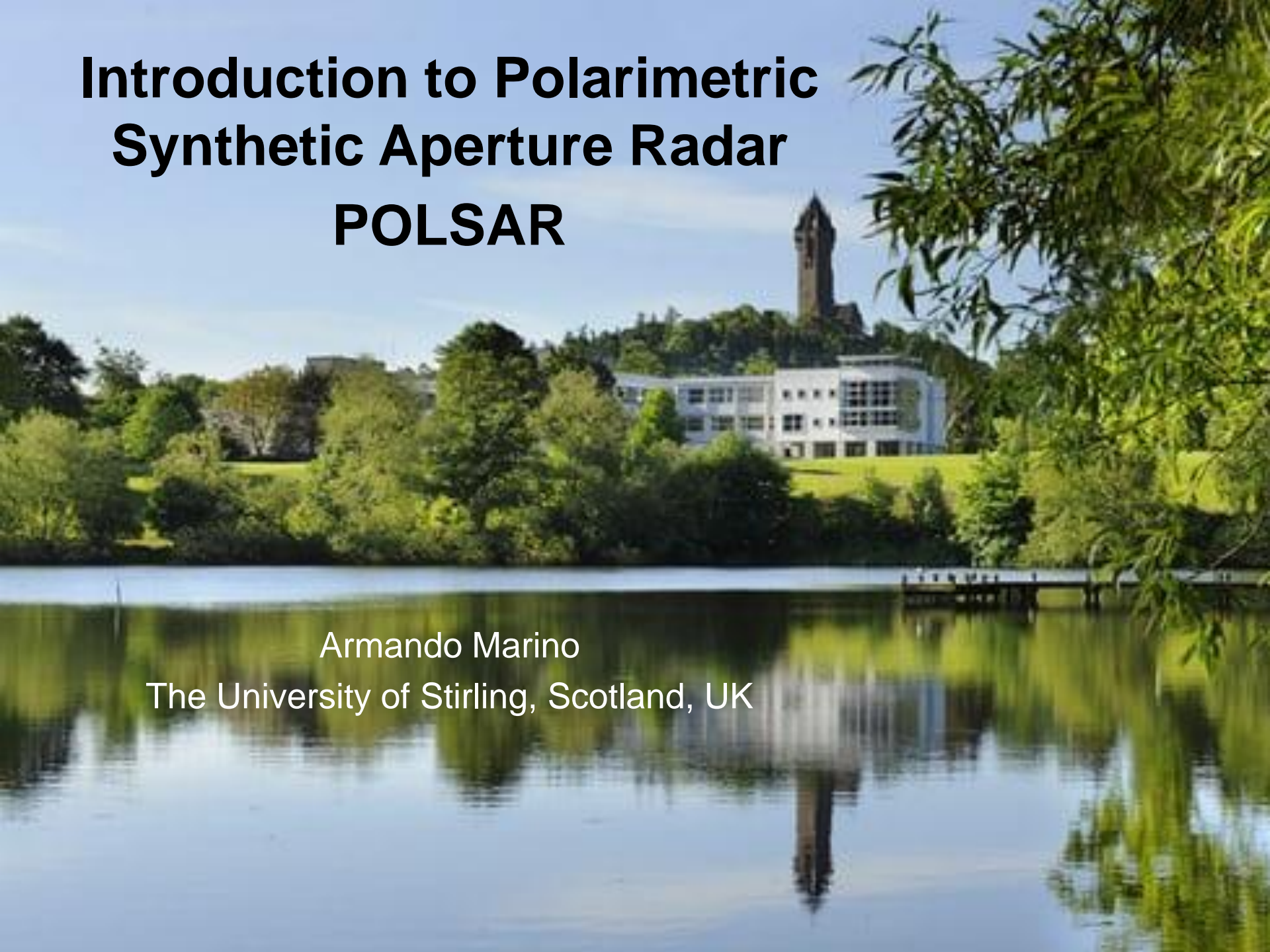


# Introduction to Polarimetric Synthetic Aperture Radar POLSAR

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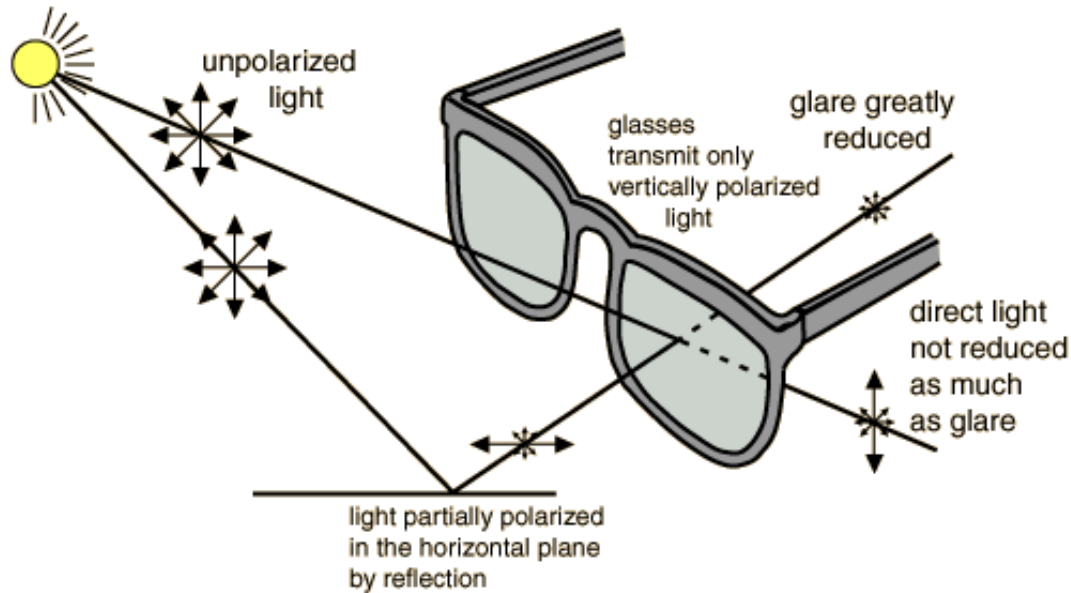
# Outline

- ✓ What is polarimetry?
- ✓ Quick recap:
  - ✓ Scattering
- ✓ Basic concepts in polarimetry:
  - ✓ Wave
  - ✓ Single targets
  - ✓ Partial targets
- ✓ Target decomposition:
  - ✓ Coherent
  - ✓ Incoherent
    - ✓ Non-model based
    - ✓ Model based



# What is Polarimetry?

## Polaroid Glasses



## 3D Cinema



You may know how polarimetry can be exploited in optics:

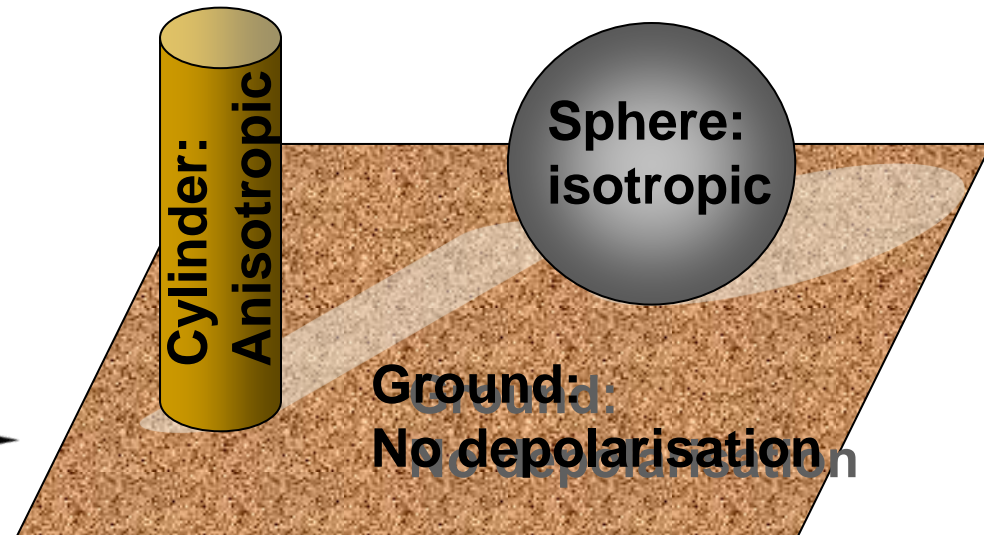
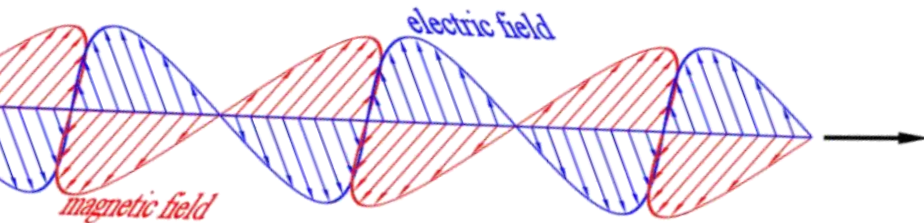
1. Polaroid glasses
2. Modern 3D Cinema

# Why Polarimetry in radar remote sensing?

✓ Different targets *generally* interact in a different way when illuminated by differently polarised plane waves

✓ We can **use** polarimetry to:

- ✓ Classify
- ✓ Detect
- ✓ Separate returns



Few **definition**... (they will be treated in details later on):

- ✓ **Isotropic**: the target interacts at the same way with any polarisation (the interaction does NOT depend on the direction of the Electric field vector)
- ✓ **Anisotropic**: the scatterer has a different behaviour for different polarisations
- ✓ **Depolarisation**: the tendency of a target to change the polarisation of the incident wave, but in some contexts is only referred to the lost of polarimetric purity (i.e. the polarisation changes in time/space)

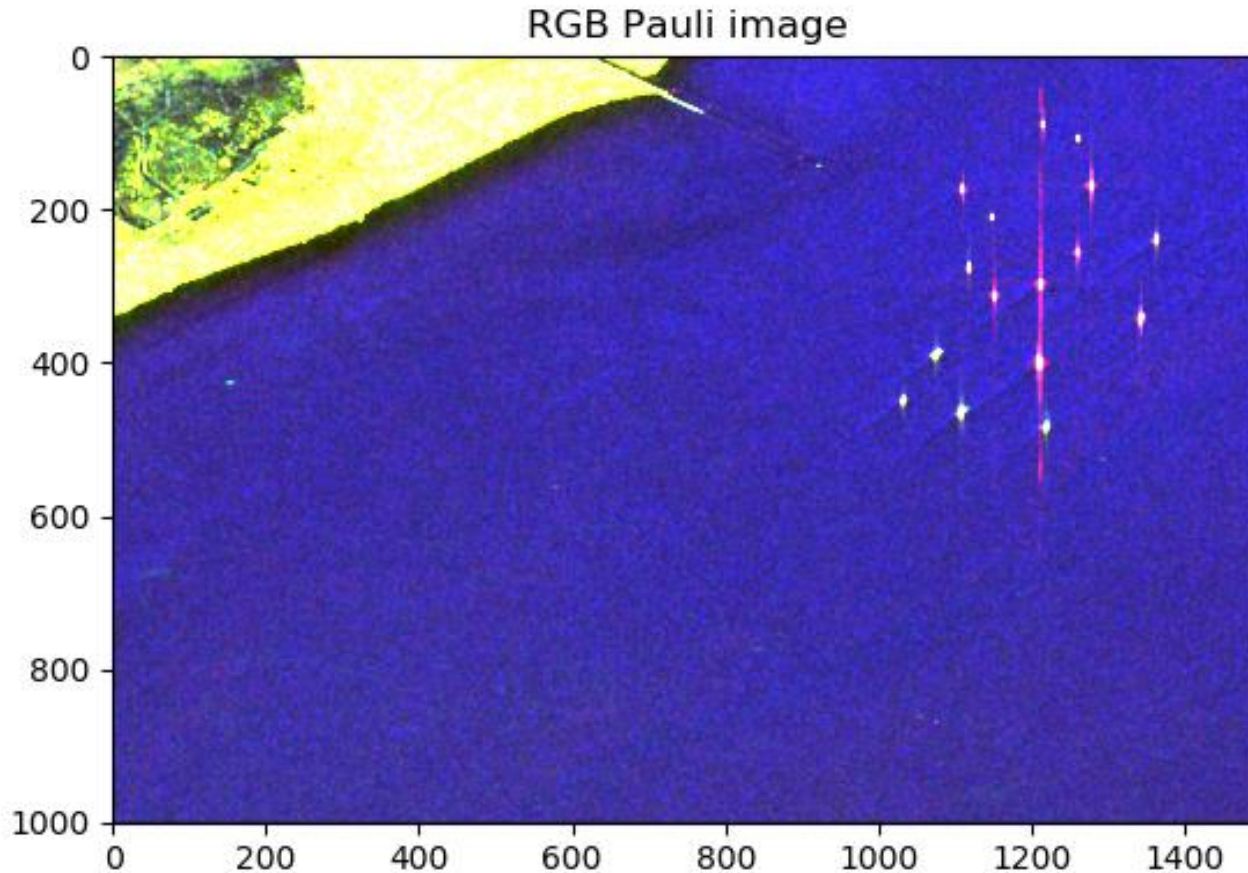


# Why Polarimetry in radar remote sensing?



Pauli RGB image of San Francisco Bay (AIRSAR). The polarimetric information is coded in the colours. As you can notice we can use colours to differentiate between targets. Data courtesy of MDA and Canadian Space Agency.

