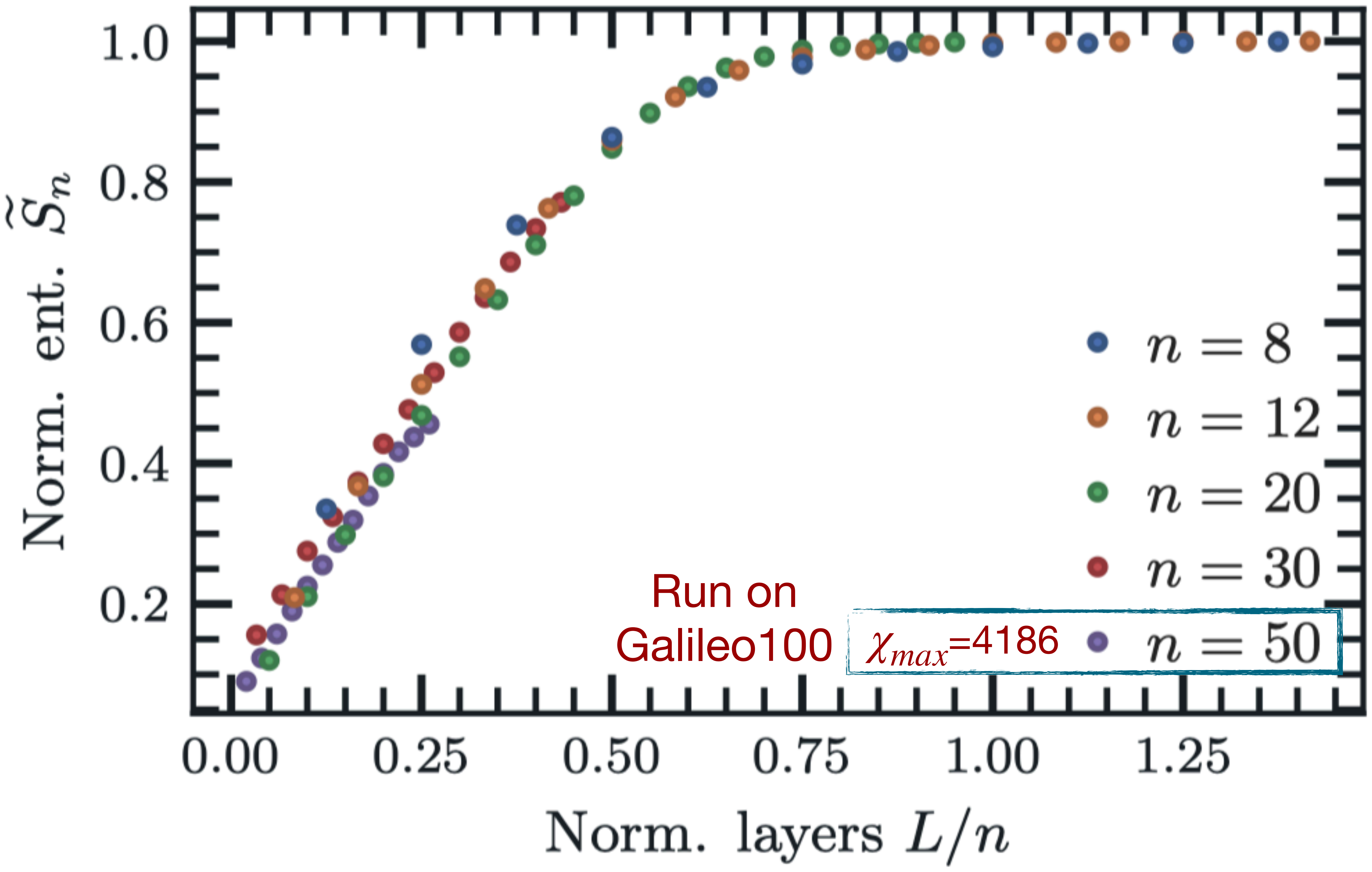
- MPS: efficient representation of the state
- Simulation of quantum circuits: gates are applied as matrices
- Measurement of local observables
- Efficient sampling of the final state

•Benchmarking quantum algorithms

https://baltig.infn.it/quantum_matcha_tea

Entanglement generation in QNN





M. Ballarin, …,S. Montangero, Quantum 7, 1023 (2023)

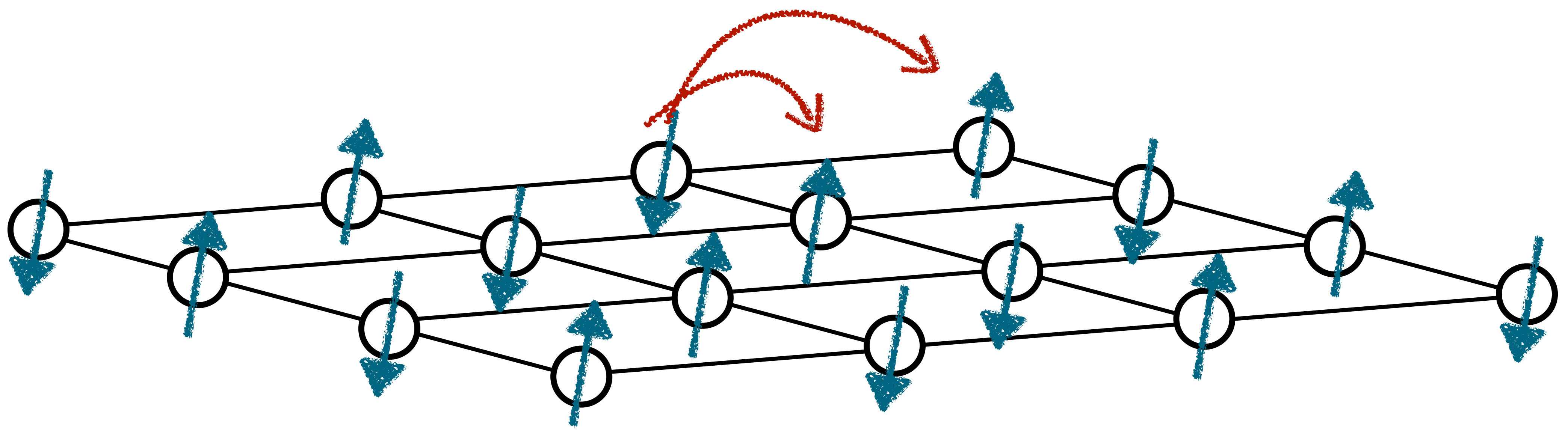
Digital quantum simulation: 2D Fermi-Hubbard

ENCODING

Fermionic theories

↓

Computational qubits



$$H_{Hubb} = -t \sum_{f=\uparrow,\downarrow} \sum_{j} \sum_{\mu=\mu_x,\mu_y} [\psi^{\dagger}_{j,f}\psi_{j+\mu,f} + \psi^{\dagger}_{j+\mu,f}\psi_{j,f}] + U \sum_{j} V(n_{j,f})$$

LATTICE

1D —> Jordan-Wigner (local)

2D —> Jordan-Wigner (non local)

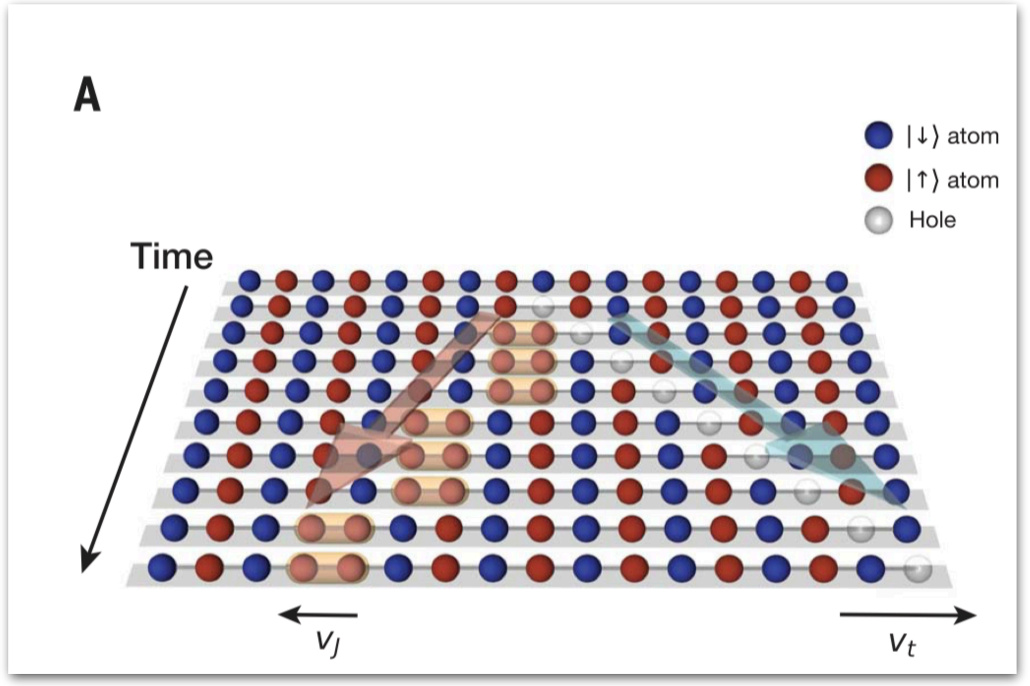
Other methods: gauge defermionization

our mapping

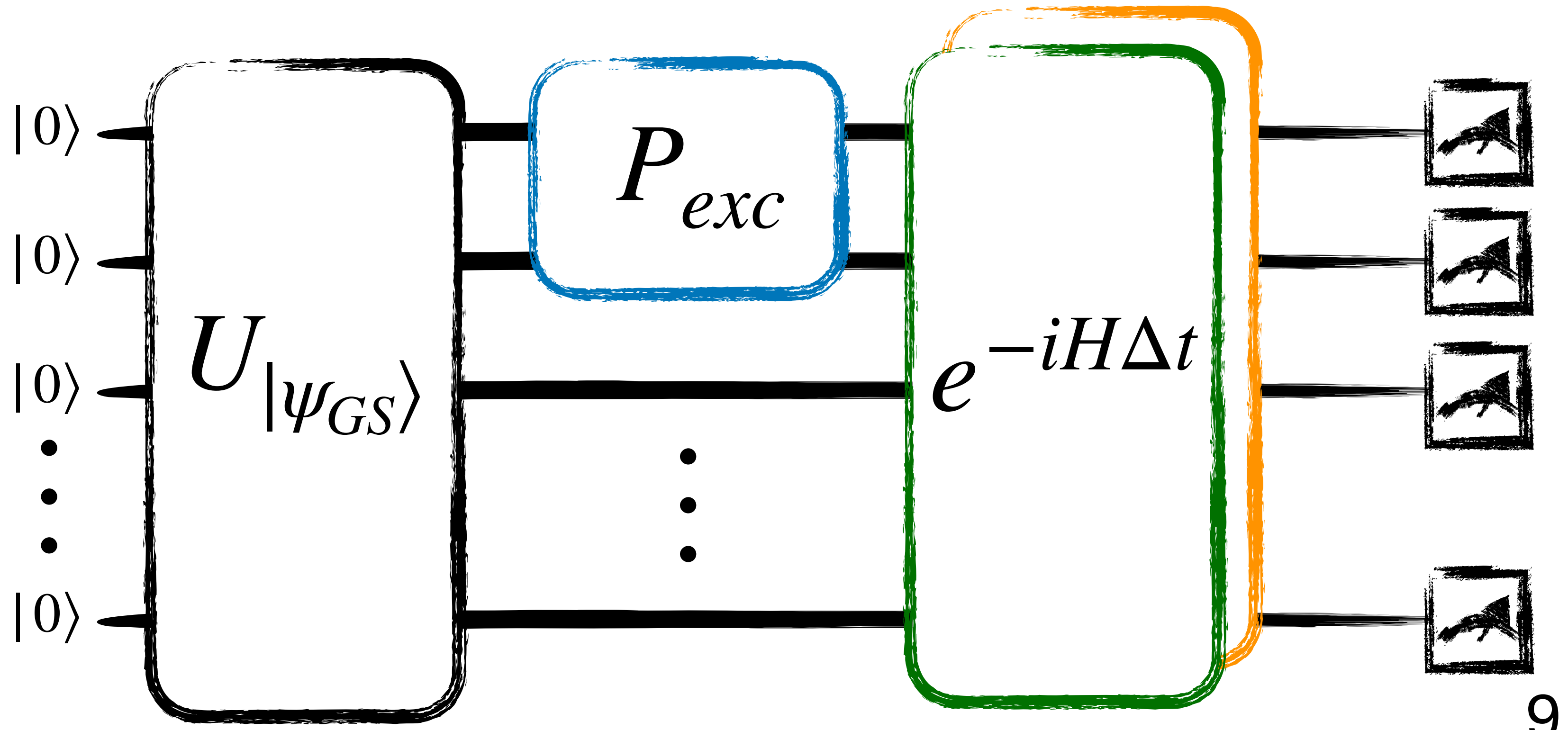
Fermions to qubit ratio	3
Fermion parity weight	1
Hopping weight	6
Stabilizer weight	6

REAL TIME DYNAMICS

Spin-charge separation over a 4x2 lattice



Vijayan et al., Science 367, 186–189 (2020)



Digital quantum simulation

GROUND STATE PREPARATION

- GS @ t=0.1, U=1 (repulsive), half-filling (Insulating regime)

