Polarimetric Target Decomposition Theorems

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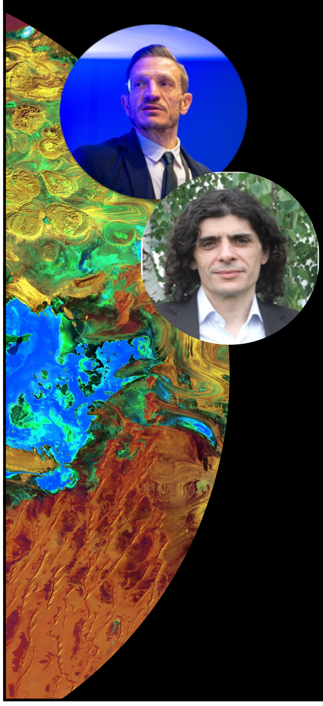

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Outline

- Polarimetric Target Decomposition Theorems
- Coherent Decompositions
- Incoherent Decompositions

Polarimetric Target Dec. Theorems

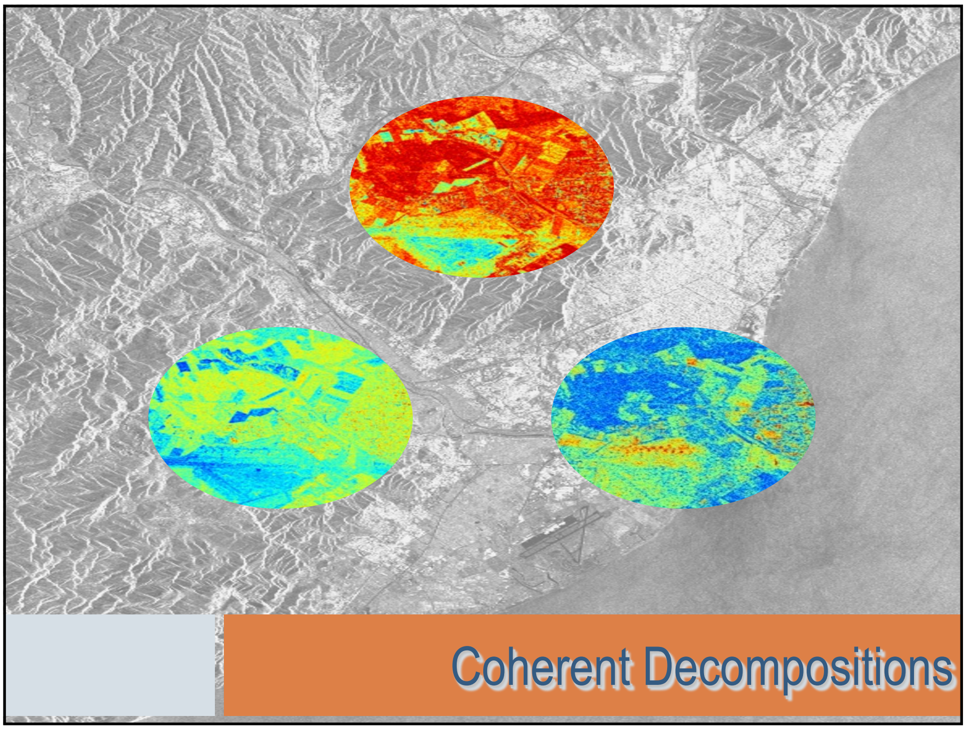
Polarimetric Target Decompositions Theorems allow the interpretation of measured PolSAR data by decomposing them

- Coherent Decompositions
  - Applied to the firs order descriptors, i.e., the Scattering Matrix
  - Valid for the interpretation of Pure or Deterministic scatters
  - Decomposition of the data into canonical scattering mechanisms

$$\mathbf{S} = \sum_{i=1}^{k} c_i \mathbf{S}_i$$

- Incoherent Decompositions
  - Applied to second order descriptors, i.e., the Covariance and Coherency matrices
  - Valid for the interpretation of Deterministic and Distributed scatters
  - Decomposition of the data into simple scattering mechanisms that may, or not, admit an equivalent scattering matrix

$$\langle \mathbf{C} \rangle = \sum_{i=1}^{k} c_i \mathbf{C}_i \qquad \langle \mathbf{T} \rangle = \sum_{i=1}^{k} c_i \mathbf{T}_i$$



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Target Decompositions — Pauli Decomposition

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Based on the decomposition of the measured scattering matrix into the orthogonal Pauli decompositions basis

- Basis components

$$\mathbf{S}_a = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{S}_b = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \mathbf{S}_c = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \mathbf{S}_d = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}$$

Single bounce — Double bounce — Volume

  - Admit a physical interpretation into simple scattering mechanisms
  - Orthogonal scattering mechanisms

- Decomposition into three orthogonal scattering mechanisms (monostatic case)

$$\mathbf{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{hv} & S_{vv} \end{bmatrix} = \alpha \mathbf{S}_a + \beta \mathbf{S}_b + \gamma \mathbf{S}_c$$

- Decomposition coefficients

$$\alpha = \frac{S_{hh} + S_{vv}}{\sqrt{2}} \qquad \beta = \frac{S_{hh} - S_{vv}}{\sqrt{2}} \qquad \gamma = \sqrt{2} S_{hv}$$

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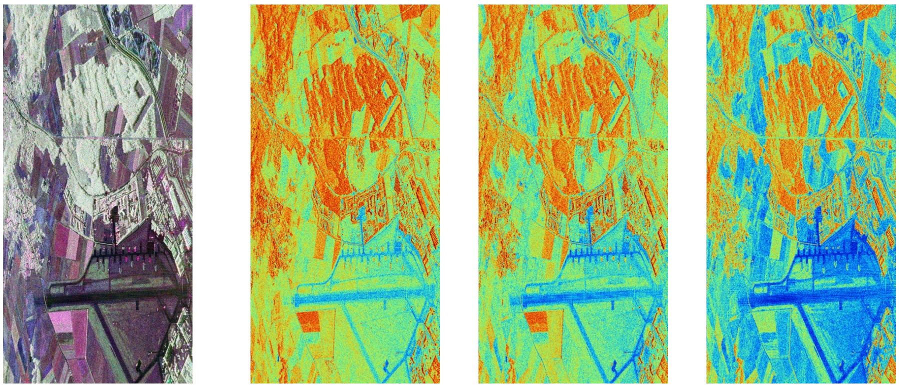
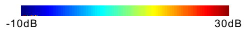
Target Decompositions

Pauli Decomposition



|Shh-Svv| 2|Shv| | Shh +Svv|

$\alpha = \frac{S_{hh} + S_{vv}}{\sqrt{2}}$ $\beta = \frac{S_{hh} - S_{vv}}{\sqrt{2}}$ $\gamma = \sqrt{2} S_{hv}$



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Target Decompositions

Krogager Decomposition

Based on the decomposition of the measured scattering matrix into the coherent scattering mechanisms: sphere, diplane and helix

- Decomposition components

$$\mathbf{S}_s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{S}_d = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \qquad \mathbf{S}_s = \begin{bmatrix} 0 & \pm j \\ \pm j & 0 \end{bmatrix}$$

Sphere
Single bounce

Diplane rotated θ
about line of sight
Double bounce

Helix

  - Non orthogonal components
- Decomposition into three orthogonal scattering mechanisms (monostatic case)

$$\mathbf{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{hv} & S_{vv} \end{bmatrix} = e^{j\varphi} \left\{ e^{j\varphi_s} k_s \mathbf{S}_s + k_d \mathbf{S}_d + k_h \mathbf{S}_h \right\}$$

$$= e^{j\varphi} \left\{ e^{j\varphi_s} k_s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + k_d \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} + k_h e^{\mp j 2\theta} \begin{bmatrix} 0 & \pm j \\ \pm j & 0 \end{bmatrix} \right\}$$

  - Roll-invariant decomposition

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Target Decompositions

Krogager Decomposition

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- Decomposition coefficients

$$\mathbf{S}=\begin{bmatrix} S_{rr} & S_{rl} \\ S_{rl} & S_{ll} \end{bmatrix}=\begin{bmatrix} |S_{rr}|e^{j\varphi_{rr}} & |S_{rl}|e^{j\varphi_{rl}} \\ |S_{rl}|e^{j\varphi_{rl}} & |S_{ll}|e^{j\varphi_{ll}} \end{bmatrix}=e^{j\varphi}\left\{e^{j\varphi_s}k_s\mathbf{S}_s+k_d\mathbf{S}_d+k_h\mathbf{S}_h\right\}$$

$$=e^{j\varphi}\left\{e^{j\varphi_s}k_s\begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix}+k_d\begin{bmatrix} e^{j2\theta} & 0 \\ 0 & -e^{-j2\theta} \end{bmatrix}+k_h\begin{bmatrix} e^{j2\theta} & 0 \\ 0 & 0 \end{bmatrix}\right\}$$

- Sphere: $k_s=|S_{rl}|$

- Diplane and Helix: Two options

$\|S_{rr}\|>\|S_{ll}\|$	$\|S_{rr}\|<\|S_{ll}\|$
$k_d^+=\|S_{ll}\|$	$k_d^-=\|S_{rr}\|$
$k_h^+=\|S_{rr}\|-\|S_{ll}\|$	$k_h^-=\|S_{ll}\|-\|S_{rr}\|$
Right sense helix	Left sense helix