# Why Polarimetry in radar remote sensing?



Pauli RGB image around Buenos Aires (ALOS-1). The polarimetric information is coded in the colours. As you can notice we can use colours to differentiate between targets.

Data courtesy of JAXA.

**Quick recap:**

**Scattering**

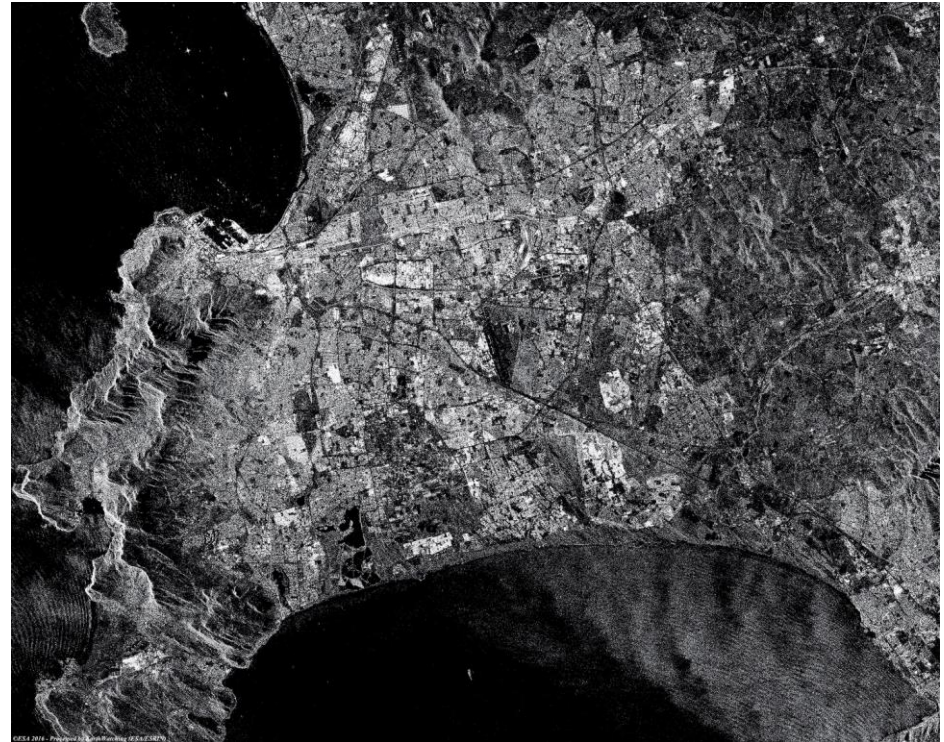


# Radar backscattering, sigma nought $\sigma$

Pixel values equate to the intensity of the backscattered microwave signal

The physical unit is the **normalised radar cross section, or sigma nought  $\sigma$**  [dBm<sup>2</sup>]

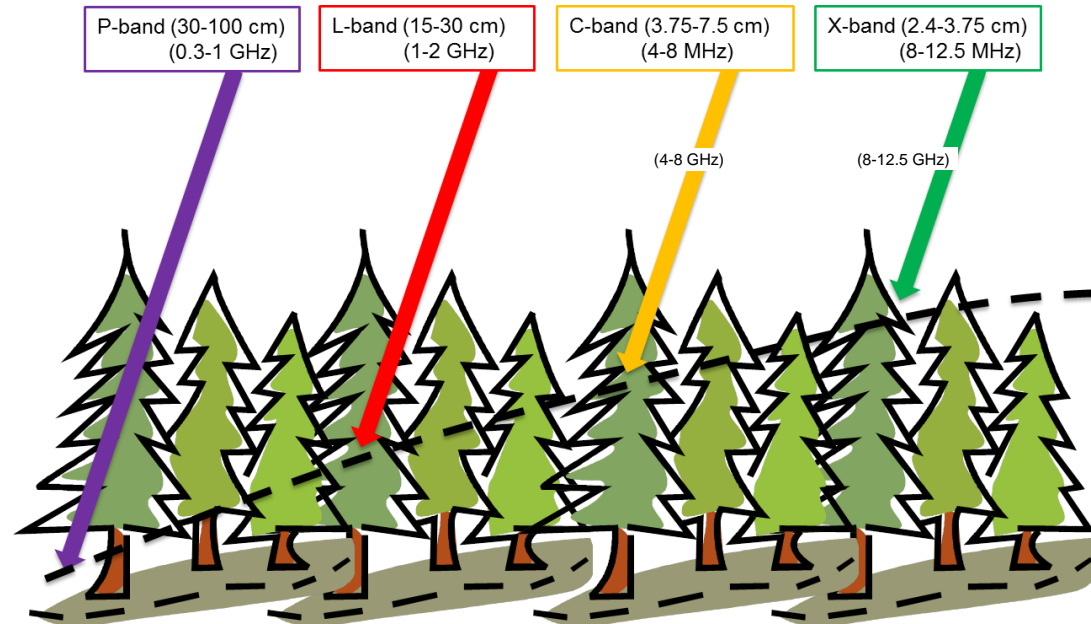
$\sigma$  ranges generally from +5 (very bright) to -40 dB (very dark) depending on Noise Floor





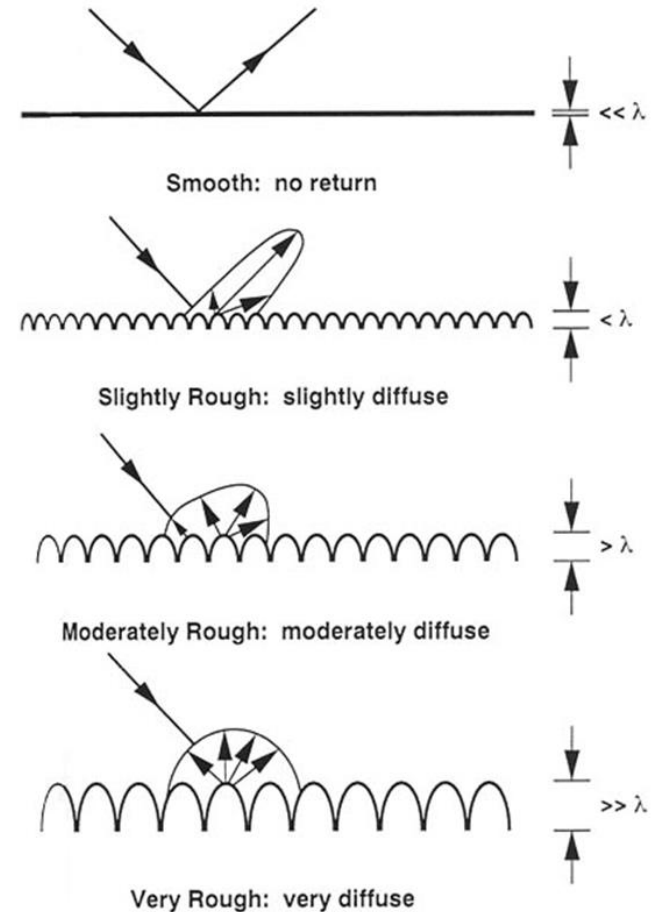
# What frequency?

- ✓ Choice of frequency governed by size of features to be imaged.
- ✓ As a rule of thumb the energy interacts more with objects of **size equal or bigger than the wavelength**.
- ✓ The **penetration** is also higher when the frequency is lower (e.g. P-band 0.3-1 GHz)



# Interactions with objects and surface

- ✓ Intensity of the backscattered microwave signal depends on **size**, **shape** and **orientation** of objects.
- ✓ Surfaces whose **roughness** is much less than the radar wavelength exhibit specular (forward) scattering (e.g. calm water).
- ✓ **Rough** surfaces scatter energy in all directions, but proportionally more in backwards direction towards the receiving antenna (e.g. rough soil).







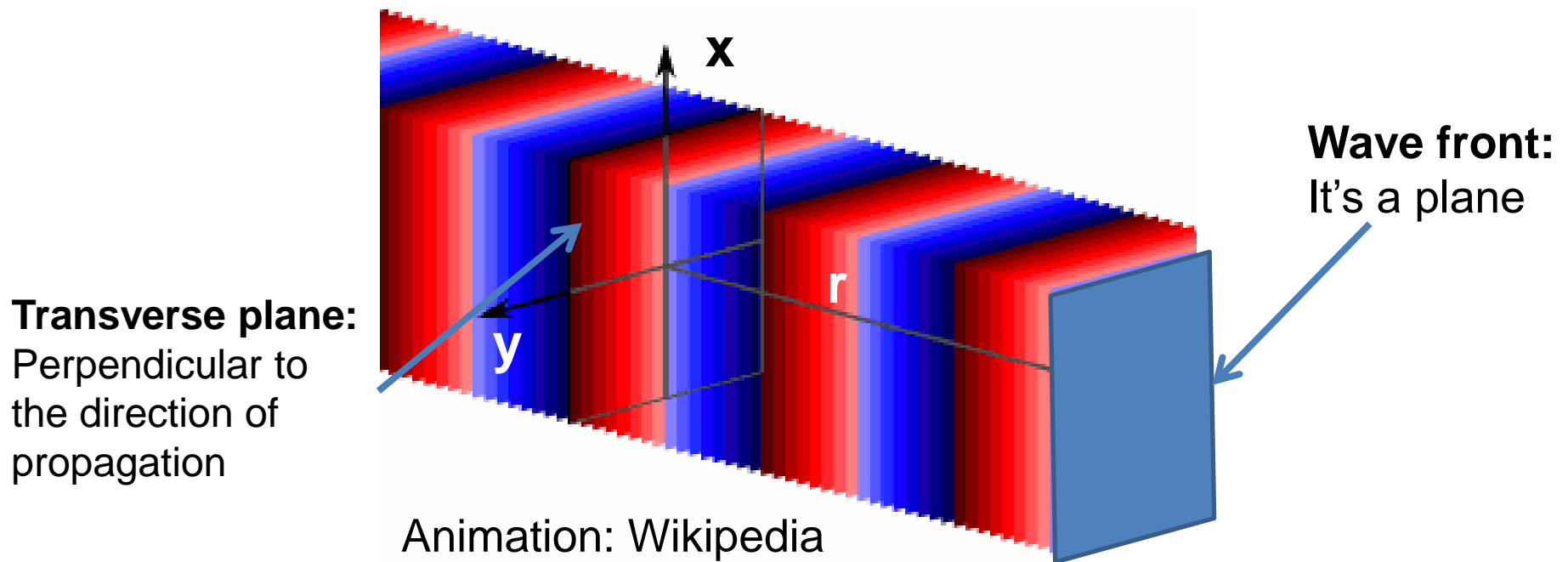
**Three core concepts you should  
remember:**

**Idea1: Wave polarimetry**

# Wave Polarimetry: Plane waves

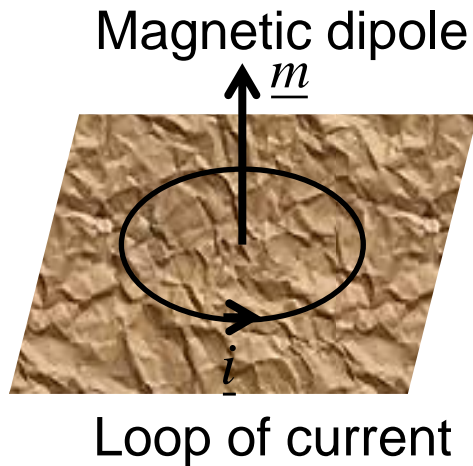
The most **general** way to describe any (macroscopic) electromagnetic phenomenon is by using the legendary **Maxwell equations**

After a series of hypothesis (i.e. monochromatic or narrowband signal, homogeneous, stationary and isotropic medium) we end up with a **Plane Wave** that can be “*easily*” described knowing the currents over the surface of the target



