SAR and PolSAR Data Statistical Description

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ESA PolSAR Training Course
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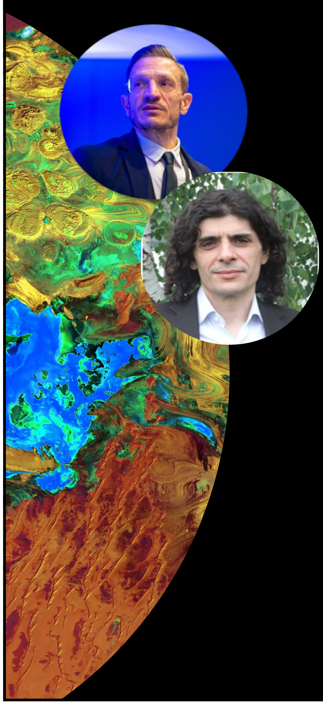

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Speckle Noise

Outline

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- SAR Data Statisitical Characterization
- PolSAR Data Statistical Characterization
- Information Estimation/Filtering
- PolSAR Data Speckle Noise Characterization

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Speckle Noise

Speckle Noise

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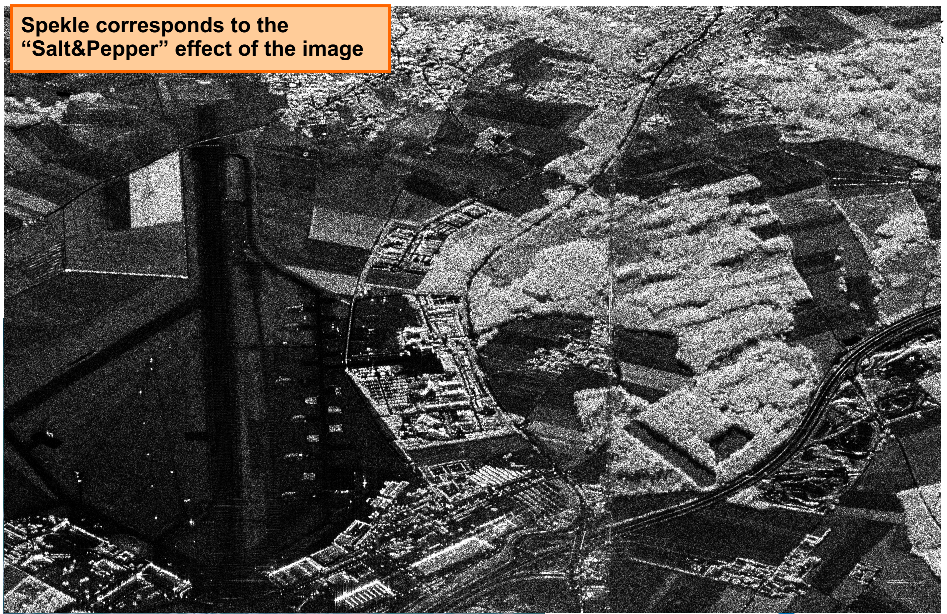

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Speckle Noise

Speckle Noise

On the basis of the discrete scatter description

$$S(x,r)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}u(x',r')h(x-x',r-r')\mathrm{d}x'\mathrm{d}r' \Rightarrow S(x,r)=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}\sqrt{\sigma_k}e^{j\theta_k}h(x-x_k,r-r_k)$$

Normalizing factor

L: Number of point scatters embraced by the resolution cell

- L as a deterministic quantity
  - L = 1: or a dominating point scatter: Deterministic scattering
    - Rice/Rician model
  - L >1: Partially developed speckle
    - Not solved model. Even numerical solution difficult
  - L >>1: Fully developed speckle
    - Gaussian model
- L as a stochastic quantity
  - L characterized by a pdf: Image texture
    - K-distribution model

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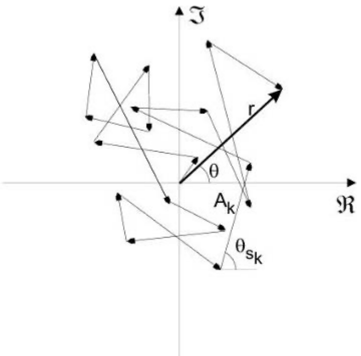
Speckle Noise

Fully Developed Speckle Noise

- SAR image formation process

$$S(x,r)=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}\sqrt{\sigma_k}e^{j\theta_k}h(x-x_k,r-r_k)$$



Random Walk Process

- Complex SAR data for L>>1

$$S(r(x,r),\theta(x,r))=\Re\{S\}+j\Im(S)$$
$$=r(x,r)\exp(j\theta(x,$$

- Real part

$$\Re\{S\}=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}A_k\cos(\theta_{s_k})$$