- Phases: $\varphi=\frac{1}{2}(\varphi_{rr}+\varphi_{ll}+\pi)$

$$\theta=\frac{1}{4}(\varphi_{rr}-\varphi_{ll}-\pi)$$

$$\varphi_s=\varphi_{rl}-\frac{1}{2}(\varphi_{rr}+\varphi_{ll}+\pi)$$

Target Decompositions

Krogager Decomposition

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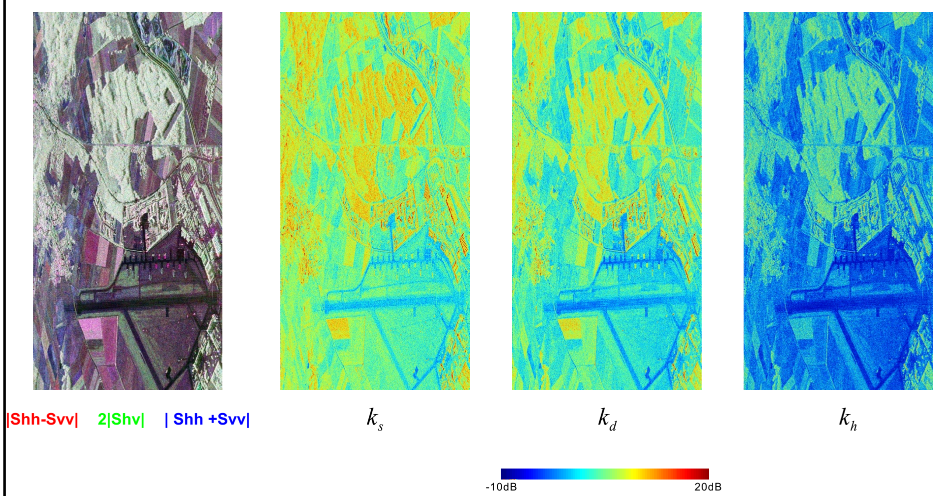
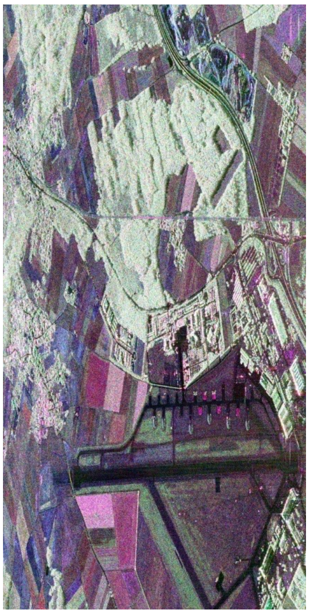


Target Decompositions

Krogager Decomposition



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Target Decompositions

Coherent Decompositions

Only valid when the assumption of pure or deterministic targets is valid, that is, data is not affected by the presence of speckle noise

- The presence of speckle noise makes the scattering matrix and its decomposition to be random

Distributed targets can not be represented by a single scattering matrix but the average covariance or coherency matrices

- Incoherent target decomposition theorems are applied to the average target represented by the average matrices

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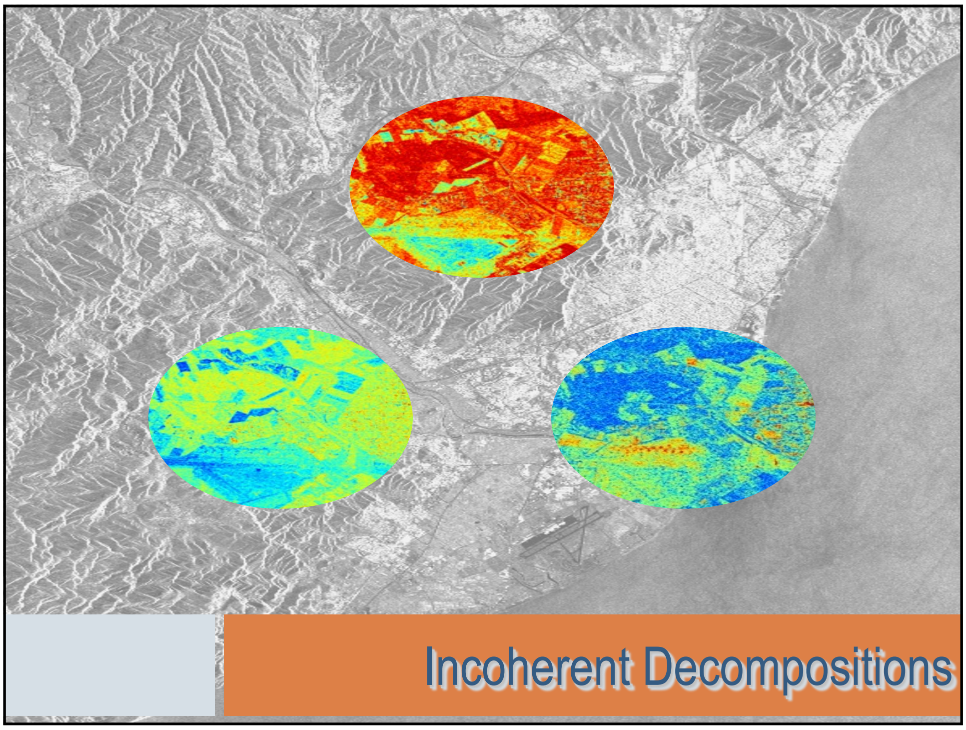

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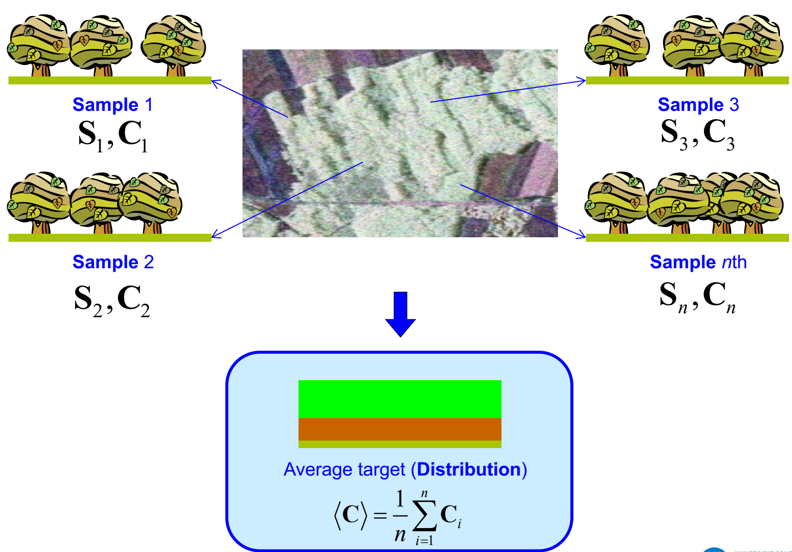
Incoherent Decompositions

Physical interpretation and analysis of distributed targets



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Target Decompositions

Incoherent Decompositions

Two type of decompositions

- Model-based decompositions: The decomposition of the covariance and the coherency matrices is based on decomposition components that model physical scattering mechanisms
  - Simple physical interpretation
  - Assumptions concerning the type of scattering are necessary
- Mathematical-based decompositions: The decomposition of the covariance and the coherency matrices is based on mathematical decompositions, normally those based on the eigenvalue/eigenvectors concept
  - Simple computation of the decomposition
  - No assumptions about the type of scattering are necessary
  - Physical interpretation of the different decomposition components are necessary

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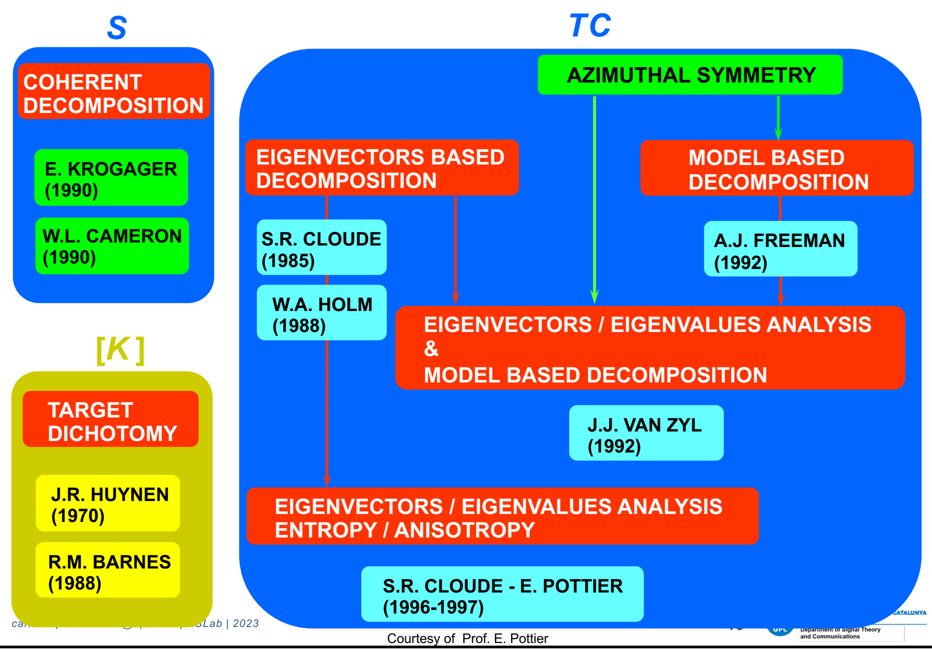
Target Decompositions

Target Decomposition Theorems



Courtesy of Prof. E. Pottier

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Target Decompositions

Freeman Decomposition

Coherency matrix form with respect to the target symmetries

General Case

$$\langle \mathbf{T} \rangle = \begin{bmatrix} T_1 & T_2 & T_3 \\ T_2^* & T_4 & T_5 \\ T_3^* & T_5^* & T_6 \end{bmatrix}$$