# Wave Polarimetry: mathematical expression

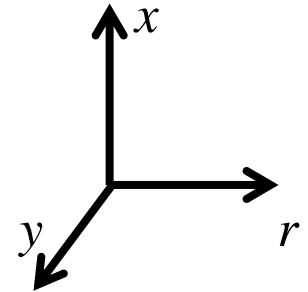
The mathematical expression of the plane wave is the following



$$\underline{E}(\underline{r}) = -\frac{\beta\omega e^{-j\beta R}}{4\pi\mu_0 R}(\underline{r} \times \underline{m})$$

$$\underline{H}(\underline{r}) = \frac{\beta^2 e^{-j\beta R}}{4\pi\mu_0 R} \underline{r} \times (\underline{r} \times \underline{m})$$

$$\beta = \sqrt{\omega^2 \varepsilon_0 \mu_0} \quad \omega = 2\pi f$$



$\underline{E}$ : electric field, it's a complex vector (when  $\underline{E}$  is cleaned by the dependences on the distance is sometime refereed as **Jones** vector)

$\underline{H}$ : magnetic field, it's a complex vector and can be derived from  $\underline{E}$

$\varepsilon_0$ : electric permittivity of vacuum

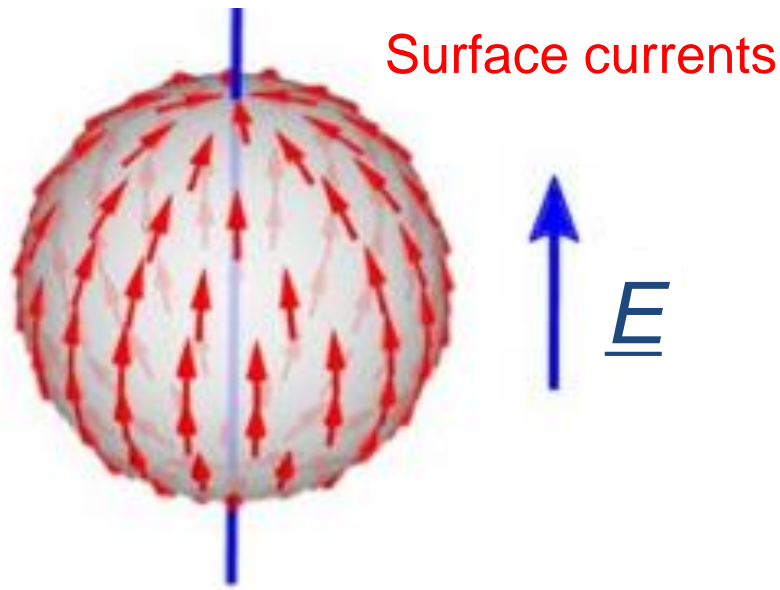
$\mu_0$ : magnetic permeability of vacuum

$f$ : frequency of monocromatic (or narrowband) wave

$R$ : distance from the source (generator) of the wave

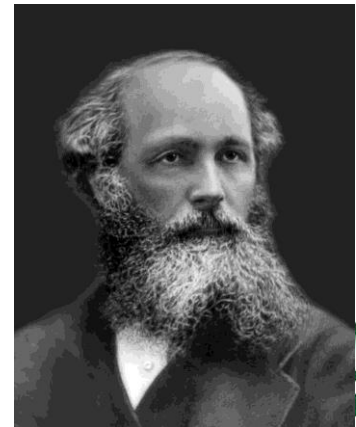


# Currents on surface generating



Modified from: Simon Horsley,  
Tutorial: Topology, waves, and  
the refractive index, 2022

- ✓ An impinging electromagnetic pulse of energy will produce current on the surface and inside the layer (depending on discontinuities).
- ✓ These currents are “alternating” and therefore they will scatter an electromagnetic field.
- ✓ The field gets tied up in a wave as the old good Maxwell was saying.



# Where are the current in the case of volume scattering?



# Wave Polarimetry: useful abstraction

4 or 2

Parameters needed

- ✓ **Problem:** It is complicated to study the wave polarimetry starting from the **Jones** vectors.
- ✓ **Solution:** we use a geometrical abstraction and wave polarimetry becomes an ellipse. We need:
  - ✓ 2 parameters for the ellipse shape
  - ✓ 1 for the amplitude
  - ✓ 1 for the phase

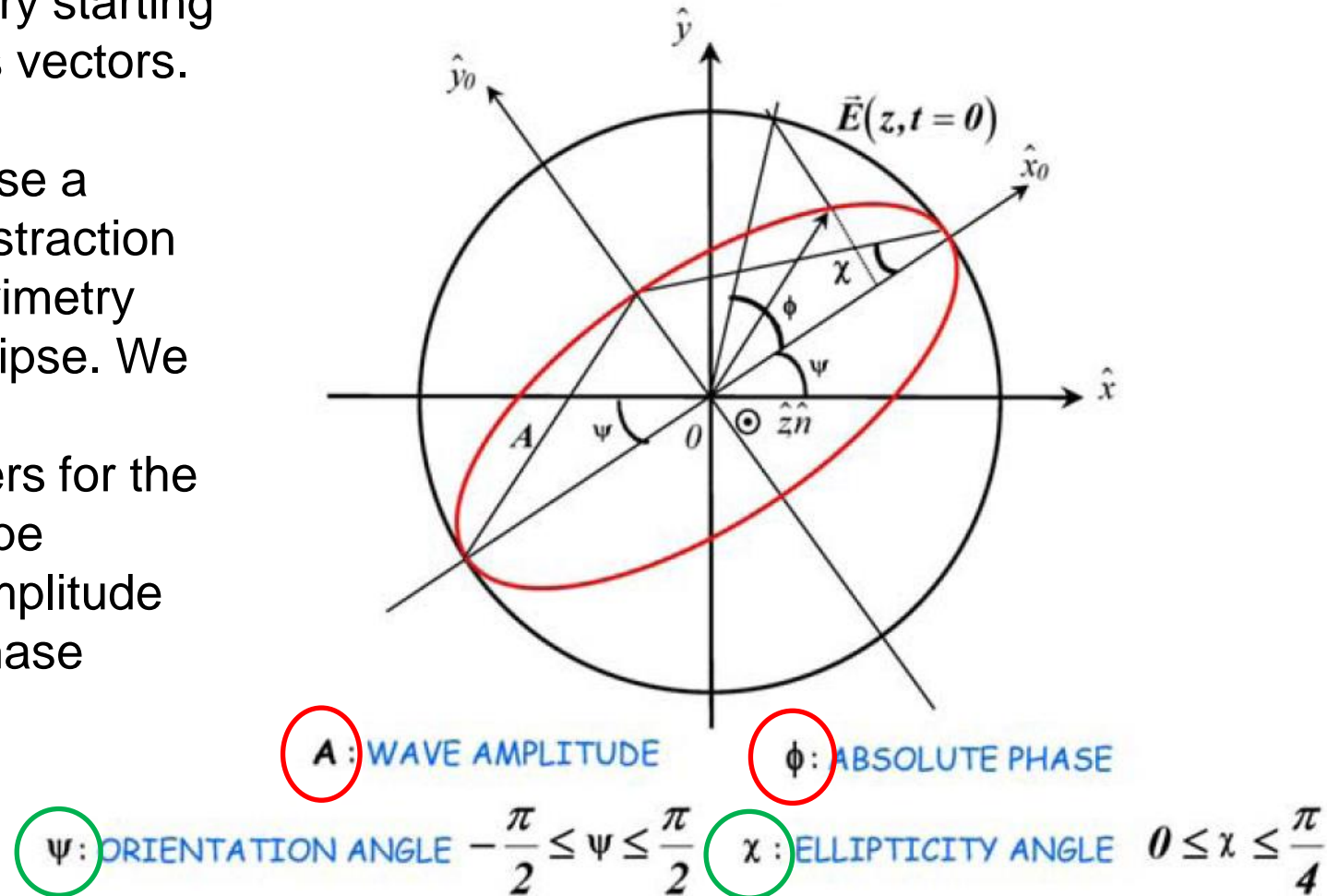


Fig. 2.2 Polarization Ellipse Relations (Courtesy of Prof. E. Pottier)



**Three different concepts you must  
remember:**

**Idea2: Scattering polarimetry  
Deterministic targets**

# Single targets?!?! What is that?!?!?

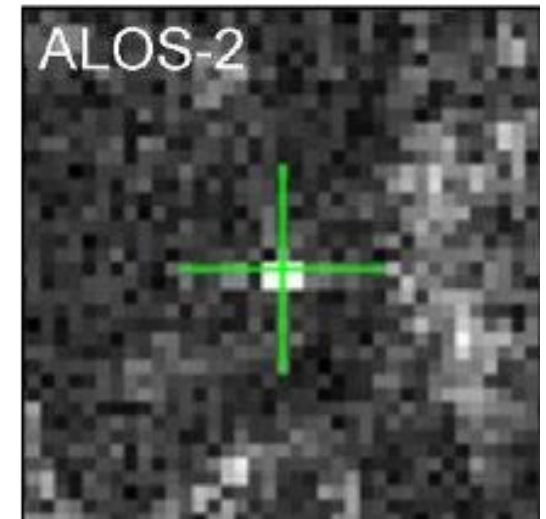
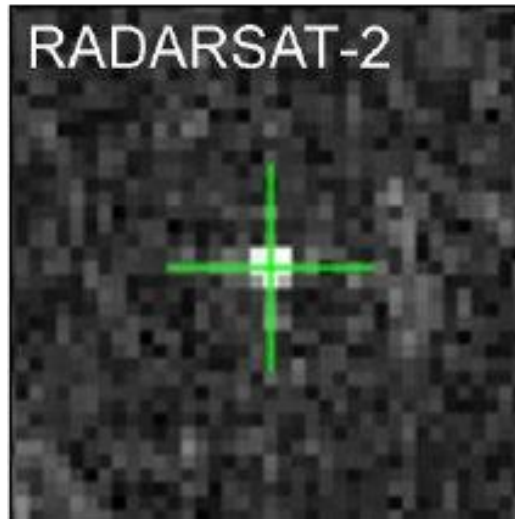
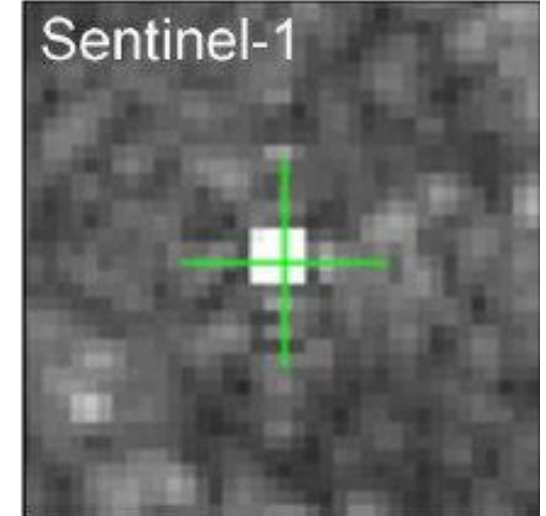
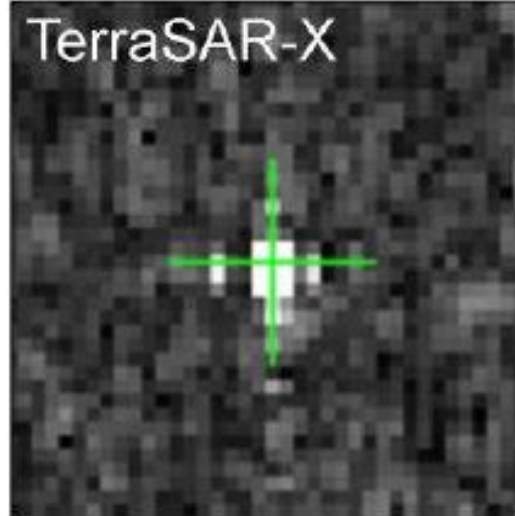
- ✓ A single target is a target that does NOT change its polarimetric signature in time/space: it is a **deterministic** target
- ✓ Examples:
  - ✓ calibration targets: corner reflectors
  - ✓ Some metallic or man-made targets (but not all of them): a car, a wall
  - ✓ Some natural target: a rock



Trihedral  
corner reflector

# How do they look like?

Corner reflectors in X-,  
C- and L-band SAR  
imagery.



