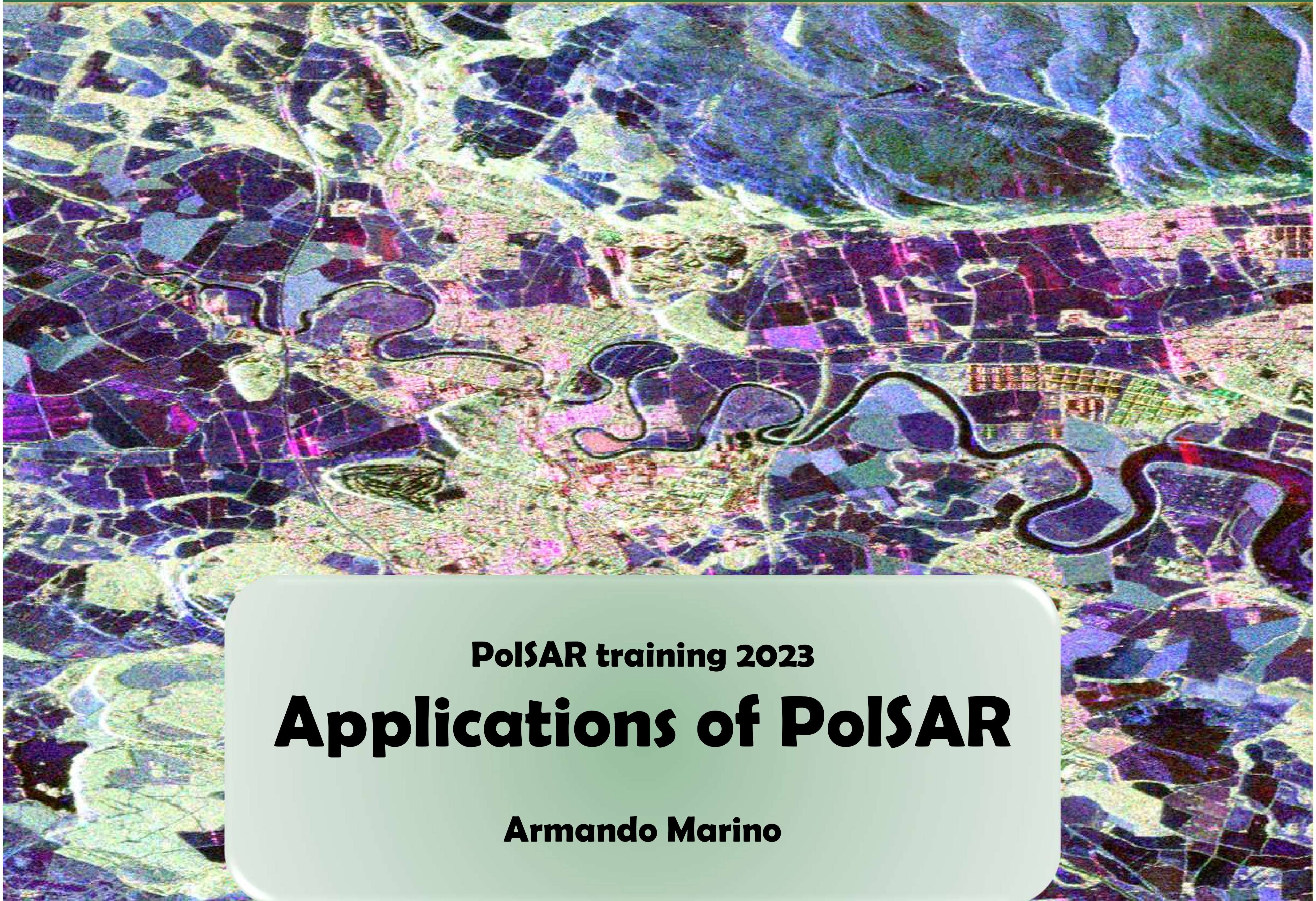
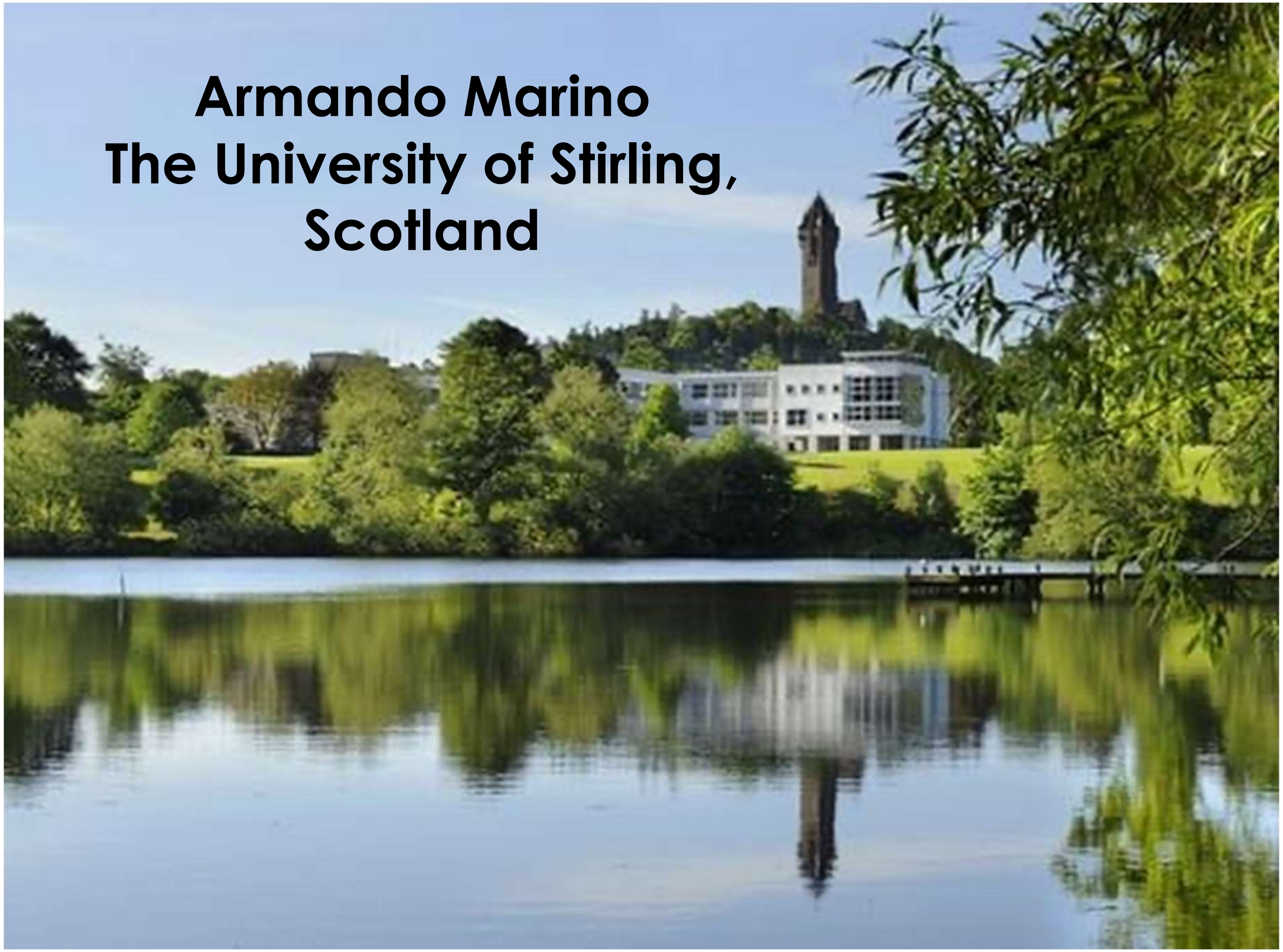


PolSAR training 2023

Applications of PolSAR

Armando Marino

Introduction



Outline

- ✓ Quick recap
- ✓ Maritime applications
  - ✓ Icebergs and acquaculture
  - ✓ Target detectors
- ✓ Agriculture/Urban/hydrology applications
  - ✓ Flooding
  - ✓ Physical change detectors
  - ✓ Statistical change detectors

Armando.marino@stir.ac.uk

Naming conventions & Warming up

Partial vs Single targets

Single Targets

Scattering matrix:

$$[S]=\begin{pmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{pmatrix}$$

Scattering vector:

$$\underline{k}=\frac{1}{2}Trace([S]\Psi)=[k_1,k_2,k_3,k_4]^T$$

Backscattering & reciprocity

$$\underline{k}=\frac{1}{2}Trace([S]\Psi)=[k_1,k_2,k_3]^T$$

Scattering mechanism, Projection vector:

$$\underline{\omega}=\underline{k}/|\underline{k}|$$

Partial Targets

The second order statistics are necessary.

$$[C_3]=\langle \underline{k}\cdot\underline{k}^+\rangle$$

Covariance matrix:

$$[C_3]=\begin{bmatrix} \langle |k_1|^2\rangle & \langle k_1k_2^*\rangle & \langle k_1k_3^*\rangle \\ \langle k_2k_1^*\rangle & \langle |k_2|^2\rangle & \langle k_2k_3^*\rangle \\ \langle k_3k_1^*\rangle & \langle k_3k_2^*\rangle & \langle |k_3|^2\rangle \end{bmatrix}$$

Quadratic forms

- ✓ The **projection vector** represents idealised targets.
- ✓ Once we have covariance matrices we can evaluate the power over any projection vector by calculating its **quadratic form**

$$\underline{\omega}^{*T}[C]\underline{\omega}$$

For the curious ones

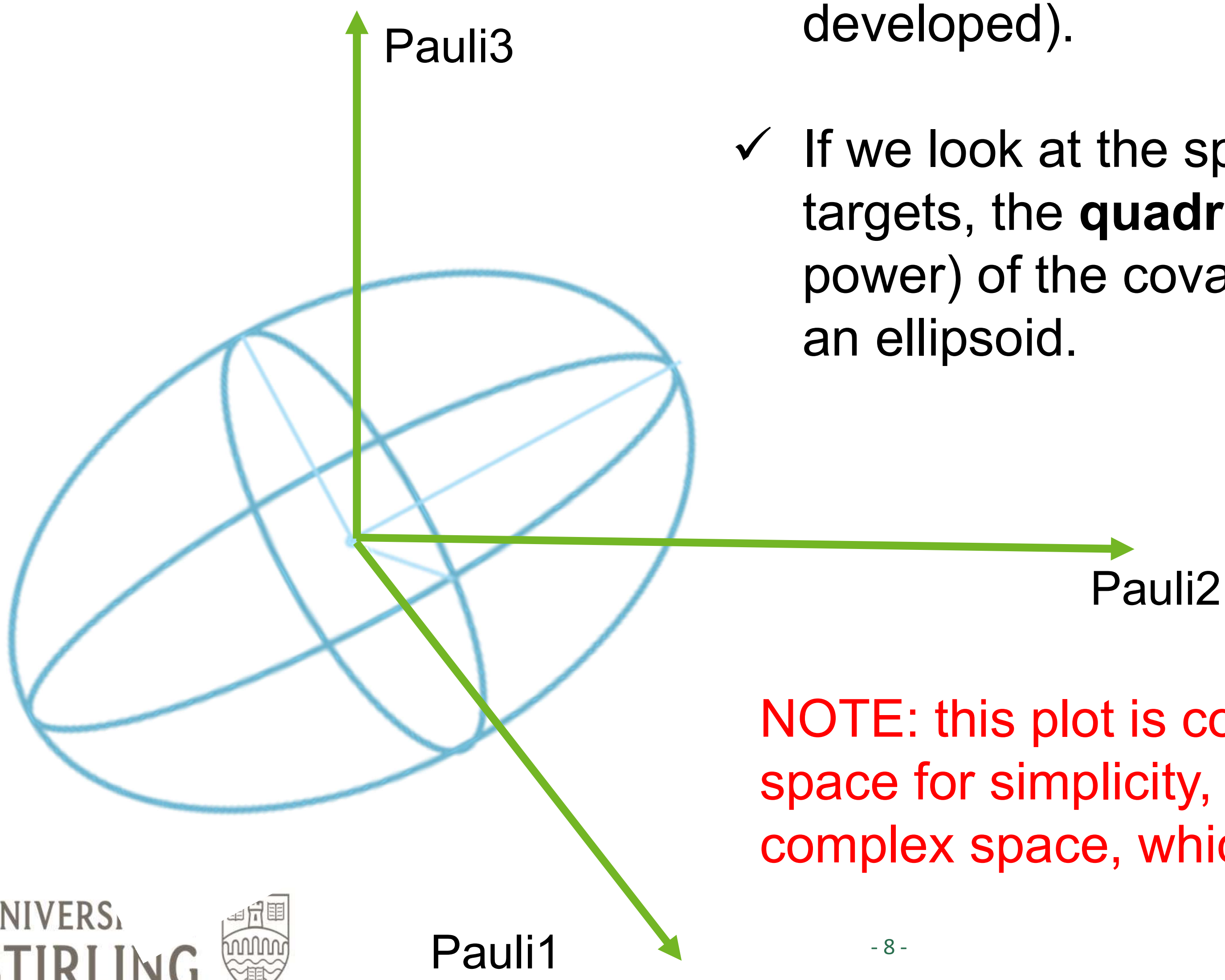
$$\mathrm{img}_1(\underline{\omega}) * \mathrm{img}_1(\underline{\omega})^*$$
$$= \underline{\omega}^{*T}\underline{k} * \left(\underline{\omega}^{*T}\underline{k}\right)^{*T}$$
$$= \underline{\omega}^{*T}\underline{k} * k^{*T}\underline{\omega} = \underline{\omega}^{*T}[C]\underline{\omega}$$

What shape do covariance matrices have in the intensity space?

https://PollEv.com/multiple_choice_polls/jAEsp8PqF9B6OgSarLfGr/respond

Not fully developed speckle

$$[C_3] = \langle \underline{k} \cdot \underline{k}^{+} \rangle$$

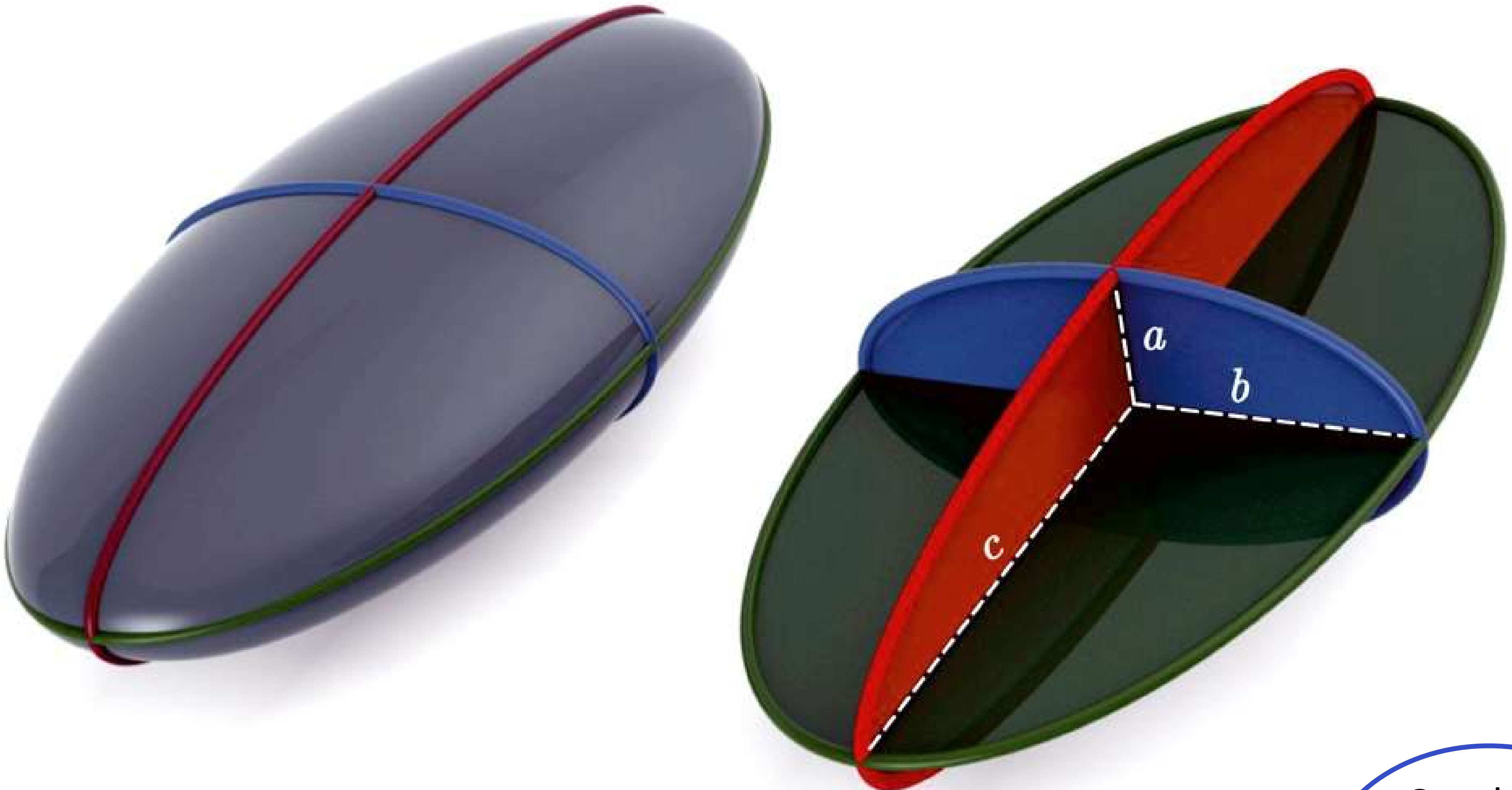


- ✓ The outer product forces the output matrix to be **convex** (which is a fantastic news when speckle is fully developed).
- ✓ If we look at the space of polarimetric targets, the **quadratic forms** (i.e. the power) of the covariance matrix shapes an ellipsoid.