Reflection Symmetry

$$\langle \mathbf{T} \rangle = \begin{bmatrix} T_1 & T_2 & 0 \\ T_2^* & T_4 & 0 \\ 0 & 0 & T_6 \end{bmatrix}$$

Rotation Symmetry

$$\langle \mathbf{T} \rangle = \begin{bmatrix} T_1 & 0 & 0 \\ 0 & T_4 & T_5 \\ 0 & T_5^* & T_4 \end{bmatrix}$$

Azimuthal Symmetry

$$\langle \mathbf{T} \rangle = \begin{bmatrix} T_1 & 0 & 0 \\ 0 & T_4 & 0 \\ 0 & 0 & T_4 \end{bmatrix}$$

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Target Decompositions

Freeman Decomposition

Covariance matrix form with respect to the target symmetries

General Case

$$\langle \mathbf{C} \rangle = \begin{bmatrix} C_1 & C_2 & C_3 \\ C_2^* & C_4 & C_5 \\ C_3^* & C_5^* & C_6 \end{bmatrix}$$

Reflection Symmetry

$$\langle \mathbf{C} \rangle = \begin{bmatrix} C_1 & 0 & C_3 \\ 0 & C_4 & 0 \\ C_3^* & 0 & C_6 \end{bmatrix}$$

Rotation Symmetry

$$\langle \mathbf{C} \rangle = \begin{bmatrix} C_1 & C_2 & C_3 \\ C_2^* & C_4 & -C_2^* \\ C_3^* & -C_2 & C_1 \end{bmatrix}$$

Azimuthal Symmetry

$$\langle \mathbf{C} \rangle = \begin{bmatrix} C_1 & 0 & C_3 \\ 0 & C_4 & 0 \\ C_3^* & 0 & C_1 \end{bmatrix}$$

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Target Decompositions

Freeman Decomposition

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Incoherent decomposition proposed by A. Freeman (1992)

- Decomposition

$\langle \mathbf{T} \rangle \quad \langle \mathbf{C} \rangle$ ➡ Three components scattering mechanism model

$\mathbf{T}_S$, $\mathbf{C}_S$: Single bounce scattering

$\mathbf{T}_D$, $\mathbf{C}_D$: Double bounce scattering

$\mathbf{T}_V$, $\mathbf{C}_V$: Volume scattering

$$\langle \mathbf{T} \rangle = f_S \mathbf{T}_S + f_D \mathbf{T}_D + f_V \mathbf{T}_V$$

$$\langle \mathbf{C} \rangle = f_S \mathbf{C}_S + f_D \mathbf{C}_D + f_V \mathbf{C}_V$$

Decomposition coefficients

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Target Decompositions

Freeman Decomposition

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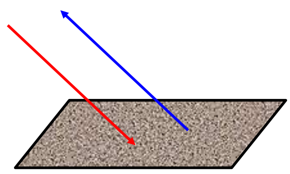
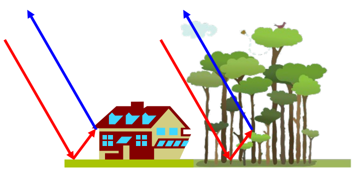
- Single bounce scattering (rough surface)



$$\mathbf{S}_s = \begin{bmatrix} R_H & 0 \\ 0 & R_V \end{bmatrix} \Rightarrow \mathbf{k} = \begin{bmatrix} R_H \\ 0 \\ R_V \end{bmatrix}$$

$$\mathbf{C}_S = f_S \begin{bmatrix} \beta^2 & 0 & \beta \\ 0 & 0 & 0 \\ \beta & 0 & 1 \end{bmatrix} \quad f_S = |R_V|^2 \quad \beta = \frac{R_H}{R_V}$$

- Double bounce scattering



$$\mathbf{S}_D = \begin{bmatrix} R_{GH}R_{TH} & 0 \\ 0 & -R_{GV}R_{TV} \end{bmatrix} \Rightarrow \mathbf{k}_D = \begin{bmatrix} R_{GH}R_{TH} \\ 0 \\ -R_{GV}R_{TV} \end{bmatrix}$$

$$\mathbf{C}_D = f_D \begin{bmatrix} \alpha^2 & 0 & -\alpha \\ 0 & 0 & 0 \\ -\alpha & 0 & 1 \end{bmatrix} \quad f_D = |R_{GV}R_{TV}|^2 \quad \alpha = \frac{R_{GH}R_{TH}}{R_{GV}R_{TV}}$$

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Target Decompositions

Freeman Decomposition

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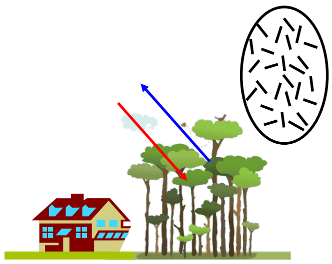
- Volume scattering (Randomly oriented very thin cylinder-like scatters)



Mechanism (cylinder) $\mathbf{S} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Oriented mechanism $\mathbf{S}_\theta = \mathbf{U}_2(\theta)^T \mathbf{S} \mathbf{U}_2(\theta)$

Uniform Orientation $P(\theta) = \frac{1}{2\pi}$

Second-Order statistics

$$\mathbf{C}_V = \langle \mathbf{C}_\theta \rangle = \int_0^{2\pi} \mathbf{C}_V P(\theta) d\theta$$

Covariance matrix

Thin cylinders

$a \mapsto 1 \quad b \mapsto 0$

$$\mathbf{C}_V = f_V \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & 1 \end{bmatrix}$$

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Target Decompositions

Freeman Decomposition

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3 Components Scattering Mechanism Model

$$\langle \mathbf{C} \rangle = \begin{bmatrix} f_S\beta^2 + f_D\alpha^2 + f_V & 0 & f_S\beta - f_D\alpha + \frac{f_V}{3} \\ 0 & \frac{2f_V}{3} & 0 \\ f_S\beta - f_D\alpha + \frac{f_V}{3} & 0 & f_S + f_D + f_V \end{bmatrix}$$

- 5 Unknown real coefficients
- 4 Observed equations
- Negative power is observed

Assumptions must be considered !!!

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Target Decompositions

Freeman Decomposition

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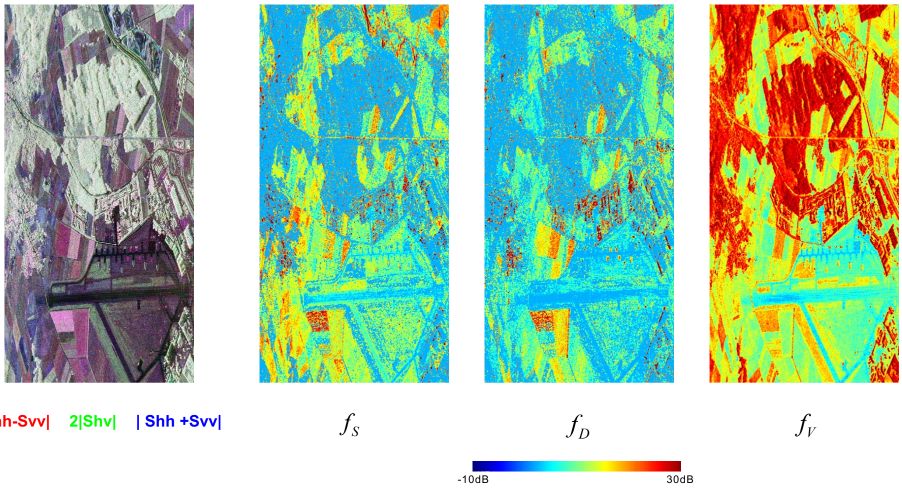
7x7 Multilook



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Target Decompositions

Freeman Decomposition

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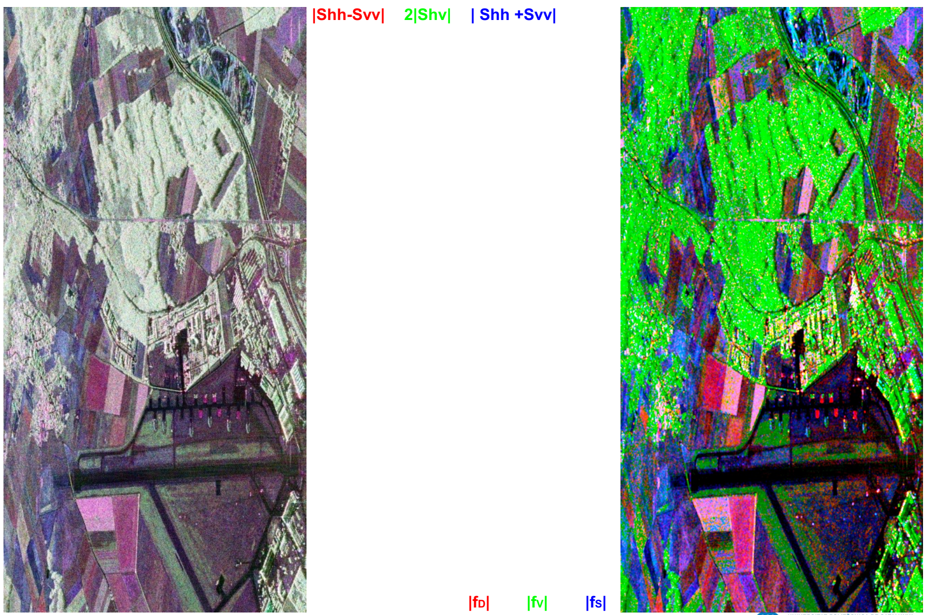