Corner Reflectors as the tie between  
InSAR and GNSS measurements: Case  
Study of Resource Extraction in Australia  
March 2015

DOI: 10.5270/Fringe2015.pp60

# Single targets: how to study polarimetric targets

- ✓ We want to use polarimetry to detect single targets and we can transmit and receive polarised waves:
  - ✓ **How many polarimetric acquisitions would we need?**
- ✓ To characterise **any polarised wave we need 2 polarisations**, since the plane wave is 2 dimensional (2-D).
- ✓ If we send a polarised wave (e.g. linear horizontal), this will generate currents on our target and these currents will scatter a wave with some polarisation. Therefore **we need to collect 2 polarisations to characterise such scattered wave.**
- ✓ But what happen if we **change the polarisation of the transmitted wave** (the wave that we send from the satellite)?...
  - ✓ Well, we will have different currents on the surface.
- ✓ In order to cover each possible transmitter waves, **we need to send two polarimetrically orthogonal waves.**
- ✓ Summarising, **we transmit 2 orthogonal waves and collect 2 orthogonal waves:**  
2x2=4 channels/acquisitions needed



# Single targets: same as before, but with math

- ✓ We can arrange the 4 acquisitions discussed before in a matrix: the **Scattering** or **Sinclair matrix**
  - ✓ **H: horizontal linear**
  - ✓ **V: vertical linear**
- ✓ The matrix will represent a **transformation from transmitted polarised waves to received waves**: i.e. it describe the polarimetric behaviour of the target

## Cross-polarisations

Scattering matrix

$$S = \begin{pmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{pmatrix}$$

## Co-polarisations

Scattered  
(received)  
wave

Incident  
(transmitted)  
wave

$$\underline{E}_r = C(r) [S] \underline{E}_i$$

Complex scalar depending on distance and medium where the wave propagates (e.g. air)

# Single Look Complex

Data are stored in complex form, that is real plus imaginary part

The screenshot shows the Spyder Python IDE interface. The top part displays the Variable explorer with a list of variables and their types and sizes. Below it, a NumPy array viewer window is open, showing a 2D array of complex numbers. The array has 12 columns and 23 rows of data. The first column is labeled '6' and the second '7'. The data is presented in a grid with alternating red and white rows.

Name	Type	Size	Value
HHFull	complex64	(1248, 18432)	array([[ 1.70937300e-01-0.07887045j, 2.42148161e-01+0.09545995j, ...
HVFull	complex64	(1248, 18432)	array([[ 0.04068684+5.5295121e-02j, -0.07140828+2.2488926e-02j, ...
VHFull	complex64	(1248, 18432)	array([[ 0.03976065+0.05898558j, -0.03031691+0.03130209j, ...
VVFull	complex64	(1248, 18432)	array([[ 0.23054181+0.04004149j, 0.2626125 +0.18533355j, ...
envi	module	1	module object of builtins module
filei_HH	str	1	i_HH
filei_HV	str	1	i_HV

	6	7	8	9	10	11	12
208	(0.07107788+0.0001118965j)	(-0.2165731-0.11895645j)	(-0.037696168+0.027938405j)	(0.16672957-0.25383106j)	(0.09106964-0.16733629j)	(-0.2292527+0.13909335j)	(-0.11490008+0.11884244j)
209	(-0.28057334-0.042206895j)	(0.38021272+0.029810535j)	(0.43548566+0.26694945j)	(-0.138532+0.40491346j)	(-0.5769423+0.1847391j)	(-0.015036544-0.05962692j)	(0.26831898-0.1743832j)
210	(0.030044155+0.34601098j)	(-0.5461137-0.025732089j)	(-0.29259968-0.14295235j)	(0.20367235-0.5841537j)	(0.37775946-0.262635j)	(-0.21820979+0.16362272j)	(-0.17708418-0.09521227j)
211	(-0.41205642+0.051513102j)	(0.7546207+0.32724288j)	(1.0687426+0.5270206j)	(0.054809444+0.82367045j)	(-0.3507389+0.1318827j)	(0.21761155-0.41316128j)	(0.08479531-0.22683923j)
212	(-1.1572694-0.38786665j)	(1.9590037+0.3208726j)	(1.9236386+1.241693j)	(-1.1034379+2.4863272j)	(-2.1077378+0.6184698j)	(0.90856504-0.7555746j)	(1.2048659-0.31549478j)
213	(0.03483691+0.24929173j)	(-0.06724173-0.19649513j)	(-0.052806515+0.14856093j)	(-0.33817375+0.041899033j)	(-0.325384-0.35427806j)	(0.15580273+0.25812522j)	(0.3568068+0.18579605j)
214	(0.05983149+0.14247231j)	(0.09123571-0.019802434j)	(-0.0056780856-0.08619735j)	(0.0276807+0.026372923j)	(0.004342114+0.07160738j)	(-0.16508183-0.13053997j)	(-0.10213288-0.1436391j)
215	(-0.23911689+0.13675046j)	(0.2510476+0.20256251j)	(0.1657779+0.29408714j)	(-0.14119186+0.3051973j)	(-0.14424676+0.11273051j)	(0.019122131-0.149513j)	(0.011736556-0.17397477j)
216	(0.14865826+0.14563574j)	(-0.14296752+0.18294339j)	(-0.08458038+0.04554746j)	(0.11610123-0.26472777j)	(0.27141842-0.2700538j)	(0.014428847+0.092346266j)	(-0.16020864+0.009083515j)
217	(0.14413048+0.050818104j)	(0.1473479+0.08817709j)	(0.023391057-0.08530626j)	(0.035040595+0.107569896j)	(-0.057849325+0.24947123j)	(0.12625532-0.06831674j)	(0.2286333-0.123649314j)
218	(0.035153415+0.10457542j)	(-0.011003431+0.120111205j)	(0.03158148+0.0710468j)	(0.17607468-0.20794402j)	(0.22993661-0.093825236j)	(-0.052824214+0.10840995j)	(-0.13797311+0.015566013j)
219	(0.03727213-0.21260321j)	(0.1176478-0.031192722j)	(0.03230082-0.16893798j)	(-0.08027681-0.031375308j)	(-0.05084166+0.05267703j)	(-0.06616261-0.09878135j)	(-0.100596376-0.028747175j)
220	(0.26260537-0.008604212j)	(0.15181771-0.14325972j)	(-0.21277122-0.3136897j)	(-0.06775208-0.13144417j)	(0.060304996+0.14034642j)	(-0.038857333+0.015634881j)	(0.023326596+0.09556501j)
221	(-0.042971827+0.23937722j)	(0.13073908+0.1270226j)	(0.048672948-0.06346875j)	(0.021206522-0.088813424j)	(0.1901794-0.2169216j)	(-0.029284136+0.024858948j)	(-0.29060984+0.03422946j)
222	(-0.037735436+0.0066418382j)	(-0.099647336-0.12131322j)	(-0.05169738-0.36016986j)	(-0.14253348+0.10819008j)	(-0.116834626+0.09651822j)	(0.056414068-0.17803384j)	(0.1667285-0.24515238j)
223	(0.070434734-0.07451398j)	(-0.1756553-0.12745978j)	(-0.08949843-0.15957025j)	(0.03456987-0.07367534j)	(-0.032520145+0.09075477j)	(0.0029854525+0.062434208j)	(0.08751949-0.08023951j)

# Single targets: same as before, but with vectors

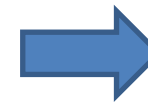
8 or 6

Parameters needed

- ✓ An easier way to see the polarimetric information is **vectorising** the scattering matrix
- ✓ Also, we are often interested in polarimetry alone and not how much the target scatter: so we **normalise** the scattering vector and obtain the **scattering mechanism** (this is sometime called projection vector).
- ✓ How many parameters we need:
  - ✓ In general 8 since we have 4 complex numbers
  - ✓ But for scattering mechanism we remove the overall power (length of vector)
  - ✓ It can also be showed that the “absolute phase” (a phase term that we can put as overall factor) does not keep information.
  - ✓ We end up with 6 parameters for a scattering vector

Scattering vector

$$\underline{k} = \frac{1}{2} \text{Trace}([S]\Psi) = [k_1, k_2, k_3, k_4]^T$$



Scattering Mechanism

$$\underline{\omega} = \underline{k} / |\underline{k}|$$

# Single targets: same as before, but with a simplification

6 or 4

Parameters needed

- ✓ In case the system is **monostatic** (one antenna as transmitter-receiver), and the medium observed is **reciprocal** (it behaves the same way independently by the direction of propagation of the wave) the two cross-polarisations are the same.
- ✓ We need less parameters to characterise targets in such situation.
- ✓ *Please note*, HV and VH are exactly the same except for **thermal noise**.
- ✓ *Please note*, at low frequencies (P and sometime L band) the **ionosphere** is not reciprocal introducing non-reciprocity (i.e. Faraday rotation). This is a problem only for satellites.

$$\underline{k} \in \mathbb{C}^3$$

$$S_{HV} = S_{VH}$$



Scattering vector

$$\underline{k} = \frac{1}{2} \text{Trace}([S]\Psi) = [k_1, k_2, k_3]^T$$



# Single targets: some physical interpretation

- ✓ The idea of using polarimetry is based on the physical concept that different targets will be excited with different currents. Therefore, *it has a narrow relation with some physical interpretation.*
- ✓ The scattering vector (as any vector) has to be expressed in some **basis**. The one that list the elements of the S matrix is called **Lexicographic basis**.
- ✓ There are also other basis that helps having some physical interpretation of the target. An example is the **Pauli basis**.
  - ✓ Each of the components is sensitive to a specific target (you will learn more on this when speaking about *Decompositions*).

Pauli bases: 
$$\vec{k}_P = \frac{1}{\sqrt{2}} [S_{HH} + S_{VV}, S_{HH} - S_{VV}, S_{HV} + S_{VH}]^T$$

Lexicographic bases: 
$$\underline{k}_l = [S_{HH}, S_{HV}, S_{VV}, ]^T$$

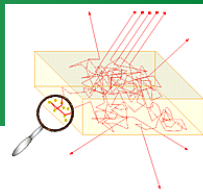
**In the polarimetric space, is 1 vector representing one unique single target?**



**Three different concepts you must  
remember:**

**Idea3: scattering polarimetry  
Distributed/partial targets**

# Problem: statistic or distributed targets



- ✓ This is a concept narrowly related with what we said in the lecture about **Speckle**
- ✓ When the target *changes* spatially it can NOT be represented by a *unique* Scattering matrix. It is a random process.
- ✓ In this image, you can imagine the squares as the **resolution cells**...
- ✓ The target under analysis is the same (the same forest)... but the objects things inside the squares are different (as you can see):
  - ✓ So, how do we deal with this variation? The scattering matrix will change pixel by pixel due to the speckle (because the targets in each resolution cell is slightly different).





# Solution: second order statistics

- ✓ In order to extract information the **second order statistics** of the target can be extracted
- ✓ In case of Gaussian complex pixels, these contain all the information about the random process
- ✓ In *Lexicographic basis*, we often talk about **COVARIANCE** matrix
- ✓ In *Pauli basis*, we often talk about **COHERENCY** matrix (the difference is only the basis and we can transform one into the other)

General formulation (any basis)

Second order statistics as outer product.

$$[C_3] = \left\langle \underline{k} \cdot \underline{k}^{*T} \right\rangle$$

Average

→

$$[C_3] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

Pauli basis

$$[T] = \begin{bmatrix} \langle |S_{HH} + S_{VV}|^2 \rangle & \langle (S_{HH} + S_{VV})(S_{HH} - S_{VV})^* \rangle & 2\langle (S_{HH} + S_{VV})S_{HV}^* \rangle \\ \langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* \rangle & \langle |S_{HH} - S_{VV}|^2 \rangle & 2\langle (S_{HH} - S_{VV})S_{HV}^* \rangle \\ 2\langle S_{HV}(S_{HH} + S_{VV})^* \rangle & 2\langle S_{HV}(S_{HH} - S_{VV})^* \rangle & 4\langle |S_{HV}|^2 \rangle \end{bmatrix}$$

# Properties of Target Coherency Matrix **9 or 8**

Parameters needed

It is  $[C] = [C]^{*T}$   
**Hermittian:**  
**Semi-positive**  
**Definite**  $I = \underline{\omega}^{*T} [C] \underline{\omega} \geq 0$

Rank 1 [C] matrices has a unique representation as [S] matrices (unless one absolute phase)... i.e. they are built with one single scattering vector

Real positive      Complex

$$[C] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

Lower and upper triangular parts are complex conjugate

- ✓ How many **parameters** we need?
  - ✓ 3 for the diagonal real positive terms
  - ✓ 6 for the off diagonal complex terms
  - ✓ If we neglect the overall amplitude (trace of the matrix) we reduce one parameter

# How do we practically use this?

- ✓ An easy way to use this is by creating one image each element of the covariance matrix
- ✓ Pay attention that cross-diagonal elements are Complex numbers.