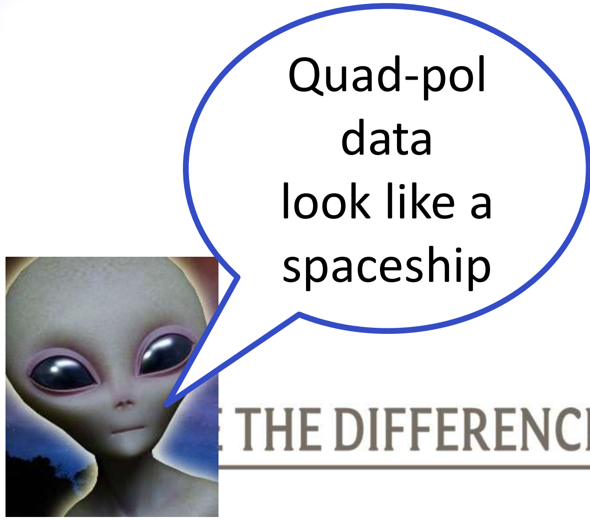
NOTE: this plot is considering a 3-D real space for simplicity, in reality we have a 3D complex space, which we cannot visualise

Armando.marino@stir.ac.uk

Not fully developed speckle



https://commons.wikimedia.org/wiki/File:Triaxial_Ellipsoid.jpg



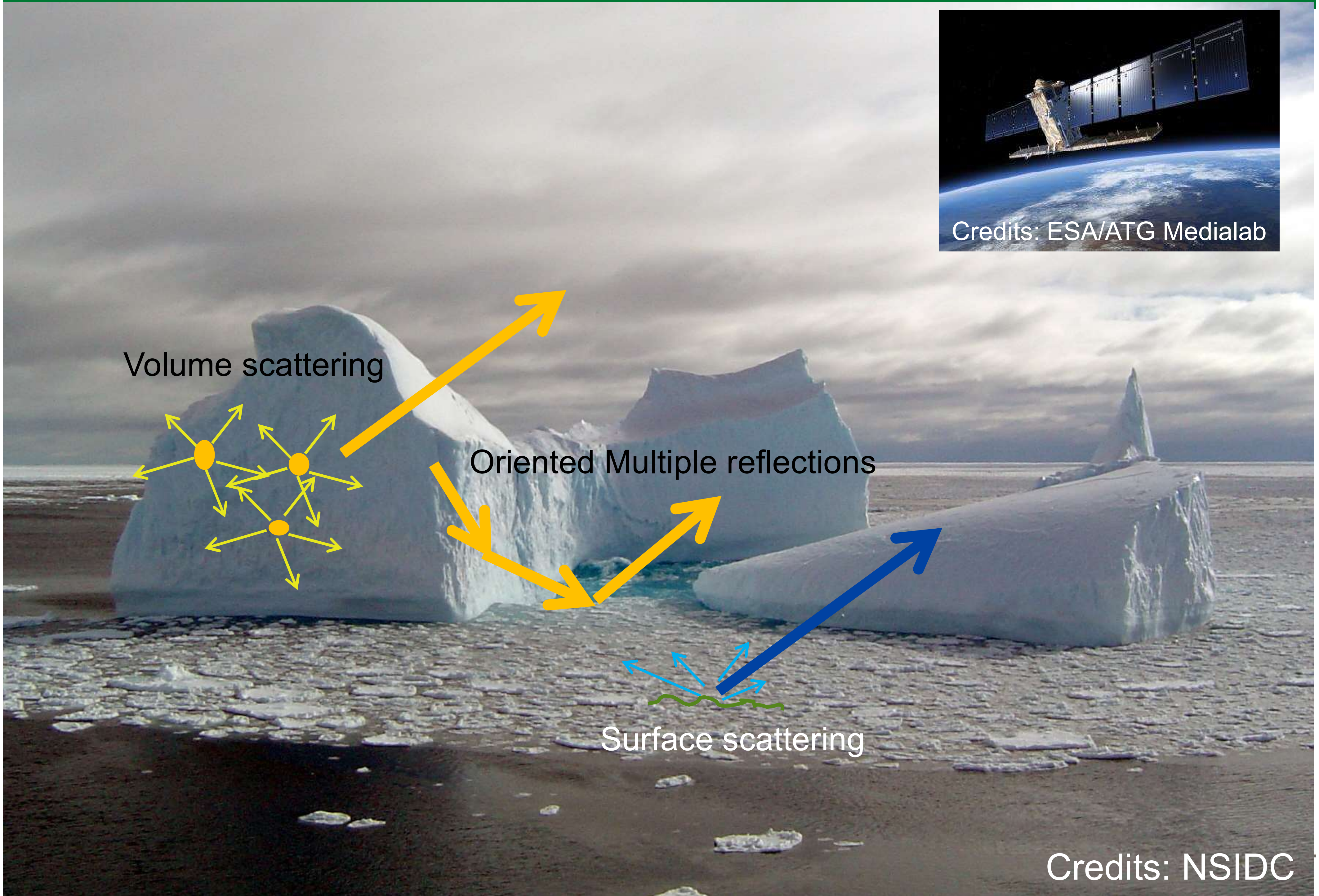

Maritime applications

Detecting objects in maritime domain

Why monitoring icebergs?

- ✓ Icebergs are generated from the calving of glaciers or ice shelves.
  - ✓ The are a **danger** for navigation
  - ✓ They play a role in **ocean circulation**.
  - ✓ They are indicator of **currents**.


Scattering from icebergs

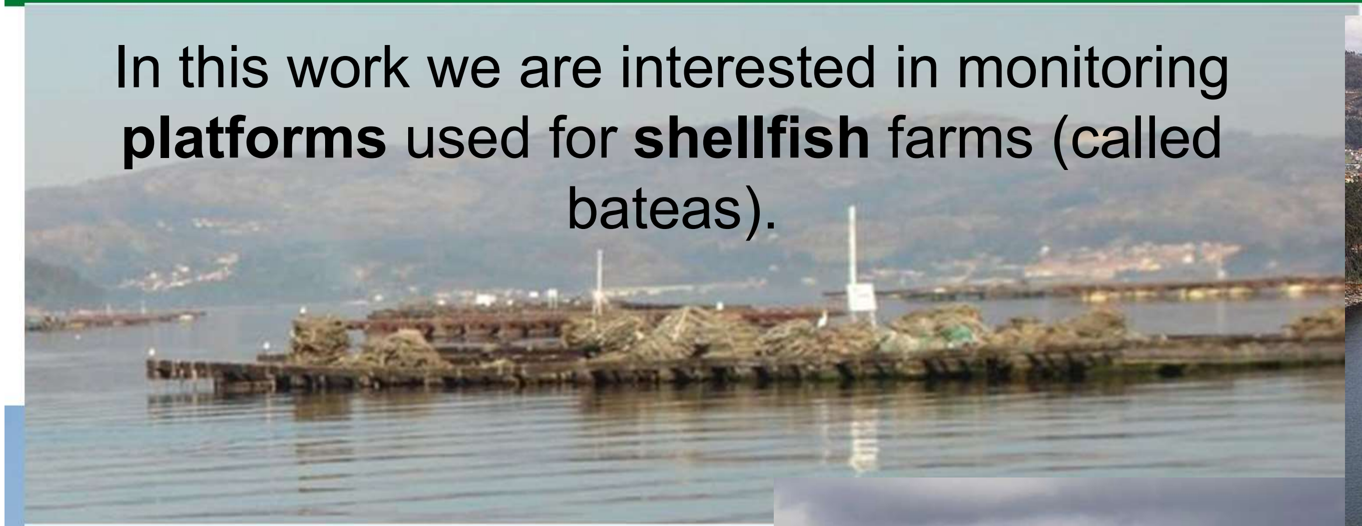


Aquaculture

- ✓ Aquaculture are a very **valuable asset** for many coastal countries
  - ✓ The industry is worth $150bn in 2017 (Financial Times).
- ✓ In the future they will play an important role in **food security**.
- ✓ **Satellite remote sensing** can improve the temporal and geo-spatial analysis of such marine facilities.
- ✓ Detecting platforms used for fish and shellfish farming provides a way to **monitor assets** and check they do not get damaged by **storms**.
- ✓ It also allows to identify **illegal placement** of structures in areas which should not host farms.
  - ✓ As the most of human enterprises aquaculture is not immune to illegal activities: e.g. the illegal bluefin tuna market is double the legal market (Europol)

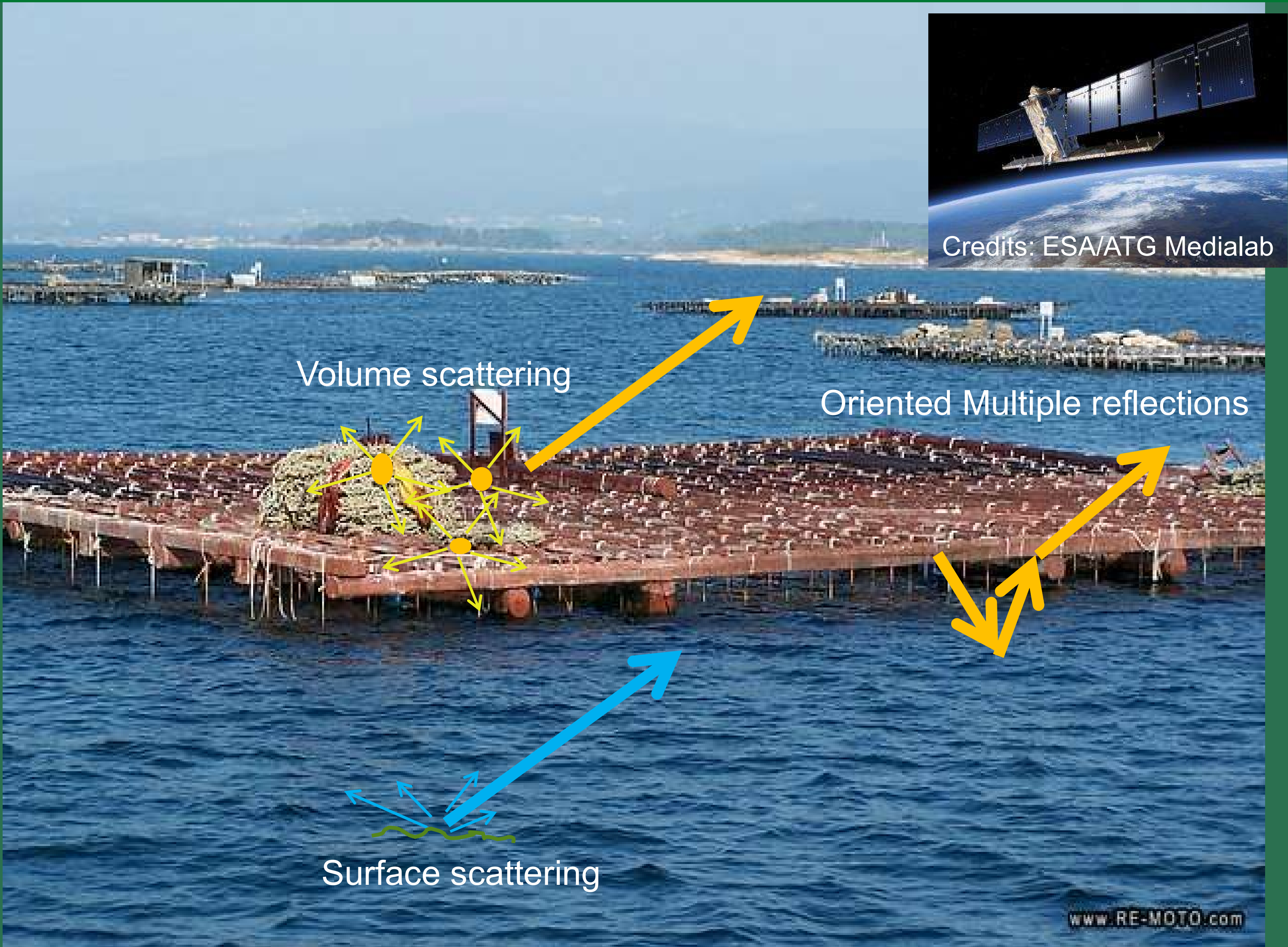
Aquaculture

In this work we are interested in monitoring **platforms** used for **shellfish** farms (called bateas).

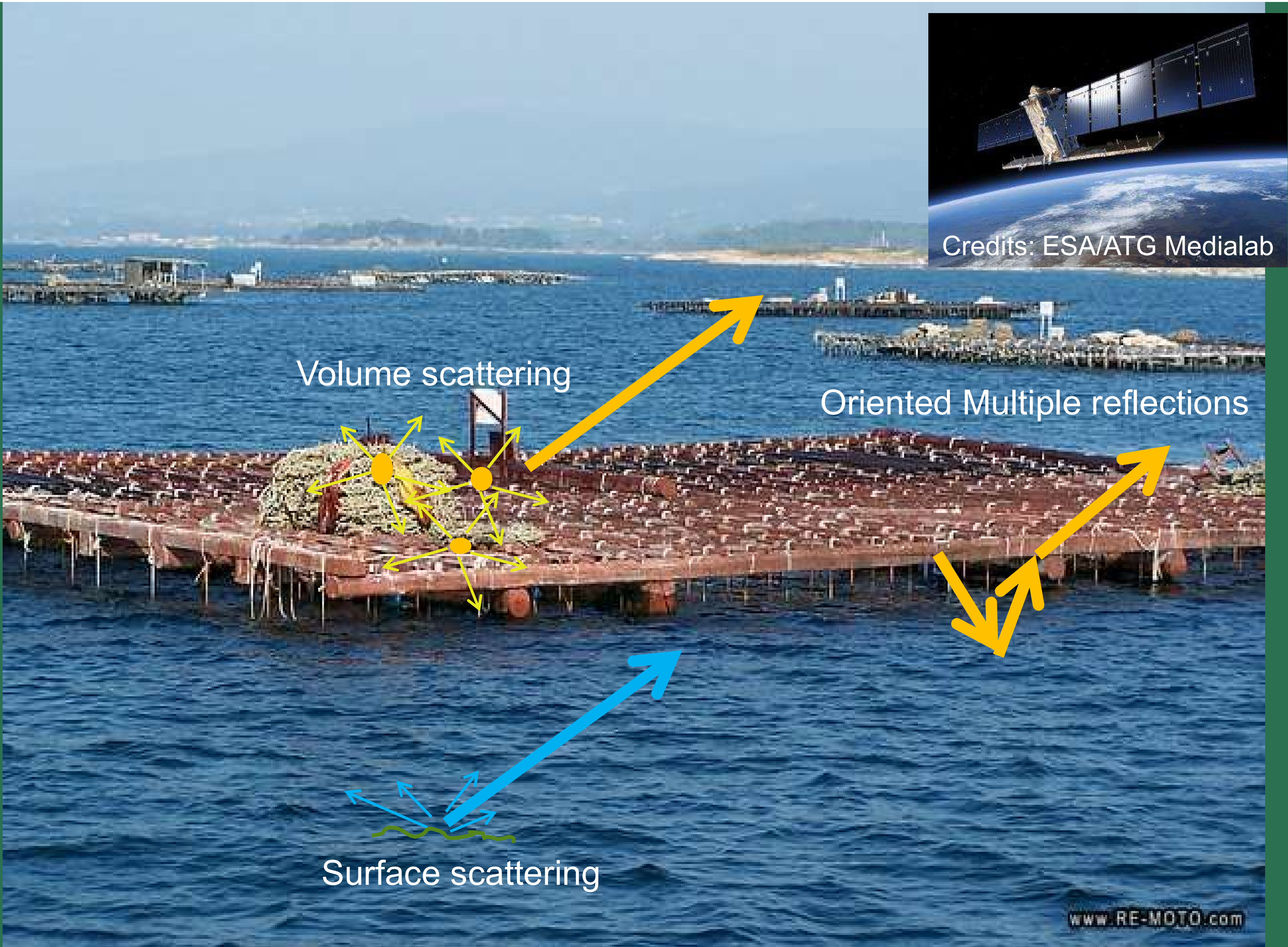


We also focus on the aria of **Vigo, Spain**.