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Target Decompositions

Yamaguchi Decomposition

4-component scattering model (Yamaguchi, 2005)

$$\langle \mathbf{T} \rangle = \frac{f_s}{1+|\alpha|^2}\begin{bmatrix}1&\beta^*&0\\ \beta&|\beta|^2&0\\ 0&0&0\end{bmatrix}+\frac{f_d}{1+|\alpha|^2}\begin{bmatrix}|\alpha|^2&\alpha&0\\ \alpha^*&1&0\\ 0&0&0\end{bmatrix}+\frac{f_v}{4}\begin{bmatrix}2&0&0\\ 0&1&0\\ 0&0&1\end{bmatrix}+\frac{f_c}{2}\begin{bmatrix}0&0&0\\ 0&1&\pm j\\ 0&\mp j&1\end{bmatrix}$$

Surface scattering | Double bounce | Volume scattering | Helix scattering

- T_{13} is not accounted for. (Lee, 2009)
  - 5-component scattering model decomposition?
- Negative power issue
  - Orientation compensation reduces HV, that reduce negative power pixels (Lee, 2009, An, 2009)
  - New volume scattering model (Yamaguchi, 2005)
  - New scattering models and non-negative eigenvalues (van Zyl and Arii, 2009, 2010)

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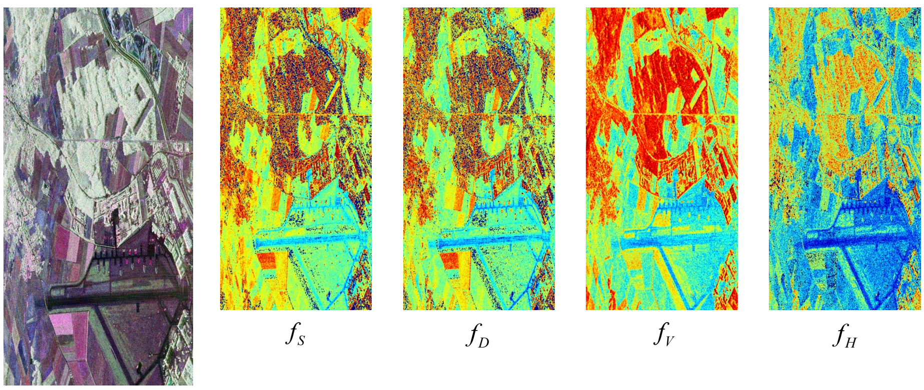
Target Decompositions

Yamaguchi Decomposition

7x7 Multilook



|Shh-Svv| 2|Shv| | Shh +Svv|

-10dB 30dB

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Target Decompositions

Eigenvalue/vectors Decomposition

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Decomposition proposed by Shane Cloude, based on the mathematical decomposition of the coherency matrix on its eigenvalue and eigenvectors

$$\mathbf{k}_p = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{hh}+S_{vv} \\ S_{hh}-S_{vv} \\ 2S_{hv} \end{bmatrix}$$

Sample

$$\langle \mathbf{T} \rangle = \frac{1}{N}\sum_{i=1}^{N}\mathbf{k}_i\mathbf{k}_i^H = \frac{1}{N}\sum_{i=1}^{N}\mathbf{T}_i$$

Estimation of the covariance matrix

- Decomposition (i)

$$\langle \mathbf{T} \rangle = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^{-1} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}\begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}^H$$

  - The eigenvectors $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$ are orthonormal
  - The eigenvalues are real $\lambda_1 > \lambda_2 > \lambda_3 \geq 0$

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Target Decompositions

Eigenvalue/vectors Decomposition

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- Decomposition components
  - The eigenvectors represent rank 1 scattering mechanisms that are related with a scattering matrix
  - Parametrization of the SU(3) unitary matrix

$$\mathbf{U} = \begin{bmatrix} \cos(\alpha_1) & \cos(\alpha_2) & \cos(\alpha_3) \\ \sin(\alpha_1)\cos(\beta_1)e^{j\delta_1} & \sin(\alpha_2)\cos(\beta_2)e^{j\delta_2} & \sin(\alpha_3)\cos(\beta_3)e^{j\delta_3} \\ \sin(\alpha_1)\sin(\beta_1)e^{j\gamma_1} & \sin(\alpha_2)\sin(\beta_2)e^{j\gamma_2} & \sin(\alpha_3)\sin(\beta_3)e^{j\gamma_3} \end{bmatrix}$$

Target 1 Target 2 Target 3

Decomposition basis

  - The eigenvalues represent the power associated to every target

λ_1 λ_2 λ_3

Decomposition coefficients

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Target Decompositions

Eigenvalue/vectors Decomposition

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- Decomposition (ii)

$$\langle \mathbf{T} \rangle = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^{-1} = \lambda_1 \mathbf{u}_1 \mathbf{u}_1^H + \lambda_2 \mathbf{u}_2 \mathbf{u}_2^H + \lambda_3 \mathbf{u}_3 \mathbf{u}_3^H = \lambda_1 \mathbf{T}_1 + \lambda_2 \mathbf{T}_2 + \lambda_3 \mathbf{T}_3$$

- The coherency matrices $\mathbf{T}_k$ are rank 1 matrices associated with a single scattering matrix
- It is possible to associate a probability to every scattering mechanism

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3} \lambda_k} \qquad Span = \sum_{k=1}^{3} \lambda_k$$

- Definition of the mean dominant scattering mechanism
  - Mean parameters

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3 \quad \underline{\beta} = P_1\beta_1 + P_2\beta_2 + P_3\beta_3$$
$$\underline{\gamma} = P_1\gamma_1 + P_2\gamma_2 + P_3\gamma_3 \quad \underline{\delta} = P_1\delta_1 + P_2\delta_2 + P_3\delta_3$$

  - Mean unitary dominant scattering mechanism

$$\mathbf{u}_0 = \left[\cos(\underline{\alpha}) \quad \sin(\underline{\alpha})\cos(\underline{\beta})\, e^{j\underline{\delta}} \quad \sin(\underline{\alpha})\sin(\underline{\beta})\, e^{j\underline{\gamma}} \right]^T$$

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Target Decompositions

Eigenvalue/vectors Decomposition

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- Mean dominant scattering mechanism

$$\mathbf{k}_0 = \sqrt{Span} \begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha})\cos(\underline{\beta})\, e^{j\underline{\delta}} \\ \sin(\underline{\alpha})\sin(\underline{\beta})\, e^{j\underline{\gamma}} \end{bmatrix} = \sqrt{\sum_{k=1}^{3} \lambda_k} \begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha})\cos(\underline{\beta})\, e^{j\underline{\delta}} \\ \sin(\underline{\alpha})\sin(\underline{\beta})\, e^{j\underline{\gamma}} \end{bmatrix}$$

- The mean dominant scattering mechanism may be employed to interpret physically the data

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Target Decompositions

Eigenvalue/vectors Decomposition

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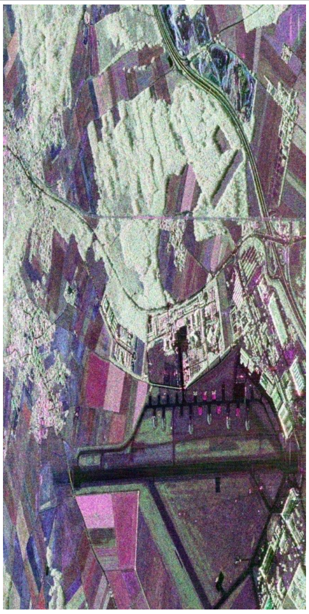

|Shh-Svv| 2|Shv| | Shh +Svv|

$\sqrt{Span}\cos(\alpha)$

$\sqrt{Span}\sin(\alpha)\cos(\beta)$

$\sqrt{Span}\sin(\alpha)\sin(\beta)$

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Target Decompositions

Eigenvalue/vectors Decomposition

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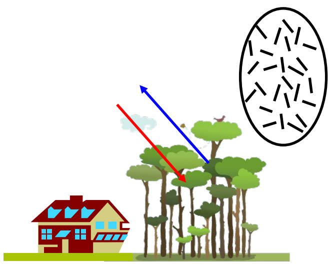
The eigendecomposition needs a physical interpretation of the decomposition basis and the decomposition coefficients

- Analysis of the eigenvectors



Mechanism (cylinder) $\mathbf{S}=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Oriented mechanism $\mathbf{S}_\theta=\mathbf{U}_2(\theta)^T\mathbf{S}\mathbf{U}_2(\theta)$

Uniform Orientation $P(\theta)=\frac{1}{2\pi}$

Second-Order statistics

$$\mathbf{T}=\langle\mathbf{T}_\theta\rangle=\int_0^{2\pi}\mathbf{T}_\theta P(\theta)d\theta$$

$$\langle\mathbf{T}\rangle=\frac{1}{2}\begin{bmatrix} 2\varepsilon & 0 & 0 \\ 0 & \nu & 0 \\ 0 & 0 & \nu \end{bmatrix}$$

Azymuthal symmetry

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Eigenvalue/vectors Decomposition

- Analysis of the azymuthal symmetric scatterer
  - Eigenvalues

$$\lambda_1 = \varepsilon \quad \Rightarrow \quad P_1 = \frac{\varepsilon}{(\varepsilon+\nu)}$$

$$\lambda_2 = \lambda_3 = \frac{\nu}{2} \quad \Rightarrow \quad P_2 = P_3 = \frac{\nu}{2(\varepsilon+\nu)}$$

  - Eigenvectors

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \Rightarrow \quad \begin{array}{c} \alpha_1 = 0 \\ \alpha_2 = \alpha_3 = \dfrac{\pi}{2} \end{array}$$

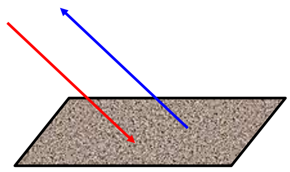
  - Average alpha angle

$$\underline{\alpha} = \frac{\pi}{2}(P_2 + P_3)$$

Target Decompositions

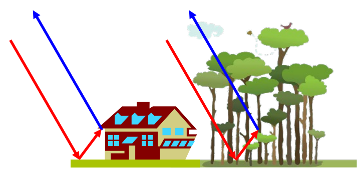
Eigenvalue/vectors Decomposition

- Physical interpretation of the average alpha angle parameter
  - Single bounce scattering, for example, a rough surface



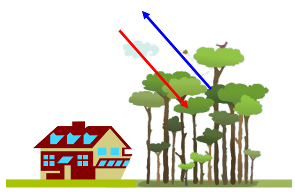
$$a \approx b, \quad \nu \approx 0 \qquad \boxed{\underline{\alpha} \approx 0}$$

  - Double bounce scattering



$$a \approx -b, \quad \varepsilon \approx 0 \qquad \boxed{\underline{\alpha} \approx \frac{\pi}{2}}$$

  - Volume scattering



$$a >> 0, \quad \varepsilon \approx \nu \qquad \boxed{\underline{\alpha} \approx \frac{\pi}{4}}$$

Same component as the Freeman dec.