The screenshot shows the Spyder Python IDE interface. The top part displays the Variable Explorer with a table of variables:

Name	Type	Size	Value
C11Full	float32	(1000, 6000)	array([[0.02843286, 0.11281456, 0.19216399, ..., 0.11451569, 0.0627168 ...
C12Full	complex64	(1000, 6000)	array([[ 1.45457825e-02+0.01128196j, 7.70821190e-03+0.00376824j, ...
C13Full	complex64	(1000, 6000)	array([[ 0.01435393-0.01395321j, 0.11613512+0.00352166j,
C22Full	float32	(1000, 6000)	array([[1.19179888e-02, 6.52541290e-04, 4.98843566e-03, ...,
C23Full	complex64	(1000, 6000)	array([[ 1.19179888e-02, 6.52541290e-04, 4.98843566e-03, ...
C33Full	float32	(1000, 6000)	array([[0.01409382, 0.11966334, 0.06784319, ..., 0.10910278, 0.0680876 ...

Below the Variable Explorer, two NumPy array viewers are shown, each with a red box around its title:

- C11Full - NumPy array**: A 7x7 matrix of float values. The diagonal elements are positive, and the off-diagonal elements are symmetric.
- C12Full - NumPy array**: A 7x7 matrix of complex numbers. The diagonal elements are real, and the off-diagonal elements are complex conjugates of each other.

**What happen if we just average the complex pixels (not the out product of the vectors)?**



# Summary of basic concepts



# Reminder: single and partial target representation

**Scattering matrix:**

$$[S] = \begin{pmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{pmatrix}$$

Scattering mechanism:

$$\underline{\omega} = \underline{k} / |\underline{k}|$$

**Scattering vector:**

$$\underline{k} = \frac{1}{2} \text{Trace}([S]\Psi) = [k_1, k_2, k_3, k_4]^T$$

Backscattering & reciprocity

$$\underline{k} = \frac{1}{2} \text{Trace}([S]\Psi) = [k_1, k_2, k_3]^T$$

The second order statistics are necessary.

$$[C_3] = \langle \underline{k} \cdot \underline{k}^+ \rangle$$

**Covariance matrix:**

$$[C_3] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

# Scattering matrix

Data are stored in complex form, that is real plus imaginary part

The screenshot shows the Spyder Python IDE interface. The Variable explorer on the right lists several variables: HHFull (complex64, size 1248x18432), HVFull (complex64, size 1248x18432), VHFull (complex64, size 1248x18432), and VFFull (complex64, size 1248x18432). Below these are 'envi' (module), 'filei\_HH' (str), and 'filei\_HV' (str).

The 'HHFull - NumPy array' window displays a 20x12 grid of complex numbers. The columns are indexed 6 through 12. The values are complex numbers in the form (real + imaginary\*j).

	6	7	8	9	10	11	12
208	(0.07107788+0.0001118965j)	(-0.2165731-0.11895645j)	(-0.037696168+0.027938405j)	(0.16672957-0.25383106j)	(0.09106964-0.16733629j)	(-0.2292527+0.13909335j)	(-0.11490008+0.11884244j)
209	(-0.28057334-0.042206895j)	(0.38021272+0.029810535j)	(0.43548566+0.26694945j)	(-0.138532+0.40491346j)	(-0.5769423+0.1847391j)	(-0.015036544-0.05962692j)	(0.26831898-0.1743832j)
210	(0.030044155+0.34601098j)	(-0.5461137-0.025732089j)	(-0.29259968-0.14295235j)	(0.20367235-0.5841537j)	(0.37775946-0.262635j)	(-0.21820979+0.16362272j)	(-0.17708418-0.09521227j)
211	(-0.41205642+0.051513102j)	(0.7546207+0.32724288j)	(1.0687426+0.5270206j)	(0.054809444+0.82367045j)	(-0.3507389+0.1318827j)	(0.21761155-0.41316128j)	(0.08479531-0.22683923j)
212	(-1.1572694-0.38786665j)	(1.9590037+0.3208726j)	(1.9236386+1.241693j)	(-1.1034379+2.4863272j)	(-2.1077378+0.6184698j)	(0.90856504-0.7555746j)	(1.2048659-0.31549478j)
213	(0.03483691+0.24929173j)	(-0.06724173-0.19649513j)	(-0.052806515+0.14856093j)	(-0.33817375+0.041899033j)	(-0.325384-0.35427806j)	(0.15580273+0.25812522j)	(0.3568068+0.18579605j)
214	(0.05983149+0.14247231j)	(0.09123571-0.019802434j)	(-0.0056780856-0.08619735j)	(0.0276807+0.026372923j)	(0.004342114+0.07160738j)	(-0.16508183-0.13053997j)	(-0.10213288-0.1436391j)
215	(-0.23911689+0.13675046j)	(0.2510476+0.20256251j)	(0.1657779+0.29408714j)	(-0.14119186+0.3051973j)	(-0.14424676+0.11273051j)	(0.019122131-0.149513j)	(0.011736556-0.17397477j)
216	(0.14865826+0.14563574j)	(-0.14296752+0.18294339j)	(-0.08458038+0.04554746j)	(0.11610123-0.26472777j)	(0.27141842-0.2700538j)	(0.014428847+0.092346266j)	(-0.16020864+0.009083515j)
217	(0.14413048+0.050818104j)	(0.1473479+0.08817709j)	(0.023391057-0.08530626j)	(0.035040595+0.107569896j)	(-0.057849325+0.24947123j)	(0.12625532-0.06831674j)	(0.2286333-0.123649314j)
218	(0.035153415+0.10457542j)	(-0.011003431+0.120111205j)	(0.03158148+0.0710468j)	(0.17607468-0.20794402j)	(0.22993661-0.093825236j)	(-0.052824214+0.10840995j)	(-0.13797311+0.015566013j)
219	(0.03727213-0.21260321j)	(0.1176478-0.031192722j)	(0.03230082-0.16893798j)	(-0.08027681-0.031375308j)	(-0.05084166+0.05267703j)	(-0.06616261-0.09878135j)	(-0.100596376-0.028747175j)
220	(0.26260537-0.008604212j)	(0.15181771-0.14325972j)	(-0.21277122-0.3136897j)	(-0.06775208-0.13144417j)	(0.060304996+0.14034642j)	(-0.038857333+0.015634881j)	(0.023326596+0.09556501j)
221	(-0.042971827+0.23937722j)	(0.13073908+0.1270226j)	(0.048672948-0.06346875j)	(0.021206522-0.088813424j)	(0.1901794-0.2169216j)	(-0.029284136+0.024858948j)	(-0.29060984+0.03422946j)
222	(-0.037735436+0.0066418382j)	(-0.099647336-0.12131322j)	(-0.05169738-0.36016986j)	(-0.14253348+0.10819008j)	(-0.116834626+0.09651822j)	(0.056414068-0.17803384j)	(0.1667285-0.24515238j)
223	(0.070434734-0.07451398j)	(-0.1756553-0.12745978j)	(-0.08949843-0.15957025j)	(0.03456987-0.07367534j)	(-0.032520145+0.09075477j)	(0.0029854525+0.062434208j)	(0.08751949-0.08023951j)

# Covariance matrix

- ✓ An easy way to use this is by creating one image each element of the covariance matrix
- ✓ Pay attention that cross-diagonal elements are Complex numbers.

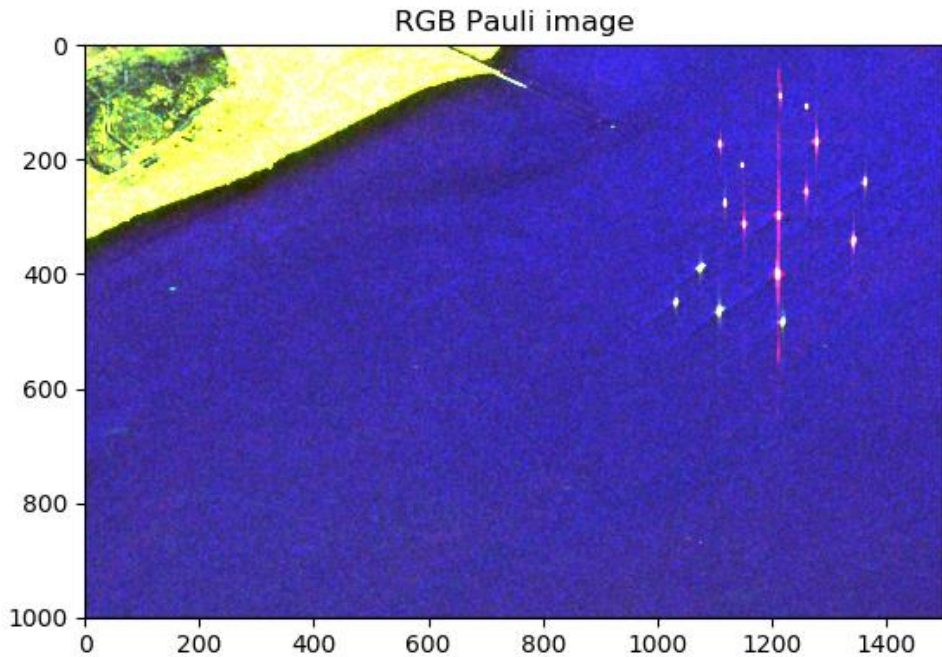
The screenshot shows the Spyder Python IDE interface. The top part displays the Variable Explorer with a table of variables:

Name	Type	Size	Value
C11Full	float32	(1000, 6000)	array([[0.02843286, 0.11281456, 0.19216399, ..., 0.11451569, 0.0627168 ...
C12Full	complex64	(1000, 6000)	array([[ 1.45457825e-02+0.01128196j, 7.70821190e-03+0.00376824j, ...
C13Full	complex64	(1000, 6000)	array([[ 0.01435393-0.01395321j, 0.11613512+0.00352166j,
C22Full	float32	(1000, 6000)	array([[1.19179888e-02, 6.52541290e-04, 4.98843566e-03, ...,
C23Full	complex64	(1000, 6000)	array([[ 1.19179888e-02, 6.52541290e-04, 4.98843566e-03, ...
C33Full	float32	(1000, 6000)	array([[0.01409382, 0.11966334, 0.06784319, ..., 0.10910278, 0.0680876 ...

Below the Variable Explorer, two NumPy array viewers are shown, each with a red box around its title:

- C11Full - NumPy array**: A 7x7 matrix of float values. The diagonal elements are approximately 0.0284, 0.1128, 0.1922, 0.0697, 0.0098, 0.0919, and 0.0766. The off-diagonal elements are real numbers.
- C12Full - NumPy array**: A 7x7 matrix of complex numbers. The diagonal elements are complex, e.g., (0.0145457825+0.0112819625j). The off-diagonal elements are also complex numbers.