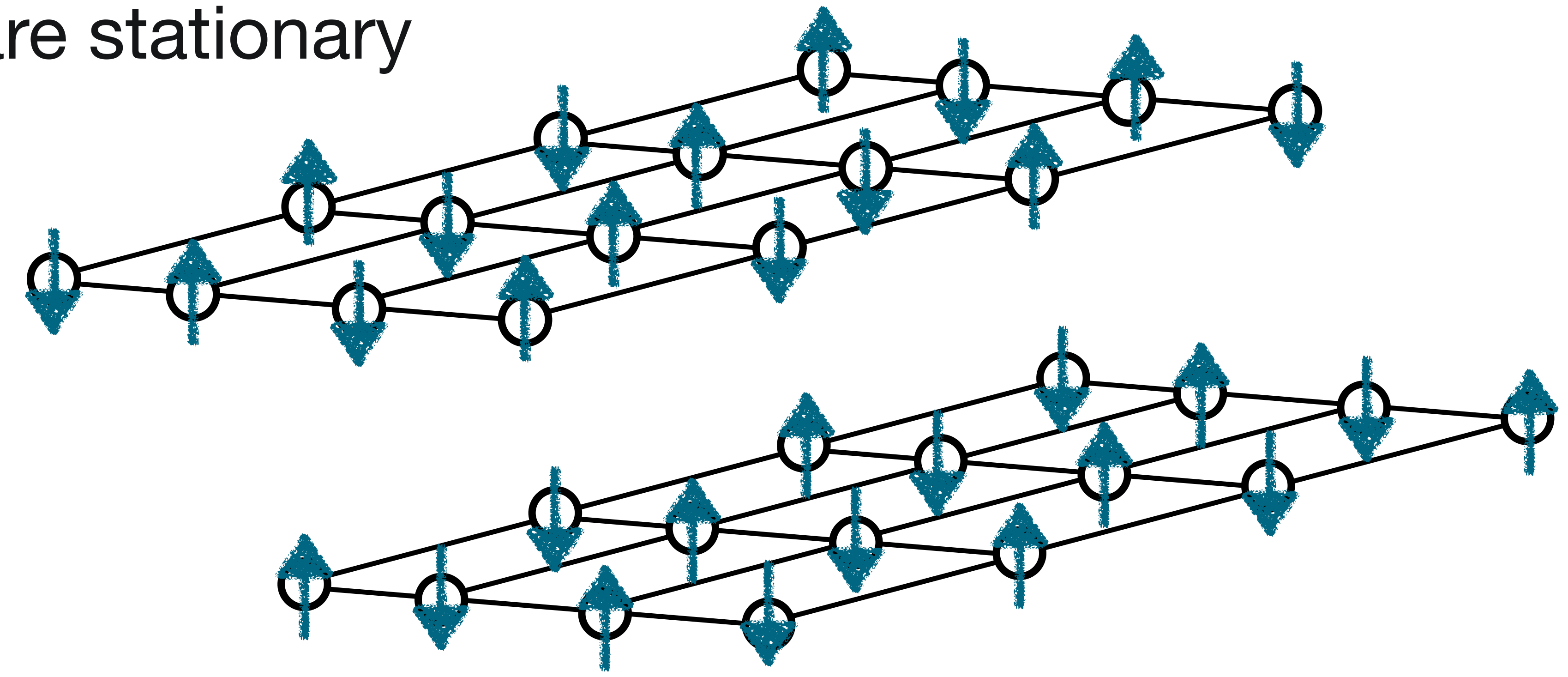
- ADIABATIC PREPARATION

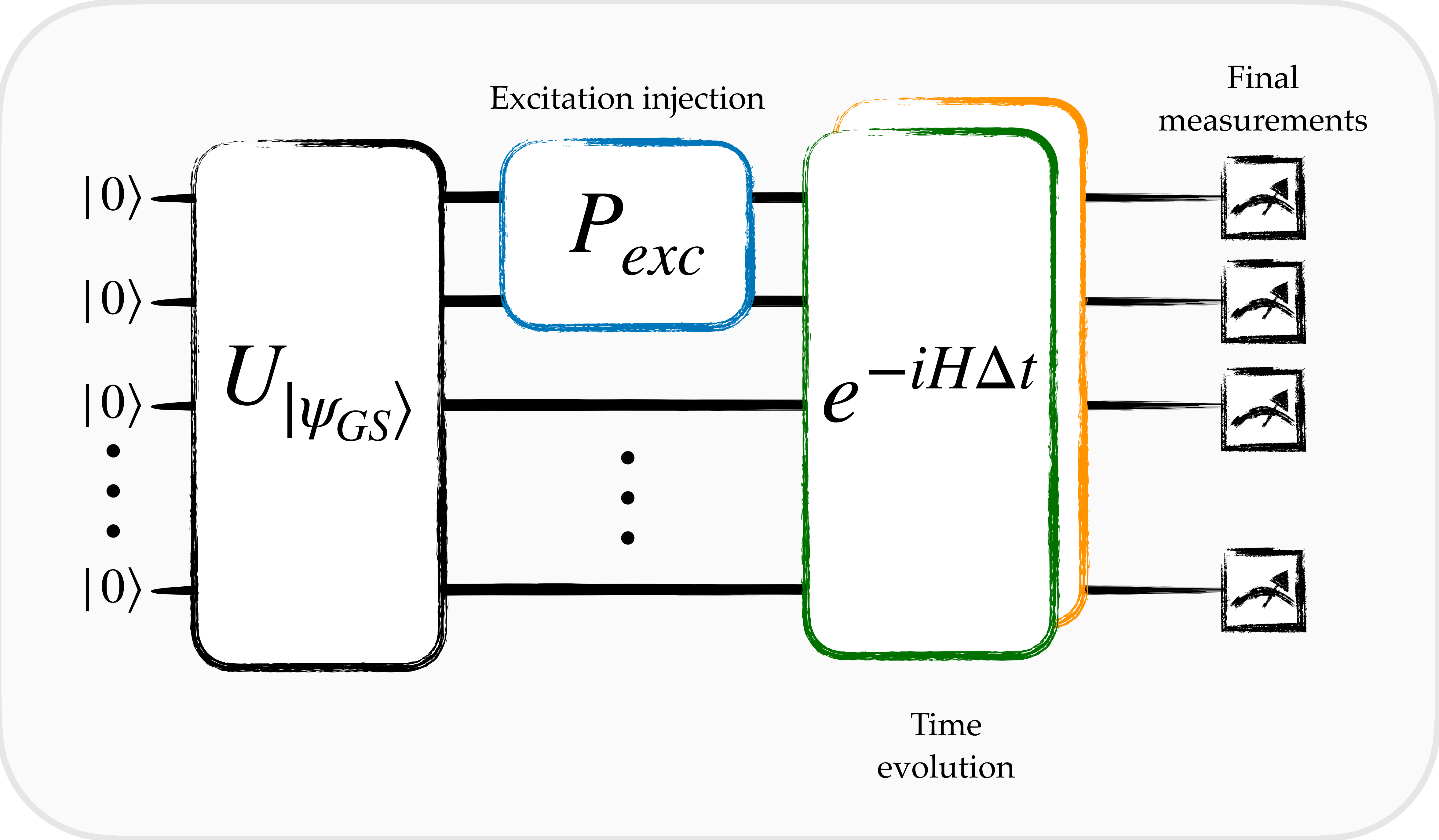
Slowly turning on hopping part

$$H = (1 - \beta)H_0 + \beta H_1$$

$$H_0 = H_{Hubb}(t = 0)$$
$$H_1 = H_{Hubb}$$

- Check local spin S_j^z and charge n_j are stationary





TIME EVOLUTION

- Trotterization of the evolution operator;

- Bond dimension 1024

- Measuring local observables;

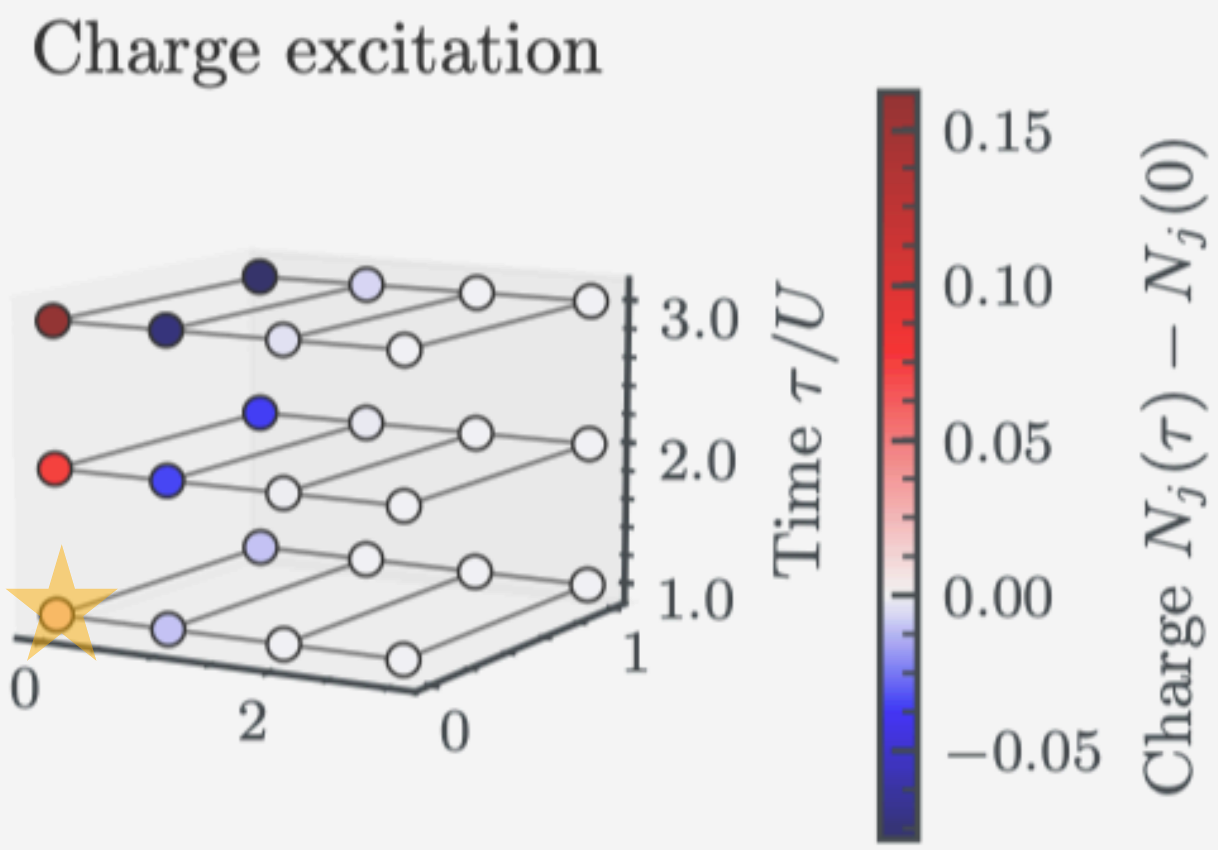
- Charge deviation from initial conditions:
  $N_j(\tau) = \langle n_{\uparrow,j}\rangle(\tau) + \langle n_{\downarrow,j}\rangle(\tau)$

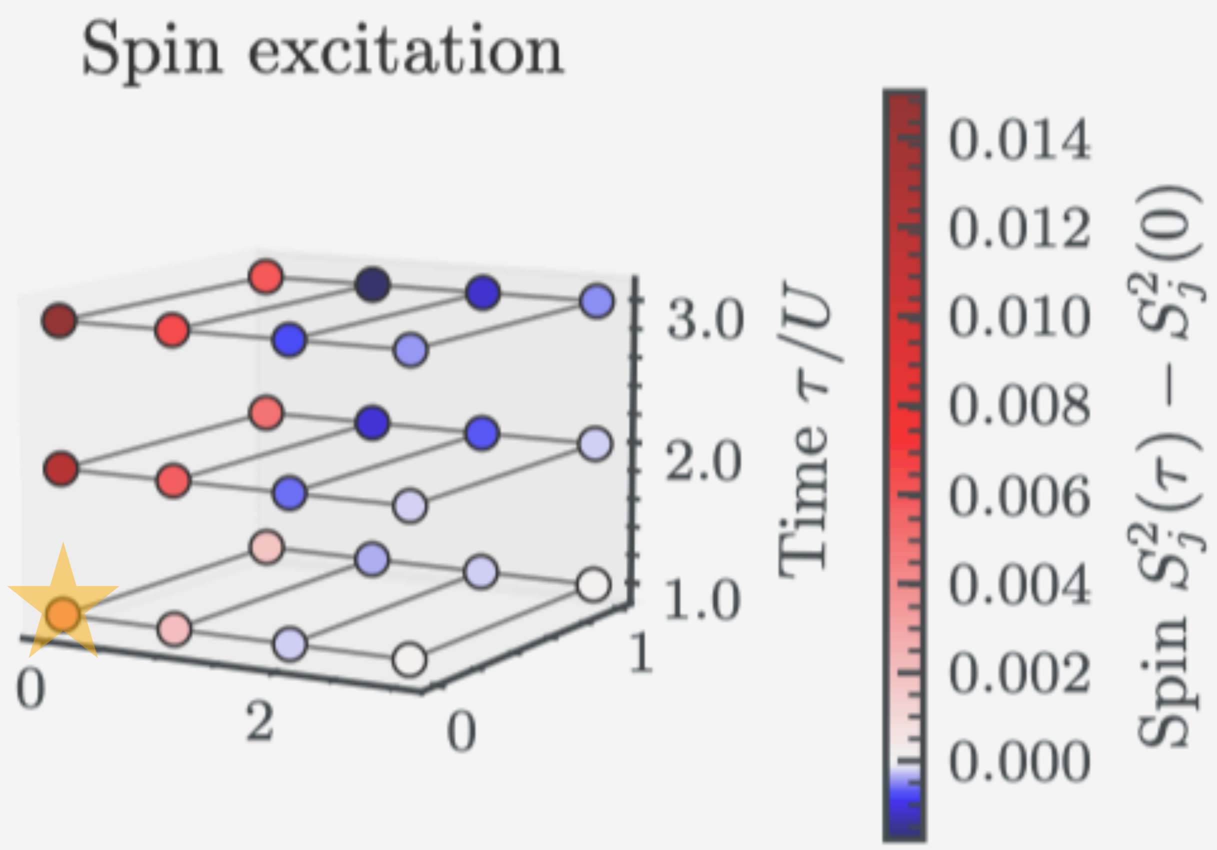
- Spin deviation from initial conditions:
  $S_j^2(\tau) = \langle n_{\uparrow,j}\rangle(\tau) + \langle n_{\downarrow,j}\rangle(\tau) - 2\langle n_{\uparrow,j} n_{\downarrow,j}\rangle(\tau)$