Target Decompositions

Eigenvalue/vectors Decomposition

- Analysis of the eigenvalues. These parameters classify the spectrum of scattering mechanisms
  - Entropy: Degree of randomness or statistical disorder

$$H = -\sum_{i=1}^{3} P_i \log_3 (P_i)$$

    - Pure target (H=0). The average coherency matrix is rank 1

$$\lambda_1 = Span \quad \lambda_2 = 0 \quad \lambda_3 = 0$$

    - Distributed target (H=1). The average coherency matrix is rank 3

$$\lambda_1 = \lambda_2 = \lambda_3 = Span / 3$$

  - Anisotropy: Scattering mechanisms discrimination for high entropies (H>0.7)

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$

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Target Decompositions

H/A/α Decomposition

Original Eigenvalue/Eigenvector decomposition

$$\langle \mathbf{T} \rangle = \mathbf{U}\Sigma\mathbf{U}^{-1} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}^H$$

H/A/α decomposition allows a physical interpretation

$$H = -\sum_{i=1}^{3} P_i \log_3 (P_i) \qquad A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3} \qquad \underline{\alpha}$$

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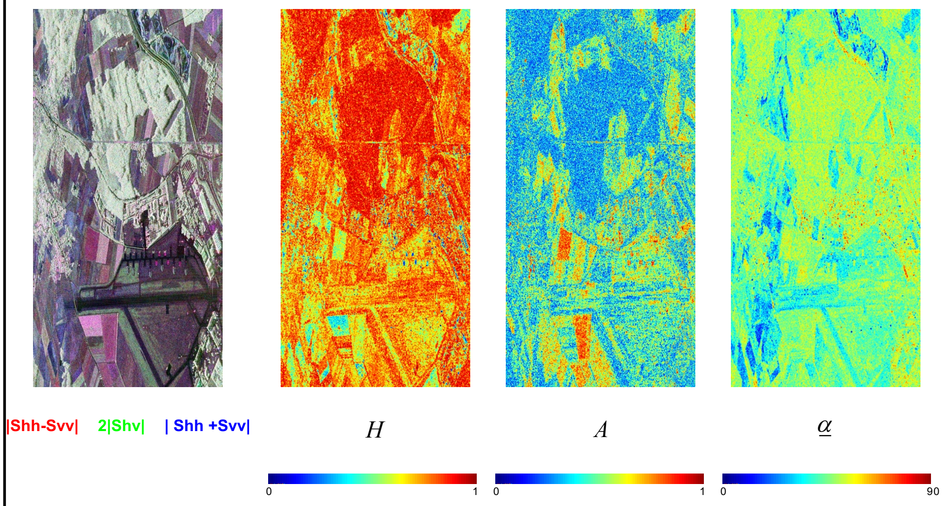
Target Decompositions

H/A/α Decomposition

esa

7x7 Multilook



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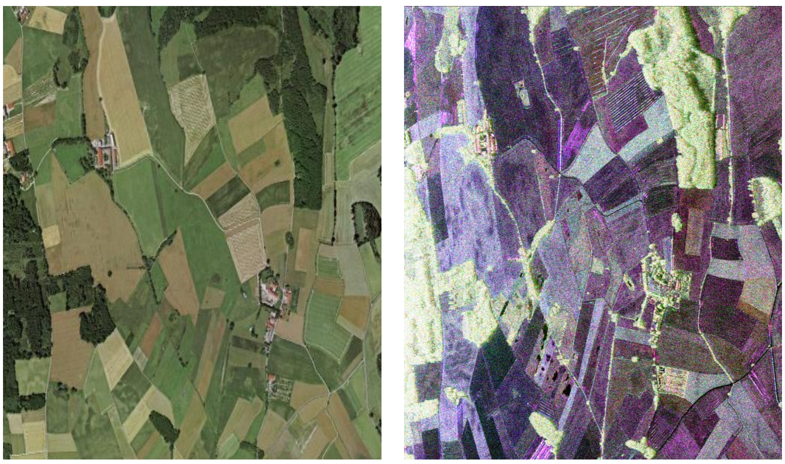
Target Decompositions

H/A/α Decomposition

esa

DLR



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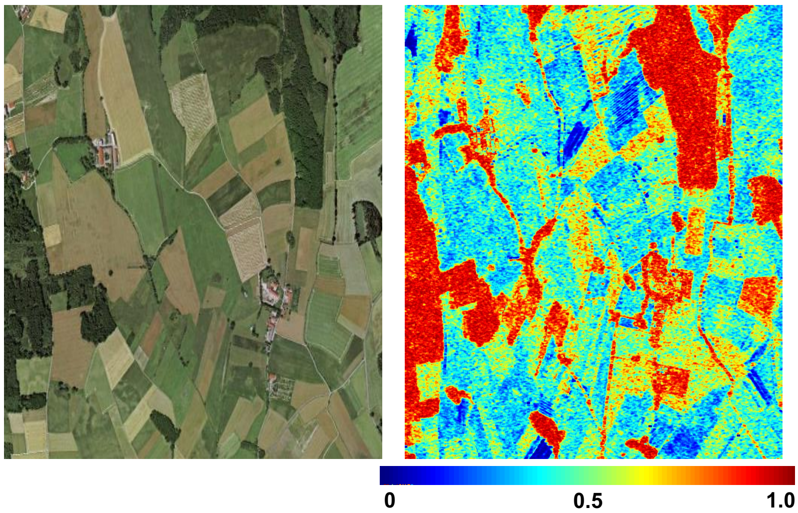
Target Decompositions

H/A/α Decomposition

Entropy (H)

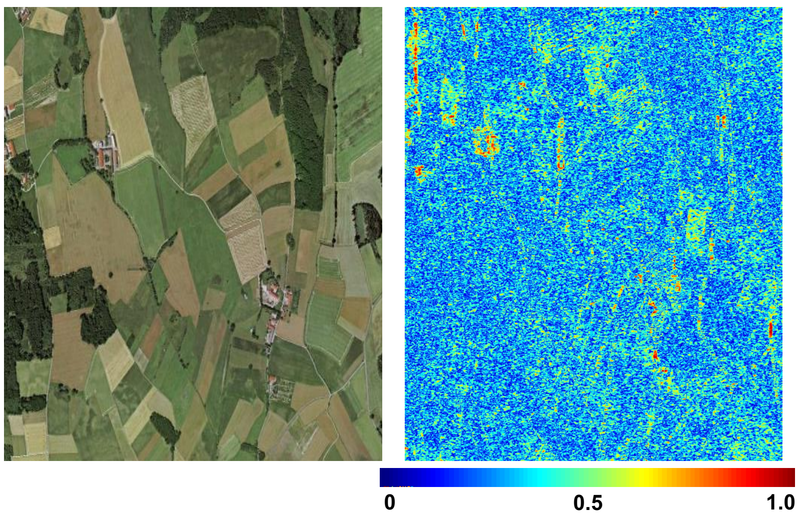


39

Target Decompositions

H/A/α Decomposition

Anisotropy (H)



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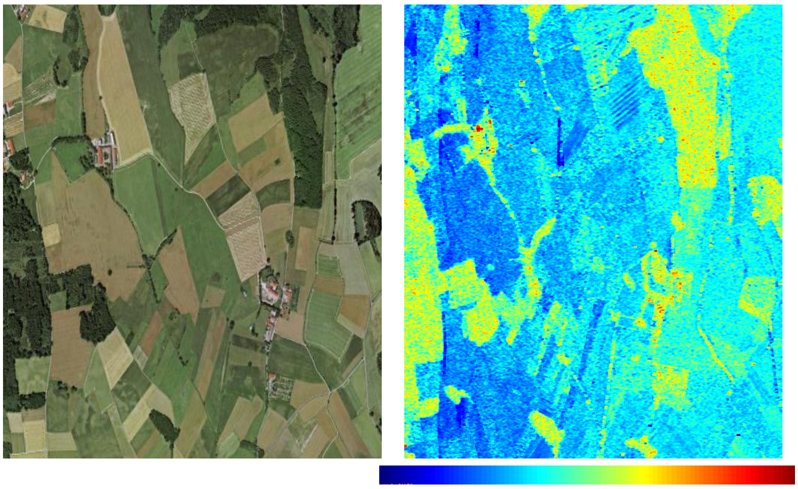
Target Decompositions

H/A/α Decomposition



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Target Decompositions

H/A/α Decomposition

A rotation about the line of sight is a unitary transformation

Rotation about the line-of-sight

$$\langle \mathbf{T}(\theta) \rangle = \mathbf{R}(\theta) \langle \mathbf{T} \rangle \mathbf{R}^{-1}(\theta) = \mathbf{R}(\theta) \mathbf{U} \mathbf{\Sigma} \mathbf{U}^{H} \mathbf{R}^{-1}(\theta) = \mathbf{U}' \mathbf{\Sigma} \mathbf{U}'^{H}$$

$$\mathbf{R}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\theta) & \sin(2\theta) \\ 0 & -\sin(2\theta) & \cos(2\theta) \end{bmatrix}$$

$$\langle \mathbf{T} \rangle = \mathbf{U} \mathbf{\Sigma} \mathbf{U}^{-1} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}^{H}$$

- Eigenvalues are roll-invariant, i.e., not affected by a rotation
- Probabilities are roll-invariant
- Average alpha angle is roll-invariant

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POLARIMETRIC TARGET SIGNATURES: An Example

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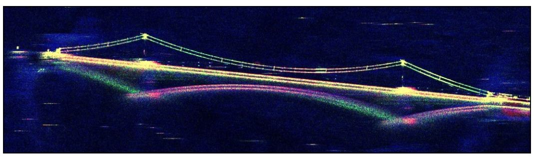
Polarimetric Scattering Characterization

High-resolution POLSAR signature of a suspension bridge under construction
The deck was not installed.