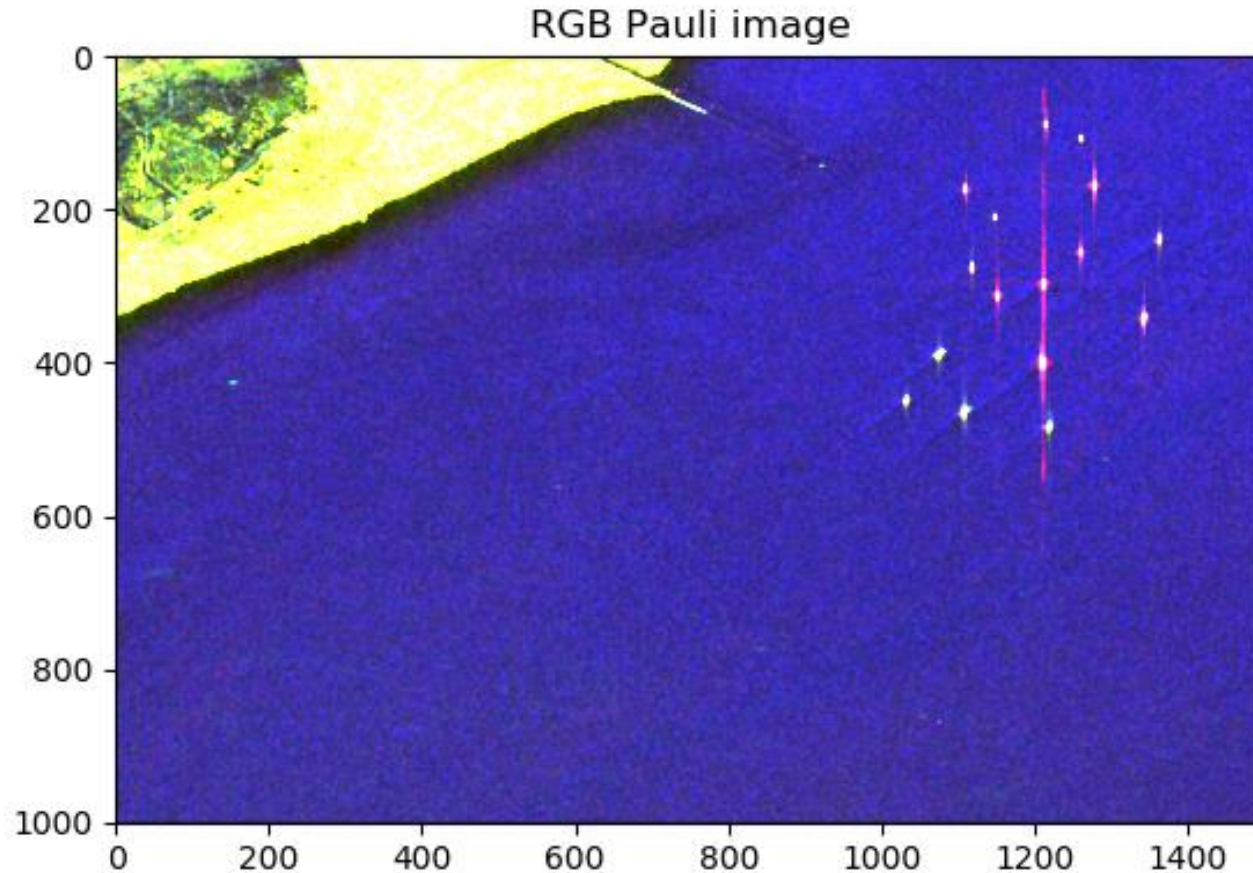
# Are ships single or partial targets?



Pauli RGB image around Buenos Aires (ALOS-1). The polarimetric information is coded in the colours. As you can notice we can use colours to differentiate between targets.

Data courtesy of JAXA.

# Are ships single targets?



It depends on the size of the ship, but generally they are a collection of several single targets.

Pauli RGB image around Buenos Aires (ALOS-1). The polarimetric information is coded in the colours. As you can notice we can use colours to differentiate between targets.

Data courtesy of JAXA.



# Target decomposition

# What is a decomposition?

Wikipedia definition: **Decomposition (or rotting)** is the process by which organic substances are broken down into simpler forms of matter.

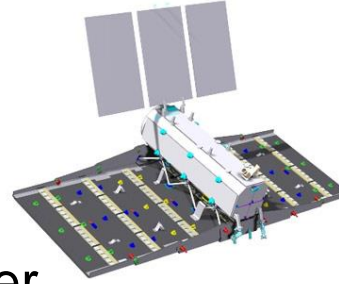
## Collins definition:

decompose (,di:kəm'pəʊz)

- 1) to break down (organic matter) or (of organic matter) to be broken down physically and chemically by bacterial or fungal action;
- 2) chem to break down or cause to break down into simpler chemical compounds
- 3) to break up or separate into constituent parts
- 4) ( tr ) maths to express in terms of a number of independent simpler components, as a set as a canonical union of disjoint subsets, or a vector into orthogonal components



# What is a decomposition?



- ✓ On the scene, several targets are **combined/mixed** each other inside the *resolution cell* AND the *averaging window*.
- ✓ It makes image *interpretation* and *retrieval* of parameters very complex
- ✓ We want to use polarimetry to **separate** (or decompose) the different contributors and extract some physical interpretation.

# What shall we decompose?

As for the basic **concepts** of polarimetry, we should separate **deterministic** and **statistical** targets

Scattering matrix

$$S = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

1) Coherent Decompositions

2) Incoherent decompositions

Covariance matrix

$$[T] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

# Coherent decompositions: Scattering matrix

# Coherent decompositions

$$[S] = c_1 [S_1] + c_2 [S_2] + c_3 [S_3]$$

$$c_1, c_2, c_3 \in \mathbb{C}$$

$$S = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

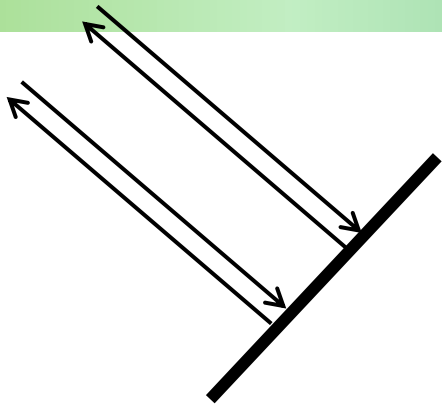
$$S = e^{j\varphi_{HH}} \begin{bmatrix} A_{HH} & A_{HV} e^{j(\varphi_{HV} - \varphi_{HH})} \\ A_{VH} e^{j(\varphi_{VH} - \varphi_{HH})} & A_{VV} e^{j(\varphi_{VV} - \varphi_{HH})} \end{bmatrix}$$

Absolute phase

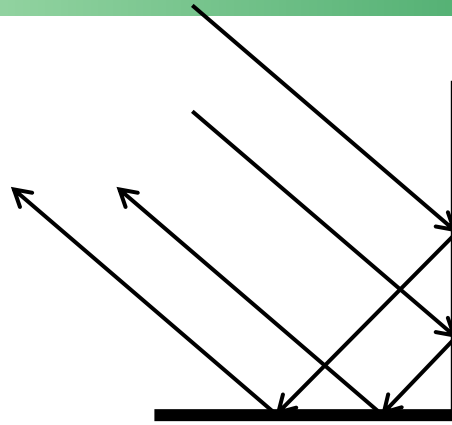
- ✓ Definition: they are called **COHERENT** because they separate the contributions at the sub-pixel level starting from the **scattering matrix** and the contributors sum “coherently” (i.e. with the phase)



# Pauli coherent decomposition



Odd-bounce



Even-bounce

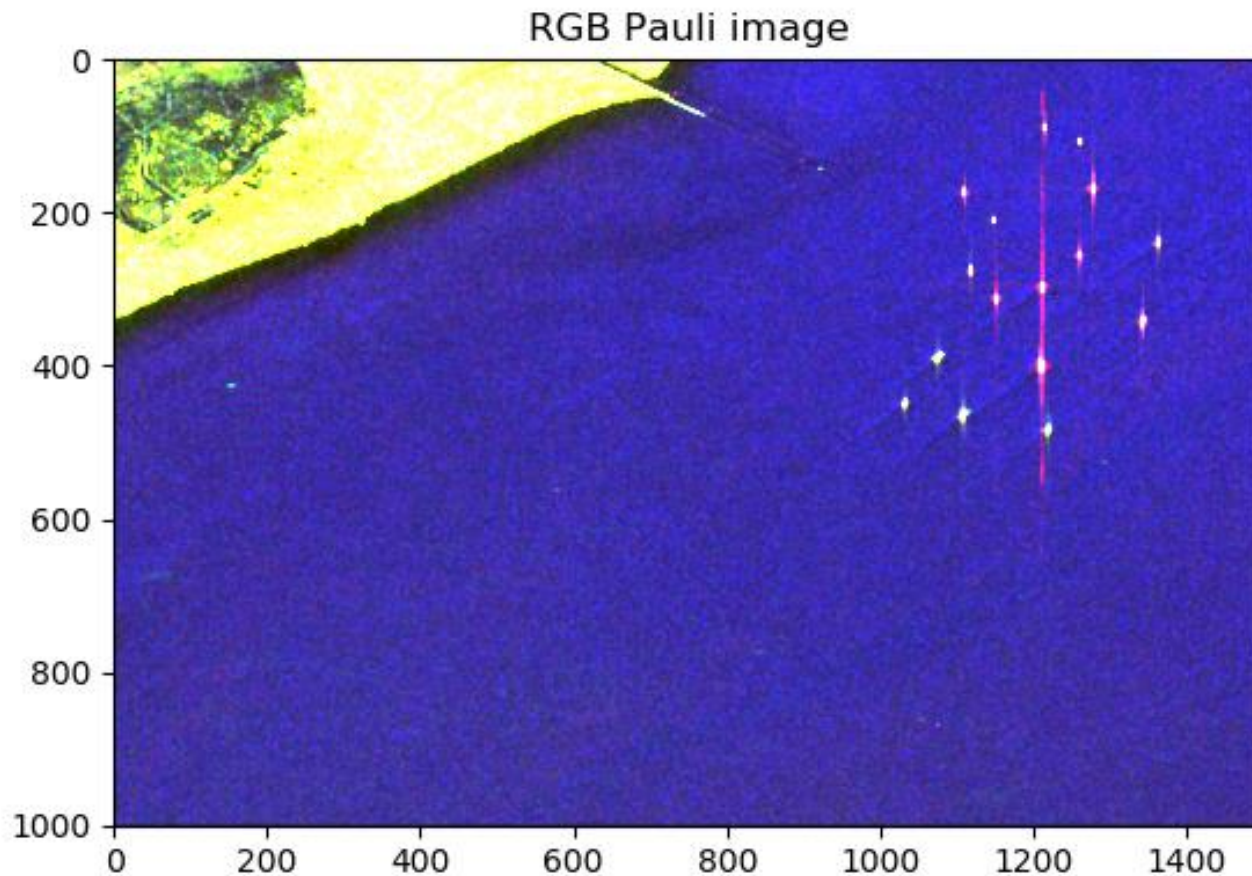
Even-bounce  
45° oriented

$$\underline{k}_p = \frac{1}{\sqrt{2}} [S_{HH} + S_{VV}, S_{HH} - S_{VV}, S_{HV} + S_{VH}]^T$$



The corner  
is 45° oriented

# Pauli decomposition



Pauli RGB image around Buenos Aires (ALOS-1). The polarimetric information is coded in the colours. As you can notice we can use colours to differentiate between targets.

Data courtesy of JAXA.

# Can we use coherent decompositions for ships?



# Incoherent decompositions: Covariance matrix

# Incoherent decompositions

$$[T] = c_1 [T_1] + c_2 [T_2] + c_3 [T_3]$$

$$c_1, c_2, c_3 \in R$$

$$[T] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

- ✓ Definition: they are defined incoherent because they separate the contribution starting from the **coherency matrix**, therefore the components sum each other **WITHOUT the phase**
- ✓ This is based on the assumption that the components/contributors are **independent** of each other and therefore they sum incoherently (without phase).

**Incoherent decompositions:  
Non-model based**



# Diagonalising the coherency matrix: Cloude-Pottier

It is based on the **diagonalisation** of the coherency matrix which is *Hermittian positive semi-definite*

$$I = \underline{\omega}^{*T} [T] \underline{\omega} \geq 0$$

$$[T] = [U][\Sigma][U]^{*T} = \sum_{i=1}^3 \lambda_i \underline{u}_i \underline{u}_i^{*T} = \lambda_1 \underline{u}_1 \underline{u}_1^{*T} + \lambda_2 \underline{u}_2 \underline{u}_2^{*T} + \lambda_3 \underline{u}_3 \underline{u}_3^{*T}$$

$$[U]^{*T} [U] = [I] \Rightarrow [U]^{*T} = [U]^{-1}$$

Unitary matrix

Eigenvalues

Eigenvectors

$$[T] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

- ✓ Each component represents a **deterministic** target (it could be expressed by a single scattering matrix): i.e. each component is a rank one matrix.

# Cloude-Pottier: interpreting eigenvalues

Nice math, but what all this eigenvalues tells us?

We can define a **probability** of each eigenvalue  $P_i = \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}$

We can calculate the **Entropy**:  
of the scattering process

$$H = \sum_{i=1}^3 (-P_i \log_3 P_i)$$

We can also calculate the **Anisotropy**:

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$

An interesting property is that the parameters on this slide are **basis invariant**:  
i.e. the same results are obtained independently on the basis used to represent  
the scattering vector. This is a property of diagonalisations... and we like it,  
since it makes the result more general.

# Cloude-Pottier: interpreting eigenvalues

- ✓ The entropy tells us the **confusion** of the scattering process. If there is a component (i.e. one eigenvector) that is much stronger than the other, then the entropy is **LOW** (close to 0) and we know there is only one dominant target in the scene (i.e. this is a more deterministic problem that could be treated with a single scattering matrix). An example is a man-made target.
- ✓ If the entropy is **HIGH** (close to 1) there are three or more equally strong scattering processes in the scene that they confuse a lot the polarisation of the pixels. An example is a forested area.
- ✓ The anisotropy tells about the **imbalance** of second and third scattering mechanisms (eigenvalues). It is used to complement the entropy... you will learn more next lecture.



