# SAR POLARIMETRY

## Polarimetric Basics

***Laurent Ferro-Famil***

***ISAE-SUPAERO & CESBIO, France***

***Eric Pottier***

***IETR, University of Rennes 1, France***


**Laurent FERRO-FAMIL**
laurent.ferro-famil@isae-supaero.fr

**Eric POTTIER**
eric.pottier@univ-rennes1.fr

Institut Supérieur de l'Aéronautique et de l'Espace
SUPAERO

CESBIO

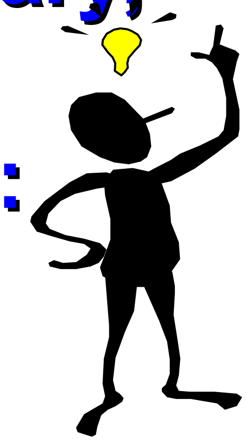
Université Fédérale
Toulouse
Midi-Pyrénées

IETR
INSTITUT D'ÉLECTRONIQUE ET DE TÉLÉCOMMUNICATIONS DE RENNES

UNIVERSITÉ DE RENNES 1

# Objective

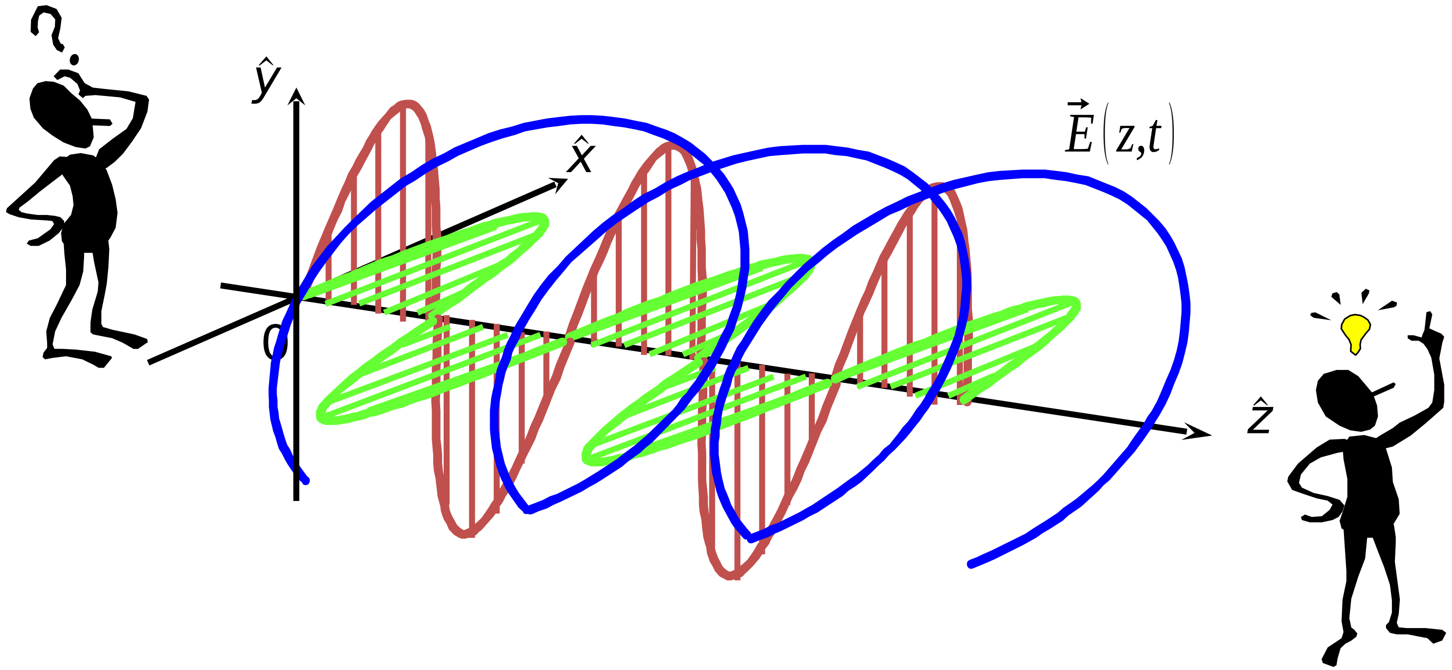
**To provide the minimum, but necessary, amount of knowledge required to understand scientific works on :**



**SAR Polarimetry (PolSAR)**
**SAR Polarimetry + Interferometry (Pol-InSAR)**
**SAR Polarimetry + Tomography (Pol-TomSAR)**