Scattering from platforms



Scattering from platforms



Are ships single or partial targets?

https://PollEv.com/multiple_choice_polls/gksc6EbPAirjF4Ssn3Xzb/respond

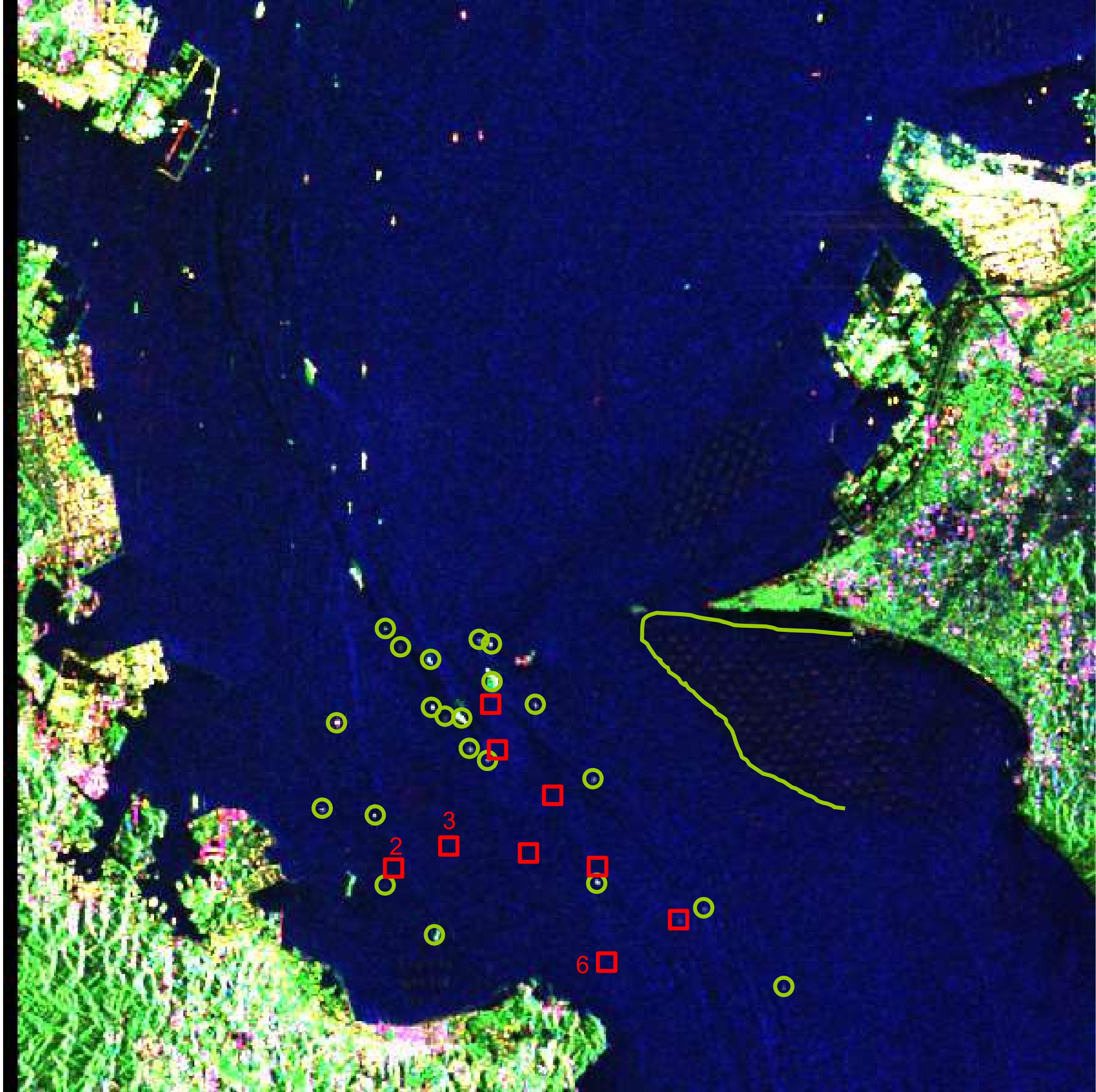
Detecting objects:
PolSAR algorithms

Radar polarimetric for ship detection

We will see the following detectors, but many more were proposed in the literature

Detector	Imaging Mode	Methods
Entropy	Quad-pol	Detecting depolarised targets
PMF	Quad-pol	Optimising the contrast ship/sea
Liu et al	Quad-pol	GLRT for covariance matrix
GP-PNF (quad)	Quad-pol	Detecting targets orthogonal to sea
Symmetry	Dual-pol (co/cross)	Detecting non-symmetric targets
GP-PNF (dual)	Dual-pol	Detecting targets orthogonal to sea in the dual-pol subset
Dihedral	Dual-pol (HH/VV)	Detecting horizontal dihedrals
HV intensity	Single-pol	Detecting high Backscattering in HV

Armando.marino@stir.ac.uk



RGB PAULI

Red: HH-VV
Green: 2*HV
Blue: HH+VV
Sigma nought
1000x1000 pixels
Multi-look 1x5
Image size ~ 30x30km.

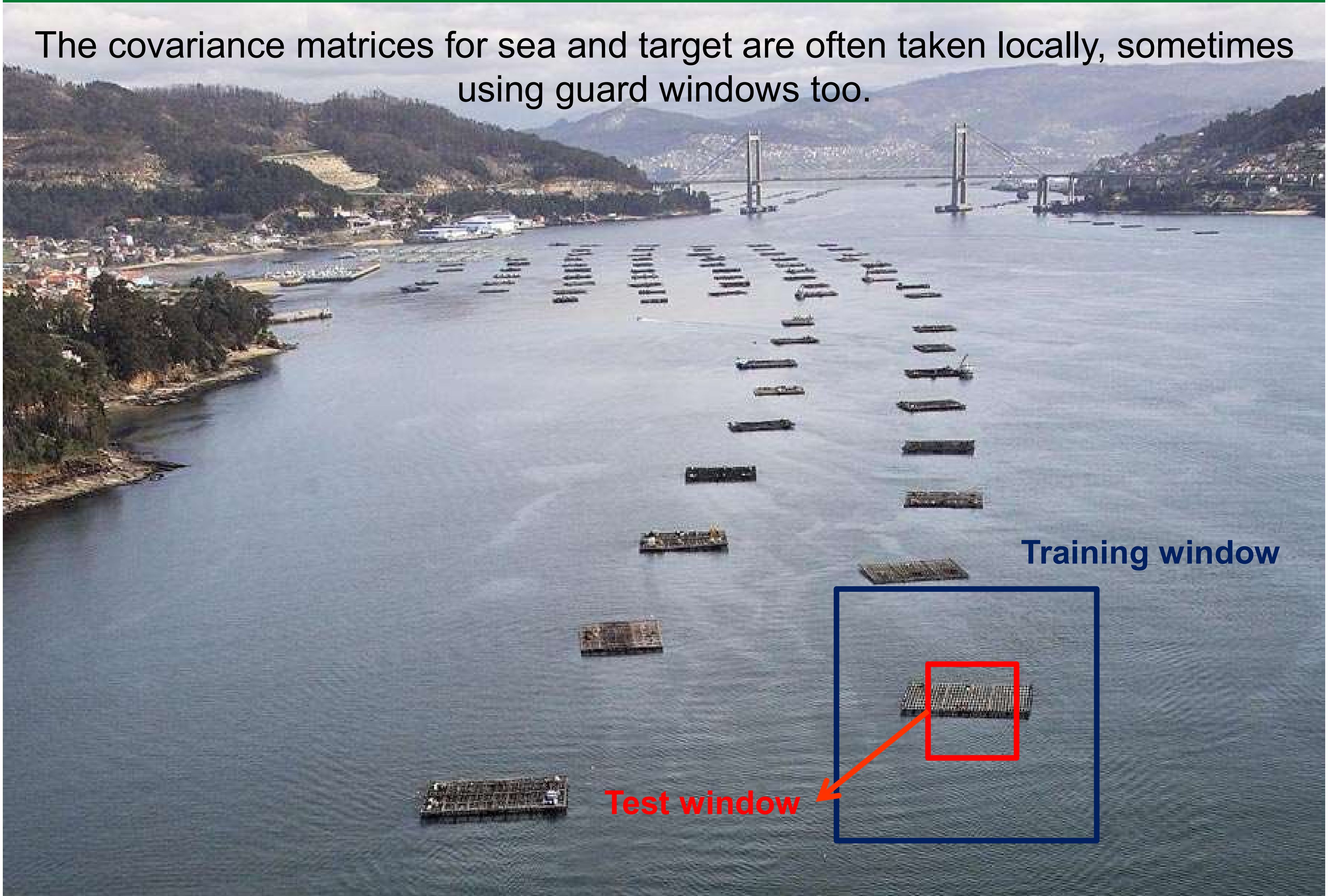
Circles: vessels observed in the video survey and in the RGB Pauli image.

Rectangles: vessels visible in the video survey but NOT in the RGB Pauli

Polarimetry detections: quad-pol

Local algorithms

The covariance matrices for sea and target are often taken locally, sometimes using guard windows too.



Entropy

It is the **Entropy** from the **Cloude-Pottier eigendecomposition** of the coherency matrix. One of the first physical quad-pol ship detectors. The sea has a small entropy while ships have high entropy (the collect different polarimetric signatures).

Many other modifications were proposed, with different measurements of "depolarisation" or degree of polarisation.

$$[T] = [U]^{*T}[\Sigma][U] = \sum_{i=1}^{3} \lambda_i \underline{u}_i \underline{u}_i^{*T} = \lambda_1 \underline{u}_1 \underline{u}_1^{*T} + \lambda_2 \underline{u}_2 \underline{u}_2^{*T} + \lambda_3 \underline{u}_3 \underline{u}_3^{*T}$$

$$P_i = \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3} \qquad H = \sum_{i=1}^{3} \left(-P_i \log_3 P_i\right)$$

Surface scattering

H_{sea} small

H_{ship} high

Volume and multiple scattering

Cloude, S. R., Pottier, E., 1996. "A review of target decomposition theorems in radar polarimetry", IEEE TGRS, 34(2), 498–518.
Touzi, R., "On the use of polarimetric SAR data for ship detection", IGARSS vol. 2, pp. 812–814, 1999.

Polarimetric Match Filter (PMF)

The detector finds the **scattering mechanism** that providers the **highest contrast** between see clutter and observed target. Then set a threshold on this contrast.

1) Power ratio between sea and target:

$\underline{\omega}$: projection vector

$$\rho_c = \frac{\underline{\omega}^{*T}[T_t]\underline{\omega}}{\underline{\omega}^{*T}[T_{sea}]\underline{\omega}} = \frac{P_t}{P_{sea}}$$

2) Optimisation with **Lagrange method**:

$$\underset{\underline{\omega}}{opt}\,\rho_c = \underset{\underline{\omega}}{opt}\left[\frac{\underline{\omega}^{*T}[T_{11}]\underline{\omega}}{\underline{\omega}^{*T}[T_{22}]\underline{\omega}}\right] \qquad L = \underline{\omega}^{*T}[T_{11}]\underline{\omega} - \lambda\left(\underline{\omega}^{*T}[T_{22}]\underline{\omega} - C\right) \qquad \frac{\partial L}{\partial \underline{\omega}^{*T}} = [T_{11}]\underline{\omega} - \lambda[T_{22}]\underline{\omega} = 0$$

$$[T_{sea}]^{-1}[T_t]\underline{\omega} = \lambda\underline{\omega}$$

$$\lambda_1 > T_{PMF}$$

Novak, L. M., Sechtin, M. B. and Cardullo, M. J., "Studies of target detection algorithms which use polarimetric radar data," IEEE Trans. Aerospace and Electric Systems, vol. 165, 1989.

Liu et al. detector

It assumes that ocean and target backscatter follow a **multi-variate Gaussian** distribution with zero mean. The **Neyman-Pearson** likelihood ratio test**.**

Decision role:

$$\Lambda = \underline{k}_L^{*T} \left[[C]_{sea}^{*T} - [C]_s^{-1} \right] \underline{k}_L$$

Lexicographic scattering vector

$$\underline{k}_L = [HH, HV, VH, VV]^T$$

If the statistics of the expected targets are unknown (as it usually is)

$$\Lambda = \underline{k}_L^{*T} [C]_{sea}^{-1} \underline{k}_L > T_{liu}$$

Based on the assumption of Gaussian statistics - **not optimal.**

Physical meaning (asymptotically): it evaluate a weighted product of the observed target. The weights are higher where the contributions of the sea are smaller… therefore they intensify targets that are different from the sea.
If the sea is completely depolarised, the test becomes a simple test on the SPAN.

Liu, C., Vachon, P.W. and Geling, G.W., “Improved ship detection with airborne polarimetric SAR data”, Canadian Journal of Remote Sensing, Vol 31, No 1, pp 122–131, 2005.

Polarimetric Notch Filter (PNF)

The algorithm is based on the *Geometrical Perturbation - Partial Target Detector,* however here, it is reversed and focused on the complementary space.

The **sea** is the **clutter** and we go looking at the **complementary** space (the rest**)** where we expect our **target of interest**

$$P_{Sea} = \left| \underline{t}^{*T} \cdot \hat{\underline{t}}_{Sea} \right|^2$$

$$\gamma_n = \frac{1}{\sqrt{1 + \frac{RedR}{P_{tot} - P_{Sea}}}} > T_n$$

$$P_{tot} = \left| \underline{t}^{*T} \cdot \underline{t} \right|^2$$

Partial scattering vector:

$$\underline{t} = \left[\underline{\omega}_1^{*T}[C]\underline{\omega}_1, \underline{\omega}_2^{*T}[C]\underline{\omega}_2, \underline{\omega}_3^{*T}[C]\underline{\omega}_3, \underline{\omega}_1^{*T}[C]\underline{\omega}_2, \underline{\omega}_1^{*T}[C]\underline{\omega}_3, \underline{\omega}_2^{*T}[C]\underline{\omega}_3 \right]^T$$

$\hat{\underline{t}}_{sea} = \frac{\underline{t}_{sea}}{\left\| \underline{t}_{sea} \right\|}$: target to reject (Null)

Marino, A., Cloude, S. R. and Woodhouse, I. H., “Detecting depolarized targets using a new geometrical perturbation filter,” IEEE TGRS, Vol. 50(10), pp 3787-3799, 2012.

Marino, A., "A Notch Filter for Ship Detection With Polarimetric SAR Data," IEEE JSTARS, early access, pp.1-14

Polarimetry detections: dual- and single-pol

Symmetry detector

The sea is expected to have **reflection symmetry along the vertical axis** and therefore its Lexicographic Covariance matrix can be written as:

$$[C_{sea}] = \begin{bmatrix} \langle |S_{HH}|^2 \rangle & 0 & \langle S_{HH} S_{VV}^* \rangle \\ 0 & \langle |S_{HV}|^2 \rangle & 0 \\ \langle S_{VV} S_{HH}^* \rangle & 0 & \langle |S_{VV}|^2 \rangle \end{bmatrix}$$

$$\langle S_{HH} S_{HV}^* \rangle \geq T$$

A ship is NOT supposed to have such property, we do not expect a vertical axis of symmetry. Therefore the inner product of co-pol and cross-pol is not supposed to be zero.

Nunziata, F., Migliaccio, M. and Brown, C.E., “Reflection symmetry for polarimetric observation of man-made metallic targets at sea,” IEEE Journal of Oceanic Engineering, vol. 37(3), pp. 384–394, 2012.

HV intensity

The sea is expected to have a **very low backscattering in the cross-polarisation** channels HV or VH (assuming Bragg model, theoretically zero). Ships on the other hand should NOT present this property. Also orientations adds up to the HV return backscattering.

A threshold on the intensity of HV returns a detector.

$$\left\langle \left| S_{HV} \right|^2 \right\rangle \geq T_{HV}$$

The **pdf** of the intensity of a single channel is known, therefore a **statistical test** can be devised. The pdf exploited here is the **K-distribution**.

Crisp, D. J., "The state-of-the-art in ship detection in synthetic aperture radar imagery," Australian Government Department of Defence, 2004.

Setting thresholds

Definition of the Problem

- ✓ In order to set a **statistical test** on your observable γ, we need to derive its probability density function, **pdf**.

- ✓ Initially we define the two detection **hypotheses**:
  - ✓ H_0: only sea clutter
  - ✓ H_1: target

Statistical Tests

1. CFAR on pdf: The threshold is set with a Constant False Alarm Rate test

$$\int_T^\infty f_\Gamma(\gamma|H_0)\, d\gamma = P_f$$

The threshold T is set in order to have a defined P_f.

2. Likelihood Ratio: $$LR = \frac{f_\Gamma(\gamma \mid H_1)}{f_\Gamma(\gamma \mid H_0)}$$

✓ The statistics of the sea clutter can be extracted on a ring window around a **guard area**.

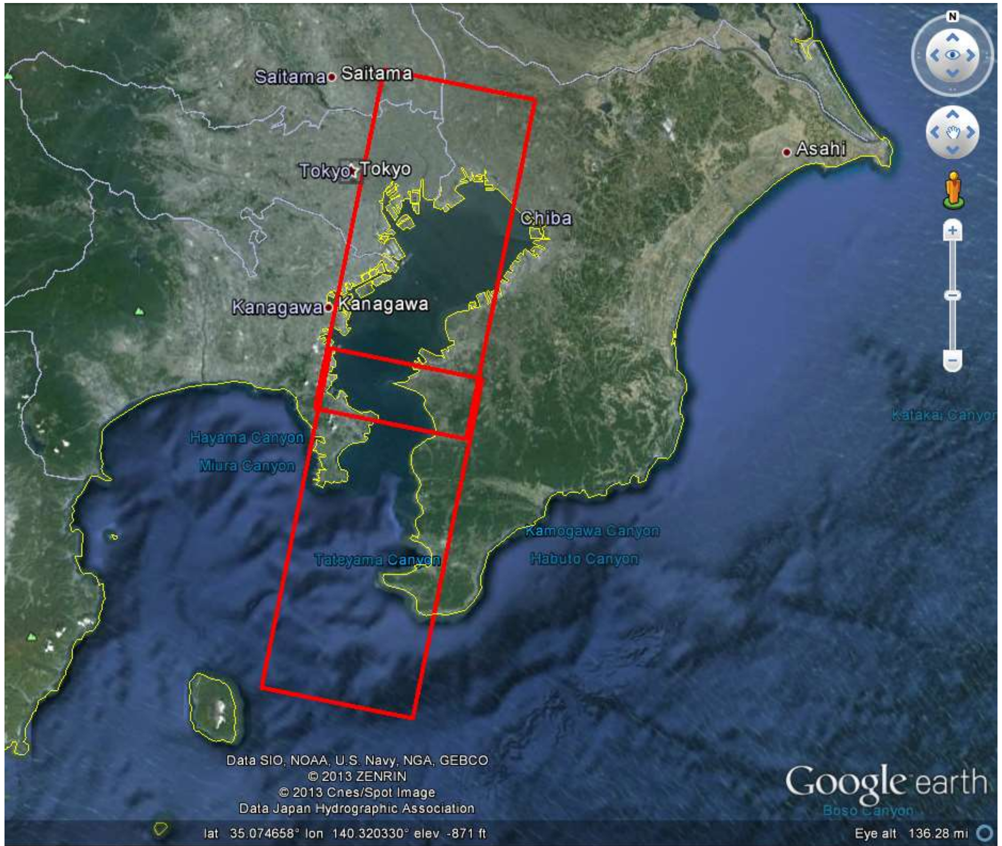
✓ The test area is a window inside the guard area.