- Imaginary part

$$\Im\{S\}=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}A_k\sin(\theta_{s_k})$$

$$r(x,r)\exp(j\theta(x,r))=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}A_k\exp(j\theta_{s_k})$$

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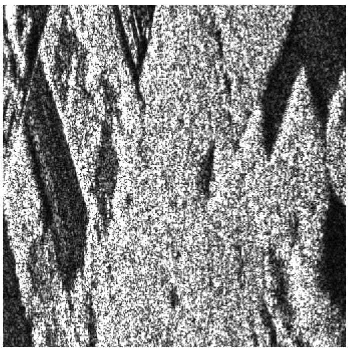
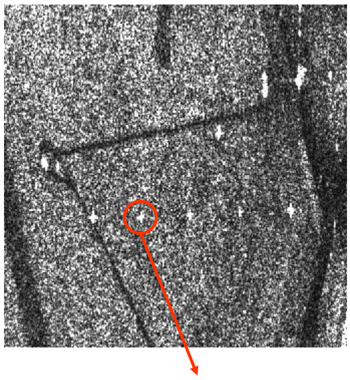
Speckle Noise

Fully Developed Speckle Noise





Fully Developed speckle

Bright points: Points where the interference is constructive

Dark points: Points where the interference is destructive

Corner reflector
Dominant scatter
No speckle

Speckle is the interference or fading pattern

S_{hh} amplitude
E-SAR L-band system

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Speckle Noise

Fully Developed Speckle Noise

- Completely developed Speckle (large L and no dominant scatterer)
  - Hypotheses
    - The amplitude A_k and the phase θ_{s_k} of the *k*th scattered wave are statistically independent of each other and from the amplitudes and phases of all other elementary waves (Uncorrelated point scatterers)
    - The phases of the elementary contributions θ_{s_k} are equally likely to lie anywhere in the primary interval *[-π, π)*
- Central Limit Theorem

$$S = \mathcal{N}_{C^2}\left(0, \sigma^2/2\right)$$

  - Real Part

$$p_{\Re\{S\}}\left(\Re\{S\}\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\left(\frac{\Re\{S\}}{\sigma}\right)^2\right) \quad \Re\{S\} \in (-\infty, \infty) \quad \text{Gaussian pdf}$$

  - Imaginary Part

$$p_{\Im\{S\}}\left(\Im\{S\}\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\left(\frac{\Im\{S\}}{\sigma}\right)^2\right) \quad \Im\{S\} \in (-\infty, \infty) \quad \text{Gaussian pdf}$$

  - Real and imaginary parts are uncorrelated $E\{\Re\{S\}\Im\{S\}\} = 0$

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Speckle Noise

Fully Developed Speckle Noise

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- Amplitude: Rayleigh pdf

$$p_r(r)=\frac{r}{\sigma^2}\exp\left(-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2\right) \quad r\in[0,\infty)$$

$$E\{r\}=\sqrt{\frac{\pi}{2}}\sigma$$

$$E\{r^2\}=2\sigma^2$$

$$\sigma_r^2=E\{r^2\}-E^2\{r\}=\left(2-\frac{\pi}{2}\right)\sigma^2$$

- Intensity ($I=r^2$): Exponential pdf

$$p_I(I)=\frac{1}{2\sigma^2}\exp\left(-\frac{I}{2\sigma^2}\right) \quad I\in[0,\infty)$$

$$E\{I\}=2\sigma^2\equiv\sigma$$

$$E\{I^2\}=2\left(2\sigma^2\right)^2$$

$$\sigma_I^2=E\{I^2\}-E^2\{I\}=\left(2\sigma^2\right)^2$$

- Phase: Uniform pdf. Contains NO information

$$p_\theta(\theta)=\frac{1}{2\pi} \quad \theta\in[-\pi,\pi)$$

- Amplitude and phase are uncorrelated

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Speckle Noise

Fully Developed Speckle Noise

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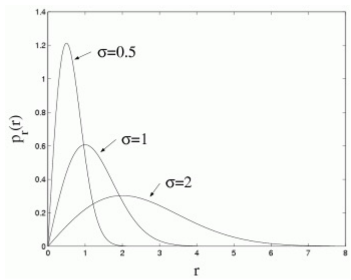
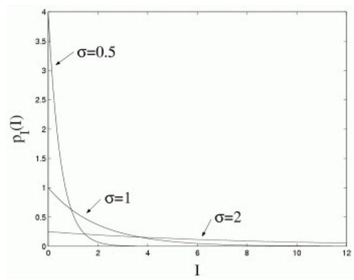
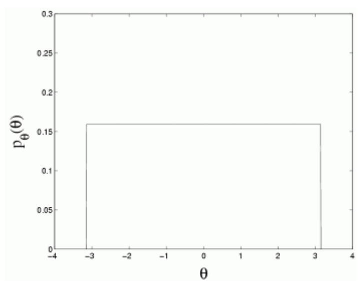


Amplitude: Rayleigh pdf



Intensity ($I=r^2$): Exponential pdf



Phase: Uniform pdf

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Speckle Noise

Information

What does it mean information in the presence of Speckle?