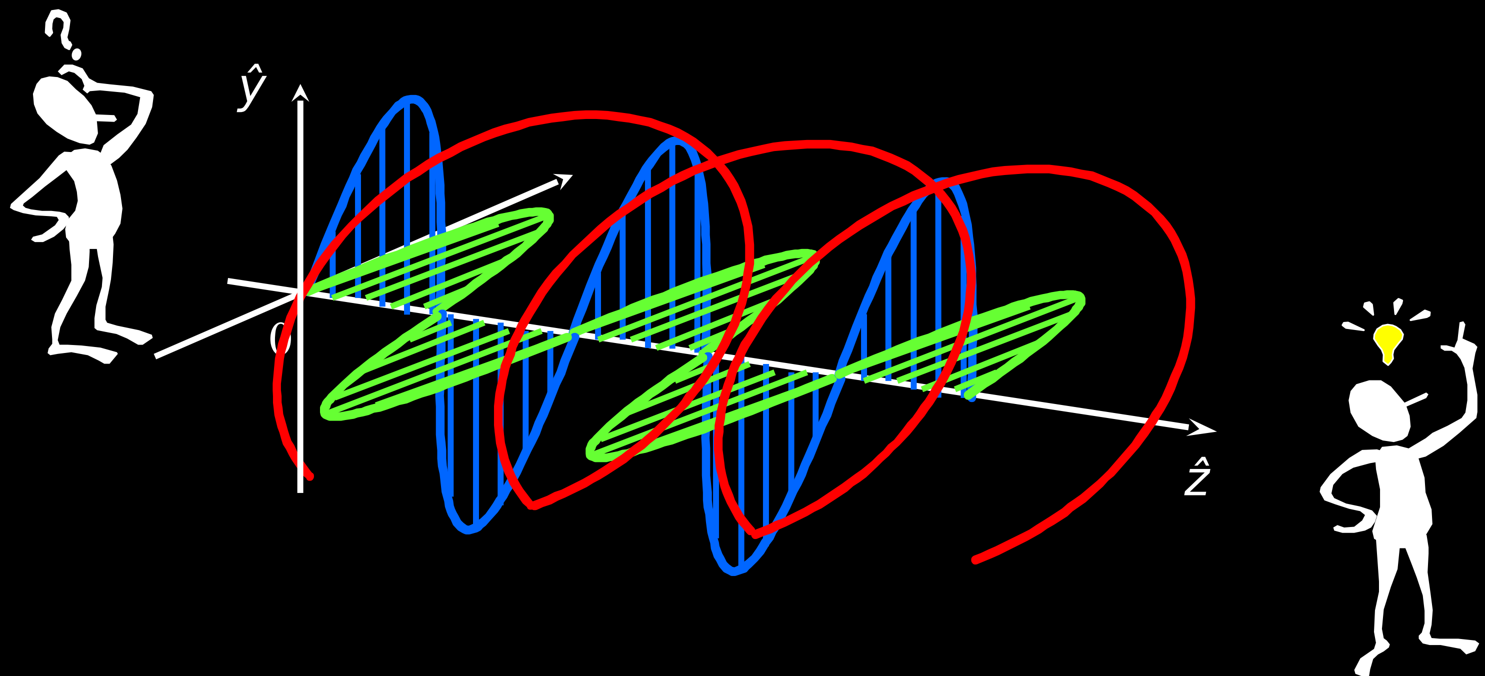


IETR ISAE SUPAERO CESBIO




# GENERAL INTRODUCTION

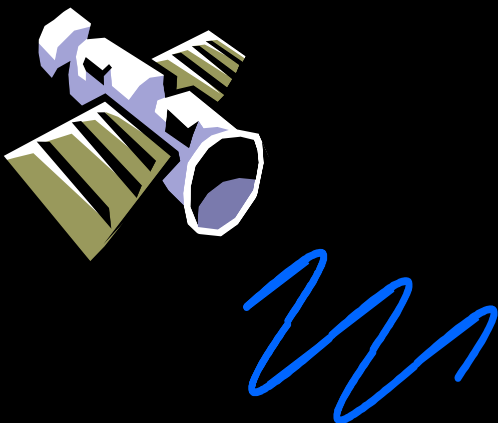
# Radar Polarimetry



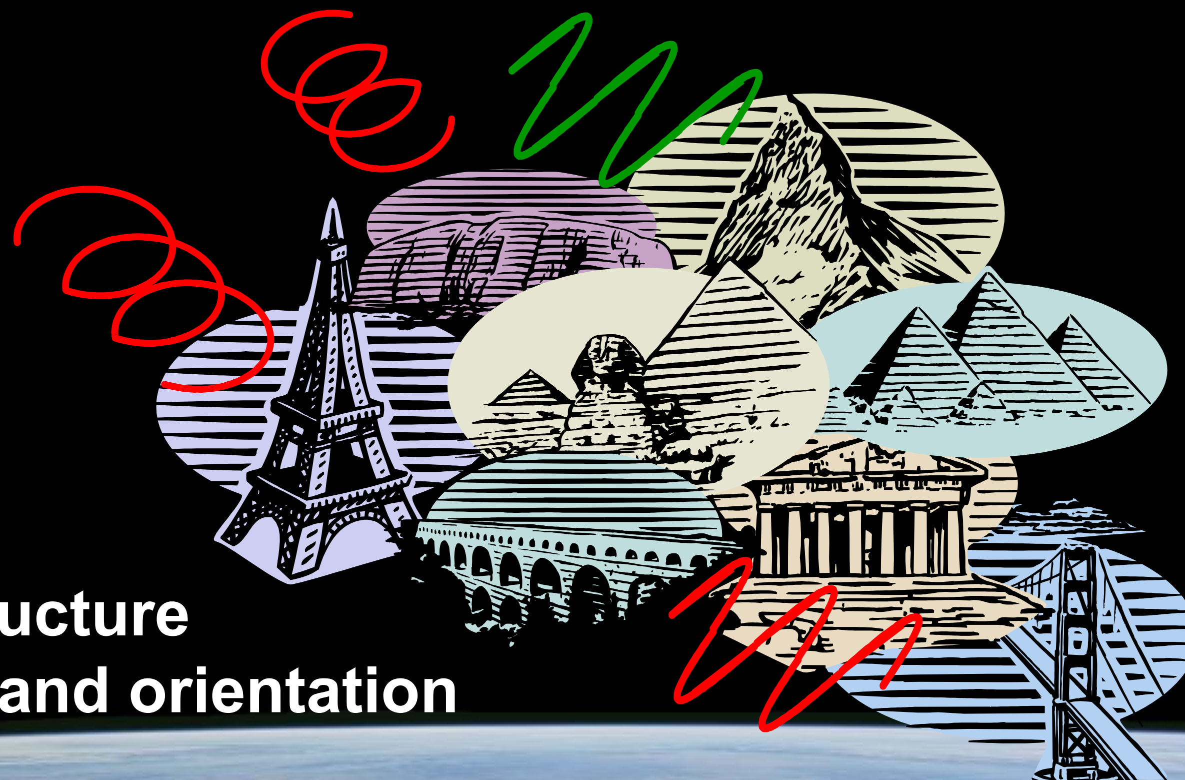

**Radar Polarimetry (Polar : polarisation Metry: measure) is the science of acquiring, processing and analysing the polarization state of an electromagnetic field**

**Radar Polarimetry deals with the full vector nature of polarized electromagnetic waves**

# Radar Polarimetry



The POLARISATION information
Contained in the waves backscattered
from a given medium is highly related to:



its geometrical structure
reflectivity, shape and orientation

its geophysical properties such as humidity, roughness, …

# SAR Polarimetry Applications

**Forest Vegetation**

- Forest Height
- Forest Biomass
- Forest Structure
- Canopy Extinction
- Underlying Topography

- Forest Ecology
- Forest Management
- Ecosystem Change
- Carbon Cycle

**Agriculture**

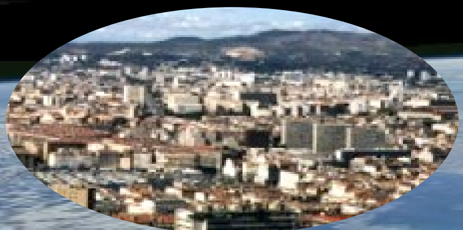
- Soil Moisture Content
- Soil roughness
- Height of Vegetation Layer
- Extinction of Vegetation Layer
- Moisture of Vegetation Layer

- Farming Management
- Water Cycle
- Desretification

**Snow and Ice**

- Topography
- Penetration Depth / Density
- Snow Ice Layer
- Snow Ice Extinction
- Water Equivalent

- Ecosystem Change
- Water Cycle
- Water Management



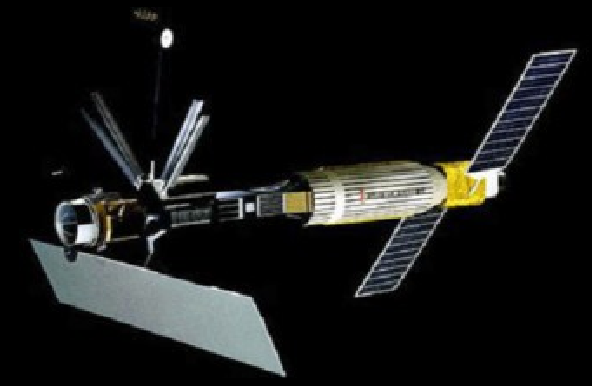
**Urban Areas**

- Geometric Properties
- Dielectric Properties

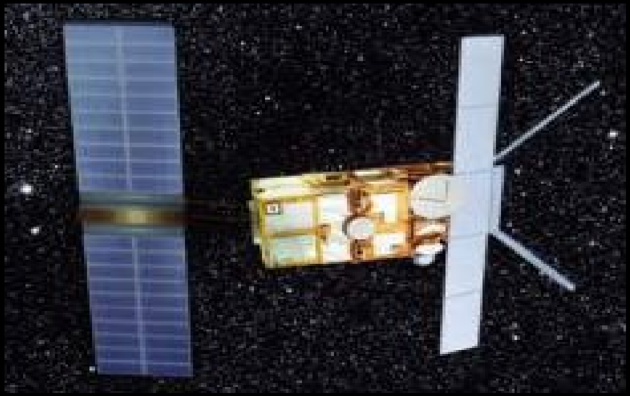
- Urban Monitoring

Courtesy of Dr. I. Hajnsek

# Space-borne Sensors



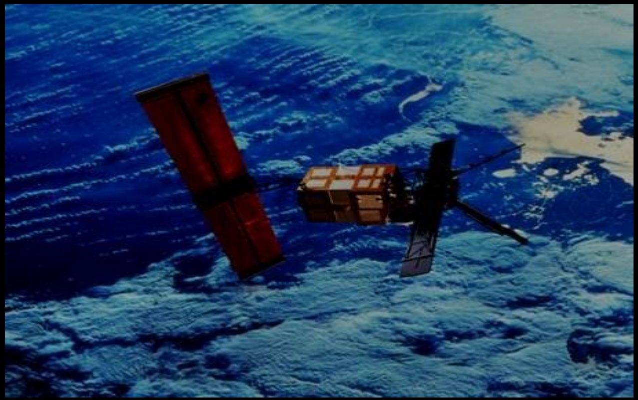
**SEASAT**
**NASA/JPL (USA)**
L-Band, 1978