Armando.marino@stir.ac.uk

Examples

Dataset: ALOS-PALSAR QUAD-POL

ALOS-PALSAR QUAD-POL data over Tokyo Bay (9th of October 2008). *Data courtesy of JAXA (Japanese Aerospace and Exploration Agency)*

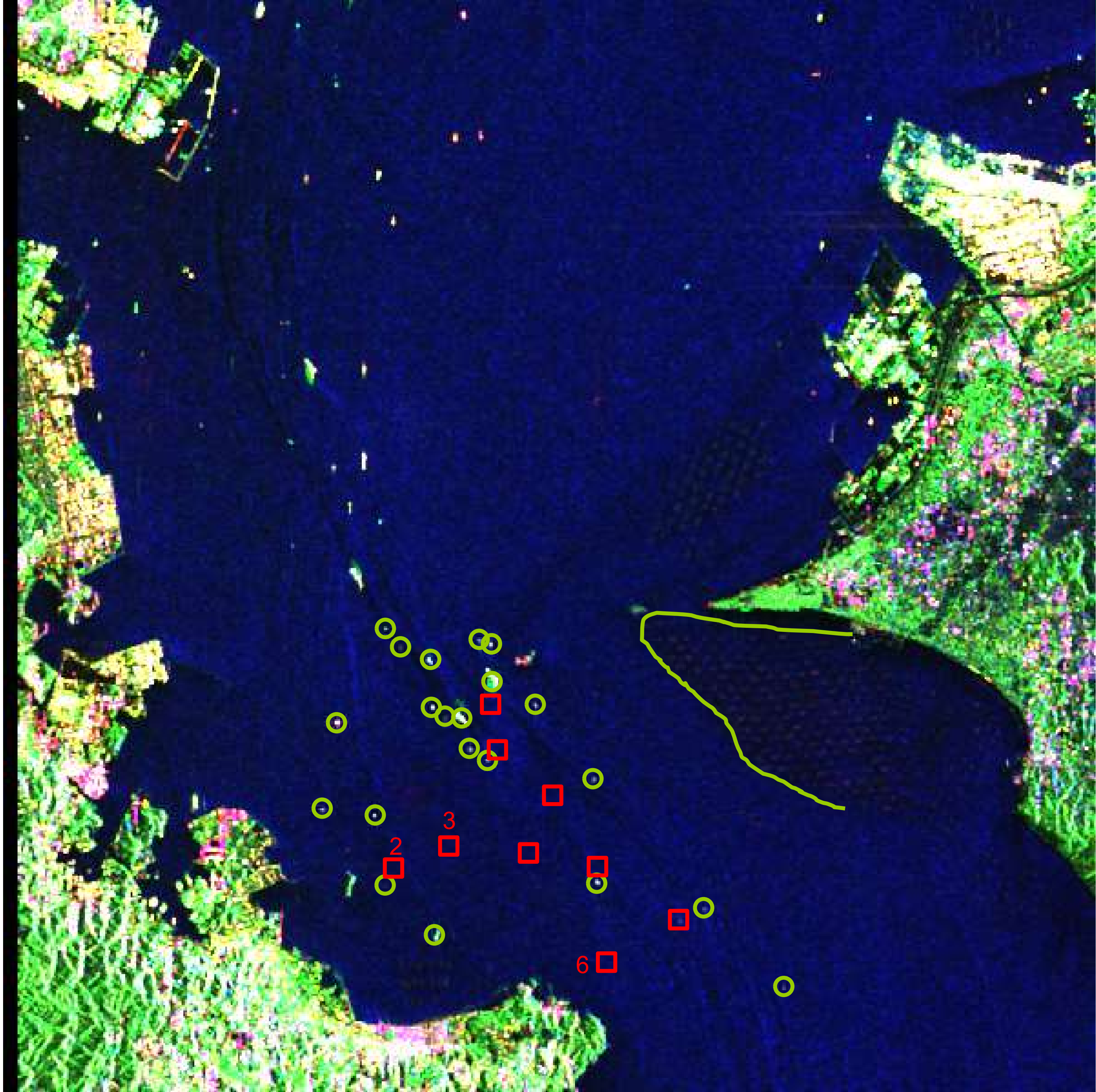


A ground survey was carried out during the acquisition in front of NDA (100m.a.s.l):

- Video survey with video camera
- Ground-based X-band radar
- AIS positioning

- There were also a sea weed farm on the coastline

Armando.marino@stir.ac.uk



RGB PAULI

Red: HH-VV
Green: 2*HV
Blue: HH+VV
Sigma nought
1000x1000 pixels
Multi-look 1x5
Image size ~ 30x30km.

Circles: vessels observed in the video survey and in the RGB Pauli image.

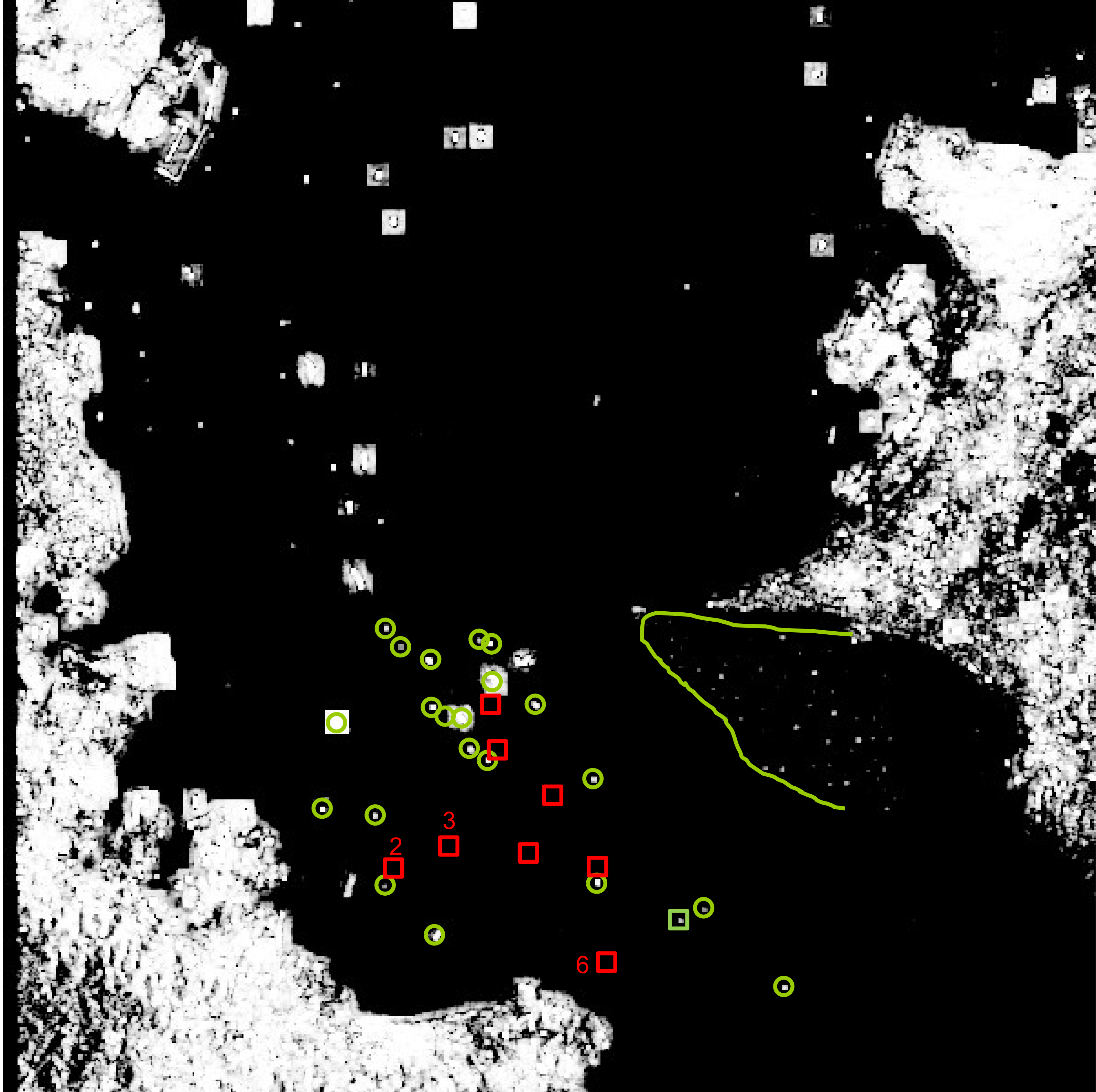
Rectangles: vessels visible in the video survey but NOT in the RGB Pauli

What is the target inside the green line?

UNIVERSITY of STIRLING

Armando.marino@stir.ac.uk

BE THE DIFFERENCE



GP-PNF

Detected: 22

Missed: 8 (15)

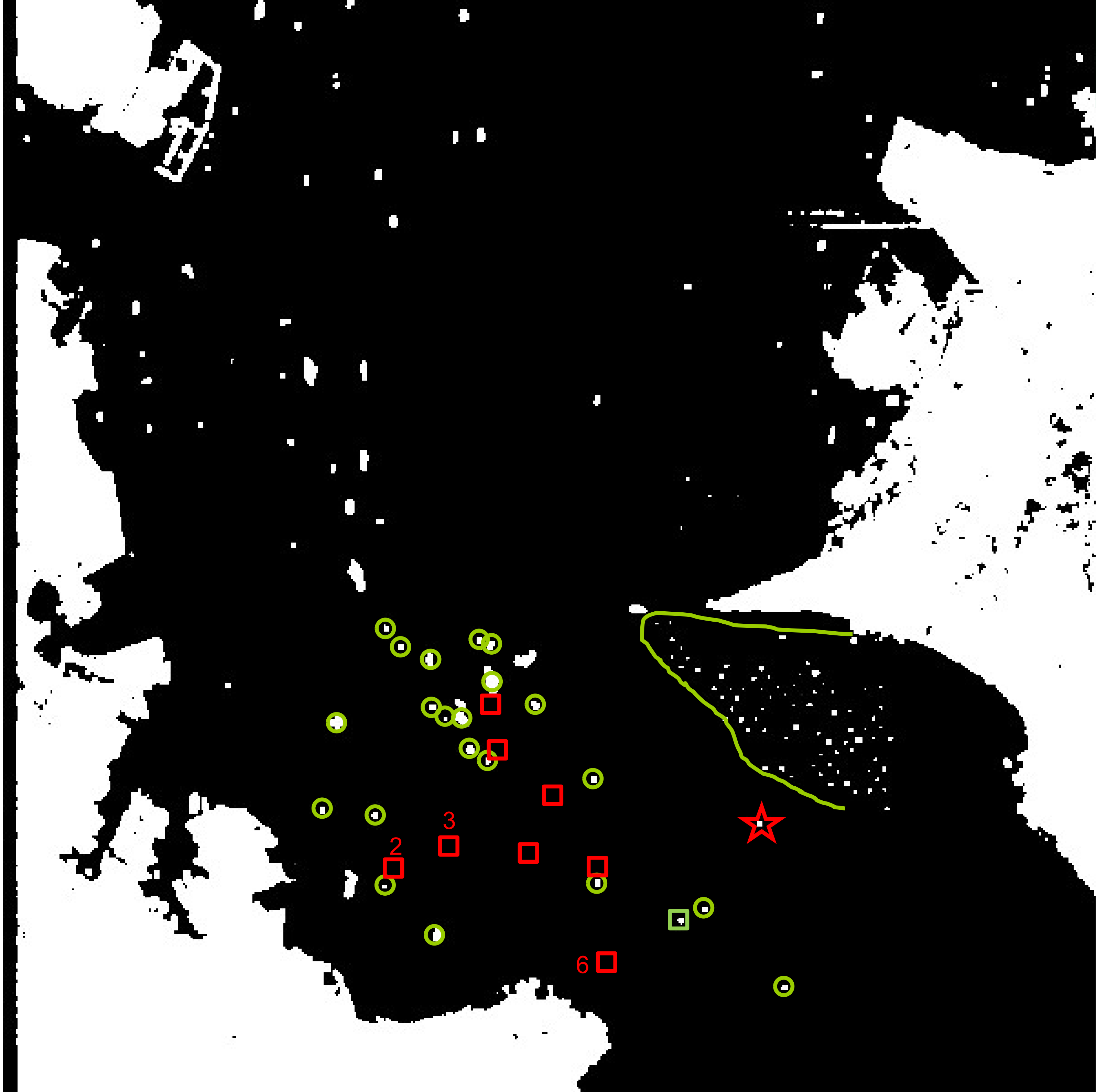
False: 0

$$\text{RedR} = 2 \cdot 10^{-4}$$
$$\text{T} = 0.9$$

Average for test: 5x25, corresponding to about 50 ENL

Circles: vessels observed in the survey and in the RGB Pauli image.

Rectangles: vessels visible in the survey but **NOT** in the RGB Pauli



PMF

Detected: 22

Missed: 8 (15)

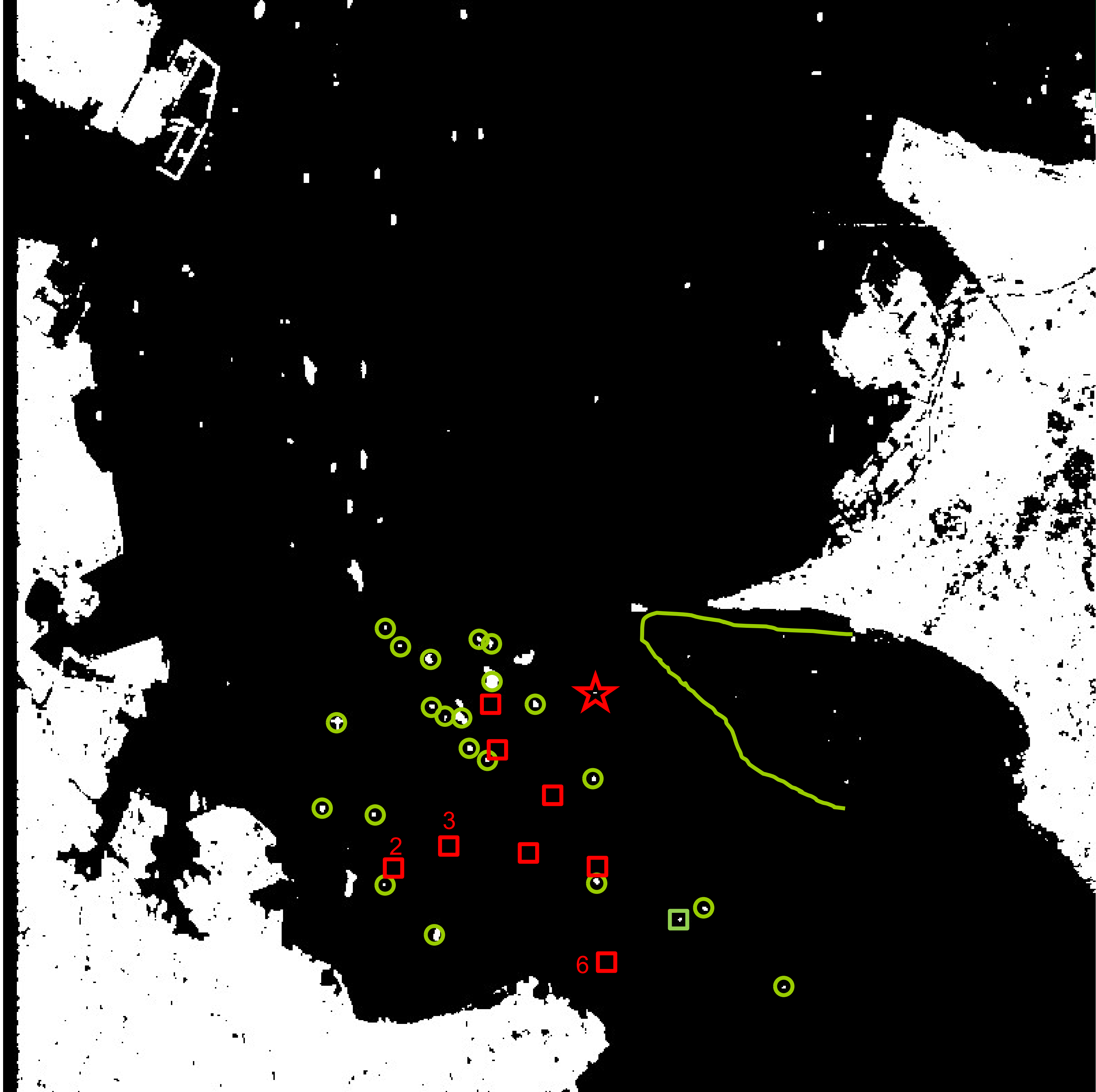
False: 1?

T = 4

Average for test: 5x25, corresponding to about 50 ENL

Circles: vessels observed in the survey and in the RGB Pauli image.

