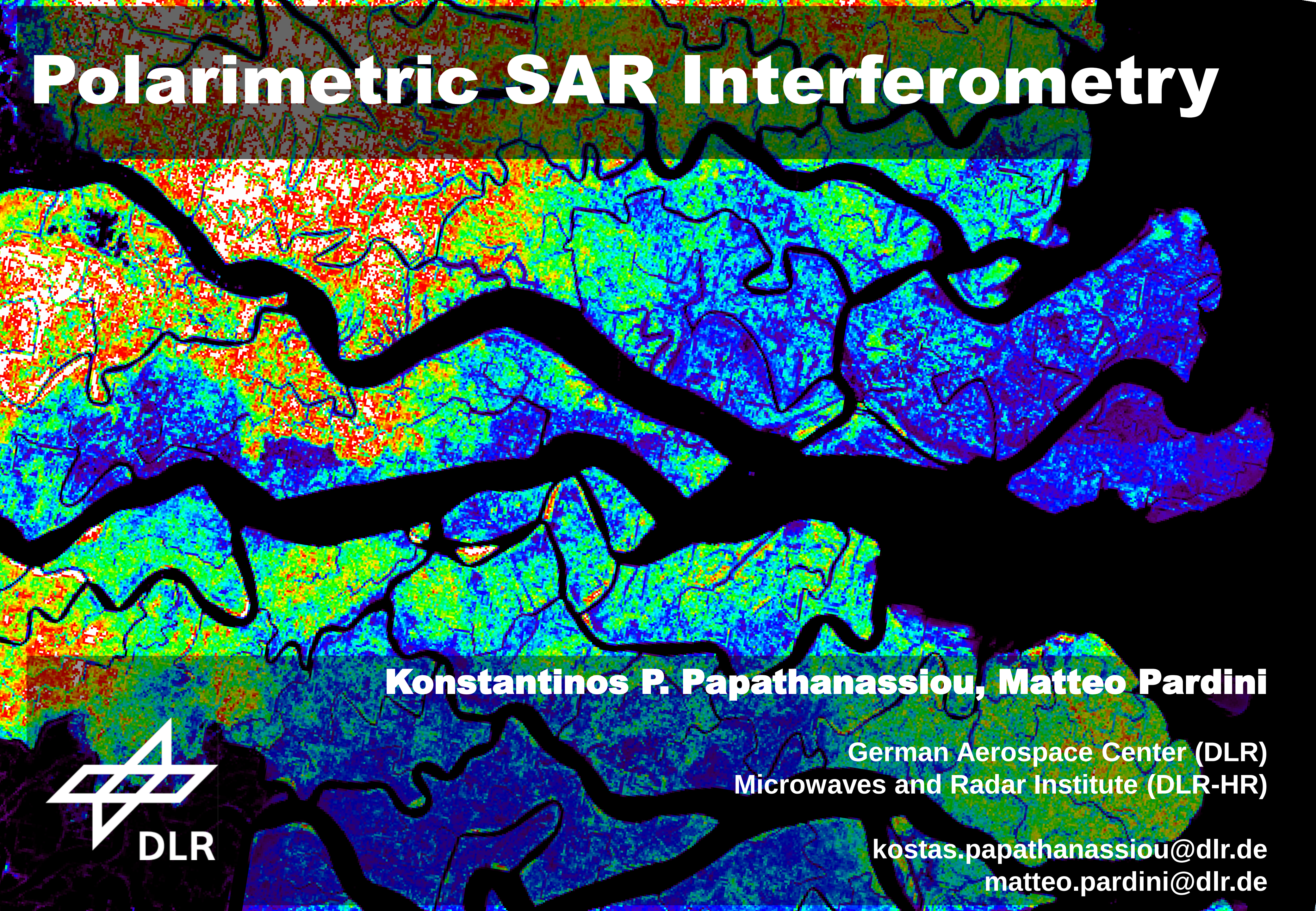

Polarimetric SAR Interferometry

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Microwaves and Radar Institute (DLR-HR)

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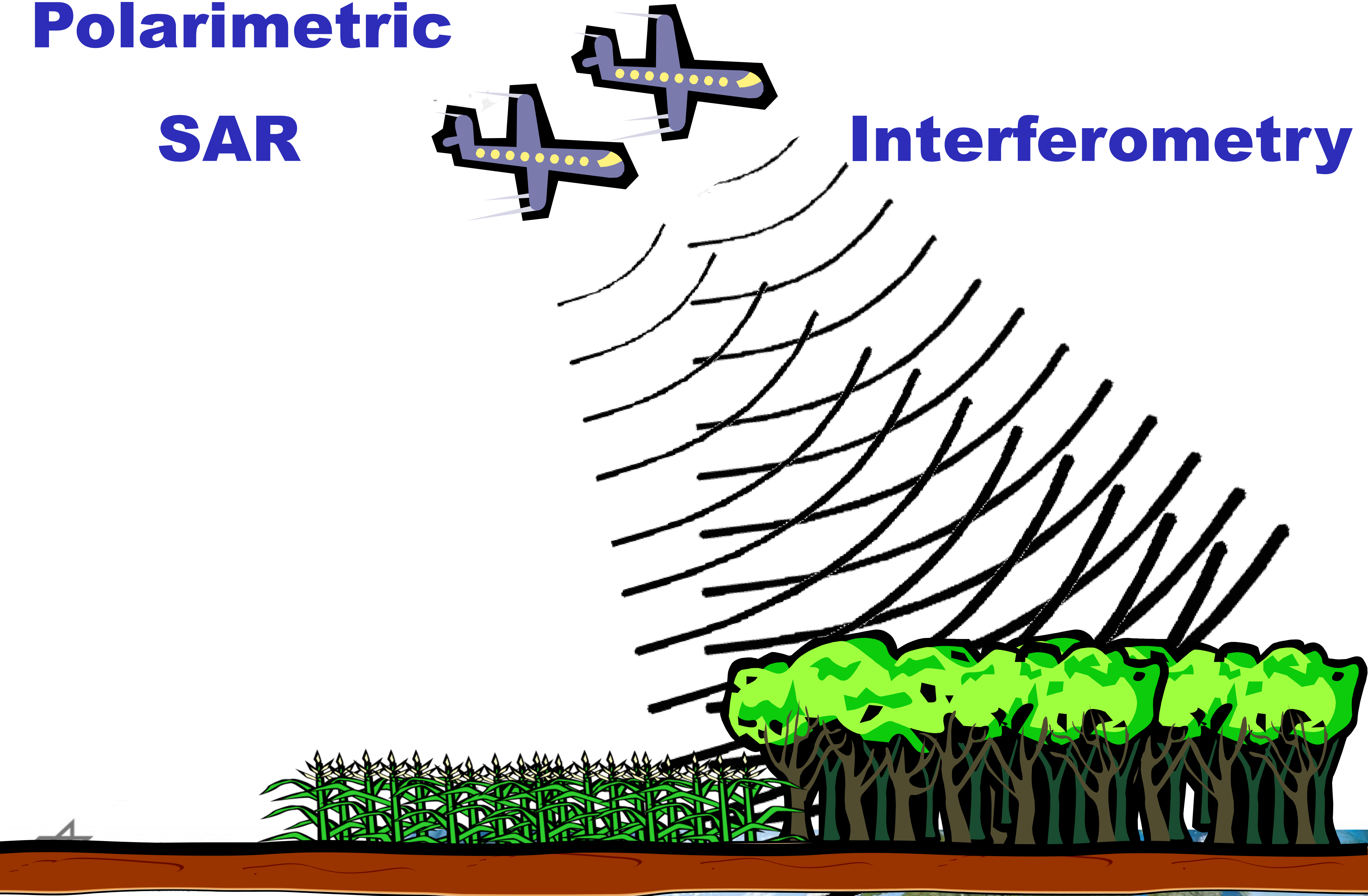
Polarimetric

SAR

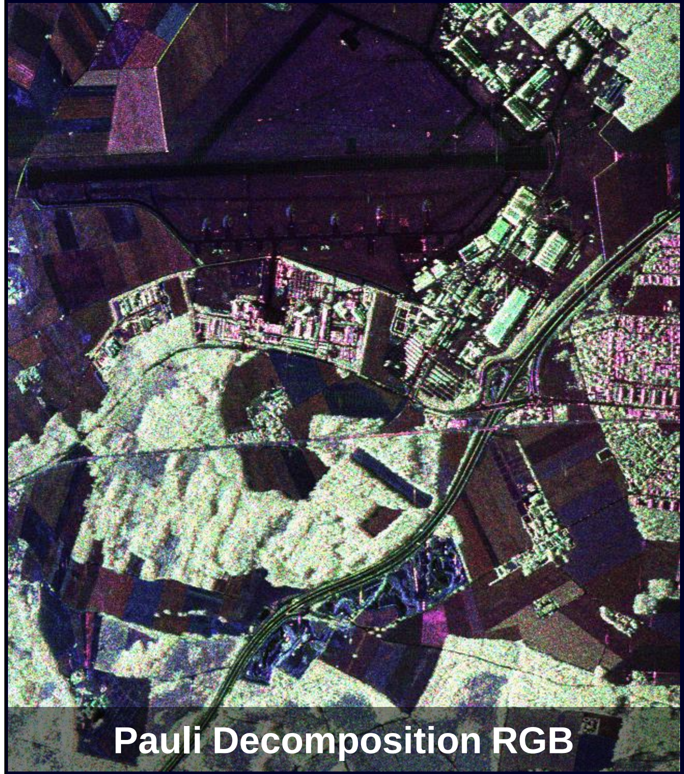
Interferometry



VV Channel Image

SAR Polarimetry (PolSAR)

Allows the identification / decomposition of different scattering processes occurring inside the resolution cell



Pauli Decomposition RGB

VV Channel Image

SAR Interferometry (InSAR)

Allows the location of the effective scattering center inside the resolution cell

VV Channel Image

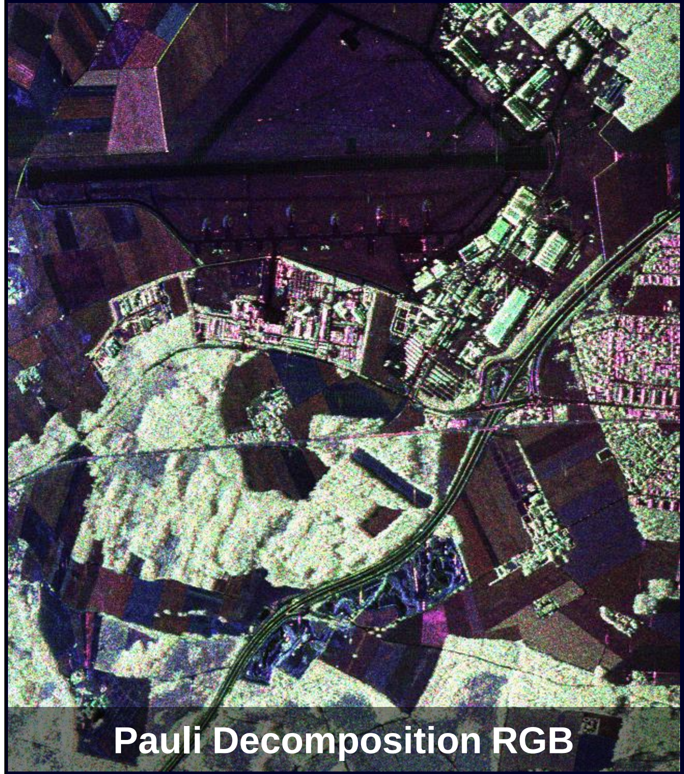
InSAR DEM

SAR Polarimetry (PolSAR)

Allows the identification / decomposition of different scattering processes occurring inside the resolution cell

SAR Interferometry (InSAR)

Allows the location of the effective scattering center inside the resolution cell



Pauli Decomposition RGB

VV Channel Image

InSAR DEM

Polarimetric SAR Interferometry (Pol-InSAR)

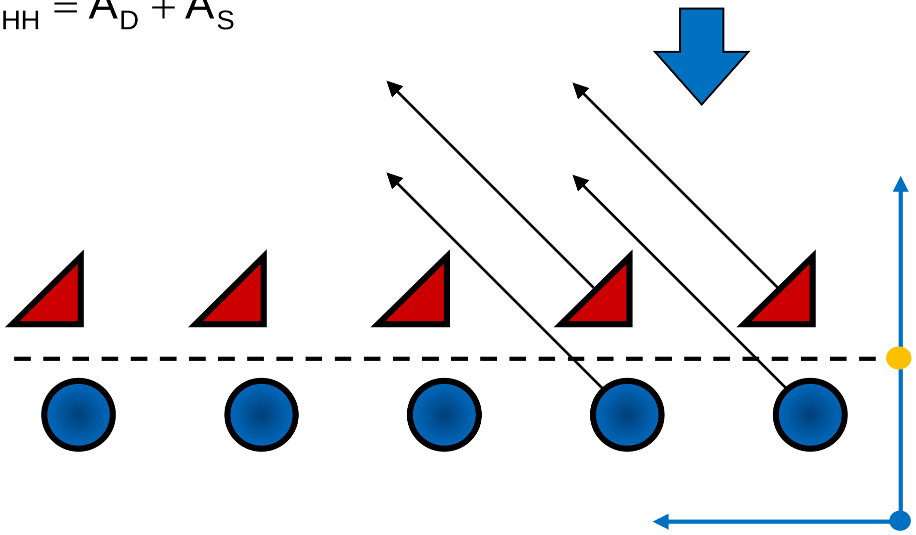
Potential to separate in height different scattering processes occurring inside the resolution cell.

Interferometry vs. Polarimetry

$$S^1_{HH} = A^1_D + A^1_S$$

$$S^2_{HH} = A^2_D + A^2_S$$

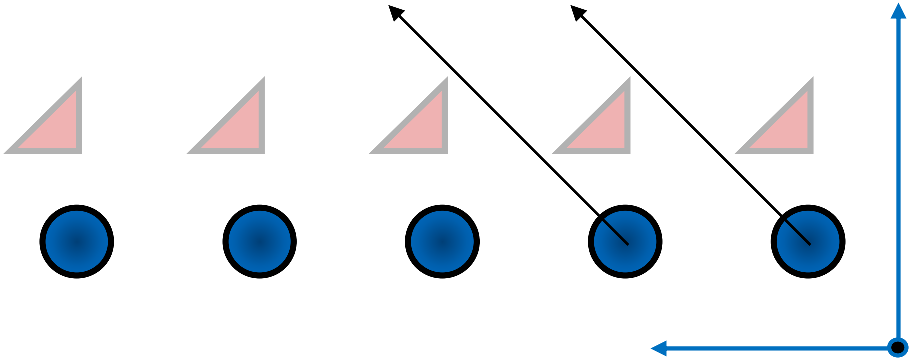
$$\varphi = \arg\{\ S^1_{HH}\ \ S^{2*}_{HH}\ \}$$



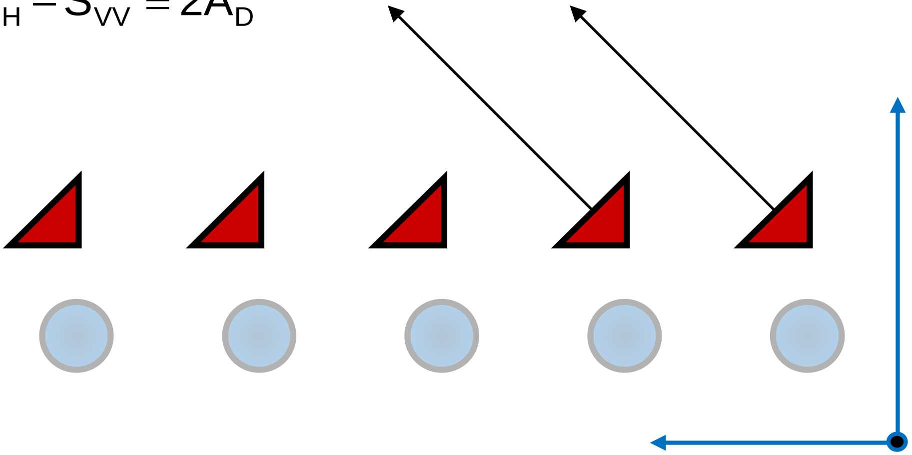
◣ $[S_D] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} = A_D \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

● $[S_S] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} = A_S \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$S_{HH} + S_{VV} = 2A_S$$



$$S_{HH} - S_{VV} = 2A_D$$

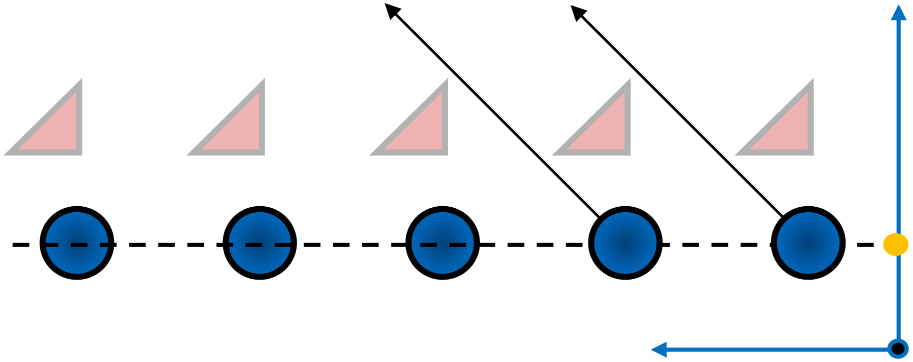


Polarimetric Interferometry

$$i_{1S} = S_{HH}^{1} + S_{VV}^{1} = 2A_{S}^{1}$$

$$i_{2S} = S_{HH}^{2} + S_{VV}^{2} = 2A_{S}^{2}$$

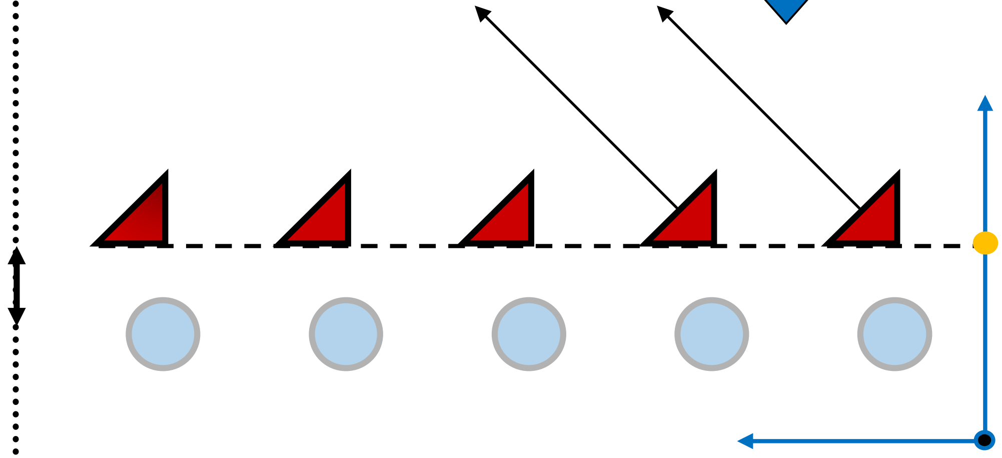
$$\varphi_{S} = \arg\{\ i_{1S}\ \ i_{2S}^{*}\ \}$$



$$i_{1D} = S_{HH}^{1} - S_{VV}^{1} = 2A_{D}^{1}$$

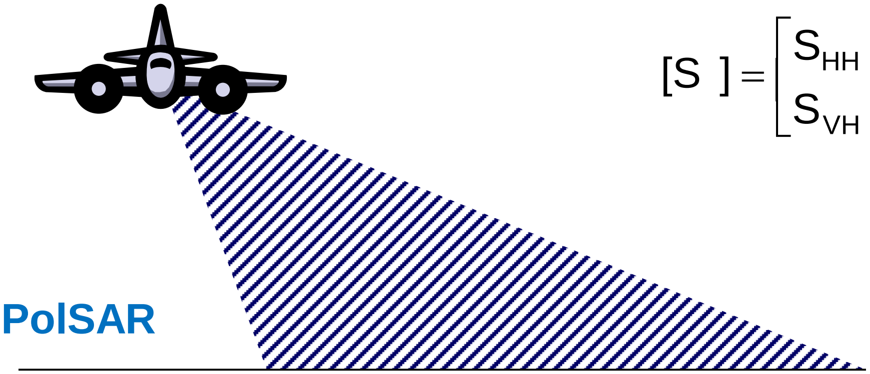
$$i_{2D} = S_{HH}^{2} - S_{VV}^{2} = 2A_{D}^{2}$$

$$\varphi_{D} = \arg\{\ i_{1D}\ \ i_{2D}^{*}\ \}$$



◢ $$[S_{D}] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} = A_{D}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

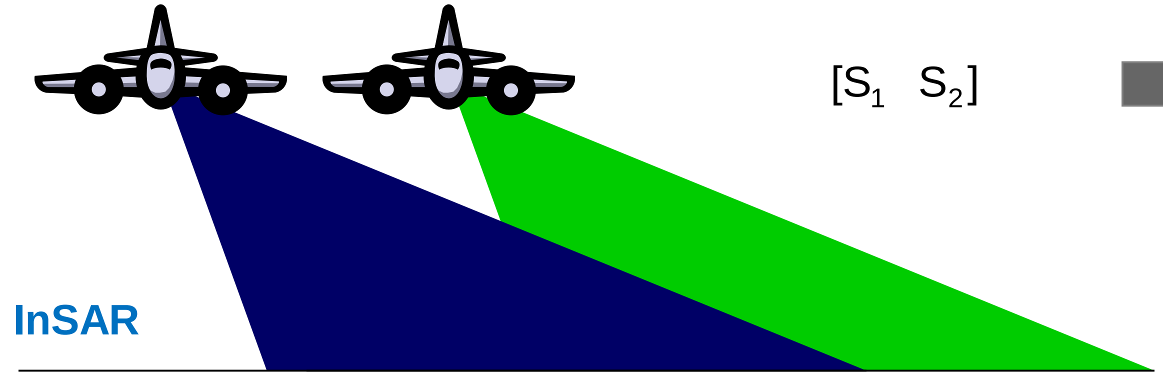
● $$[S_{S}] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} = A_{S}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$[S\ ] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

Polarimetric Coherences

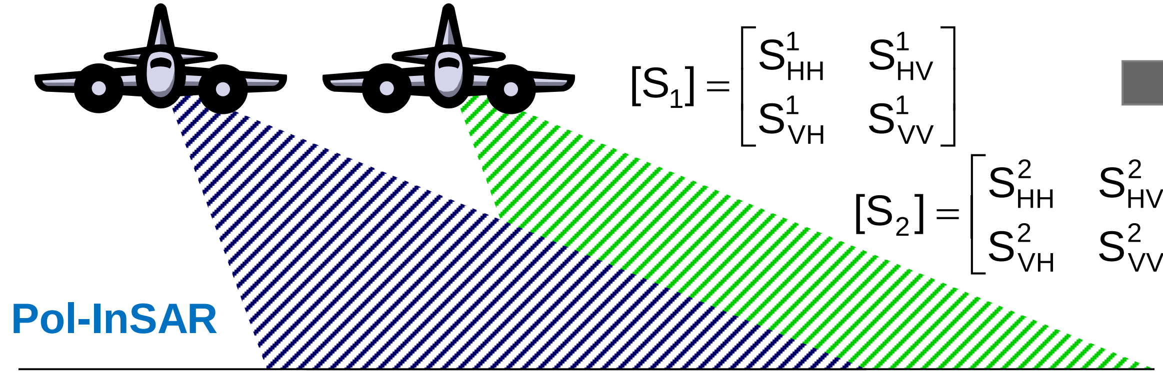
$$\tilde{\gamma}(S_{ij}\ S_{mn}) = \frac{< S_{ij}\ S^*_{mn} >}{\sqrt{< S_{ij}\ S^*_{ij} > < S_{mn}\ S^*_{mn} >}}$$



$$[S_1\quad S_2]$$

Interferometric Coherences

$$\tilde{\gamma}(S_1\ S_2) = \frac{< S_1\ S^*_2 >}{\sqrt{< S_1\ S^*_1 > < S_2\ S^*_2 >}}$$



$$[S_1] = \begin{bmatrix} S^1_{HH} & S^1_{HV} \\ S^1_{VH} & S^1_{VV} \end{bmatrix}$$

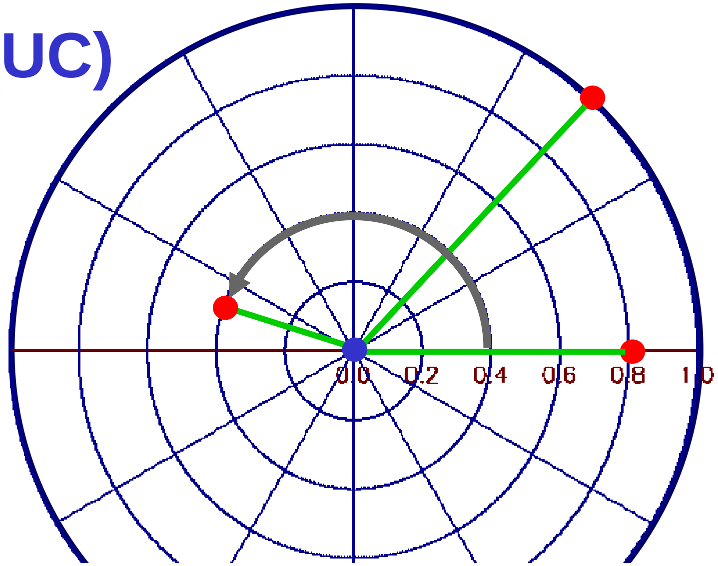
$$[S_2] = \begin{bmatrix} S^2_{HH} & S^2_{HV} \\ S^2_{VH} & S^2_{VV} \end{bmatrix}$$

Polarimetric / Interferometric Coherences

$$\tilde{\gamma}(S^1_{ij}\ S^2_{mn}) = \frac{< S^1_{ij}\ S^{2\ *}_{mn} >}{\sqrt{< S^1_{ij}\ S^{1*}_{ij} > < S^2_{mn}\ S^{2\ *}_{mn} >}}$$

Complex Coherences on the Unit Circle (UC)

$$\tilde{\gamma} := \frac{\sum_{k=1}^{N} S_1(k)S_2^*(k)}{\sqrt{\sum_{k=1}^{N} S_1(k)S_1^*(k)\ \sum_{k=1}^{N} S_2(k)S_2^*(k) >}} = \exp(i\ \mathrm{Arg}(\tilde{\gamma})\ )\cdot|\tilde{\gamma}|$$



Correlation Coefficient $0 \leq |\tilde{\gamma}| = \gamma \leq 1$

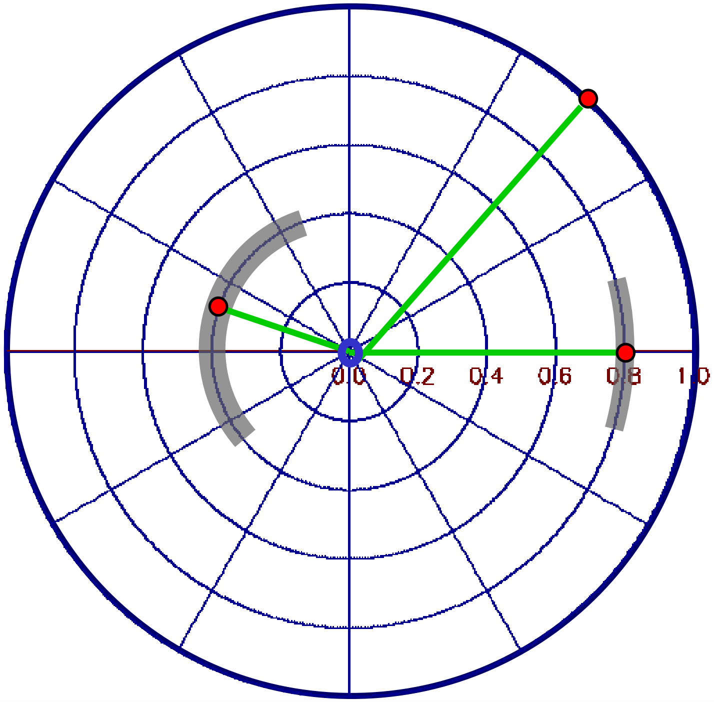
Interferometric Phase $0 \leq \mathrm{Arg}(\tilde{\gamma}) = \varphi \leq 2\pi$

Cramer Rao Bounds:

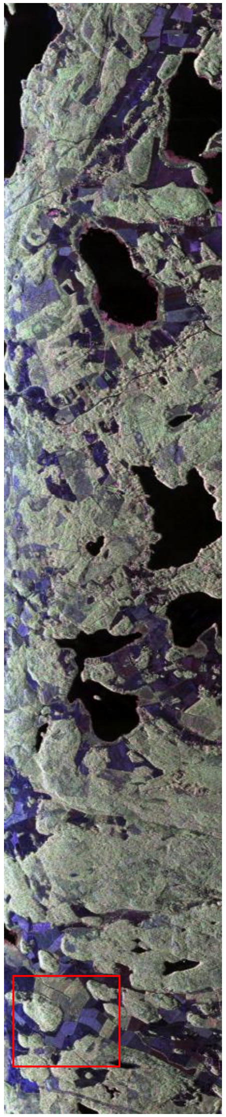
Correlation Coefficient $\mathrm{VAR}(|\tilde{\gamma}|)_{CR} = \frac{(1-|\gamma|^2)^2}{2N}$

Interferometric Phase $\mathrm{VAR}(\varphi)_{CR} = \frac{1-|\gamma|^2}{2N|\gamma|^2}$

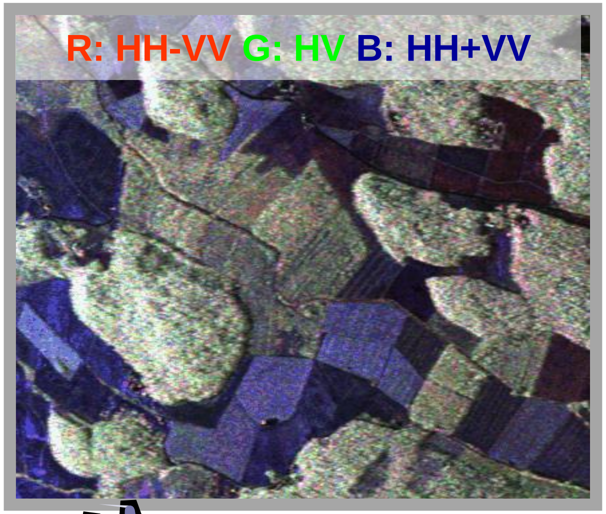
$\varphi = \arg(\tilde{\gamma})$ **and N is the number of Looks**



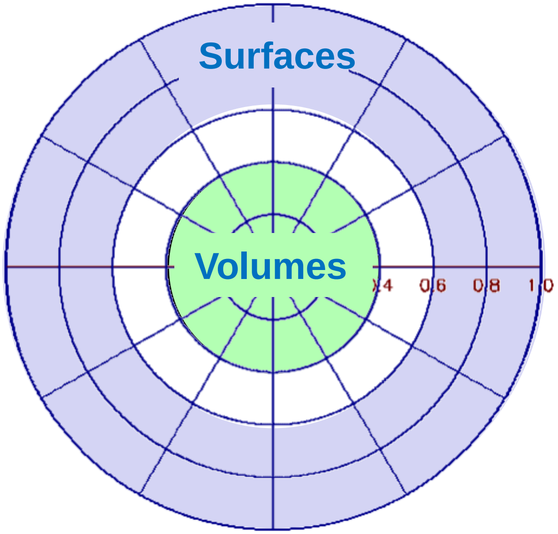
Why is Interferometry important for Volume Scatterers?



E-SAR / Test Site: Helsinki, Finland

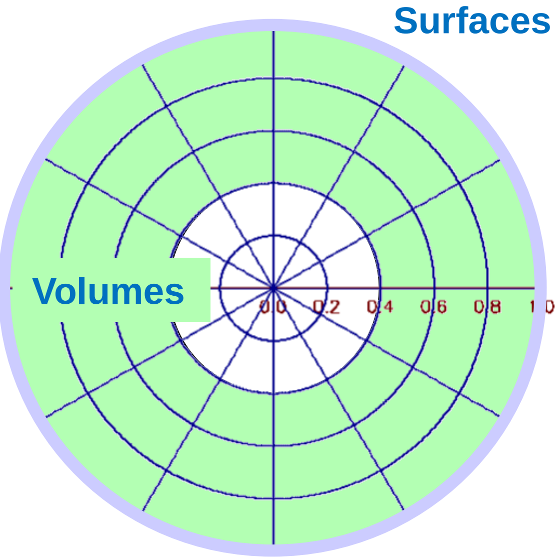


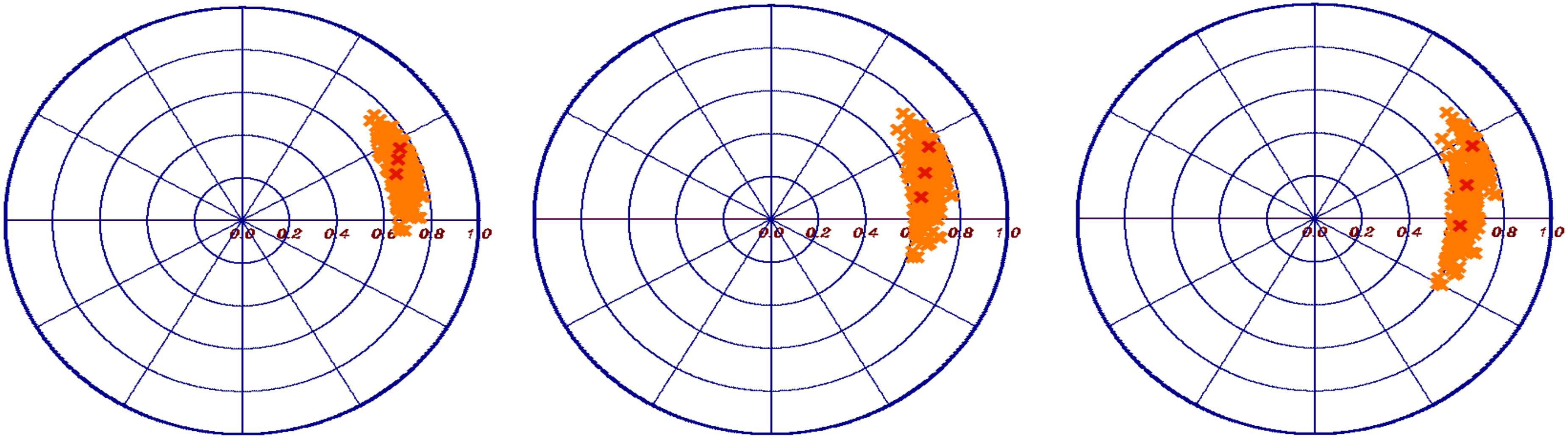
HH-VV Coherence



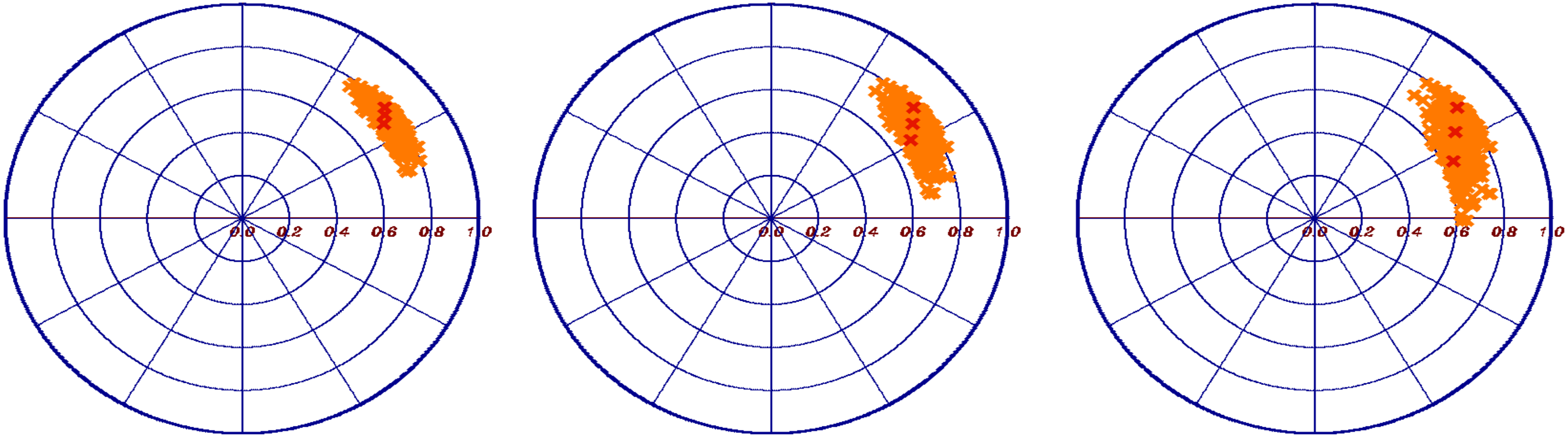
HH-HH Coherence

HH-HH Coherence

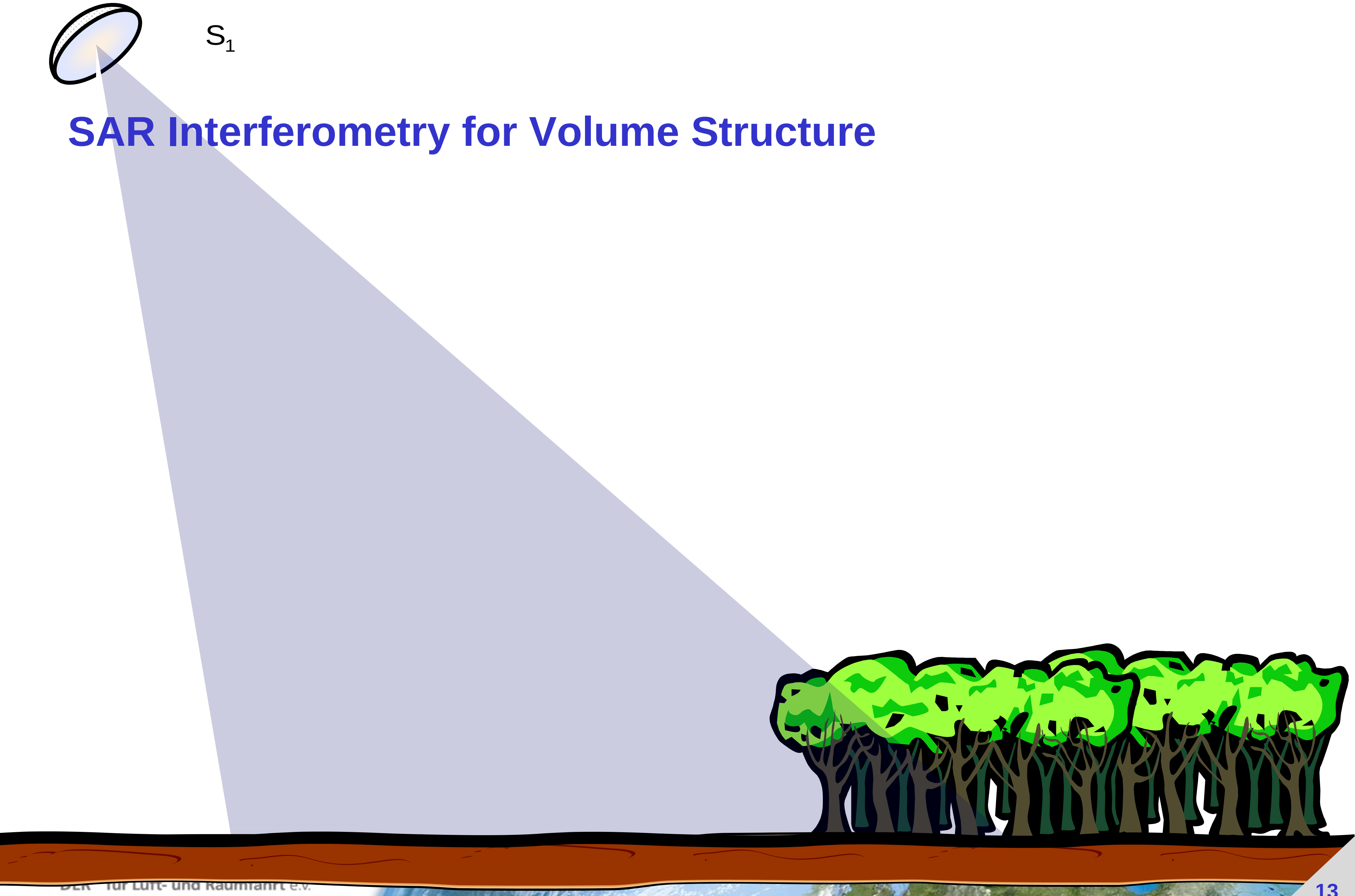


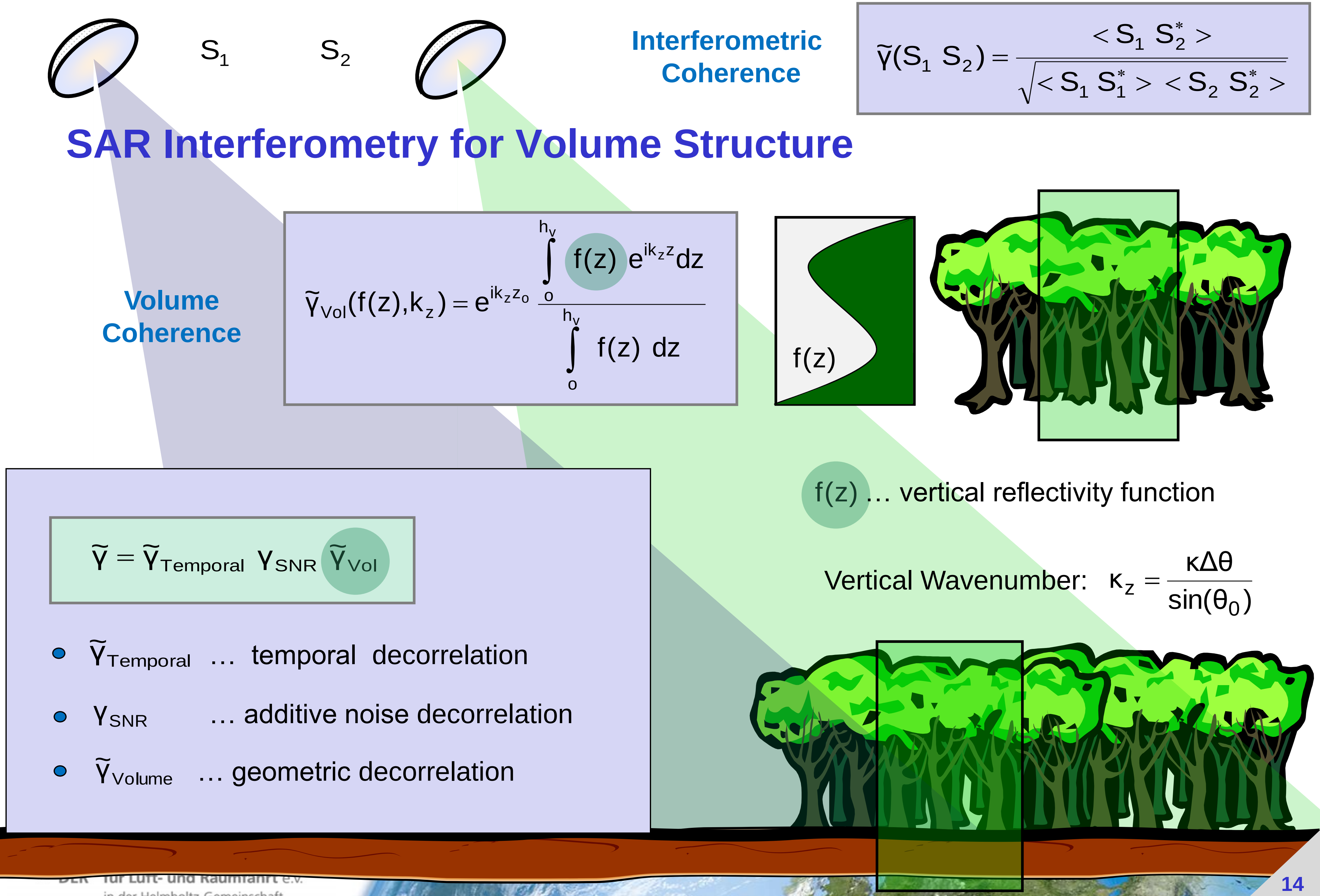


Pol-InSAR: Basic Principles & Ideas



SAR Interferometry for Volume Structure





Interferometric Coherence

$$\tilde{\gamma}(S_1\ S_2) = \frac{< S_1\ S_2^* >}{\sqrt{< S_1\ S_1^* > < S_2\ S_2^* >}}$$

SAR Interferometry for Volume Structure

Volume Coherence

$$\tilde{\gamma}_{Vol}(f(z),k_z) = e^{ik_z z_o} \frac{\int_o^{h_V} f(z)\ e^{ik_z z} dz}{\int_o^{h_V} f(z)\ dz}$$

f(z) … vertical reflectivity function

Vertical Wavenumber: $\kappa_z = \dfrac{\kappa\Delta\theta}{\sin(\theta_0)}$

$$\tilde{\gamma} = \tilde{\gamma}_{Temporal}\ \gamma_{SNR}\ \tilde{\gamma}_{Vol}$$

- $\tilde{\gamma}_{Temporal}$ … temporal decorrelation
- γ_{SNR} … additive noise decorrelation
- $\tilde{\gamma}_{Volume}$ … geometric decorrelation

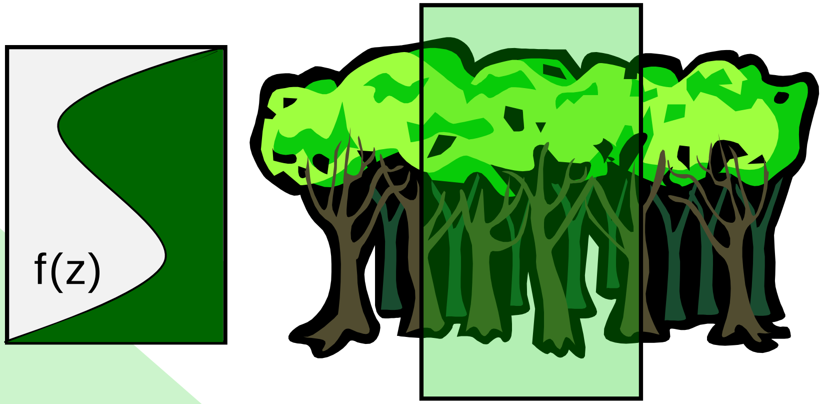
Interferometric Coherence

$$\tilde{\gamma}(S_1\ S_2) = \frac{< S_1\ S_2^* >}{\sqrt{< S_1\ S_1^* > < S_2\ S_2^* >}}$$

SAR Interferometry for Volume Structure

Volume Coherence

$$\tilde{\gamma}_{Vol}(f(z),k_z) = e^{ik_z z_o}\frac{\int_o^{h_V} f(z)\ e^{ik_z z}dz}{\int_o^{h_V} f(z)\ dz}$$



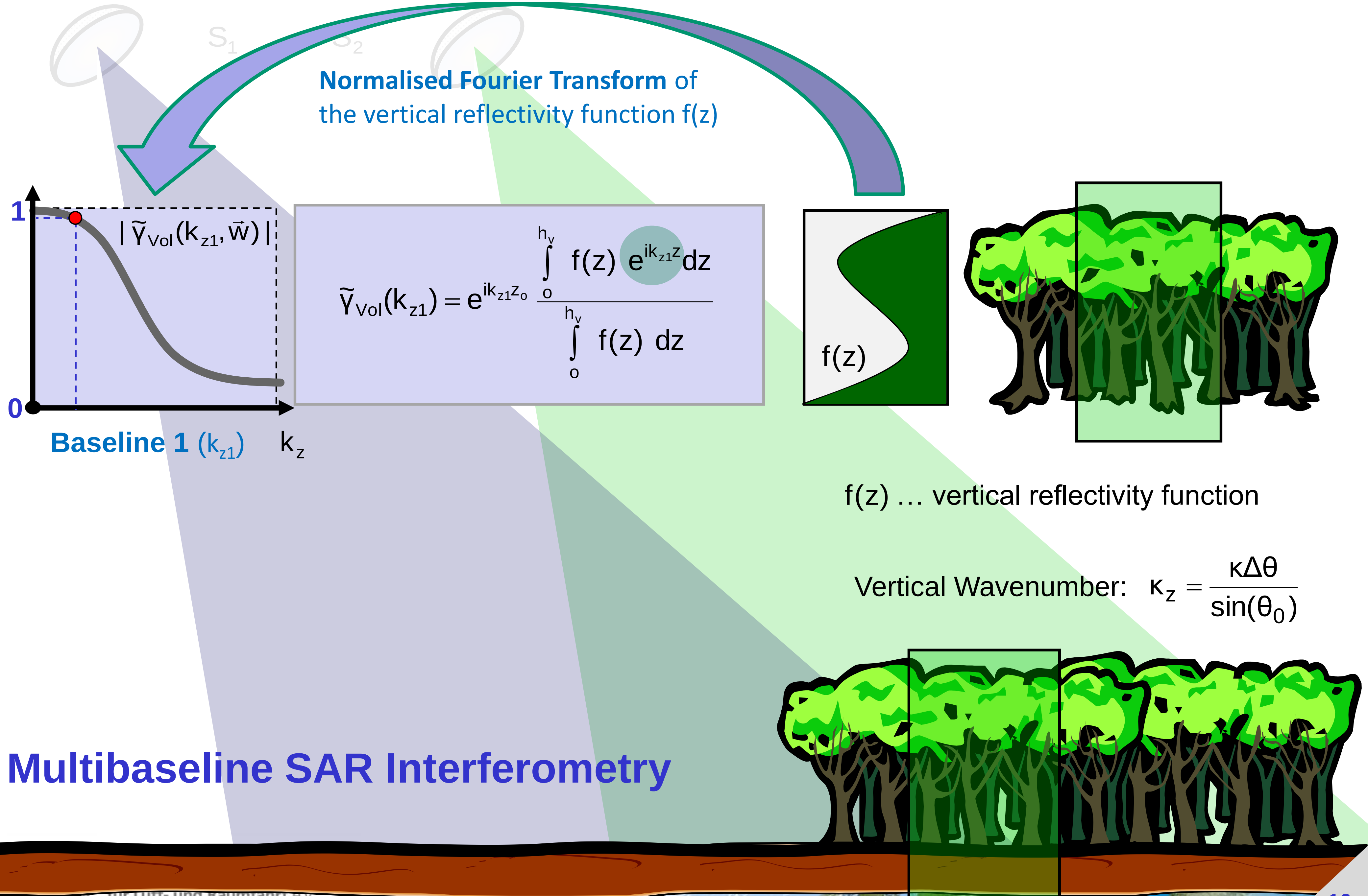
$f(z)$ … vertical reflectivity function

Vertical Wavenumber: $\kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$

$$\tilde{\gamma} = \tilde{\gamma}_{Temporal}\ \gamma_{SNR}\ \tilde{\gamma}_{Vol}$$

- $\tilde{\gamma}_{Temporal}$ … temporal decorrelation
- γ_{SNR} … additive noise decorrelation
- $\tilde{\gamma}_{Volume}$ … geometric decorrelation

SAR interferometry allows to reconstruct the vertical reflectivity function f(z) of a volume scatterer by means of interferometric (volume) coherence measurements at different vertical wavenumbers κ_Z, i.e. at different spatial baselines.