**ERS-1**
**European Space Agency (ESA)**
C-Band, 1991-2000

**J-ERS-1**
**Japanese Space Agency (NASDA)**
L-Band, 1992-1998

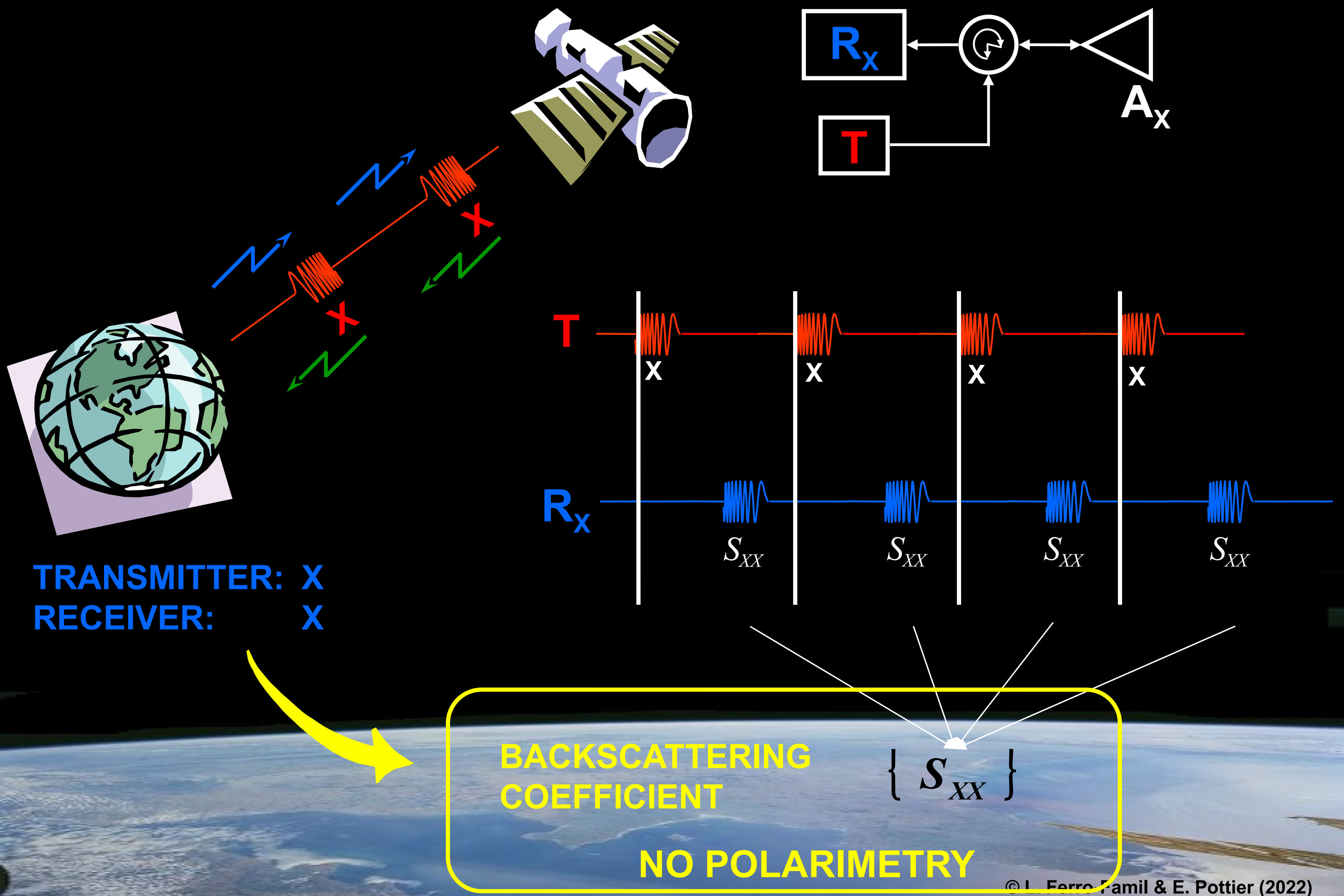
**RadarSAT-1**
**Canadian Space Agency (CSA)**
C-Band, 1995



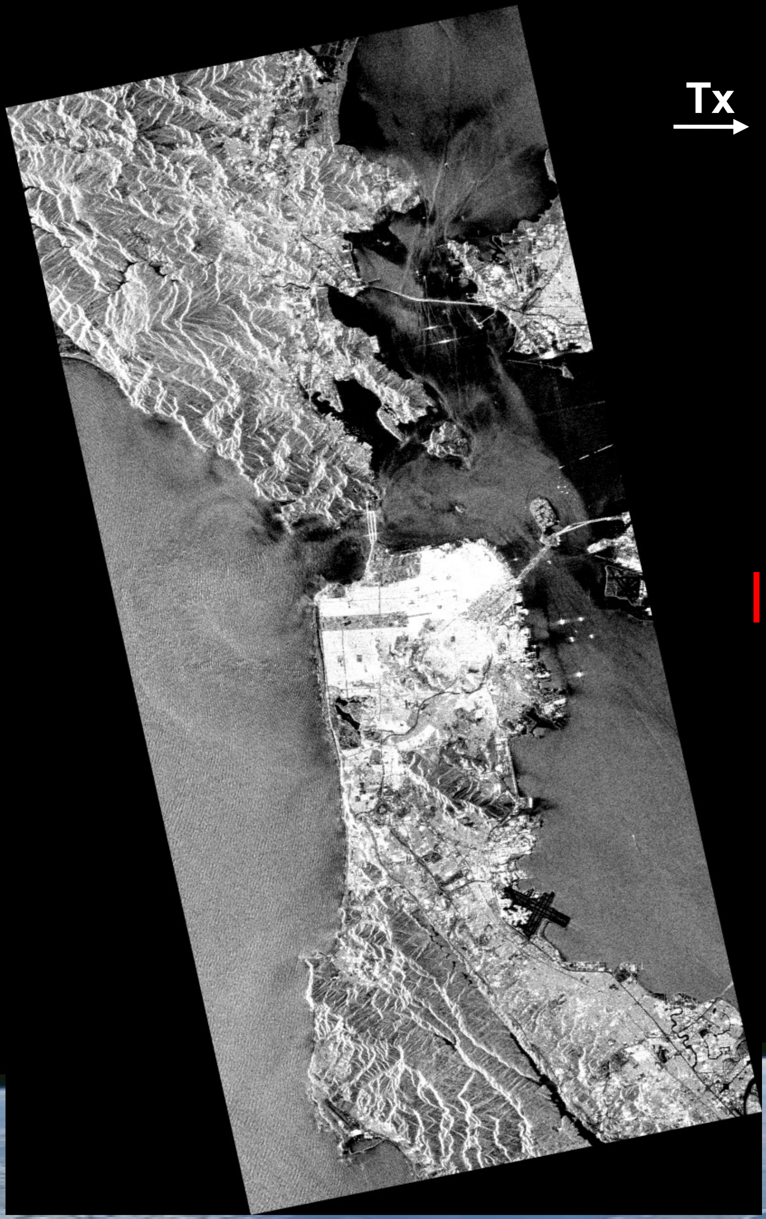
**ERS-2**
**European Space Agency (ESA)**
C-Band, 1995