- Phase contains no information
- Intensity exponentially distributed

$$p_I(I)=\frac{1}{2\sigma^2}\exp\left(-\frac{I}{2\sigma^2}\right) \quad I\in[0,\infty) \quad \Rightarrow \quad E\{I\}=2\sigma^2 \quad \sigma_I=2\sigma^2$$

Exponential pdf — First and second order moments

- Intensity, under the previous hypotheses, is completely determined by the exponential pdf
  - Pdf completely determined by the pdf shape
  - Pdf shape parameterized by σ ➡ INFORMATION ➡ RCS σ^0
- Not useful information is considered as NOISE

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Speckle Noise

Fully Developed Speckle Noise Model

Objectives of a Noise Model

- To embed the data distribution into a noise model, that is, a function that allows identifying of the useful information to be retrieved, the noise sources, and how these terms interact
- Optimize the information extraction process, i.e., the noise filtering process

SAR image intensity noise model

SAR image intensity ($I=r^2$) $\quad p_I(I)=\frac{1}{2\sigma^2}\exp\left(-\frac{I}{2\sigma^2}\right) \quad I\in[0,\infty) \quad E\{I\}=2\sigma^2 \quad \sigma_I=2\sigma^2$

$I=2\sigma^2 n \quad p_n(n)=\exp(-n) \quad n\in[0,\infty) \quad E\{I\}=1 \quad \sigma_I=1$

One dimensional speckle noise model (Model over the SAR image intensity - 2nd moment) ➡ $I(x,r)=\sigma(x,r)n(x,r)$

Multiplicative Speckle Noise Model

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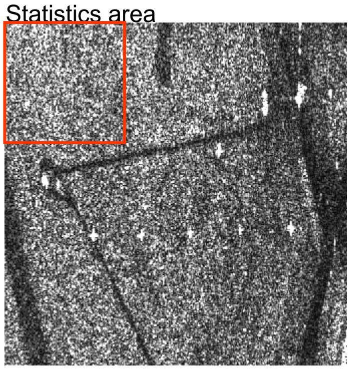
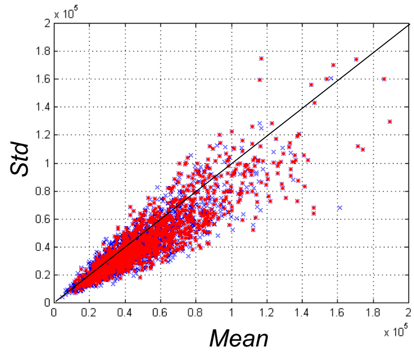
Speckle Noise — Fully Developed Speckle Noise Model

Moments calculated over local 7x7 local windows

Statistics area



Grass area



Blue: $|S_{hh}|^2$
Red: $|S_{vv}|^2$

DLR

S_{hh} amplitude
E-SAR L-band system

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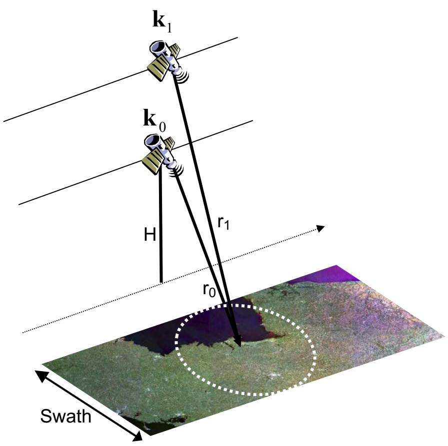
Speckle Noise — Polarimetric SAR Systems

The Polarimetric SAR system acquires 3 complex SAR images

Target vector $\mathbf{k} = [S_{hh}, 2S_{hv}, S_{vv}]^T$



The properties of the target vector follow from the properties of a single SAR image

- **k** is deterministic for point scatters. It contains all the necessary information to characterize the scatter
- **k** is a multidimensional random variable for distributed scatters due to speckle. A single sample does not characterize the scatterer

SAR images characterized through second order moments

- Second order moments in multidimensional SAR data are matrix quantities

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Speckle Noise

Fully Developed Speckle Noise

Completely developed Speckle (large L and no dominant scatter)

- Hypotheses
  - The amplitude A_k and the phase θ_{s_k} of the kth scattered wave are statistically independent of each other and from the amplitudes and phases of all other elementary waves (Uncorrelated point scatterers)
  - The phases of the elementary contributions θ_{s_k} are equally likely to lie anywhere in the primary interval *[-π, π)*