Normalised Fourier Transform of the vertical reflectivity function f(z)

Baseline 1 (k_{z1})

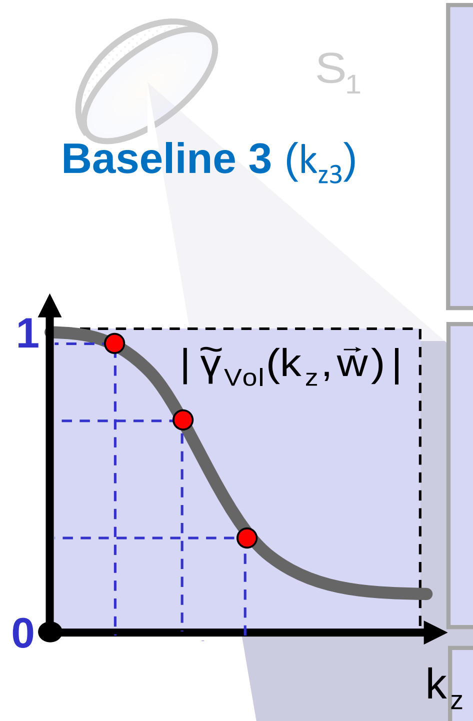
$$\tilde{\gamma}_{Vol}(k_{z1}) = e^{ik_{z1}z_o}\frac{\int_o^{h_v} f(z)\; e^{ik_{z1}z} dz}{\int_o^{h_v} f(z)\; dz}$$

f(z) … vertical reflectivity function

Vertical Wavenumber: $\kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$

Multibaseline SAR Interferometry

Multibaseline SAR Interferometry

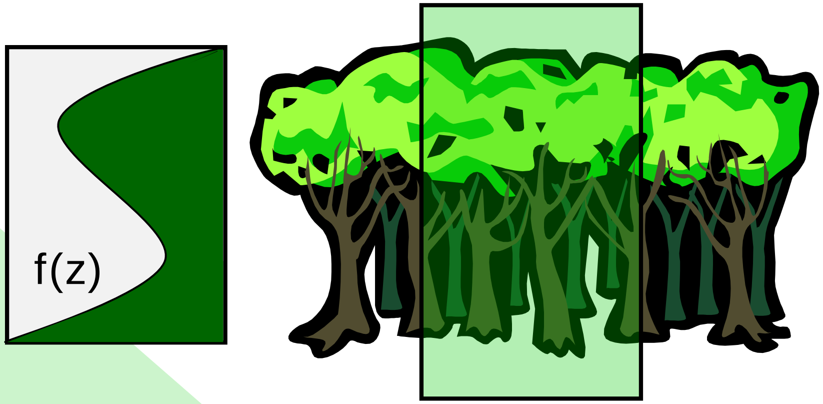


Baseline 2 (k_{z2})

$$\tilde{\gamma}_{Vol}(k_{z3}) = e^{ik_z z_o}\frac{\int_o^{h_v} f(z)\, e^{ik_{z3}z}\, dz}{\int_o^{h_v} f(z)\, dz}$$

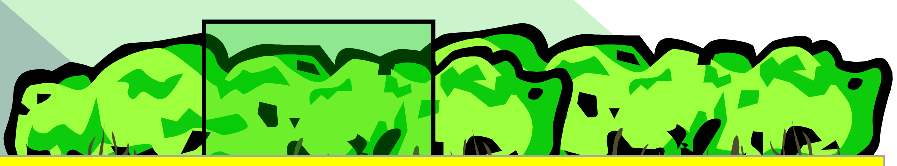
$$\tilde{\gamma}_{Vol}(k_{z1}) = e^{ik_{z1} z_o}\frac{\int_o^{h_v} f(z)\, e^{ik_{z1}z}\, dz}{\int_o^{h_v} f(z)\, dz}$$

$$\tilde{\gamma}_{Vol}(k_{z2}) = e^{ik_z z_o}\frac{\int_o^{h_v} f(z)\, e^{ik_{z2}z}\, dz}{\int_o^{h_v} f(z)\, dz}$$



$f(z)$ … vertical reflectivity function

Vertical Wavenumber: $\kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$



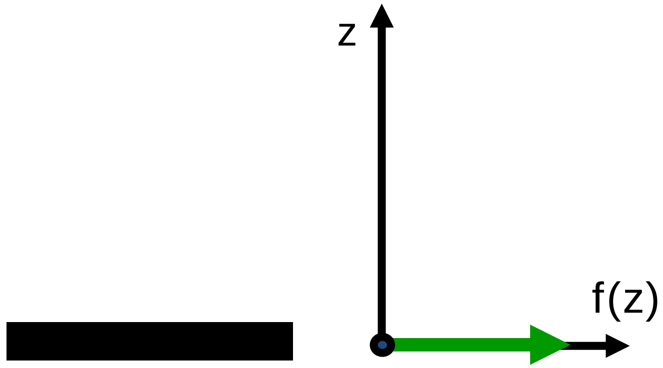
Multi-baseline measurements allow to sample **the spectrum of the vertical reflectivity** FT{f(z)} **@ different (spatial) frequencies** (k_z).

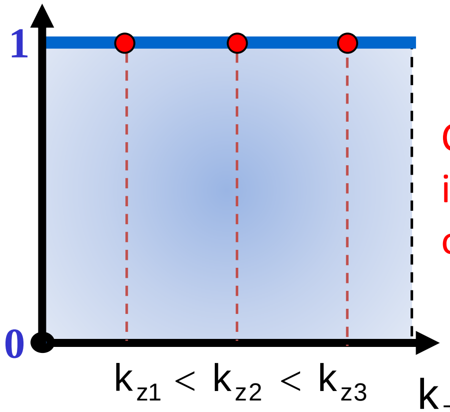
Vertical Reflectivity Function f(z)

InSAR Volume Coherence $|\tilde{\gamma}_{Vol}(k_z)|$

$$|\tilde{\gamma}_{Vol}(k_z)| = \frac{\left|\int_{0}^{h_v} f(z)\, e^{ik_z z} dz\right|}{\int_{0}^{h_v} f(z)\, dz}$$

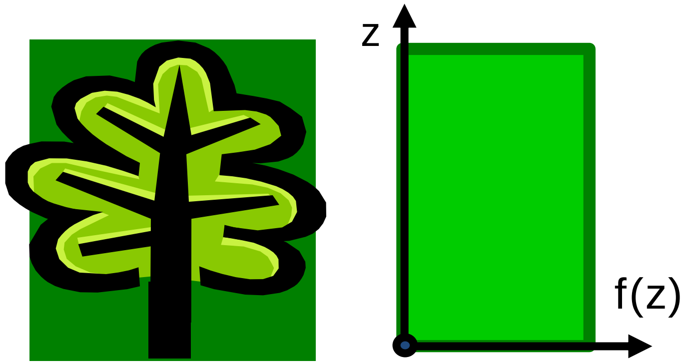
Surface Scatterer



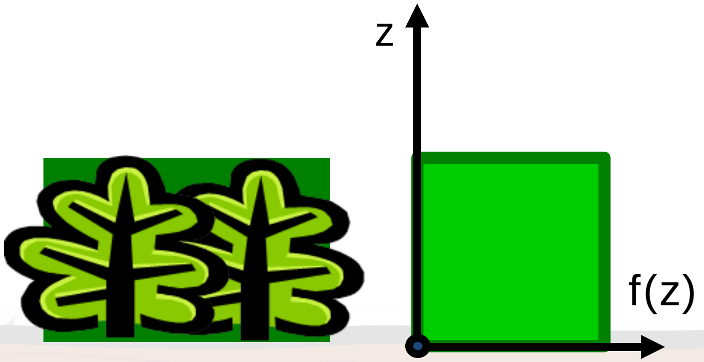


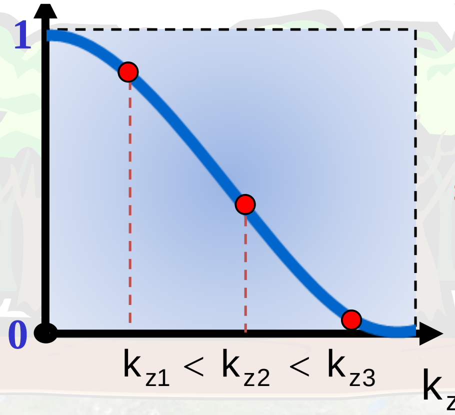
Coherence is independent of baseline

Tall Vegetation



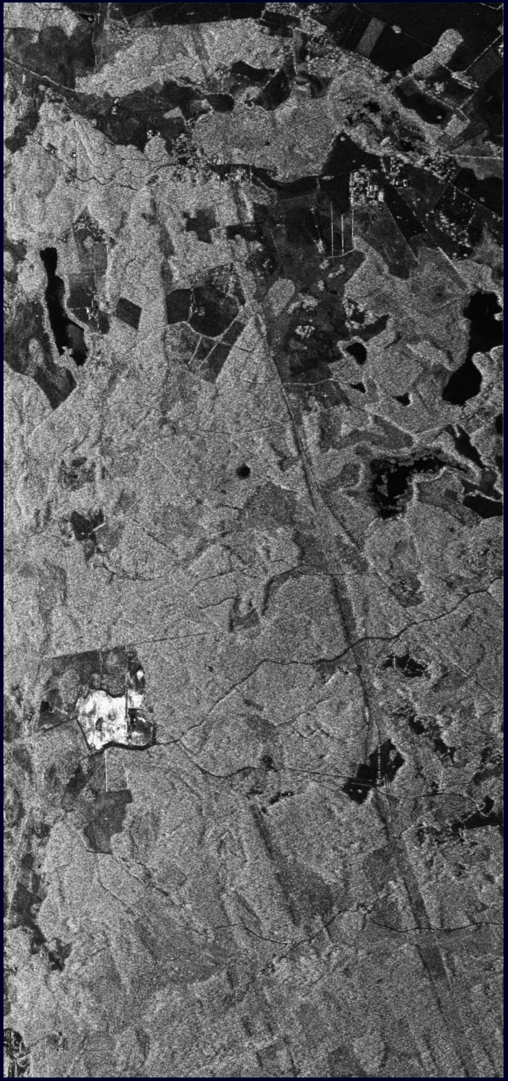
Short Vegetation





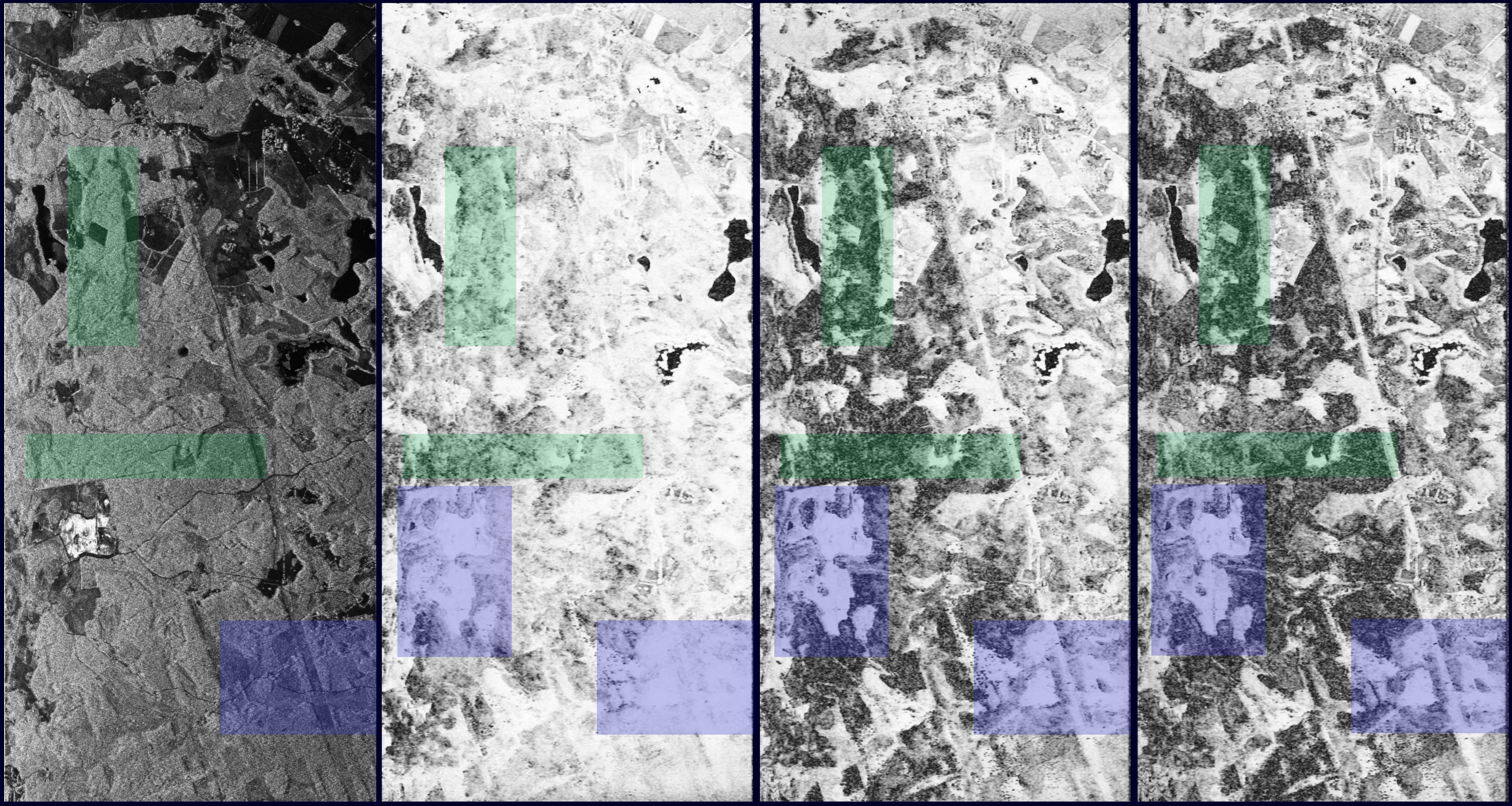
Coherence degreases **slower** with increasing baseline

Amplitude Image

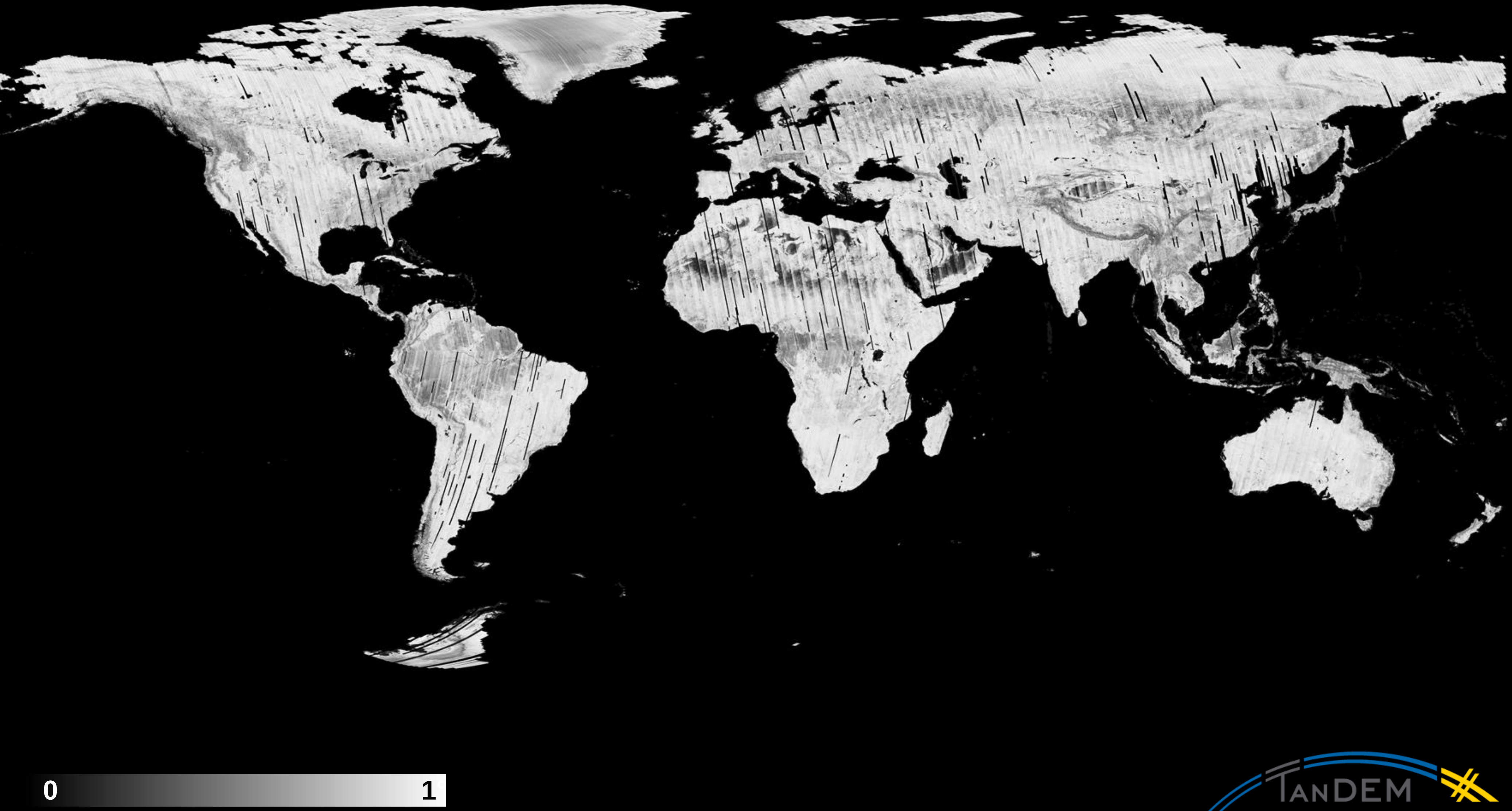


Amplitude Image HH

Interferometric Coherence: Volume Decorrelation



InSAR Coherence $\tilde{\gamma} = \tilde{\gamma}_{\text{Temporal}} \; \gamma_{\text{SNR}} \; \tilde{\gamma}_{\text{Vol}}$ For surfaces: $\tilde{\gamma}_{\text{Vol}} = 1 \rightarrow \tilde{\gamma} = \gamma_{\text{SNR}}$



By DLR-HR-STL

500x500 m^2 resolution

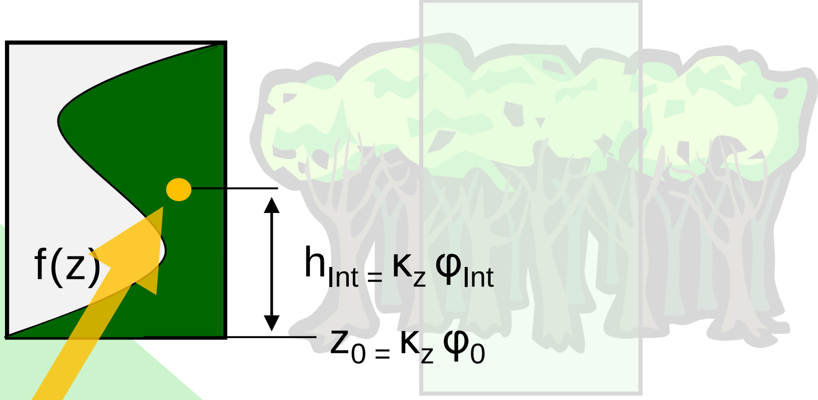
SAR Interferometry for Volume Structure: The Phase Center

Interferometric Coherence

$$\tilde{\gamma}(S_1\ S_2) = \frac{< S_1\ S_2^* >}{\sqrt{< S_1\ S_1^* > < S_2\ S_2^* >}}$$

Volume Coherence

$$\tilde{\gamma}_{Vol}(f(z),k_z) = e^{ik_z z_0} \frac{\int_{o}^{h_V} f(z)\ e^{ik_z z} dz}{\int_{o}^{h_V} f(z)\ dz}$$



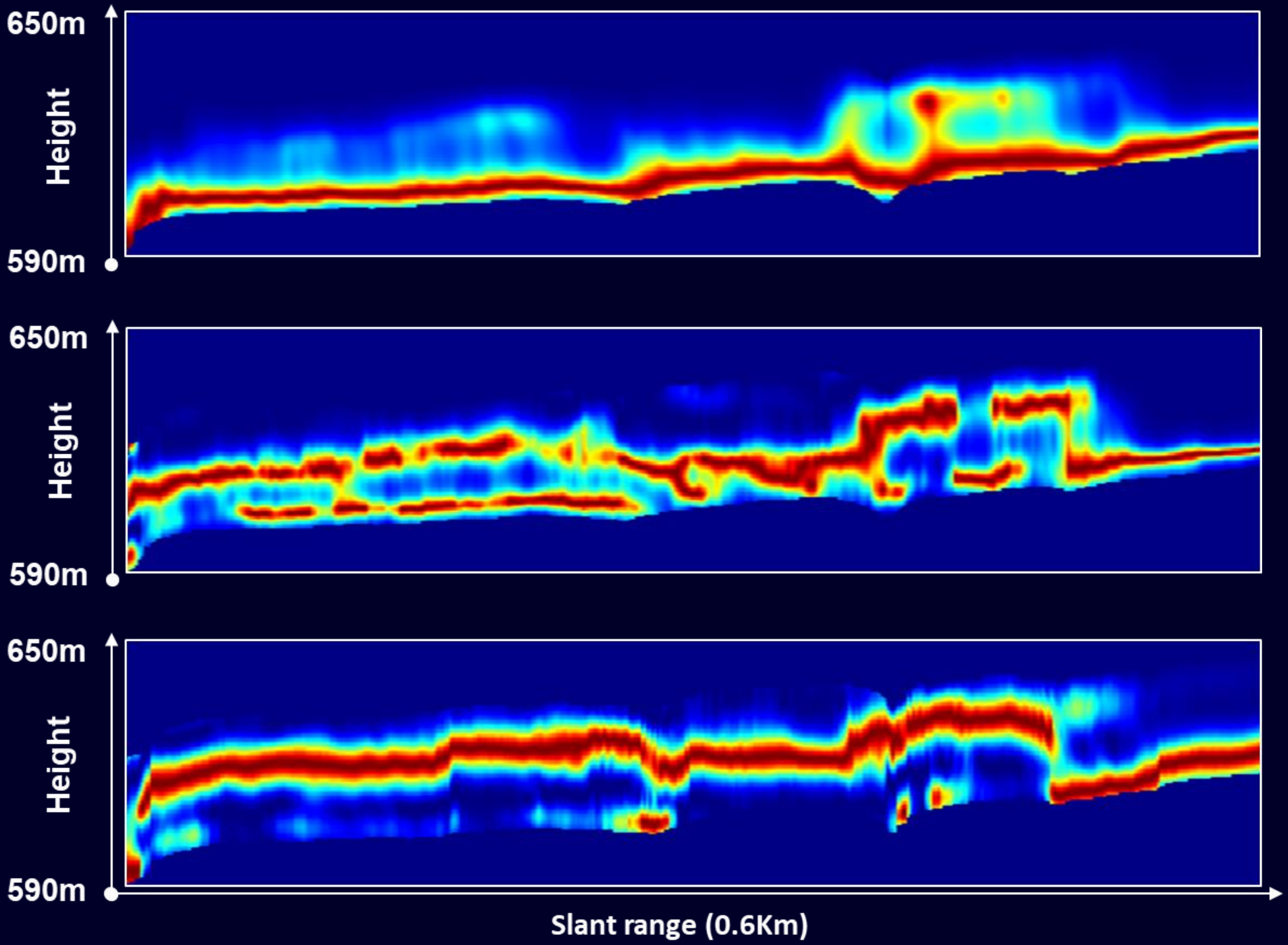
f(z) … vertical reflectivity function

Vertical Wavenumber: $\kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$

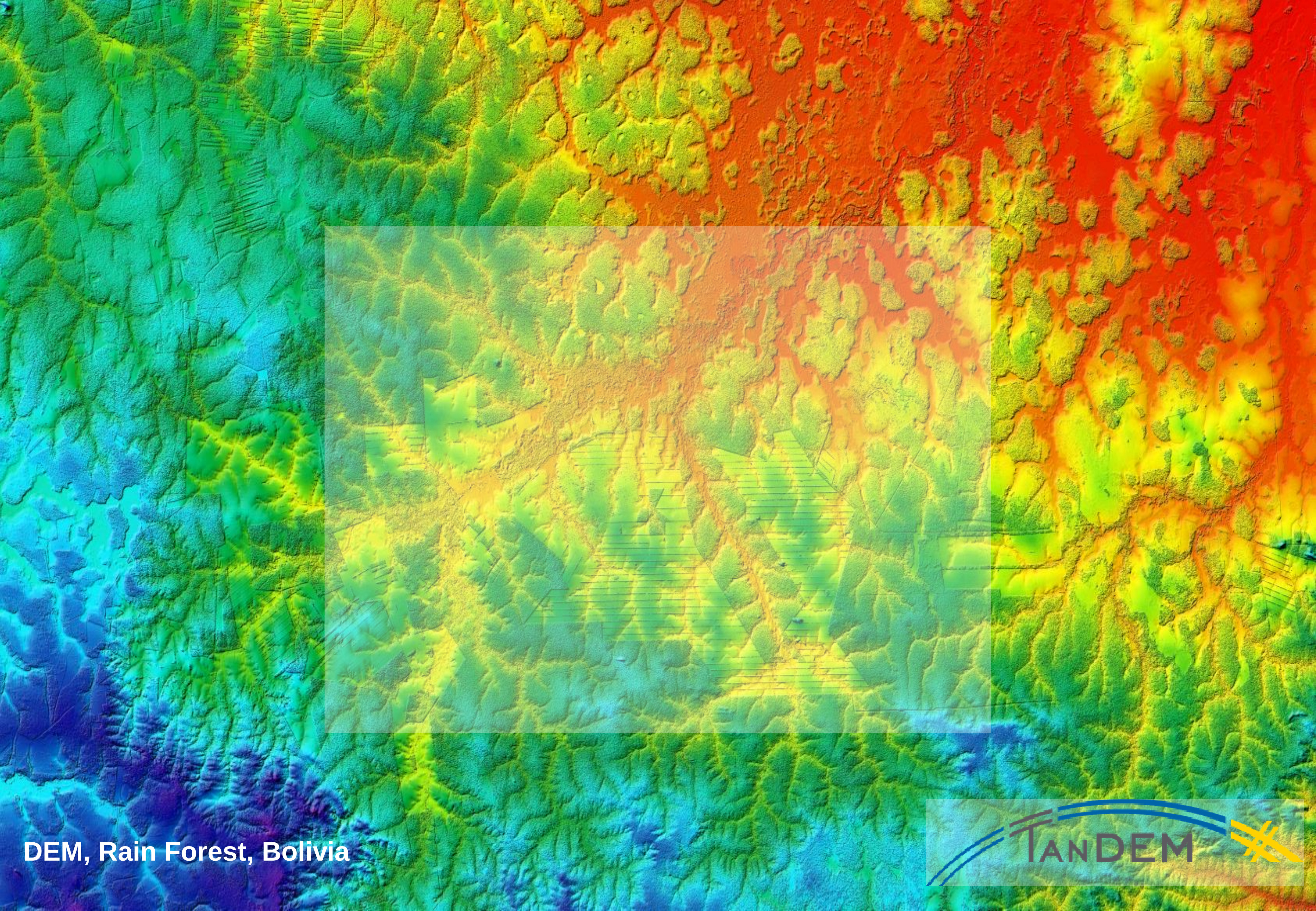
$$\tilde{\gamma} = \tilde{\gamma}_{Temporal}\ \gamma_{SNR}\ \tilde{\gamma}_{Vol}$$

- $\tilde{\gamma}_{Temporal}$ … temporal decorrelation
- γ_{SNR} … additive noise decorrelation
- $\tilde{\gamma}_{Volume}$ … geometric decorrelation

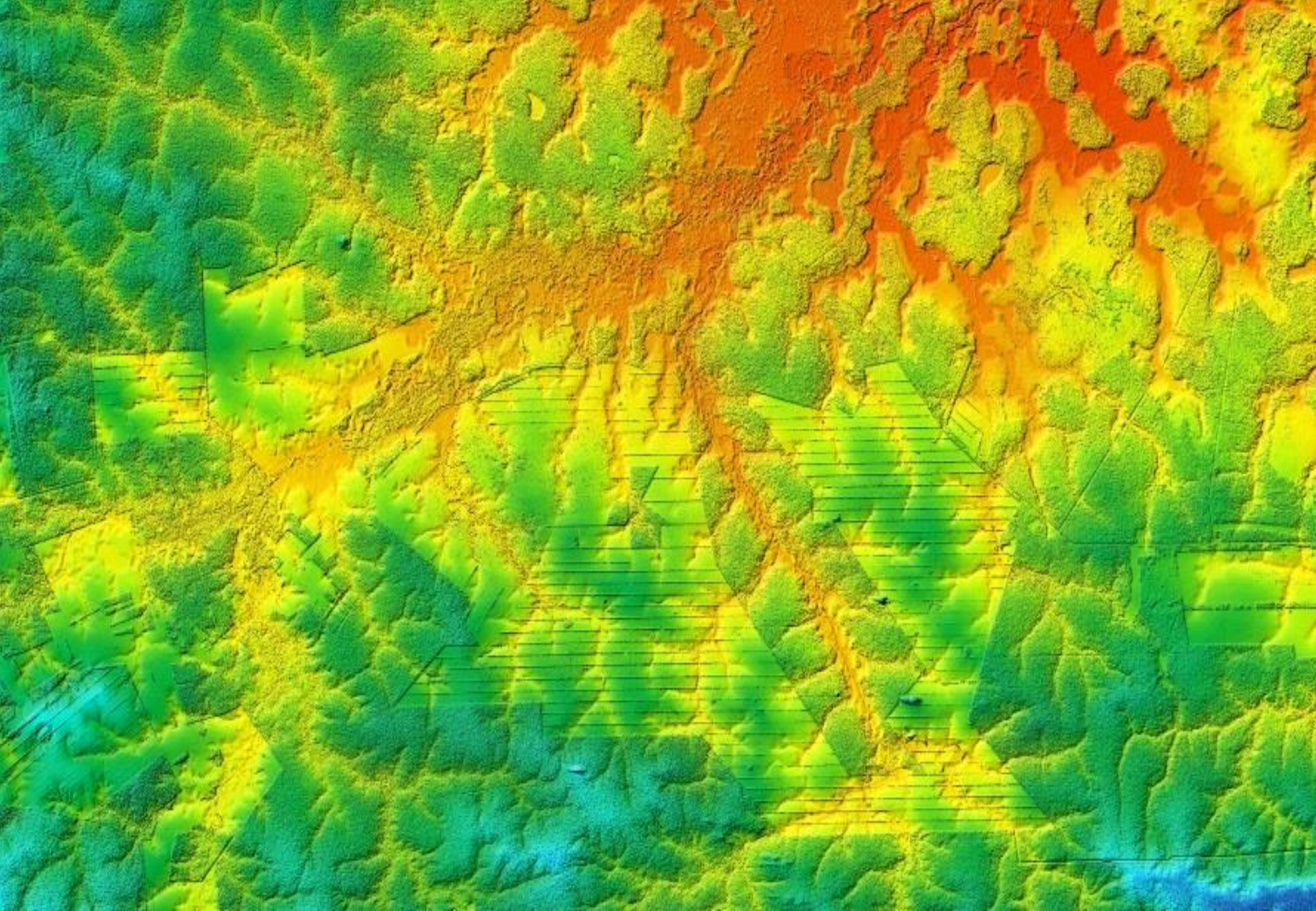
The **phase (center)** of is γ_{Vol} associated to the **center of mass of f(z)**: The phase center height $h_{Int} = \kappa_z\ \varphi_{Int}$ corresponds to the height of the center of mass of f(z) with respect to z_0 !!!



Traunstein forest (Germany) - Capon - HH



DEM, Rain Forest, Bolivia



Polarimetric SAR Interferometry

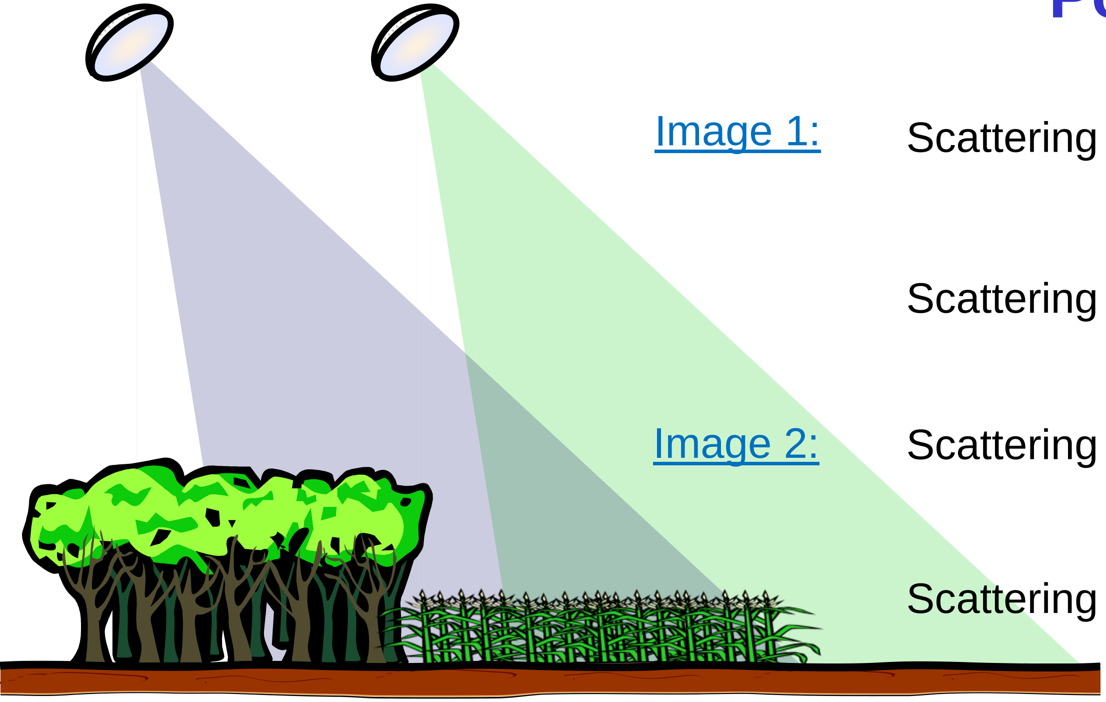


Image 1:

Scattering Matrix: $[S_1] = \begin{bmatrix} S^1_{HH} & S^1_{HV} \\ S^1_{VH} & S^1_{VV} \end{bmatrix}$

Scattering Vector: $\vec{k}_1 = \frac{1}{\sqrt{2}}[S^1_{HH} + S^1_{VV} \quad S^1_{HH} - S^1_{VV} \quad 2S^1_{HV}]^T$

Image 2:

Scattering Matrix: $[S_2] = \begin{bmatrix} S^2_{HH} & S^2_{HV} \\ S^2_{VH} & S^2_{VV} \end{bmatrix}$

Scattering Vector: $\vec{k}_2 = \frac{1}{\sqrt{2}}[S^2_{HH} + S^2_{VV} \quad S^2_{HH} - S^2_{VV} \quad 2S^2_{HV}]^T$

Image formation: $i_1 = \vec{w}_1^+ \cdot \vec{k}_1$ and $i_2 = \vec{w}_2^+ \cdot \vec{k}_2$ … projection of the scattering vector on a (complex) unitary vector $\vec{w}_i$

$\vec{w}_i$ used to select a given polarisation out of all possible polarisations provided by [S]

Example: $S_{HH} + S_{VV}$ image: $\vec{w} = [1 \quad 0 \quad 0]^T \rightarrow i = \vec{w}^+ \cdot \vec{k}_j = \frac{1}{\sqrt{2}}(S^j_{HH} + S^j_{VV})$

S_{HH} image: $\vec{w} = [1/\sqrt{2} \quad 1/\sqrt{2} \quad 0]^T \rightarrow i_j = \vec{w}^+ \cdot \vec{k}_j = S^j_{HH}$

Polarimetric SAR Interferometry

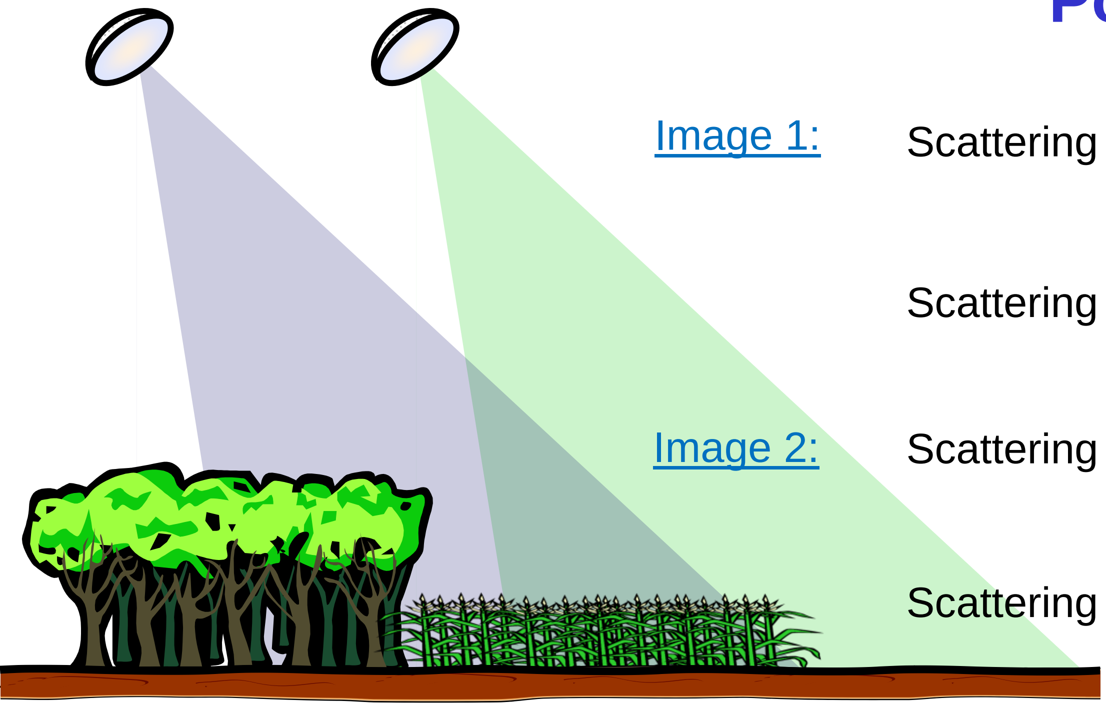


Image 1:

Scattering Matrix: $$[S_1] = \begin{bmatrix} S^1_{HH} & S^1_{HV} \\ S^1_{VH} & S^1_{VV} \end{bmatrix}$$

Scattering Vector: $$\vec{k}_1 = \frac{1}{\sqrt{2}}[S^1_{HH} + S^1_{VV} \quad S^1_{HH} - S^1_{VV} \quad 2S^1_{HV}]^T$$

Image 2:

Scattering Matrix: $$[S_2] = \begin{bmatrix} S^2_{HH} & S^2_{HV} \\ S^2_{VH} & S^2_{VV} \end{bmatrix}$$

Scattering Vector: $$\vec{k}_2 = \frac{1}{\sqrt{2}}[S^2_{HH} + S^2_{VV} \quad S^2_{HH} - S^2_{VV} \quad 2S^2_{HV}]^T$$

Image formation: $i_1 = \vec{w}_1^+ \cdot \vec{k}_1$ and $i_2 = \vec{w}_2^+ \cdot \vec{k}_2$ where $\vec{w}_i$ are complex unitary vectors*

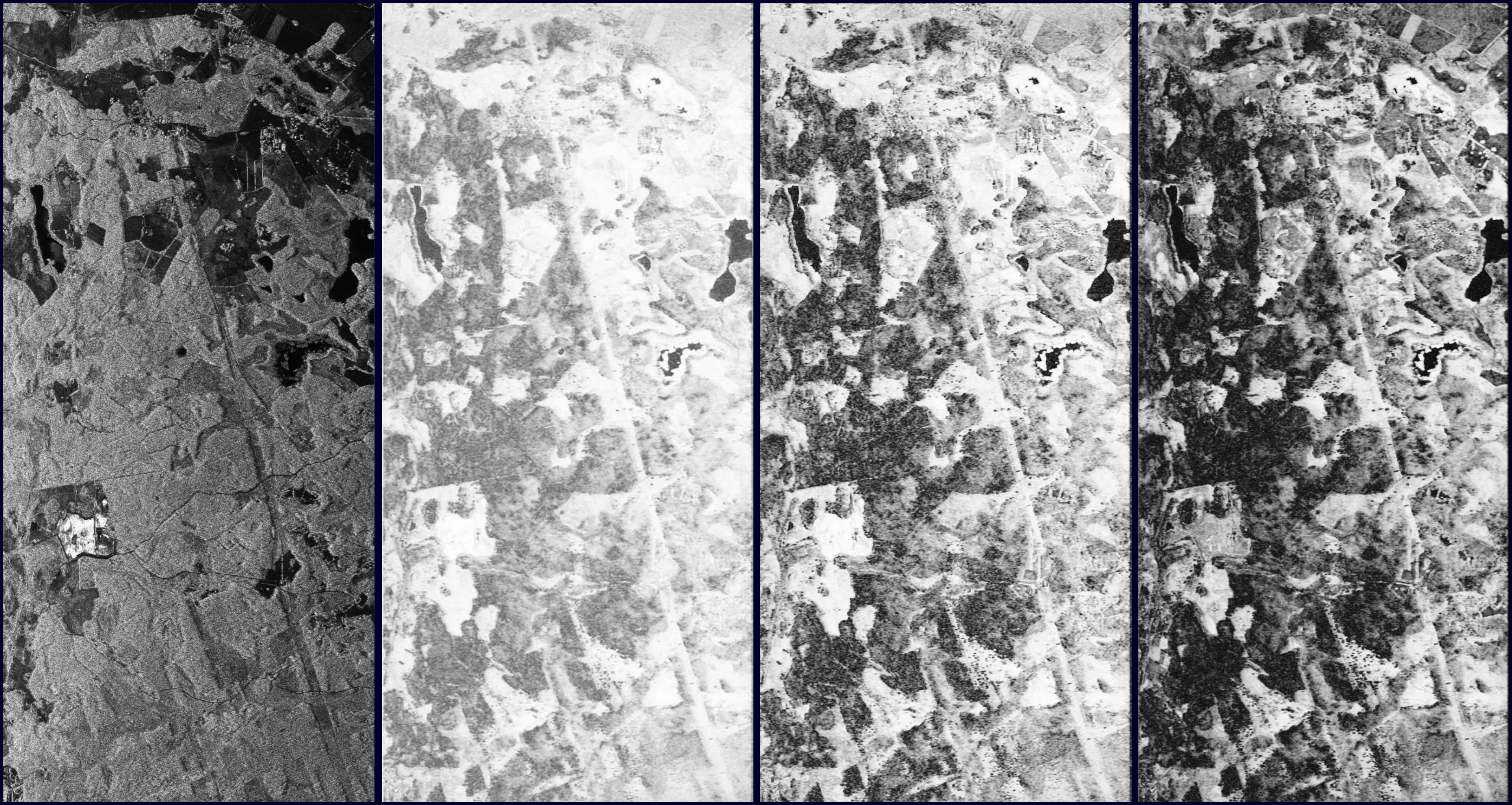
Interferogram formation: $$i_1 \; i_2^* = (\vec{w}_1^+ \cdot \vec{k}_1)(\vec{w}_2^+ \cdot \vec{k}_2)^+ = \vec{w}_1^+(\vec{k}_1 \cdot \vec{k}_2^+)\vec{w}_2 = \vec{w}_1^+[\Omega]\vec{w}_2$$

Interferometric Coherence: $$\tilde{\gamma}(\vec{w}_1, \vec{w}_2) = \frac{< i_1 \; i_2^* >}{\sqrt{< i_1 \; i_1^* > \; < i_2 \; i_2^* >}} = \frac{< \vec{w}_1^+[\Omega]\vec{w}_2 >}{\sqrt{< (\vec{w}_1^+[T_{11}]\vec{w}_1) > \; < (\vec{w}_2^+[T_{22}]\vec{w}_2) >}}$$

where $[T_{11}] = < \vec{\kappa}_1 \cdot \vec{\kappa}_1^+ >$ $[T_{22}] = < \vec{\kappa}_2 \cdot \vec{\kappa}_2^+ >$ and $[\Omega] = < \vec{\kappa}_1 \cdot \vec{\kappa}_2^+ >$