Aerial Photo

Pauli Decomposition, |HH-VV|, |HV|, |HH+VV|



Krogager SDH Decomposition
Blue= Sphere Green = Helix Red = Diplane.

EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

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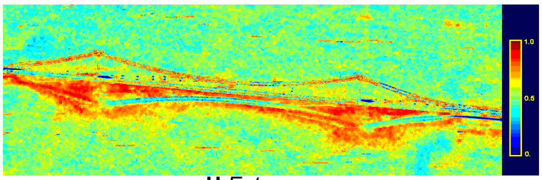
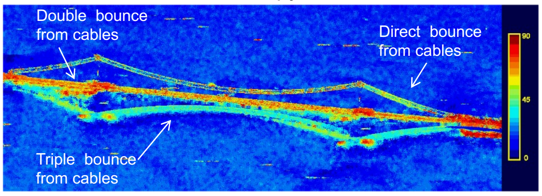
Polarimetric Scattering Characterization

High-resolution POLSAR signature of a suspension bridge under construction
The deck was not installed.

Aerial Photo



H *Entropy*



α *Angle*

Dominant scattering mechanisms are extracted by applying target decomposition:
Blue= Surface Green = Dipole Red = Double Bounce.

EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

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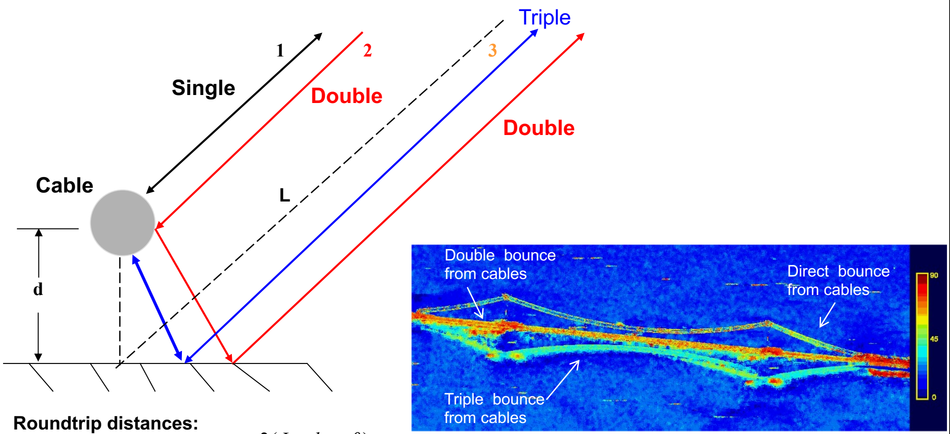
Polarimetric Scattering Characterization

Multi – Bounce Scattering mechanisms



α *Angle*

Roundtrip distances:

Single bounce return: $2(L - d\cos\theta)$

Double bounce return: $2L$ ⟸ ***Independent of height***

Triple bounce return: $2(L + d\cos\theta)$

θ is the local incidence angle.

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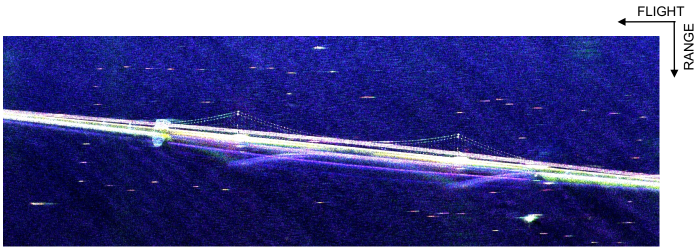
Polarimetric Scattering Characterization

High-resolution POLSAR signature of a suspension bridge after completion.
The deck is installed.



|HH-VV|, |HV|, |HH+VV|

EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

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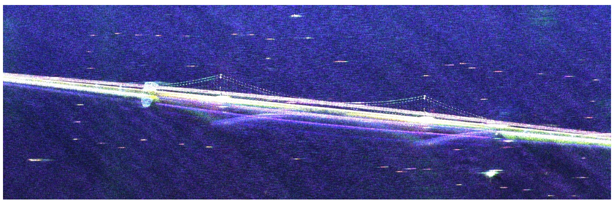
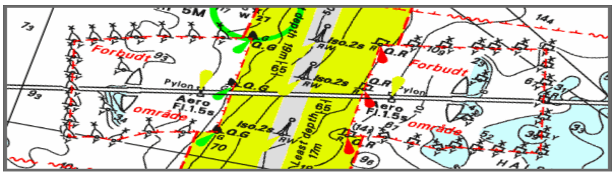
Polarimetric Scattering Characterization

Buoy



Buoy

|HH-VV|, |HV|, |HH+VV|



Navigation Map of Storebelt, Denmark

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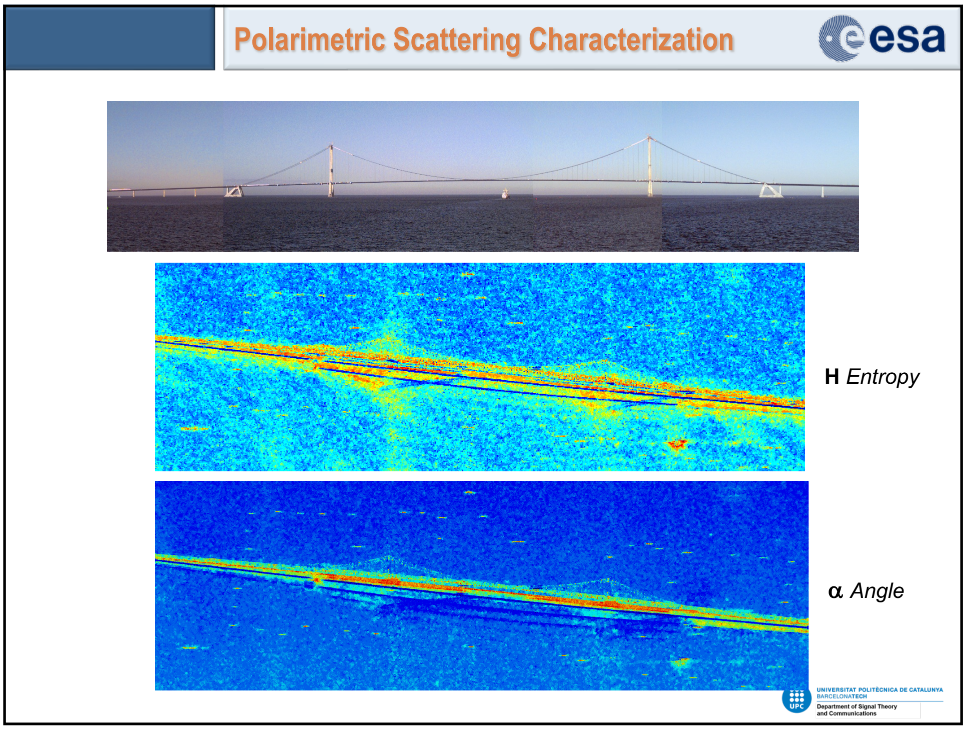
Polarimetric Scattering Characterization
esa
H Entropy
α Angle
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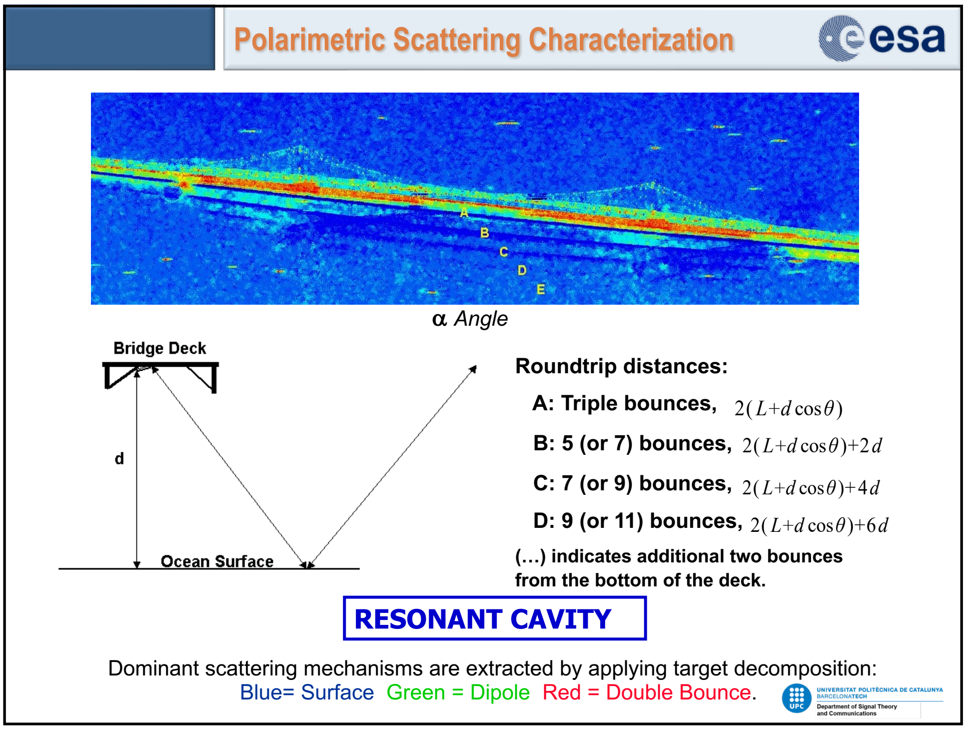
Polarimetric Scattering Characterization
esa
A
B
C
D
E
α Angle
Bridge Deck
d
Ocean Surface
Roundtrip distances:
A: Triple bounces, $2(L+d\cos\theta)$
B: 5 (or 7) bounces, $2(L+d\cos\theta)+2d$
C: 7 (or 9) bounces, $2(L+d\cos\theta)+4d$
D: 9 (or 11) bounces, $2(L+d\cos\theta)+6d$
(…) indicates additional two bounces from the bottom of the deck.
RESONANT CAVITY
Dominant scattering mechanisms are extracted by applying target decomposition:
Blue= Surface Green = Dipole Red = Double Bounce.
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Polarization Orientation Angle Estimation and Applications