**Shuttle Radar Topography Mission**
**NASA/JPL (C-Band), DLR (X-Band)**
February 2000

# Scattering Coefficient



# Space-borne Sensors

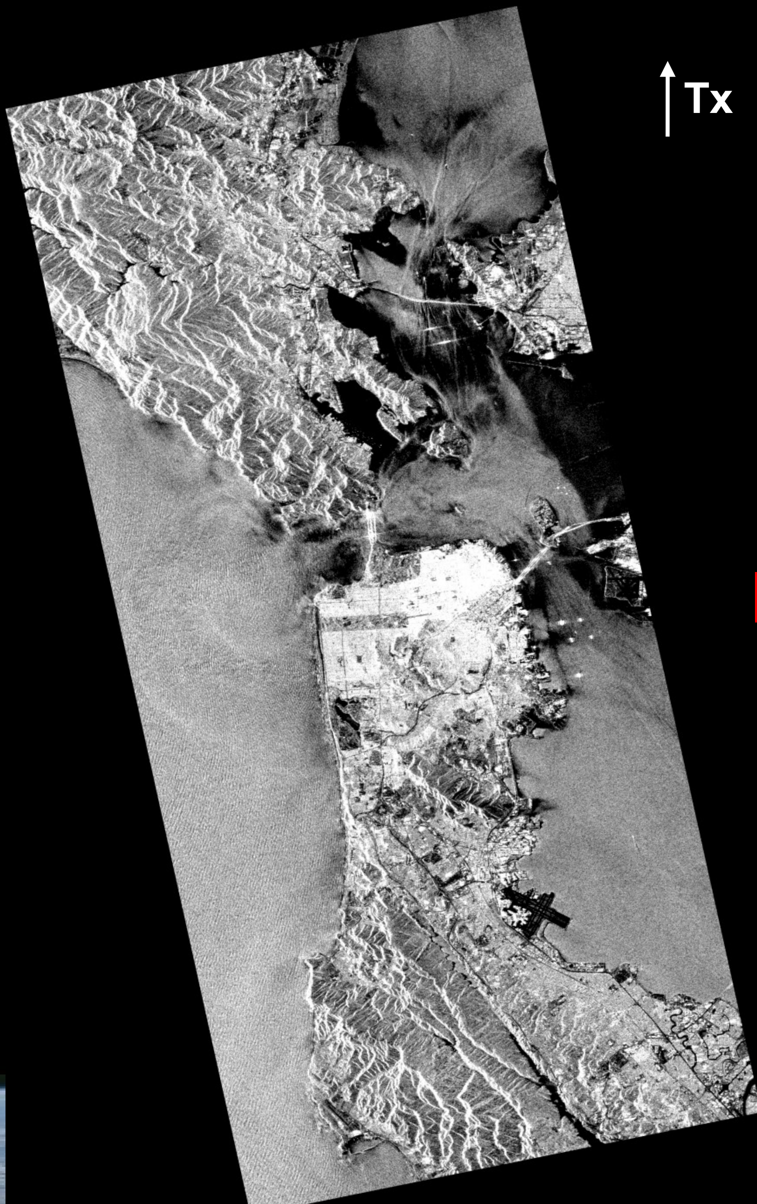



Tx → Rx →

$|HH|_{dB}$

San Francisco Bay – (L-Band)