Central Limit Theorem $S = \mathcal{N}_{C^2}\left(0, \sigma^2/2\right)$

- Real Part

$$p_{\Re\{S\}}\left(\Re\{S\}\right) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2}\left(\frac{\Re\{S\}}{\sigma}\right)^2\right) \quad \Re\{S\} \in (-\infty, \infty) \quad \text{Gaussian pdf}$$

- Imaginary Part

$$p_{\Im\{S\}}\left(\Im\{S\}\right) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{1}{2}\left(\frac{\Im\{S\}}{\sigma}\right)^2\right) \quad \Im\{S\} \in (-\infty, \infty) \quad \text{Gaussian pdf}$$

- Real and imaginary parts are uncorrelated $E\left\{\Re\{S\}\Im\{S\}\right\} = 0$

Speckle Noise

Mathematical Representation

PDF for non-correlated SAR images

- Zero-mean multidimensional complex (also circular) Gaussian pdf

$$p_{\mathbf{k}}(\mathbf{k}) = \prod_{k=1}^{3}\frac{1}{\pi\sigma^2}\exp\left(-\frac{S_k S_k^H}{\sigma^2}\right) = \frac{1}{\pi^m\sigma^{2m}}\exp\left(-\sum_{k=1}^{m}\frac{S_k S_k^H}{\sigma^2}\right) = \frac{1}{\pi^m\sigma^{2m}}\exp\left(-\frac{1}{\sigma^2}\mathrm{tr}\left(\mathbf{k}\mathbf{k}^H\right)\right)$$

⇧

Independent SAR images with the same power $S_k = \mathcal{N}_{C^2}\left(0, \sigma^2/2\right)$

- First order moment

$$E\{\mathbf{k}\} = \mathbf{0}$$

- Second order moment: Covariance matrix

$$\mathbf{C} = E\left\{\mathbf{k}\mathbf{k}^H\right\} = \sigma^2\mathbf{I}_{m\times m}$$

Speckle Noise

Multidimensional Gaussian pdf Properties

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Characterization of random variables

- Probability Density Function (pdf)
- Moment-generating function
- Statistical moments (mean, power, kurtosis, skewness…)

Zero-mean multidimensional complex Gaussian pdf

$$p_{\mathbf{k}}(\mathbf{k}) = \frac{1}{\pi^3 |\mathbf{C}|} \exp\left(-\mathbf{k}^H \mathbf{C}^{-1} \mathbf{k}\right)$$

- First order moment $E\{\mathbf{k}\} = \mathbf{0}$
- Second order moment: Covariance matrix

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} E\{|S_{hh}|^2\} & E\{S_{hh}S_{hv}^*\} & E\{S_{hh}S_{vv}^*\} \\ E\{S_{hv}S_{hh}^*\} & E\{|S_{hv}|^2\} & E\{S_{hv}S_{vv}^*\} \\ E\{S_{vv}S_{hh}^*\} & E\{S_{vv}S_{hv}^*\} & E\{|S_{vv}|^2\} \end{bmatrix}$$

$$E\{S_k S_l^*\} \neq 0 \;\; k,l \in \{1,\ldots,m\}, k \neq l$$

↓

Correlated SAR images

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Multidimensional Gaussian pdf Properties

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A zero-mean multidimensional complex Gaussian pdf is completely characterized by the second order moments, i.e., the covariance matrix

- Moment theorem for complex Gaussian processes, given Q correlated SAR images
  - For $k \neq l$, where m_k and n_l are integers from $\{1,2,\ldots,Q\}$

    $$E\{S_{m_1}S_{m_2}\cdots S_{m_k}S_{n_1}^*S_{n_2}^*\cdots S_{n_l}^*\} = 0$$

  - For $k = l$, where π is a permutation of the set of integers $\{1,2,\ldots,Q\}$

    $$E\{S_{m_1}S_{m_2}\cdots S_{m_k}S_{n_1}^*S_{n_2}^*\cdots S_{n_l}^*\} = \sum_{\pi} E\{S_{m_{\pi(1)}}S_{n_1}^*\}E\{S_{m_{\pi(2)}}S_{n_2}^*\}\cdots E\{S_{m_{\pi(l)}}S_{n_l}^*\}$$

- Considering the covariance matrix
  - Higher order moments are function of the covariance matrix

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Speckle Noise

Mathematical Representation

The covariance matrix contains the correlation structure of the set of m SAR images