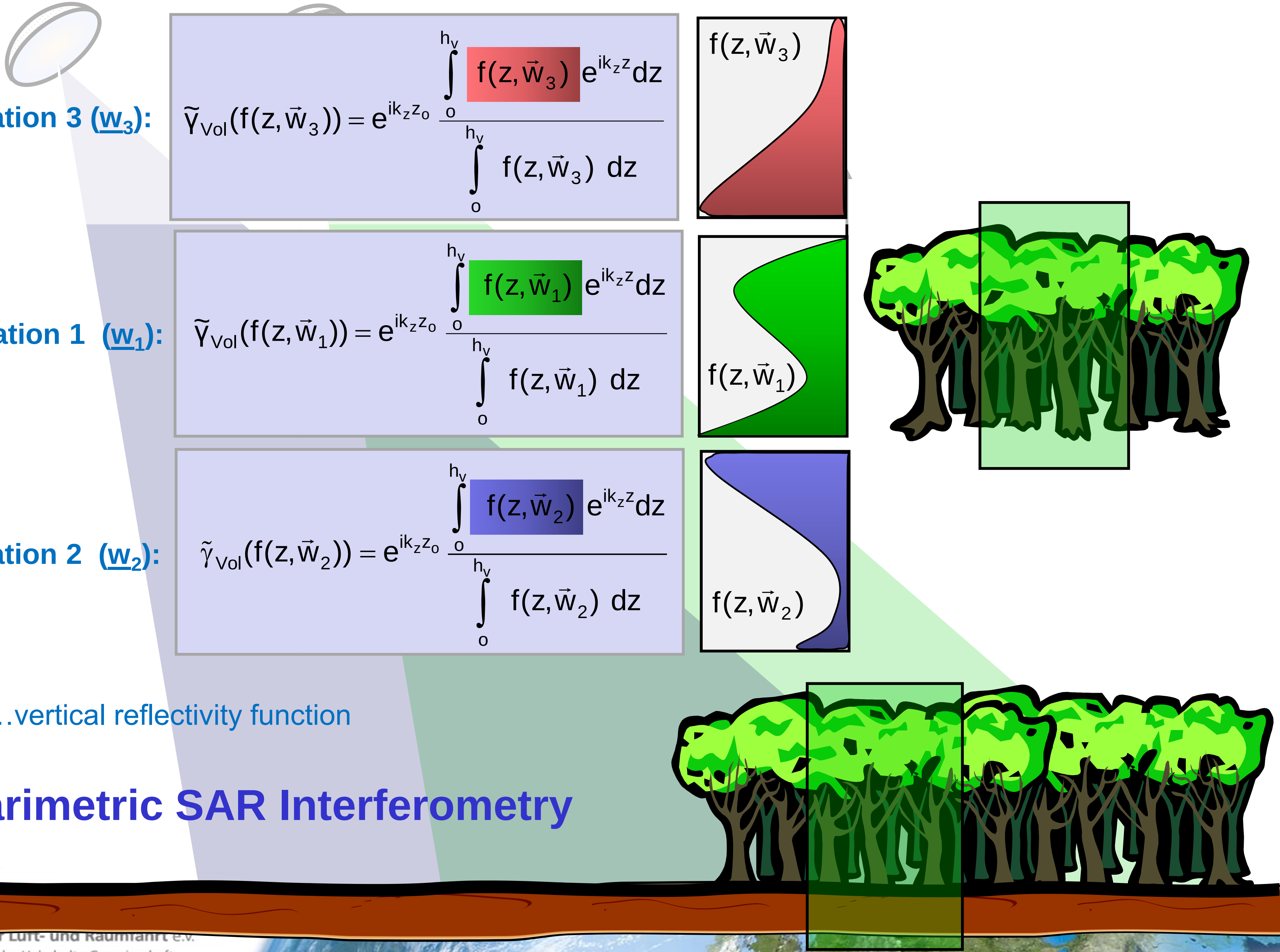
$\vec{w}_i$ used to select a polarisation state out of all possible polarisations provided by the scattering matrix [S]

Interferometric Coherence: Volume Decorrelation



Amplitude Image HH **Sp. Baseline 16m** **Opt 1** **HH** **Opt 3**

Polarimetric SAR Interferometry



Polarisation 3 ($\underline{w}_3$):

$$\tilde{\gamma}_{Vol}(f(z,\vec{w}_3)) = e^{ik_z z_o} \frac{\int_o^{h_v} \boxed{f(z,\vec{w}_3)}\, e^{ik_z z} dz}{\int_o^{h_v} f(z,\vec{w}_3)\; dz}$$

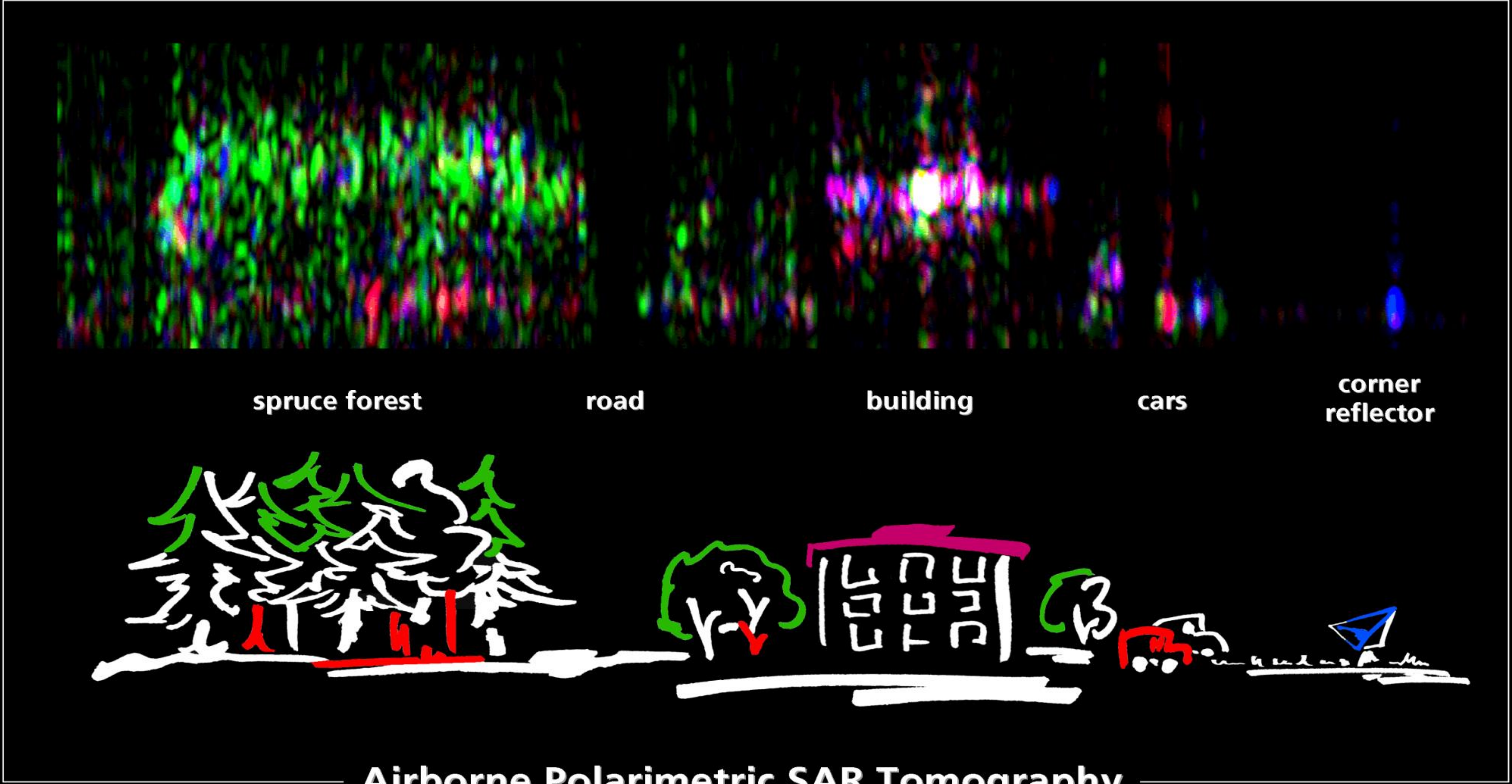
Polarisation 1 ($\underline{w}_1$):

$$\tilde{\gamma}_{Vol}(f(z,\vec{w}_1)) = e^{ik_z z_o} \frac{\int_o^{h_v} \boxed{f(z,\vec{w}_1)}\, e^{ik_z z} dz}{\int_o^{h_v} f(z,\vec{w}_1)\; dz}$$

Polarisation 2 ($\underline{w}_2$):

$$\tilde{\gamma}_{Vol}(f(z,\vec{w}_2)) = e^{ik_z z_o} \frac{\int_o^{h_v} \boxed{f(z,\vec{w}_2)}\, e^{ik_z z} dz}{\int_o^{h_v} f(z,\vec{w}_2)\; dz}$$

$f(z,\vec{w})$ …vertical reflectivity function



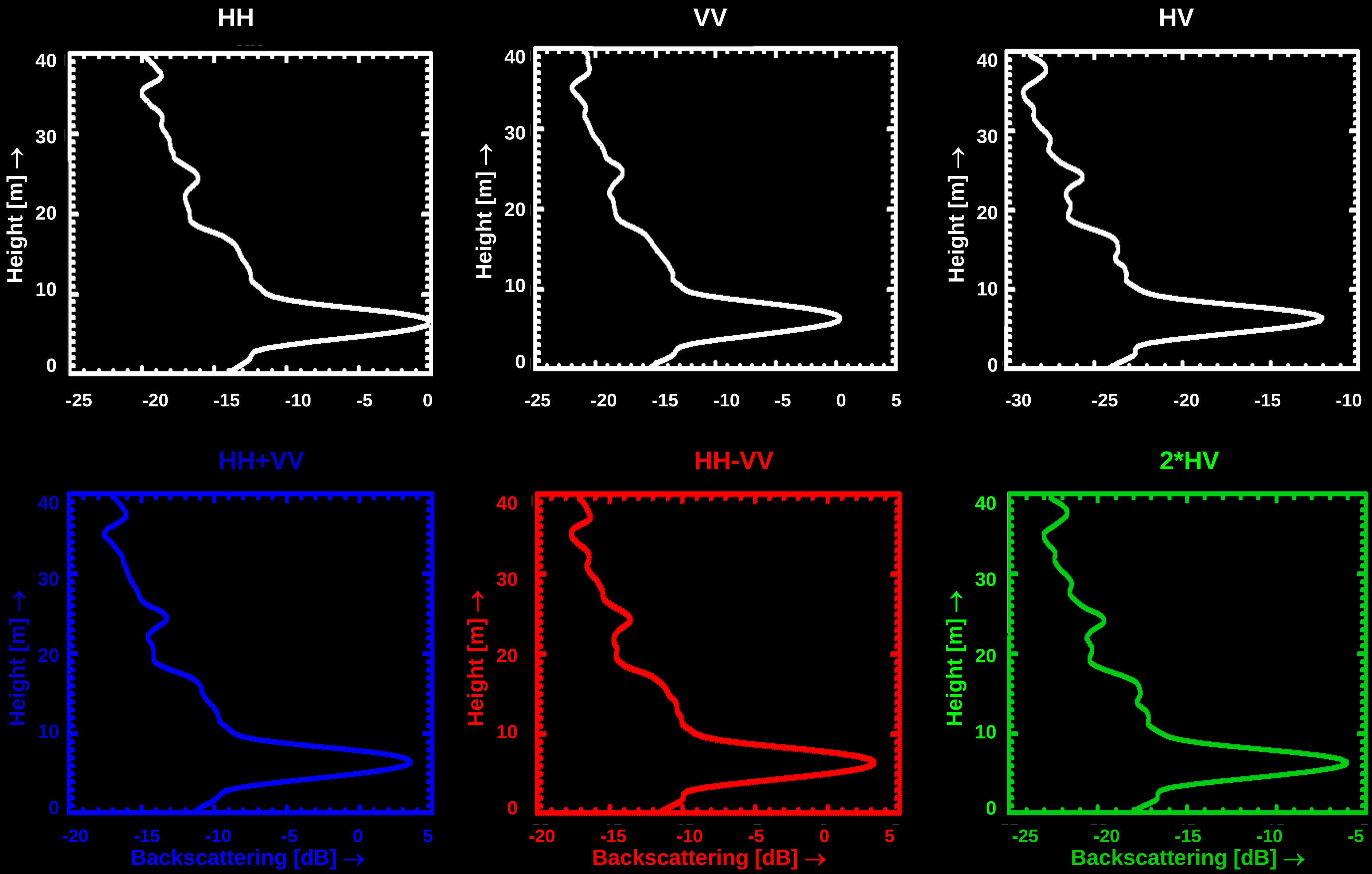
Airborne Polarimetric SAR Tomography

Upper image: Polarimetric color composite (L-band) of a tomographic slice in the height/azimuth-direction

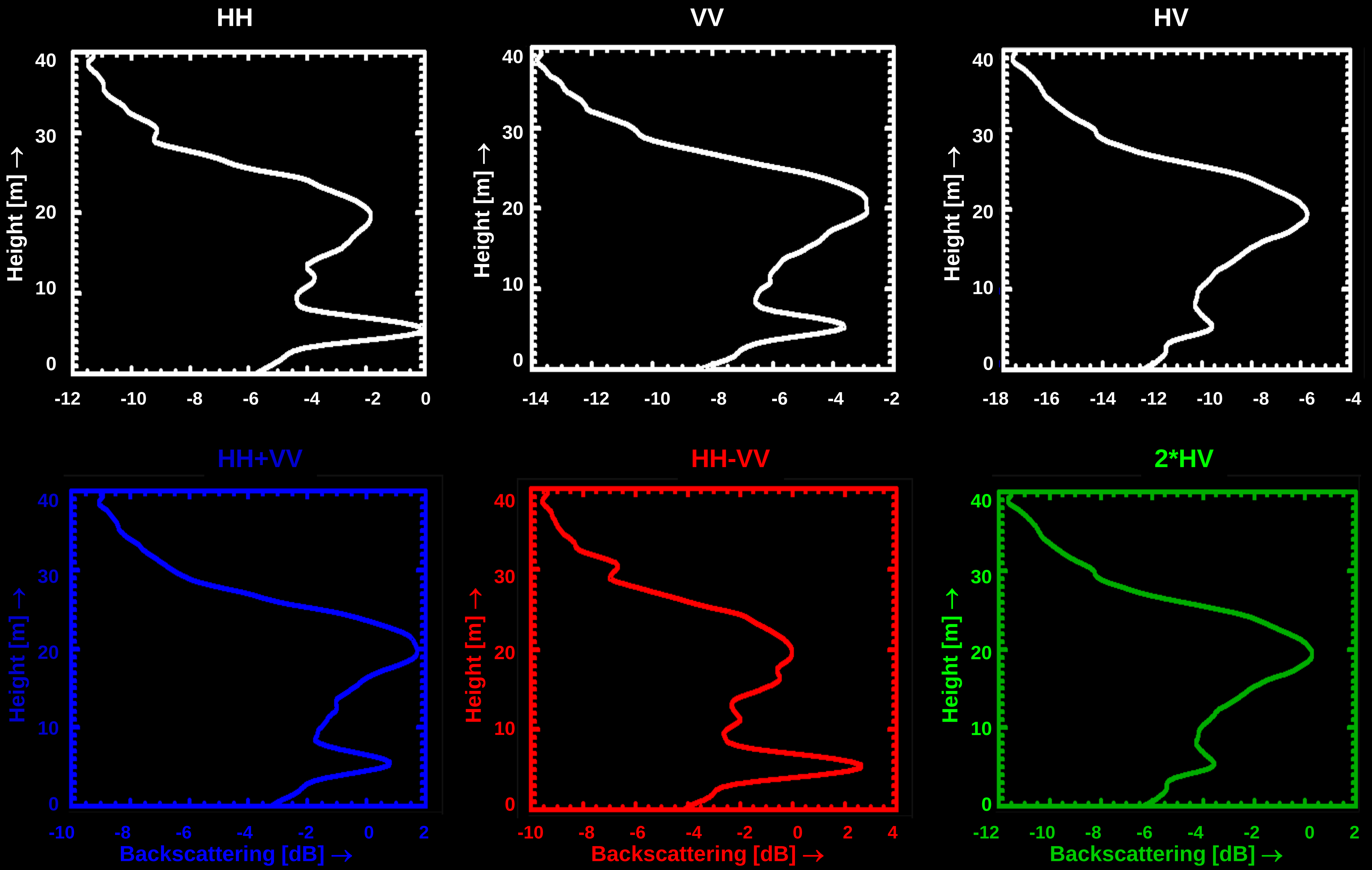
🟦 HH+VV, 🟥 HH-VV, 🟩 2*HV

Lower image: Schematic view of the imaged area

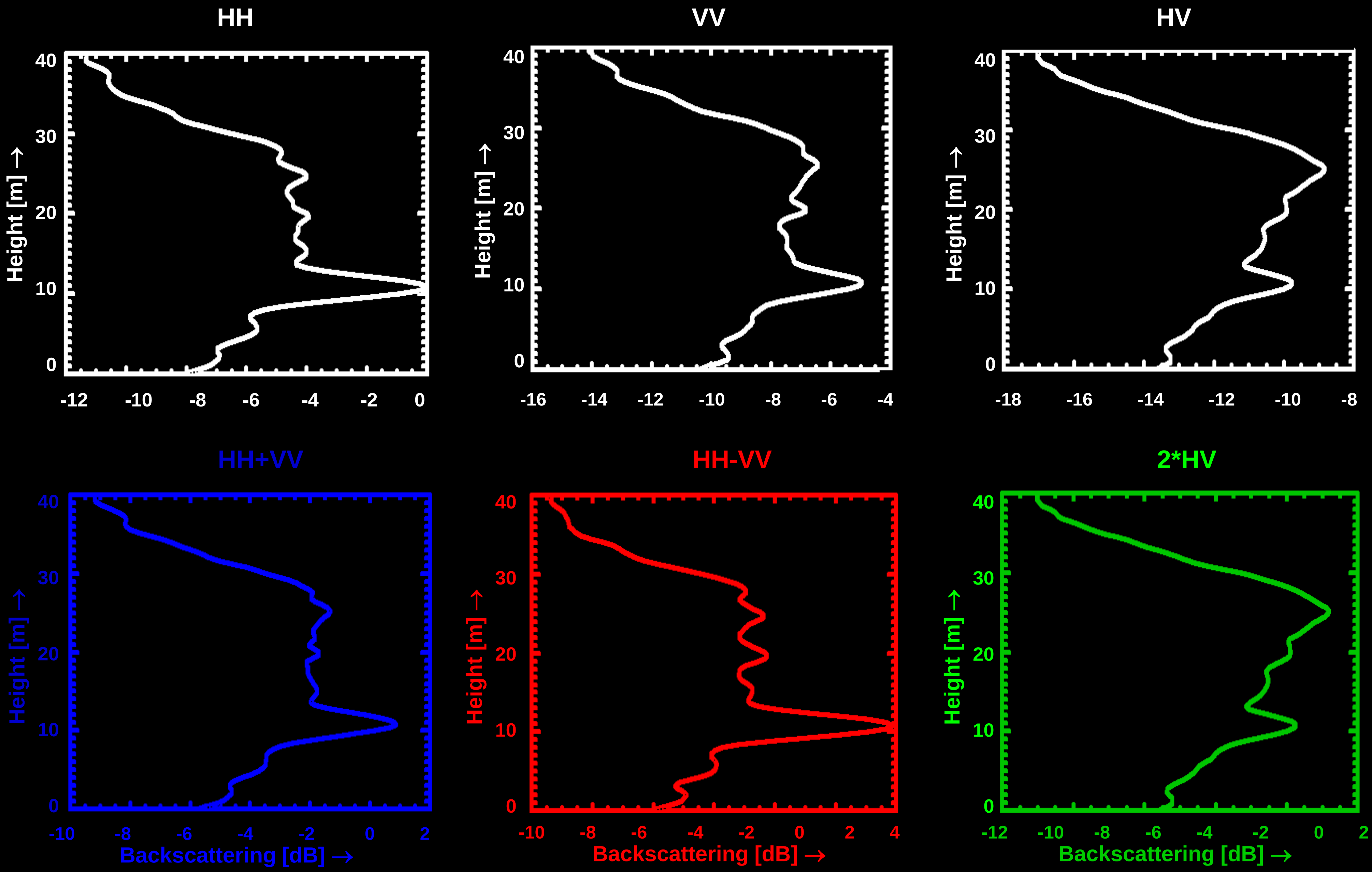
Reigber, A. et al: “First Demonstration of Airborne SAR Tomography using Multibaseline L-Band Data”. IEEE Trans. on Geosc. and Remote Sensing, Vol. 38 (5), Sept. 2000



Bare Surface Backscattering Profiles (12-20 m height)

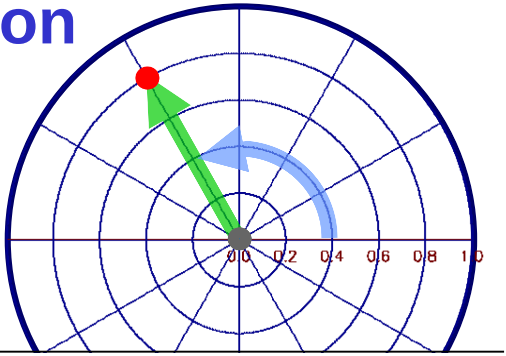


Spruce Forest Backscattering Profiles (15-20 m height)



Mixed Forest Backscattering Profiles (12-20 m height)

Geometrical Representation: The Coherence Region



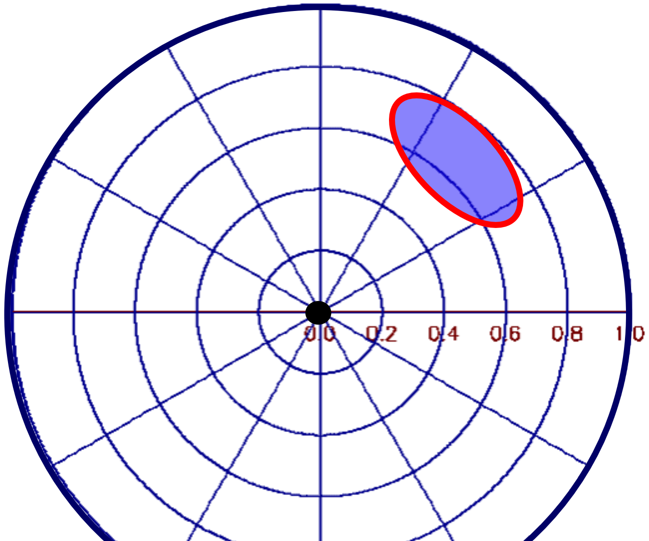
Interferometric Coherence: $\tilde{\gamma}(\vec{w}_i,\vec{w}_i) = \underbrace{|\tilde{\gamma}(\vec{w}_i,\vec{w}_i)|}_{\textbf{Radius}} \cdot \exp(i\, \underbrace{\mathrm{Arg}\{\,\tilde{\gamma}(\vec{w}_i,\vec{w}_i)\,\}}_{\textbf{Angle}})$

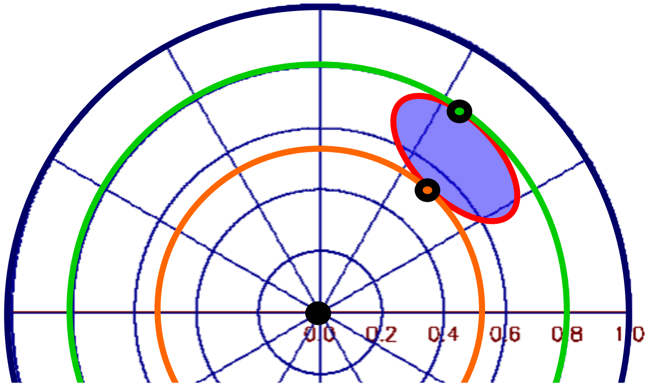
► can be represented by a single point on the unit circle (UC)

Coherence Region: $\tilde{\gamma}(\vec{w}_i,\vec{w}_i) \quad \forall \quad \vec{w}_i = \begin{bmatrix} \cos\alpha\exp(i\varphi_1) \\ \sin\alpha\cos\beta\exp(i\varphi_2) \\ \sin\alpha\sin\beta\exp(i\varphi_3) \end{bmatrix}$ with $0 \le \alpha \le \frac{\pi}{2}$ and $-\pi \le \beta \le \pi$

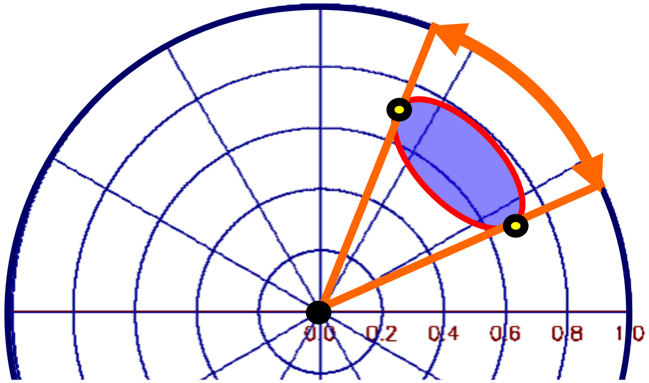
The set of interferometric coherences obtained for all the possible polarizations $\vec{w}_i$ plotted on the unit circle (UC) defines the so-called **coherence region.**

Its shape & size depend on the structure of the underlying scatterer and on acquisition parameters.



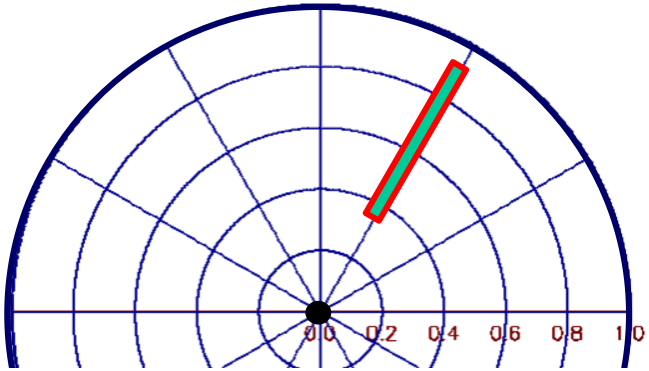


Max./ Min. Interferometric Coherence as function of $\vec{w}_i$



Max. Phase Difference between interferograms formed with $\vec{w}_i$ and $\vec{w}_j$

Coherence Region Interpretation

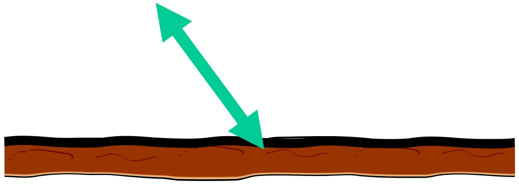


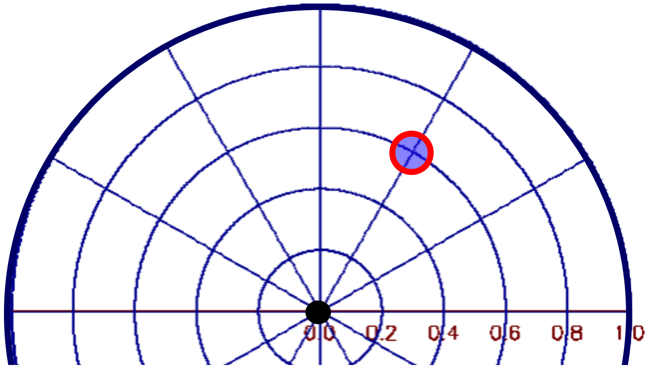
Radial Shaped CR

i.e. InSAR coherence amplitude changes with polarisation but not the location of the phase center.

Surface Scattering

$$\tilde{\gamma}(\vec{w}) = \gamma_{SNR}(\vec{w})\ \tilde{\gamma}_{Vol} \overset{\tilde{\gamma}_{Vol}:=1}{=} \gamma_{SNR}(\vec{w})$$



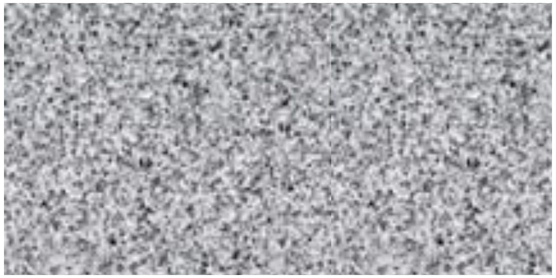


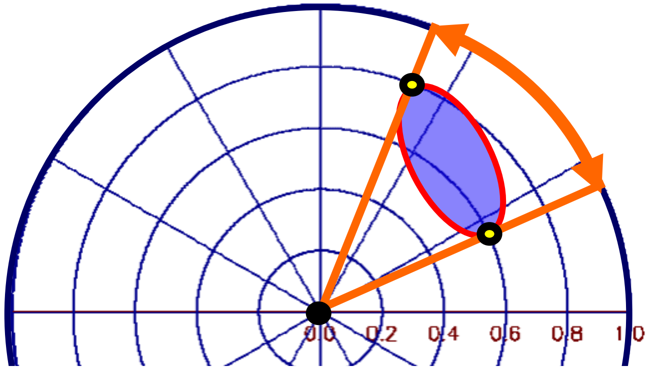
Point Like Coherence Region

i.e. InSAR Coherence and Phase are independent of polarisation.

Pol-InSAR does not provide any additional information compated to InSAR !!!

(Random) Volume scattering

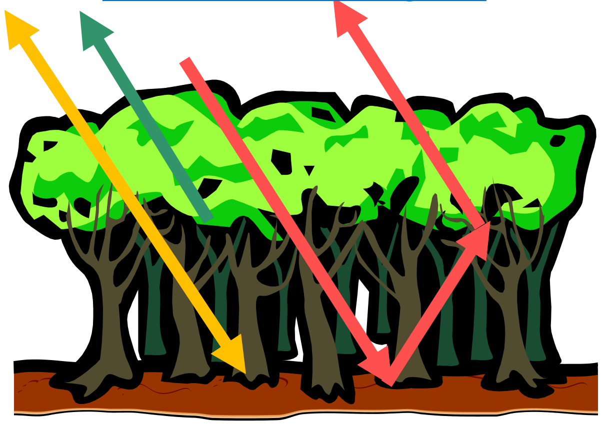




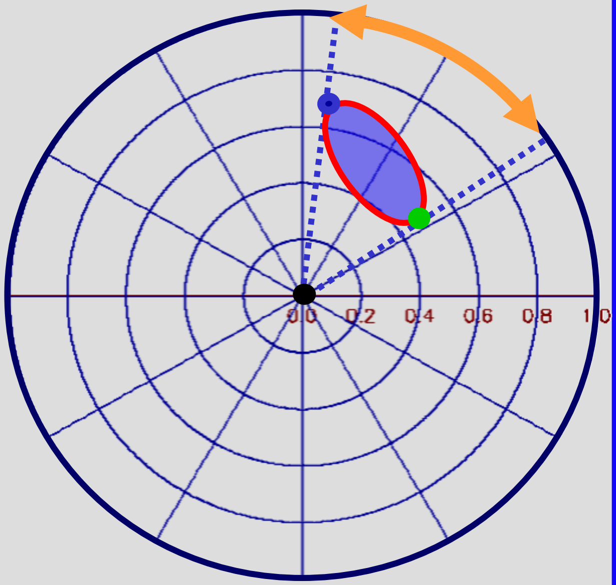
Elliptical Shaped CR

i.e. InSAR coherence magnitude and phase center location changes with polarisation.

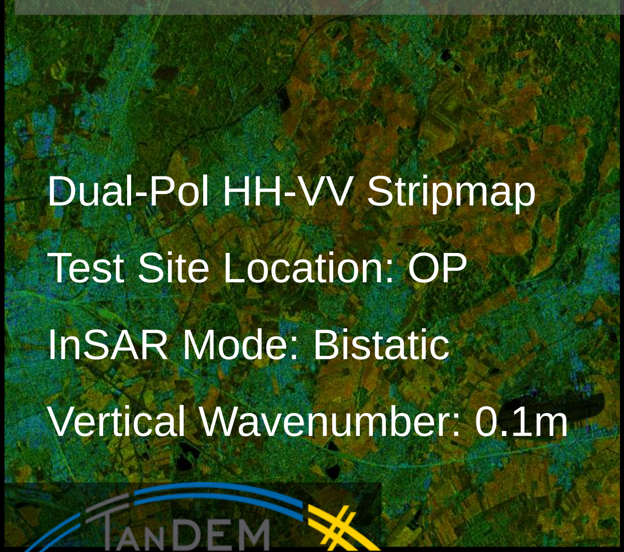
(Depolarising) Scaterrers at different heights



First Bistatic Dual Pol-InSAR Data Takes



Max. Phase Difference between Polarisations

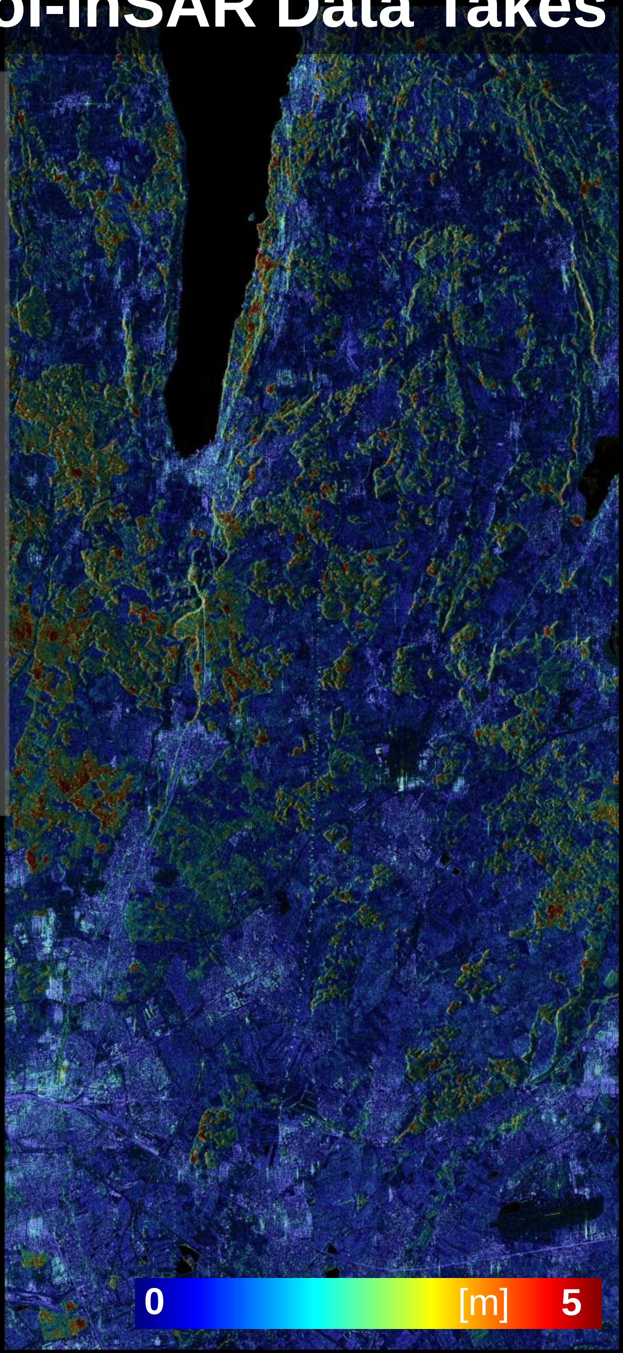


Dual-Pol HH-VV Stripmap

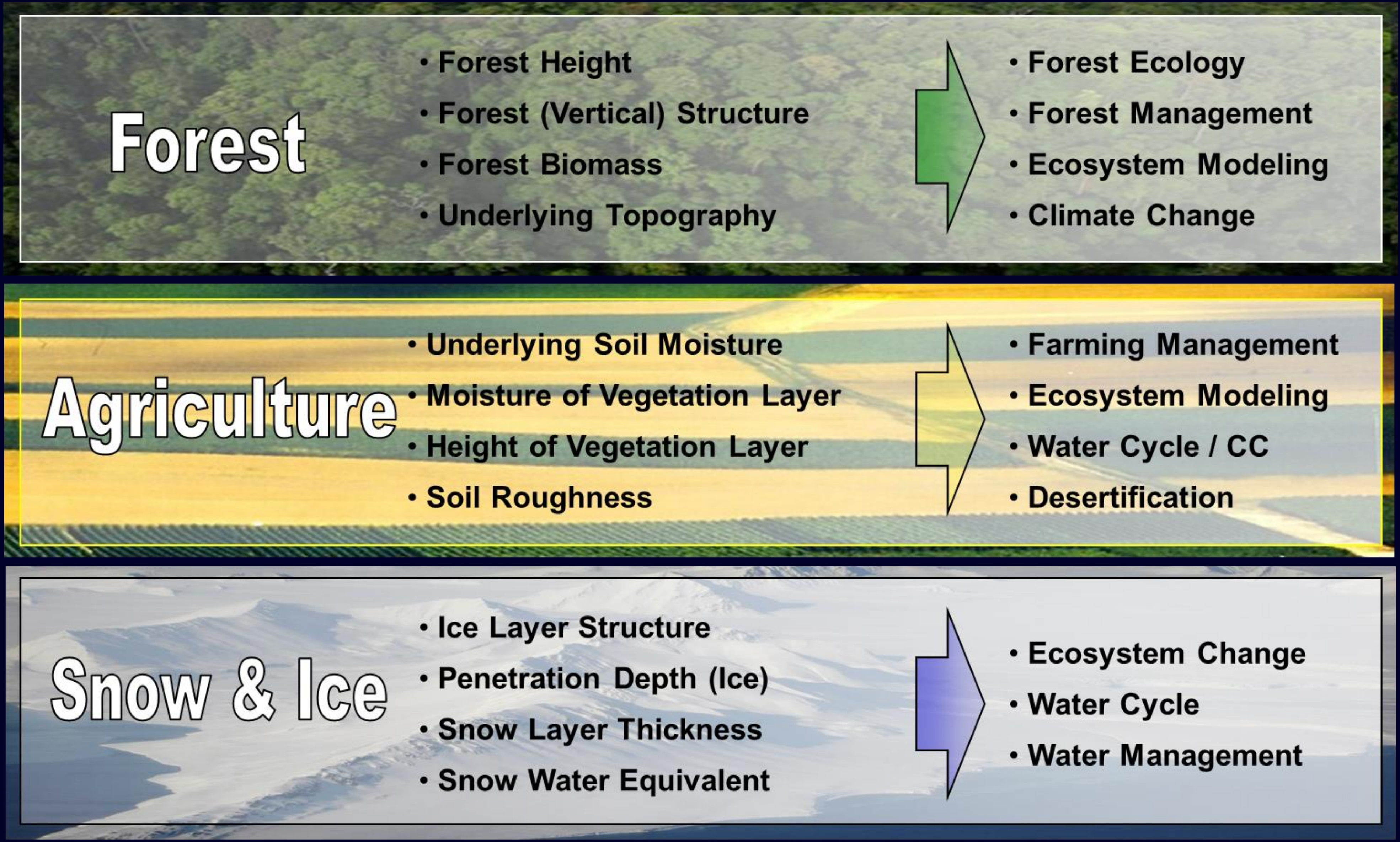
Test Site Location: OP

InSAR Mode: Bistatic

Vertical Wavenumber: 0.1m



Structure Parameters & Applications



Forest

- Forest Height
- Forest (Vertical) Structure
- Forest Biomass
- Underlying Topography

- Forest Ecology
- Forest Management
- Ecosystem Modeling
- Climate Change

Agriculture

- Underlying Soil Moisture
- Moisture of Vegetation Layer
- Height of Vegetation Layer
- Soil Roughness

- Farming Management
- Ecosystem Modeling
- Water Cycle / CC
- Desertification

Snow & Ice

- Ice Layer Structure
- Penetration Depth (Ice)
- Snow Layer Thickness
- Snow Water Equivalent

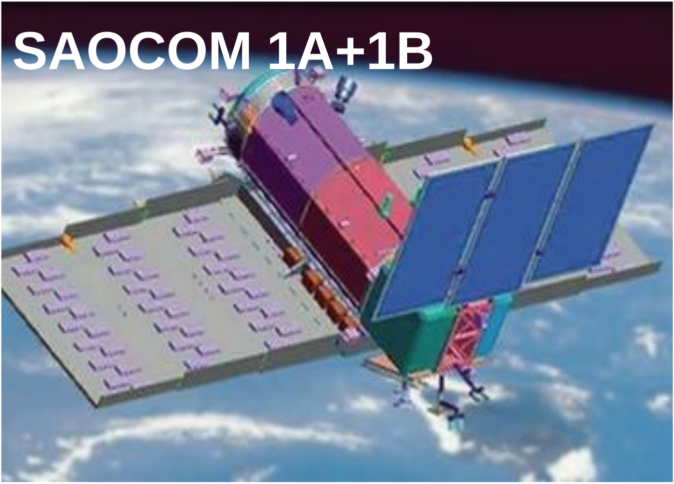
- Ecosystem Change
- Water Cycle
- Water Management

Pol-InSAR In Orbit

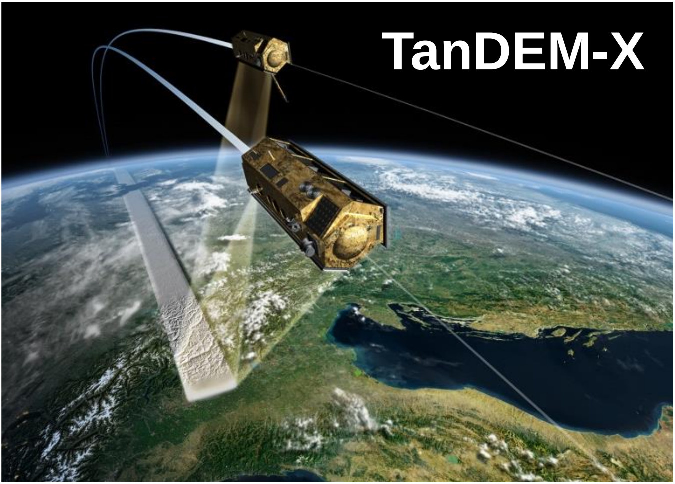
ALOS-2 (4)

RadarSAT 2

Sentinel 1a+1b (1c + 1d)



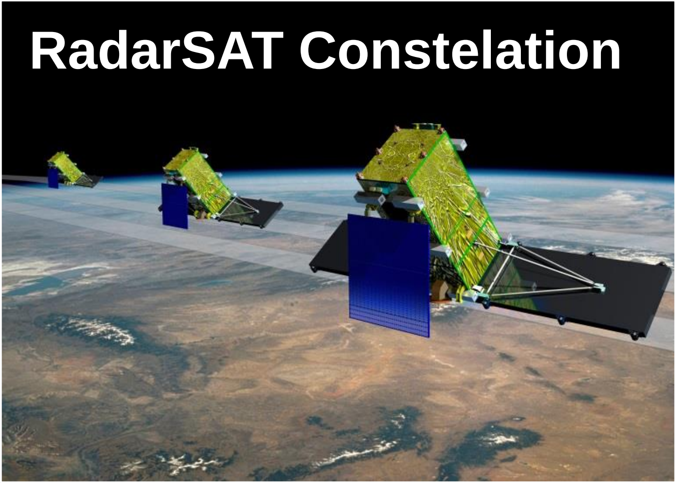
SAOCOM 1A+1B



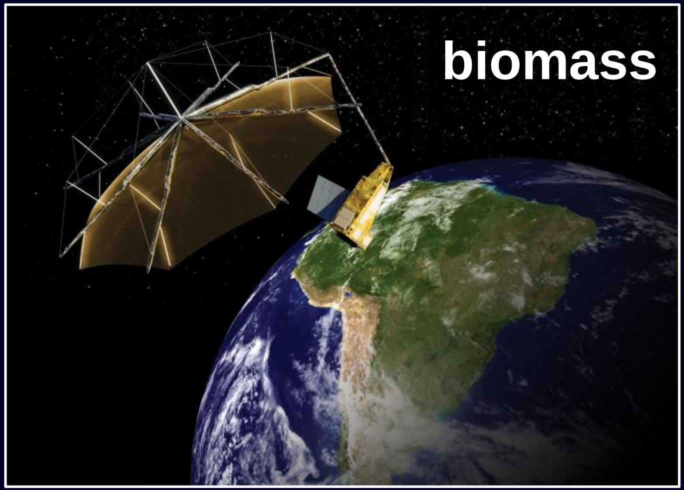
TanDEM-X

RISAT-1

Pol-InSAR In Orbit

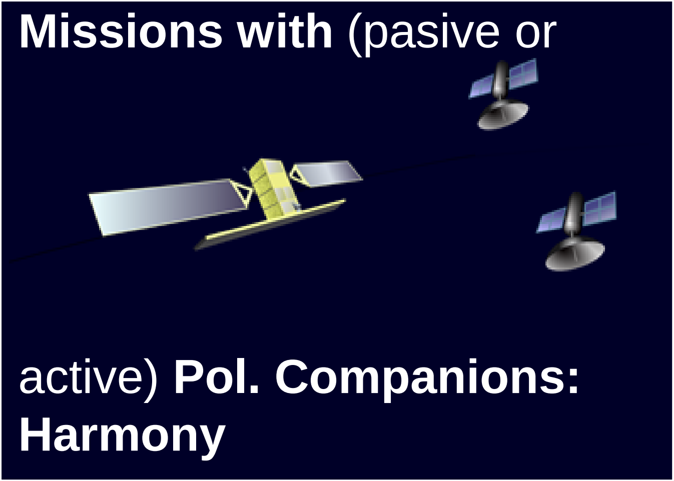


RadarSAT Constelation



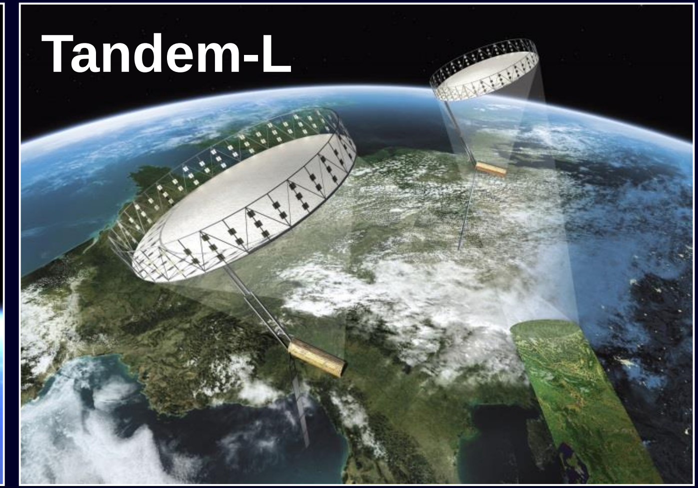
biomass

NISAR

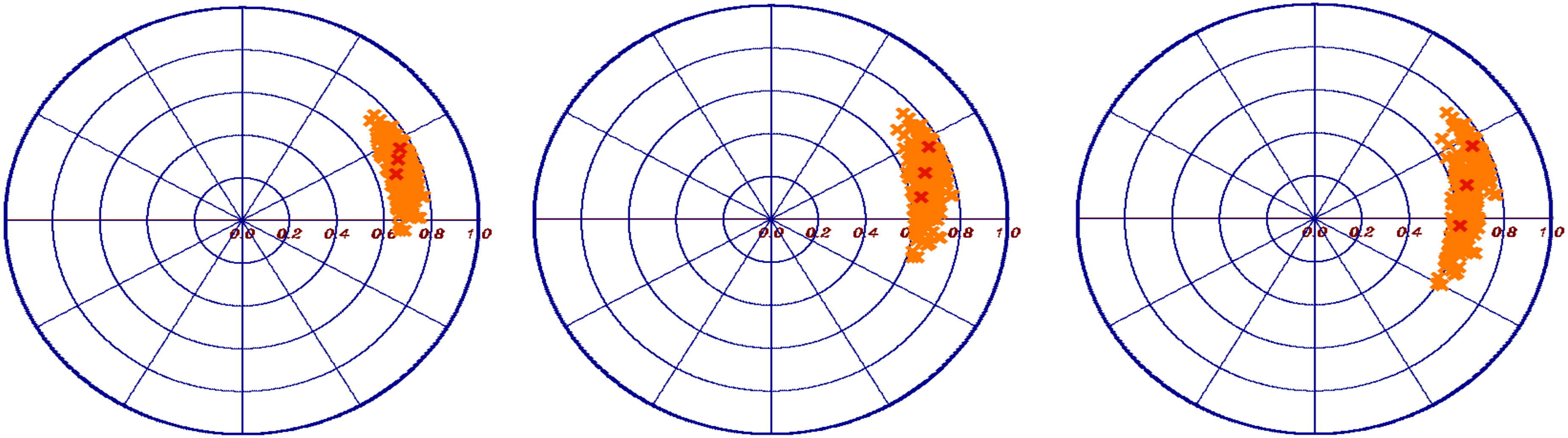


Missions with (pasive or active) **Pol. Companions: Harmony**

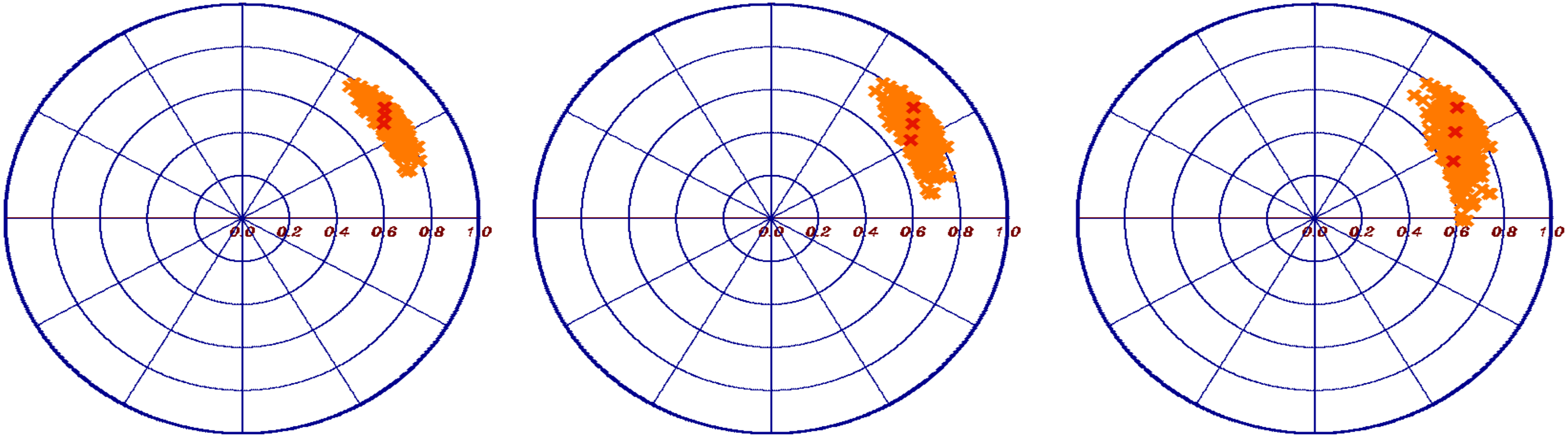
Rose-L (+)