Spin-charge separation

Charge removal in (0,0)

$|1_u 0_s\rangle \longrightarrow |0_u 1_s\rangle$



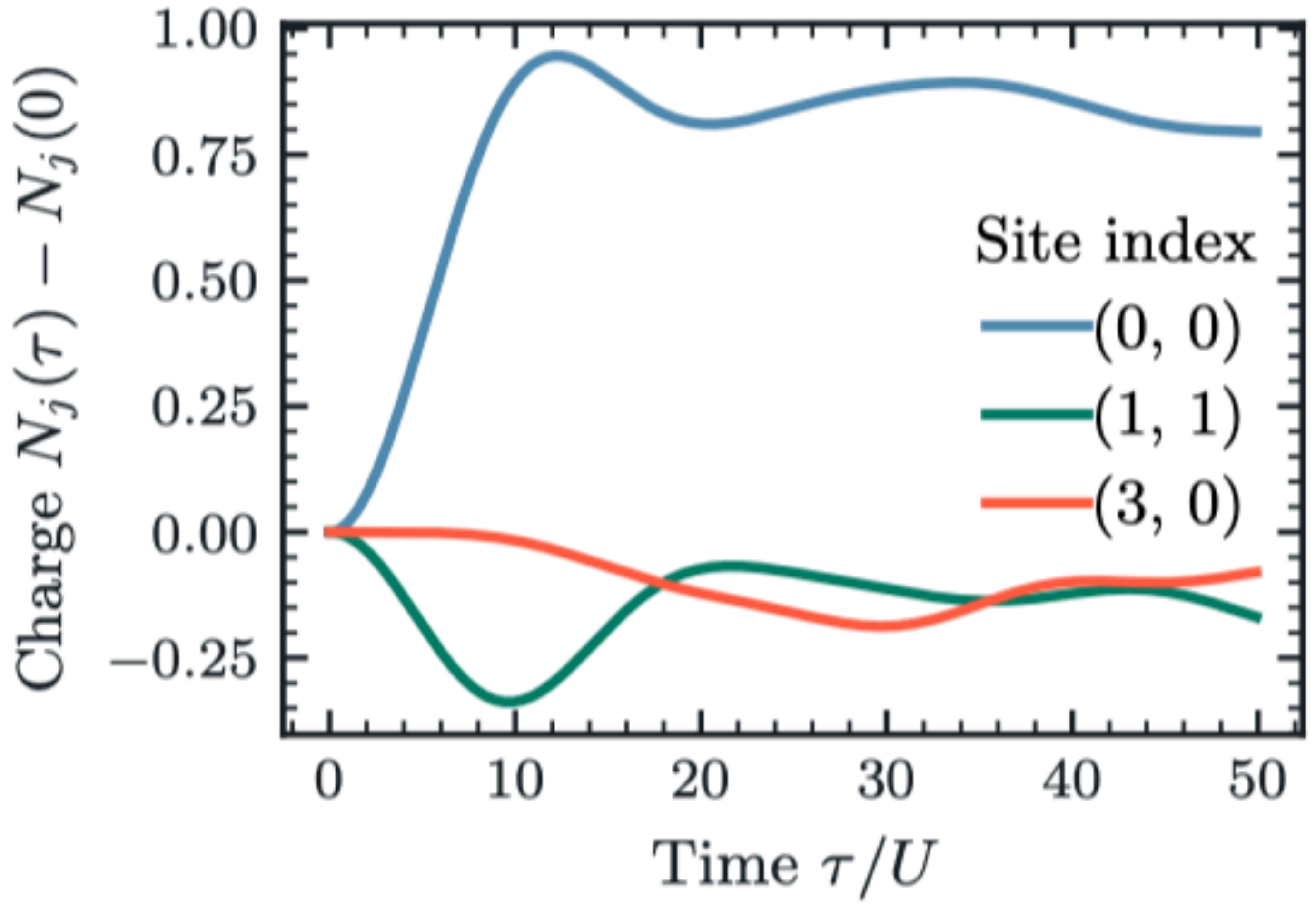


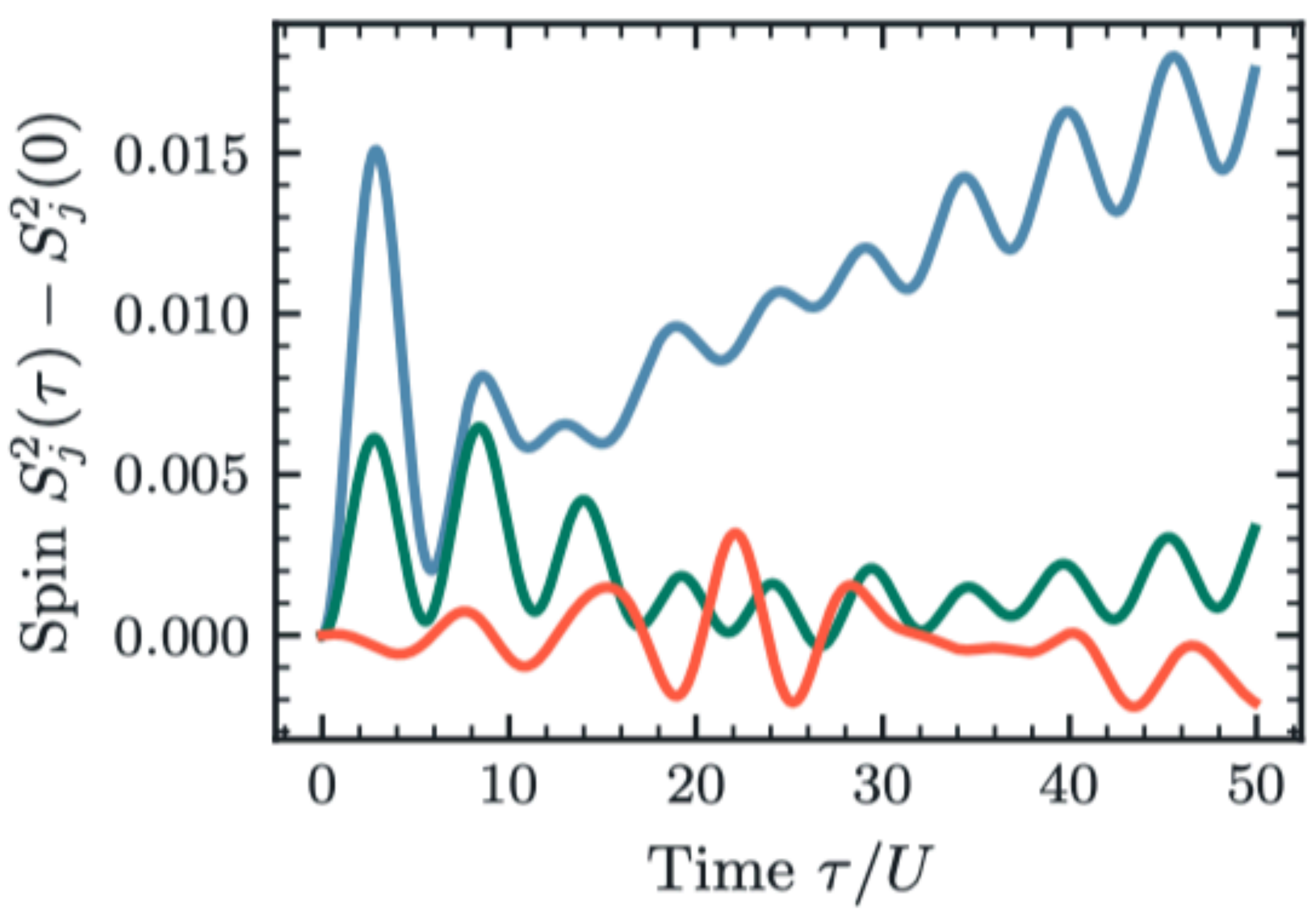
Spin excitation in (0,0)

$|\uparrow\rangle \rightarrow |\downarrow\rangle$

$|1_u 0_d\rangle \longrightarrow |0_u 1_d\rangle$

Slower dynamics





Faster dynamics

M. Ballarin, G. Cataldi, G. Magnifico, D. Jascke, IS, S. Montangero, P. SIlvi, to be submitted (2023)

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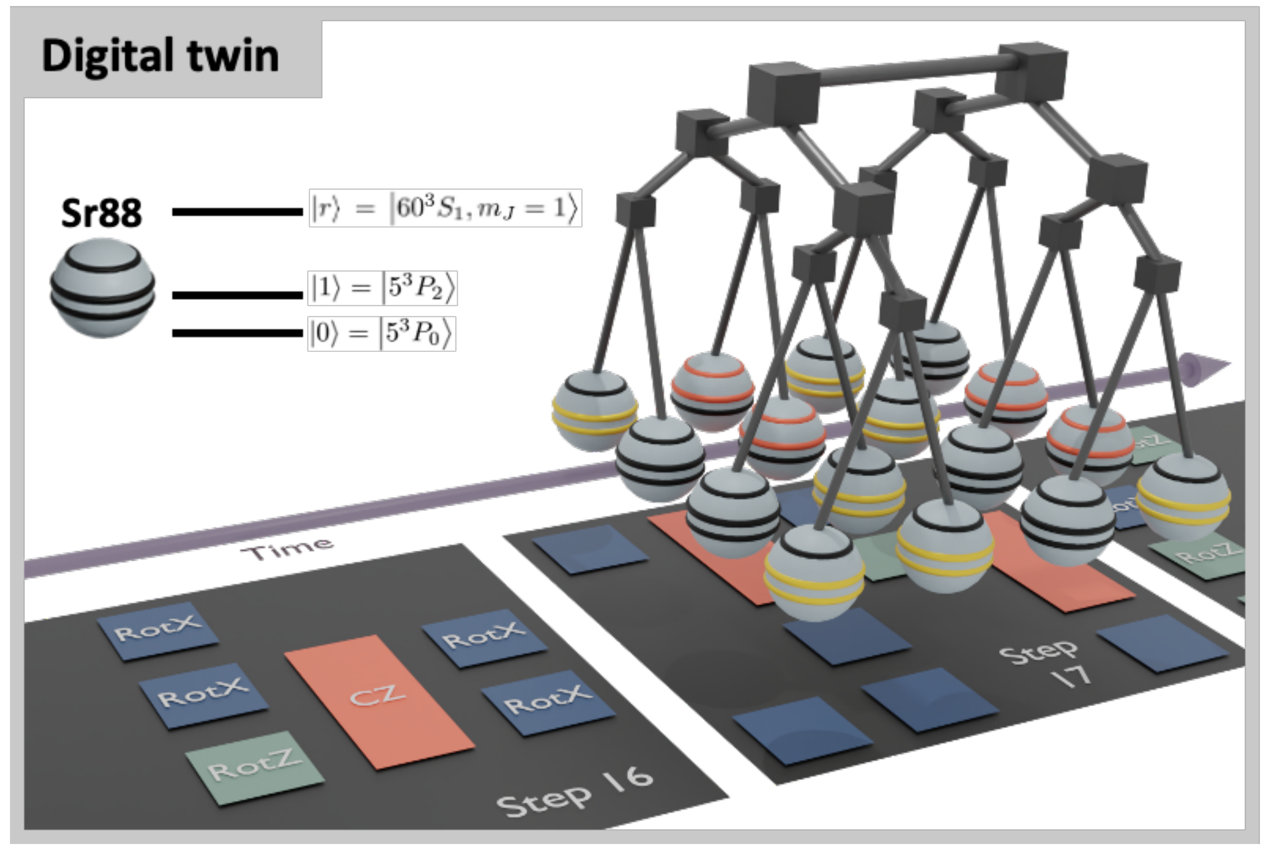
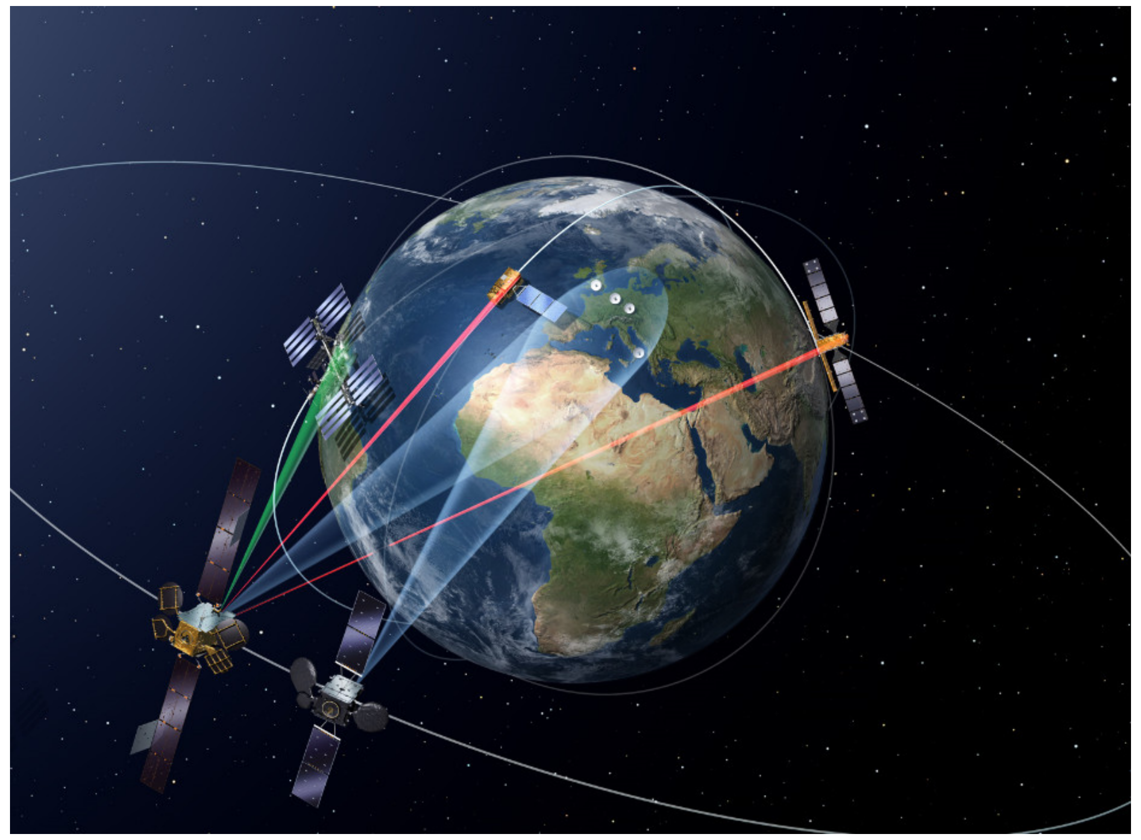
Digital Twin