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P.O.A. Estimation

POLARIZATION ORIENTATION ANGLE ESTIMATION AND APPLICATIONS

D.L. Schuler, J.S. Lee and G. De Grandi, "Measurement of Topography Using Polarimetric SAR Images," *IEEE Trans. on Geoscience and Remote Sensing*, vol. 34, no. 5, 1266-1277, September, 1996.

J.S. Lee, D.L. Schuler and T.L. Ainsworth, "Polariemtric SAR data compensation for terrain azimuth slope variations," IEEE TGRS (September, 2000)

J.S. Lee, D.L. Schuler, T.L. Ainsworth, and W. M. Boerner "A Review of Polarization orientation angle estimation and Applications," Proceedings of EUSAR 2006, E. Lueneburg Memorial Session, 2006

F. Xu, and Y.-Q. Jin, "Deorientation theory of polarimetric scattering targets and application to terrain surface classification," *IEEE Trans. on Geoscience and Remote Sensing,* vol.43, no.10, pp. 2351-2364, 2005.

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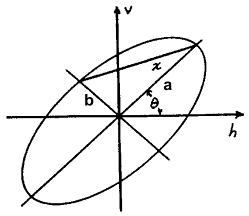
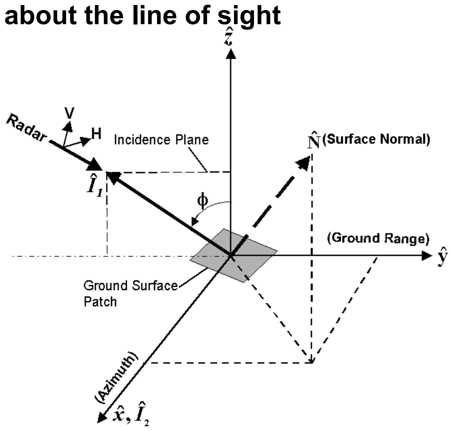
P.O.A. Estimation

Polarization Orientation Shifts

Orientation angles = rotation about the line of sight





J.S. Lee, D.L. Schuler and T.L. Ainsworth, "Polarimetric SAR data compensation for terrain azimuth slope variations," IEEE TGRS (September, 2000)

J.S. Lee, D.L. Schuler, T.L. Ainsworth, and W. M. Boerner "A Review of Polarization orientation angle estimation and Applications," Proceedings of EUSAR 2006, E. Lueneburg Memorial Session, 2006

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P.O.A. Estimation

Polarization Orientation Shifts

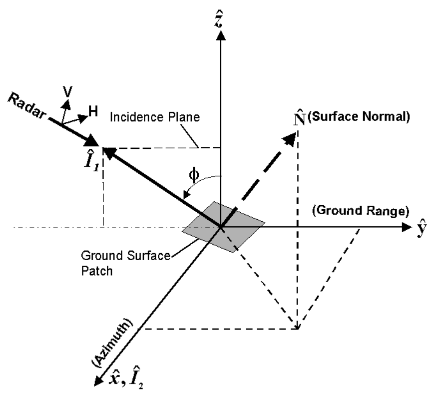
Orientation angle shifts induced by azimuthal slopes
Orientation information imbedded in Pol-SAR data

$$\tan\theta = \frac{\tan\omega}{-\tan\gamma\cos\varphi + \sin\varphi}$$

φ = Radar look angle

$\tan\omega$ = Azimuth slope

$\tan\gamma$ = Ground range slope



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P.O.A. Estimation

Orientation Estimation

Scattering Matrix

$$S^{(new)} = \begin{bmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

Coherency Matrix

$$T^{(new)} = UTU^T \qquad U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\,2\theta & sin\,2\theta \\ 0 & -sin\,2\theta & cos\,2\theta \end{bmatrix}$$

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P.O.A. Estimation

Orientation Rotation

Circular Polarizations (only phase is affected)

$$S_{LL} = (S_{hh} - S_{vv} + 2jS_{hv})/2$$
$$S = (-S_{hh} + S_{vv} + 2jS_{hv})/2$$
$$S_{LL} = j(S_{hh} + S_{vv})/2$$

ROTATION →

$$\tilde{S}_{LL} = S_{LL}\, e^{-i2\theta}$$
$$\tilde{S} = S\, e^{i2\theta}$$
$$\tilde{S}_{LL} = S_{LL}$$

Circular Covariance Matrix

$$\tilde{C} = \begin{bmatrix} \langle |S_{LL}|^2 \rangle & \sqrt{2}\langle (S_{LL}S^*_{LL_\perp})e^{-i2\theta} \rangle & \langle (S_{L_\perp L_\perp}S^*_{LL})e^{-i4\theta} \rangle \\ \sqrt{2}\langle (S_{LL_\perp}S^*_{LL})e^{i2\theta} \rangle & 2\langle |S_{LL_\perp}|^2 \rangle & \sqrt{2}\langle (S_{LL_\perp}S^*_{L_\perp L_\perp})e^{-i2\theta} \rangle \\ \langle (S_{LL}S^*_{L_\perp L_\perp})e^{i4\theta} \rangle & \sqrt{2}\langle (S_{L_\perp L_\perp}S^*_{LL_\perp})e^{i2\theta} \rangle & \langle |S_{L_\perp L_\perp}|^2 \rangle \end{bmatrix}$$

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P.O.A. Estimation

Estimation Methods

Circular Polarization Estimators

$$\tilde{S}_{LL} = S_{LL}\, e^{-i2\theta}$$

$$\Rightarrow \langle \tilde{S}_{LL} \tilde{S} \rangle = \langle (S_{LL} S)\, e^{-i4\theta} \rangle \langle (S_{LL} S) \rangle\, e^{-i4\theta}$$

$$\langle \tilde{S}_{LL} \tilde{S}_{LL} \rangle = \langle (S_{LL} S_{LL})\, e^{-i2\theta} \rangle \langle (S_{LL} S_{LL}) \rangle\, e^{-i2\theta}$$

Which estimator is the good one ?

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P.O.A. Estimation

Terrain Vegetation and Topography Representative of Camp Roberts, CA

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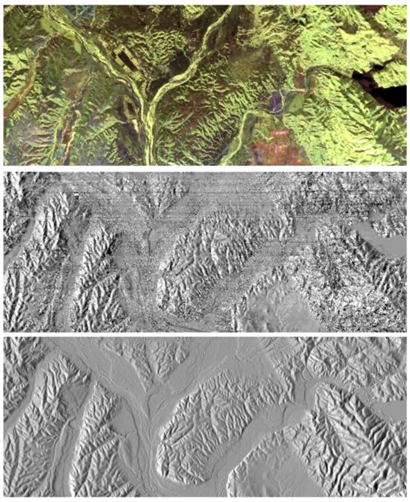
P.O.A. Estimation

FLIGHT



JPL AIRSAR L-Band (Camp Roberts)
|HH-VV| |HV|+|VH| |HH+VV|

From POLSAR data

$$-45^o \leq \theta \leq 45^o$$

From C-band Interferometry

$$\tan\theta = \frac{\tan\omega}{-\tan\gamma\cos\varphi + \sin\phi}$$

J.S. Lee, D.L. Schuler and T.L. Ainsworth, “Polarimetric SAR data compensation for terrain azimuth slope variations,” IEEE TGRS (September, 2000)

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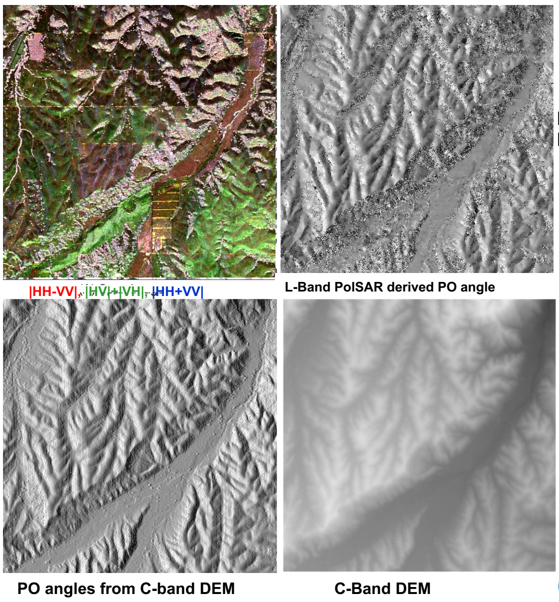
P.O.A. Estimation