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} E\{|S_{hh}|^2\} & E\{S_{hh}S_{hv}^*\} & E\{S_{hh}S_{vv}^*\} \\ E\{S_{hv}S_{hh}^*\} & E\{|S_{hv}|^2\} & E\{S_{hv}S_{vv}^*\} \\ E\{S_{vv}S_{hh}^*\} & E\{S_{vv}S_{hv}^*\} & E\{|S_{vv}|^2\} \end{bmatrix}$$

Information

- Diagonal elements: Power information

$$E\{S_k S_k^H\} = E\{|S_k|^2\} \quad k \in \{1,2,\ldots,m\}$$

- Off-diagonal elements: Correlation information

$$E\{S_k S_l^H\} \quad k,l \in \{1,2,\ldots,m\}, k \neq l$$

Speckle Noise

Mathematical Representation

PDF for correlated SAR images

- Zero-mean multidimensional complex Gaussian pdf

$$p_{\mathbf{k}}(\mathbf{k}) = \frac{1}{\pi^3 |\mathbf{C}|} \exp\left(-\mathbf{k}^H \mathbf{C}^{-1} \mathbf{k}\right)$$

- First order moment

$$E\{\mathbf{k}\} = \mathbf{0}$$

- Second order moment: Covariance matrix

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} E\{|S_{hh}|^2\} & E\{S_{hh}S_{hv}^*\} & E\{S_{hh}S_{vv}^*\} \\ E\{S_{hv}S_{hh}^*\} & E\{|S_{hv}|^2\} & E\{S_{hv}S_{vv}^*\} \\ E\{S_{vv}S_{hh}^*\} & E\{S_{vv}S_{hv}^*\} & E\{|S_{vv}|^2\} \end{bmatrix}$$

All the information characterizing the set of 3 SAR images is contained in the covariance matrix

Speckle Noise

Complex Correlation Coefficient

How to consider the correlation information

- Off-diagonal covariance matrix elements

$$E\{S_k S_l^H\} \quad k,l \in \{1,2,\ldots,m\}, k \neq l$$

  - Absolute correlation information

- Complex correlation coefficient

$$\rho_{k,l} = \frac{E\{S_k S_l^*\}}{\sqrt{E\{|S_k|^2\} \cdot E\{|S_l|^2\}}} = |\rho_{k,l}| e^{j\theta_{k,l}}$$

$$0 \leq |\rho_{k,l}| \leq 1 \quad \text{Coherence}$$

$$-\pi \leq \theta_{k,l} \leq \pi$$

  - Normalized correlation information

- The complex correlation information represents the most important observable for multidimensional SAR data. Its physical interpretation depends on the multidimensional SAR system configuration

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Speckle Noise

Mathematical Representation

Multidimensional SAR data descriptors for distributed scatterers

First order moment

$$\mathbf{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{hv}^H & S_{vv} \end{bmatrix}$$

Scattering matrix

$$\mathbf{k} = [S_1, S_2, \ldots, S_m]^T$$

Target vector

Second order moment

$$\mathbf{k}\mathbf{k}^H = \begin{bmatrix} S_{hh}S_{hh}^H & \sqrt{2}S_{hh}S_{hv}^H & S_{hh}S_{vv}^H \\ \sqrt{2}S_{hv}S_{hh}^H & 2S_{hv}S_{hv}^H & \sqrt{2}S_{hv}S_{vv}^H \\ S_{vv}S_{hh}^H & \sqrt{2}S_{vv}S_{hv}^H & S_{vv}S_{vv}^H \end{bmatrix}$$

Covariance matrix

$$\mathbf{k}\mathbf{k}^H = \begin{bmatrix} S_1S_1^H & S_1S_2^H & \cdots & S_1S_m^H \\ S_2S_1^H & S_2S_2^H & \cdots & S_2S_m^H \\ \vdots & \vdots & \ddots & \vdots \\ S_mS_1^H & S_mS_2^H & \cdots & S_mS_m^H \end{bmatrix}$$

Description for PolSAR data

Point scatterers characterization

Description for generalized multidimensional SAR data

$$E\{\mathbf{k}\} = \mathbf{0}$$

Characterizes completely the data distribution

Distributed scatterers characterization

$$E\{\mathbf{k}\mathbf{k}^H\} = \mathbf{C}$$

Covariance matrix

$$p_{\mathbf{k}}(\mathbf{k}) = \frac{1}{\pi^m |\mathbf{C}|} \exp\left(-\mathbf{k}^H \mathbf{C}^{-1} \mathbf{k}\right)$$

Speckle Noise

Information Estimation/Filtering

For multidimensional SAR data, under the hypothesis of Gaussian scattering, all the information is contained in the covariance matrix