# Space-borne Sensors

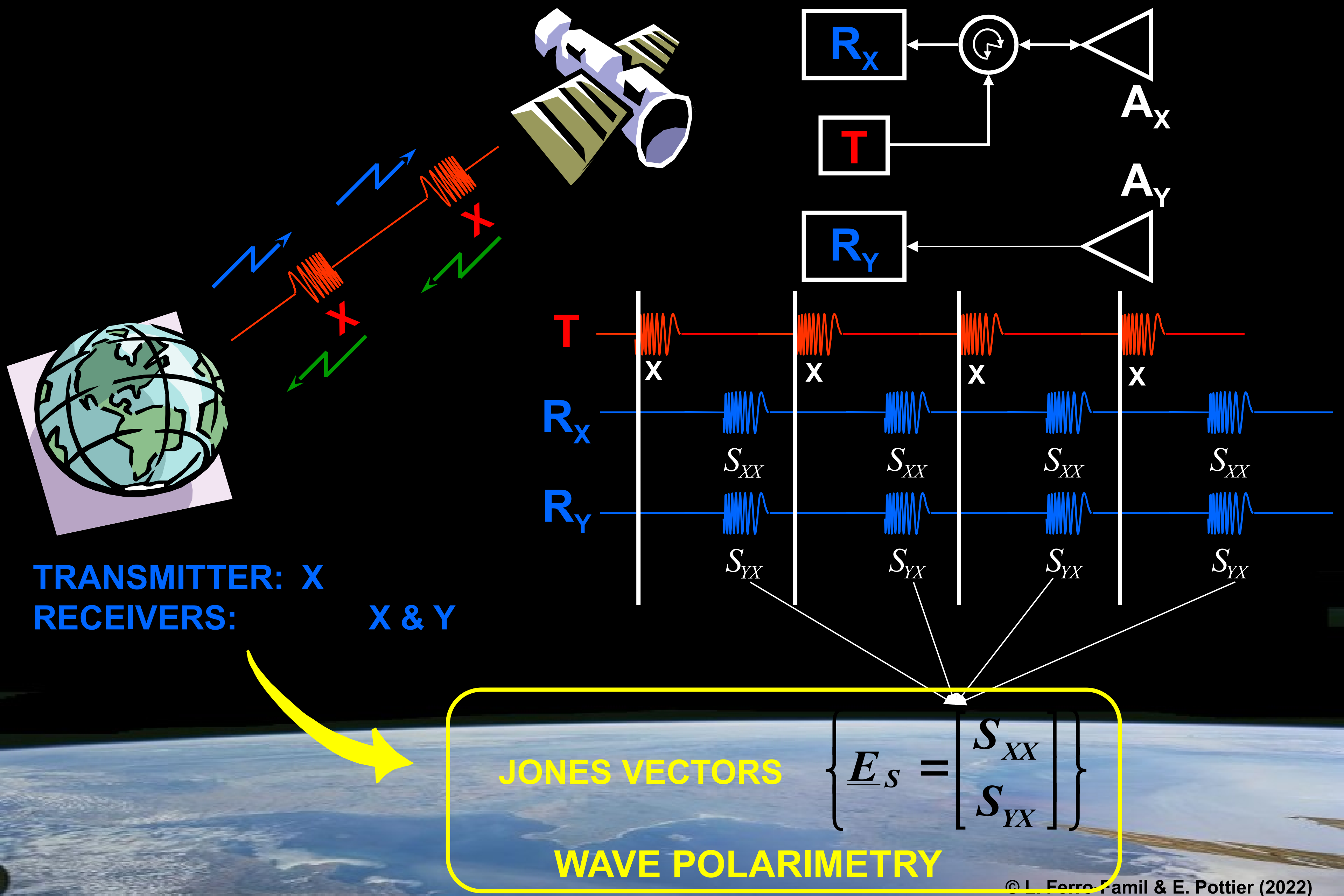


Tx Rx

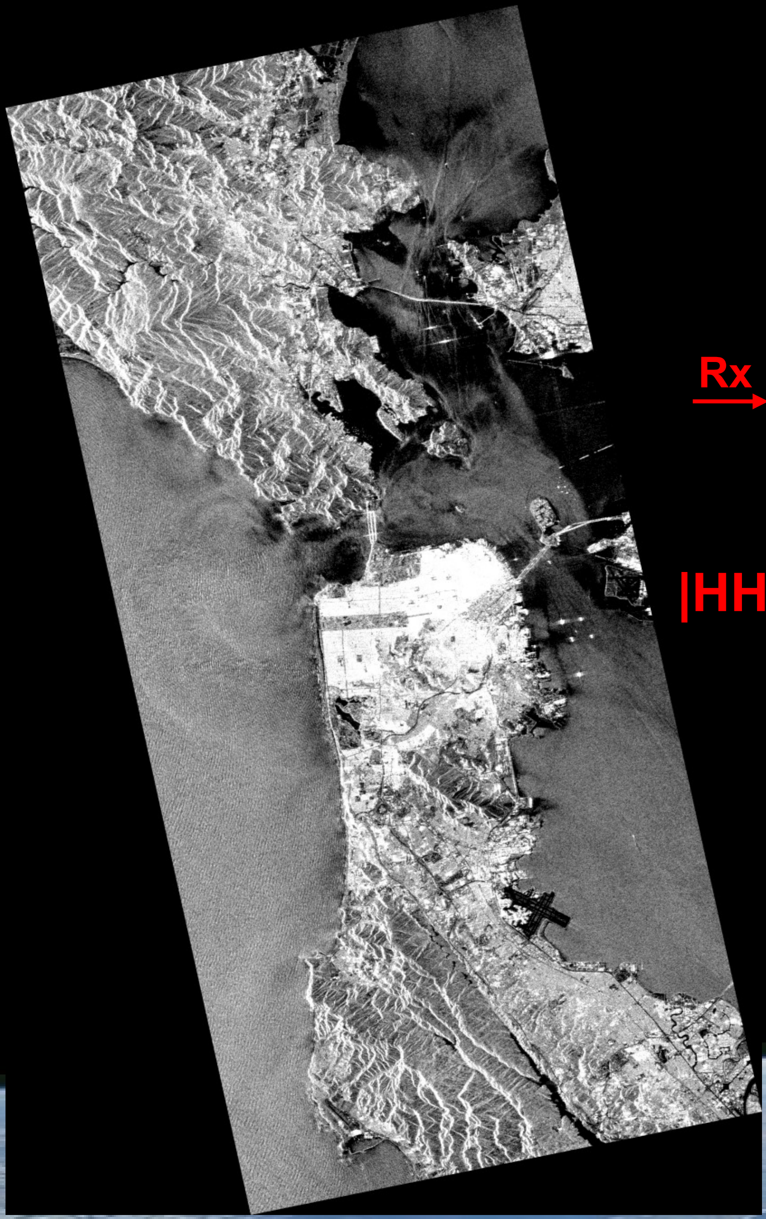
$|VV|_{dB}$

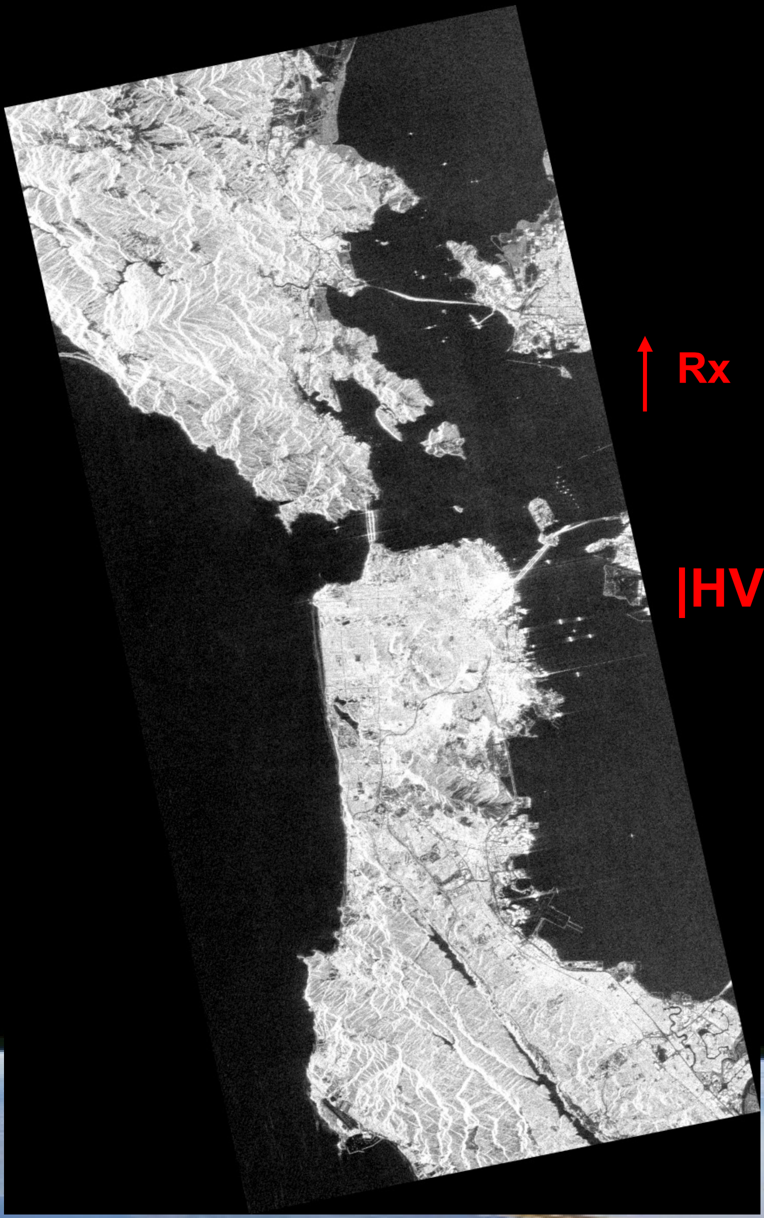

San Francisco Bay – (L-Band)

# Wave Polarimetry



# Space-borne Sensors




San Francisco Bay – (L-Band)