Rectangles: vessels visible in the survey but **NOT** in the RGB Pauli



LIU

Detected: 22

Missed: 8 (15)

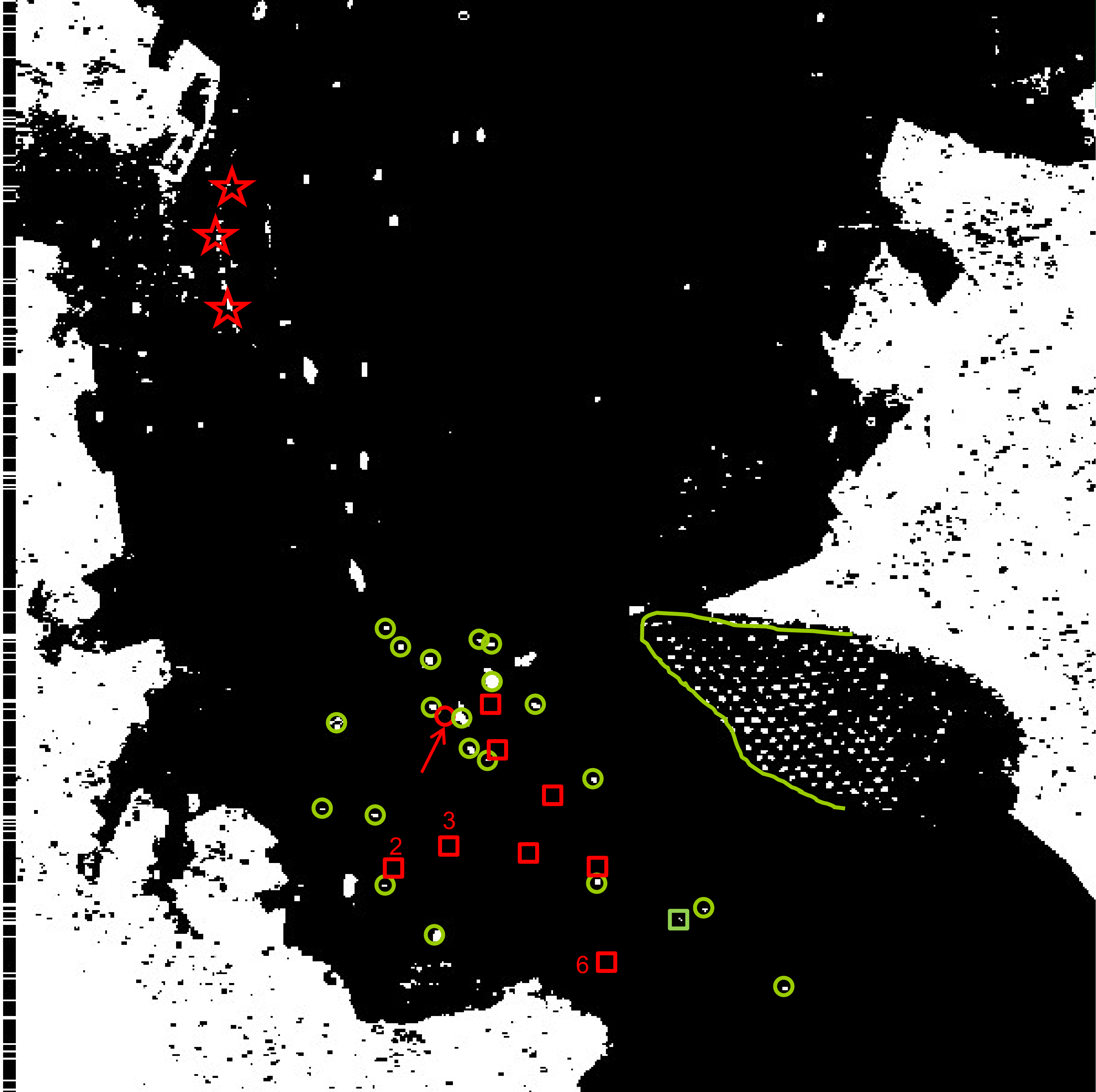
False: 1

$$P_f = 10^{-4}$$

Average for test: 5x25, corresponding to about 50 ENL

Circles: vessels observed in the survey and in the RGB Pauli image.

Rectangles: vessels visible in the survey but **NOT** in the RGB Pauli



ENTROPY

Detected: 21

Missed: 8 (15)

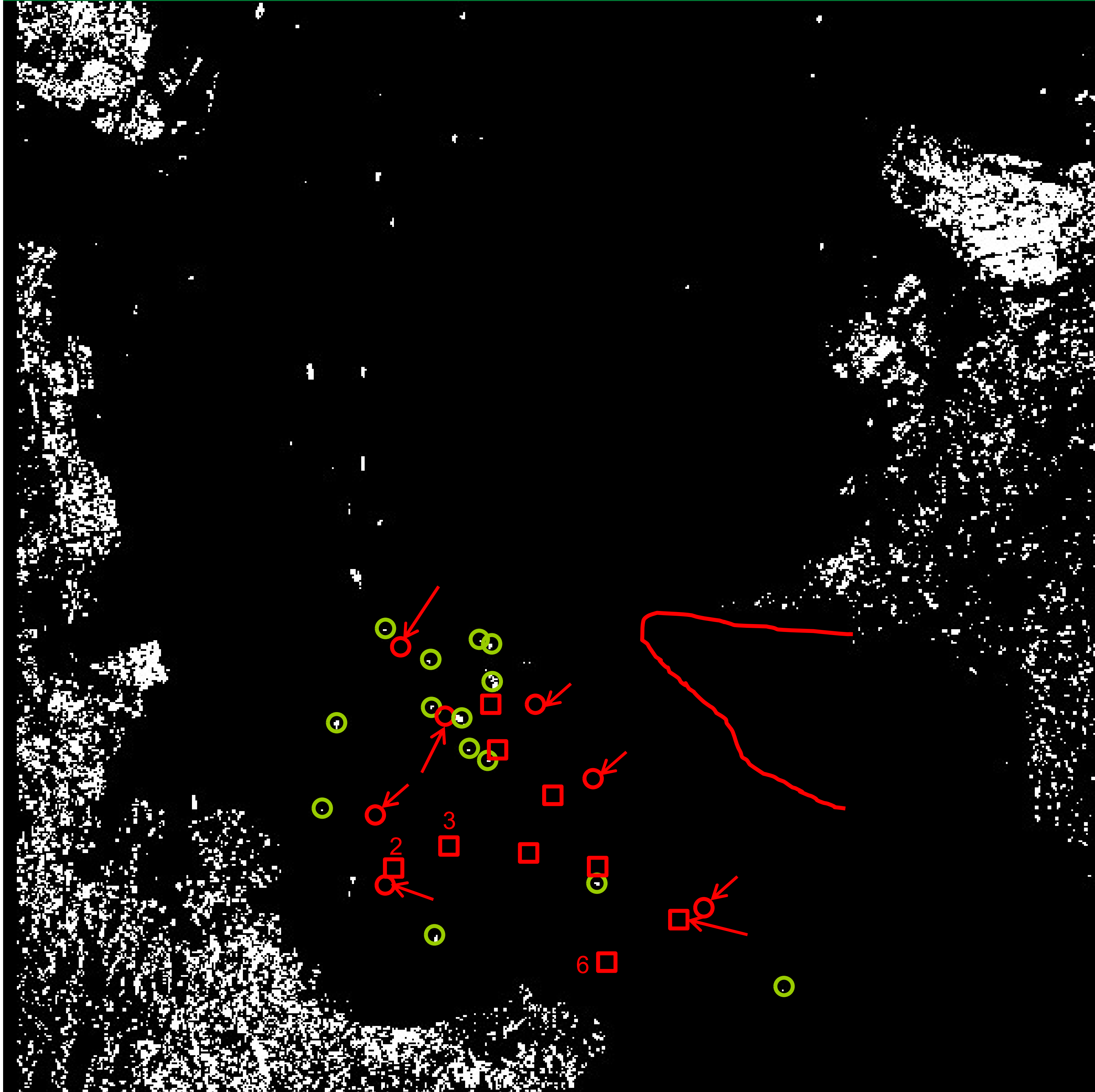
False: several

$P_f = 10^{-4}$

Average for test: 5x25, corresponding to about 50 ENL

Circles: vessels observed in the survey and in the RGB Pauli image.

Rectangles: vessels visible in the survey but **NOT** in the RGB Pauli



SYM.

Detected: 14

Missed: 14 (21)

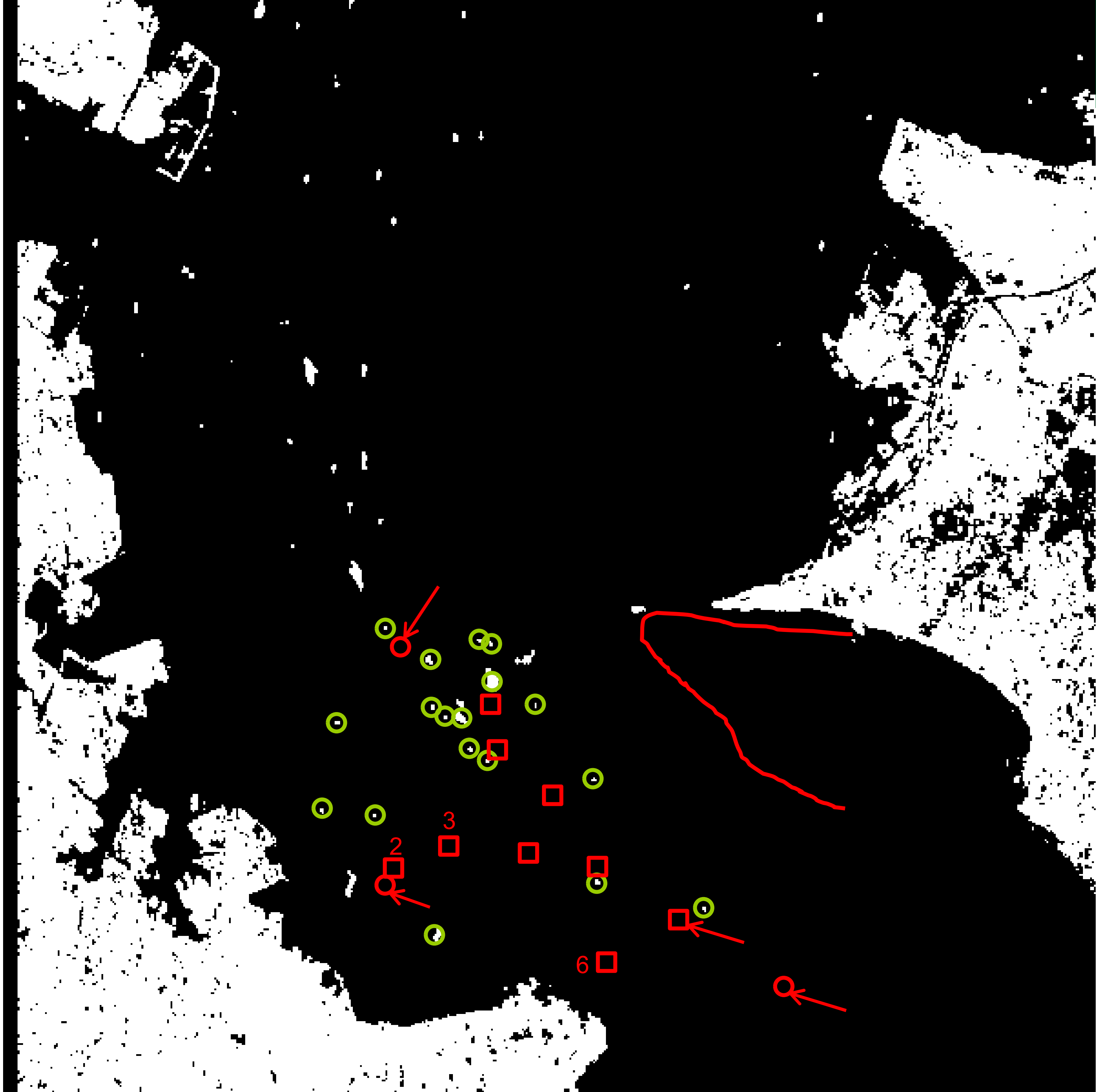
False: 0

$T = 0.1$

Average for test: 3x3, corresponding to about 3 ENL

Circles: vessels observed in the survey and in the RGB Pauli image.

Rectangles: vessels visible in the survey but **NOT** in the RGB Pauli



HV

K-DIST.

Detected: 18

Missed: 10 (17)

False: 0

Average for test:
5x25,
corresponding to
about 50 ENL

Circles: vessels observed in the survey and in the RGB Pauli image.

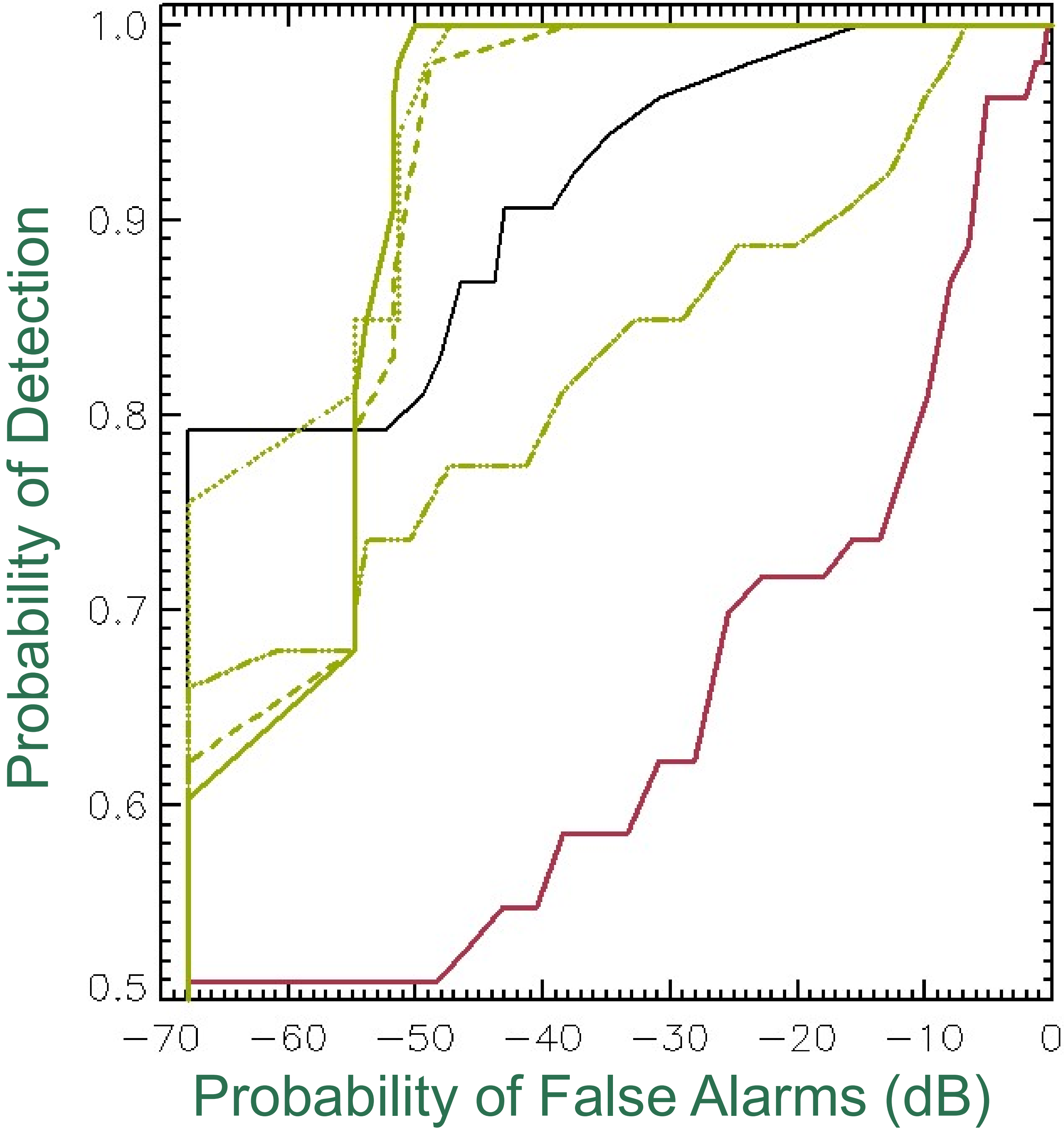
Rectangles: vessels visible in the survey but **NOT** in the RGB Pauli

Summary table

Number/ Algorithms	Quad-Pol: GB-PNF	PMF	Liu	Entropy	Single- and Dual-Pol: Symmetry	HV-k
Detected	22	22	22	21	14	18
Missed	15(8)	15(8)	15(8)	16(9)	21(14)	17(10)
False Alarms	0	1?	1	Several	0	0

The Green indicates the algorithms with best performance.

ROC curves



Green: Quad-pol
Solid: GP-PNF quad
Dot: Entropy
Das: PMF
Dot-Dash: Liu

Red: Dual-pol
Solid: Symmetry

Black: Single-pol
Solid: HV

Only visible targets in the RGB are considered (we know where they are)


Land / Urban / Hydrology
Change detection


Change detection:
Example flooding

Floodings

- ✓ Every year floods claim around **20,000 lives** and adversely affect at least **20 million people worldwide**, mostly through homelessness (Smith 2009)

- ✓ A case study in the UK, in 2014 (The Guardian, 2014)
    - ✓ 1,100 homes have been flooded.
    - ✓ 1/6 property in England are at risk.
    - ✓ 2.3 billion £ on flood fences (Cameron 2013)

- ✓ Also, they have also been associated to *Global Warming*.