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} E\{|S_{hh}|^2\} & E\{S_{hh}S_{hv}^*\} & E\{S_{hh}S_{vv}^*\} \\ E\{S_{hv}S_{hh}^*\} & E\{|S_{hv}|^2\} & E\{S_{hv}S_{vv}^*\} \\ E\{S_{vv}S_{hh}^*\} & E\{S_{vv}S_{hv}^*\} & E\{|S_{vv}|^2\} \end{bmatrix}$$

This matrix must be estimated from the available information

- The scattering vector for each pixel/sample of the SAR data

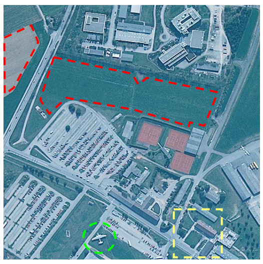
$$\mathbf{k} = [S_{hh}, 2S_{hv}, S_{vv}]^T$$

- The estimation process reduces to estimate the ensemble average (expectation operator) $E\{\cdot\}$
- The estimation process also receives the name of data filtering process.

Speckle Noise

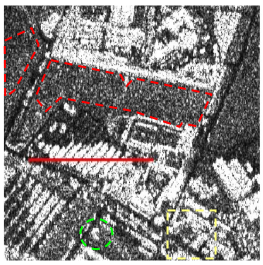
Information Estimation/Filtering

Considerations about speckle noise reduction



Optical image DLR OP

SAR images reflex the Nature's complexity



SAR image DLR OP

Homogeneous areas	Image details	Heterogeneous areas
Maintain useful information (σ)	Maintain spatial details (Shape and value)	Maintain both
RADIOMETRIC RESOLUTION	SPATIAL RESOLUTION	LOCAL ANALYSIS

Image data: S_{hh} amplitude. E-SAR L-band system

Speckle Noise

Sample Covariance Matrix

Covariance matrix estimation by means of a MultiLook (BoxCar)

- Maximum likelihood estimator: Sample covariance matrix

$$\mathbf{Z}_n = \frac{1}{n}\sum_{k=1}^{n}\mathbf{k}\mathbf{k}^H = \begin{bmatrix} \frac{1}{n}\sum_{k=1}^{n} S_1(k)S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_1(k)S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_1(k)S_m^*(k) \\ \frac{1}{n}\sum_{k=1}^{n} S_2(k)S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_2(k)S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_2(k)S_m^*(k) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n}\sum_{k=1}^{n} S_m(k)S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_m(k)S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_m(k)S_m^*(k) \end{bmatrix}$$

  - n represents the total number of samples employed to estimate the covariance matrix, taken a region (square, rectangular, adapted…)
  - $\mathbf{Z}_n$ as estimator of $\mathbf{C}$
    - Does not consider signal morphology/heterogeneity
    - Loss of spatial resolution

The sample covariance matrix $\mathbf{Z}_n$ is itself a multidimensional random variable

Speckle Noise

Sample Covariance Matrix Distribution

The sample covariance matrix $\mathbf{Z}_n$ is characterized by the complex Wishart distribution $\mathbf{Z}_n \sim \mathcal{W}(n, \mathbf{C})$

$$p_{\mathbf{Z}_n}(\mathbf{Z}_n) = \frac{n^{mn}|\mathbf{Z}_n|^{n-m}}{|\mathbf{C}|^n \tilde{\Gamma}_m(n)} \mathrm{etr}\left(-n\mathbf{C}^{-1}\mathbf{Z}_n\right) \qquad \tilde{\Gamma}_m(n) = \pi^{m(m-1)/2}\Pi_{i=1}^{m}\Gamma(n-i+1)$$

- Multidimensional data distribution
- Valid for $n \geq m$, otherwise $|\mathbf{Z}_n|^{n-m}$ is equal to zero and the Wishart pdf is undetermined
  - Equivalent to $Rank(\mathbf{Z}_n)=m$, i.e., the sample covariance matrix is a full rank matrix
  - The higher the data dimensionality m the higher the number of looks n for the Wishart pdf to be defined

Speckle Noise

Multidimensional SAR Data Description

$$S_k = \mathcal{N}_{C^2}\left(0, \sigma^2/2\right)$$

Single SAR image

$$\mathbf{k} = \left[S_1, S_2, \ldots, S_m\right]^T$$

Multidimensional SAR dataset

$$p_{\mathbf{k}}(\mathbf{k}) = \frac{1}{\pi^m |\mathbf{C}|}\exp\left(-\mathbf{k}^H \mathbf{C}^{-1}\mathbf{k}\right)$$

Multidimensional complex Gaussian pdf

$$\mathbf{Z}_n = \frac{1}{n}\sum_{i=1}^{n} \mathbf{k}_i \mathbf{k}_i^H$$

$$\mathbf{Z}_n \sim \mathcal{W}(n, \mathbf{C})$$

Sample covariance matrix

$$p_{\mathbf{Z}_n}(\mathbf{Z}_n) = \frac{n^{mn} |\mathbf{Z}_n|^{n-m}}{|\mathbf{C}|^n \tilde{\Gamma}_m(n)} \mathrm{etr}\left(-n\mathbf{C}^{-1}\mathbf{Z}_n\right)$$