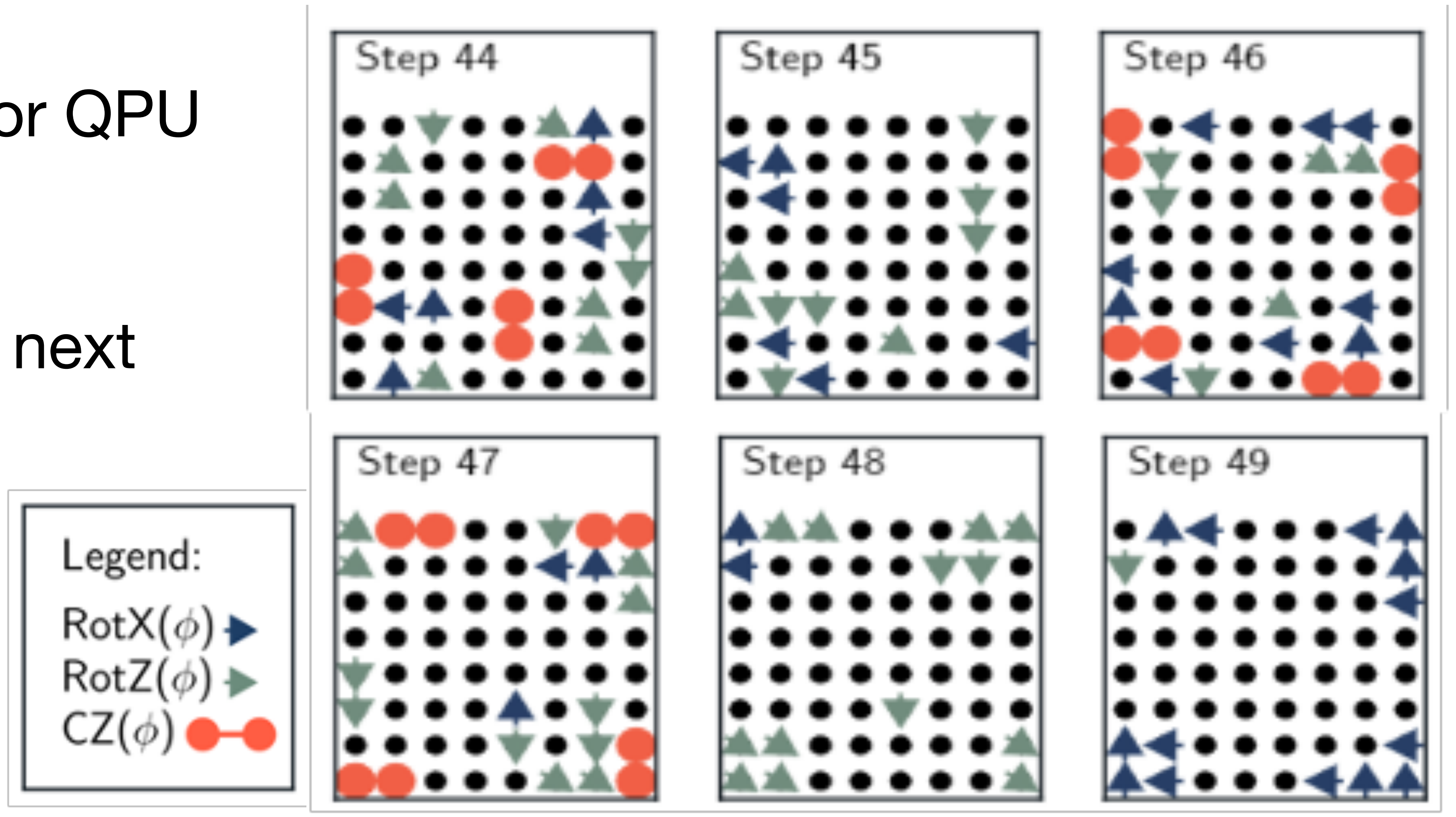
Hard-Optimization problems



Digital twin for Rydberg QPU

GOAL

- Gain insights on quantum hardware for QPU development
- Large scale simulation to support the next decades of hardware developments

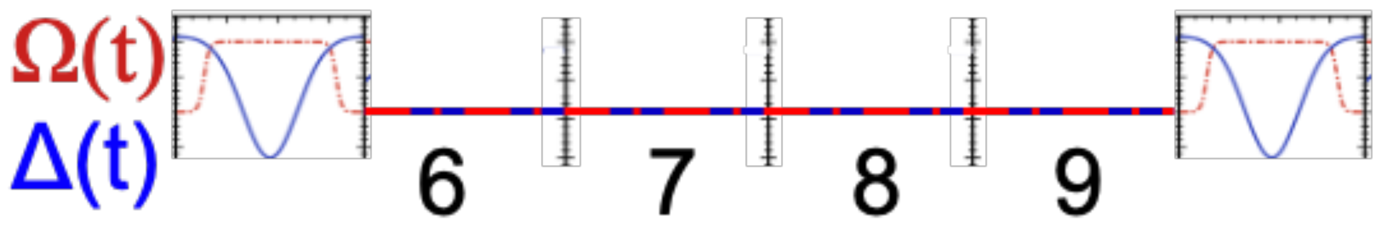


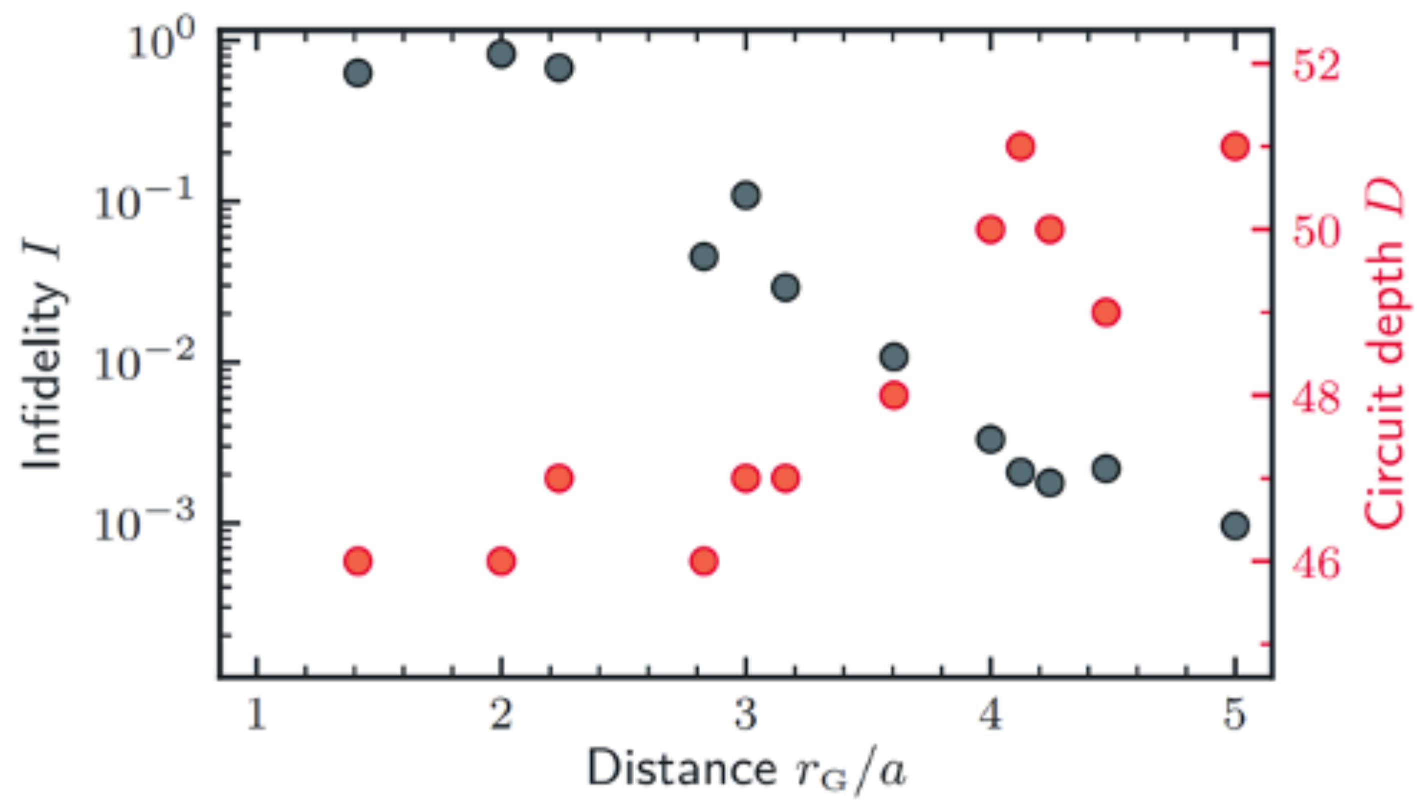
QUESTION

Quantify crosstalk between CZ gates executed in parallel during the preparation of a global GHZ state in an 8x8 Rydberg array

Find minimal distance r_g required between CZ gates in parallel to have crosstalk negligible (I~10^(-3))