Wraysbury, January 2014

Flood fences

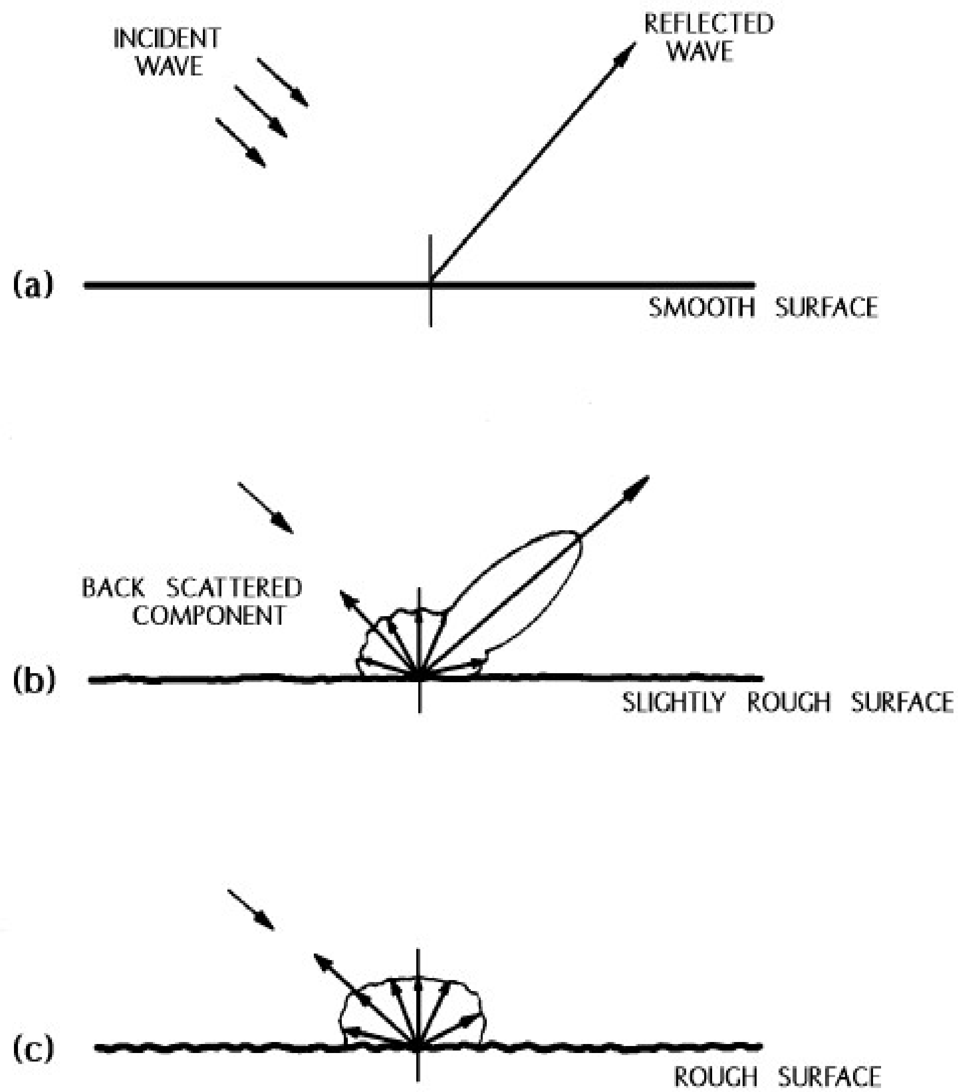
Why remote sensing?

- ✓ Mapping floods with **fieldwork** has some issue:
  - ✓ It may be **dangerous**
  - ✓ It is hard to provide **synoptic information** if the flood is large
  - ✓ It is sometime **hard to survey floods under some situations**: e.g. water under vegetation
  - ✓ What is often done is to measure the flood based on its effects, which is not always possible.

- ✓ Remote sensing can be used for **several purposes** (besides monitoring the flood itself) including gaining a better understanding of the flood basin for hydraulic models.
  - ✓ What we treat in this lecture is only the pure observation of the flood, not improving models and predictions.

How SAR sees floods? (as seen before)

- ✓ To study the interaction of microwaves with water, we need a **scattering model**.
- ✓ Water is **impenetrable** by microwaves, therefore it is seen as a pure **surface**.
  - ✓ If the surface is **smooth**, the radiation will be reflected in the specular direction
  - ✓ If there is some surface **roughness** we can expect some return.
- ✓ In SAR images, we expect floods to be dark.



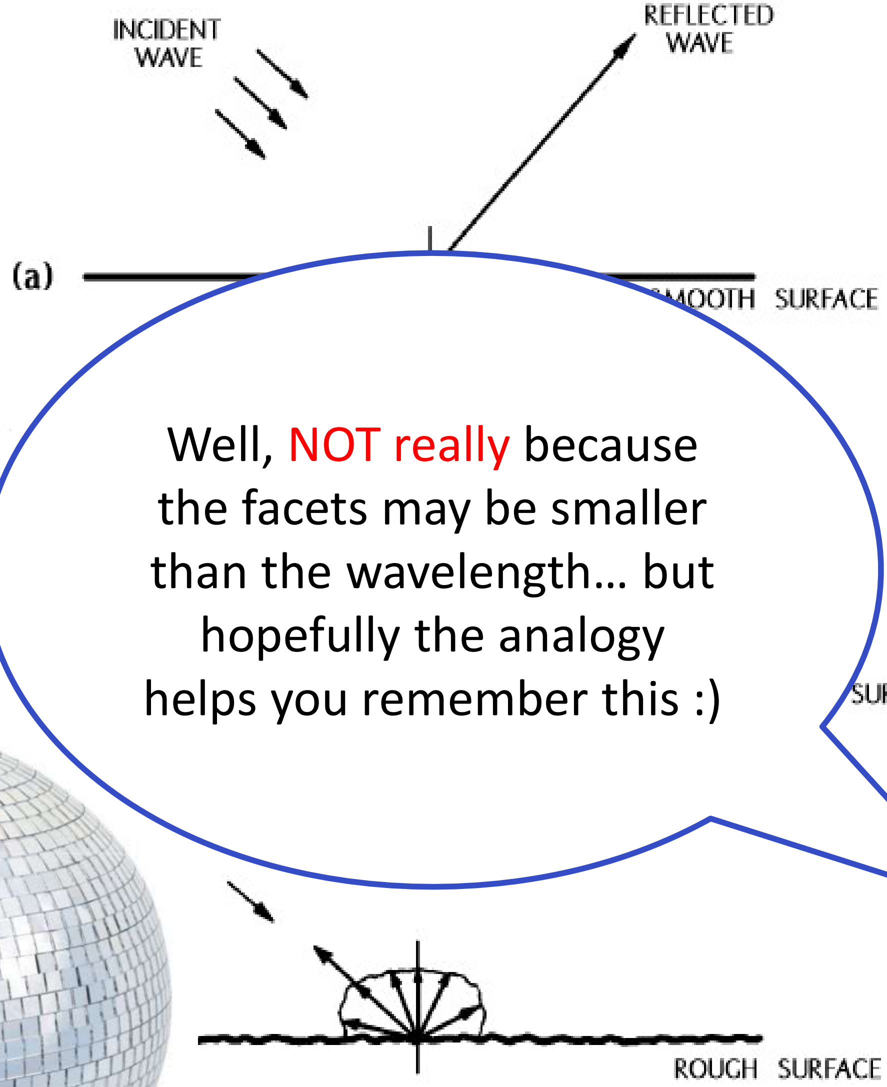
UNIVERSITY of STIRLING

BE THE DIFFERENCE

Armando.marino@stir.ac.uk

How SAR sees floods? (as seen before)

- ✓ To study the interaction of microwaves with water, we need a **scattering model**.
- ✓ Water is **impenetrable** by microwaves, therefore it is seen as a pure **surface**.
  - ✓ If the surface is **smooth**, the radiation will be reflected in the specular direction
  - ✓ If there is some s
    roughness we ca
    retu
- ✓ In
  be



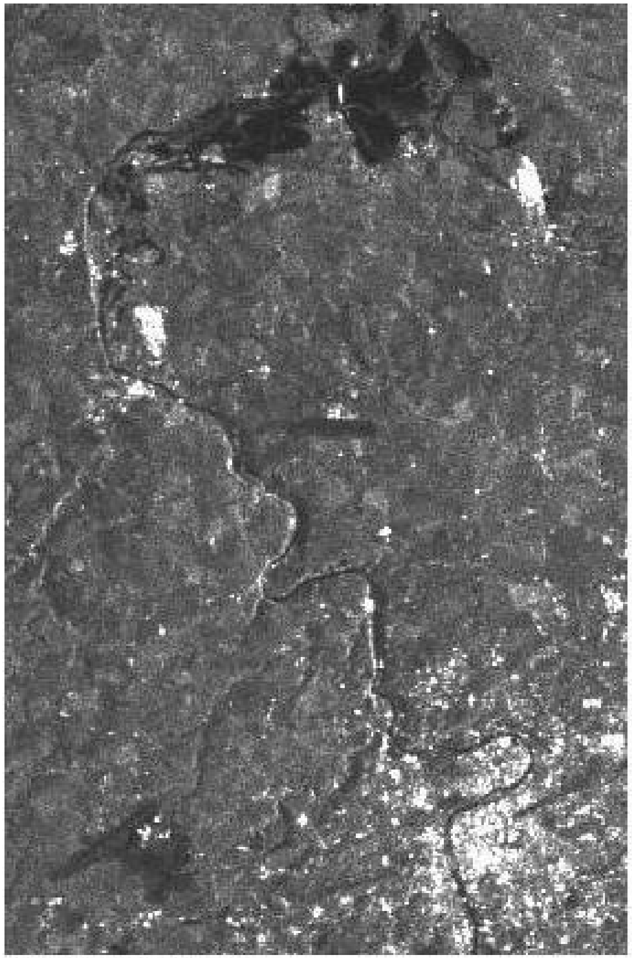
Well, NOT really because the facets may be smaller than the wavelength… but hopefully the analogy helps you remember this :)

That is like a discoball?

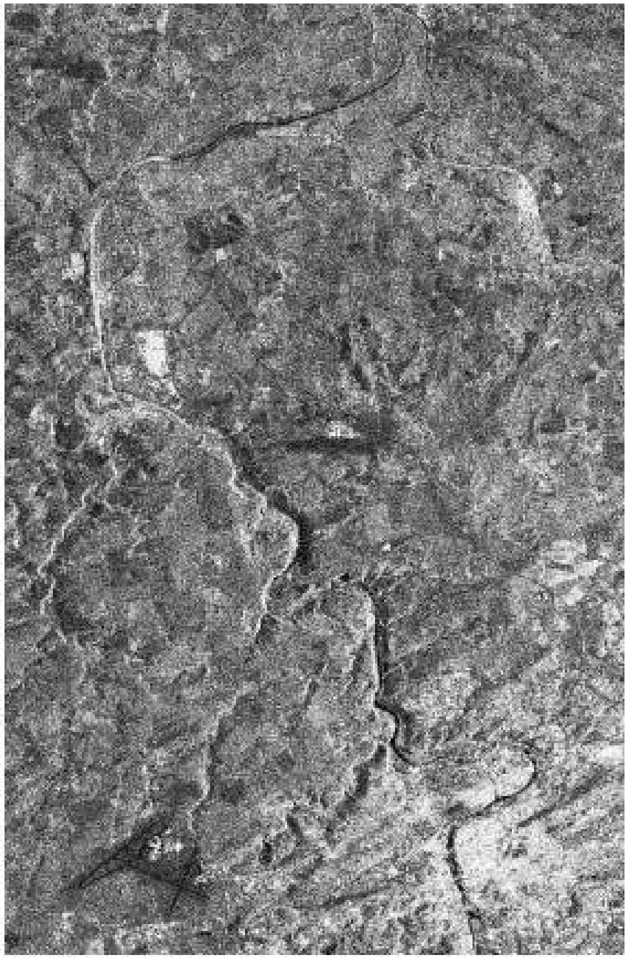
How SAR sees floods?

Flooding in Central Europe - August 2002, Moldava Flooding

ERS-2's SAR instrument documented the spread of floodwaters from the Moldava river, a confluent of the Elbe river, after heavy rains during August flooded the cities along its banks.



16 August 2002



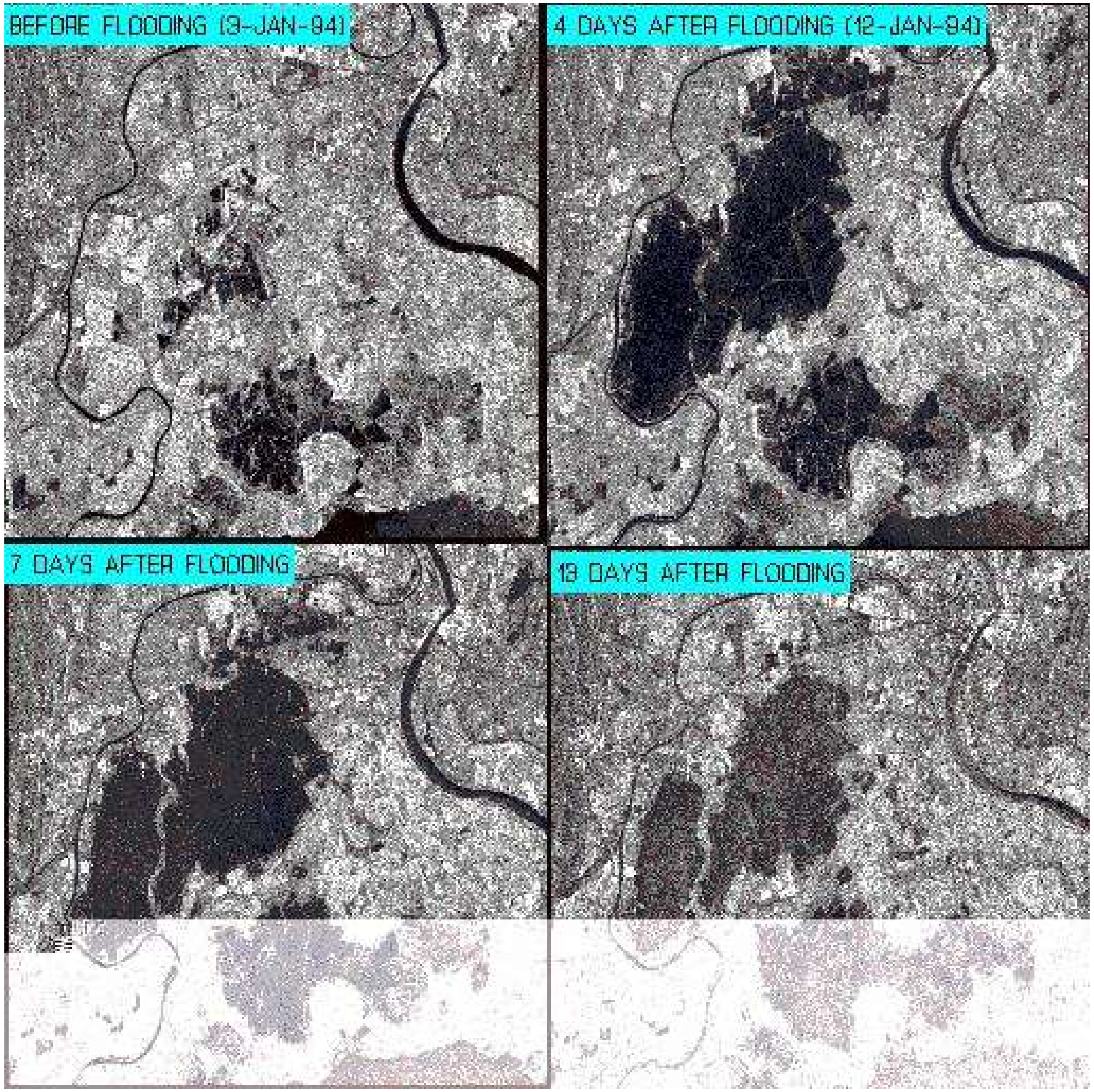
2 January 2002

How SAR sees floods?

Flood monitoring in the Camargue, France

Subsection of an ERS-1 SAR scene at 4 different dates.

The following datas are:
3 January 1994
12 January 1994
18 January 1994
21 January 1994



Are floodings in SAR always dark? If not when they are not dark?

https://PollEv.com/free_text_polls/toGHnbFPsEsS2DdITW4jX/respond

UNIVERSITY of STIRLING

Armando.marino@stir.ac.uk

BE THE DIFFERENCE

How can we detect floods?

Single SAR acquisition and single polarisation channel
We can use the fact that floods appear dark in intensity images

Two SAR acquisitions and single polarisation
We can use the fact that floods change the intensity of the image

Two SAR acquisitions and multiple polarisations
We can use the fact that floods change the polarimetric signature of the target

Armando.marino@stir.ac.uk


Change detection:
Methodologies

Change detection

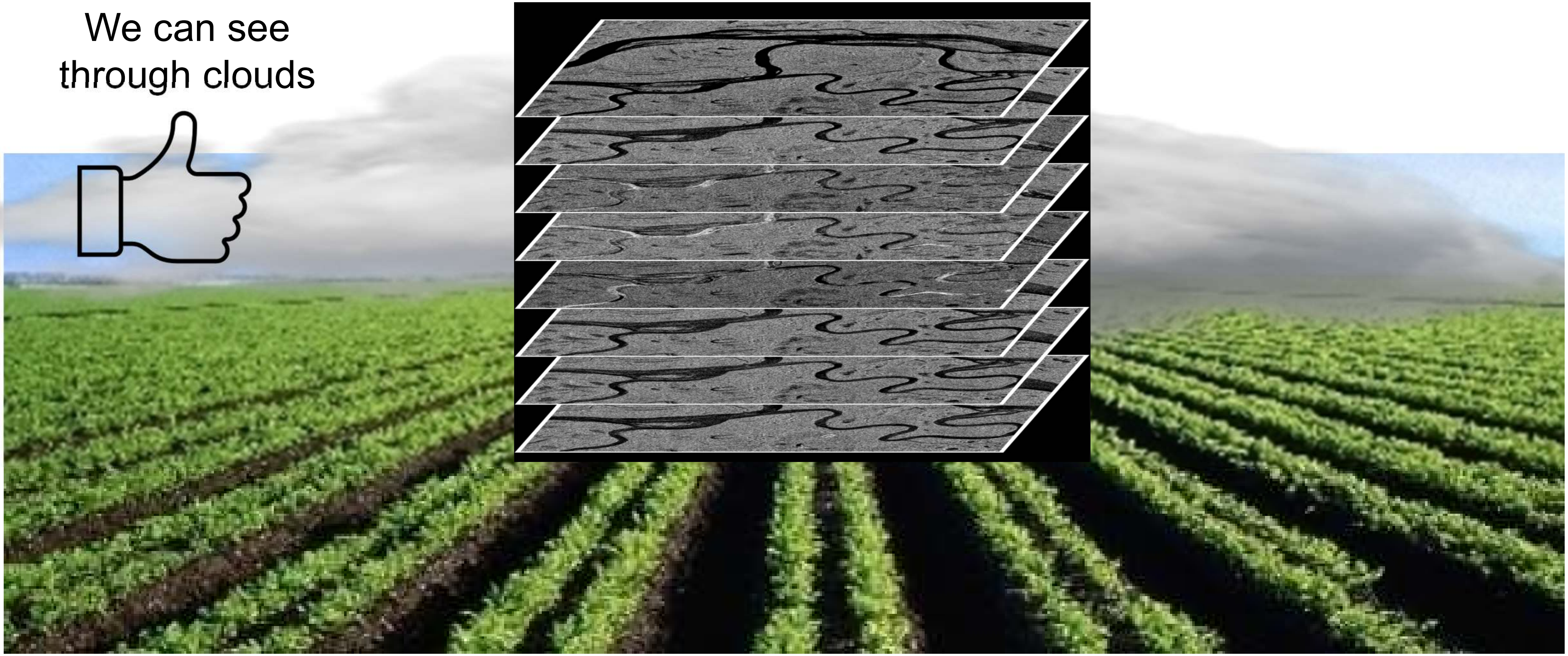
- ✓ Besides detecting dark areas, the idea is to take an image when the flood was not there and one with the flood and see the differences (as in the game **Spot the Difference**).