FLIGHT



PO angles derived By L-Band PolSAR

PO angles derived from DEM of C-Band interferometry

$$\tan\theta = \frac{\tan\omega}{-\tan\gamma\cos\varphi + \sin\phi}$$

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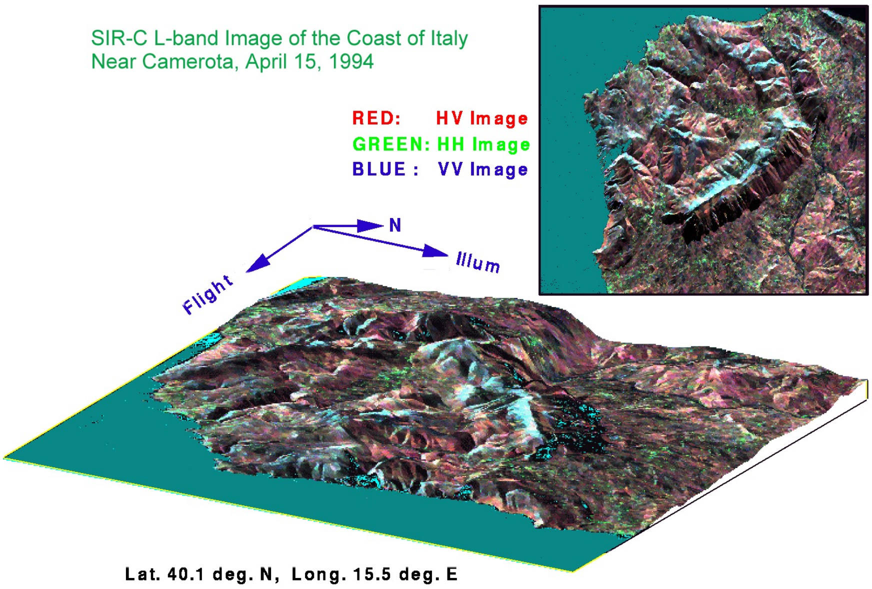
POLARIMETRIC SAR DERIVED TOPOGRAPHY

SIR-C L-band Image of the Coast of Italy
Near Camerota, April 15, 1994



Lat. 40.1 deg. N, Long. 15.5 deg. E

Courtesy of Dr. J. S. Lee, US NRL

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P.O.A. Estimation

The effect of POA Compensation

The Polarization Orientation (PO) angle effect :

- Azimuth slopes and buildings induce PO angle shift

- Model based decompositions based on uncompensated data may mis-interpret scattering mechanisms

- High relief terrain = Forest (volume scattering)

- Buildings = Forest

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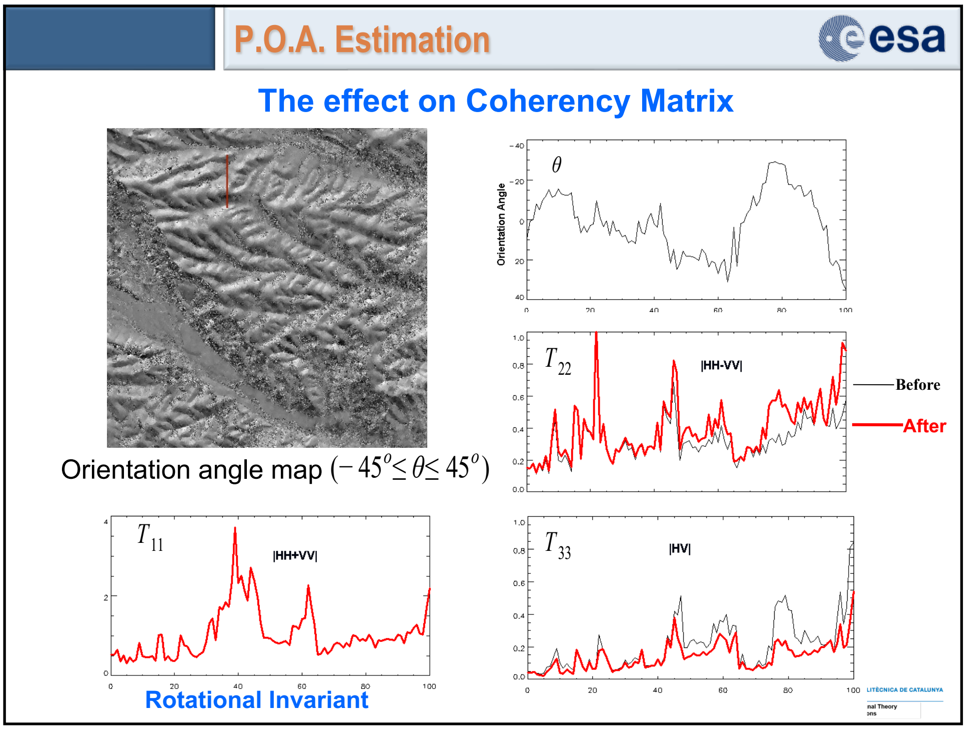
P.O.A. Estimation
esa
The effect on Coherency Matrix
Orientation angle map $(-45^o \leq \theta \leq 45^o)$
Orientation Angle
θ
T_{22}
|HH-VV|
Before
After
T_{11}
|HH+VV|
T_{33}
|HV|
Rotational Invariant

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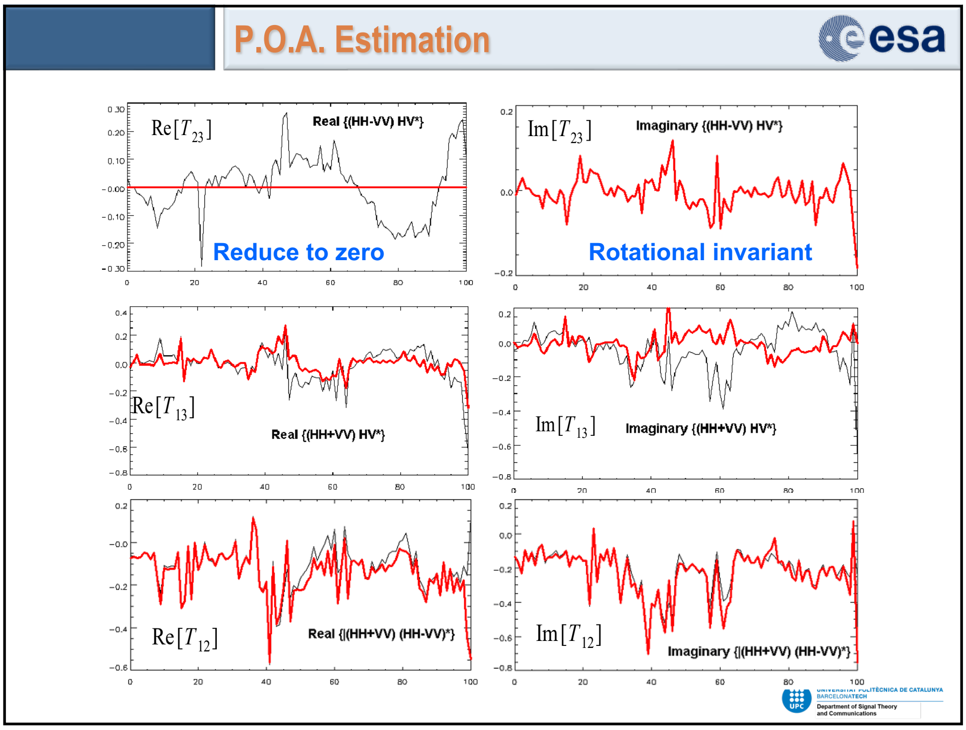
P.O.A. Estimation
esa
$\mathrm{Re}[T_{23}]$
Real {(HH-VV) HV*}
Reduce to zero
$\mathrm{Im}[T_{23}]$
Imaginary {(HH-VV) HV*}
Rotational invariant
$\mathrm{Re}[T_{13}]$
Real {(HH+VV) HV*}
$\mathrm{Im}[T_{13}]$
Imaginary {(HH+VV) HV*}
$\mathrm{Re}[T_{12}]$
Real {(HH+VV) (HH-VV)*}
$\mathrm{Im}[T_{12}]$
Imaginary {(HH+VV) (HH-VV)*}

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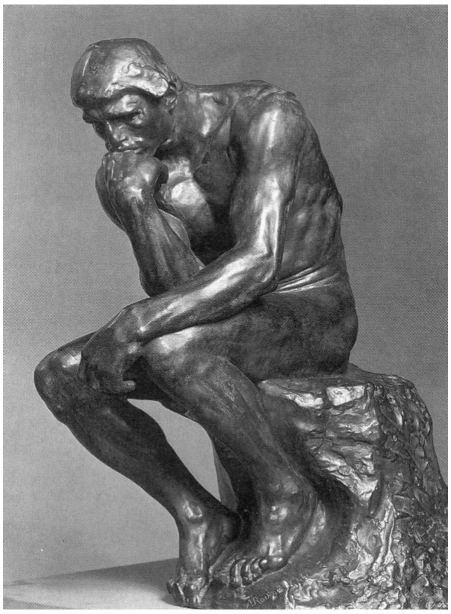

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