$$F = |\langle \psi(\tau) | \psi_{\mathrm{GHZ}} \rangle|^2 \qquad I = 1 - F$$





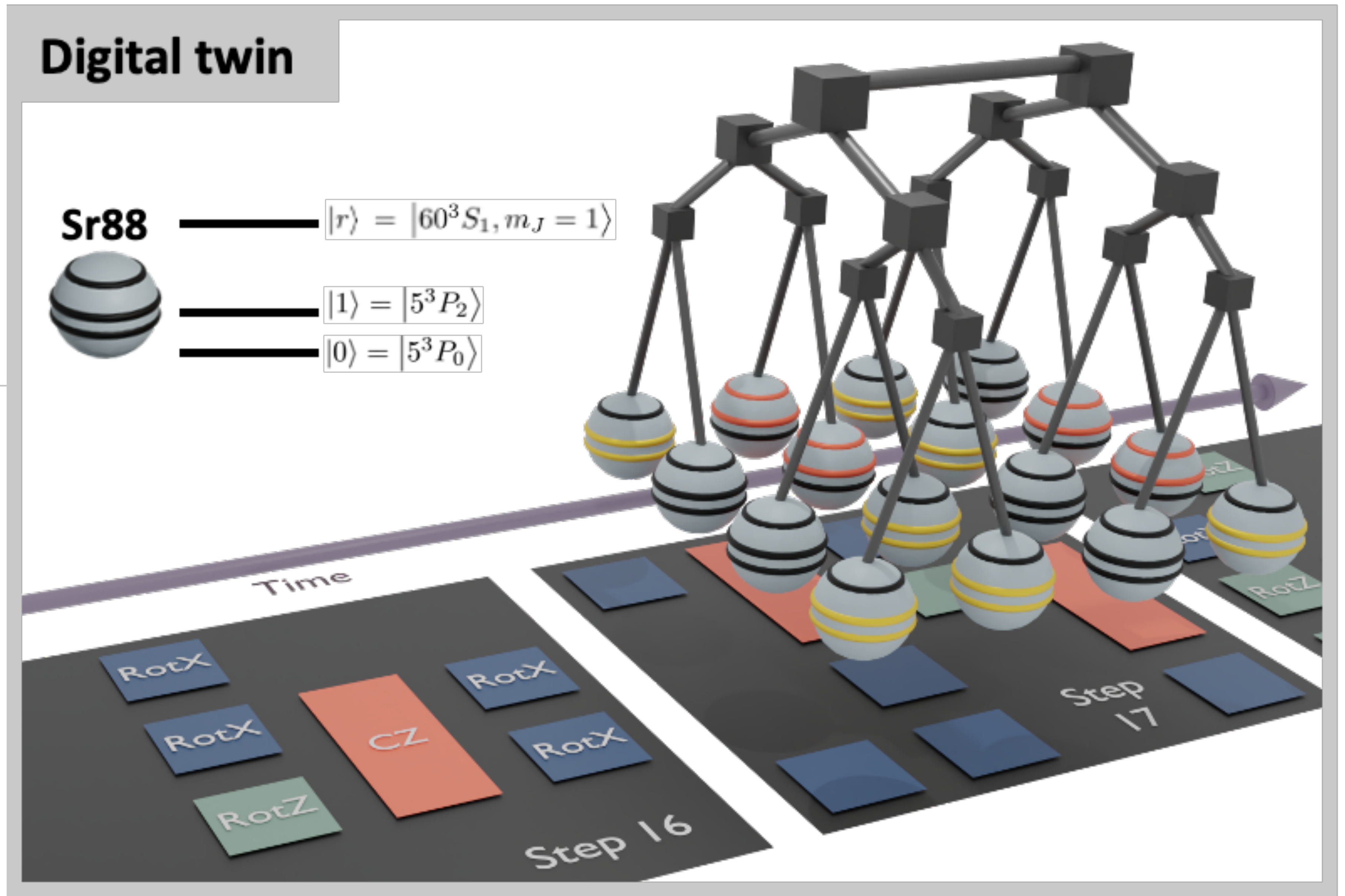
Hamiltonian description

Optimal pulses

\+

Compiler & Scheduler

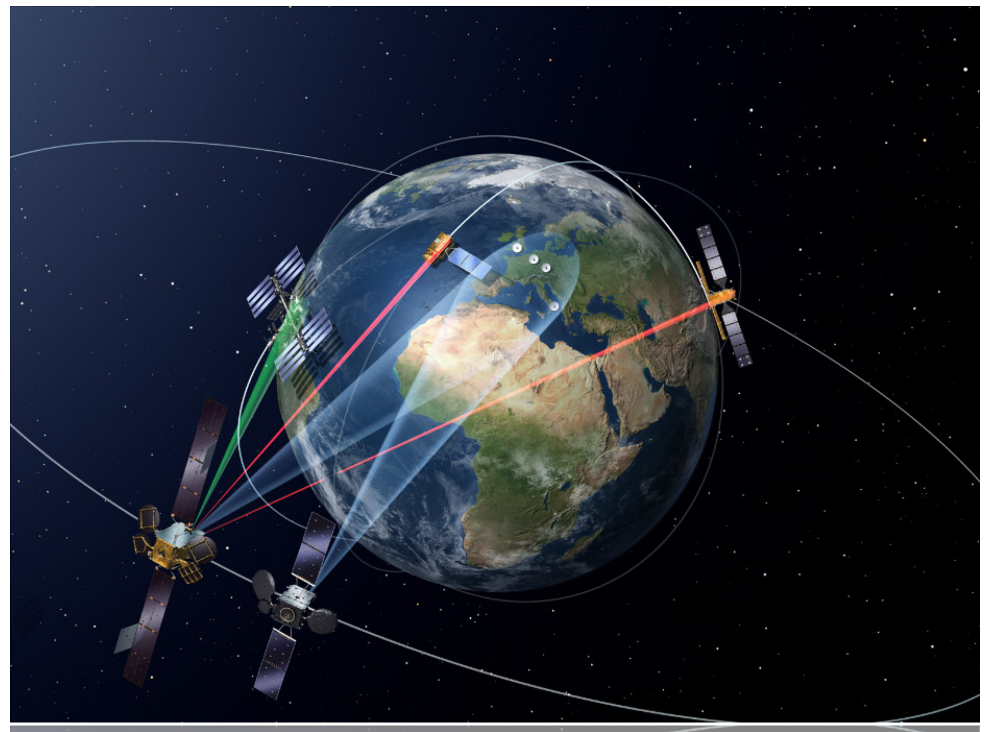
TTN simulation



D. Jaschke, ..., A. Pagano et al, arxiv 2210.03763 (2022)

Mission planning for earth observation

ThalesAlenia Space
a Thales / Leonardo company



Knapsack problem

$$\min_{\{\vec{x},\vec{y}\}} \mathscr{C}_{EO}(\vec{x},\vec{y})$$

$$\mathscr{C}_{EO}(\vec{x},\vec{y}) = \sum_{i,j,k,r}^{n,M,\theta_{i,j},\sigma_i} \left(\alpha + \frac{\gamma}{M}\right) t_{i,k}^{(k)} x_{i,j}^{(k)} + \beta s_{i,r}^{(r)} y_{i,j}^{(r)}$$

+ constraints

QUBO
(Quadratic Unconstrained Binary Optimization)

$$\mathscr{C}_{QUBO}(\vec{x}) = \sum_{j,k} Q_{j,k} x_j x_k$$

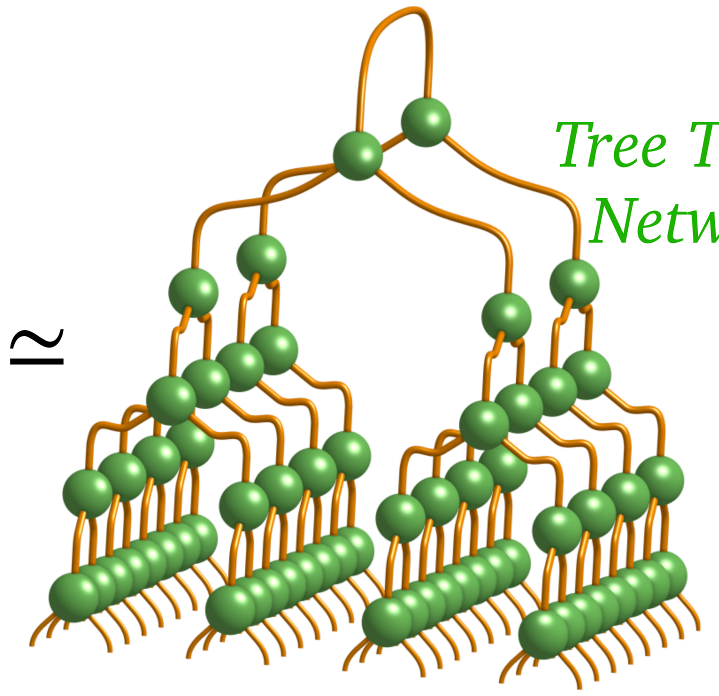
$$\vec{x} = \boxed{0\,0\,1\,0\,1\,1\,1\,0} \cdots \boxed{0\,1\,0\,0}$$

Binaries $10^6 - 10^9$

$$\vec{s} = \downarrow \downarrow \uparrow \downarrow \uparrow \uparrow \uparrow \cdots \downarrow \downarrow$$

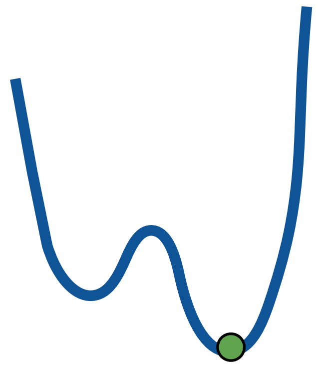
$$\min_{\vec{s}} \mathscr{H}_{EO}(\bullet \bullet \bullet \bullet \cdots \bullet \bullet)$$

$|\psi\rangle \simeq$



Quantum GREEN TEA

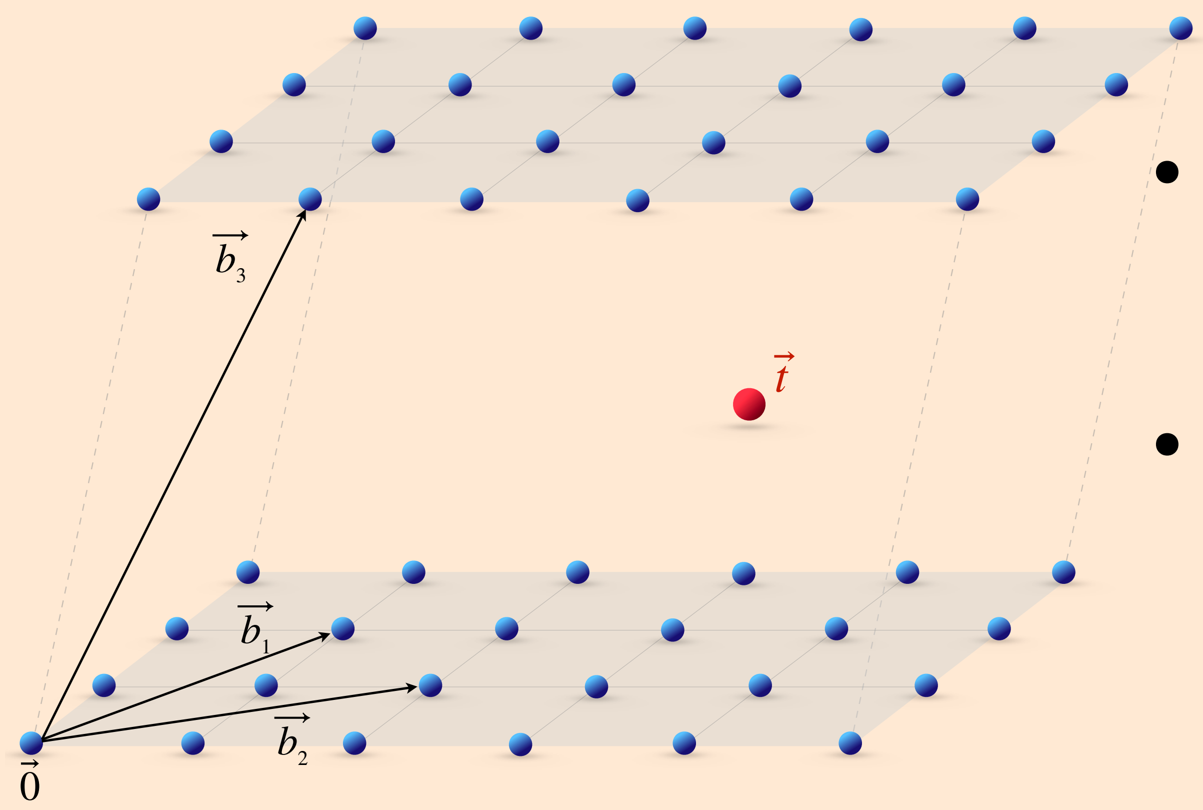
$$|\psi_{GS}\rangle = \downarrow \uparrow \downarrow \uparrow \downarrow \cdots \uparrow \downarrow$$



$$\vec{x} = \boxed{0\,1\,0\,1\,0} \cdots \boxed{1\,0}$$

EO mission planning problem — QUBO reformulation — Mapping onto Ising — Ground-state search via TNs — Read-out and decoding

Closest Vector Problem



- CVP is NP-hard
- Approximated CVP is polynomial (Babai algorithm) $\overrightarrow{b}_{op}$

Schnorr's lattice sieving

- An integer N and prime factoring basis define a family of CVP problems:

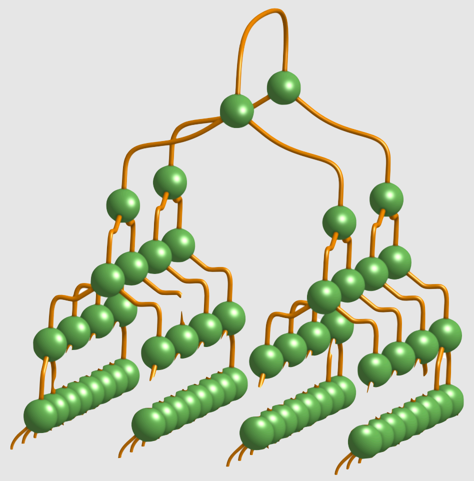
$$(p^{(1)}, p^{(2)}, p^{(3)}, \cdots, p^{(m)})$$

- Approximated solutions of CVPs provide congruence relations for factorization

$$\hat{\mathcal{H}} = \left\| \vec{t} - \sum_{k=1}^{n} \hat{x}_k \vec{d}_k - \overrightarrow{b}_{op} \right\|^2 + h_i \sigma_i^x$$