- ✓ This is more properly called **Change Detection**.

Change detection with SAR

The satellite pass over a scene periodically (e.g. Sentinel-1 pass **every 6 or 12 days. I**t produces an image every time.

We can see through clouds



Armando.marino@stir.ac.uk

Two images

If img_1 is one image acquired before the flood (archive image) and img_2 is acquired after, we can use a "change detector".

Change detector: an algorithm that detects "changes" between two images acquired at different moments in time.

Two very easy detectors can be devised considering the difference or the ratio of the intensities

$$\Delta I = \left| \left\langle |img_1|^2 \right\rangle - \left\langle |img_2|^2 \right\rangle \right| > T_1 \qquad \rho_I = \frac{\left\langle |img_1|^2 \right\rangle}{\left\langle |img_2|^2 \right\rangle} > T_2$$

The difference can also be normalised as

$$\Delta I_n = \frac{\left| \left\langle |img_1|^2 \right\rangle - \left\langle |img_2|^2 \right\rangle \right|}{\left\langle |img_1|^2 \right\rangle + \left\langle |img_2|^2 \right\rangle} > T_3$$

Two images: the role of normalisation

- ✓ It is interesting to understand the role played by the normalisation (i.e. difference vs normalised difference).

- ✓ If we DO NOT normalise, **differences over bright areas appear stronger**.
  - ✓ *The 1% difference over intensities around 1000 is 10; the 1% difference over intensities around 10 is 0.1.* The same difference in percentage produces very dissimilar outputs of the "difference change detector"

- ✓ Adv. of normalisation: It treats differences on bright and darker areas more equally.
- ✓ Dis. of normalisation: The noise is enhanced, especially on dark targets
  - ✓ If the SNR is low, additive noise can produce large changes to the pixel value: e.g. with SNR=1, noise can easily modify the pixel of 100%

Summary: normalised indexes on the intensities are great, but we need to take care when applied to noisy images.

Armando.marino@stir.ac.uk

Two images: coregistration

- ✓ One issue in change detection, is that the two images have to **overlap** perfectly.
  - ✓ Each pixel of each image has to be located at the same geographical point. If this is not true, we may detect changes just because we are looking at different areas.

- ✓ The process of making two images overlapping is often called **Co-registration.**

Two multidimensional acquisitions

SAR

✓ Foods change the **polarimetric behaviour** of the observed targets.

✓ Several **detectors** were proposed:

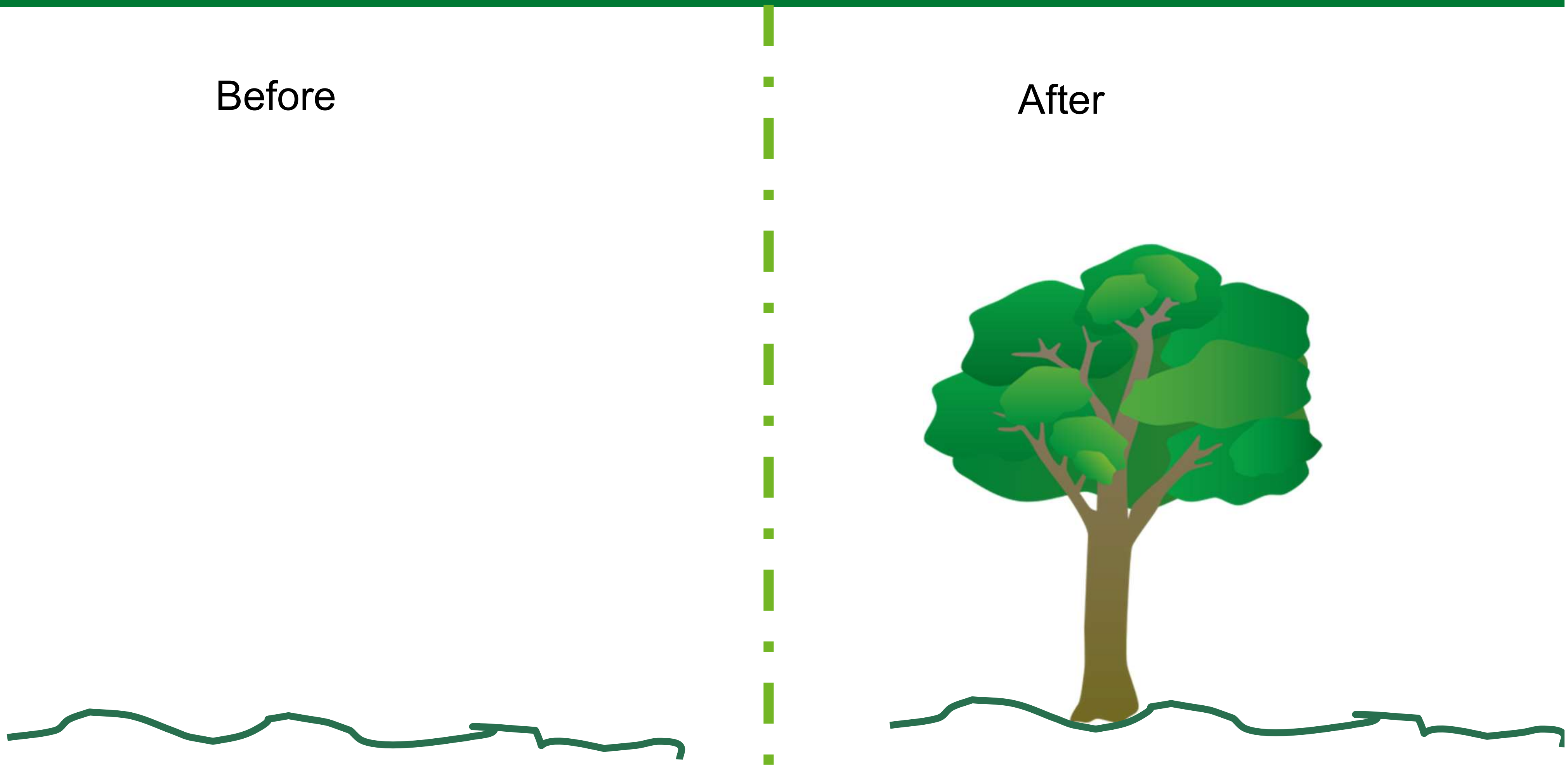
1. Physically based
2. Statistically based

✓ Adv.: Better discrimination; Dis.: More data to acquire and process.

Armando.marino@stir.ac.uk

Change detection: Physically based

Signal models: 1) additive model



Additive model: when a change is produced by adding or subtraction a target. Change detectors are generally obtained considering differences.

Additive model

Lagrange Method

$$L = \underline{\omega}^{*T}([T_2] - [T_1])\underline{\omega} - \lambda(\underline{\omega}^{*T}\underline{\omega} - C)$$

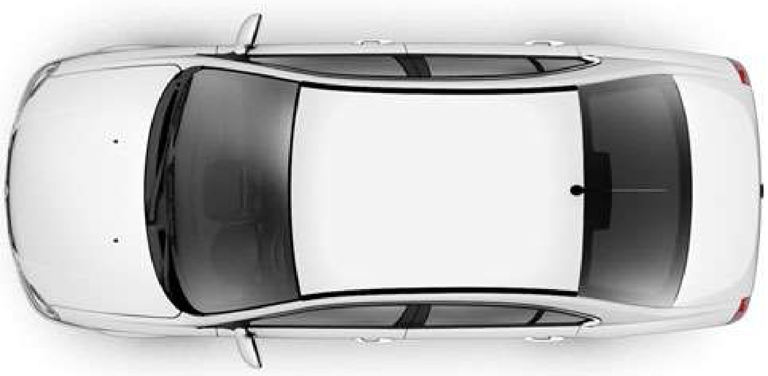
$$\frac{\partial L}{\partial \underline{\omega}^{*T}} = ([T_2] - [T_1])\underline{\omega} = 0$$

$$([T_2] - [T_1])\underline{\omega} = \lambda\underline{\omega}$$

- ✓ We can perform an **eigenproblem of the difference matrix**
- ✓ Eigenvalues will tell the maximum/minimum amount of change for the scattering mechanisms
- ✓ The eigenvector will tell which projection vector and in some instances scattering mechanism is suffering maximally/minimally

Signal models: 2) multiplicative model

Before



After

Multiplicative model: when a change is produced by transforming the target. If we still assume linearity this transformation is done by multiplying by a matrix.

For which application would you use a multiplicative model

https://PollEv.com/free_text_polls/TUDXRDjOZw9bXutcK7WOd/respond

2) Multiplicative model

We already saw a detector based on the power ratio (the Generalised Rayleigh Quotient):

$$\rho_c = \frac{\underline{\omega}^{*T}[T_1]\underline{\omega}}{\underline{\omega}^{*T}[T_2]\underline{\omega}} = \frac{P_1}{P_2}$$

We can optimize it using a **Lagrange** constrained optimization:

$$\frac{\partial L}{\partial \underline{\omega}^{*T}} = [T_1]\underline{\omega} - \lambda[T_2]$$

$$L = \underline{\omega}^{*T}[T_1]\underline{\omega} - \lambda(\underline{\omega}^{*T}[T_2]\underline{\omega} - C)$$

$$[T_2]^{-1}[T_1]\underline{\omega} = \lambda\underline{\omega}$$

Marino, A. and Hajnsek, I. "A Change Detector Based on an Optimization With Polarimetric SAR Imagery," IEEE Transactions on Geoscience and Remote Sensing, 8(52), 4781-4798, 2014
Alonso-Gonzalez, A. and Jagdhuber, T. and Hajnsek, I. "Exploitation of agricultural Polarimetric SAR time series with Binary Partition Trees." POLinSAR 2015.

Signal model comparison

- ✓ Additive model:
  - ✓ a canopy cover that grows over ground
  - ✓ a car that moves away
- ✓ Multiplicative model:
  - ✓ a car that rotates (if resolution bigger than the car)
  - ✓ Stems that tilt


Change detection:
Results

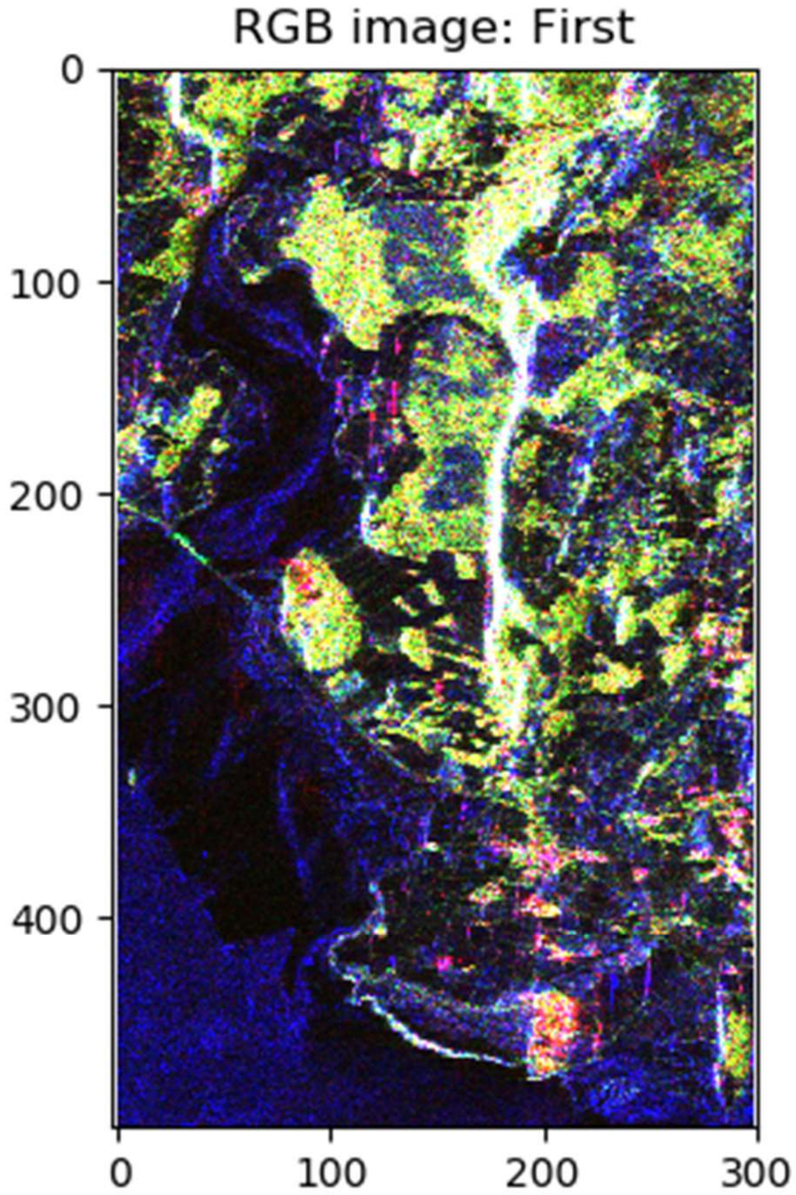
ALOS data

- ✓ The data were acquired by ALOS (JAXA) and are L-band quad-polarimetric.
- ✓ We use here the **L-band quad polarimetric**
- ✓ The data were provided by a call of opportunity with project number 1151.

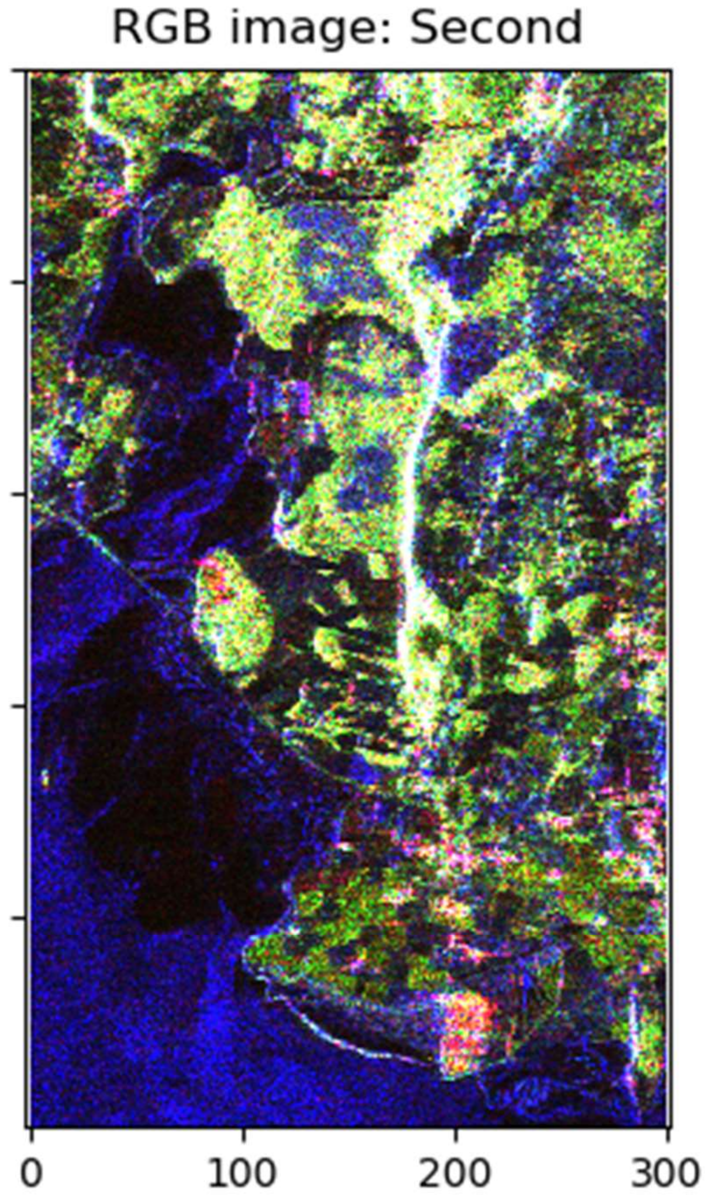
Morecambe Bay (England)