# Space-borne PolSAR Sensors

## ENVISAT - ASAR

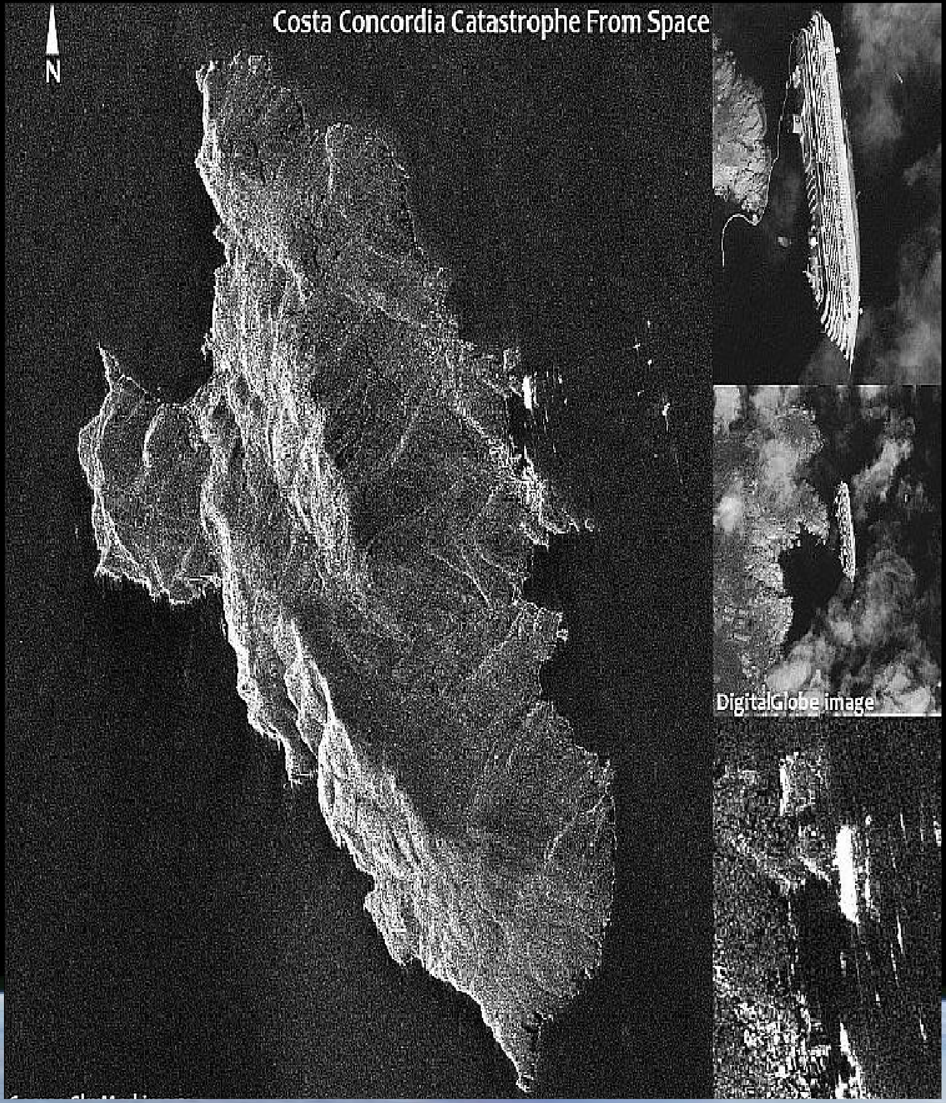
October 2001
C-Band (Sngl / Dual Inc)

# Space-borne PolSAR Sensors

## COSMO - SkyMed

agenzia spaziale italiana — TELESPAZIO, A Finmeccanica / Thales Company

**June 2007, Dec. 2007**
**Oct. 2008, Nov. 2010**
**X-Band (Sngl / Dual)**
**Revisit : 1 day**



# Space-borne PolSAR Sensors

## TerraSAR - X

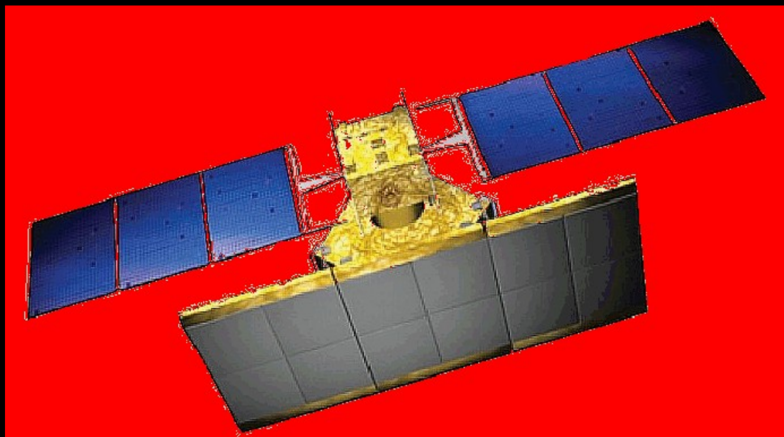

**June 2007**

**X-Band (Sngl / Twin HH-VV / Quad Exp.)**

DLR EADS ASTRIUM

# Space-borne PolSAR Sensors

## RISAT-1A




**26 April 2012**

**C-Band (Sngl, Dual, Hybrid)**

***Operational since 2015***

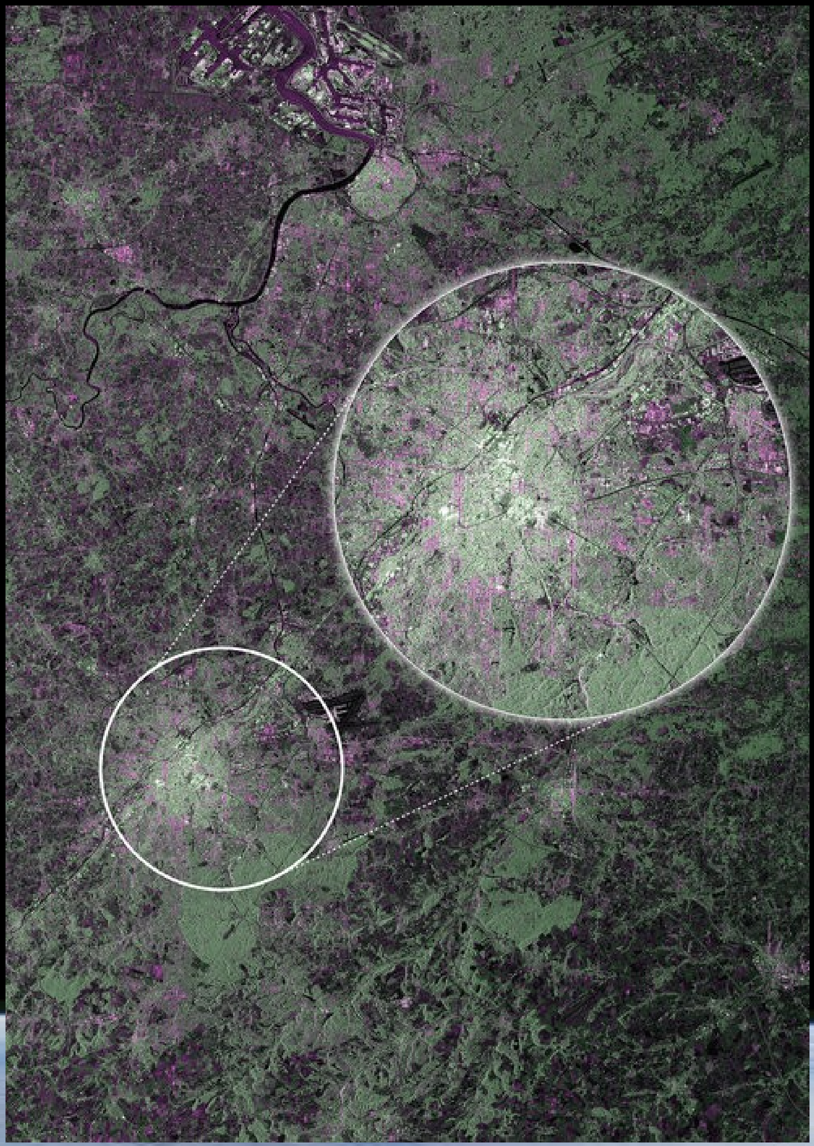
# Space-borne PolSAR Sensors

## SENTINEL – 1A

S1A : April 2014 S1B : April 2016

C-Band (Sngl, Dual)