All-to-all Ising like Hamiltonian — small perturbation

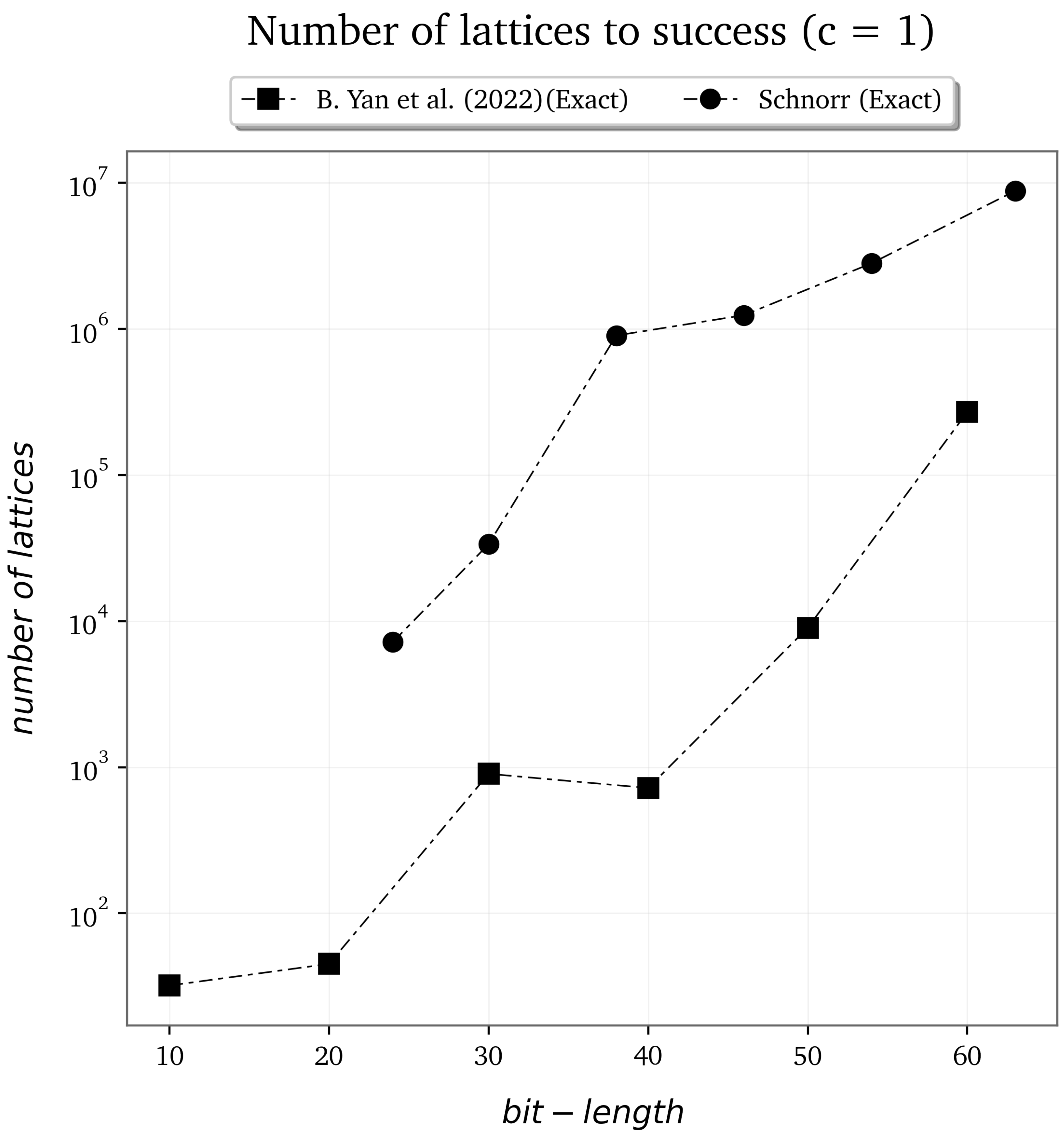
n=#qubits=lattice size $\quad n \approx \dfrac{\log_2 N}{\log_2 \log_2 N}$

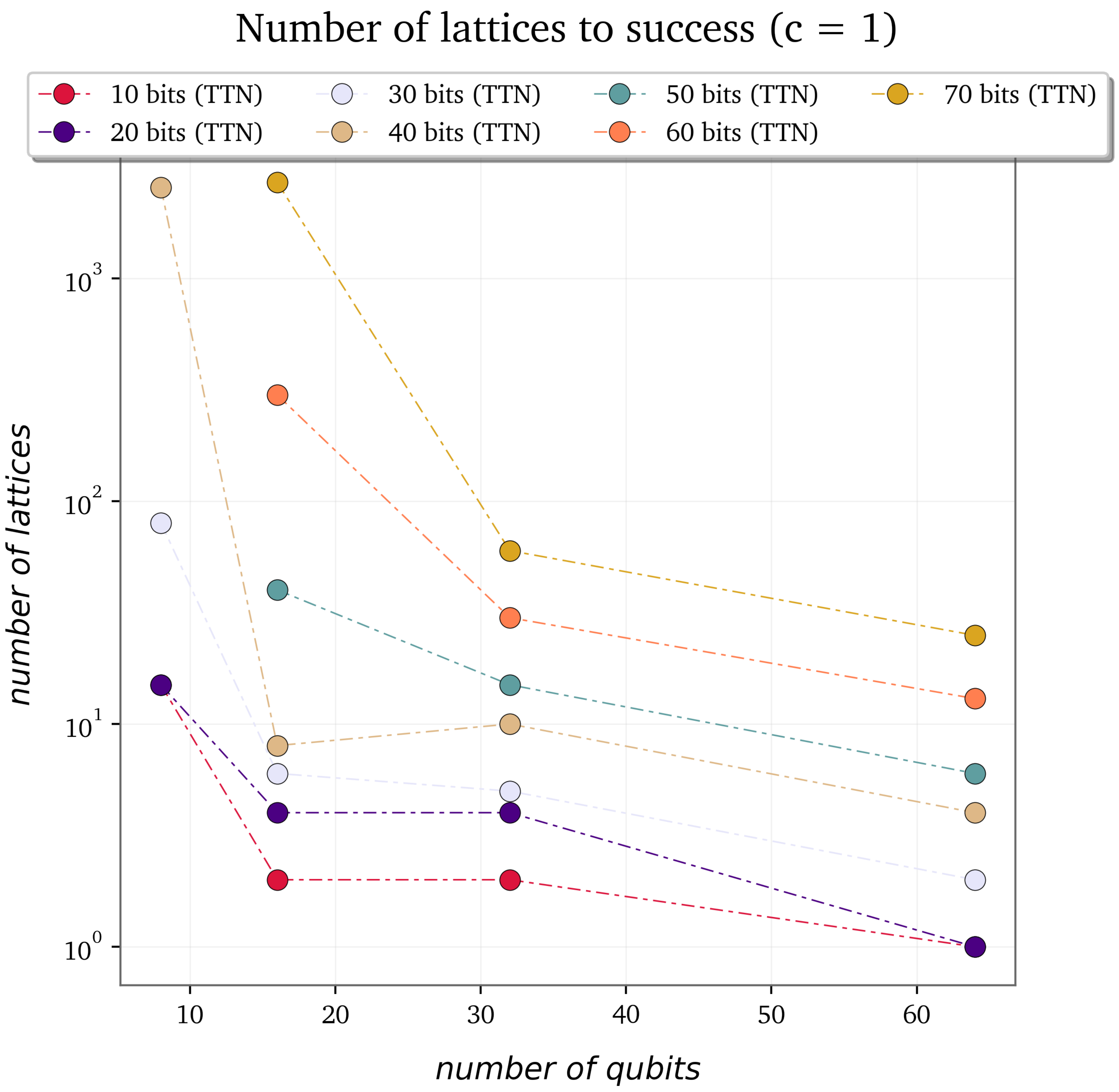


TTN ground state search
+
Sampling bit-strings from TTN state

Quantum GREEN TEA

Closest Vector Problem





Quantum Circuit Emulator

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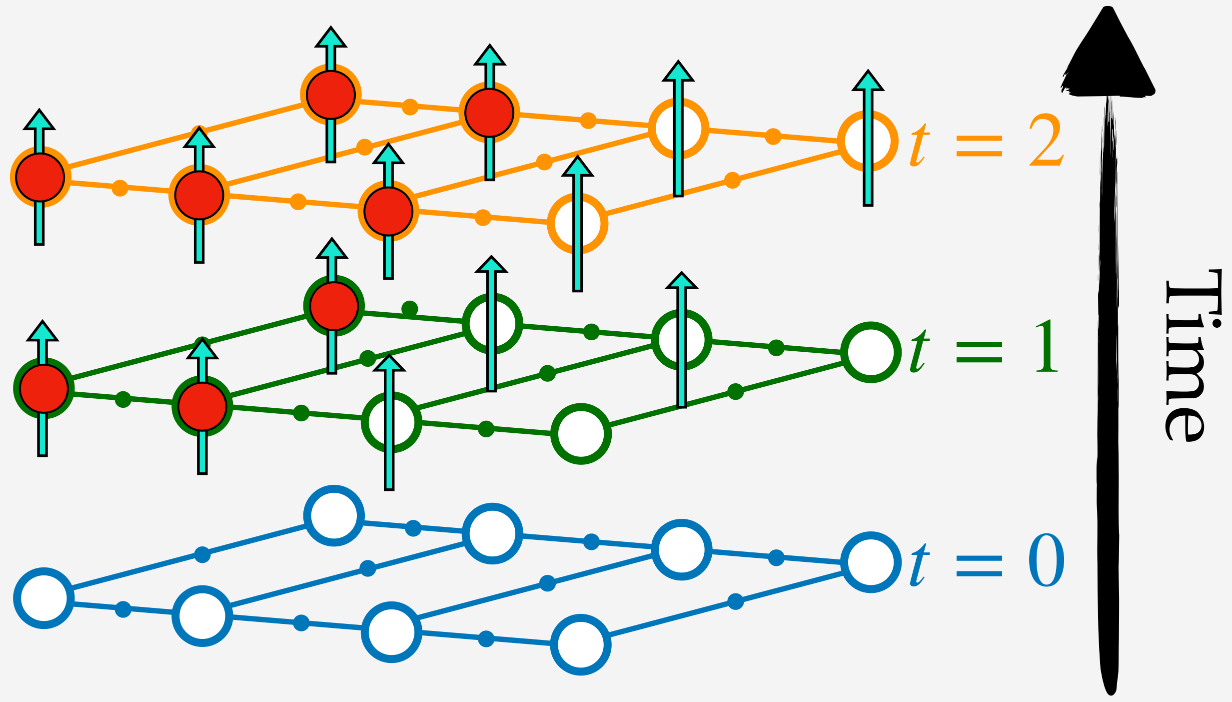
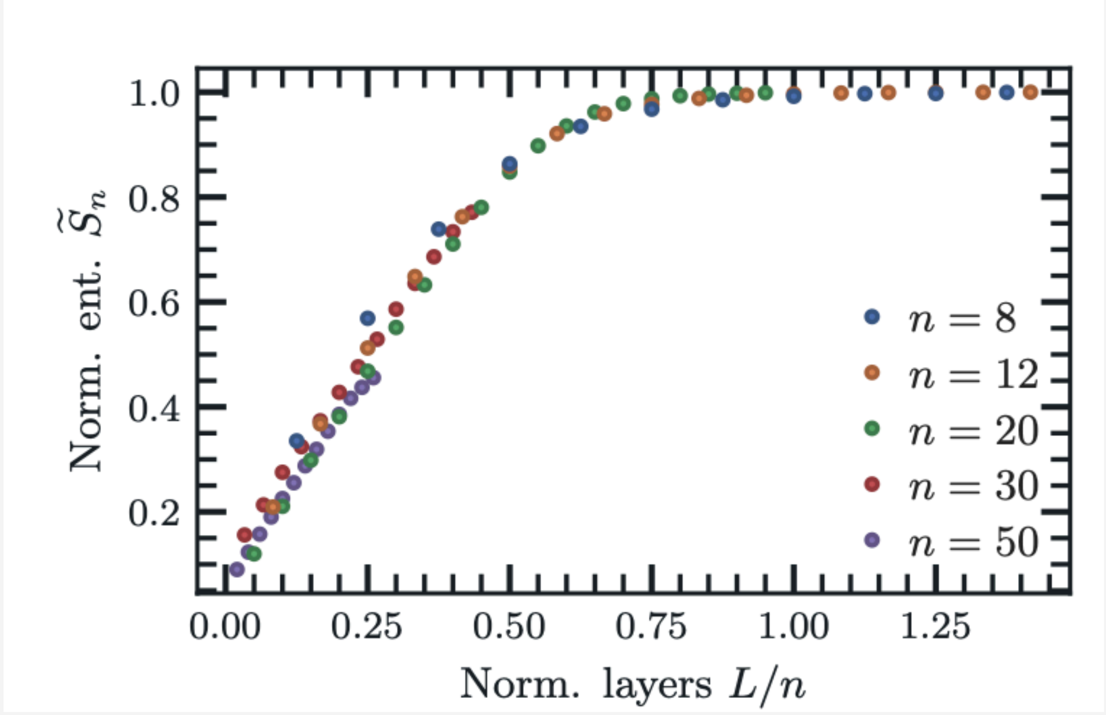
2D Fermi-Hubbard model & spin-charge separation