Marecombe Bay: RGB Pauli

1 April 2007



17 May 2007



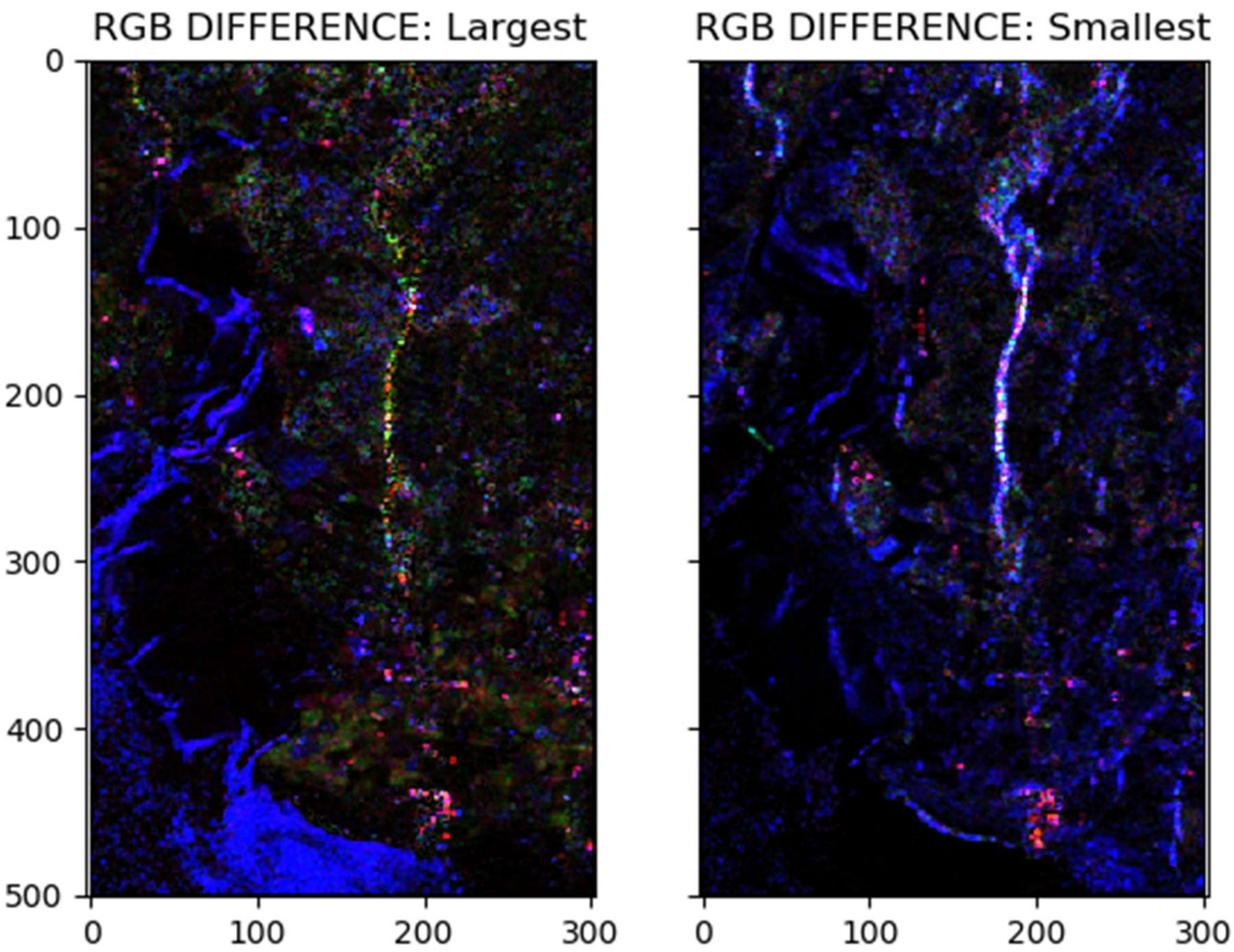
Morecambe Bay

Armando.marino@stir.ac.uk

BE THE DIFFERENCE

1) Morecambe Bay: additive RGB composite

$$([T_2] - [T_1])\underline{\omega} = \lambda\underline{\omega}$$

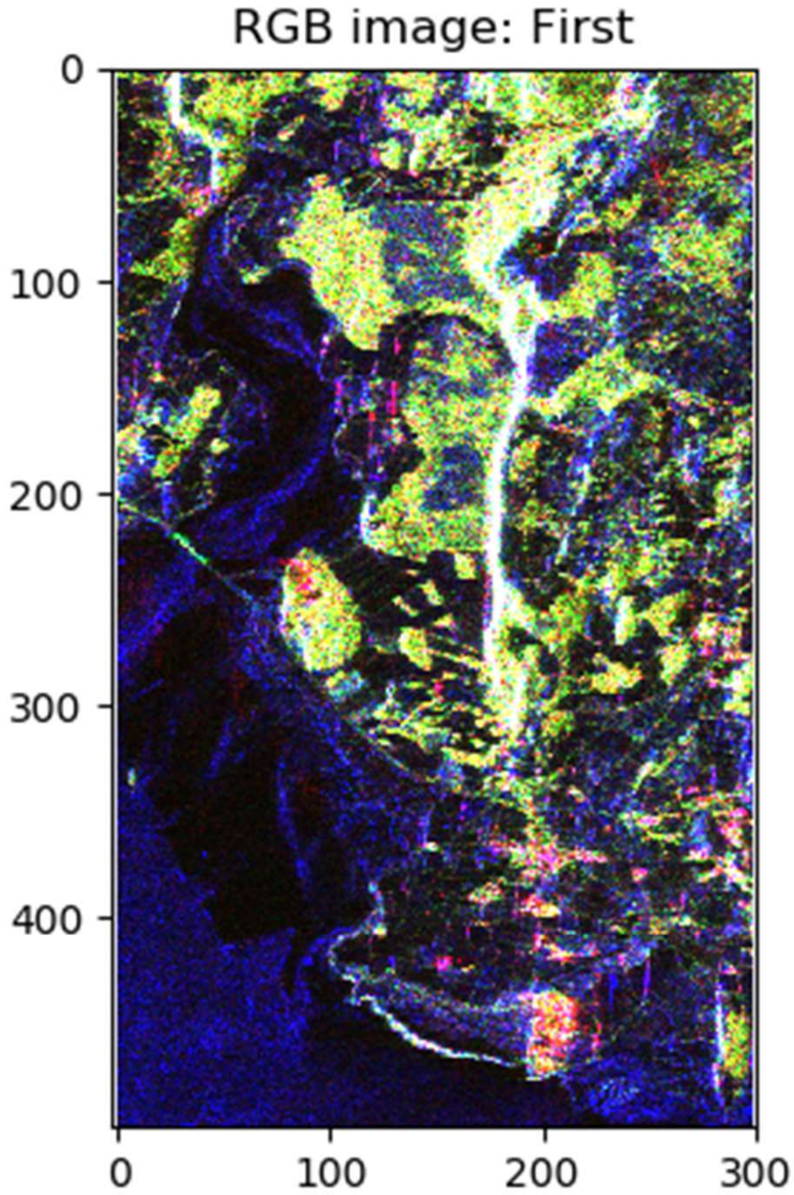


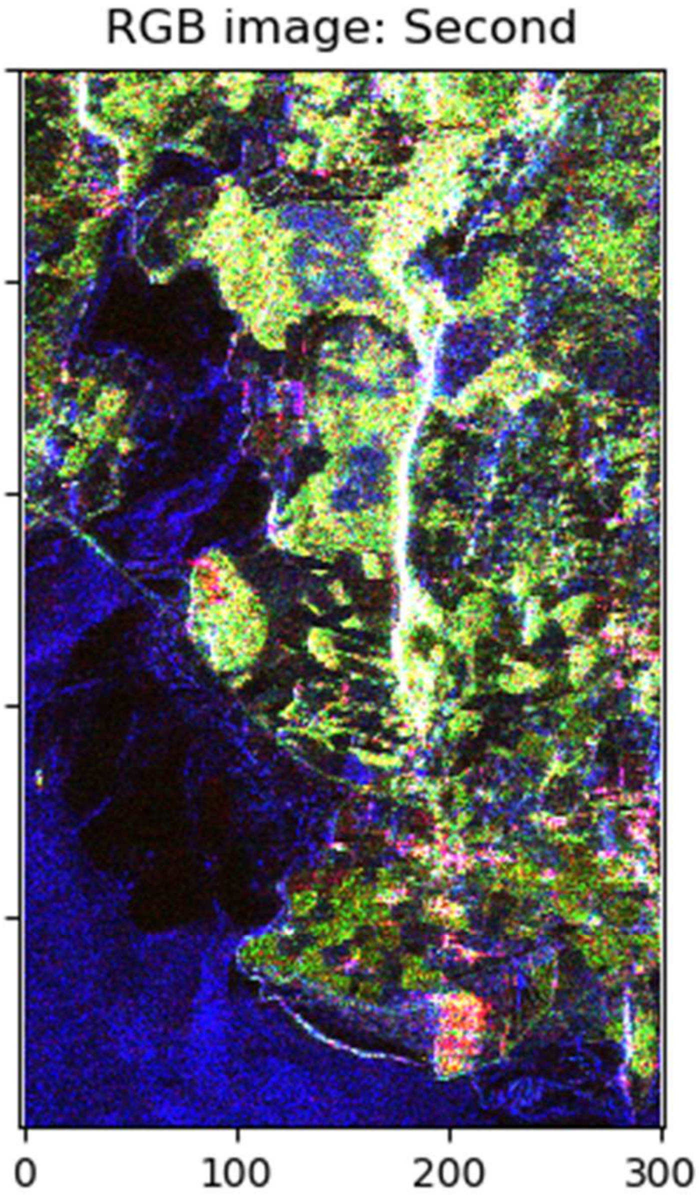
The value of the RGB is modulated by the eigenvalue

Morecambe Bay: Pauli RGB

1 April 2007

17 May 2007

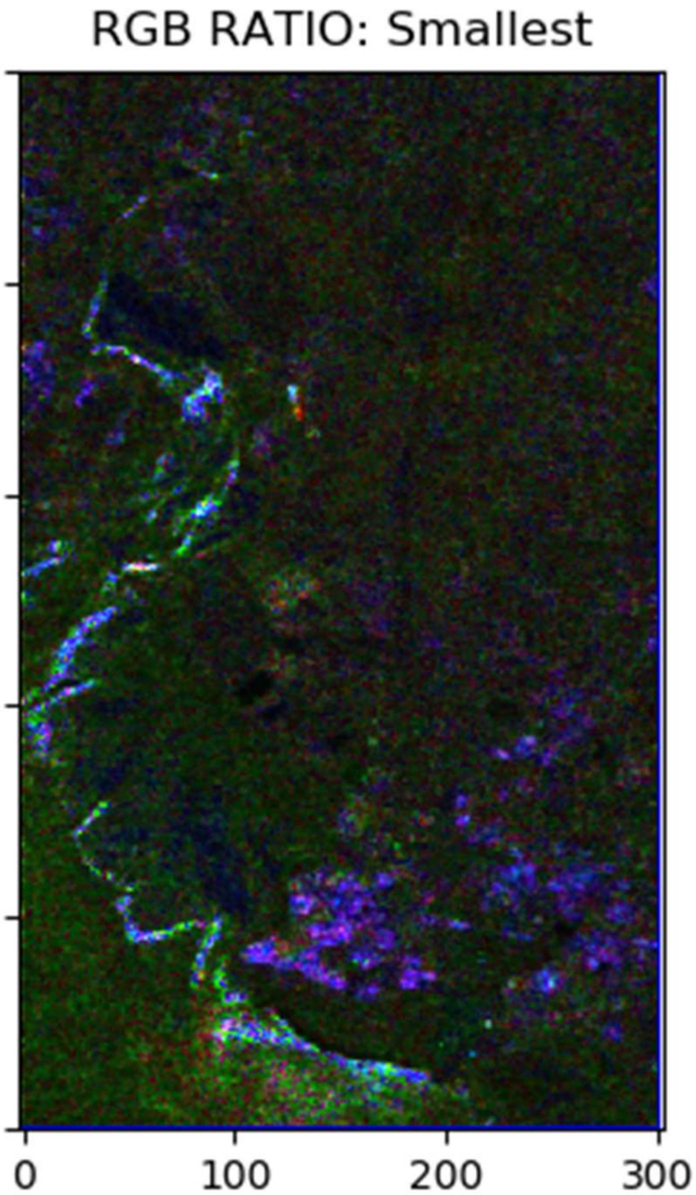




FFERENCE

2) Morecambe Bay: mult. RGB composite

$$[T_{22}]^{-1}[T_{11}]\underline{\omega} = \lambda\underline{\omega}$$



The value of the RGB is modulated by the eigenvalue

Change detection: Statistically based

Hypothesis

✓ In order to set a statistical test from a known distribution, we need to define the hypothesis first:

$$H_0 : \Sigma_1 = \Sigma_2$$
$$H_1 : \Sigma_1 \neq \Sigma_2$$

Σ_1: expected value of covariance matrix for acquisition 1

Σ_2: expected value of covariance matrix for acquisition 2

C_1: sample mean of covariance matrix for acquisition 1

C_2: sample mean of covariance matrix for acquisition 2

Wishart

- ✓ The classifier proposed previously is considering the physical behaviour of scatterers, but it does not take into account the statistical variation of the image pixels
- ✓ In order to do this, we need to know the pdf of the covariance (or coherency) matrix.
- ✓ The simplest case (no texture) consider a **Wishart** distribution.

L: number of independent looks
p: number of polarisation channels

$$f_T\left([T]/[T_m]\right) = \frac{L^{Lp} \left|[T]\right|^{L-p} e^{-L Tr\left([T_m]^{-1}[T]\right)}}{\pi^{\frac{p(p-1)}{2}} \Gamma(L) \ldots \Gamma(L-p+1) \left|[T_m]\right|^{L}}$$

pdf of the covariance matrix

Matrix Trace — Tr

Conditional to a specific class — $[T]/[T_m]$

Gamma function — Γ

Matrix determinant — $\left|[T_m]\right|$

Likelihood ratio test

- ✓ If [C1] and [C2] are the same, then their sum will still be Wishart
- ✓ We can therefore set a Likelihood Ratio Test to check if both covariance matrices are Wishart and they have the same variance.
  - ✓ The likelihoods product will be equal to the likelihood of the sum
- ✓ The ratio results in the following:

$$Q = \frac{(n+m)^{p(m+n)}}{n^{pn} m^{pm}} \frac{Det([C_1])^n Det([C_2])^m}{Det([C_1]+[C_2])^{n+m}}$$

n: number of looks for $[C_1]$, first acquisition
m: number of looks for $[C_2]$, second acquisition
P: number of polarimetric channels
Det: matrix determinant

K. Conradsen, A. A. Nielsen, J. Schou and H. Skriver, "A test statistic in the complex Wishart distribution and its application to change detection in polarimetric SAR data," in IEEE Transactions on Geoscience and Remote Sensing, vol. 41, no. 1, pp. 4-19, Jan. 2003, doi: 10.1109/TGRS.2002.808066.

Complex Hotelling–Lawley Trace

- ✓ The Hotelling–Lawley Trace has a known distribution called the FS.
- ✓ Setting a threshold on it can be done rigorously.

$$HLT = Trace([T_2]^{-1}[T_1])$$

V. Akbari, S. N. Anfinsen, A. P. Doulgeris, T. Eltoft, G. Moser and S. B. Serpico, "Polarimetric SAR Change Detection With the Complex Hotelling–Lawley Trace Statistic," in *IEEE Transactions on Geoscience and Remote Sensing*, vol. 54, no. 7, pp. 3953-3966, July 2016, doi: 10.1109/TGRS.2016.2532320.

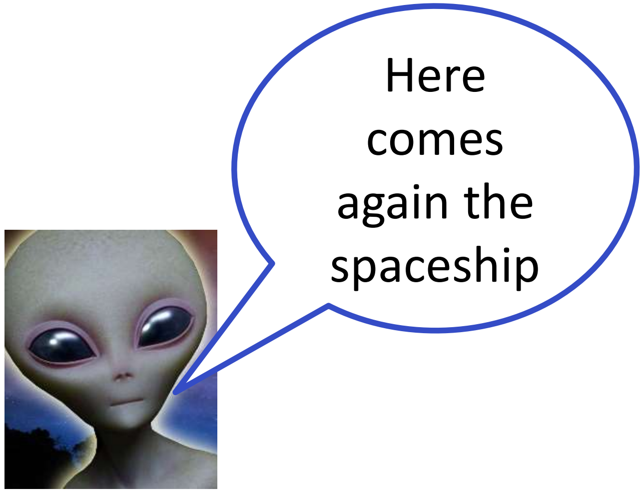
Armando.marino@stir.ac.uk

HLT and Power ratio

$$[T_2]^{-1}[T_1]\underline{\omega} = \lambda\underline{\omega}$$

- ✓ The term ($[T_2]^{-1}[T_1]$) may remind you the searching space of the power ratio optimisation.

- ✓ Taking the trace of a matrix allow us to go to its integral over the full domain of $\underline{\omega}$



Note the ratio is not Hermitian and therefore it may not look like a regular ellipsoid, but it is still convex, please read the paper for more info

A. Marino, "Trace Coherence: A New Operator for Polarimetric and Interferometric SAR images," in *IEEE Transactions on Geoscience and Remote Sensing*, vol. 55, no. 4, pp. 2326-2339, April 2017, doi: 10.1109/TGRS.2016.2641742.

What is the hardest concept you have learned today?

UNIVERSITY of STIRLING

Armando.marino@stir.ac.uk

BE THE DIFFERENCE

What would you like me to explain more right now?

UNIVERSITY of STIRLING

Armando.marino@stir.ac.uk

BE THE DIFFERENCE

Thank you for your attention!