Complex Wishart pdf

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Speckle Noise

Multilook/Boxcar Example

|Shh-Svv| 2|Shv| | Shh +Svv|

Original data

7x7 MLT data

L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)

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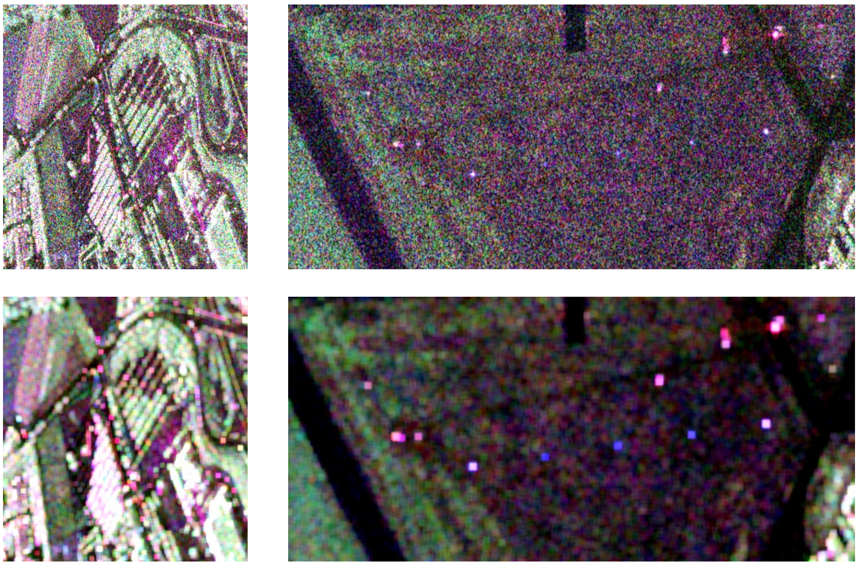
Speckle Noise

Multilook/Boxcar Example

Original data

7x7 MLT data



|Shh-Svv| 2|Shv| | Shh +Svv|

Speckle Noise

Local Statistics Linear Filter

Local statistics linear filter (Lee filter)

Filter form $\hat{I}(x,r) = a\,E\{I(x,r)\} + bI(x,r)$

Signal noise model $I(x,r) = \sigma(x,r)n(x,r)$

Minimization criteria (MMSE) $\min_{(a,b)} J = E\{(\hat{I}(x,r) - I(x,r))\}$

MMSE gives $a = \frac{1}{E\{n\}} - b$ $\quad b = E\{n\}\frac{\operatorname{var}(\sigma)}{\operatorname{var}(I)}$

$$\hat{I}(x,r) = \frac{E\{I(x,r)\}}{E\{n\}} + b\left(I(x,r) - E\{I(x,r)\}\right)$$

Statistics need to be derived from noisy data

$a = \frac{1}{E\{n\}} - b = 1 - b$ ($E\{n\} = 1$)

$b = E\{n\}\frac{\operatorname{var}(\sigma)}{\operatorname{var}(I)} = \frac{\operatorname{var}(I) - E^2\{I\}\sigma_n^2}{\operatorname{var}(I)(1+\sigma_n^2)}$

Information estimated from data

$E\{n\} = 1$

$$\boxed{\hat{I}(x,r) = E\{I(x,r)\} + b\left(I(x,r) - E\{I(x,r)\}\right)}$$

Local statistics $E^2\{I(x,r)\}$ $\quad \operatorname{var}\{I\}$

A priori information $\sigma_n^2 = \operatorname{var}(n) = \frac{1}{N}$

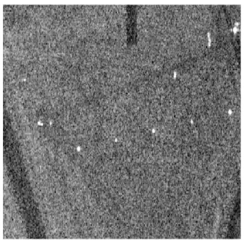
Speckle Noise

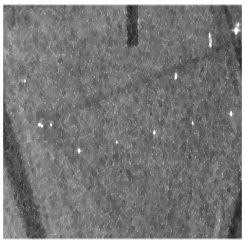
Local Statistics Linear Filter

$$\hat{I}(x,r) = E\{I(x,r)\} + b\left(I(x,r) - E\{I(x,r)\}\right)$$

$\mathrm{var}(I) >> E^2\{I\} \Rightarrow b \to 1$ Multiplicative noise model can not explain data variability

$\mathrm{var}(I) \approx E^2\{I\}\sigma_n^2 \Rightarrow b \to 0$ Multiplicative noise model can explain data variability