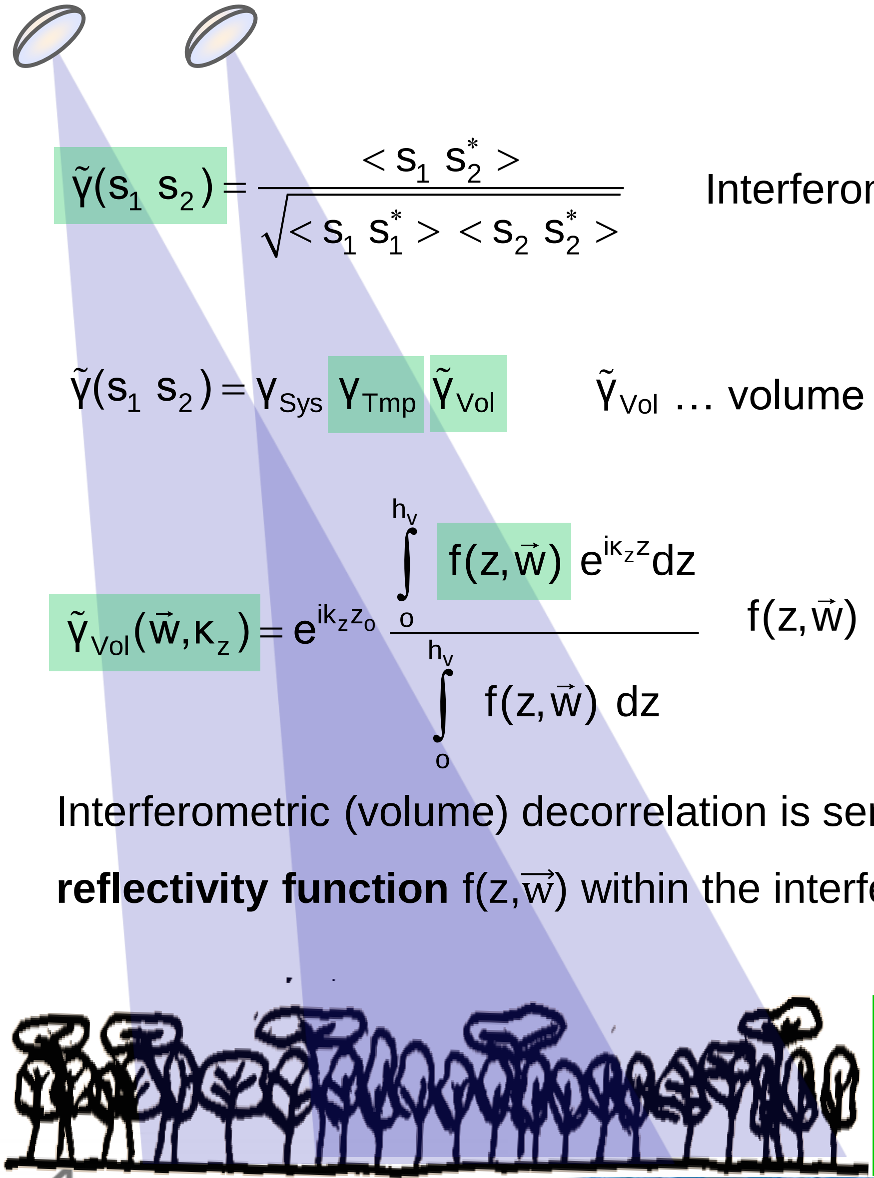
Tandem-L



Pol-InSAR: Modelling and Inversion



Forest Height inversion from InSAR Data



$$\tilde{\gamma}(s_1\ s_2) = \frac{< s_1\ s_2^* >}{\sqrt{< s_1\ s_1^* > < s_2\ s_2^* >}}$$

Interferometric Coherence (complex)

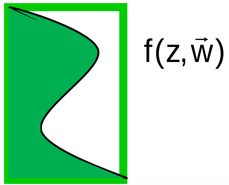
$$\tilde{\gamma}(s_1\ s_2) = \gamma_{Sys}\ \gamma_{Tmp}\ \tilde{\gamma}_{Vol}$$

$\tilde{\gamma}_{Vol}$ … volume decorrelation $\tilde{\gamma}_{Tmp}$ … temporal decorrelation

$$\tilde{\gamma}_{Vol}(\vec{w},\kappa_z) = e^{ik_z z_o} \frac{\int_o^{h_v} f(z,\vec{w})\ e^{i\kappa_z z} dz}{\int_o^{h_v} f(z,\vec{w})\ dz}$$

$f(z,\vec{w})$ … vertical reflectivity function κ_z … vertical wavenumber

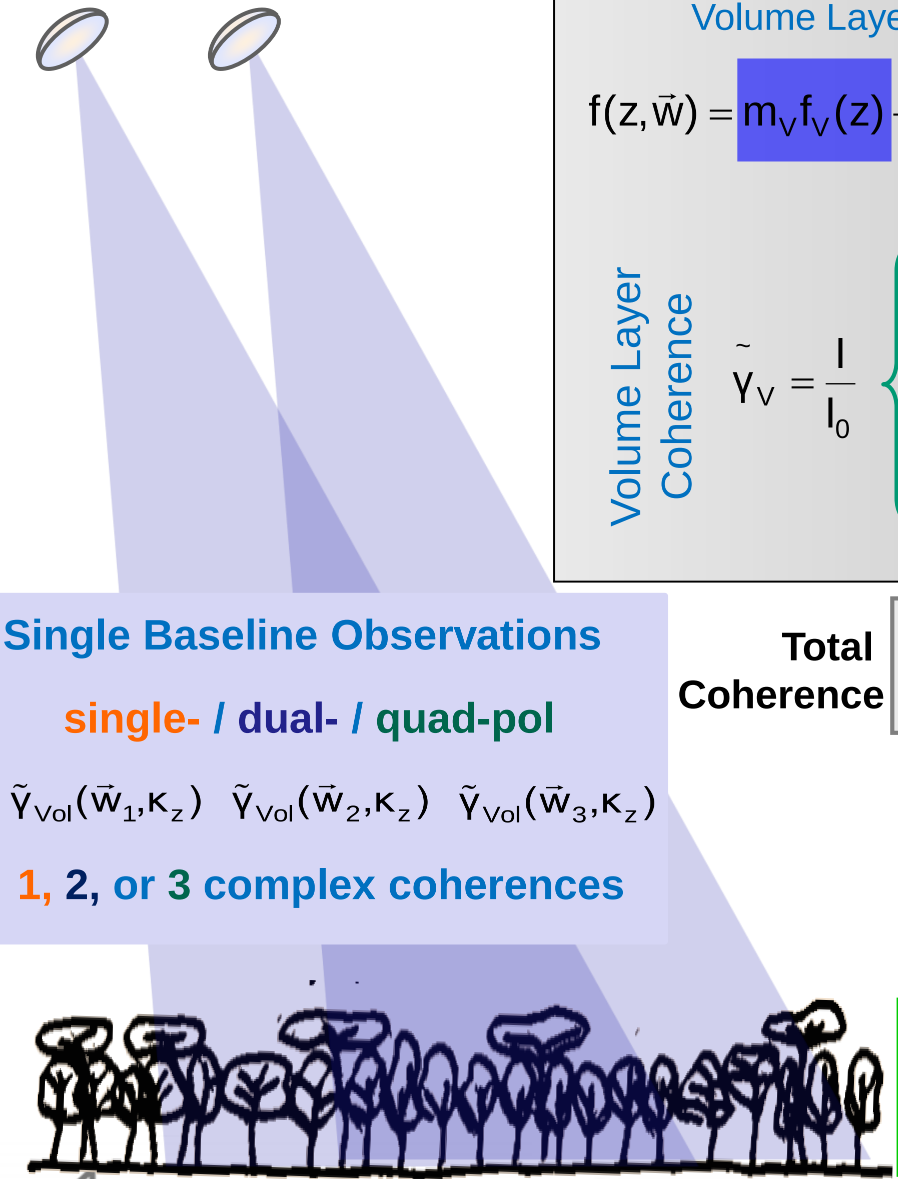
Interferometric (volume) decorrelation is sensitive to the „visible“ (forest) **height** and to the **vertical reflectivity function** $f(z,\vec{w})$ within the interferometric resolution cell.



Forest Height inversion challenges:

- The parameterisation / description of $f(z,\vec{w})$
- The presence of $\tilde{\gamma}_{Tmp}$

2 Layer Inversion Model: (Random) Volume over Ground (RVoG)



Volume Layer Ground Layer

$$f(z,\vec{w}) = m_V f_V(z) + m_G(\vec{w})\delta(z-z_0)$$

$$\tilde{\gamma}_{Vol}(\vec{w},\kappa_z) = \exp(i\varphi_0)\frac{\tilde{\gamma}_V(\kappa_z)+m(\vec{w})}{1+m(\vec{w})}$$

Volume Layer Coherence

$$\tilde{\gamma}_V = \frac{I}{I_0} \quad \begin{cases} I = \int_0^{h_V} \exp(i\kappa_z z')f_V(z')dz' \\ I_0 = \int_0^{h_V} f_V(z')dz' \end{cases}$$

$$m(\vec{w}) = \frac{m_G(\vec{w})}{m_V(\vec{w})I_0} \qquad \kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$$

$f_V(z)$ … volume reflectivity function

$\varphi_0 = k_z z_0$ … underlying topography

Total Coherence

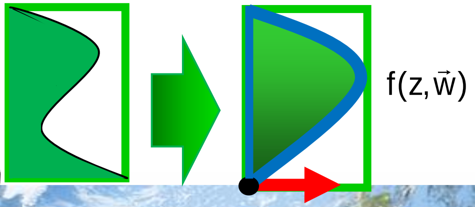
$$\tilde{\gamma}(\vec{w},\kappa_z) = \gamma_{Temp}(\kappa_z)\ \tilde{\gamma}_{Vol}(\vec{w},\kappa_z)$$

Single Baseline Observations

single- / dual- / quad-pol

$\tilde{\gamma}_{Vol}(\vec{w}_1,\kappa_z)$ $\tilde{\gamma}_{Vol}(\vec{w}_2,\kappa_z)$ $\tilde{\gamma}_{Vol}(\vec{w}_3,\kappa_z)$

1, 2, or 3 complex coherences



For a Single Baseline

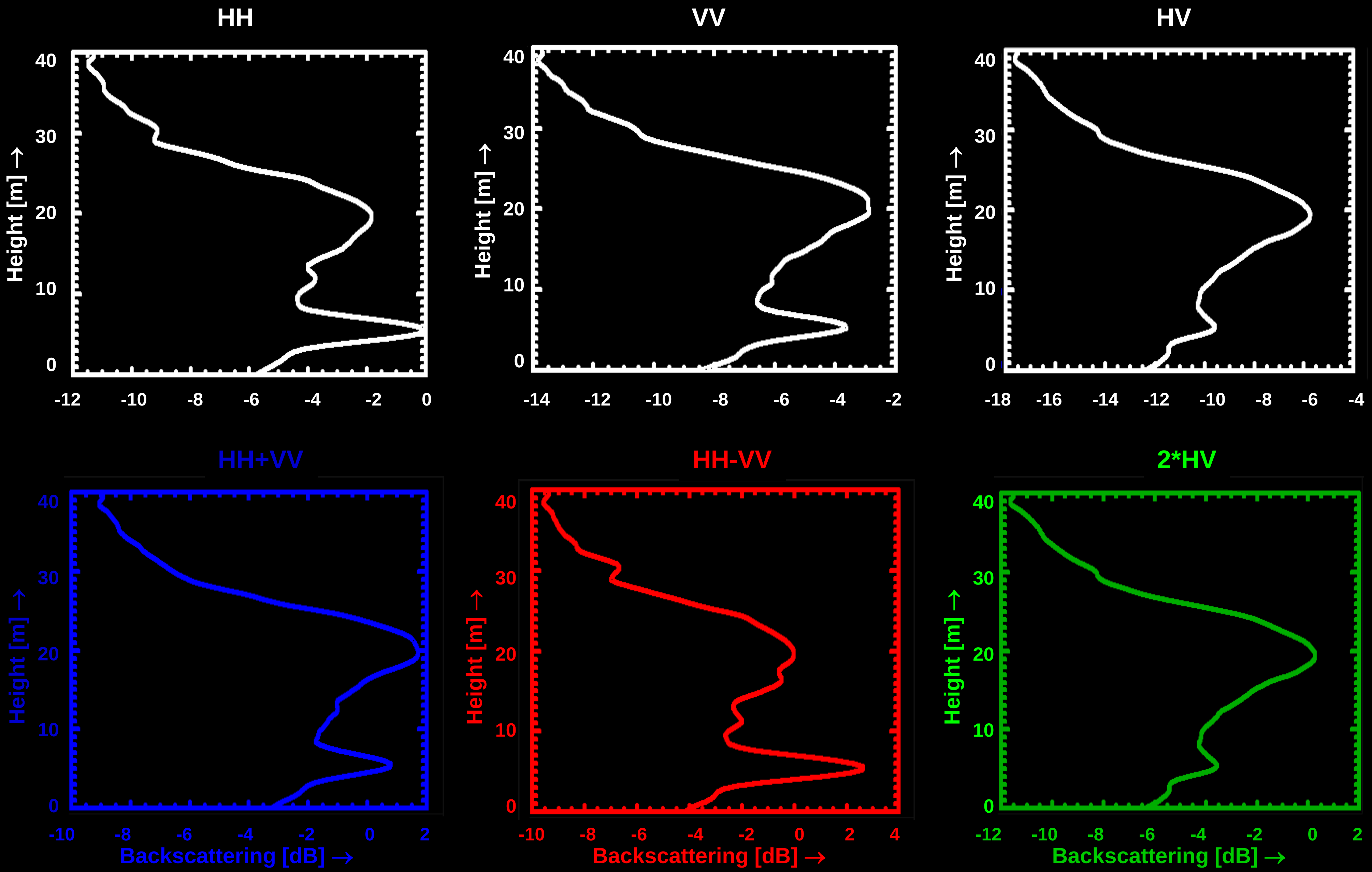
3+N unknown parameters

Volume Height	h_V
Topography	φ_0
G/V Ratio	$m(\vec{w}) = f(pol)$
$f_V(z)$	… N parameters

plus one more

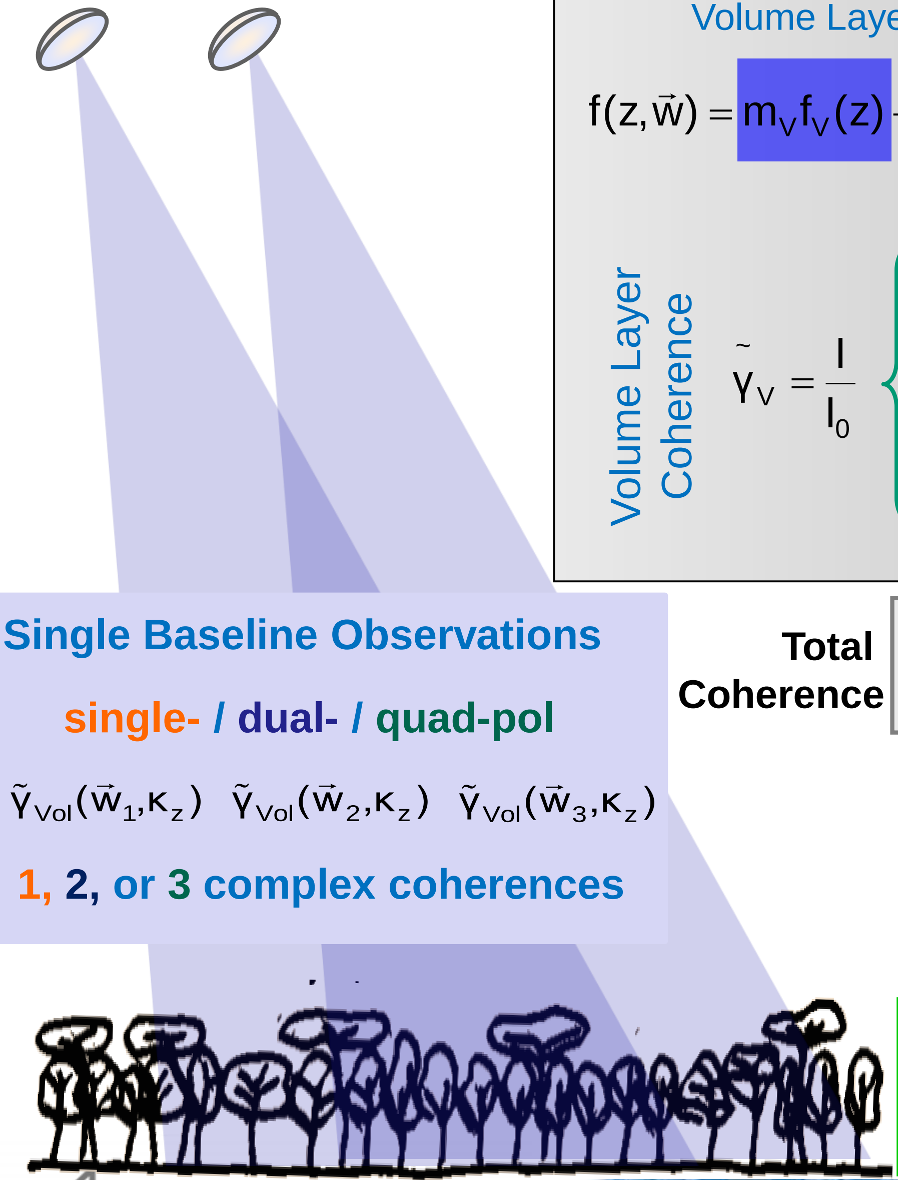
Temporal Deco	γ_{Temp}

for repeat-pass implementations



Spruce Forest Backscattering Profiles (15-20 m height)

2 Layer Inversion Model: (Random) Volume over Ground (RVoG)



Volume Layer Ground Layer

$$f(z,\vec{w}) = m_V f_V(z) + m_G(\vec{w})\delta(z - z_0)$$

$$\tilde{\gamma}_{Vol}(\vec{w},\kappa_z) = \exp(i\varphi_0)\frac{\tilde{\gamma}_V(\kappa_z) + m(\vec{w})}{1 + m(\vec{w})}$$

Volume Layer Coherence

$$\tilde{\gamma}_V = \frac{I}{I_0} \quad \begin{cases} I = \int_0^{h_V} \exp(i\kappa_z z') f_V(z') dz' \\ I_0 = \int_0^{h_V} f_V(z') dz' \end{cases}$$

$$m(\vec{w}) = \frac{m_G(\vec{w})}{m_V(\vec{w}) I_0} \qquad \kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$$

$f_V(z)$ … volume reflectivity function

$\varphi_0 = k_z z_0$ … underlying topography

Total Coherence

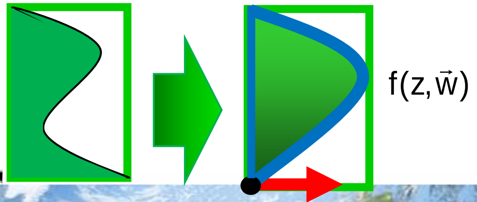
$$\tilde{\gamma}(\vec{w},\kappa_z) = \gamma_{Temp}(\kappa_z)\ \tilde{\gamma}_{Vol}(\vec{w},\kappa_z)$$

Single Baseline Observations

single- / dual- / quad-pol

$\tilde{\gamma}_{Vol}(\vec{w}_1,\kappa_z)$ $\tilde{\gamma}_{Vol}(\vec{w}_2,\kappa_z)$ $\tilde{\gamma}_{Vol}(\vec{w}_3,\kappa_z)$

1, 2, or 3 complex coherences



For a Single Baseline

3+N unknown parameters

Volume Height	h_V
Topography	φ_0
G/V Ratio	$m(\vec{w}) = f(pol)$
$f_V(z)$	… N parameters

plus one more

Temporal Deco	γ_{Temp}

for repeat-pass implementations

2 Layer Inversion Model with exponential volume reflectivity

Volume Layer Ground Layer

$$f(z,\vec{w}) = m_V f_V(z) + m_G(\vec{w})\delta(z-z_0)$$

$$\tilde{\gamma}_{Vol}(\vec{w},\kappa_z) = \exp(i\varphi_0)\frac{\tilde{\gamma}_V(\kappa_z)+m(\vec{w})}{1+m(\vec{w})}$$

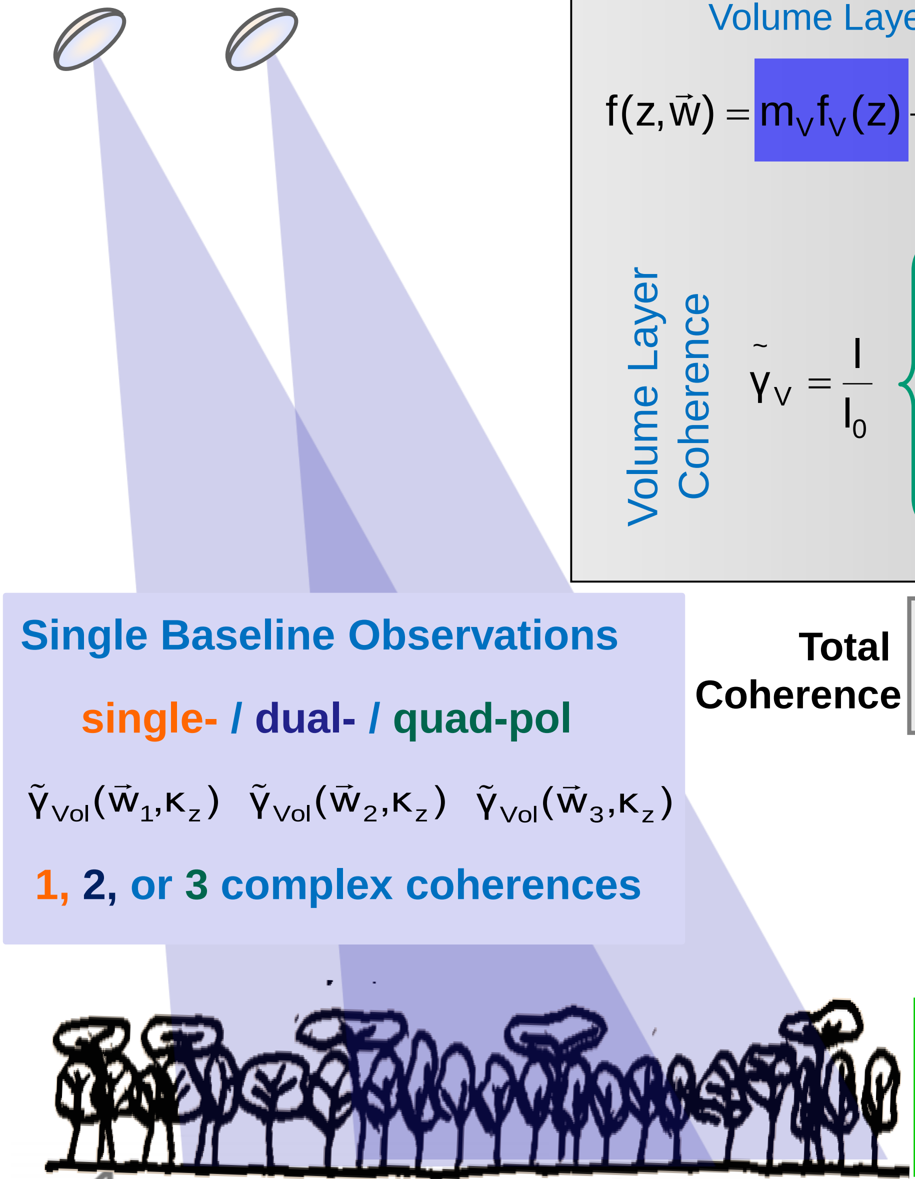
Volume Layer Coherence

$$\tilde{\gamma}_V = \frac{I}{I_0} \quad \begin{cases} I = \int_0^{h_V} \exp(i\kappa_z z') e^{\left(\frac{2\ \sigma\ z'}{\cos\theta_0}\right)} dz' \\ I_0 = \int_0^{h_V} e^{\left(\frac{2\ \sigma\ z'}{\cos\theta_0}\right)} dz' \end{cases}$$

$$m(\vec{w}) = \frac{m_G(\vec{w})}{m_V(\vec{w}) I_0} \qquad \kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$$

$$f_V(z) = \exp\left(\frac{2\ \sigma\ z}{\cos\theta_0}\right)$$

exponential volume reflectivity



Single Baseline Observations

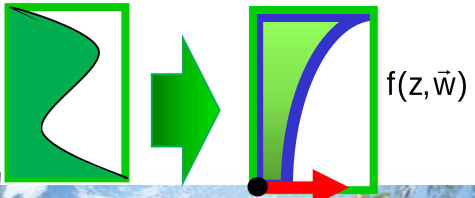
single- / dual- / quad-pol

$\tilde{\gamma}_{Vol}(\vec{w}_1,\kappa_z)$ $\tilde{\gamma}_{Vol}(\vec{w}_2,\kappa_z)$ $\tilde{\gamma}_{Vol}(\vec{w}_3,\kappa_z)$

1, 2, or 3 complex coherences

Total Coherence

$$\tilde{\gamma}(\vec{w},\kappa_z) = \gamma_{Temp}(\kappa_z)\ \tilde{\gamma}_{Vol}(\vec{w},\kappa_z)$$



For a Single Baseline

4 unknown parameters

Volume Height h_V

Topography φ_0

G/V Ratio $m(\vec{w})$ = f(pol)

$f_V(z)$ … 1 parameter (σ)

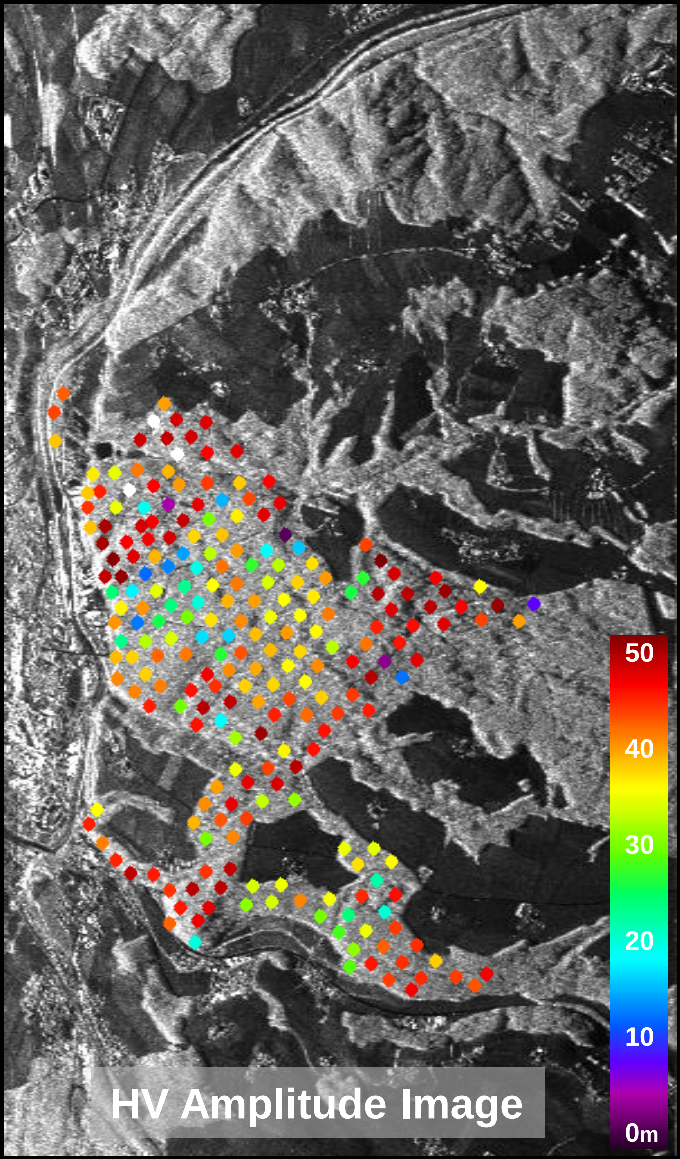
plus one more

Temporal Deco γ_{Temp}

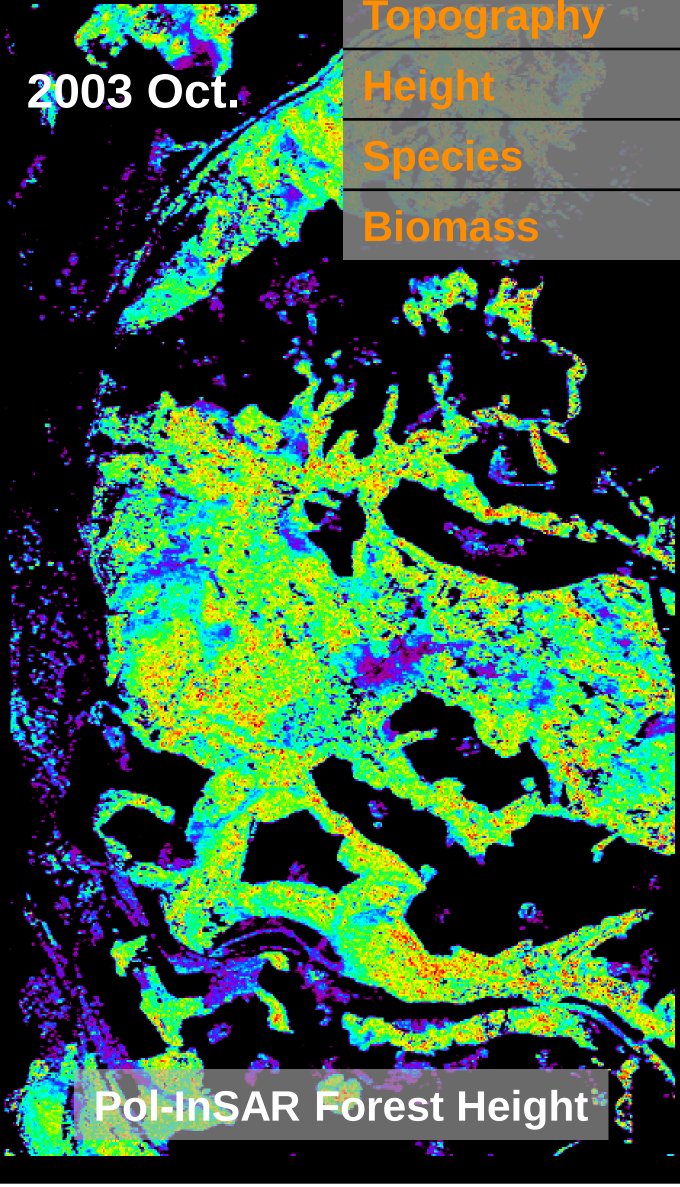
for repeat-pass implementations

Traunstein Test Site

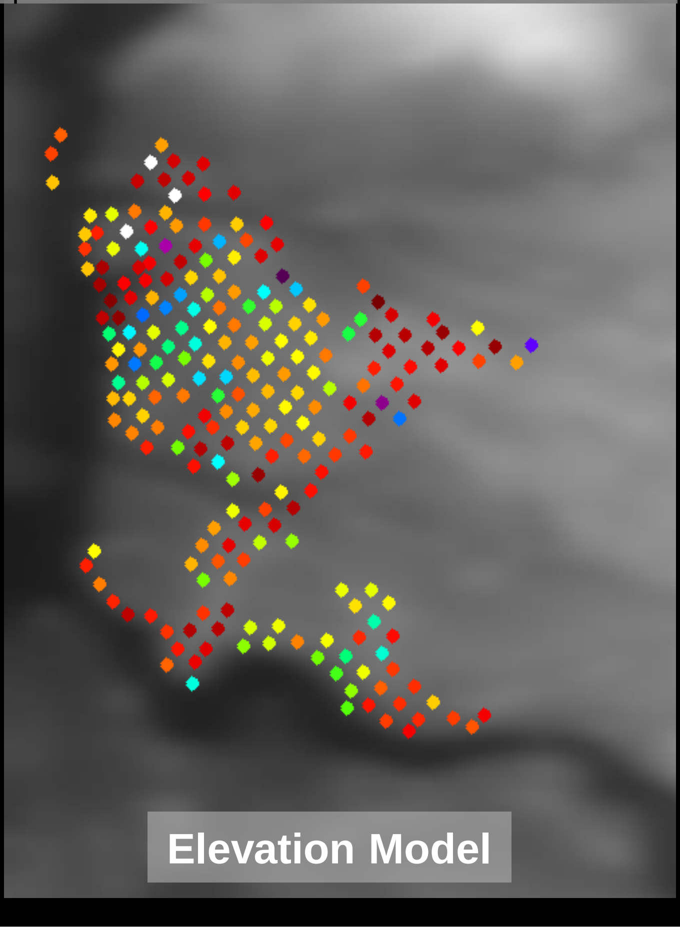
Forest type	Temperate
Topography	Moderate slopes
Height	25 ~ 35m
Species	N. Spruce, E. Beech, White Fir
Biomass	40 ~ 450 t/ha



HV Amplitude Image

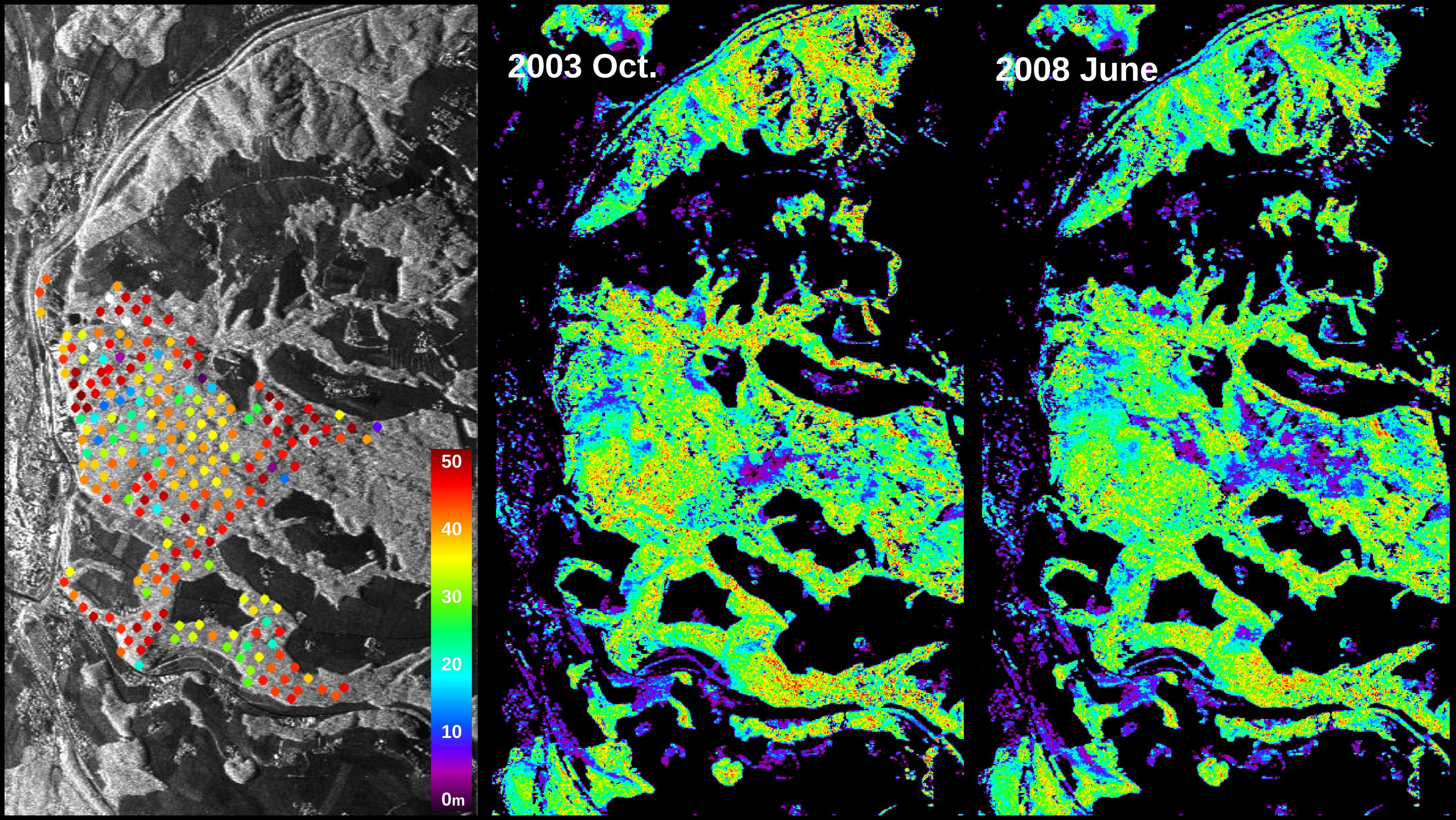


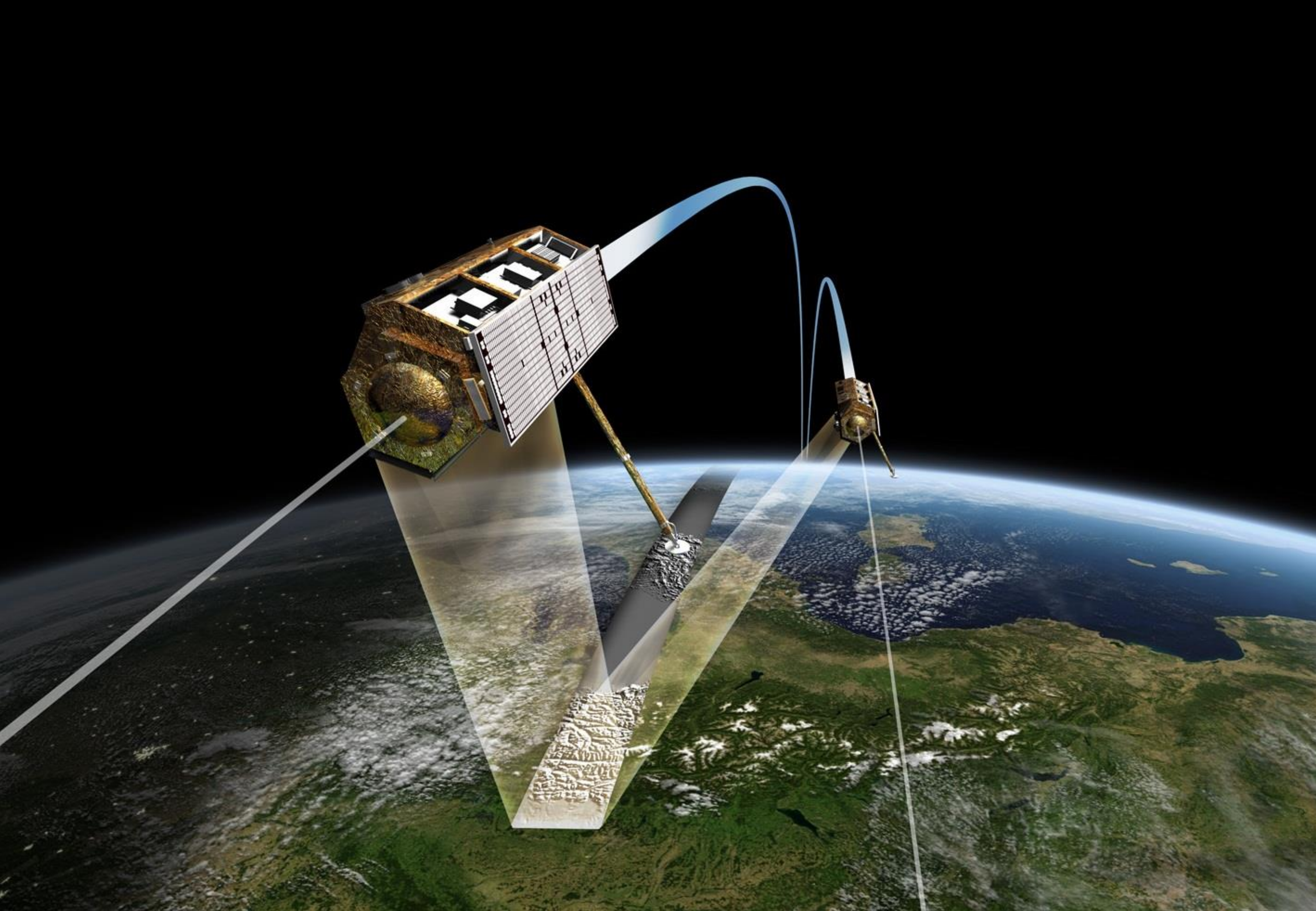
Pol-InSAR Forest Height



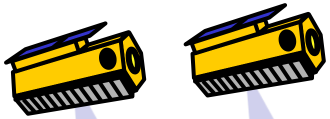
Elevation Model

Traunstein Test Site





TanDEM-X: 2 Layer model with exp. volume reflectivity



Volume Layer Ground Layer

$$f(z,\vec{w}) = \boxed{m_V f_V(z)} + \boxed{m_G(\vec{w})\delta(z-z_0)}$$

$$\boxed{\tilde{\gamma}_{Vol}(\vec{w},\kappa_z) = \exp(i\varphi_0)\frac{\tilde{\gamma}_V(\kappa_z)+m(\vec{w})}{1+m(\vec{w})}}$$

Volume Layer Coherence

$$\tilde{\gamma}_V = \frac{I}{I_0} \quad \begin{cases} I = \int_0^{h_V} \exp(i\kappa_z z')e^{\left(\frac{2\,\sigma\,z'}{\cos\theta_0}\right)}dz' \\ I_0 = \int_0^{h_V} e^{\left(\frac{2\,\sigma\,z'}{\cos\theta_0}\right)}dz' \end{cases}$$

$$m(\vec{w}) = \frac{m_G(\vec{w})}{m_V(\vec{w})I_0} \qquad \kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$$

$$f_V(z) = \exp\left(\frac{2\,\sigma\,z}{\cos\theta_0}\right)$$

exponential volume reflectivity

Single Baseline Observation(s)

single-pol

$\tilde{\gamma}_{Vol}(\vec{w}_1,\kappa_z)$

1 complex coherence

Total Coherence

$$\tilde{\gamma}(\vec{w},\kappa_z)$$ [redacted] $$\tilde{\gamma}_{Vol}(\vec{w},\kappa_z)$$

For a Single Baseline

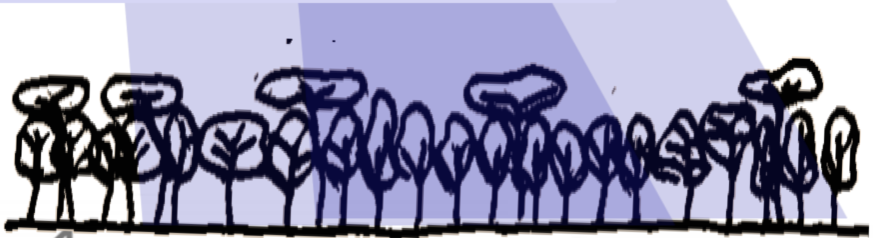
4 unknown parameters

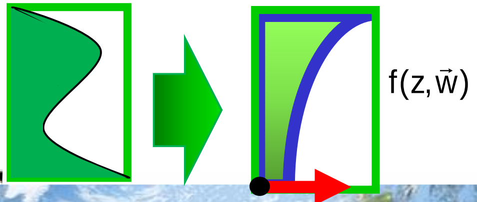
Volume Height h_V

Topography φ_0

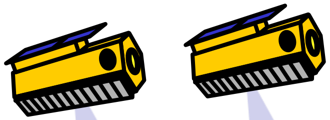
G/V Ratio $m(\vec{w})$ = f(pol)

$f_V(z)$ … 1 parameter (σ)





TanDEM-X: 2 Layer model with exp. volume reflectivity + no ground



Volume Layer Ground Layer

$$f(z,\vec{w}) = m_V f_V(z) + \cancel{m_G(\vec{w})\delta(z - z_0)}$$

$$\tilde{\gamma}_{Vol}(\vec{w},\kappa_z) = \exp(i\varphi_0)\frac{\tilde{\gamma}_V(\kappa_z) + \cancel{m(\vec{w})}}{\cancel{1 + m(\vec{w})}}$$

Volume Layer Coherence

$$\tilde{\gamma}_V = \frac{I}{I_0} \quad \begin{cases} I = \int_0^{h_V} \exp(i\kappa_z z') e^{\left(\frac{2\ \sigma\ z'}{\cos\theta_0}\right)} dz' \\ I_0 = \int_0^{h_V} e^{\left(\frac{2\ \sigma\ z'}{\cos\theta_0}\right)} dz' \end{cases}$$

$$\cancel{m(\vec{w}) =} \frac{m_G(\vec{w})}{m_V(\vec{w}) I_0} \qquad \kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$$

$$f_V(z) = \exp\left(\frac{2\ \sigma\ z}{\cos\theta_0}\right)$$

exponential volume reflectivity

Invertible with single-pol data only if the ground topography φ_0 is known !!!