Entanglement generation in QNN



Ground state search & time evolution Many Body systems

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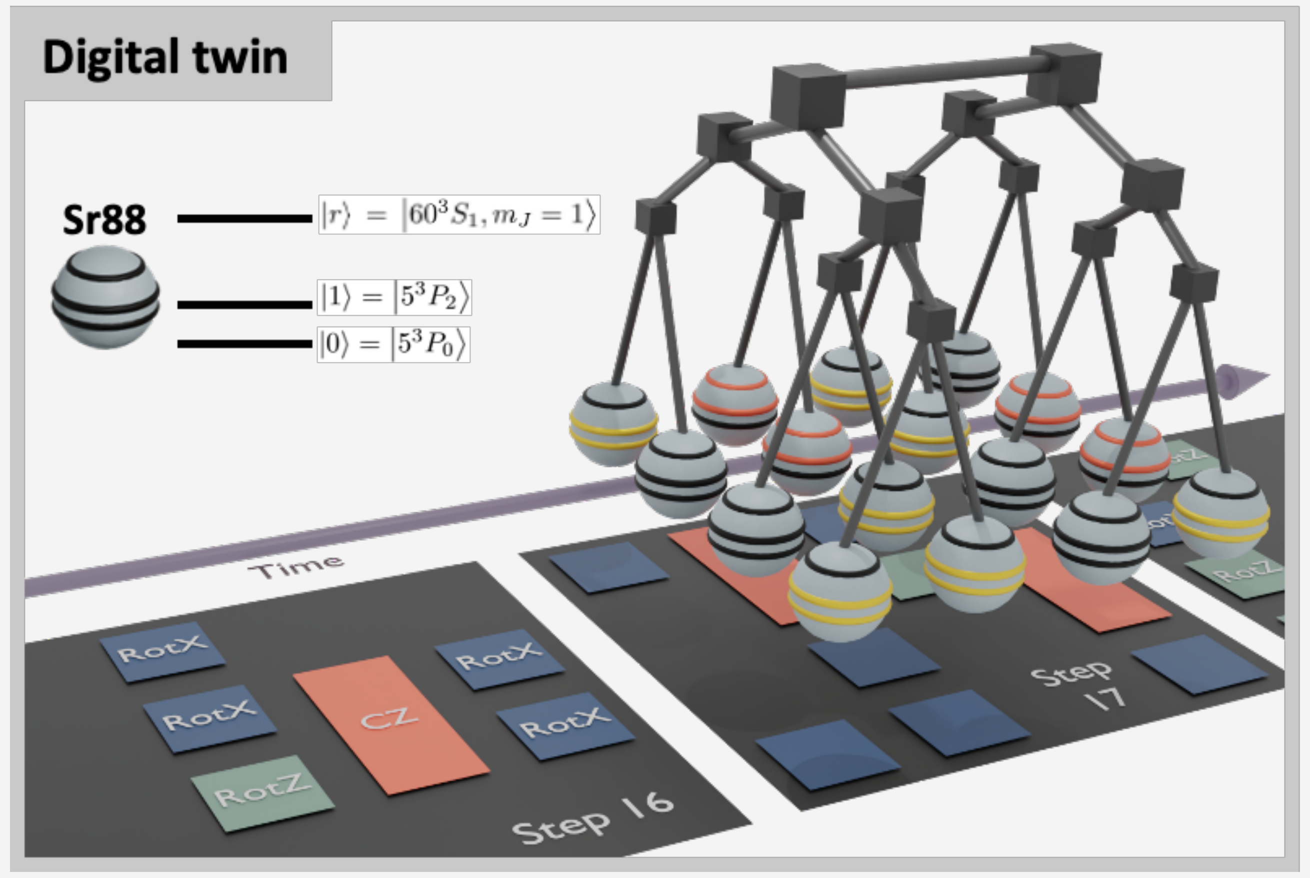
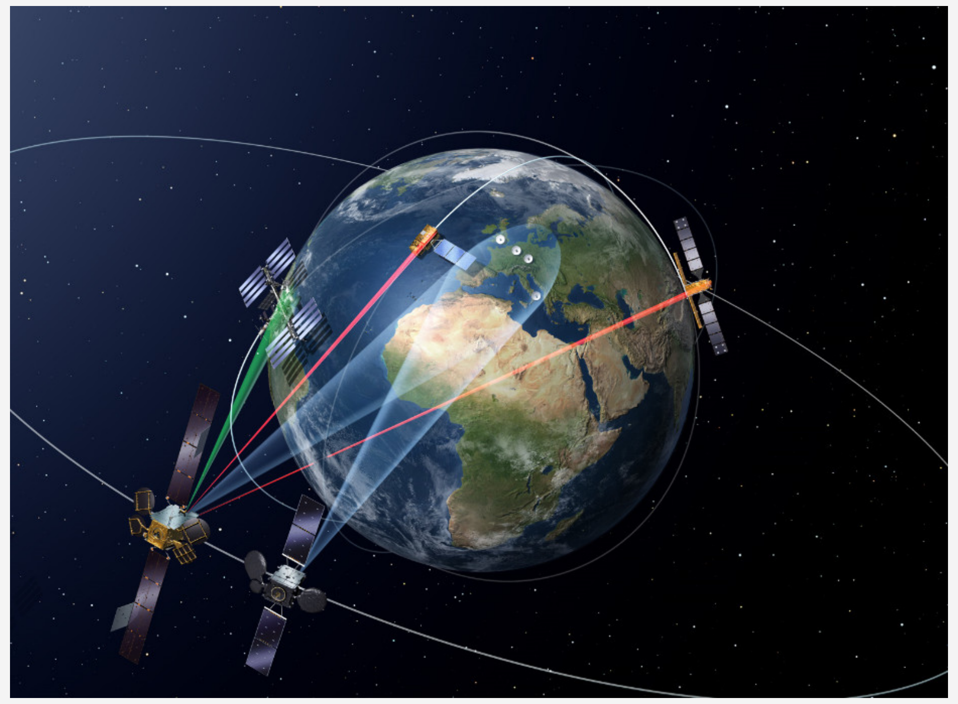
Digital Twin for Rydberg QPU



Mission planning for Earth Observation

Closest Vector Problem



Quantum Information and Matter

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ICSC
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Big Data and Quantum Computing

AQTIVATE

T-NiSQ
Tensor Networks in Simulation of Quantum Matter

QUANTERA
ERA-NET Cofund in Quantum Technologies

QuantHEP

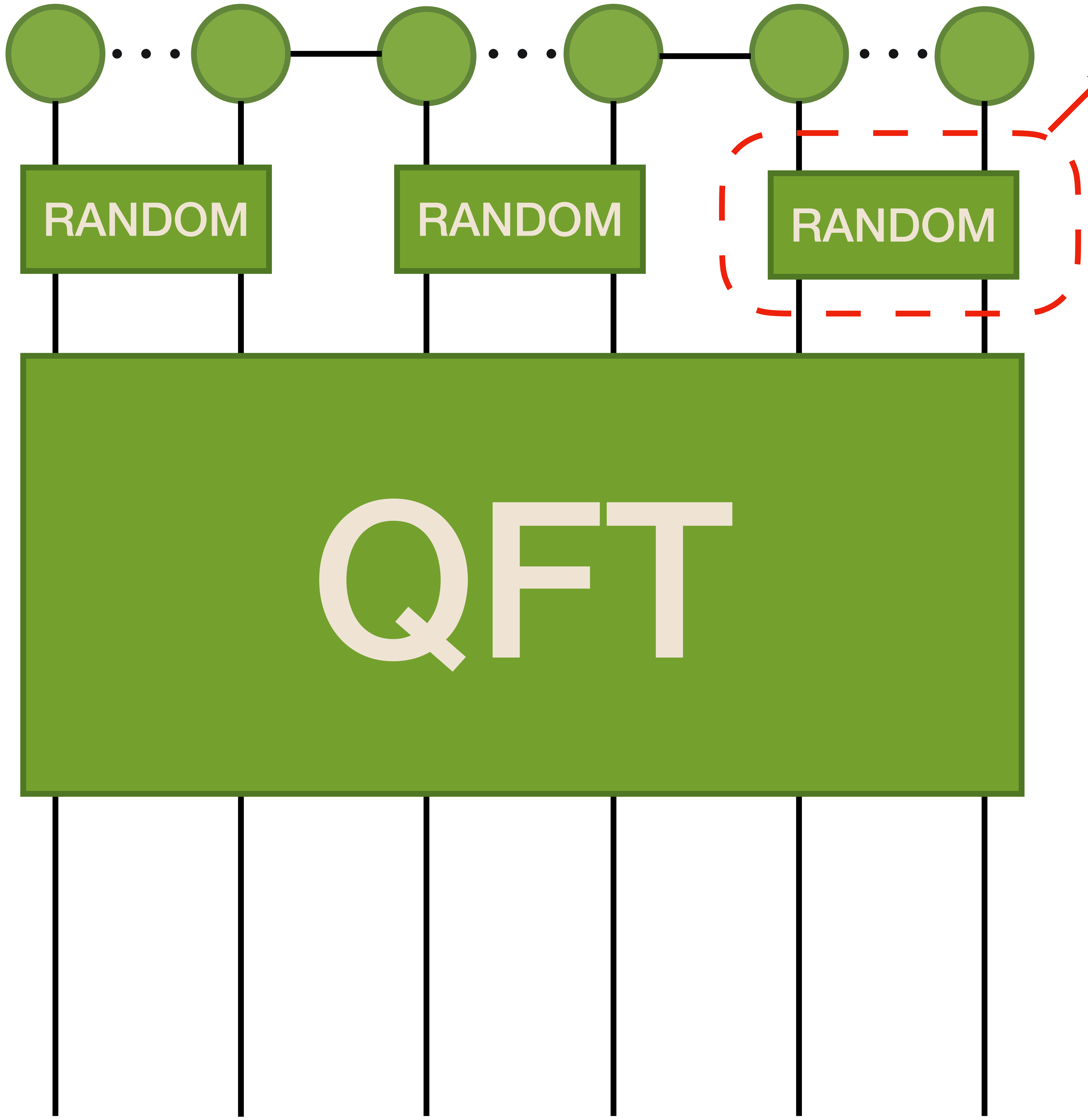
ENGAGE
Marie Sklodowska-Curie PhD

EuRyQa

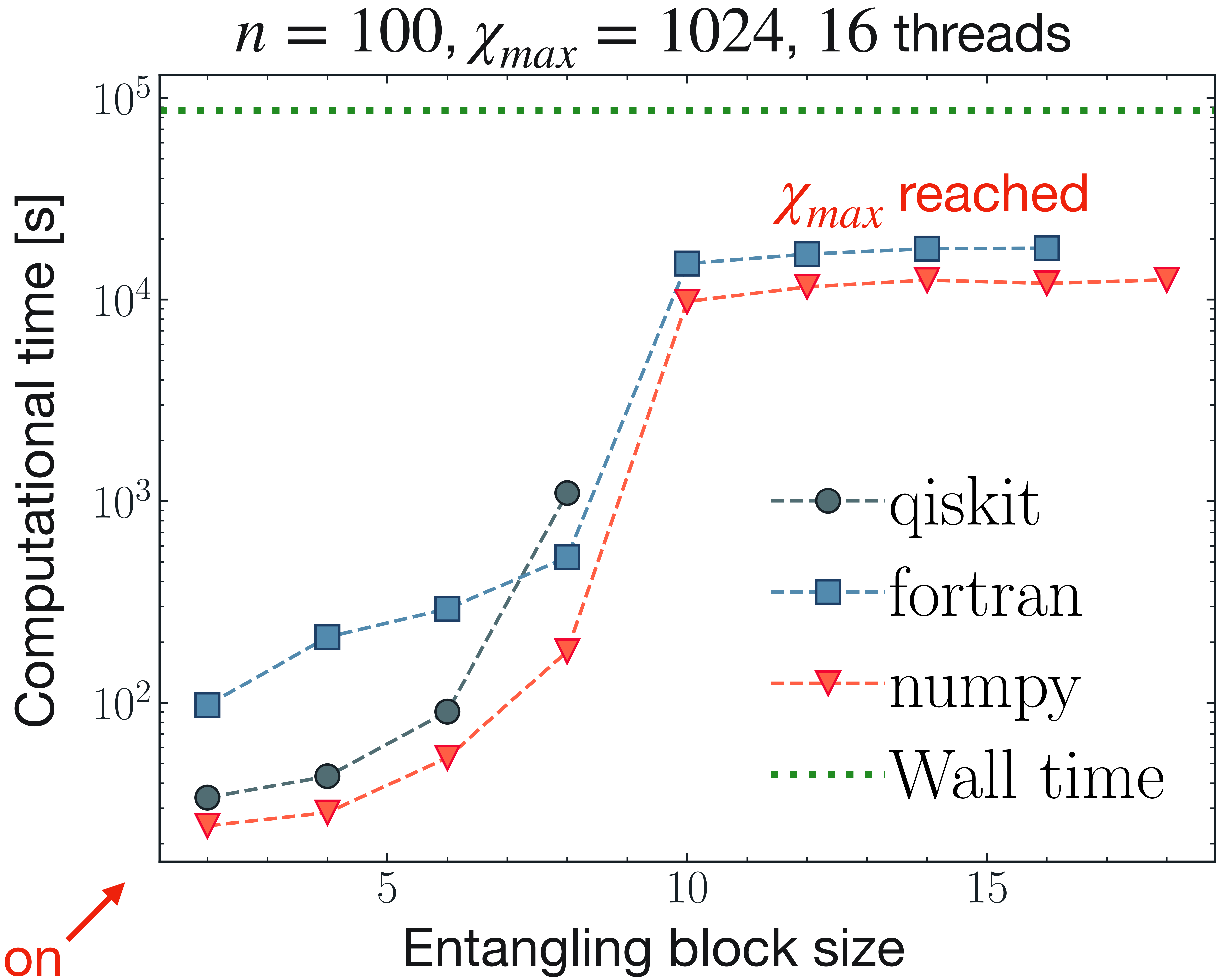
MUR
Ministero dell'Università e della Ricerca
PRIN 2022