# Cloude-Pottier: interpreting eigenvectors

- ✓ What about the eigenvectors? They are **3 scattering mechanisms orthogonal each other**
- ✓ Their **representation** (i.e. the numbers in the vector components) is not basis invariant and we need to select a basis to visualise them (since they are vectors)
- ✓ The Cloude-Pottier decomposition consider using the **Pauli** basis and perform a parameterisation based on spherical coordinates (with unitary radius)

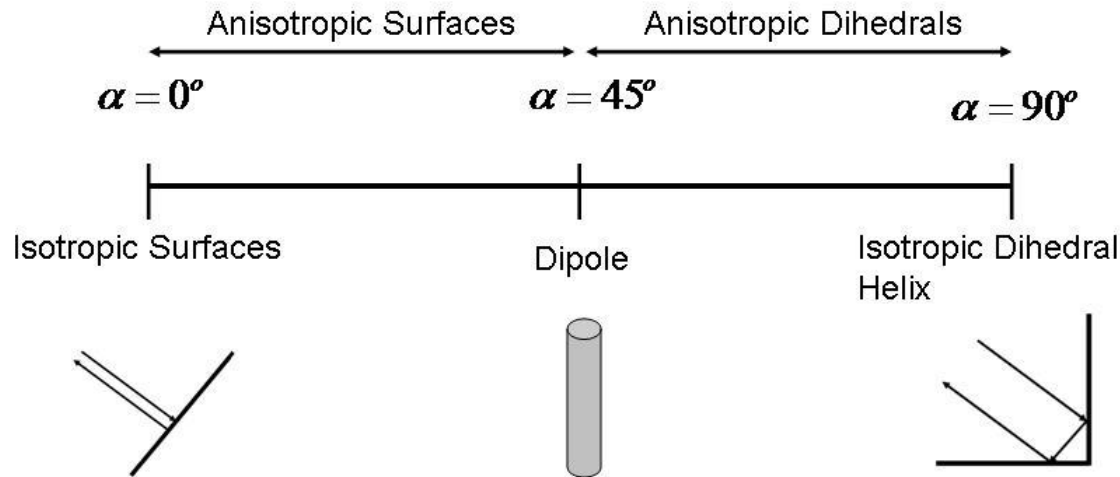
Scattering vector in Pauli basis with spherical coordinates

$$\underline{u}_i = \left[ \cos \alpha_i, \sin \alpha_i \cos \beta_i \cdot e^{j\varepsilon_i}, \sin \alpha_i \sin \beta_i \cdot e^{j\eta_i} \right]^T, \quad i=1,2,3$$

Each one of the eigenvectors can be represented this way

# Cloude-Pottier: interpreting eigenvectors

1) The parameter  $\alpha$  is related to the type of scattering mechanism (it can be easily proved substituting the values of alpha in the previous parameterisation)



2) The parameter  $\beta$  is related to the orientation of the scattering mechanism (also can be easily proved substituting the values in the previous parameterisation)

3) The parameters  $\epsilon$  and  $\eta$  are phases with complicated physical interpretation (but they stay the same once decided the target to represent)

# Cloude-Pottier: interpreting eigenvectors

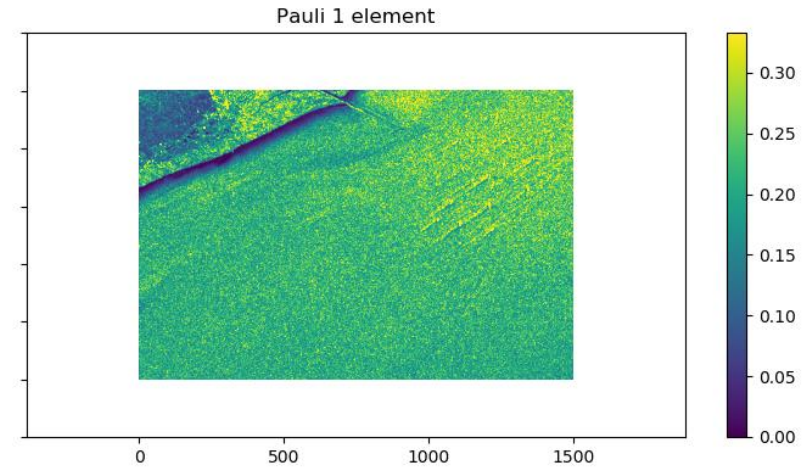
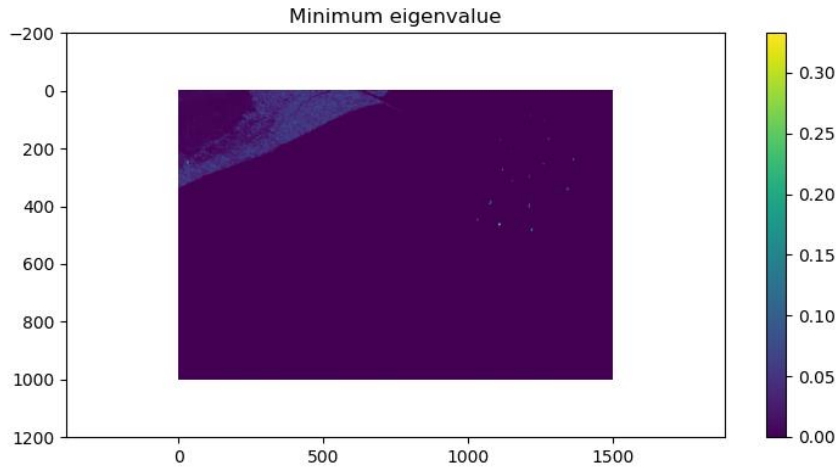
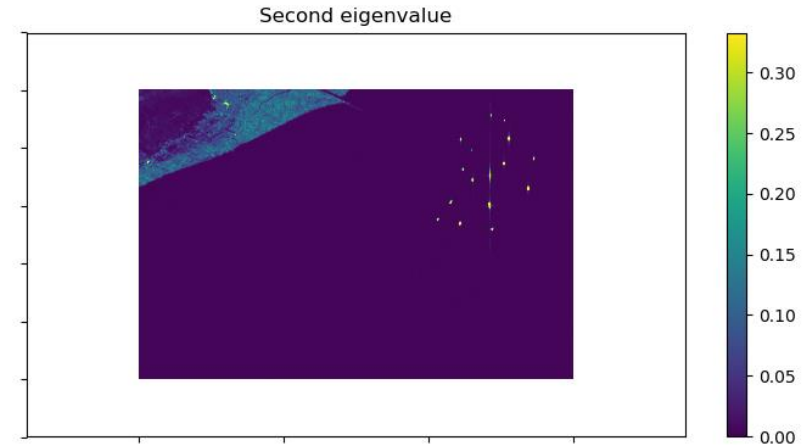
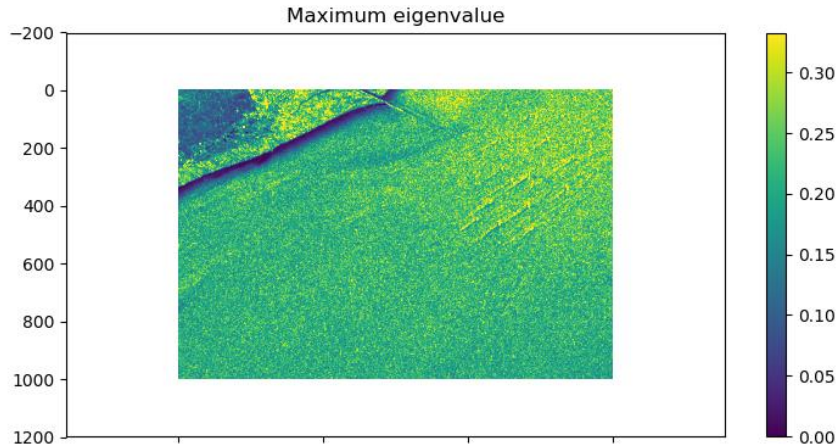
- ✓ We have three  $\alpha$  angles in the decomposition (one from each scattering mechanism). Which one shall we use?
  - ✓ If the entropy is **low** (one dominant target) we can use the dominant  $\alpha$
  - ✓ If the entropy is **high**, the process is very confused and it is better to use an **averaged** value for  $\alpha$ .
  - ✓ We consider a **Bernulli** process to average the  $\alpha$  (i.e. we do a weighted average where the weights are the probability of the eigenvalues).
- ✓ The same is for  $\beta$ , we can consider dominant or averaged values

It is useful to calculate an average  $\alpha$ ,  
obtained as the result of a Bernulli process

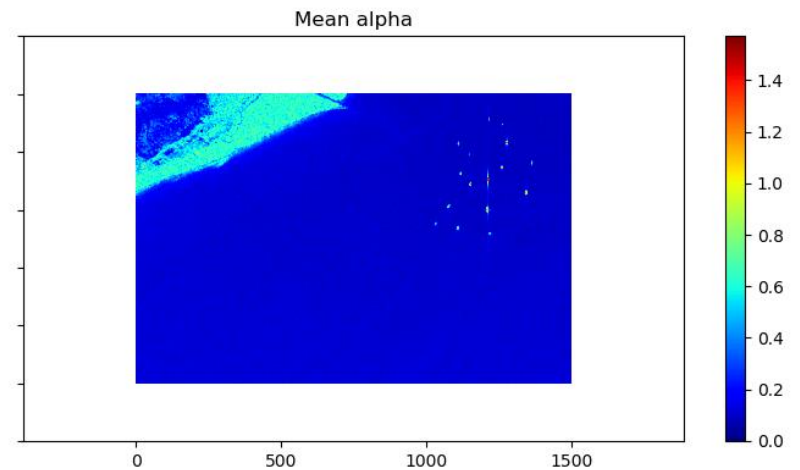
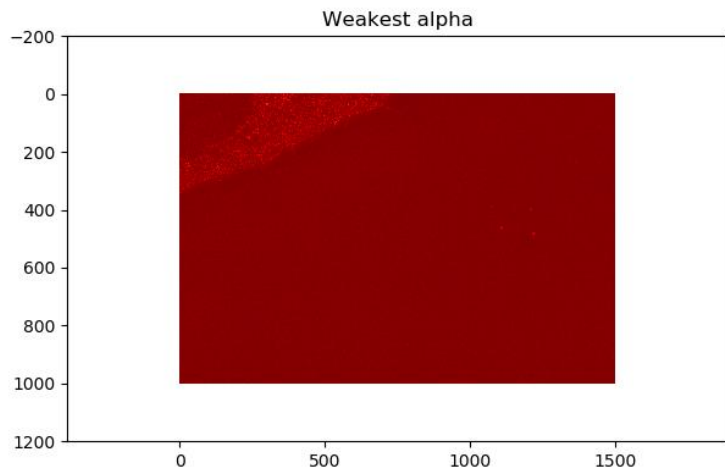
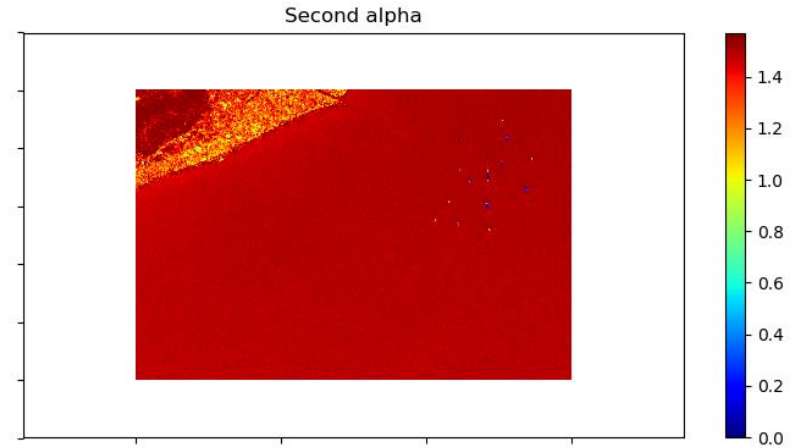
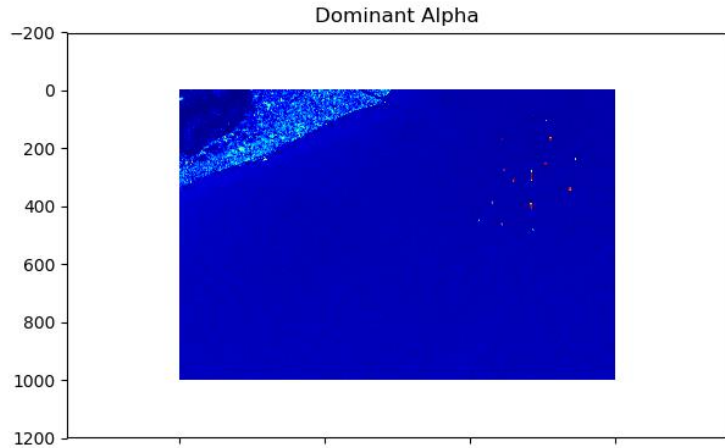
$$\hat{\alpha} = \sum_{i=1}^3 (P_i \alpha_i)$$