Single Baseline Observation(s)

single-pol

$\tilde{\gamma}_{Vol}(\vec{w}_1,\kappa_z)$

1 complex coherence

Total Coherence

$$\tilde{\gamma}(\vec{w},\kappa_z) = \cancel{\gamma_{Temp}(\kappa_z)}\ \tilde{\gamma}_{Vol}(\vec{w},\kappa_z)$$

For a Single Baseline

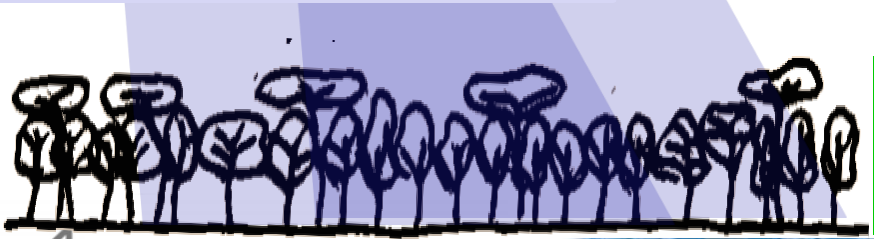
3 unknown parameters

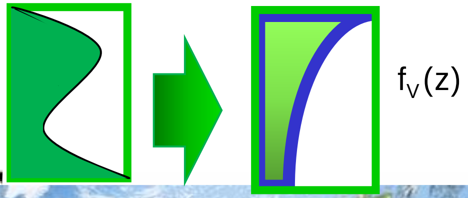
Volume Height h_V

Topography φ_0

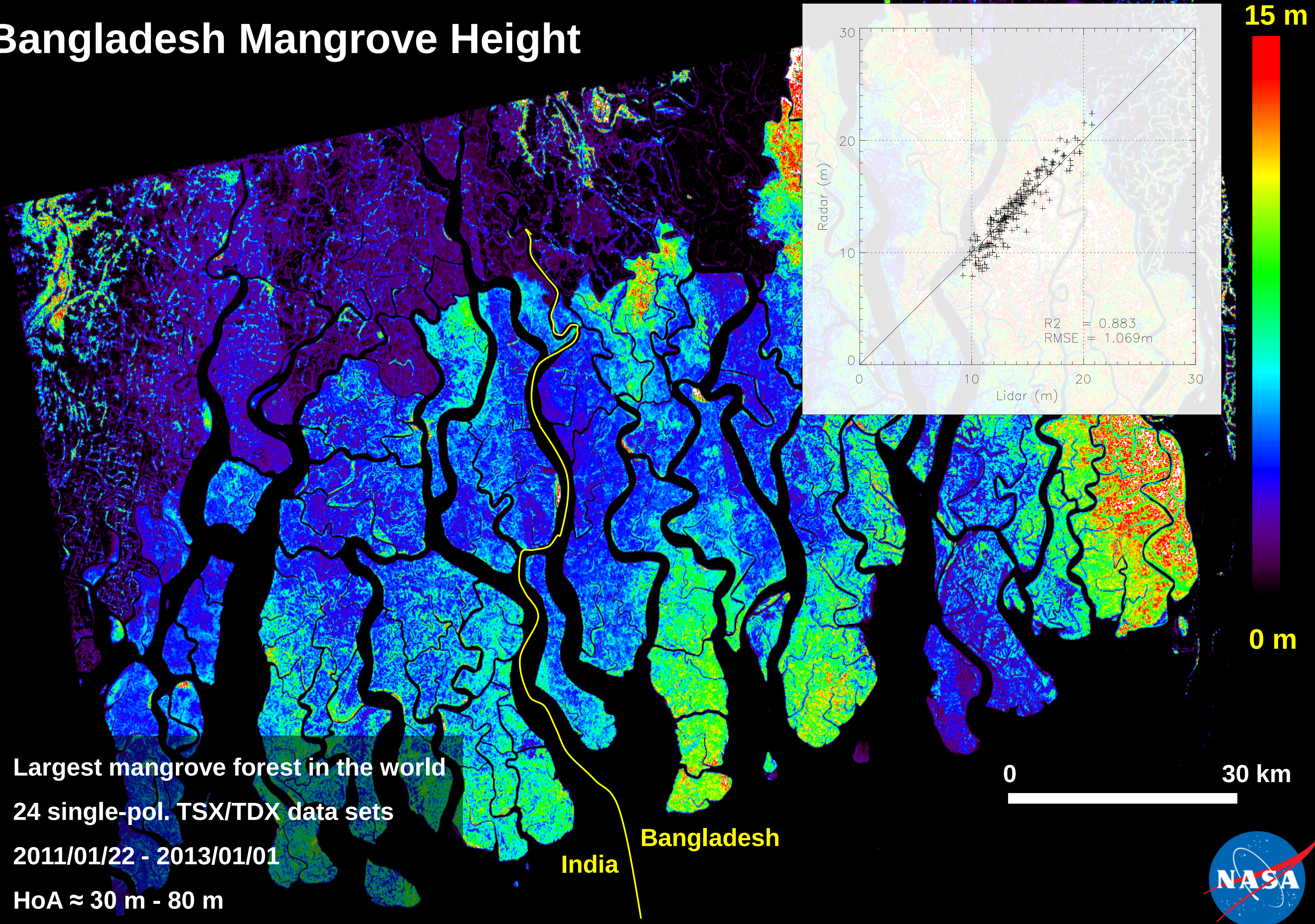
~~G/V Ratio~~ $\cancel{m(\vec{w}) = f(pol)}$

$f_V(z)$ … 1 parameter (σ)

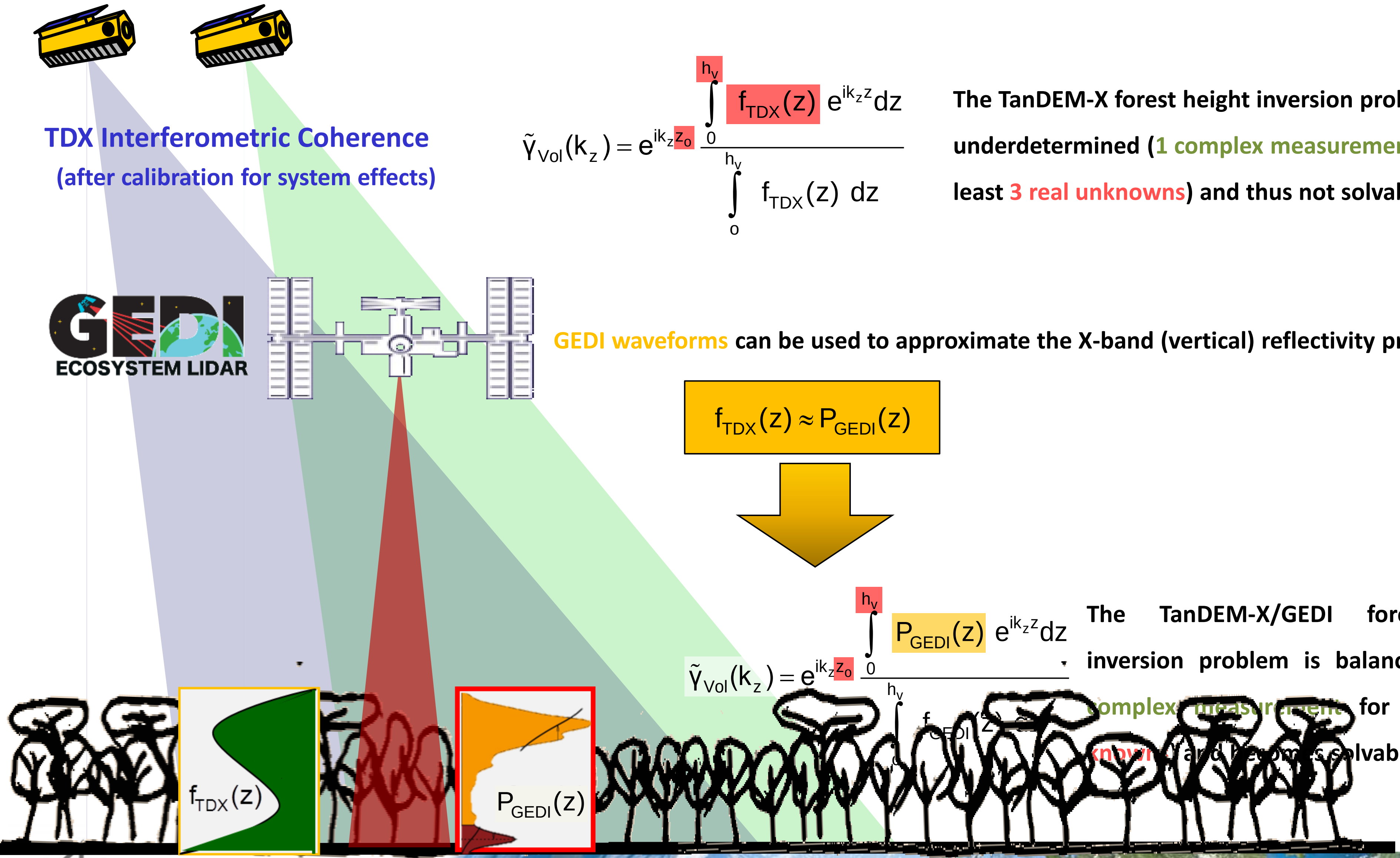




Bangladesh Mangrove Height



- Largest mangrove forest in the world
- 24 single-pol. TSX/TDX data sets
- 2011/01/22 - 2013/01/01
- HoA ≈ 30 m - 80 m



TDX Interferometric Coherence

(after calibration for system effects)

$$\tilde{\gamma}_{Vol}(k_z) = e^{ik_z z_0} \frac{\int_0^{h_v} f_{TDX}(z)\, e^{ik_z z} dz}{\int_0^{h_v} f_{TDX}(z)\, dz}$$

The TanDEM-X forest height inversion problem is underdetermined (1 complex measurement for at least 3 real unknowns) and thus not solvable !

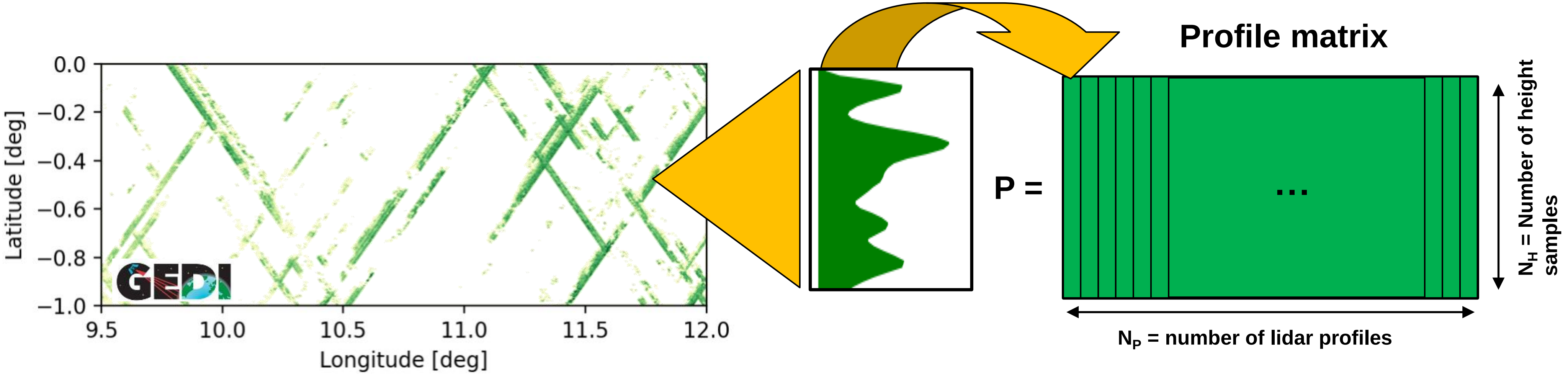
GEDI waveforms can be used to approximate the X-band (vertical) reflectivity profiles

$$f_{TDX}(z) \approx P_{GEDI}(z)$$

$$\tilde{\gamma}_{Vol}(k_z) = e^{ik_z z_0} \frac{\int_0^{h_v} P_{GEDI}(z)\, e^{ik_z z} dz}{\int_0^{h_v} f_{GEDI}(z)\, dz}$$

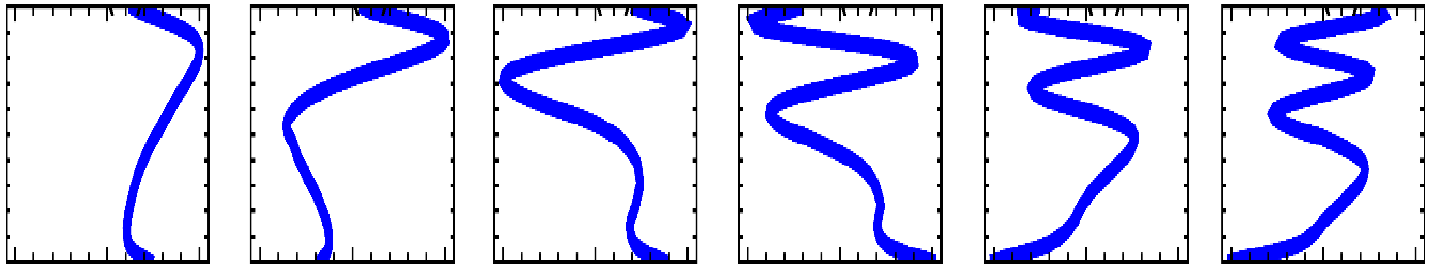
The TanDEM-X/GEDI forest height inversion problem is balanced (with 1 complex measurement for 2 real unknowns) and becomes solvable !

TanDEM-X GEDI Fusion: Common Reflectivity Profile Estimation

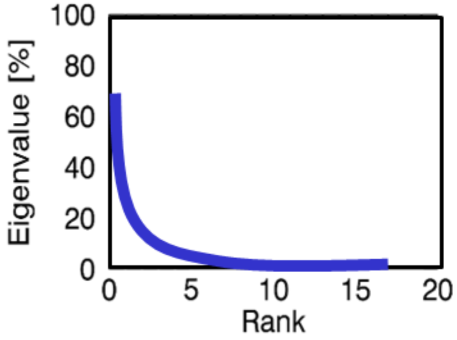


Profile covariance matrix: $R = P\,P^T = U\,\Lambda\,U^T$ **Eigen-decomposition of R**

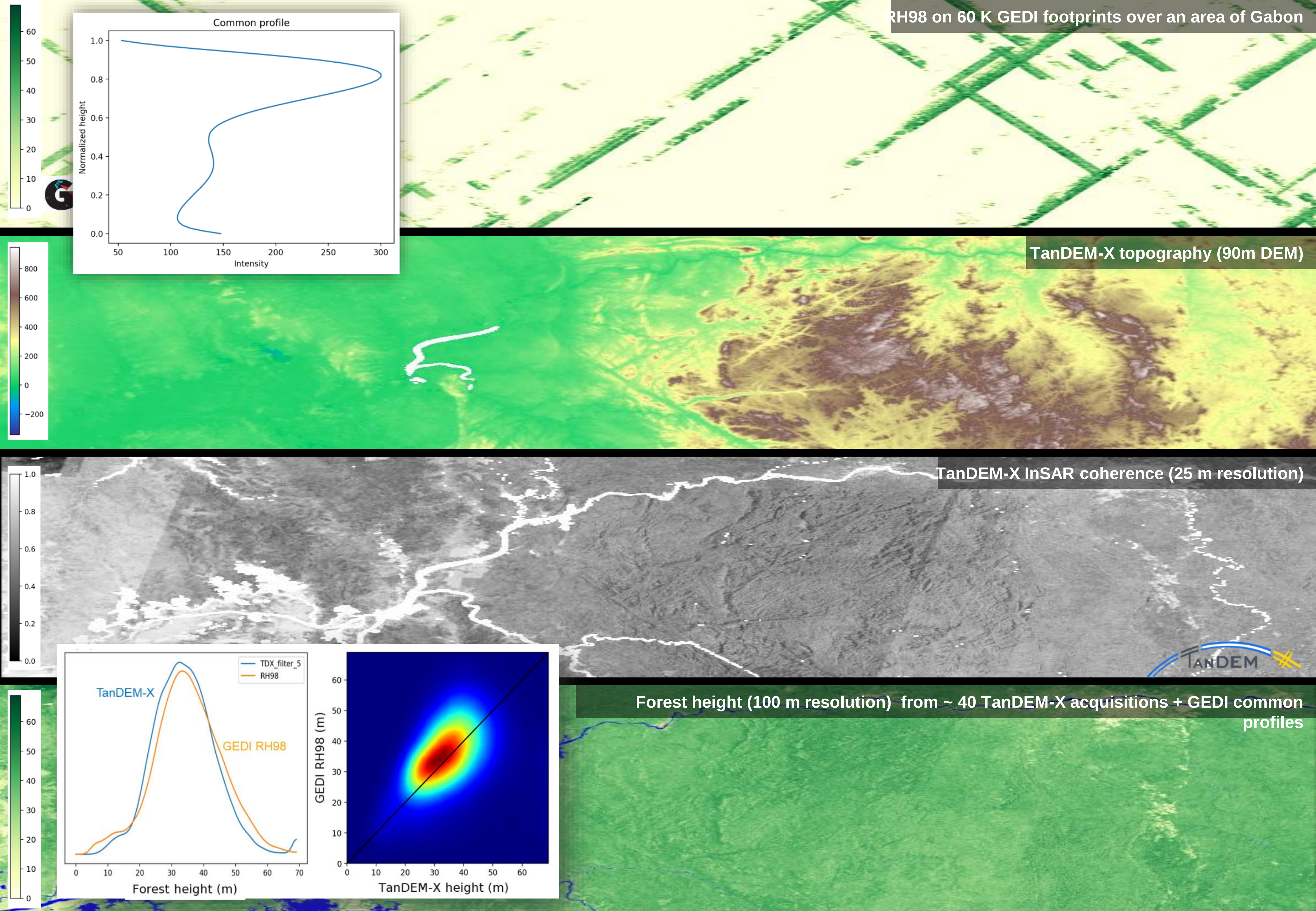
Eigen-Vectors (e.g. Eigen-Functions)



Eigen-Values



The Eigen-Vectors can be used derive a “mean” vertical profile

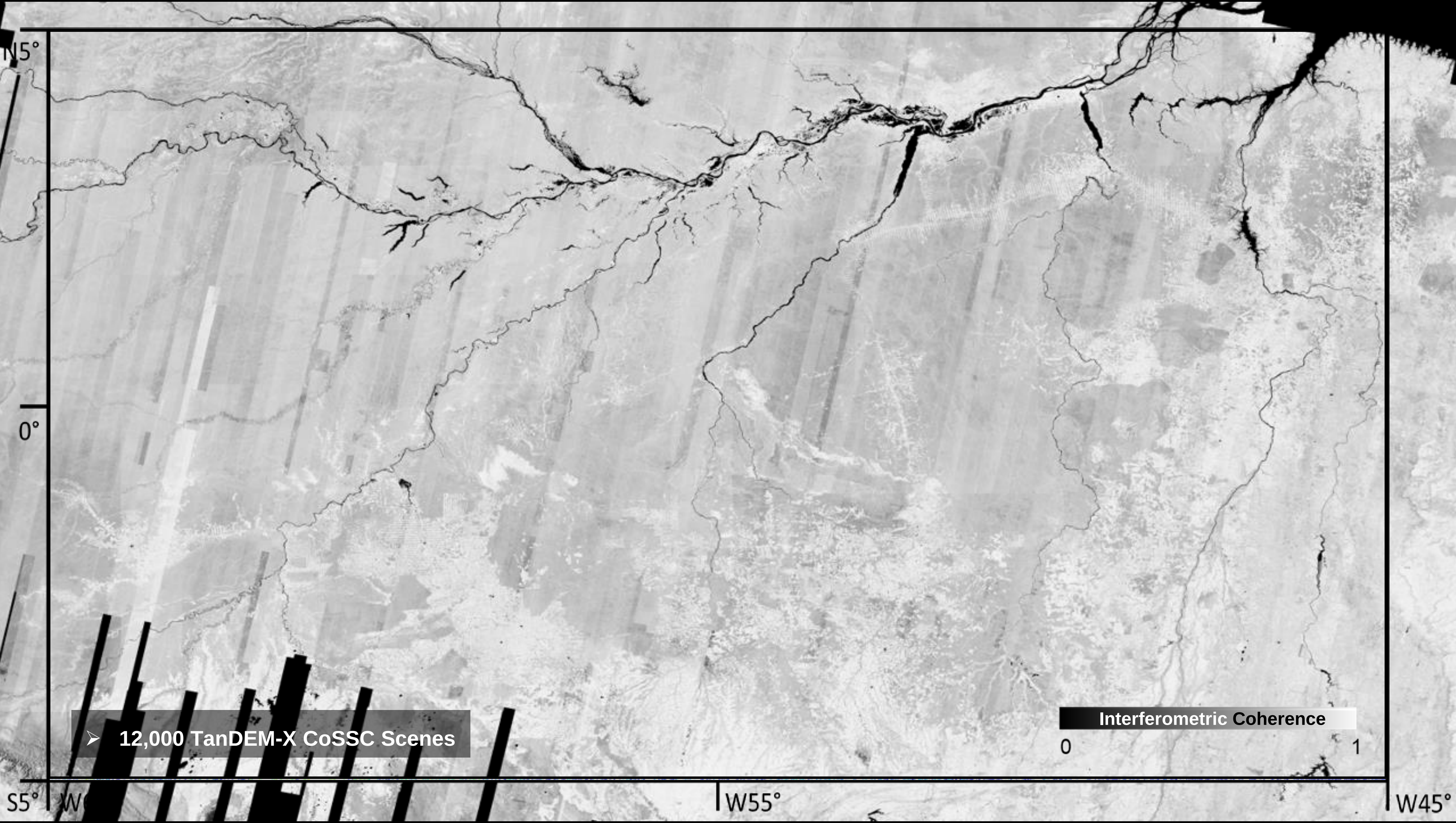


RH98 on 60 K GEDI footprints over an area of Gabon

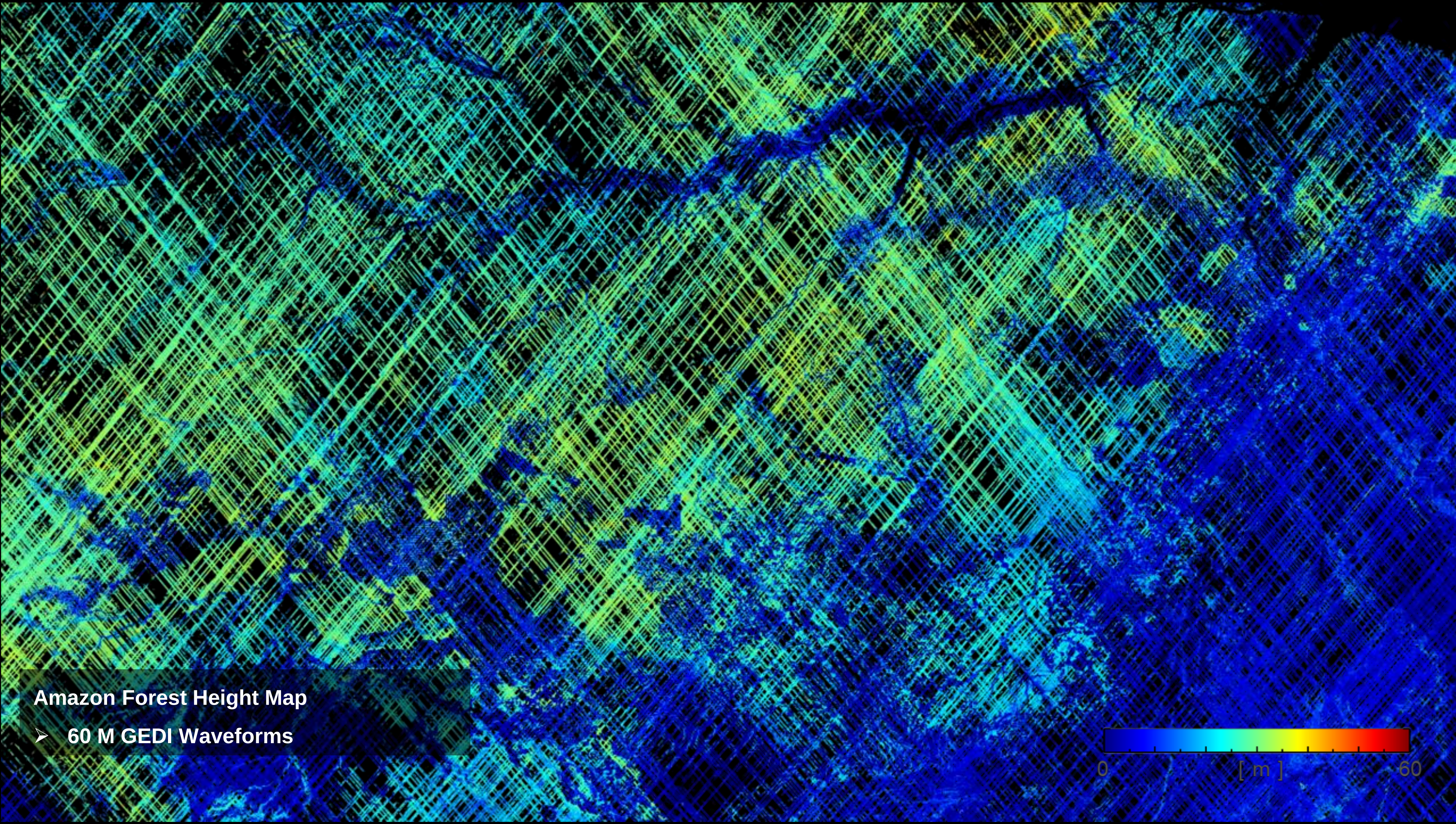
TanDEM-X topography (90m DEM)

TanDEM-X InSAR coherence (25 m resolution)

Forest height (100 m resolution) from ~ 40 TanDEM-X acquisitions + GEDI common profiles

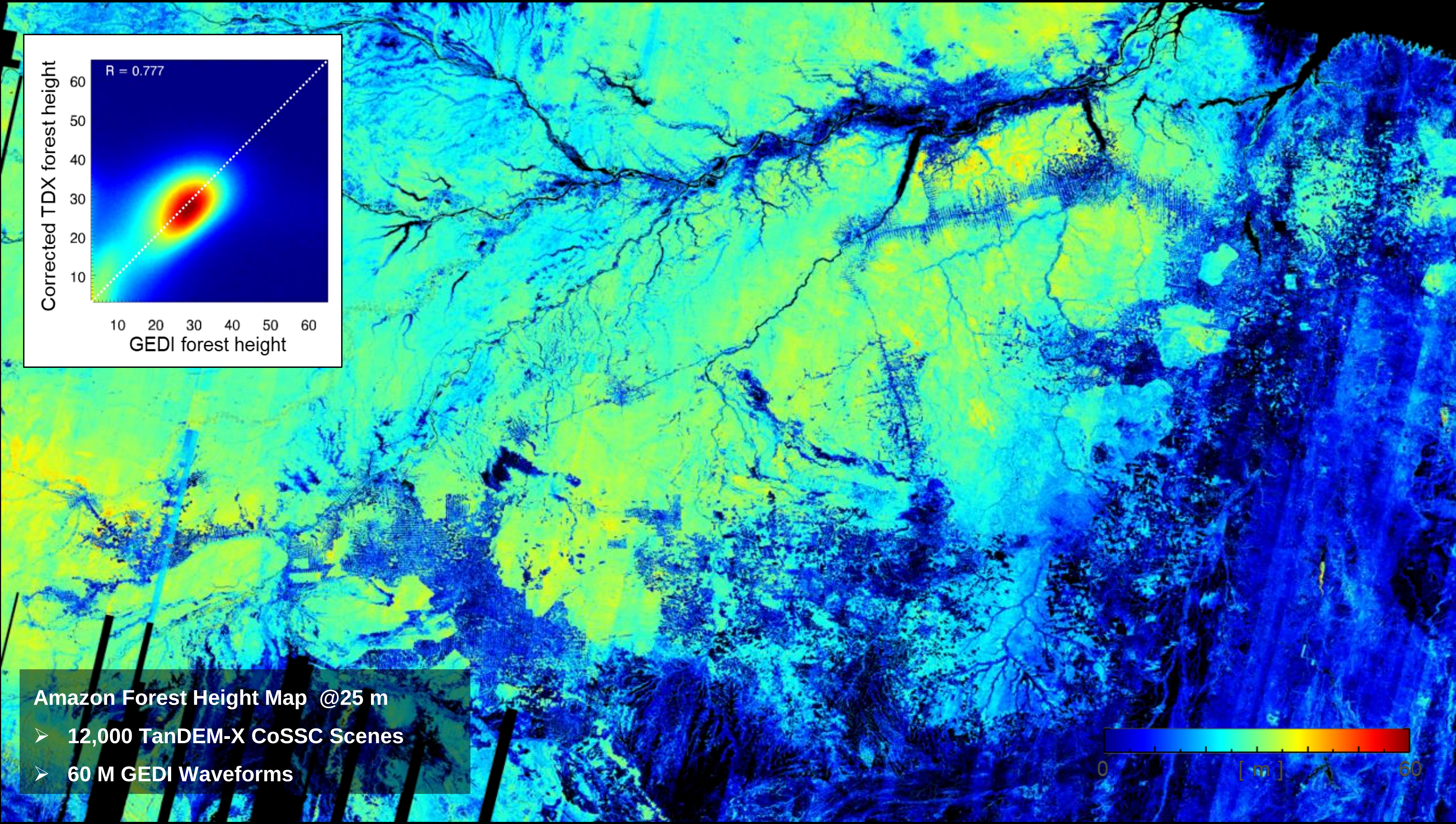


- 12,000 TanDEM-X CoSSC Scenes



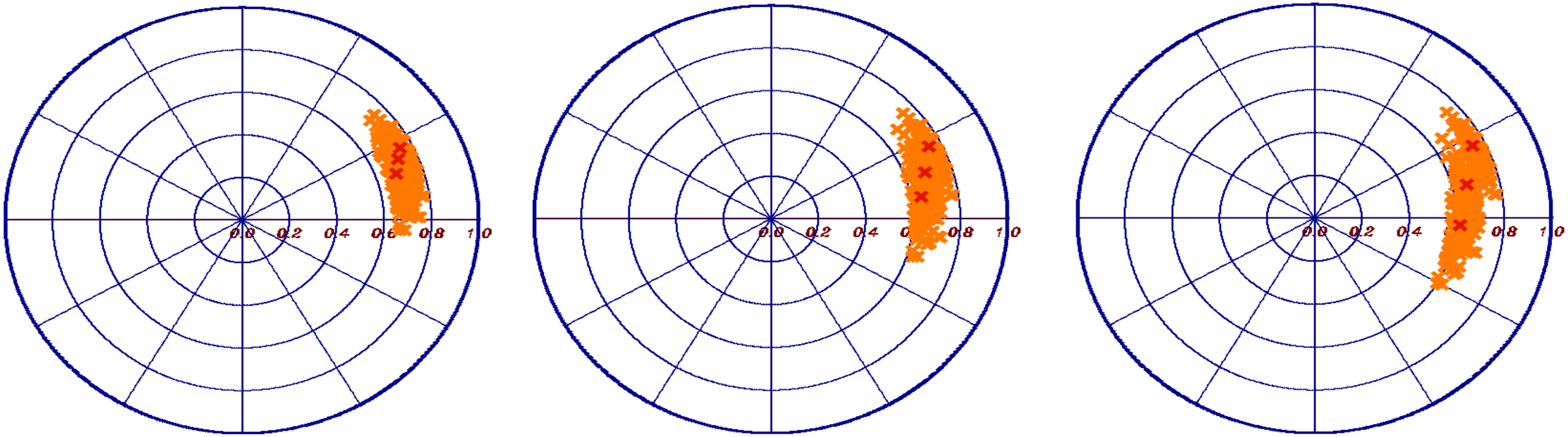
Amazon Forest Height Map

- **60 M GEDI Waveforms**

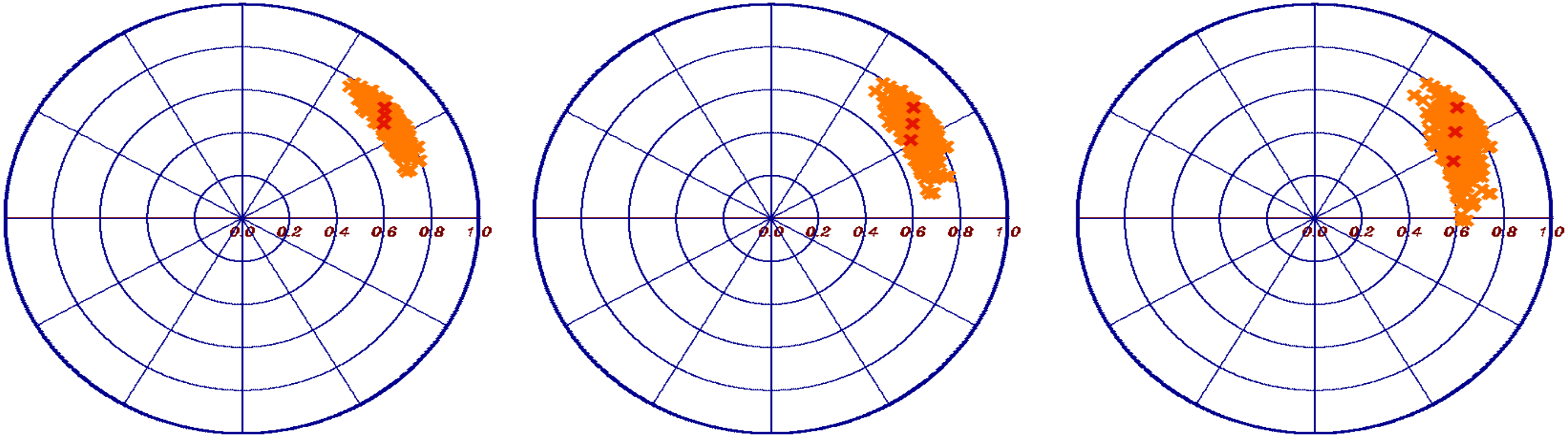


Amazon Forest Height Map @25 m

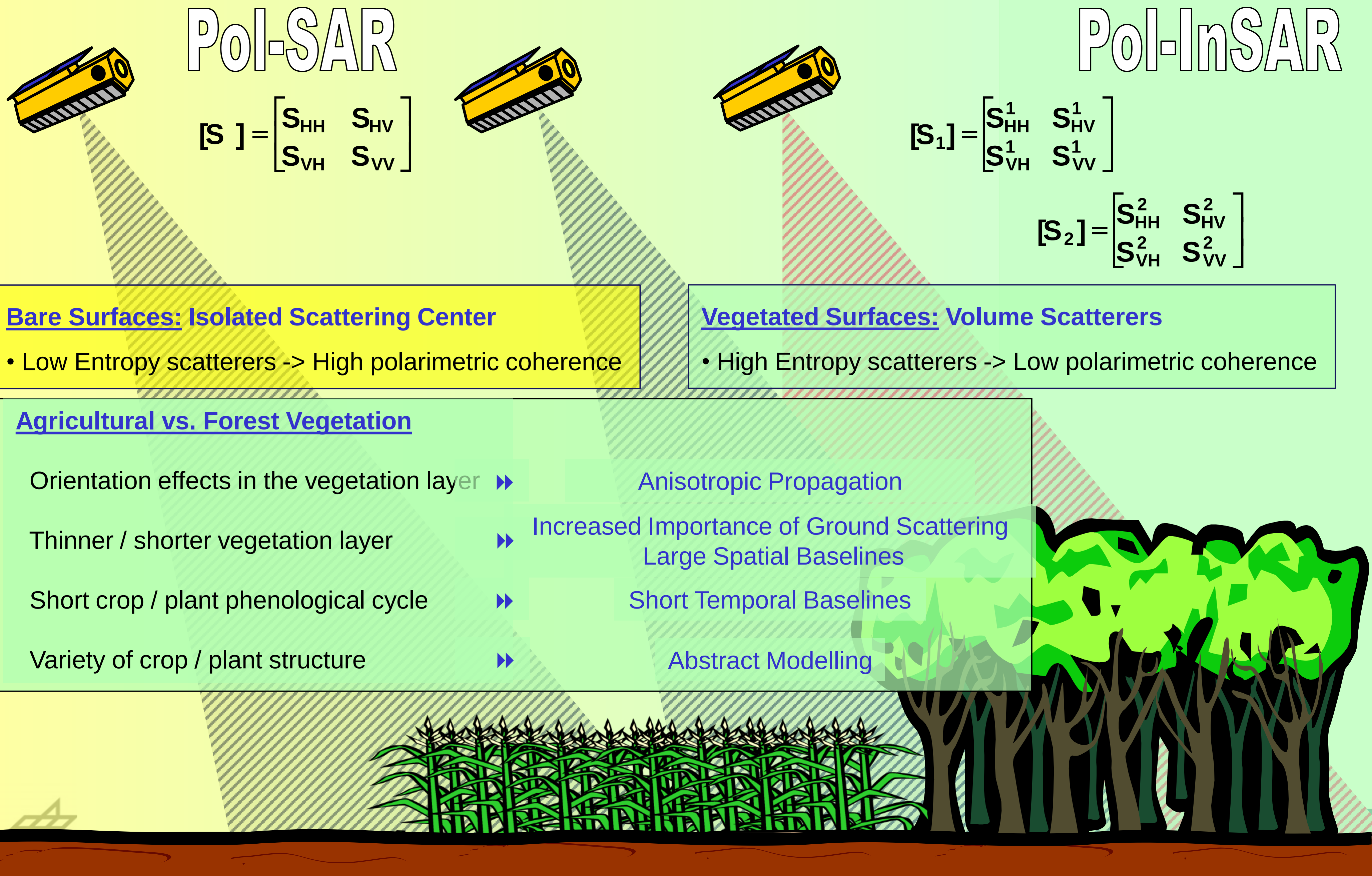
- 12,000 TanDEM-X CoSSC Scenes
- 60 M GEDI Waveforms



Agriculture Vegetation



Agriculture Pol-InSAR Applications



Pol-SAR

$$[S\ ] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

Pol-InSAR

$$[S_1] = \begin{bmatrix} S^1_{HH} & S^1_{HV} \\ S^1_{VH} & S^1_{VV} \end{bmatrix}$$

$$[S_2] = \begin{bmatrix} S^2_{HH} & S^2_{HV} \\ S^2_{VH} & S^2_{VV} \end{bmatrix}$$

Bare Surfaces: Isolated Scattering Center

- Low Entropy scatterers -> High polarimetric coherence

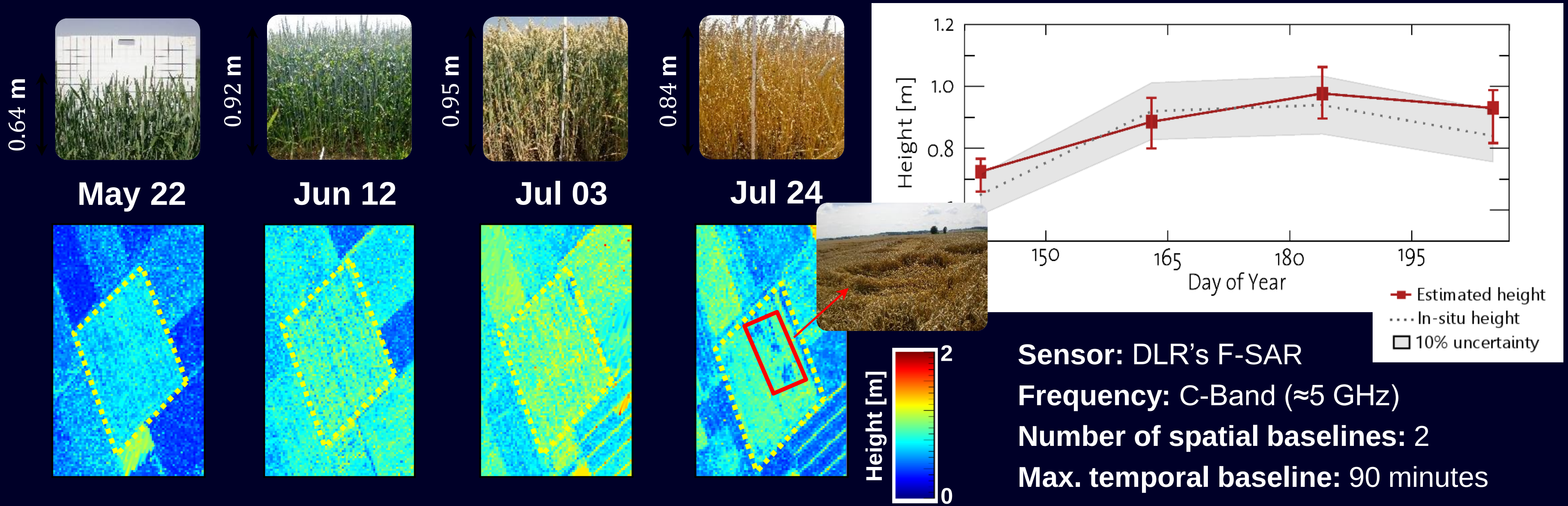
Vegetated Surfaces: Volume Scatterers

- High Entropy scatterers -> Low polarimetric coherence

Agricultural vs. Forest Vegetation

Orientation effects in the vegetation layer	⏩	Anisotropic Propagation
Thinner / shorter vegetation layer	⏩	Increased Importance of Ground Scattering Large Spatial Baselines
Short crop / plant phenological cycle	⏩	Short Temporal Baselines
Variety of crop / plant structure	⏩	Abstract Modelling

Wheat



Sensor: DLR's F-SAR

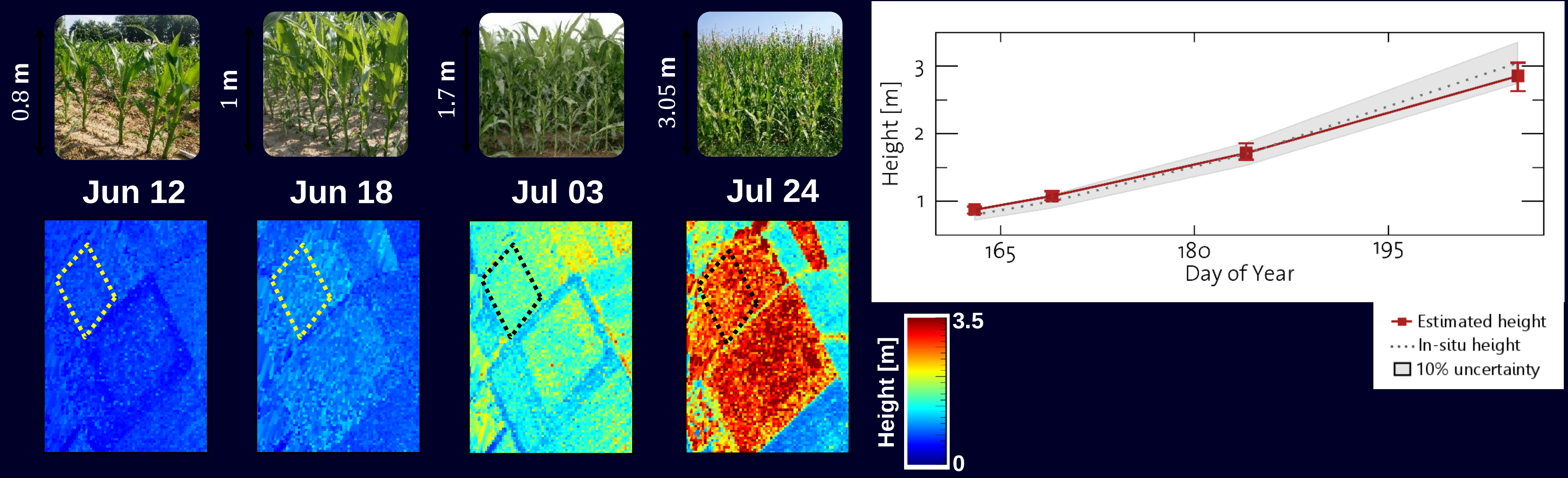
Frequency: C-Band (≈5 GHz)

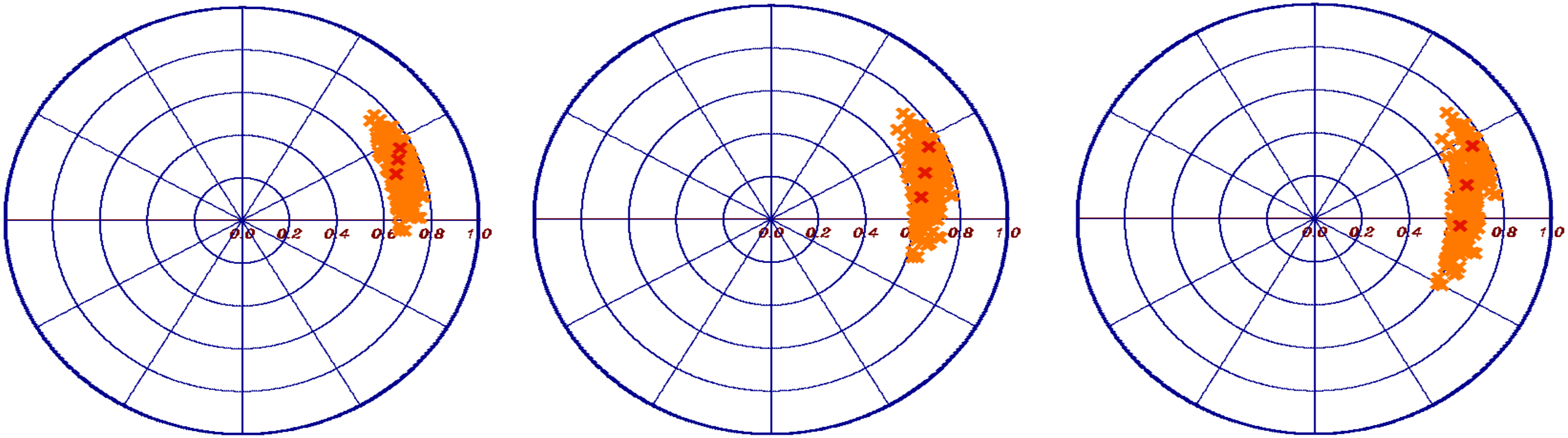
Number of spatial baselines: 2

Max. temporal baseline: 90 minutes

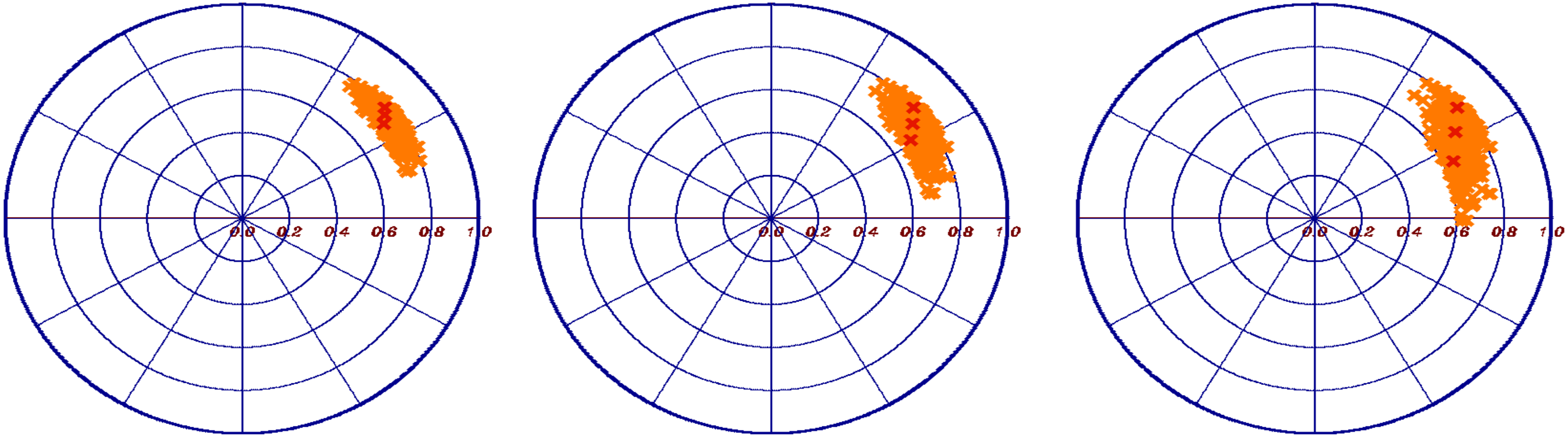
Equivalent Number of Looks: 100

Corn (Maize)

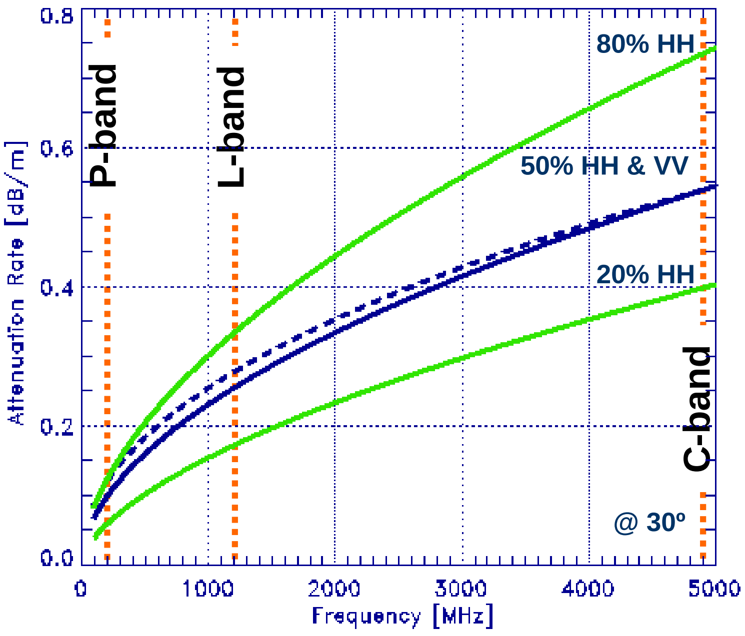




Pol-InSAR: Frequency Effects

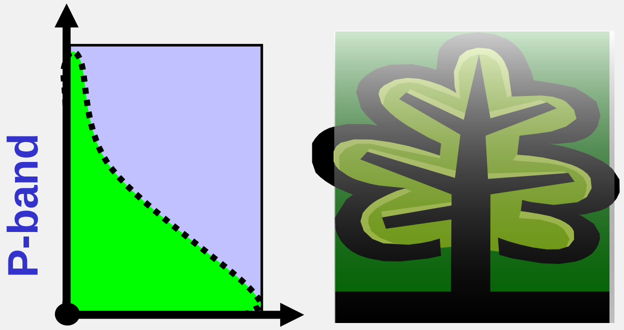


Frequency Dependency

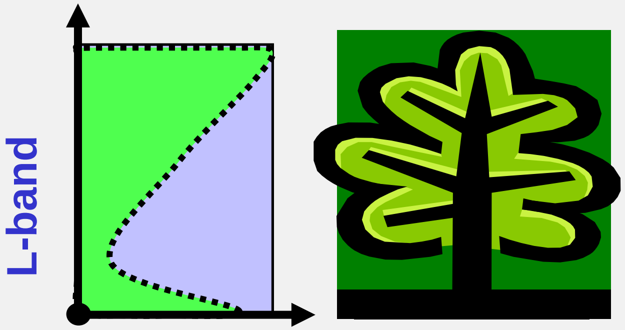


With degreasing frequency:

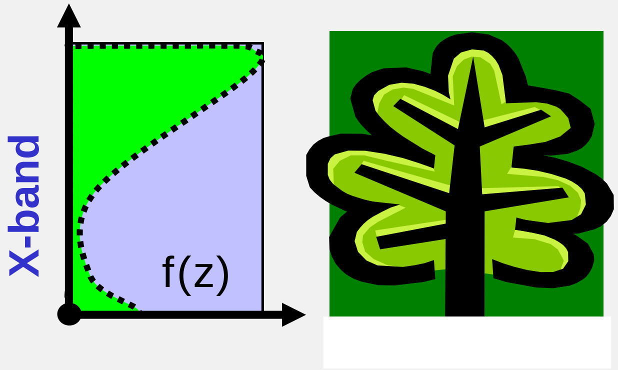
- •The attenuation through the vegetation degreases;
- •The relative importance of the volume decreases;
- •The relative importance of the ground increases;
- •The effective scatterers and their distribution changes.



$$\tilde{\gamma}(\vec{w}) = \exp(i\varphi_0)\frac{\tilde{\gamma}_V + m(\vec{w})}{1+m(\vec{w})}$$



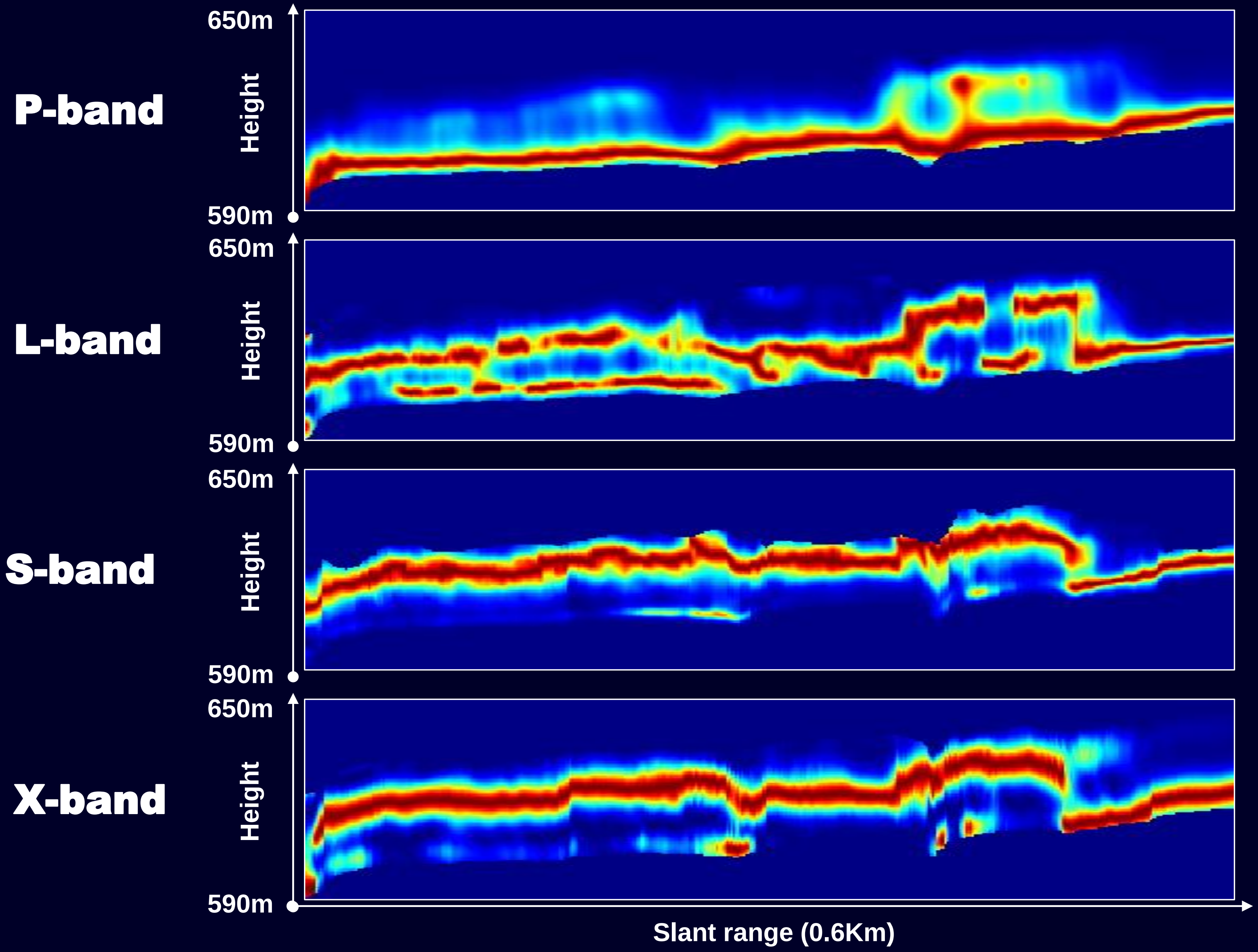
$$\tilde{\gamma}(\vec{w}) = \exp(i\varphi_0)\frac{\tilde{\gamma}_V + m(\vec{w})}{1+m(\vec{w})}$$

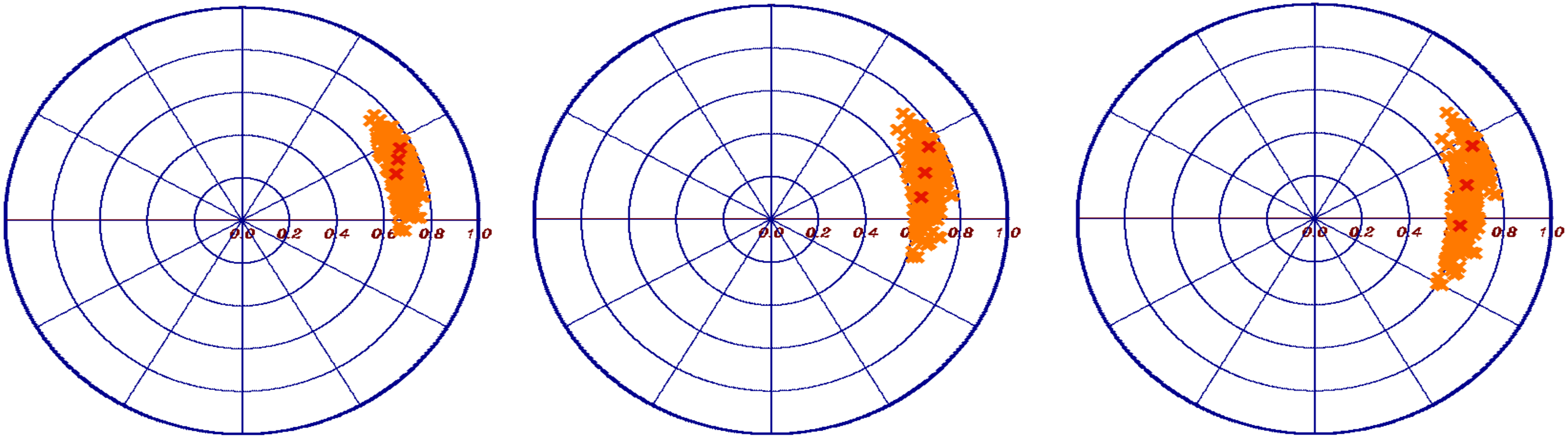


$$\tilde{\gamma}(\vec{w}) = \exp(i\varphi_0)\tilde{\gamma}_V$$

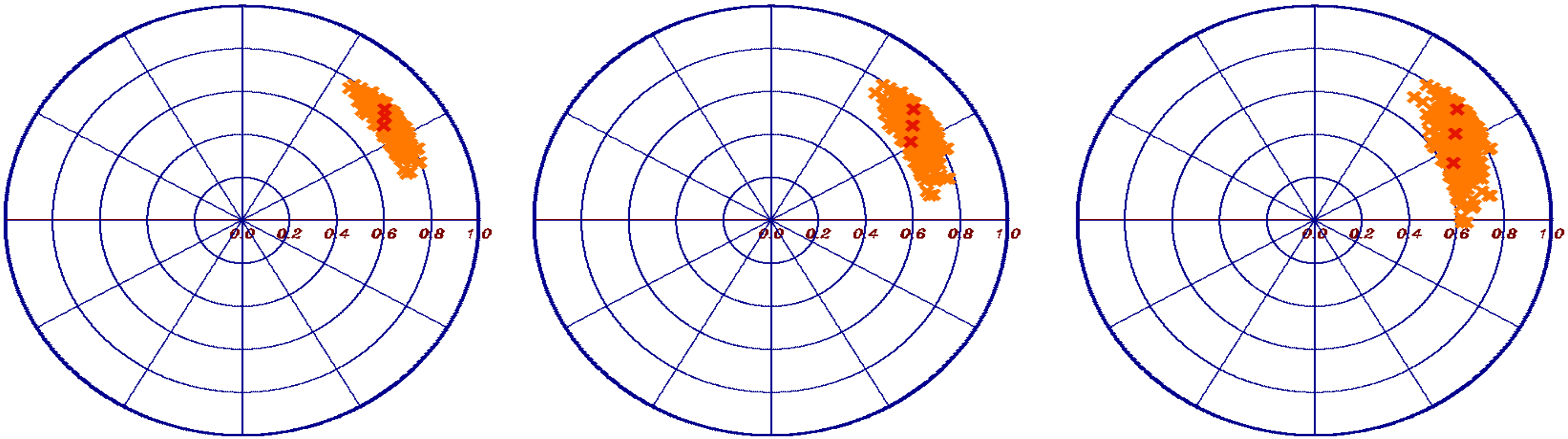
L.Bessette, S.Ayasli "Ultra Wide Band P-3 and Carabas II Foliage Attenuation and Backscatter Analysis", Proceedings of IEEE Radar Conference, 2001.

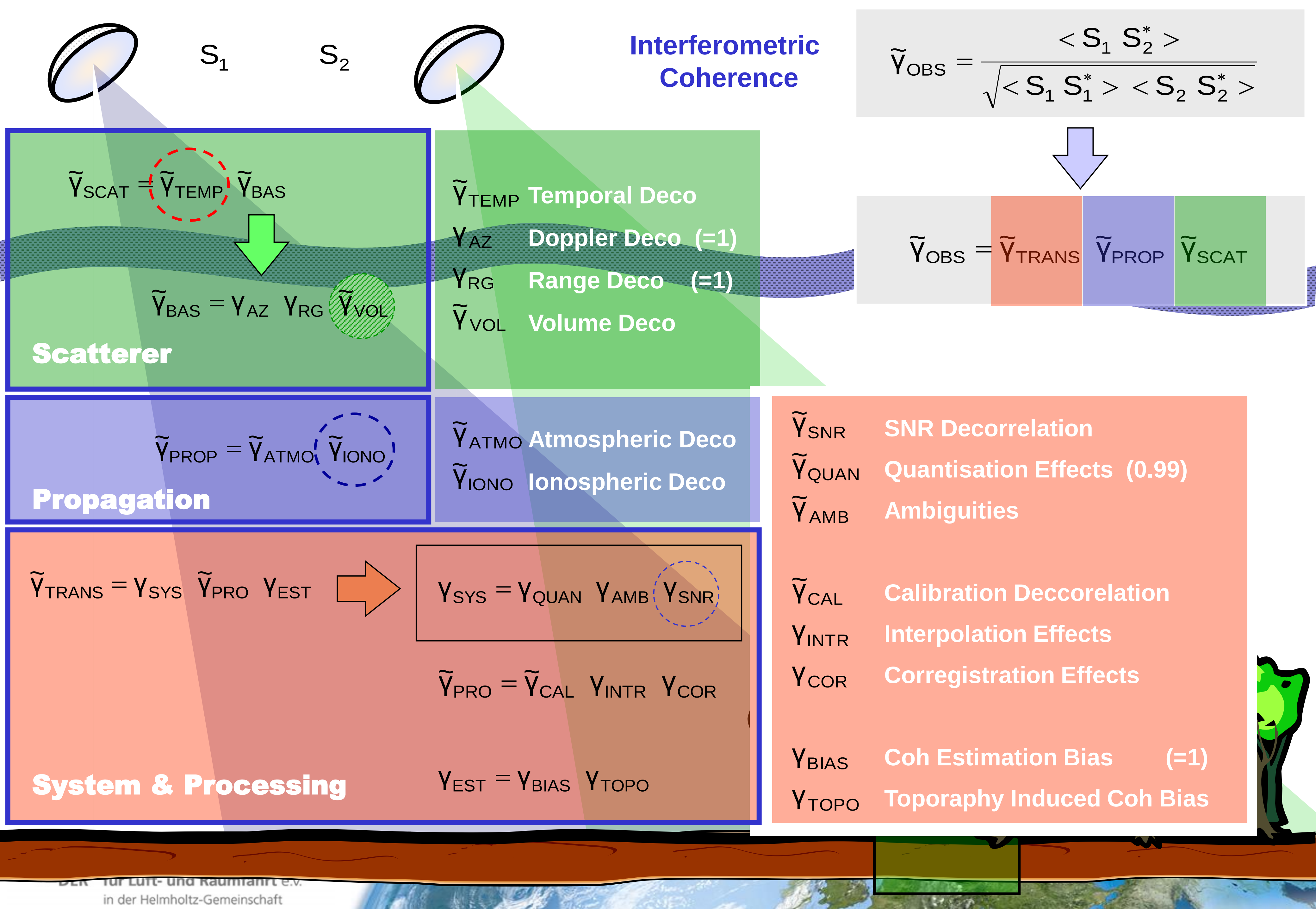
Traunstein forest (Germany) - Capon - HH





Non-Volumetric Decorrelation Effects





Interferometric Coherence

$$\tilde{\gamma}_{OBS} = \frac{< S_1 \; S_2^* >}{\sqrt{< S_1 \; S_1^* > \; < S_2 \; S_2^* >}}$$

$$\tilde{\gamma}_{OBS} = \tilde{\gamma}_{TRANS} \; \tilde{\gamma}_{PROP} \; \tilde{\gamma}_{SCAT}$$

Scatterer

$$\tilde{\gamma}_{SCAT} = \tilde{\gamma}_{TEMP} \; \tilde{\gamma}_{BAS}$$

$$\tilde{\gamma}_{BAS} = \gamma_{AZ} \; \gamma_{RG} \; \tilde{\gamma}_{VOL}$$

$\tilde{\gamma}_{TEMP}$	**Temporal Deco**
γ_{AZ}	**Doppler Deco (=1)**
γ_{RG}	**Range Deco (=1)**
$\tilde{\gamma}_{VOL}$	**Volume Deco**

Propagation

$$\tilde{\gamma}_{PROP} = \tilde{\gamma}_{ATMO} \; \tilde{\gamma}_{IONO}$$

$\tilde{\gamma}_{ATMO}$	**Atmospheric Deco**
$\tilde{\gamma}_{IONO}$	**Ionospheric Deco**

System & Processing

$$\tilde{\gamma}_{TRANS} = \gamma_{SYS} \; \tilde{\gamma}_{PRO} \; \gamma_{EST}$$

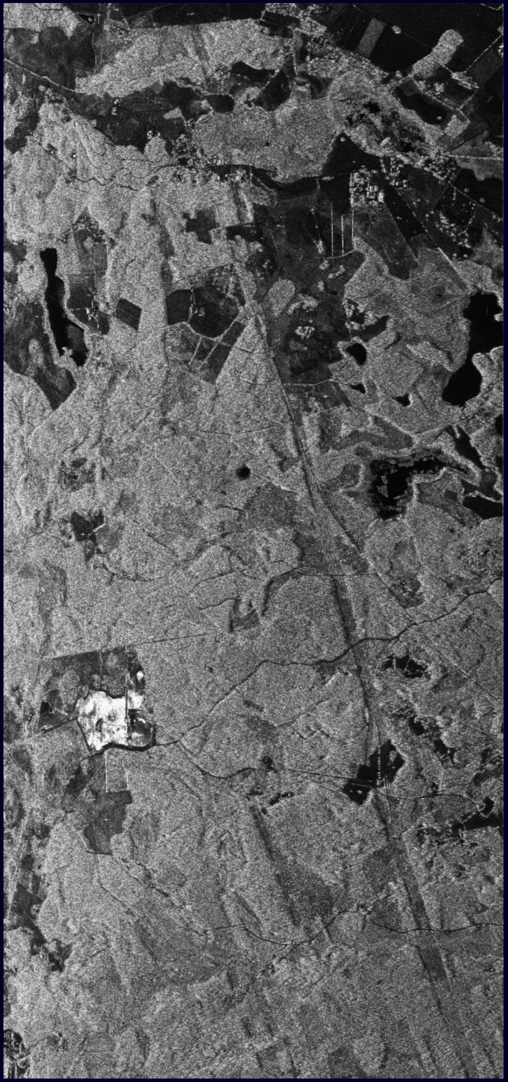
$$\gamma_{SYS} = \gamma_{QUAN} \; \gamma_{AMB} \; \gamma_{SNR}$$

$$\tilde{\gamma}_{PRO} = \tilde{\gamma}_{CAL} \; \gamma_{INTR} \; \gamma_{COR}$$

$$\gamma_{EST} = \gamma_{BIAS} \; \gamma_{TOPO}$$

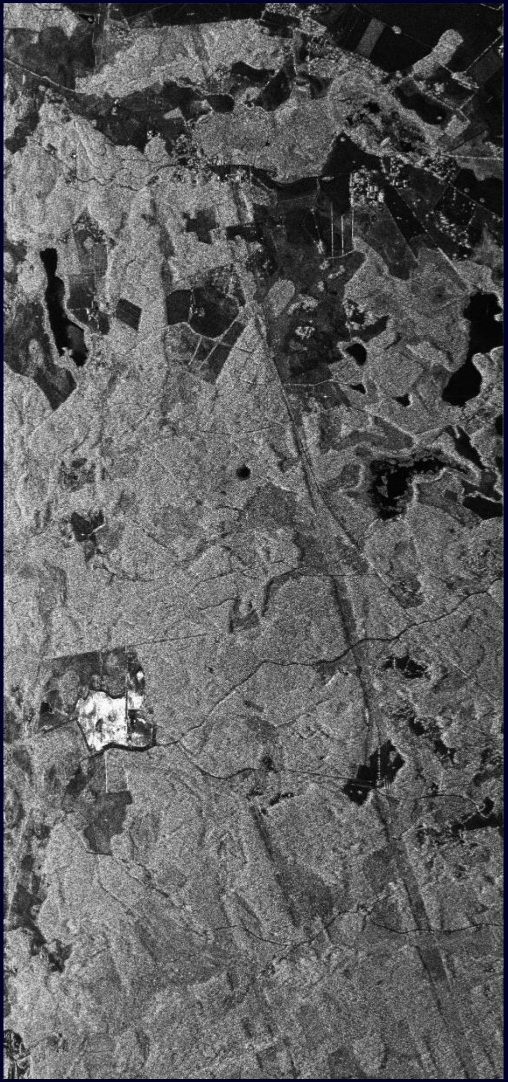
$\tilde{\gamma}_{SNR}$	**SNR Decorrelation**
$\tilde{\gamma}_{QUAN}$	**Quantisation Effects (0.99)**
$\tilde{\gamma}_{AMB}$	**Ambiguities**
$\tilde{\gamma}_{CAL}$	**Calibration Deccorelation**
γ_{INTR}	**Interpolation Effects**
γ_{COR}	**Corregistration Effects**
γ_{BIAS}	**Coh Estimation Bias (=1)**
γ_{TOPO}	**Toporaphy Induced Coh Bias**

Amplitude Image

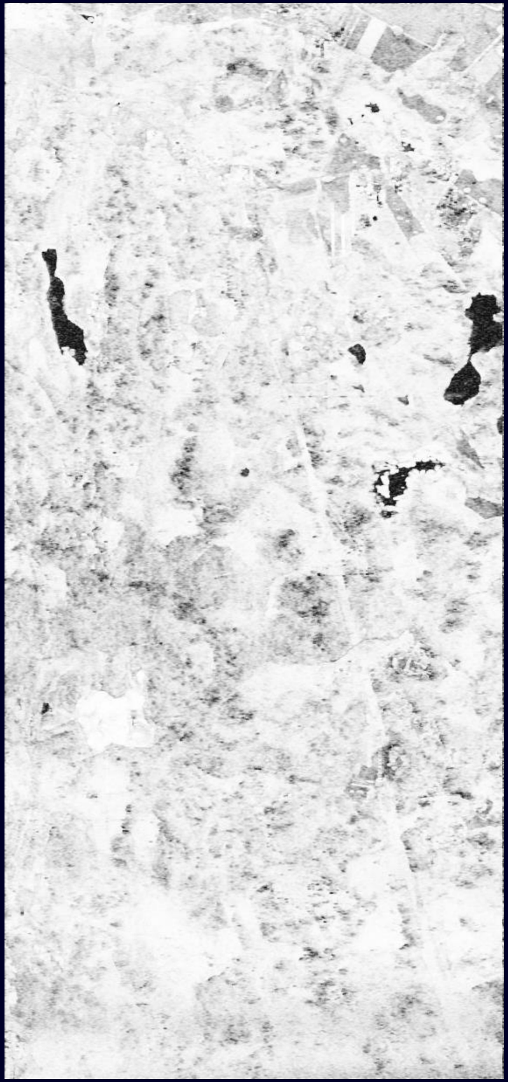


Amplitude Image HH

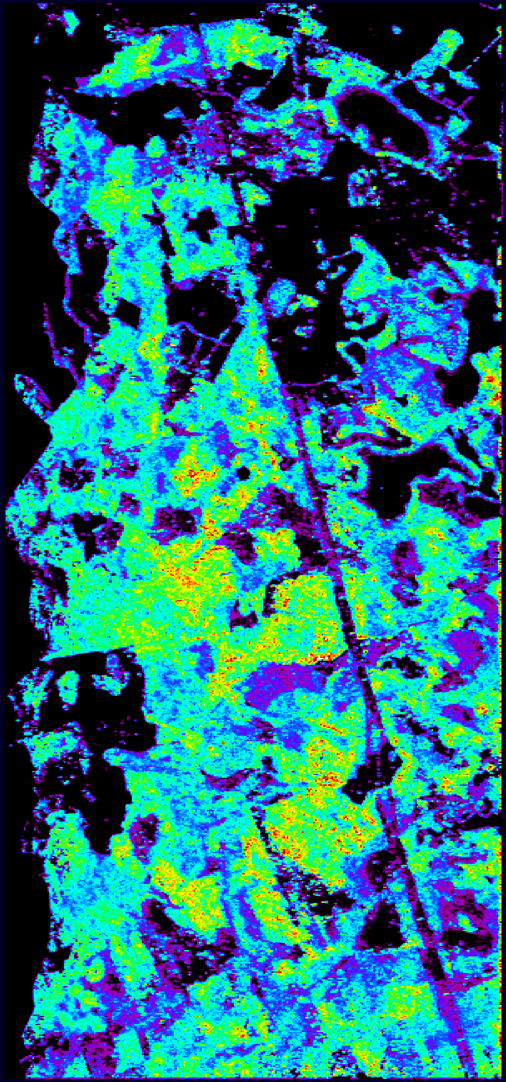
Interferometric Coherence: Volume vs Temporal Decorrelation



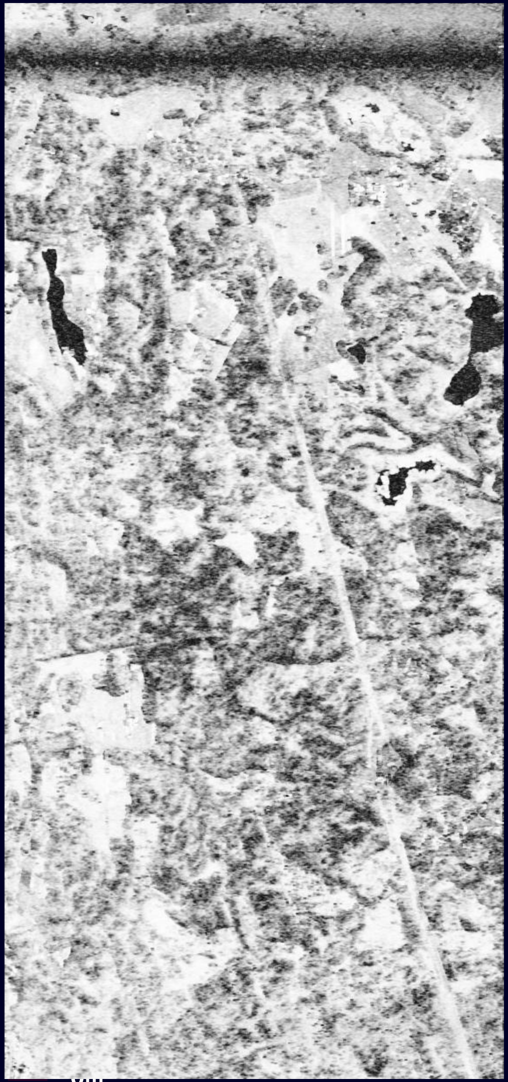
Amplitude Image HH



Volume Coherence

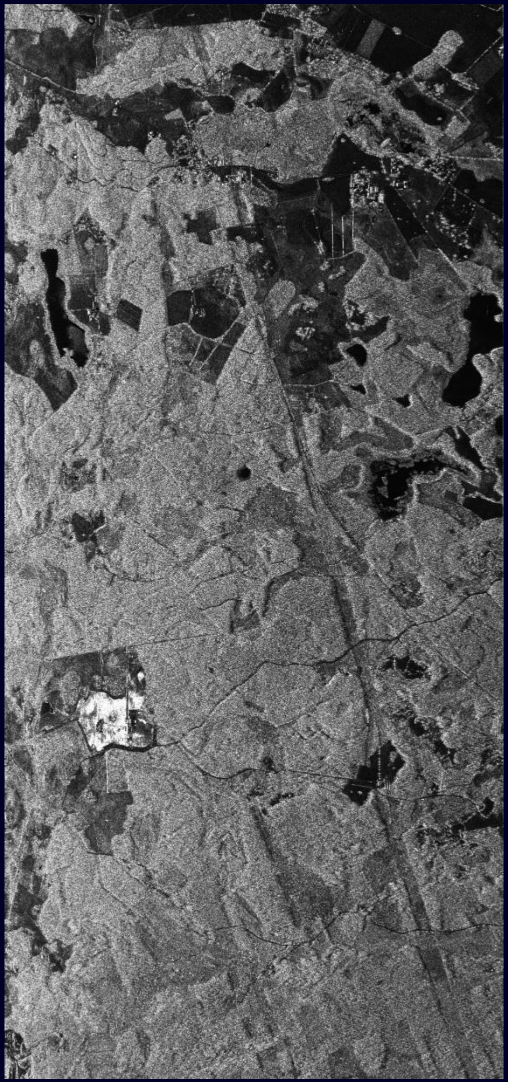


Forest Height Map

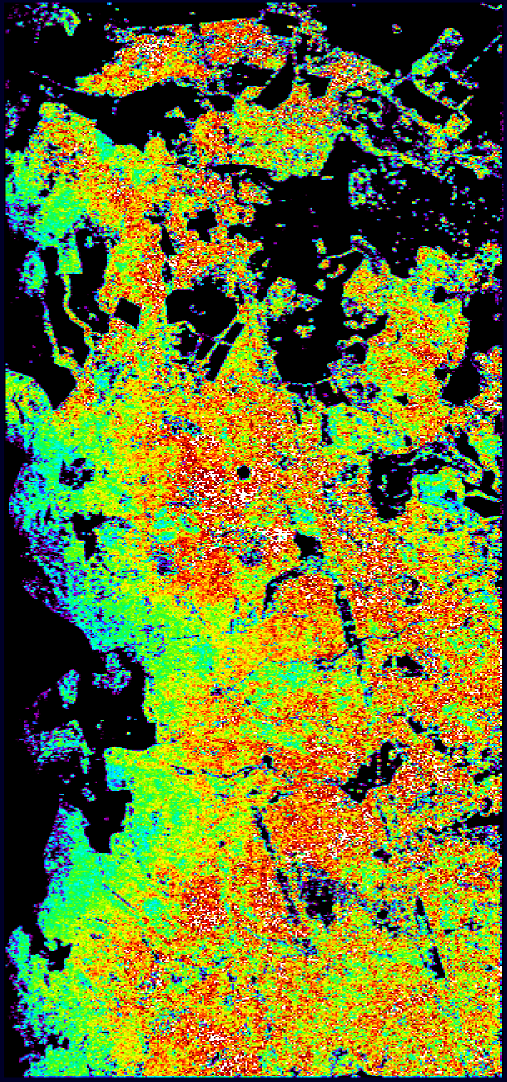


Volume + Temporal 24h

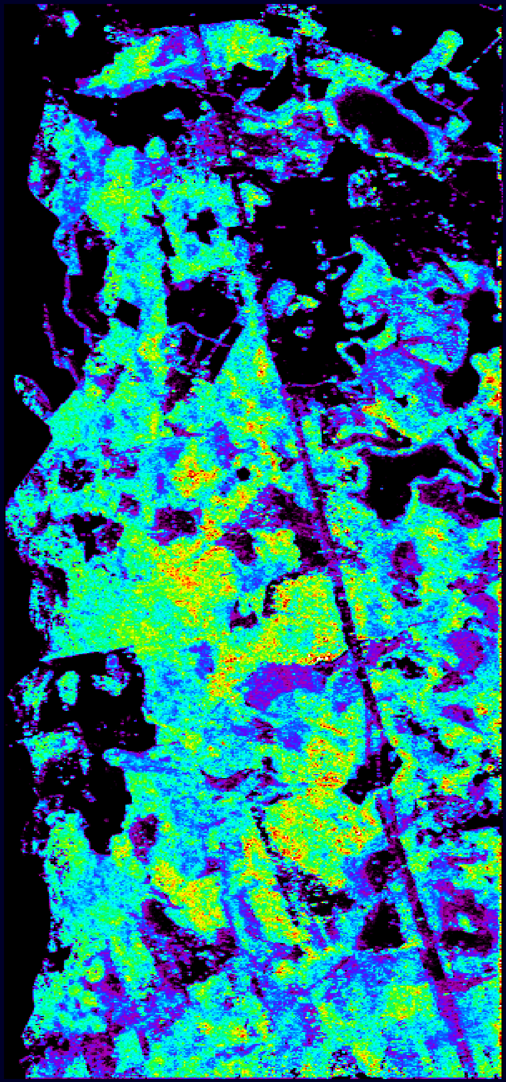
Interferometric Coherence: Volume vs Temporal Decorrelation



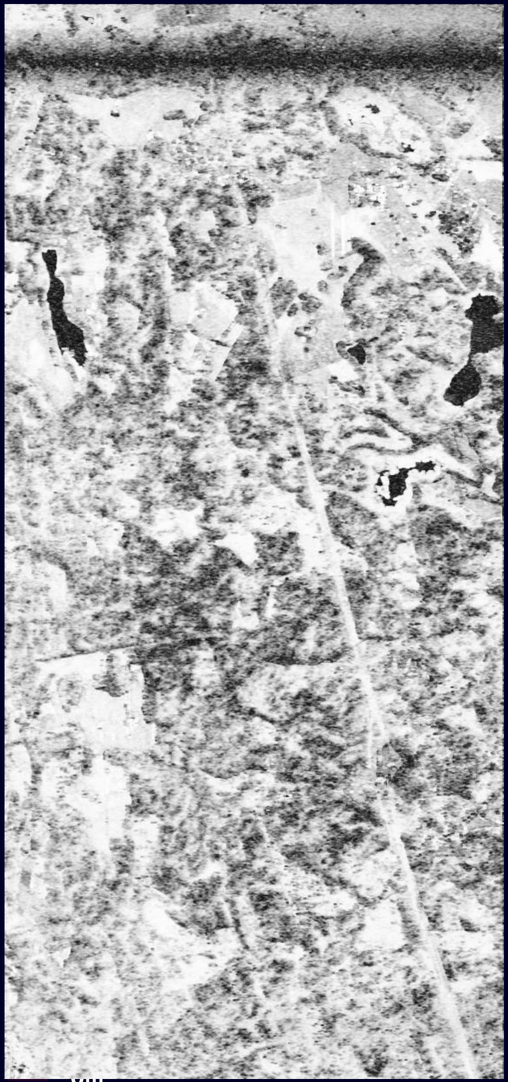
Amplitude Image HH



Forest Height Map



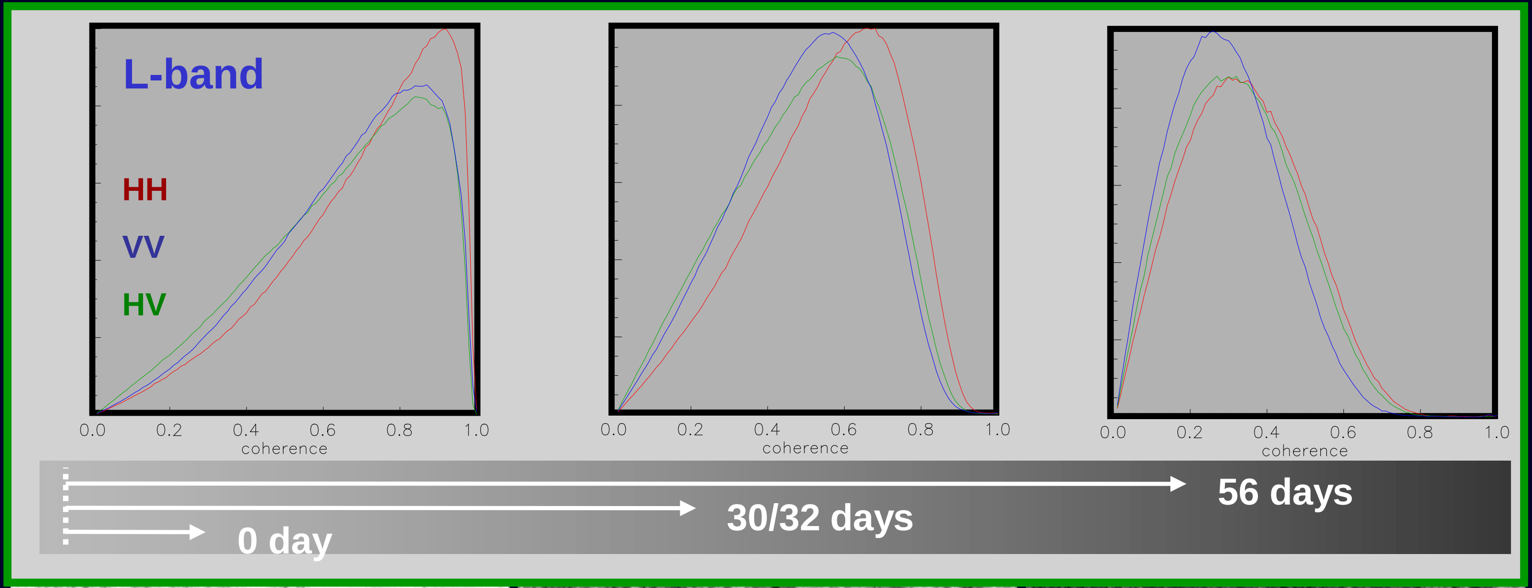
Forest Height Map



Volume + Temporal 24h

Remningstorp Test Site: Temporal Decorrelation: L-band

HV Amplitude Image

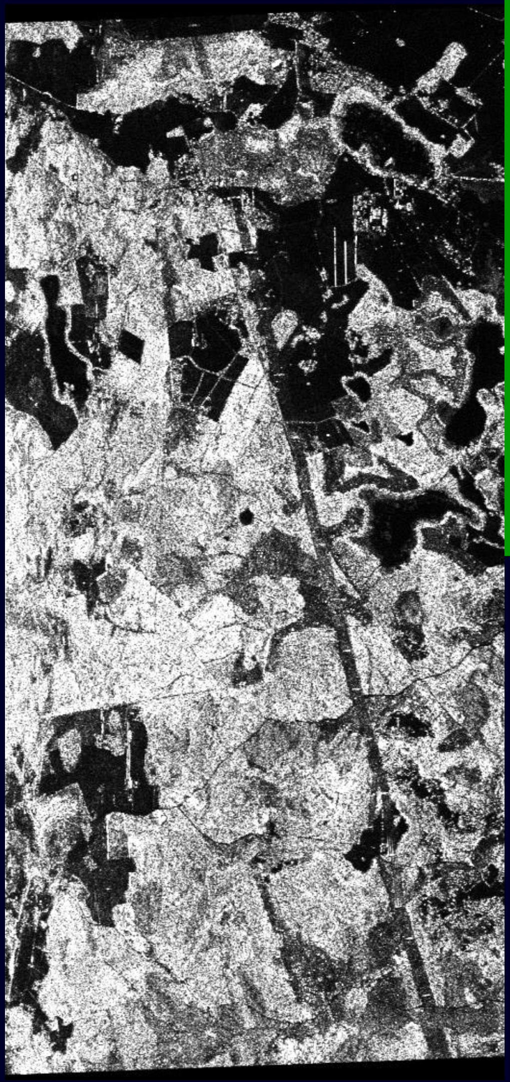


0days

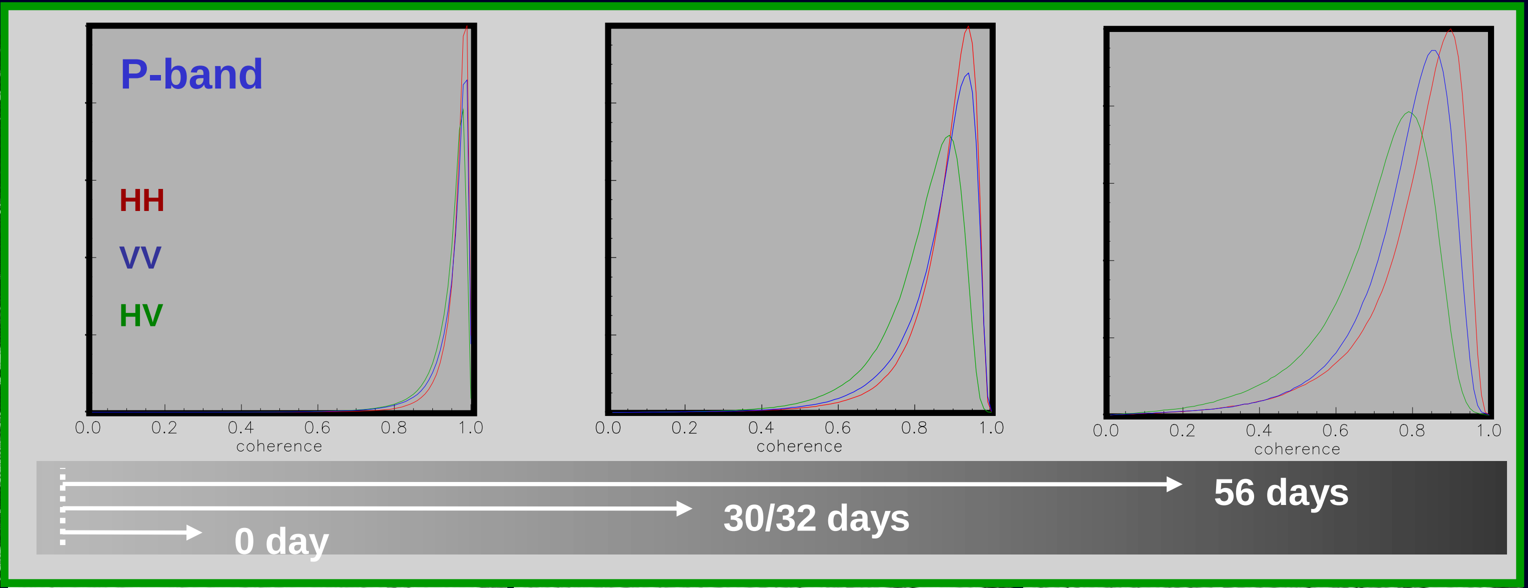
30days

58days

Remningstorp Test Site: Temporal Decorrelation: P-Band



HV Amplitude Image



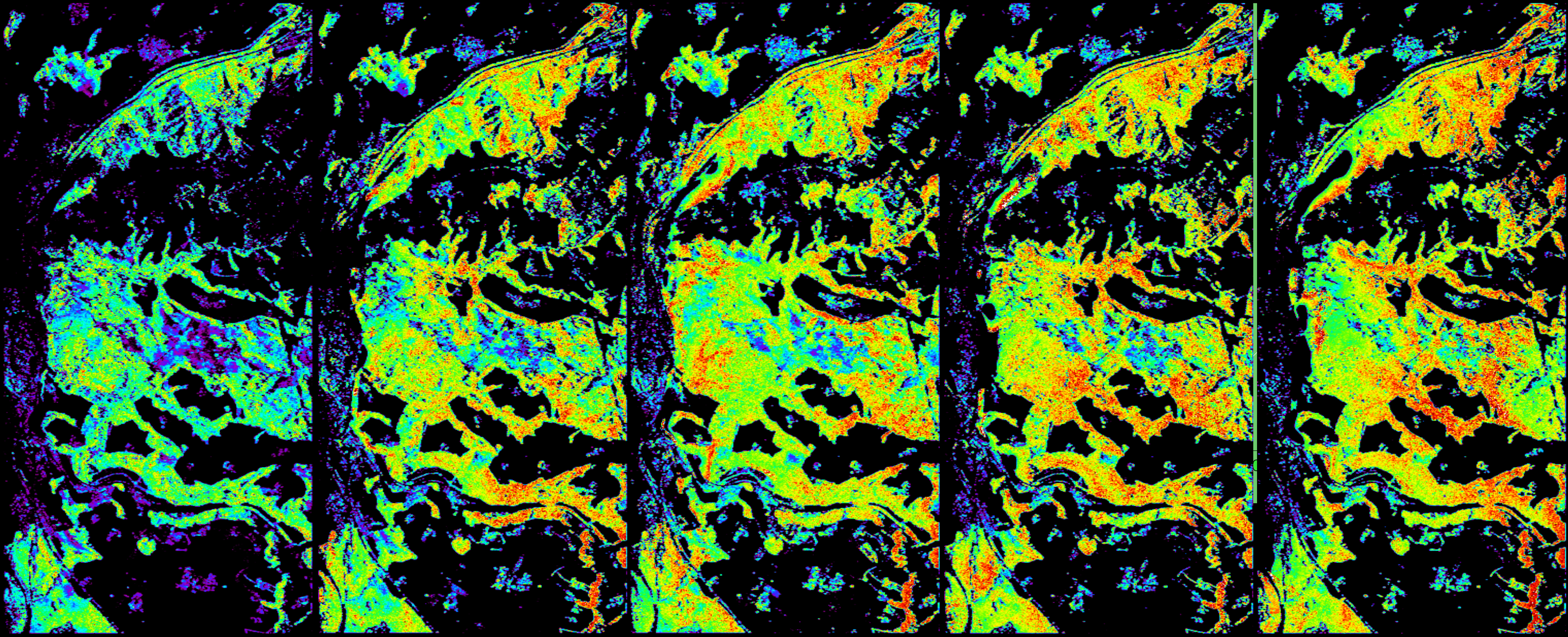
0days

30days

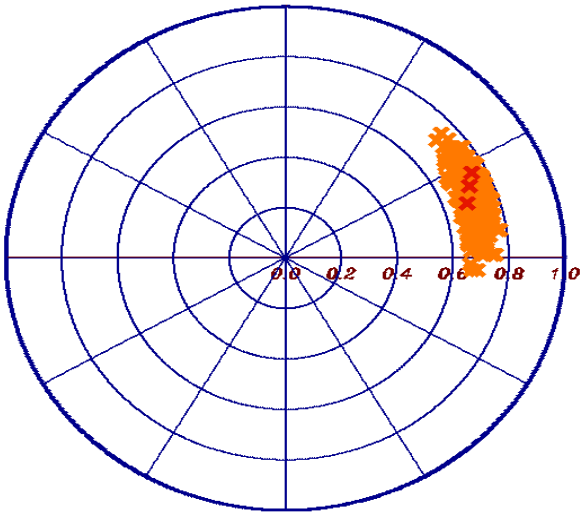
56days

Traunstein Test Site

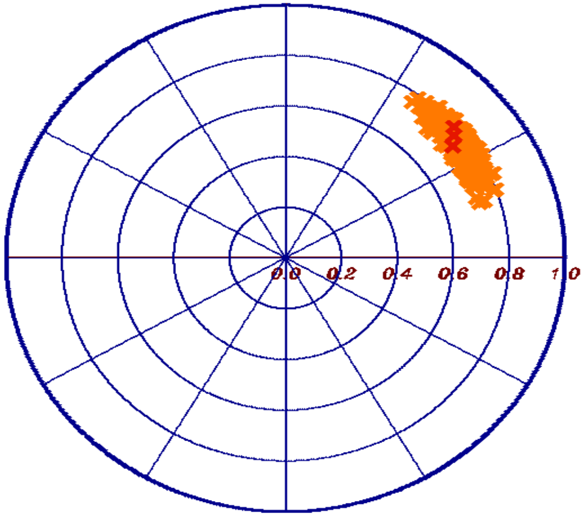
Forest Height Maps from different Temporal Baselines: 10min-13days



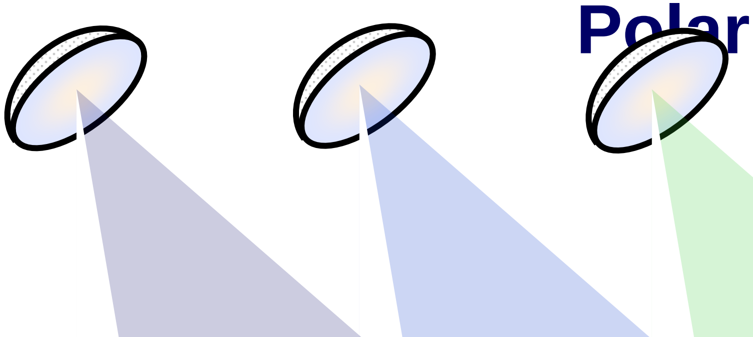
ΔT=10min ΔT=1day ΔT=5days ΔT=7days ΔT=13days



Polarimetric Coherence Tomography (PCT)



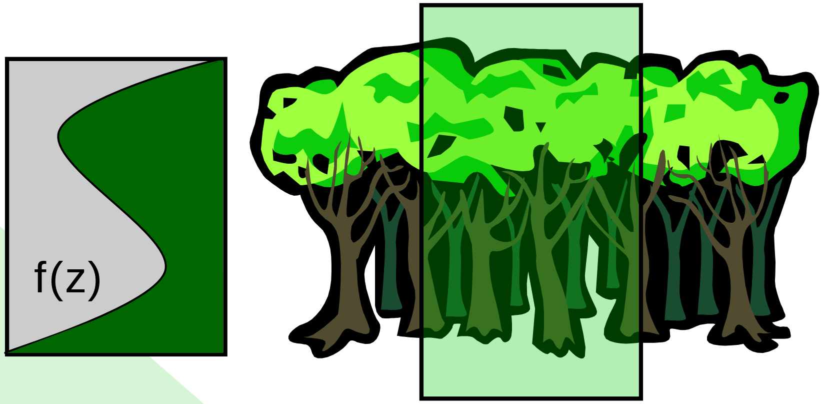
Polarimetric Coherence Tomography (PCT)