DFG Deutsche Forschungsgemeinschaft

Benchmarks & QNN



Strongly entangling

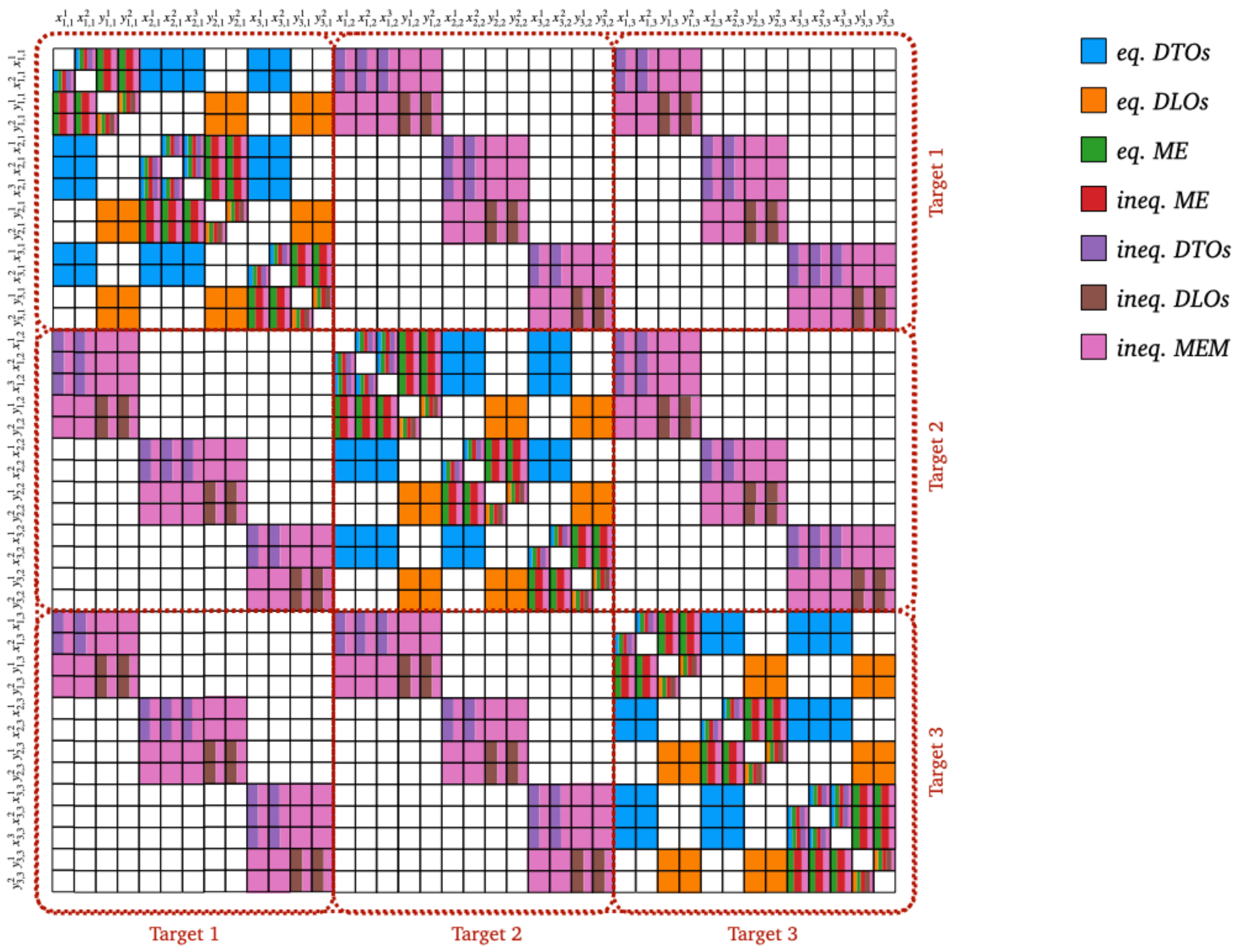


Run on Galileo100

M. Ballarin, …,S. Montangero, Quantum 7, 1023 (2023)

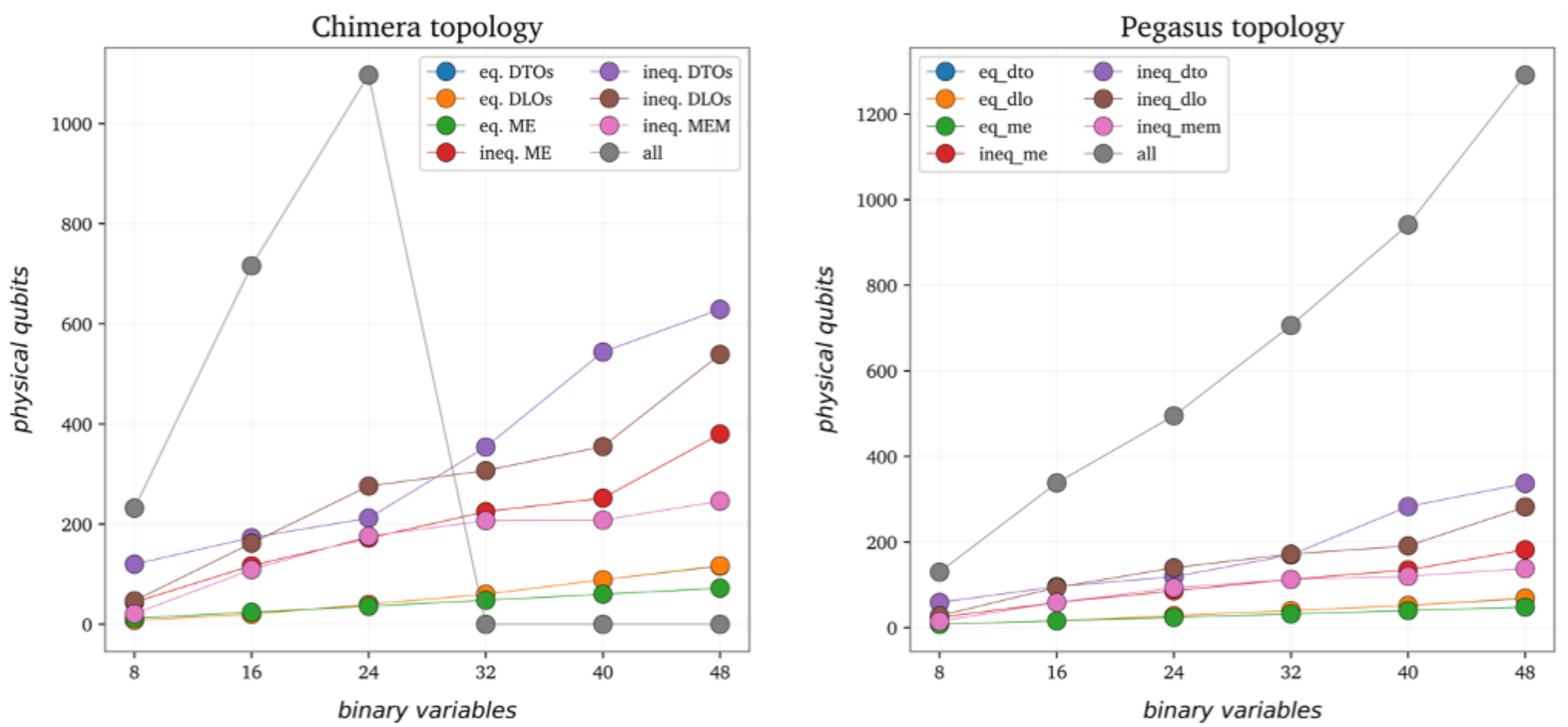
QUBO embedding

QUBO matrix representation
N=3 satellites, M=3 targets

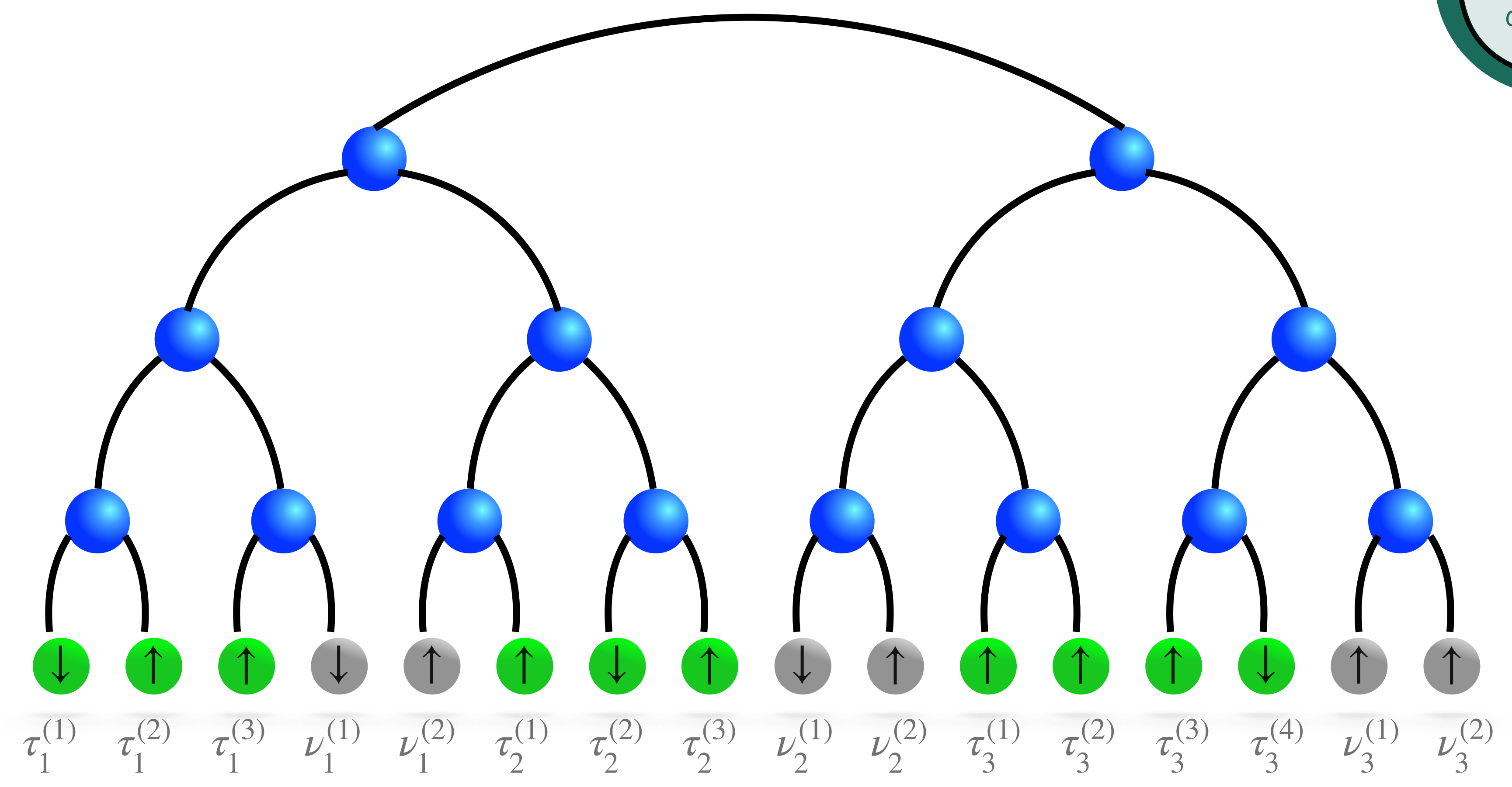


- Reducing slack variables using preprocessing

DWAVE HARDWARE



TREE TENSOR NETWORK



- Arbitrary connectivity
- Larger local dimension

Table 5
Comparative summary of lattice tailored encodings. d represents the degree of the Hamiltonian graph, v and h respectively the vertical and horizontal dimensions of a 2-dimensional lattice (we set $v \leq h$), n is the number of fermionic modes / sites on the lattice. The number of operators scales linearly with E, the number of edges. We have $E = h(v-1) + v(h-1)$ for a 2-dimensional regular lattice, and $E = \sum_i^D \left[(n_i - 1)\prod_{j\neq i}^D n_j\right]$ for an regular lattice of dimension D and n_i the number of sites along the i^{th} dimension.

Method	Pauli Weight	Qubits	Comments
Jordan-Wigner (snake pattern) [305]	$\mathcal{O}(2v)$	n	Optimal direct application of the Jordan-Wigner mapping to a 2-dimensional lattice.
Bravyi-Kitaev (Fenwick tree lattice mapping) [253]	$\mathcal{O}(\log(v))$	n	Optimal direct application of the Bravyi-Kitaev mapping to a 2-dimensional lattice.
Auxiliary Fermion Scheme [305]	4 (2-dim.)	$2n$, $(n = vh)$	Uses auxiliary fermion/qubit registers to create operators that cancel-out Z strings in the Jordan-Wigner mapping
Superfast Bravyi-Kitaev [239, 253, 255]	$\mathcal{O}(2d)$	$\mathcal{O}(nd/2)$	Relies on stabilizers formalism to define an efficient encoding with cost dependent on the degree of the Hamiltonian graph
Generalized Superfast Encoding [221]	$\mathcal{O}(\log_2(d))$	$\mathcal{O}(nd/2)$	Extension of Superfast BK, optimizing Pauli weight and offering better opportunities for error corrections
Compact encoding [224, 225]	3 (2-dim.), 4 (3-dim.)	$\mathcal{O}(1.5n)$	Modifies the stabilizer formalism used in Superfast BK to optimize the number of qubits required. So far limited to 2 and 3-dimensional lattices