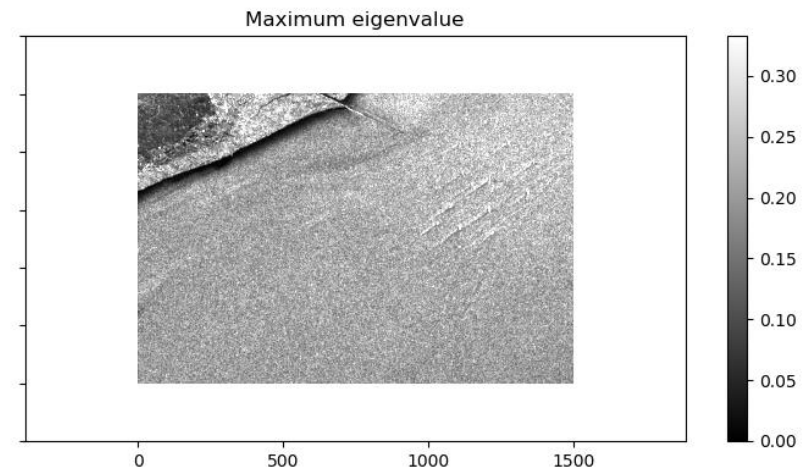
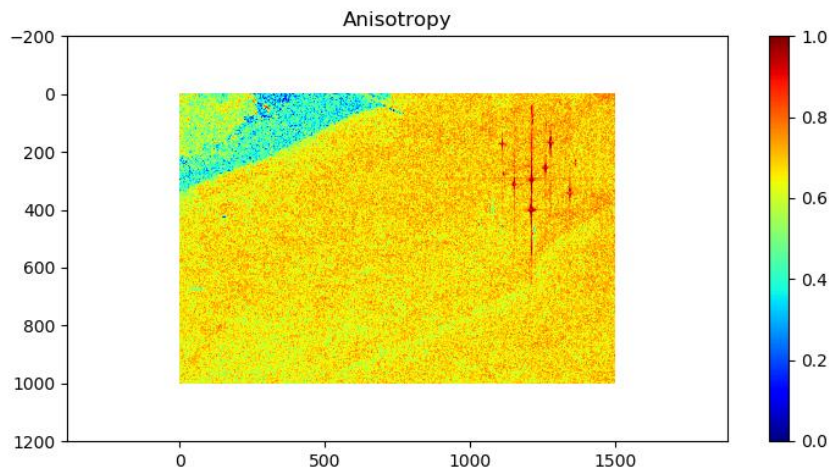
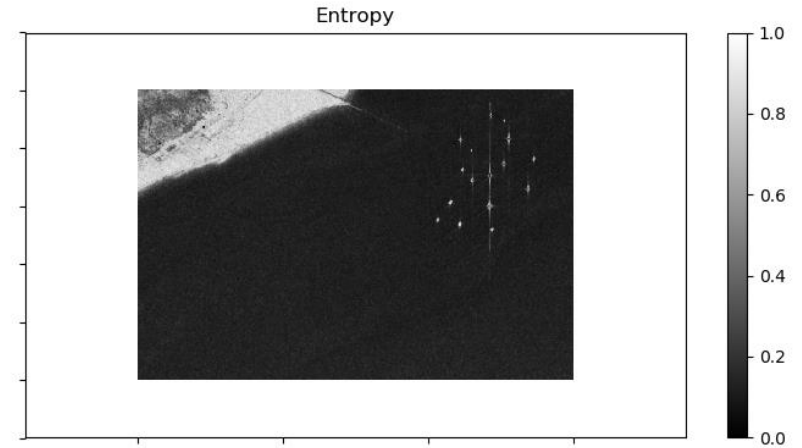
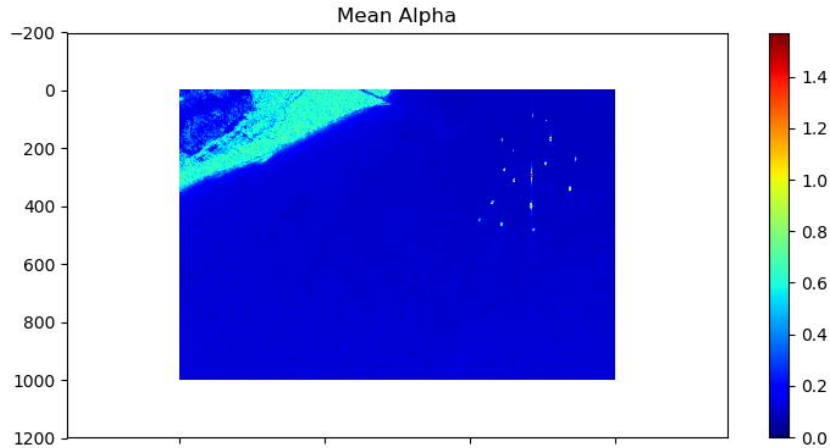
# Cloude-Pottier: Buenos Aires (ALOS-1)



# Cloude-Pottier: Buenos Aires (ALOS-1)



# Cloude-Pottier: Buenos Aires (ALOS-1)



**What do you think is a target that could produce low entropy and 45 degrees alpha?**



# **Incoherent decompositions: Model based**

# Yamaguchi decomposition

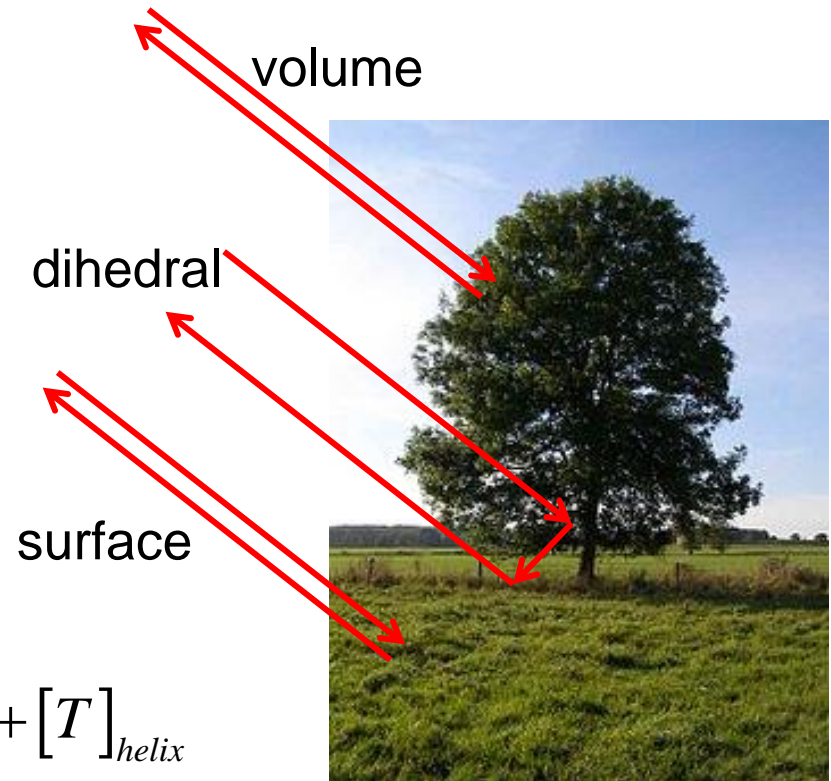
It is based on a model for the backscattering of **forested areas**.

The total return is decomposed in  
Surface, Dihedral, Volume and Helix scattering.

In order to solve the problem with the **orientation** of the dihedrals, it perform a **correction** for the orientation angle

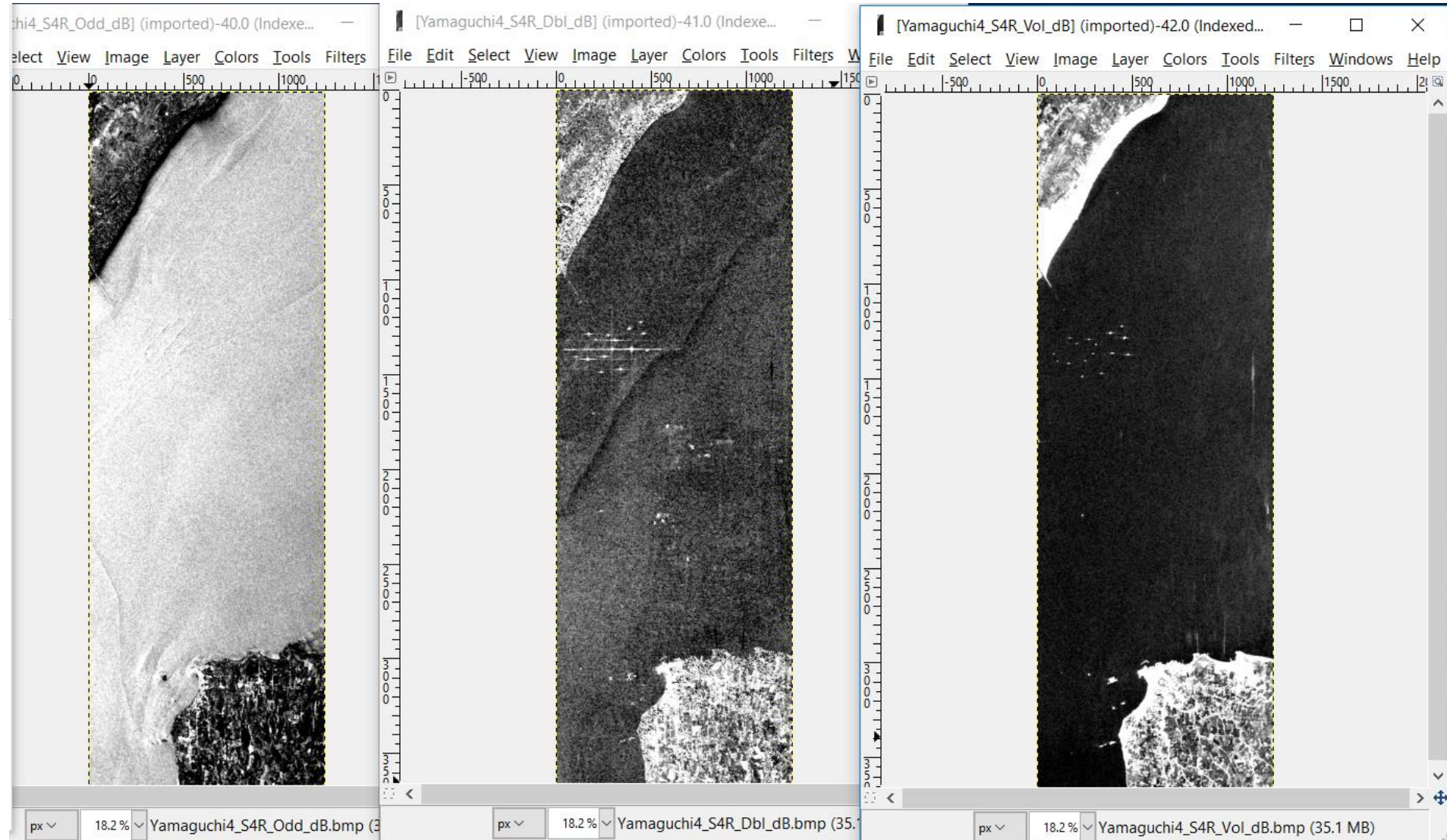
It rotates the partial target in order to give it an *overall horizontal orientation*

$$[T] = [T]_{surface} + [T]_{dihedral} + [T]_{volume} + [T]_{helix}$$





# Yamaguchi decomposition: Buenos Aires (ALOS-1)



**Practical**

# SNAP

- ✓ Today you will use the SNAP software to investigate some of the polarimetric theory you have studied.
  - ✓ Creating the covariance matrices

# Python

- ✓ Tomorrow you will use Python to process polarimetric data.
  - ✓ Pauli decomposition
  - ✓ Covariance and Coherency matrix
  - ✓ Claude Pottier decomposition
  - ✓ Ship detection
- ✓ You will be give the code with missing parts to complete.

# What is the hardest concept you have learned today?



# What would you like me to explain more right now?





**Thank you for your  
attention!**