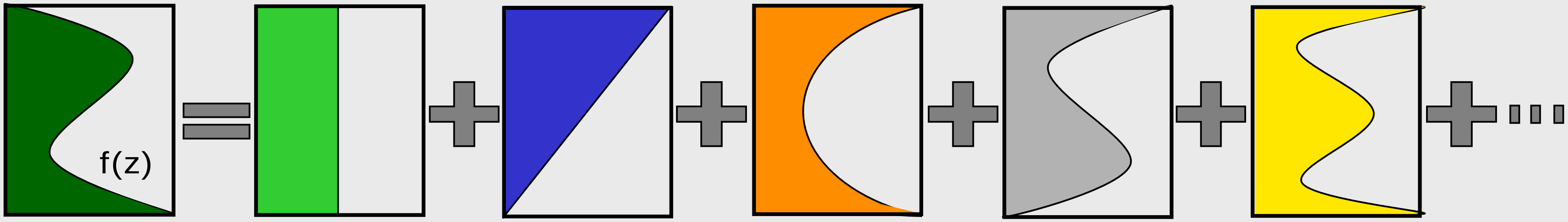


f(z) … vertical reflectivity function

Volume Coherence

$$\tilde{\gamma}_{Vol}(f(z)) = e^{ik_z z_o} \frac{\int_o^{h_v} f(z)\, e^{ik_z z} dz}{\int_o^{h_v} f(z)\; dz}$$





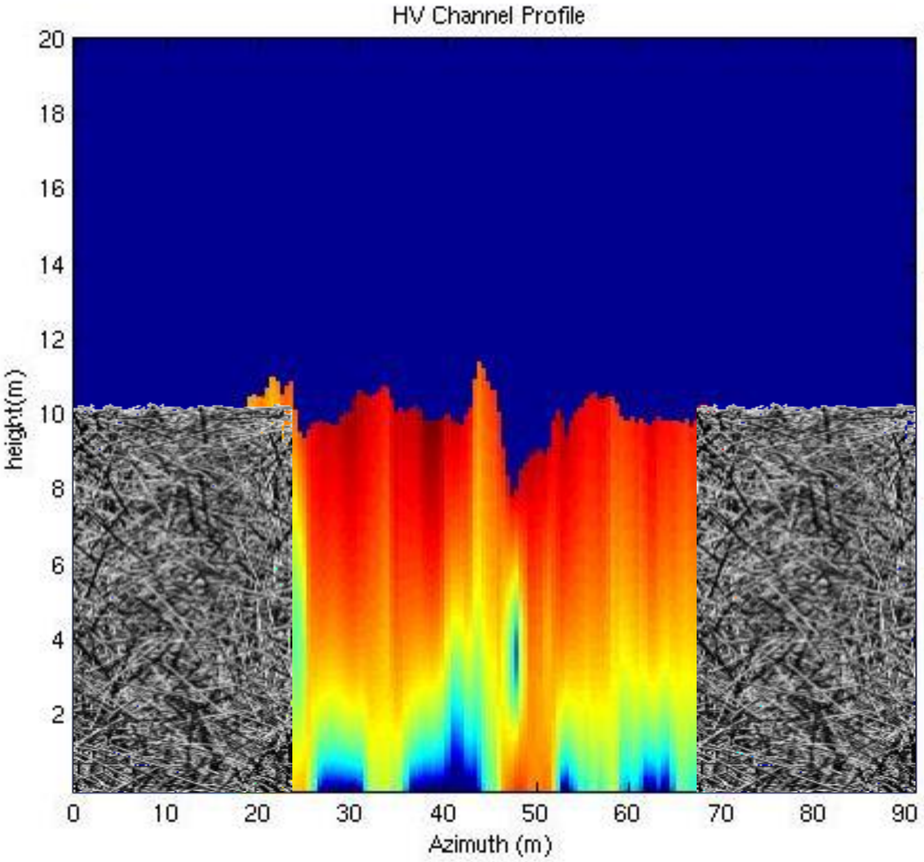
$$\tilde{\gamma}_{Vol}(f(z)) = e^{ik_z z_o} \frac{\int_o^{h_v} f(z)\, e^{ik_z z} dz}{\int_o^{h_v} f(z)\, dz}$$

$$\int_0^{h_v} f(z)\; e^{ik_z z} dz = \frac{h_v}{2} e^{i\frac{k_z h_v}{2}} \int_{-1}^{1} (1+f(z'))\; e^{i\frac{k_z h_v}{2} z'} dz'$$

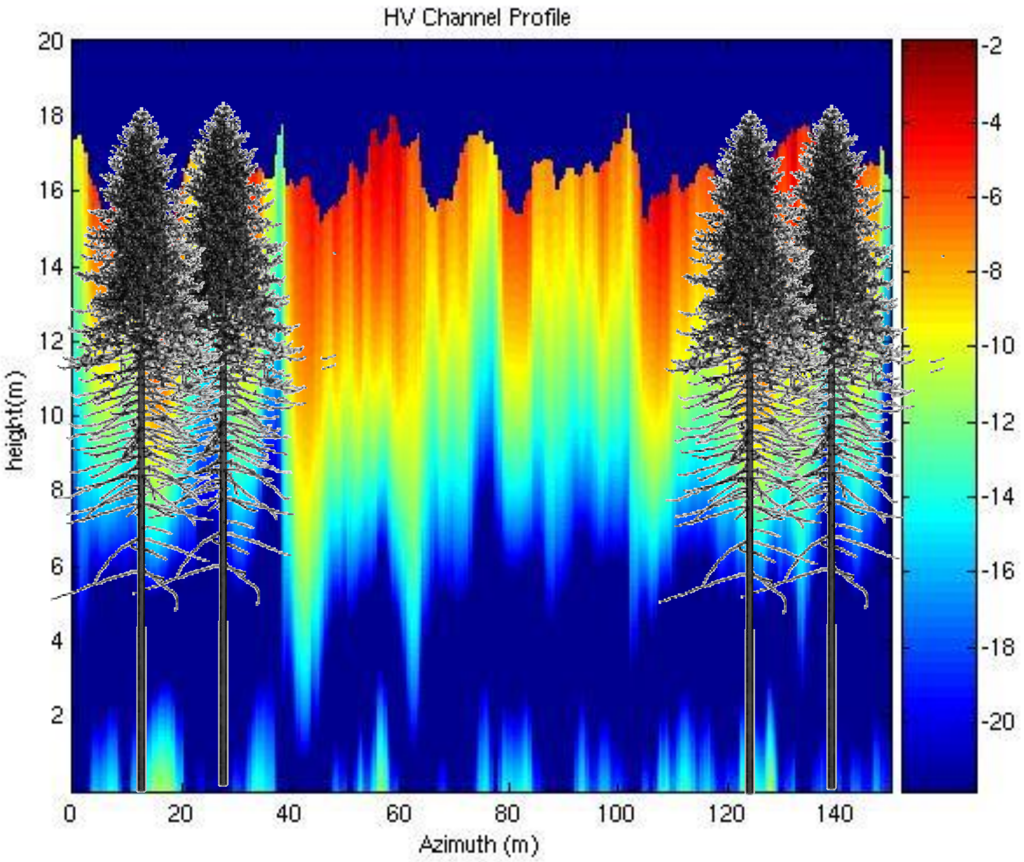
$$\int_0^{h_v} f(z)\; dz = \frac{h_v}{2} \int_{-1}^{1} (1+f(z'))\; dz'$$

Fourier Legendre Series: $f(z') = \sum_n a_n P_n(z')$ **where** $a_n = \frac{2n+1}{2} \int_{-1}^{1} f(z') P_n(z') dz'$

PCT



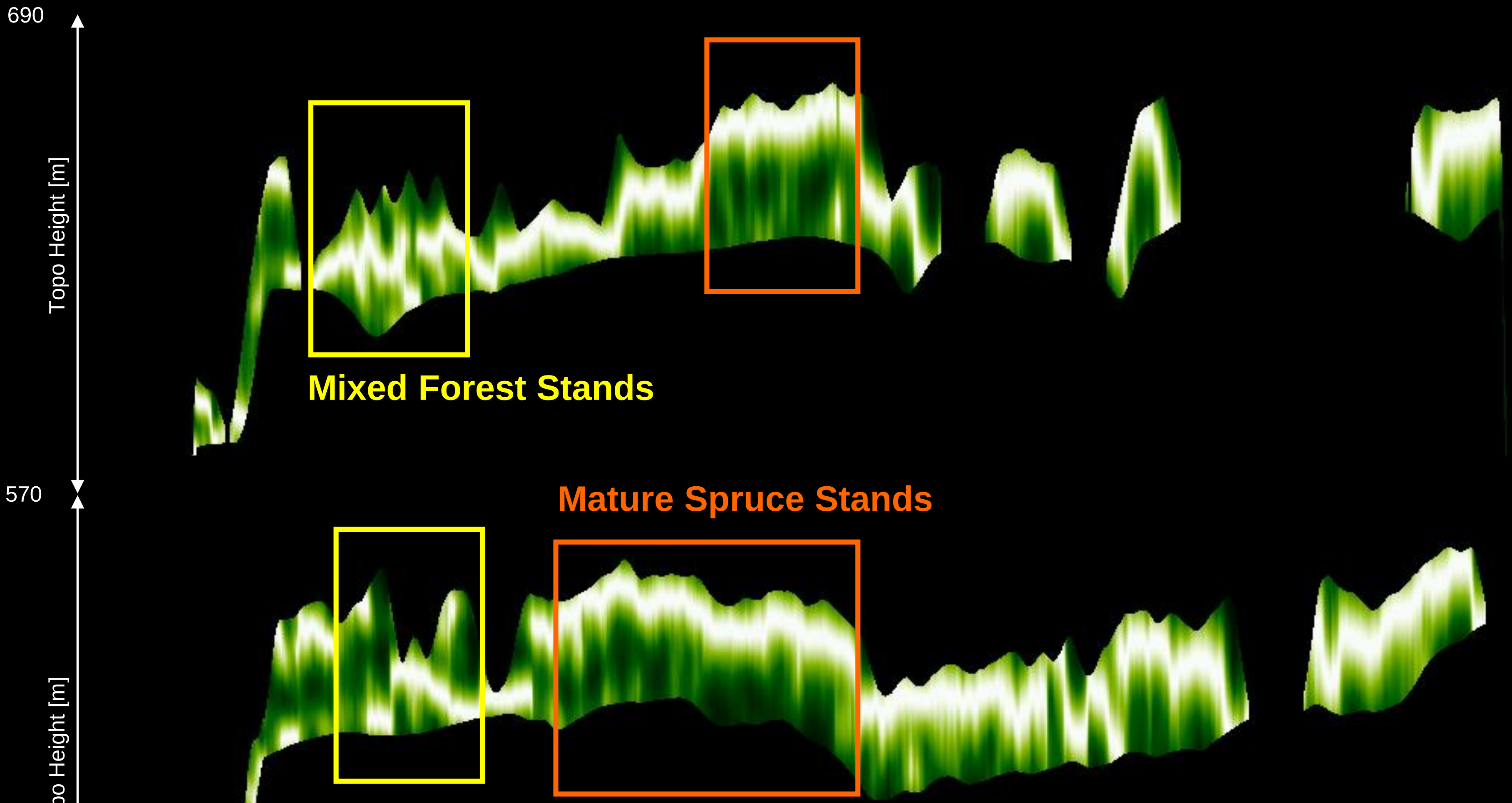
10m Uniform Hedge



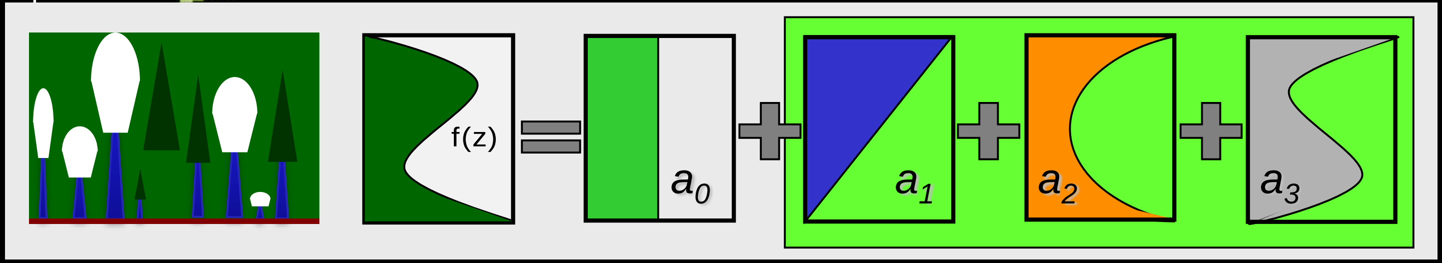
17m Scots Pine Forest

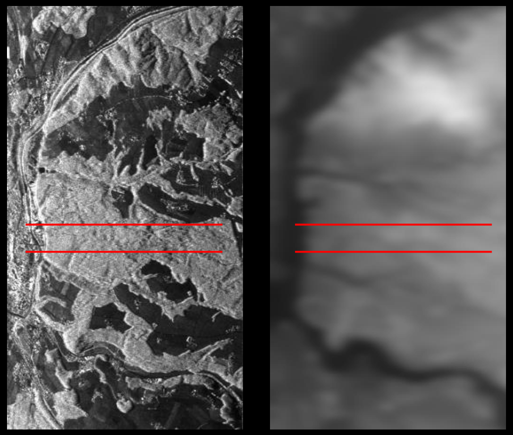
Simulations courtesy of Mark Williams:

S.R. Cloude, D.G. Corr, M.L. Williams, "Target Detection Beneath Foliage Using PolInSAR", Waves in Random Media, vol. 14, pages S393 - S414., 2004

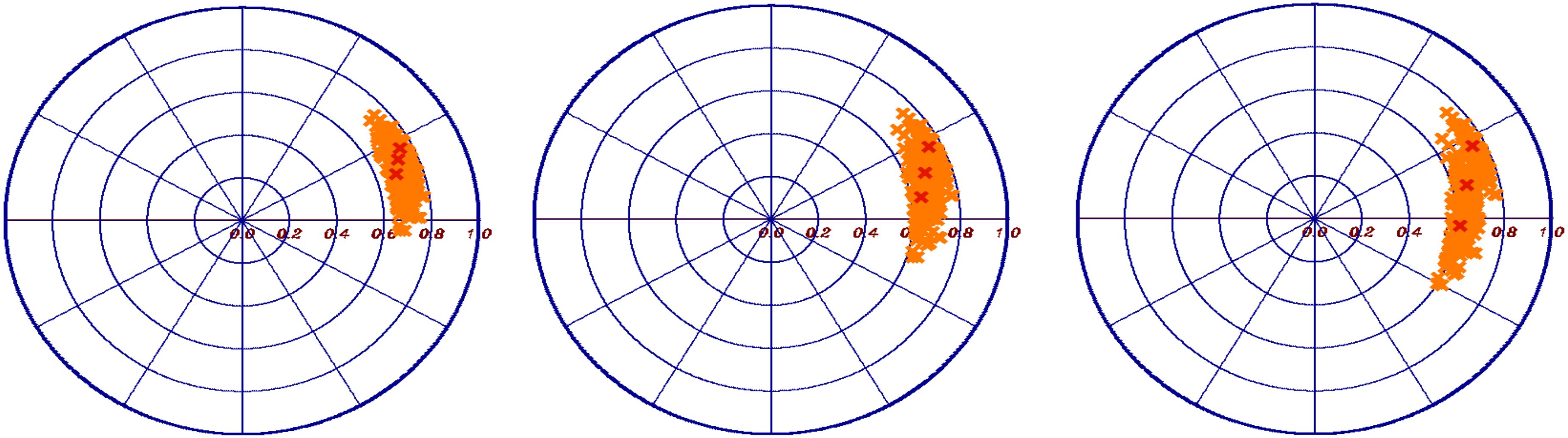


PCT Reconstruction from 2 Baselines

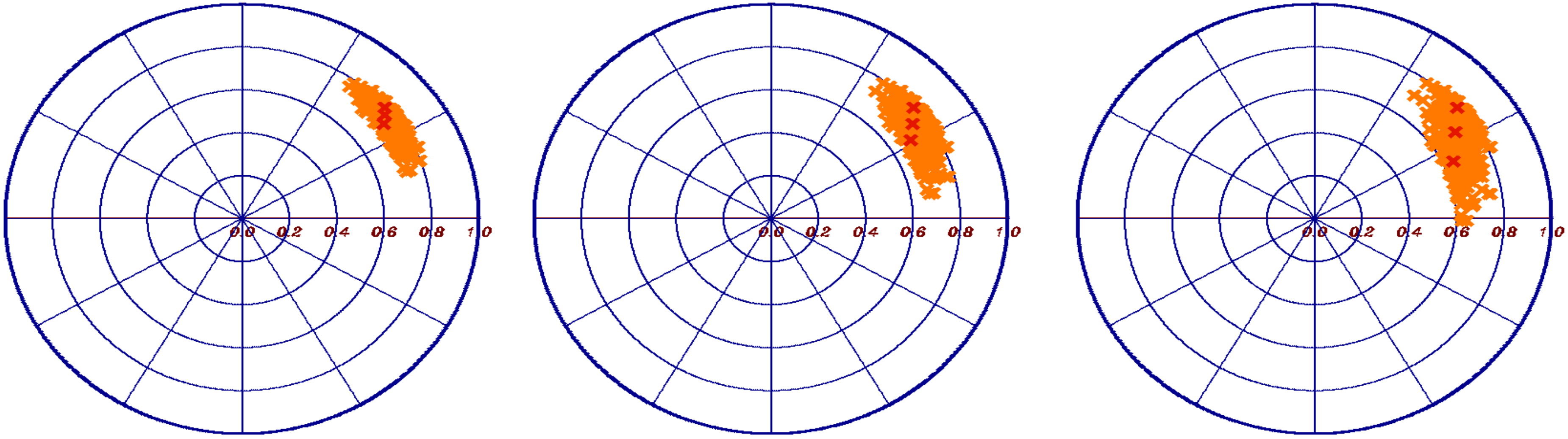




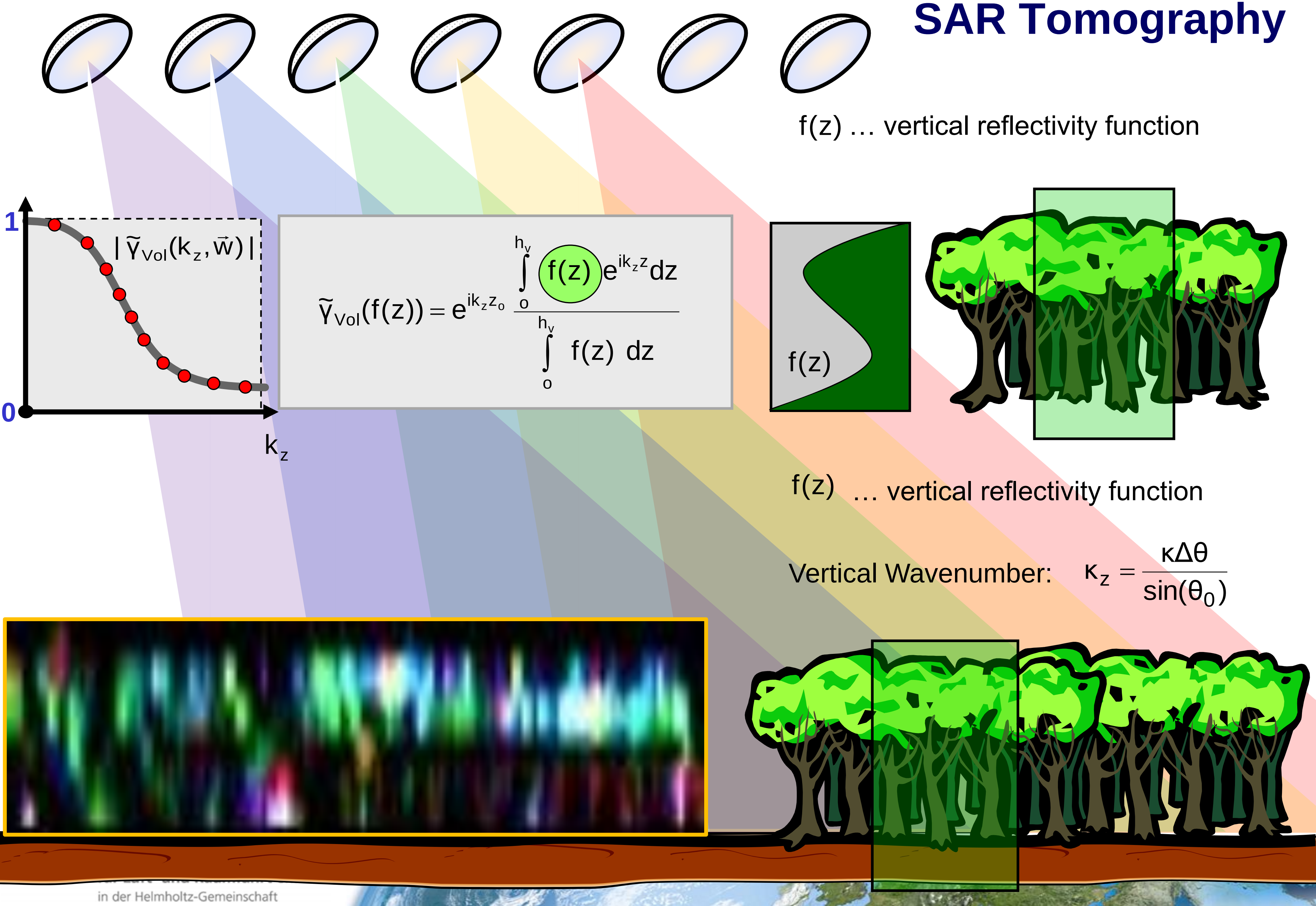
Test site: Traunstein, Germany, L-band @ HV Polarisation



Polarimetric SAR Tomography (TomoSAR)



SAR Tomography

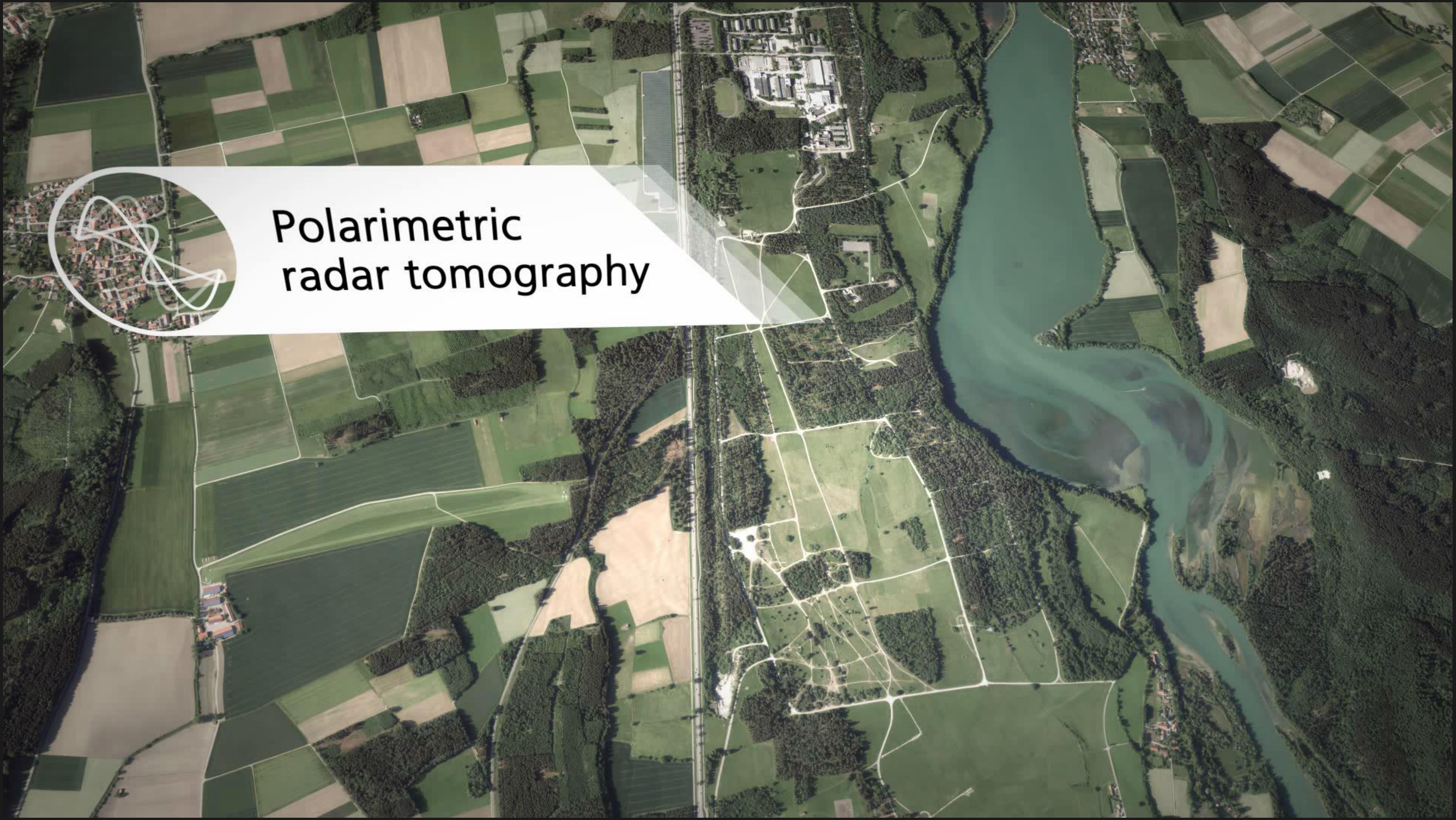


f(z) … vertical reflectivity function

$$\tilde{\gamma}_{Vol}(f(z)) = e^{ik_z z_0} \frac{\int_0^{h_v} \boxed{f(z)} e^{ik_z z} dz}{\int_0^{h_v} f(z) \; dz}$$

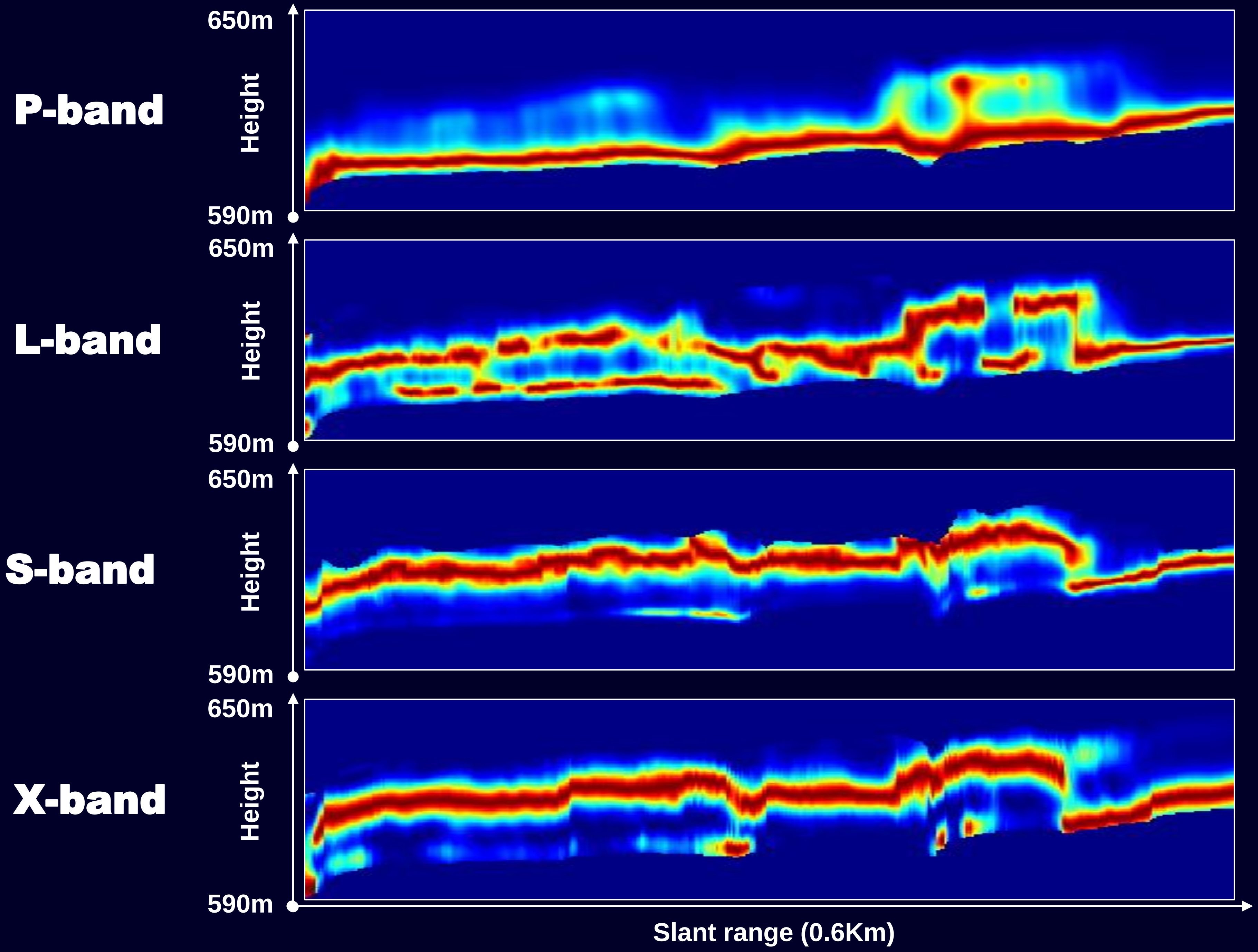
f(z) … vertical reflectivity function

Vertical Wavenumber: $\kappa_z = \frac{\kappa \Delta\theta}{\sin(\theta_0)}$

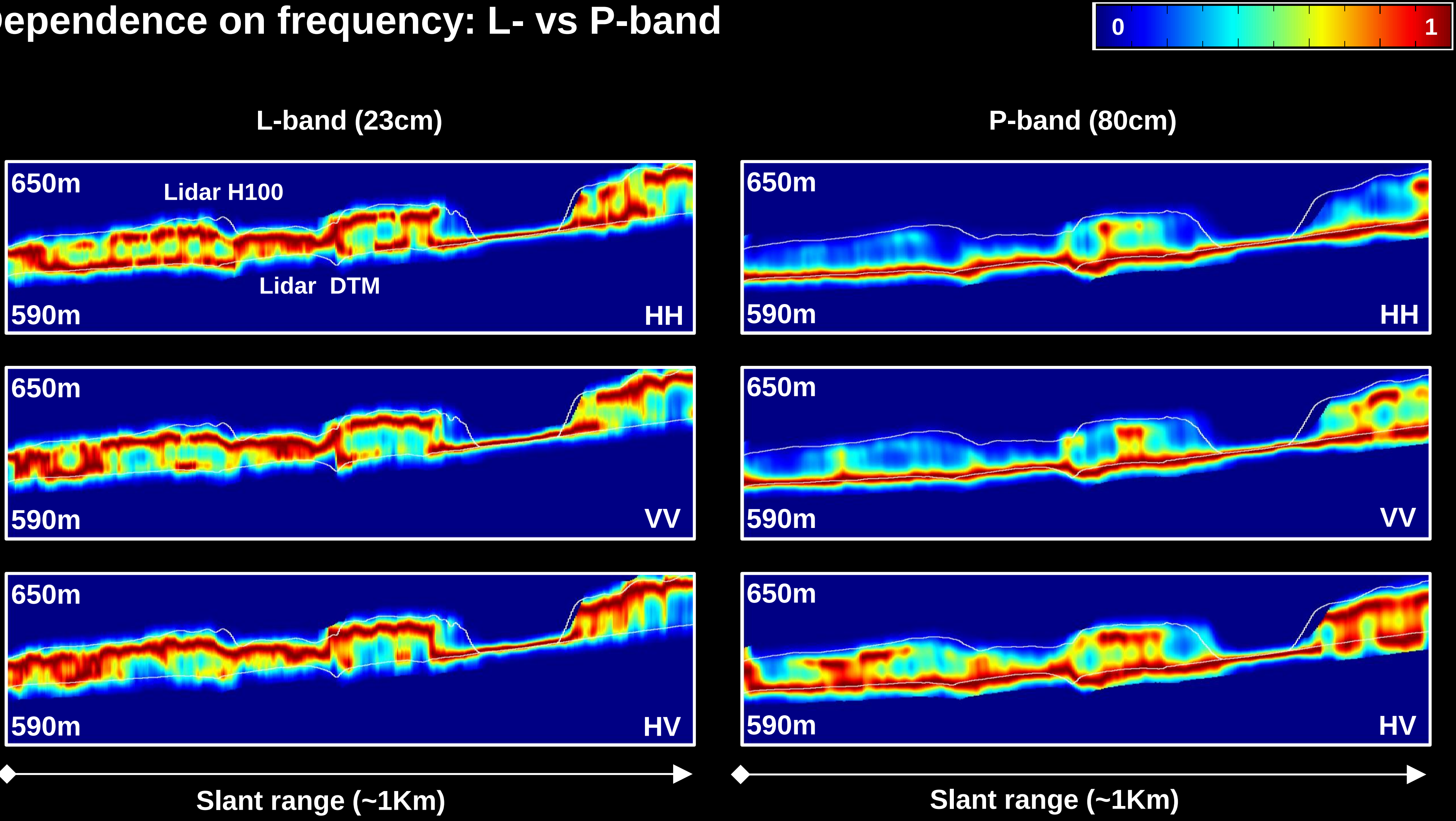


Polarimetric radar tomography

Traunstein forest (Germany) - Capon - HH

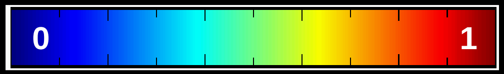


Dependence on frequency: L- vs P-band

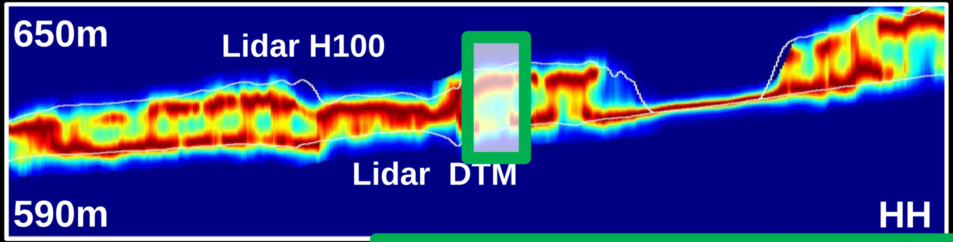


Traunstein site (Germany), 4 tracks L / 5 tracks P, E-SAR, Capon

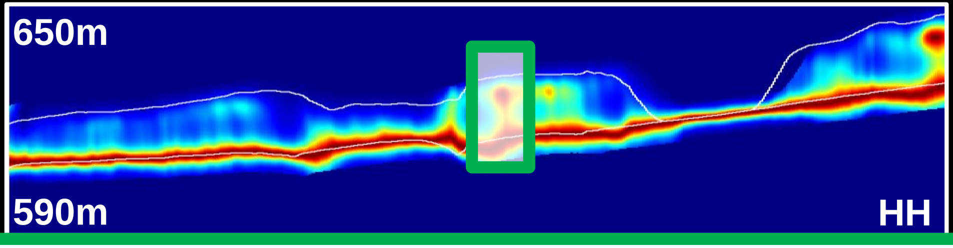
Dependence on frequency: L- vs P-band

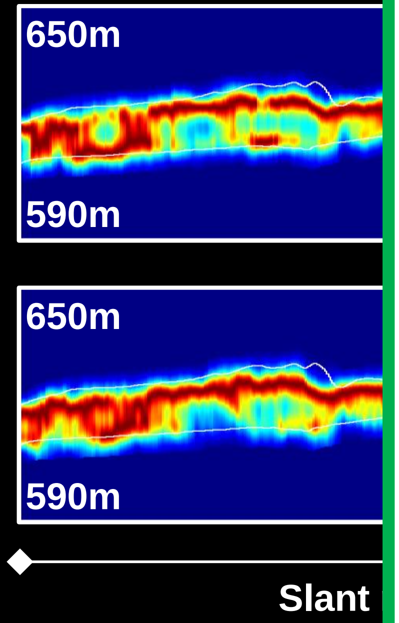


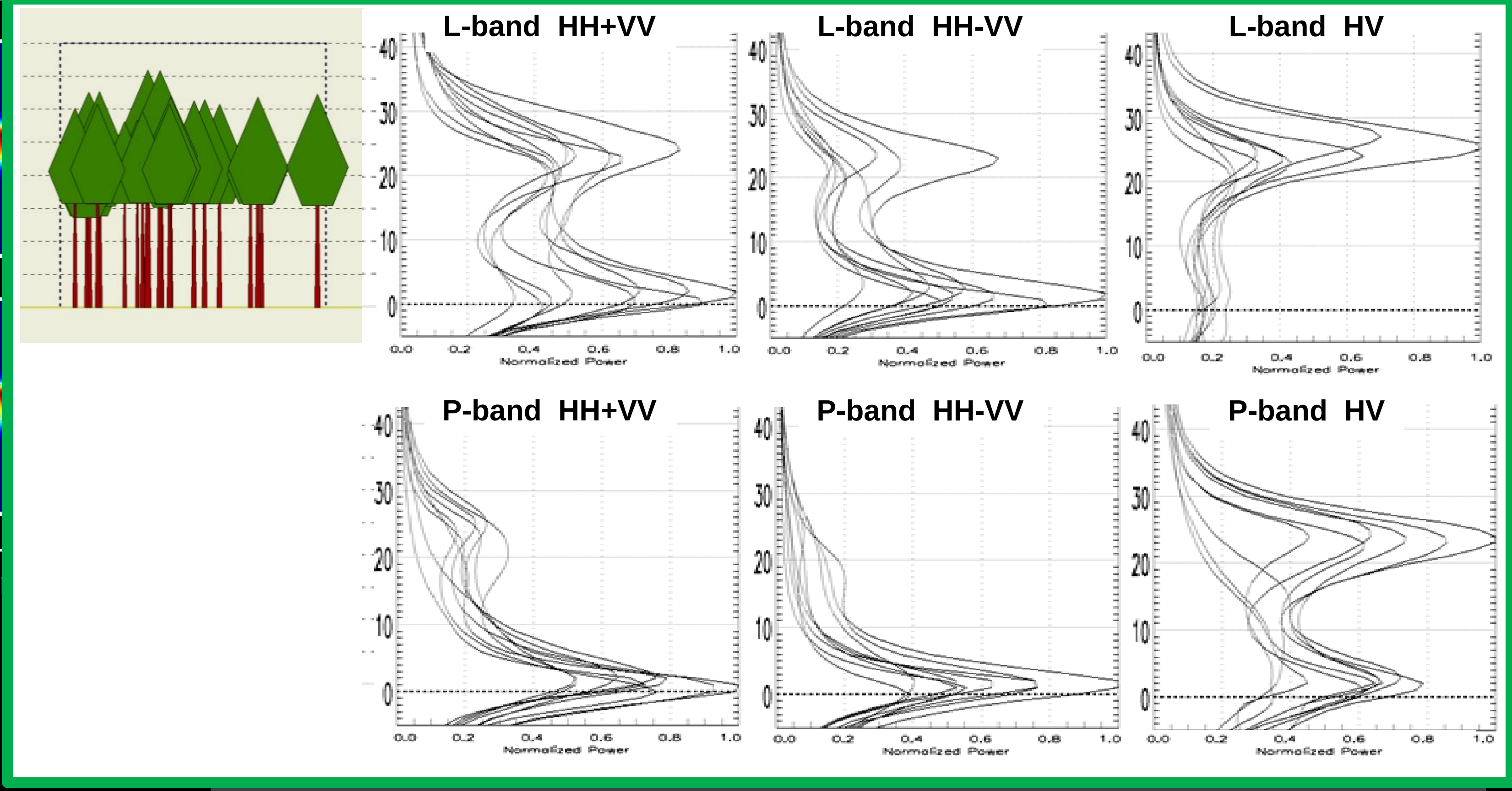
L-band (23cm)



P-band (80cm)

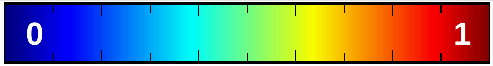


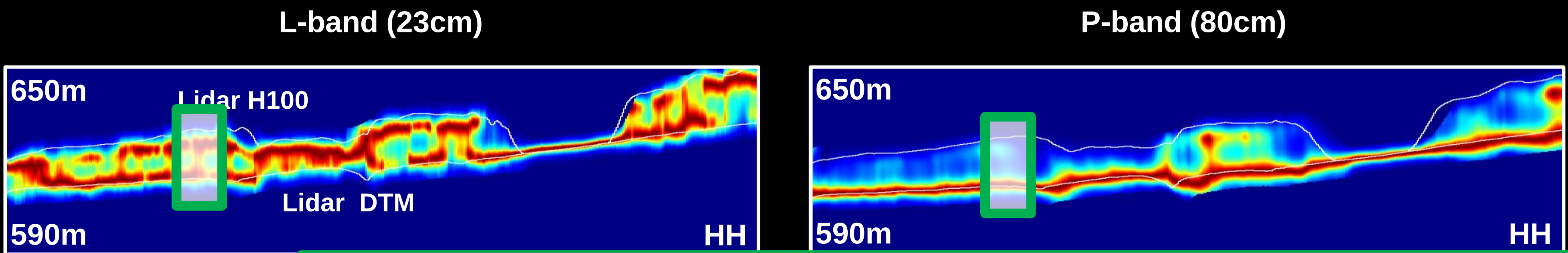


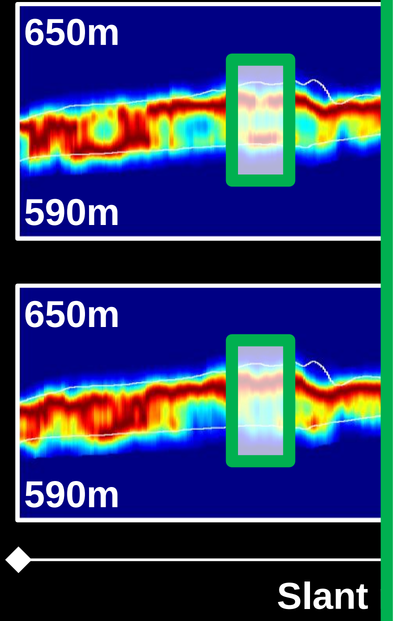


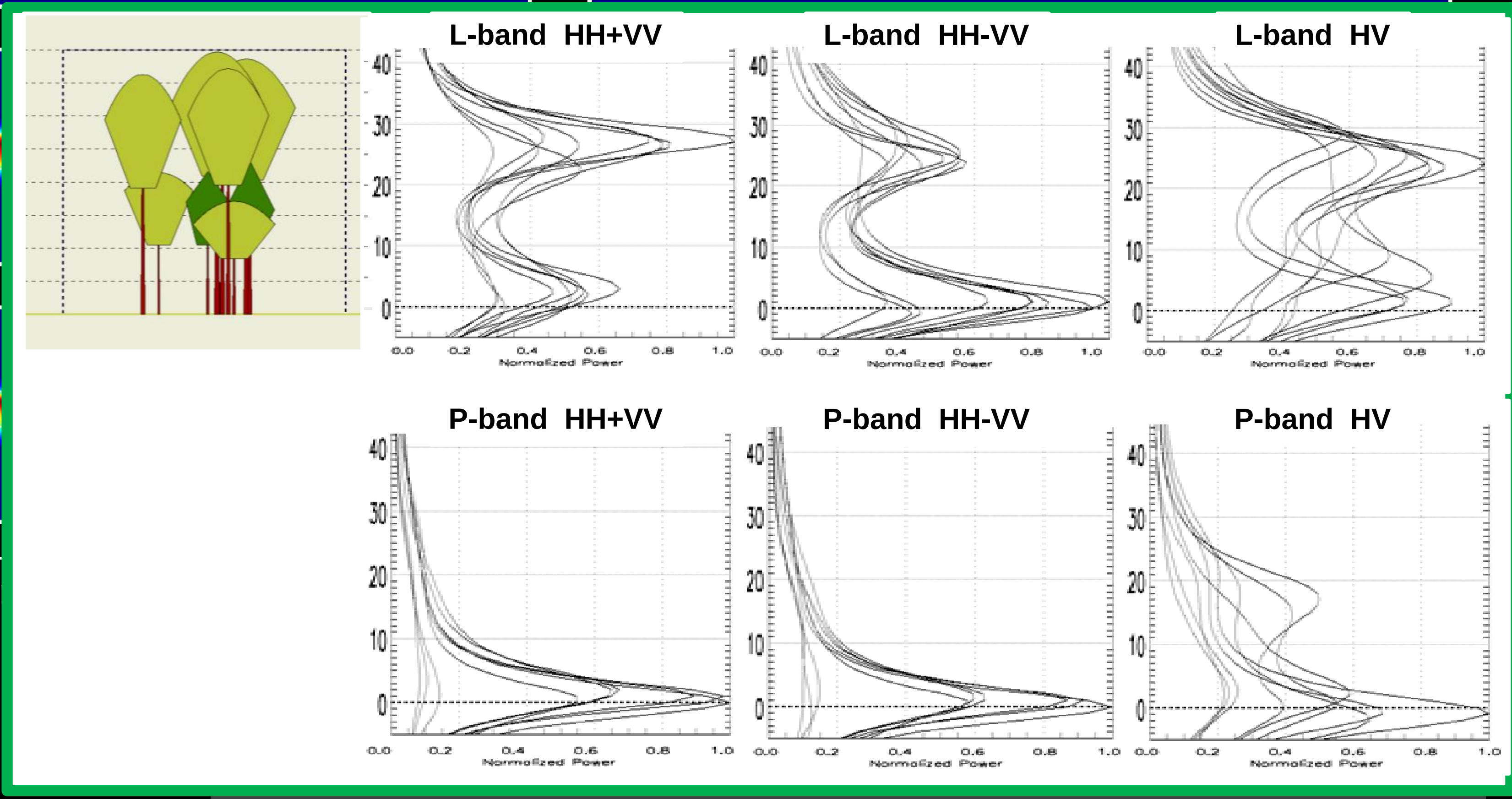
Traunstein site (Germany), 4 tracks L / 5 tracks P, E-SAR, Capon