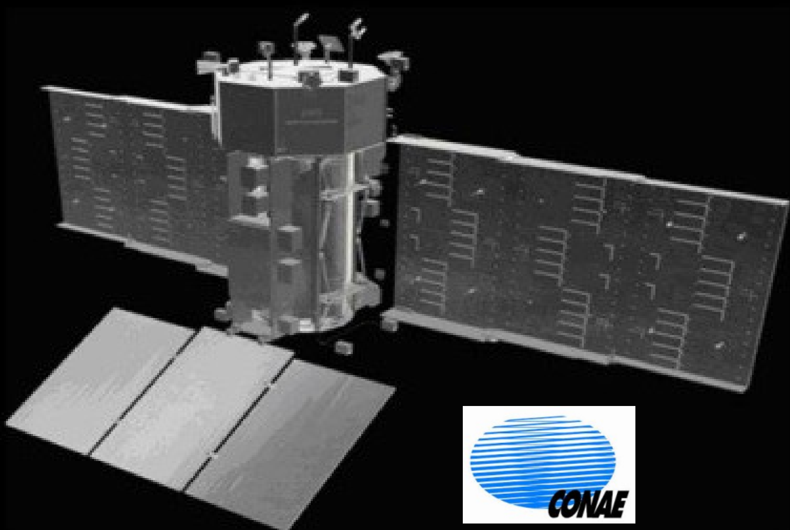
Revisit : 6 days



Brussels – 12 April 2014

# Space-borne PolSAR Sensors

## SAOCOM – SAR-L

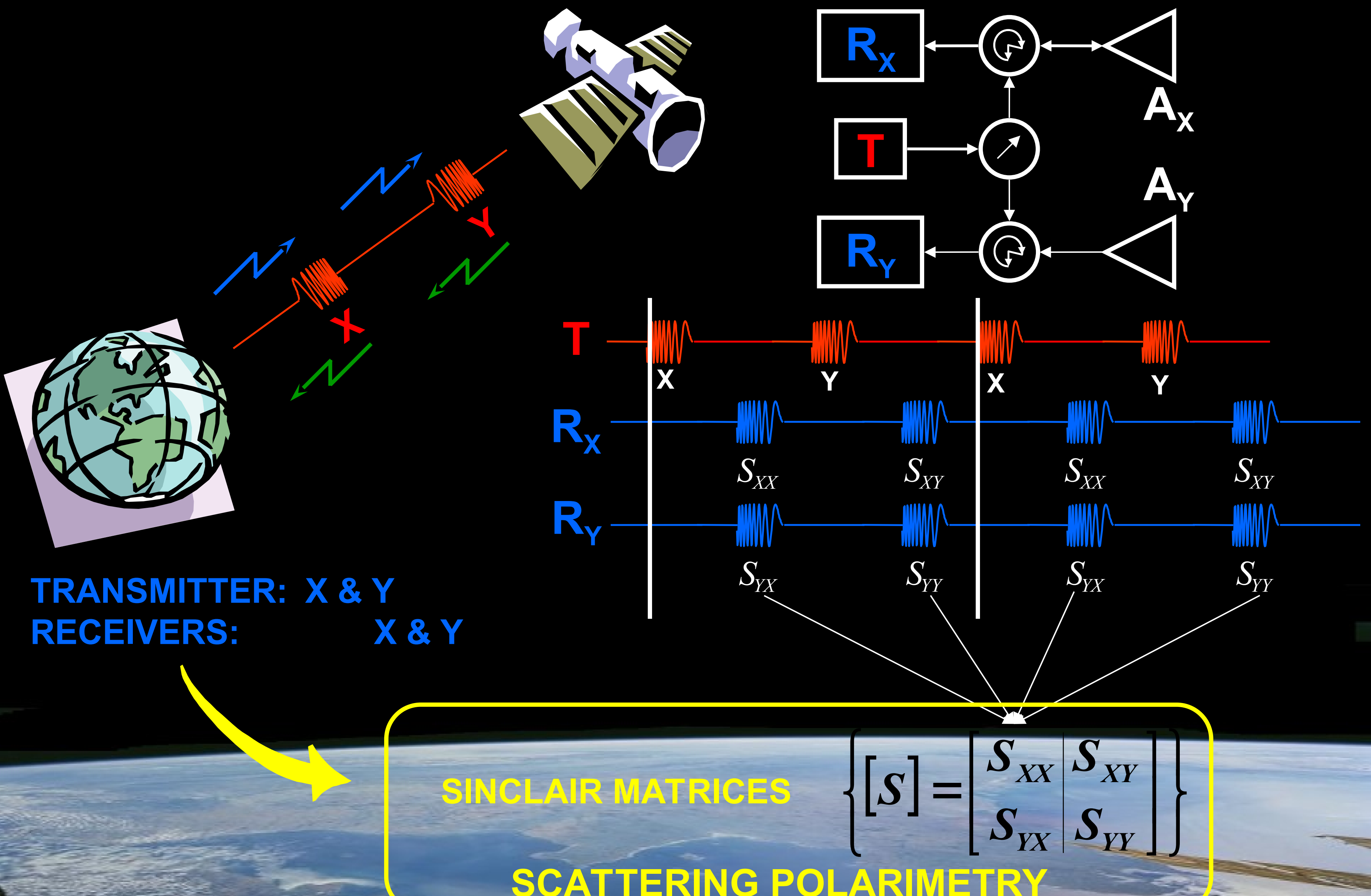



1A : 2017 1B : 2018
2A : 2019 2B : 2020
L-Band (Sngl, Dual, Twin HH-VV)
Revisit : 4 days

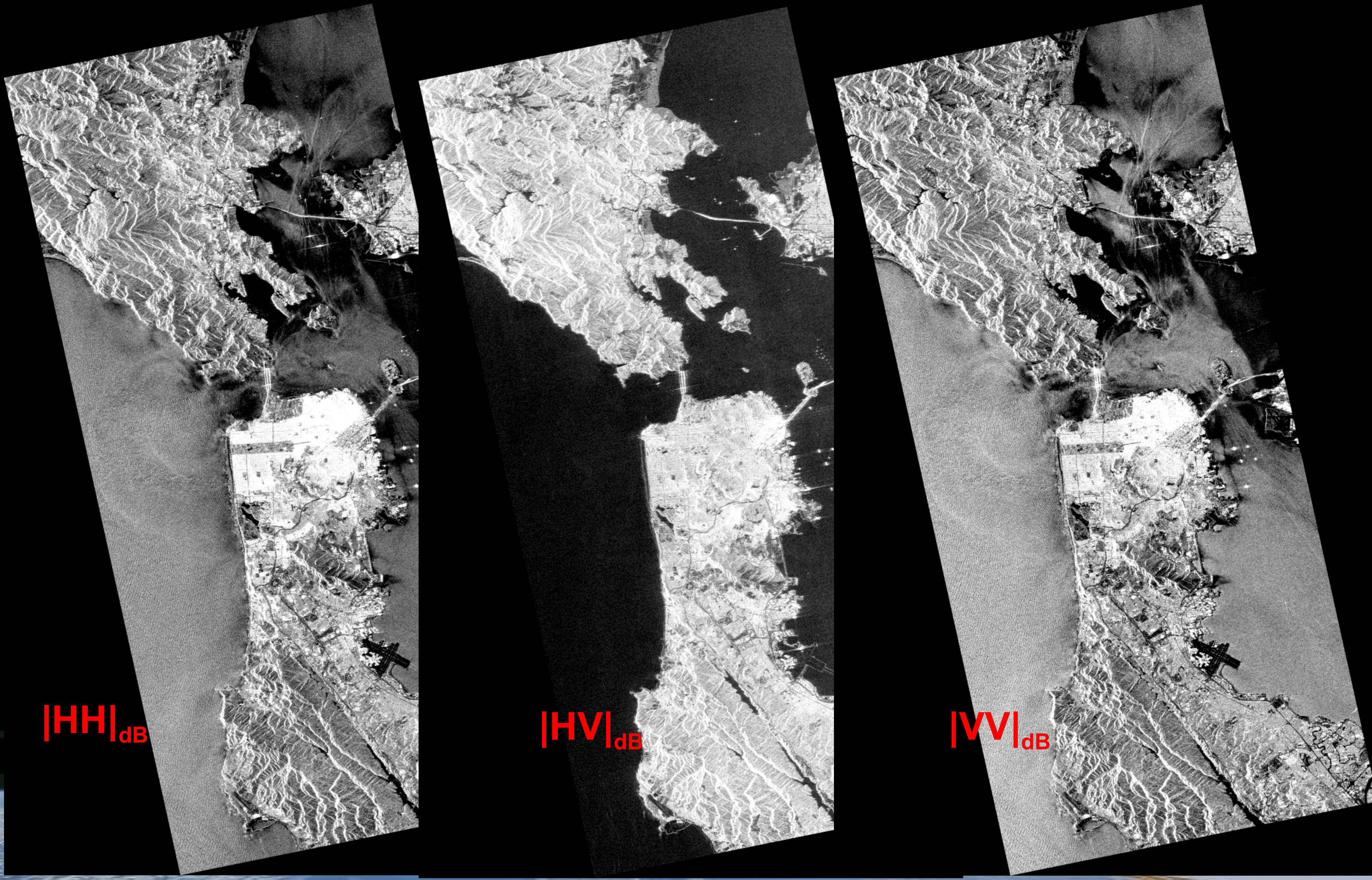
## RADARSAT Constellation Mission (RCM)