Original SAR intensity image


Filtered SAR intensity image
Lee Filter

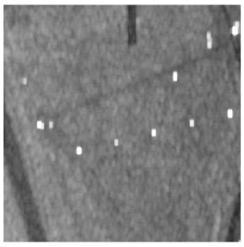

Filtered SAR intensity image
Boxcar Filter

Image data: S_{hh} amplitude. E-SAR L-band system

Speckle Noise

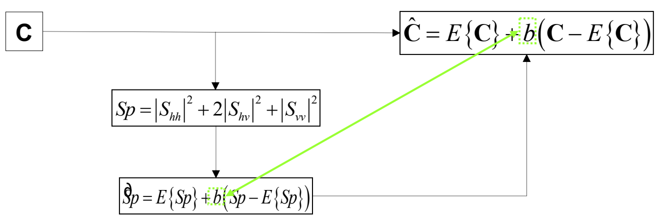
Local Statistics Linear Filter

Polarimetric Lee filter

Nowadays is the most employed polarimetric filtering solution

Extension of the linear scalar Lee filter for SAR images by considering a multiplicative speckle noise model over all the covariance matrix entries

Working principles



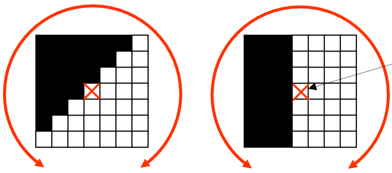
Linear scalar
Lee filter

Speckle Noise

Local Statistics Linear Filter

Refined Lee filter

Statistics estimation in windows selected according to the signal morphology in order to retain edges, spatial feature and point targets



The extension of the scalar linear Lee filter presents limitations

Not based on the multiplicative-additive speckle noise model. This limits the capacity to reduce noise in those images areas characterized by low correlation ➡ The elements of the covariance matrix can be processed differently, but according to the right speckle noise model

The a priori information in the span image σ_n^2 is no longer a constant as the noise content in span depends on the data's correlation structure

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Speckle Noise

Local Statistics Linear Filter

|Shh| |Shv| |Svv|

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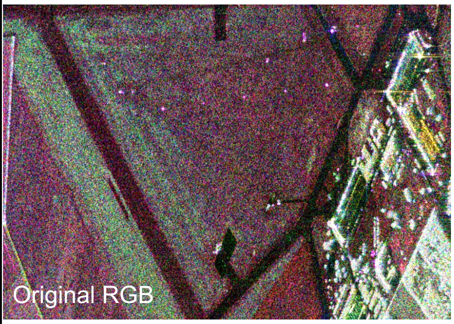
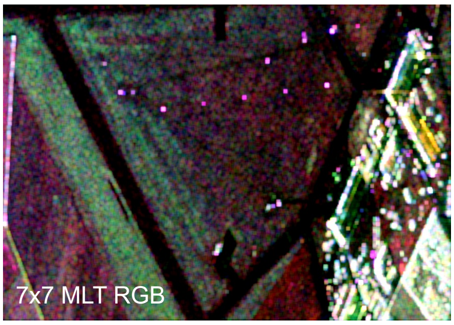
Speckle Noise

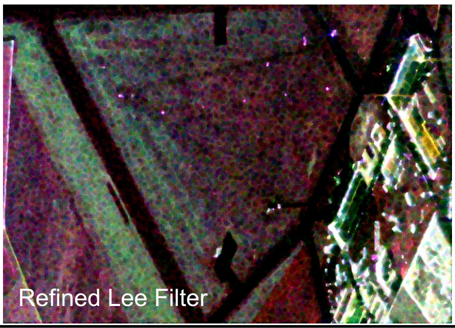
Local Statistics Linear Filter

esa





|Shh| |Shv| |Svv|



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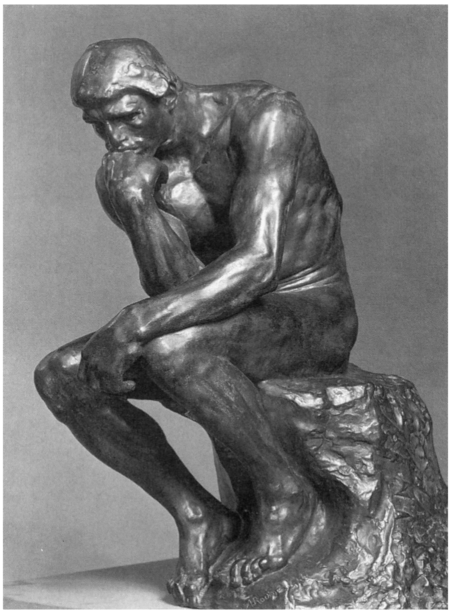
Speckle Noise

esa



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