Dependence on frequency: L- vs P-band



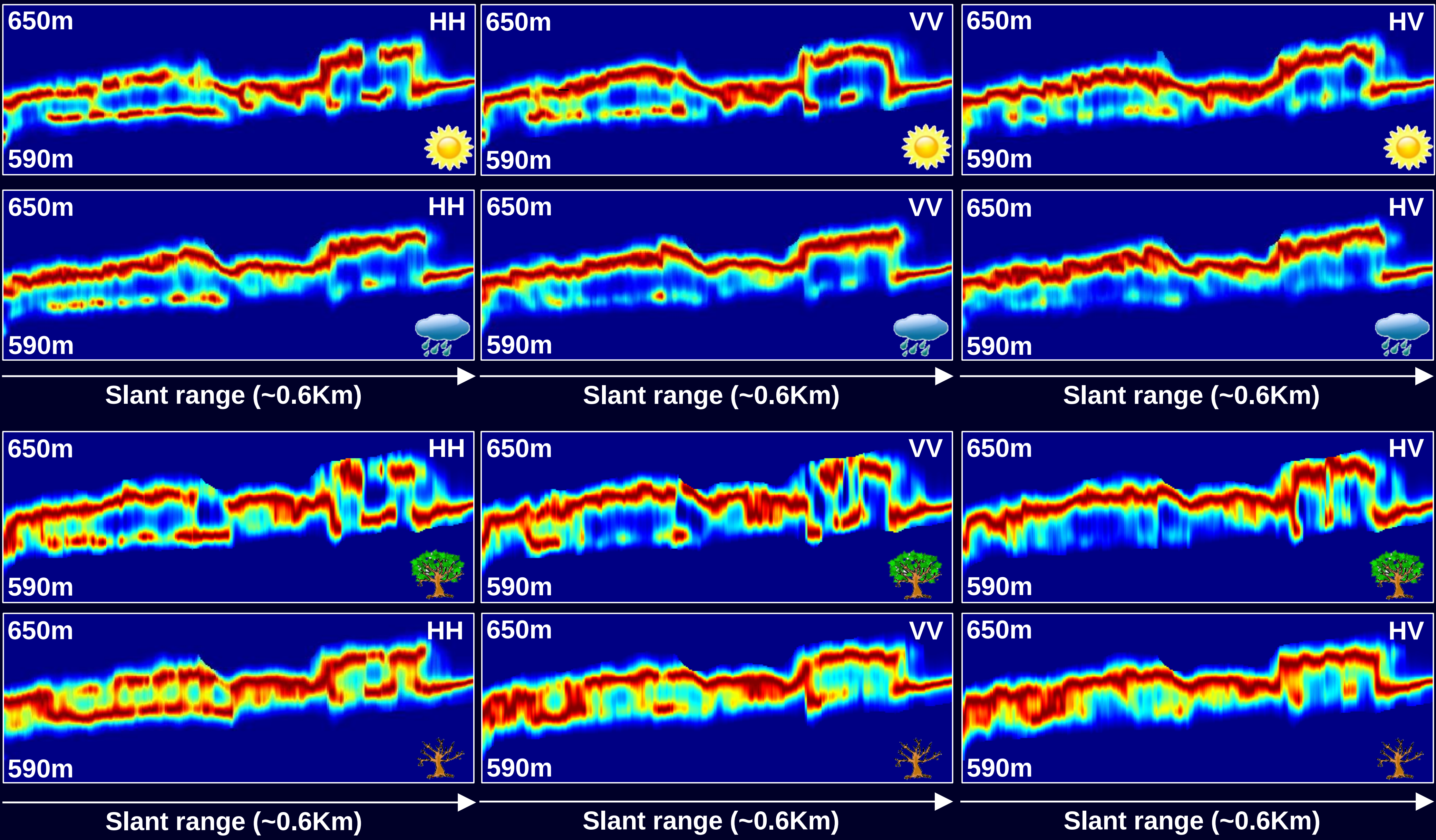






Traunstein site (Germany), 4 tracks L / 5 tracks P, E-SAR, Capon

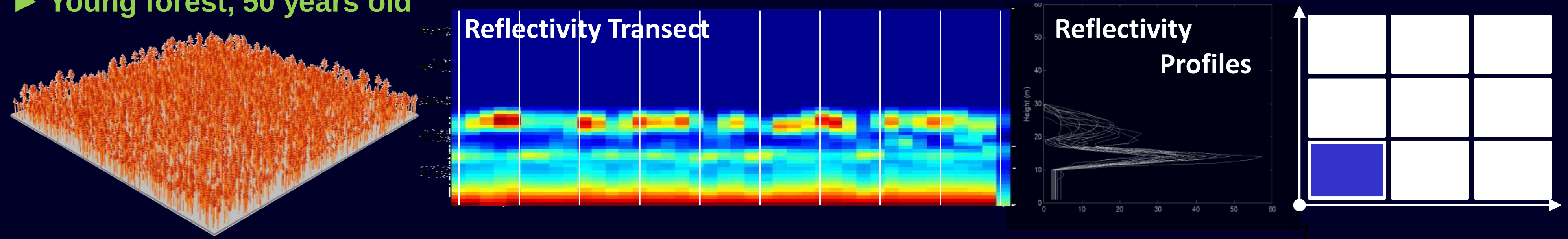
Temporal variations at L-band (Capon)



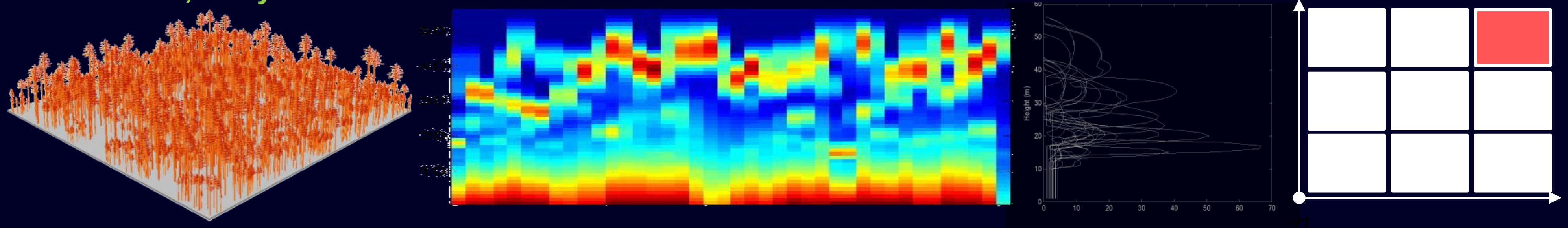
Forest Structure Characterisation

Helmholtz Alliance:
Remote Sensing and Earth System Dynamics

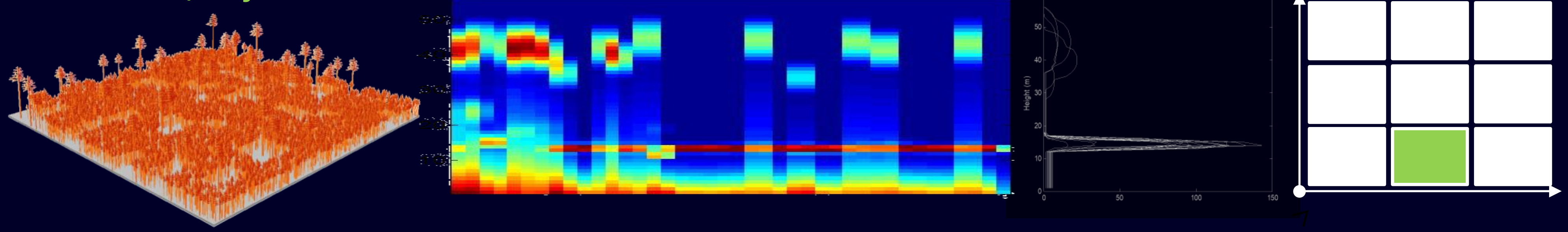
- Young forest, 50 years old



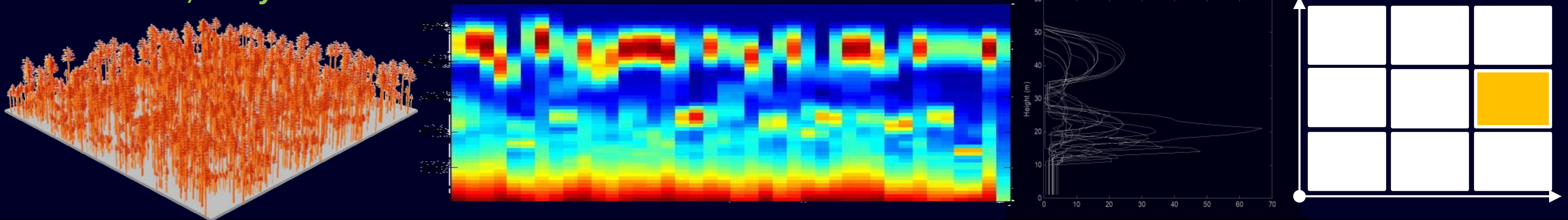
- Old forest, 500 years old

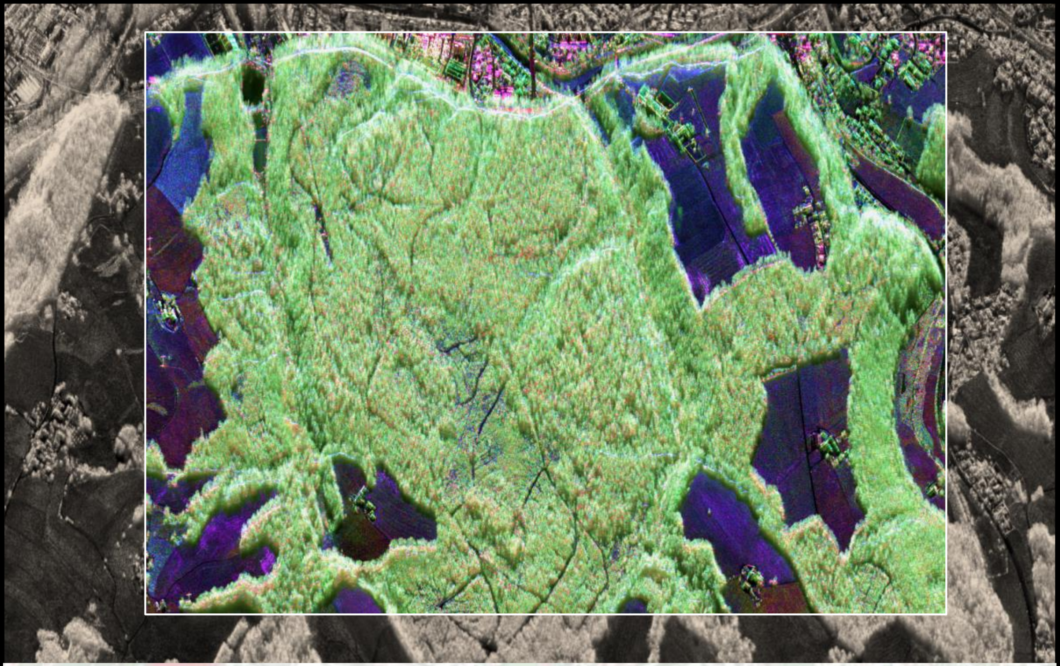


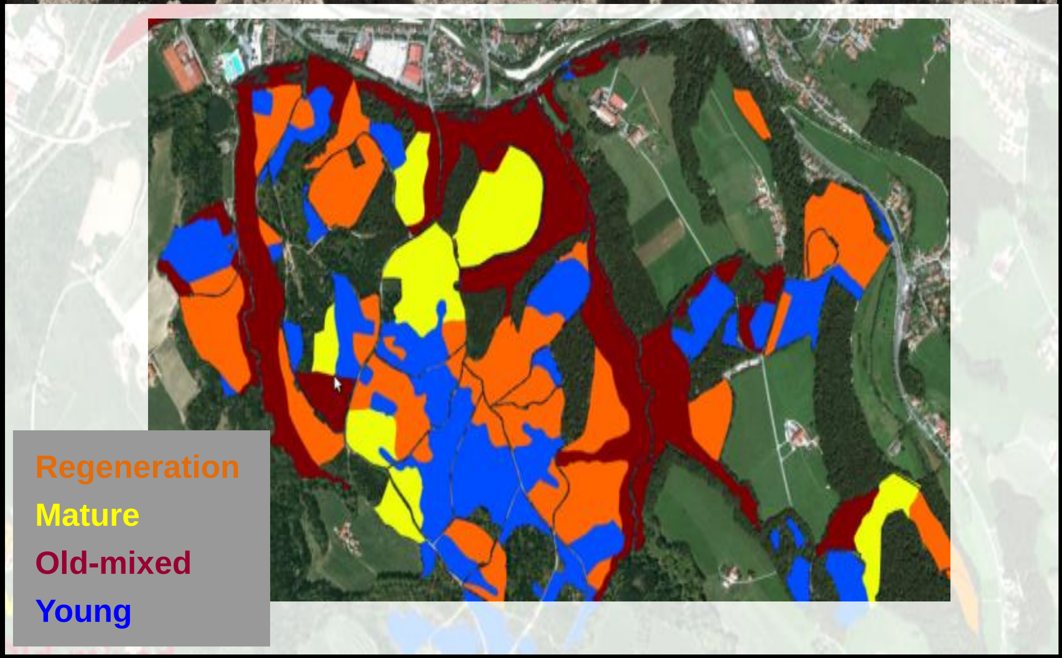
- Old forest, 10 years after a fire event



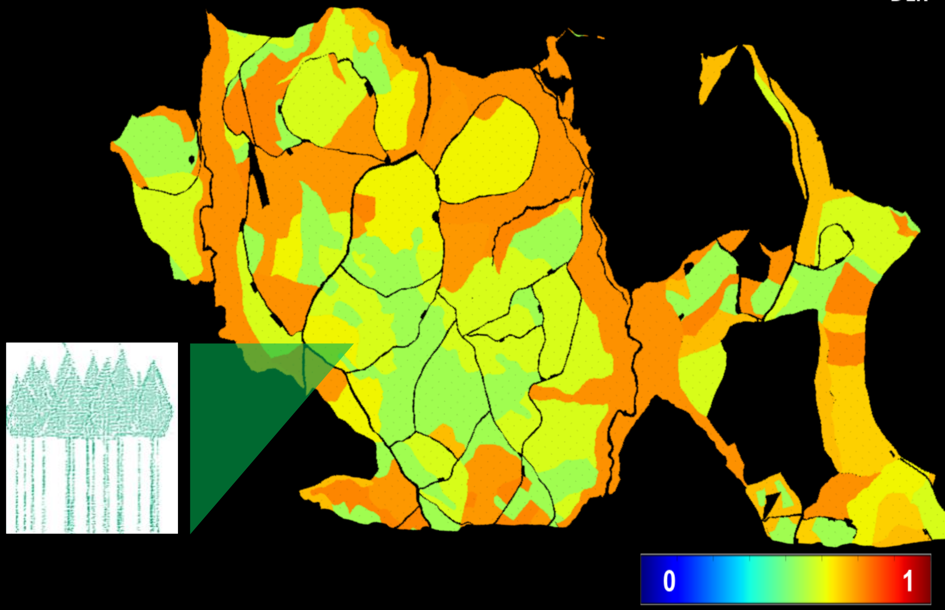
- Old forest, 200 years after a fire event



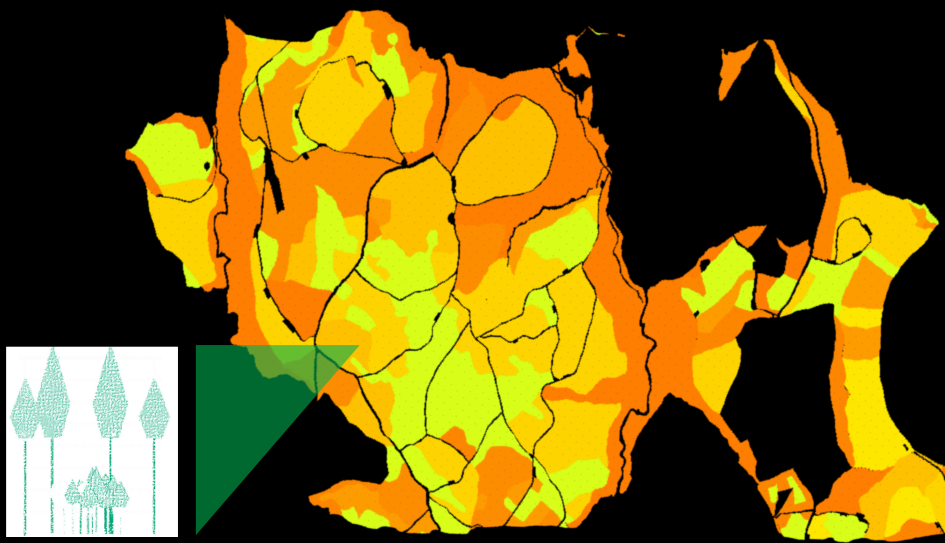




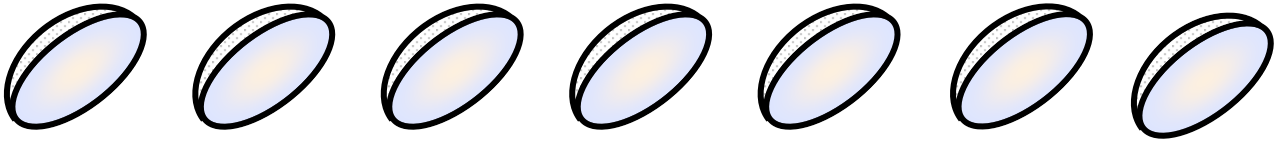
Vertical structure CM (Radar 2008)



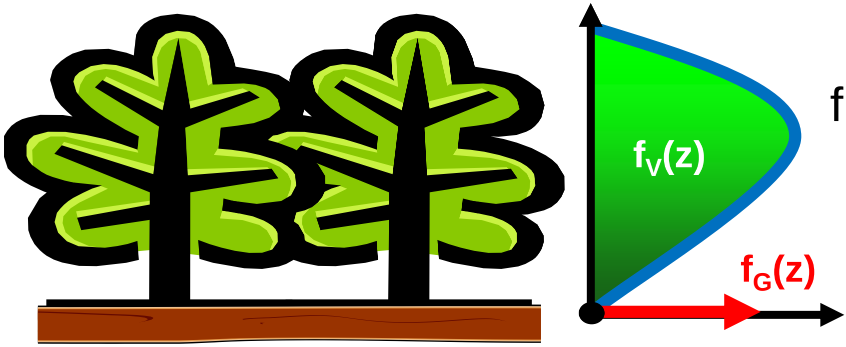
Vertical structure CM (Radar 2012)



Forest Structure Classification (25x25 m): Traunstein, Germany, 2008 / 2012



SAR Tomography
… beyond profiles



Ground Layer Volume Layer

$$f(z) = \boxed{P_G\, f_G(z)} + \boxed{P_V\, f_V(z)} \Rightarrow \mathbf{R} = \boxed{P_G\, \boldsymbol{\Gamma}_G} + \boxed{P_V\, \boldsymbol{\Gamma}_V}$$

Single-pol coherences

Assumption : $f_G(z) = \delta(z - z_G)$

P_G, P_V : backscattering powers (single-pol)

$\boldsymbol{\Gamma}_G$, $\boldsymbol{\Gamma}_V$: multi-baseline coherence matrices

From single-pol to full-pol, Random volume assumption:

$$\mathbf{R}_P = \boxed{\mathbf{C}_G \otimes \boldsymbol{\Gamma}_G} + \boxed{\mathbf{C}_V \otimes \boldsymbol{\Gamma}_V}$$

Sum of Kronecker Products (SKP)

$\mathbf{C}_G$, $\mathbf{C}_V$: polarimetric covariance matrices

In both single and multi polarimetric cases, even under an RVoG assumption and independently on the number of baselines, the separation of ground / volume InSAR coherences and polarimetric covariances does not admit a unique solution !!

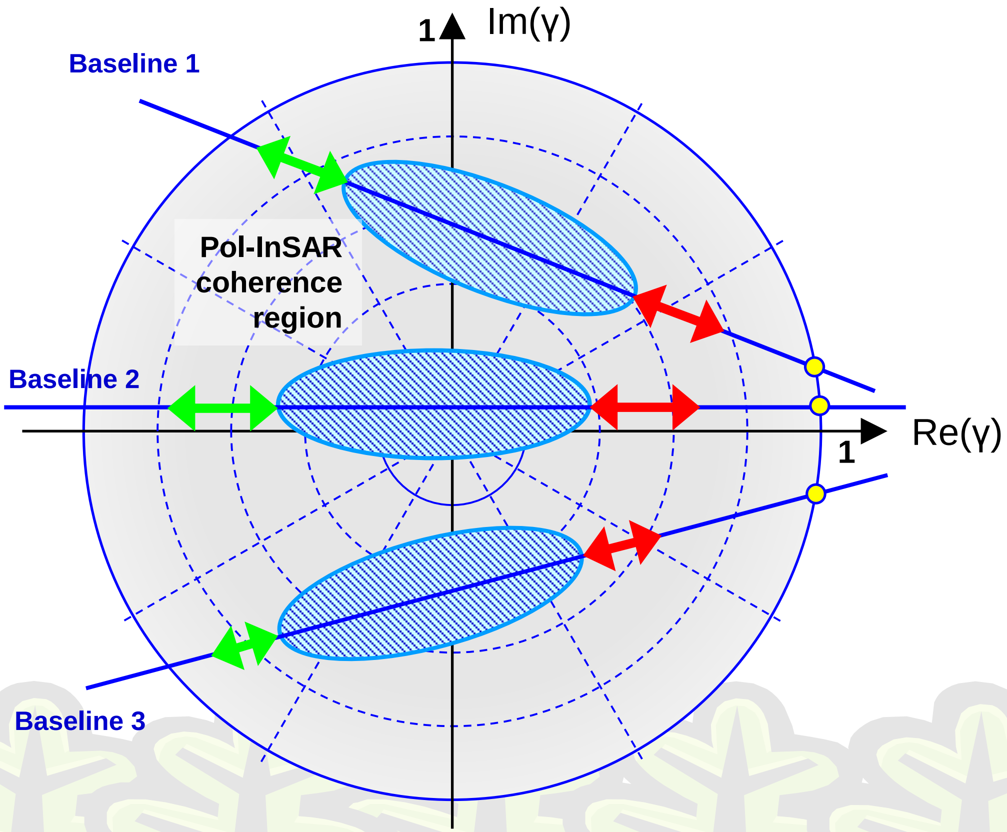
[T. Marzetta, IEEE-Proc. 1983 – S. Tebaldini, IEEE-TGARS 2009]

The Sum-of-Kronecker-Products

- It extends Pol-InSAR concepts to TomoSAR
- Based on simple algebraic tools

$$\mathbf{R_P} = \mathbf{C_G} \otimes \mathbf{\Gamma_G} + \mathbf{C_V} \otimes \mathbf{\Gamma_V}$$

Sum of Kronecker Products (SKP)



$$\hat{\mathbf{\Gamma}}_{G,V}, \hat{\mathbf{C}}_{G,V} = \underset{\mathbf{\Gamma}_{G,V},\, \mathbf{C}_{C,V}}{\arg\min} \| \mathbf{R_P} - [\, \mathbf{C_G} \otimes \mathbf{\Gamma_G} + \mathbf{C_V} \otimes \mathbf{\Gamma_V} \,] \|^2$$

$$\mathbf{\Gamma_G} = a\, \mathbf{R_1} + (1 - a)\, \mathbf{R_2}$$

$$\mathbf{\Gamma_V} = b\, \mathbf{R_1} + (1 - b)\, \mathbf{R_2}$$

$$\mathbf{C_G} = [(1 - b)\, \mathbf{C_1} - b\, \mathbf{C_2}] / (a - b)$$

$$\mathbf{C_V} = [-(1 - a)\, \mathbf{C_1} + a\, \mathbf{C_2}] / (a - b)$$

$\mathbf{R_1}$, $\mathbf{R_2}$ from SVD of a permutation of $\mathbf{R_P}$

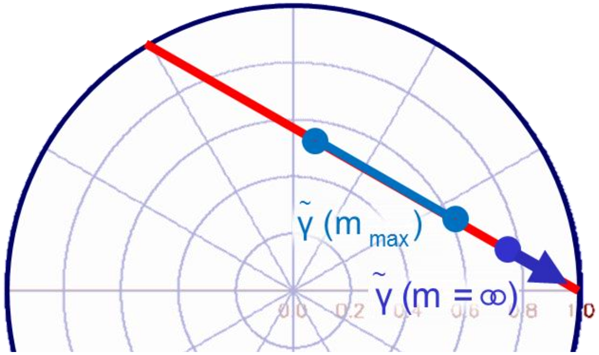
(a,b) are free parameters, bounded in order to provide positive (semi-)definite matrices

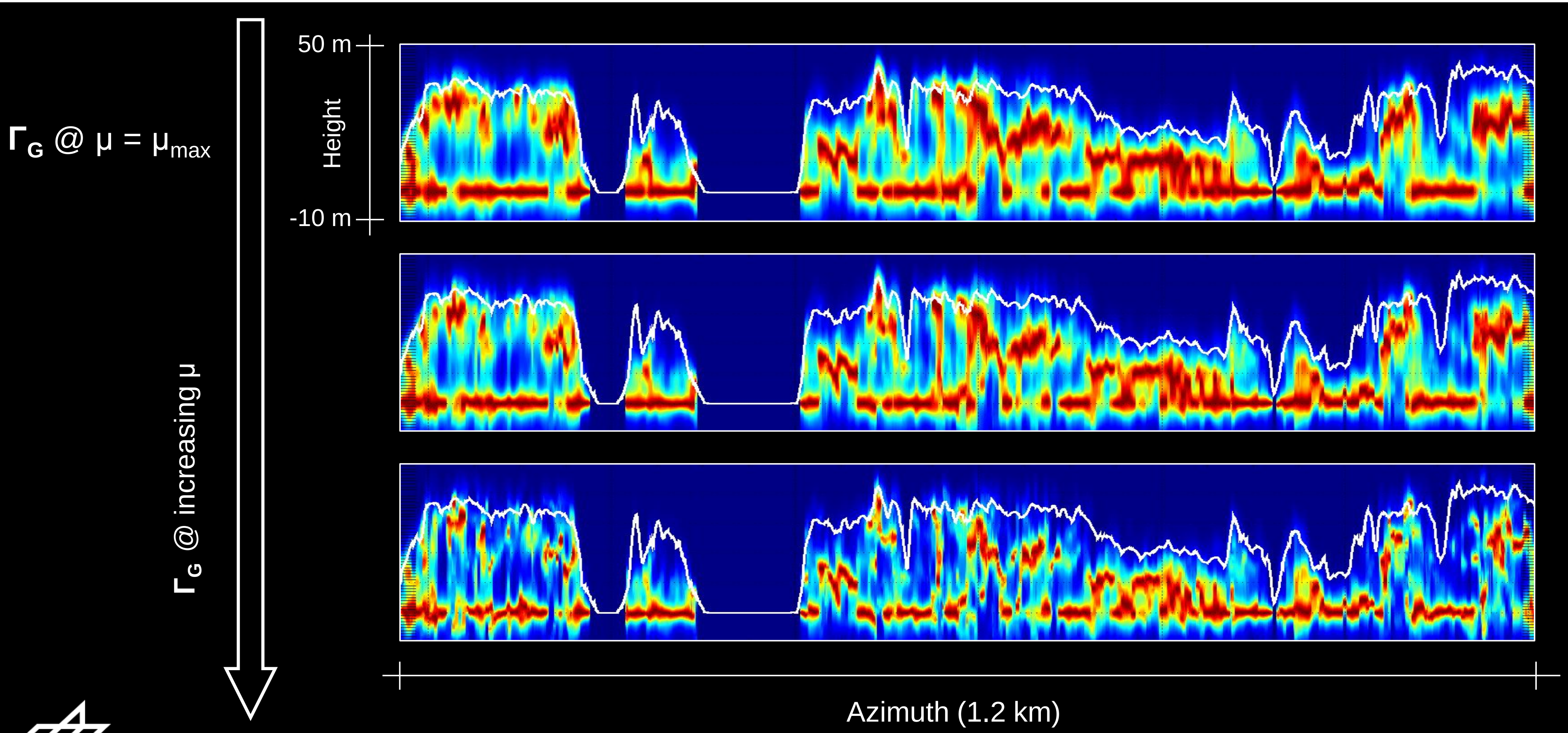
[S. Tebaldini, IEEE-TGARS 2009]

- It is an unconstrained Least Squares fitting, with no additional external knowledge
- The separation ambiguity is transferred to two unknowns scalar parameters

Ground solutions

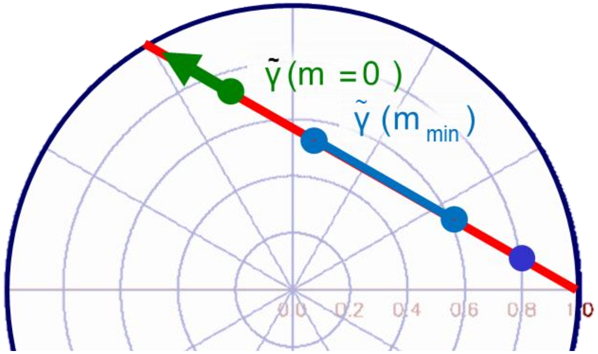
All the feasible ground coherence matrices are on the Pol-InSAR line segments outside the coherence region, under the positive (semi-)definiteness constraint.



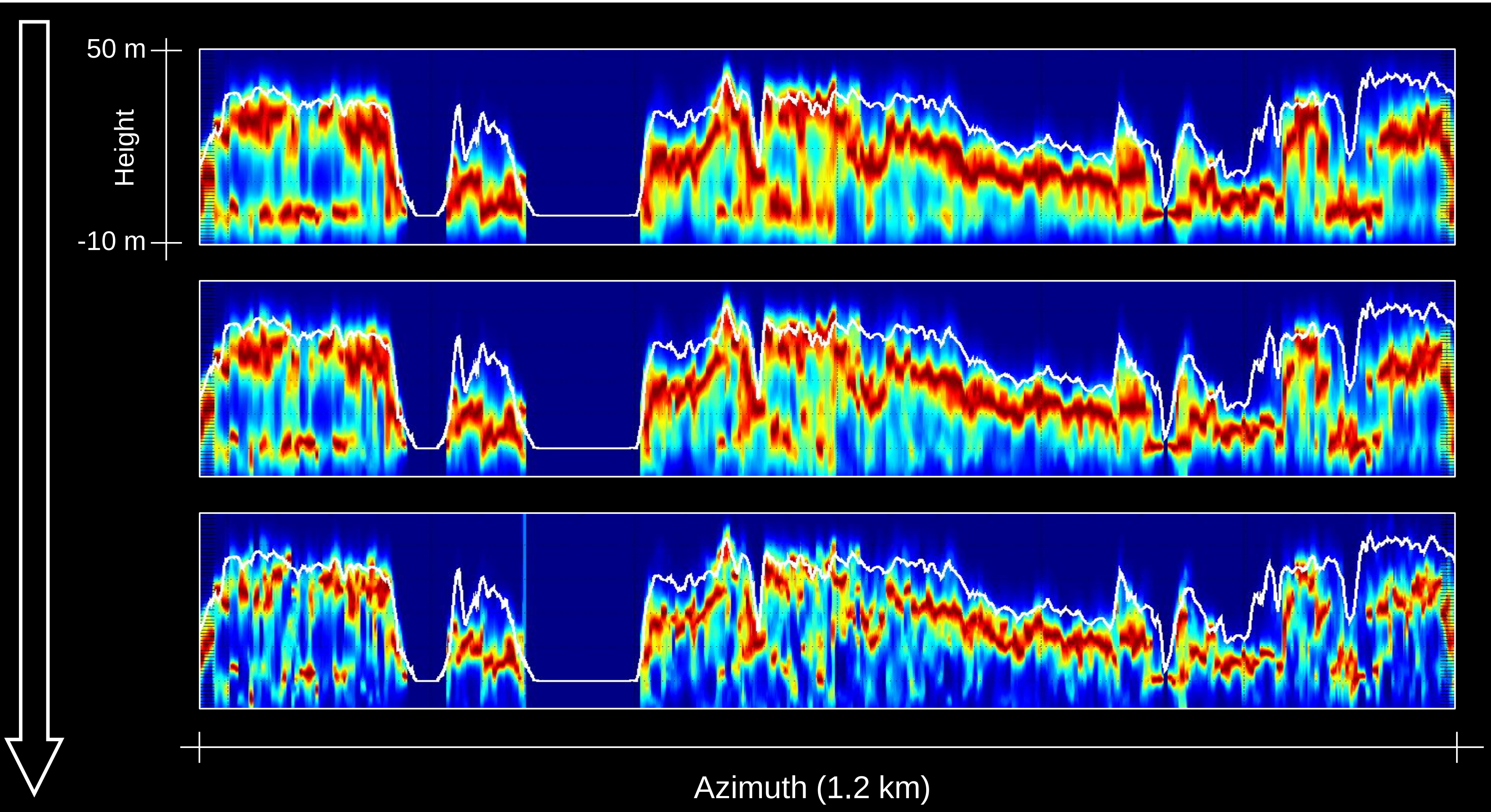


Traunstein forest (south of Germany) – L band – DLR's E-SAR, 7 tracks – Capon

Volume solutions



All the feasible volume coherence matrices are on the Pol-InSAR line segments outside the coherence region, under the positive (semi-)definiteness constraint.



Γ_V @ $\mu = \mu_{min}$

Γ_V @ decreasing μ

Traunstein forest (south of Germany) – L band – DLR's E-SAR, 7 tracks – Capon

Dependence on polarization & frequency

L-band

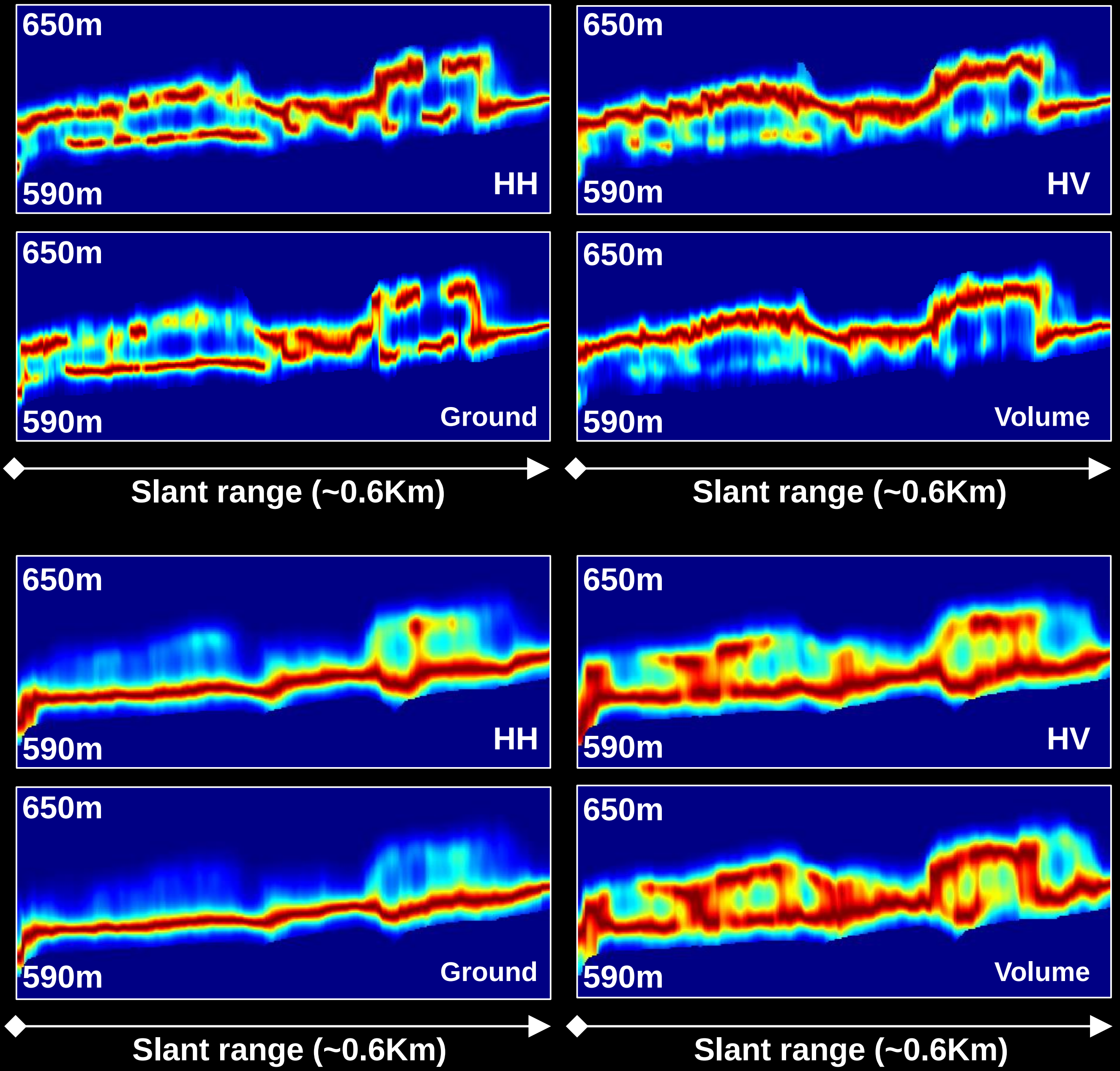
Lexicographic basis
HH vs HV

Maximum ground vs
Maximum volume

P-band

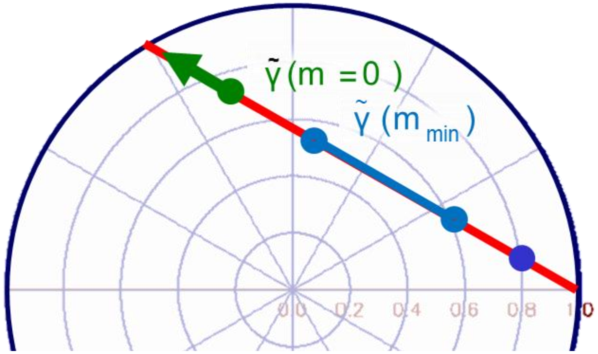
Lexicographic basis
HH vs HV

Maximum ground vs
Maximum volume



Ground powers / polarimetry solutions

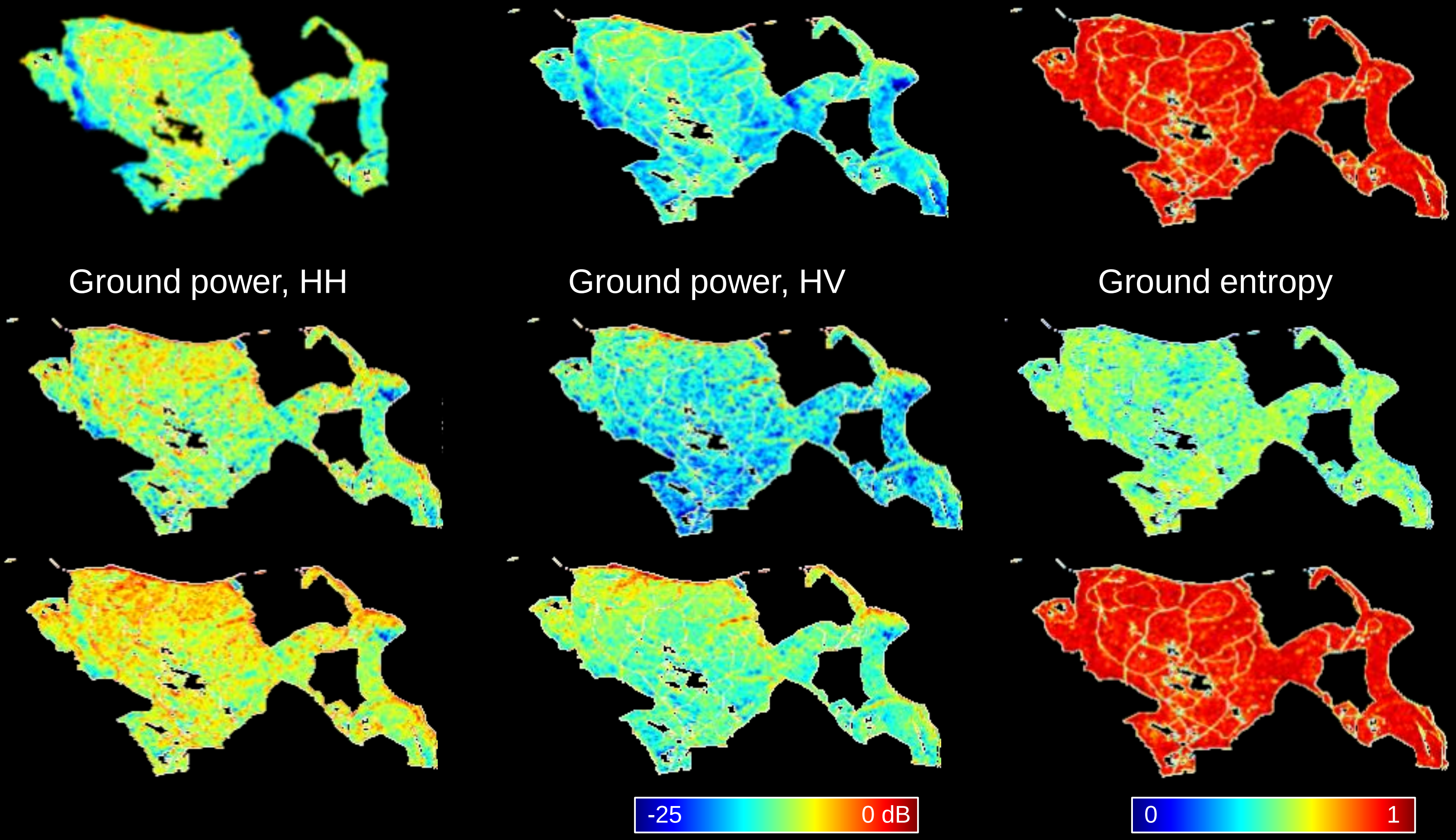
All the feasible ground covariance matrices are on the Pol-InSAR line segments outside the coherence region, under the positive (semi-)definiteness constraint.



Capon estimates
With known ground

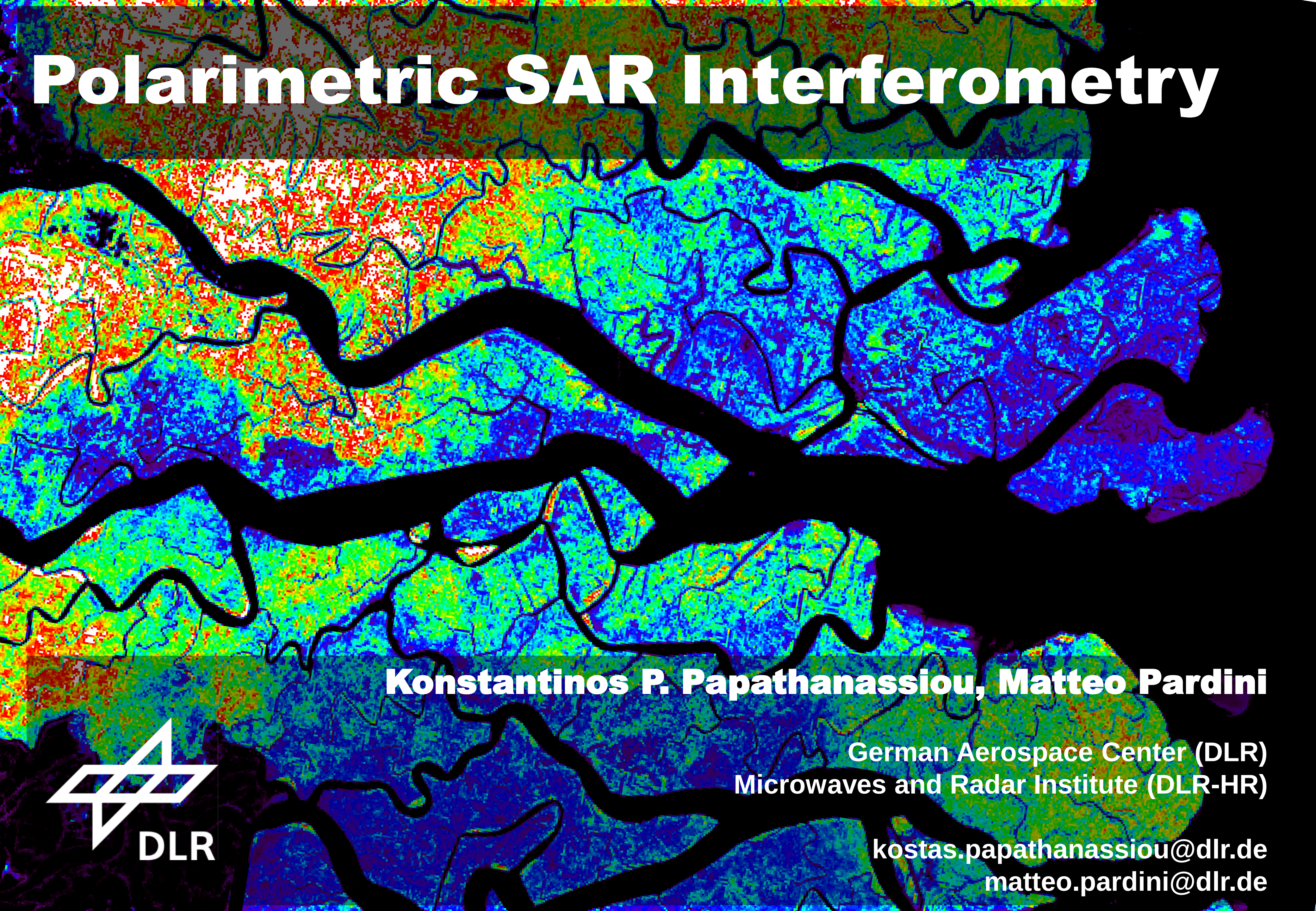
C_G @ $\mu = \mu_{min}$

C_G @ decreasing μ



Traunstein forest (south of Germany) – L band – DLR's E-SAR, 7 tracks – Capon

Polarimetric SAR Interferometry



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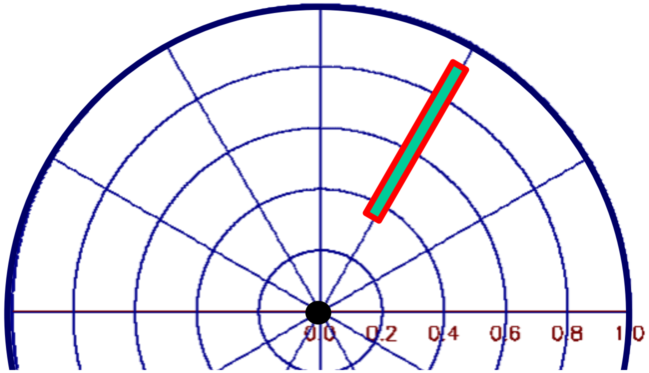
Coherence Region (CR) Interpretation

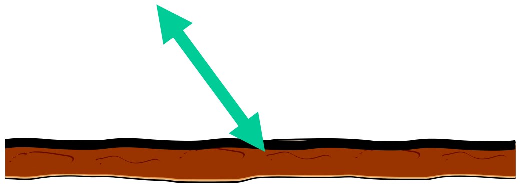


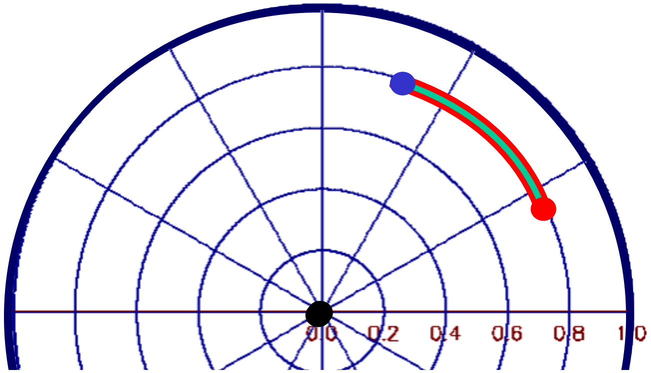
Radial Shaped CR

i.e. InSAR coherence amplitude changes with polarisation but not the location of the phase center.

Surface Scattering

$$\tilde{\gamma}(\vec{w}) = \gamma_{SNR}(\vec{w})\ \tilde{\gamma}_{Vol} \overset{\tilde{\gamma}_{Vol}:=1}{=} \gamma_{SNR}(\vec{w})$$

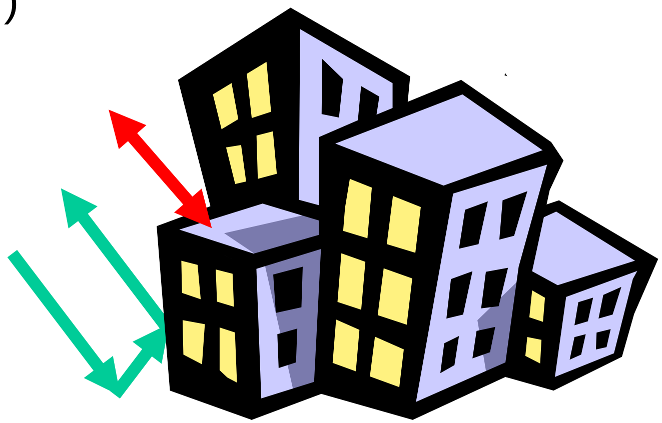


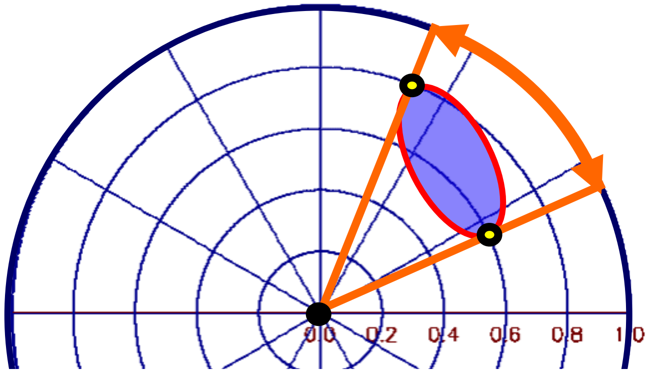


Arc Shaped CR

i.e. InSAR phase center location changes with polarisation but not the absolute value of the coherence amplitude.

(Polarised) Coherent scaterrers at different heights





Elliptical Shaped CR

i.e. InSAR coherence magnitude and phase center location changes with polarisation.

(Depolarising) Scaterrers at different heights