1A : 2017 1B / 1C : 2018
C-Band (Sngl, Dual, Hybrid)
Revisit : 4 days

# Scattering Polarimetry



# Space-borne Sensors




San Francisco Bay – (L-Band)

# Space-borne Sensors

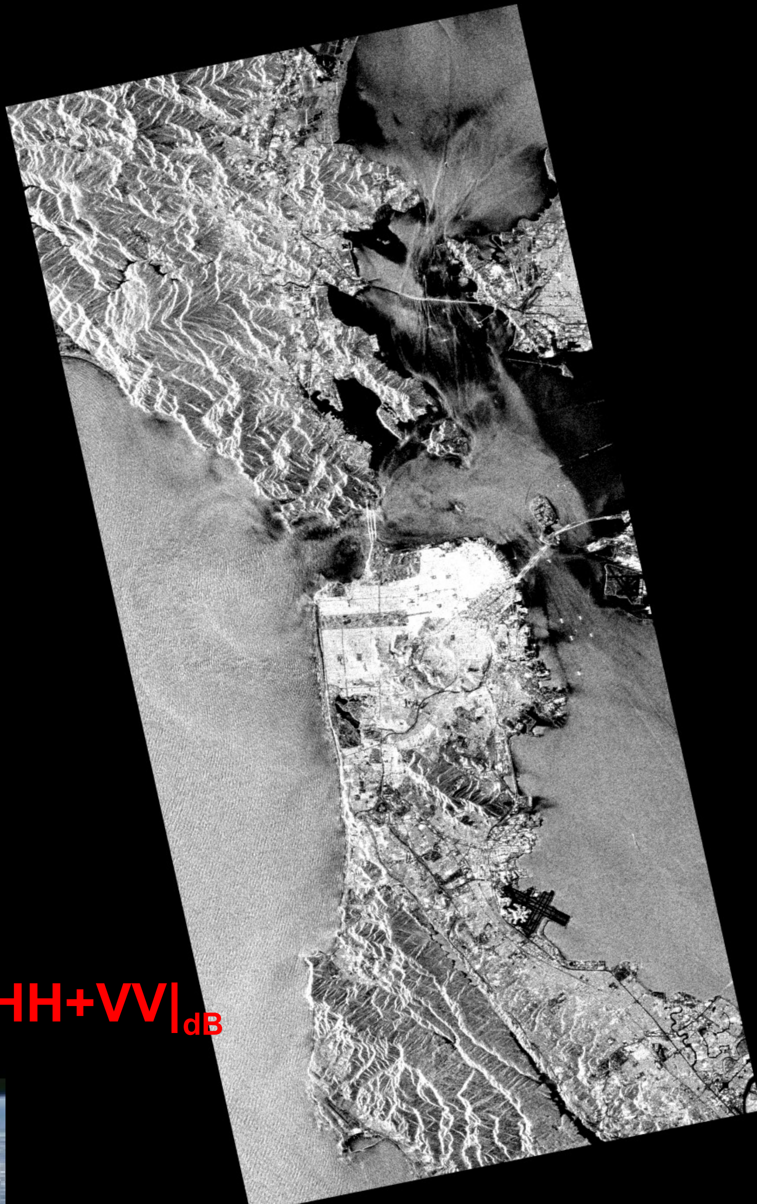
$|HH|_{dB}$

$|HV|_{dB}$

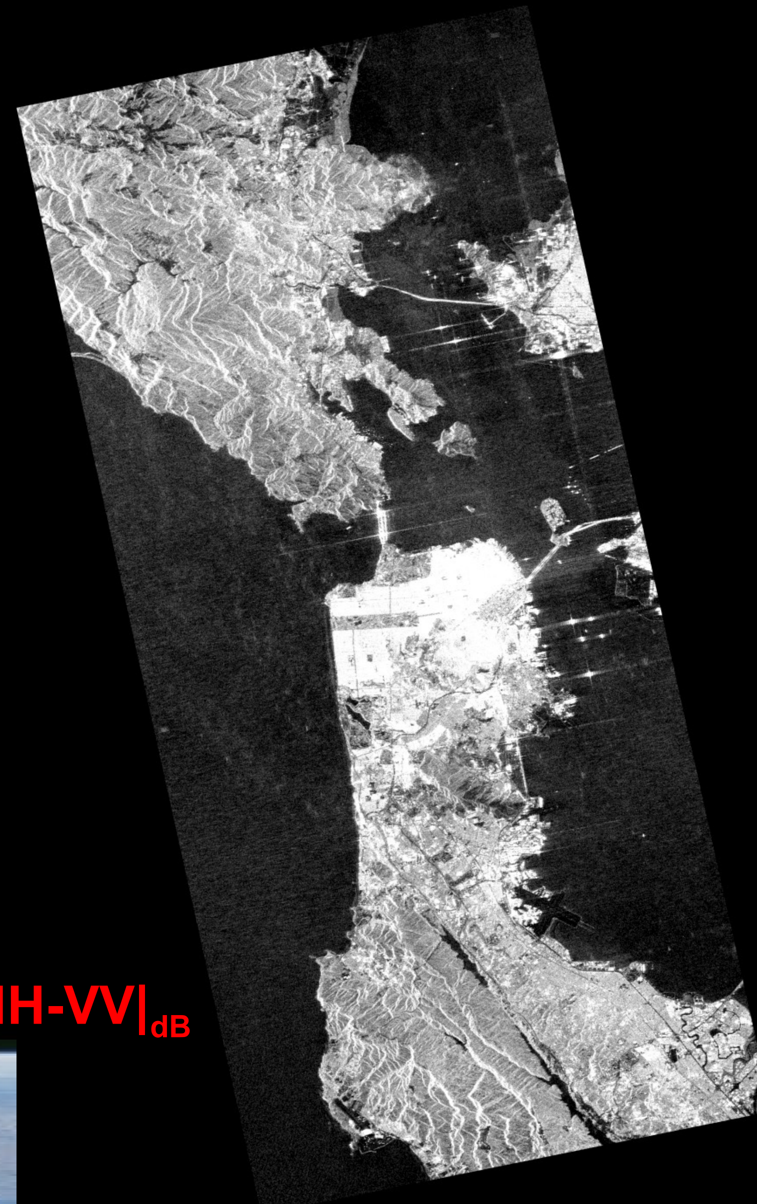
$|VV|_{dB}$


San Francisco Bay – (L-Band)

# Space-borne Sensors



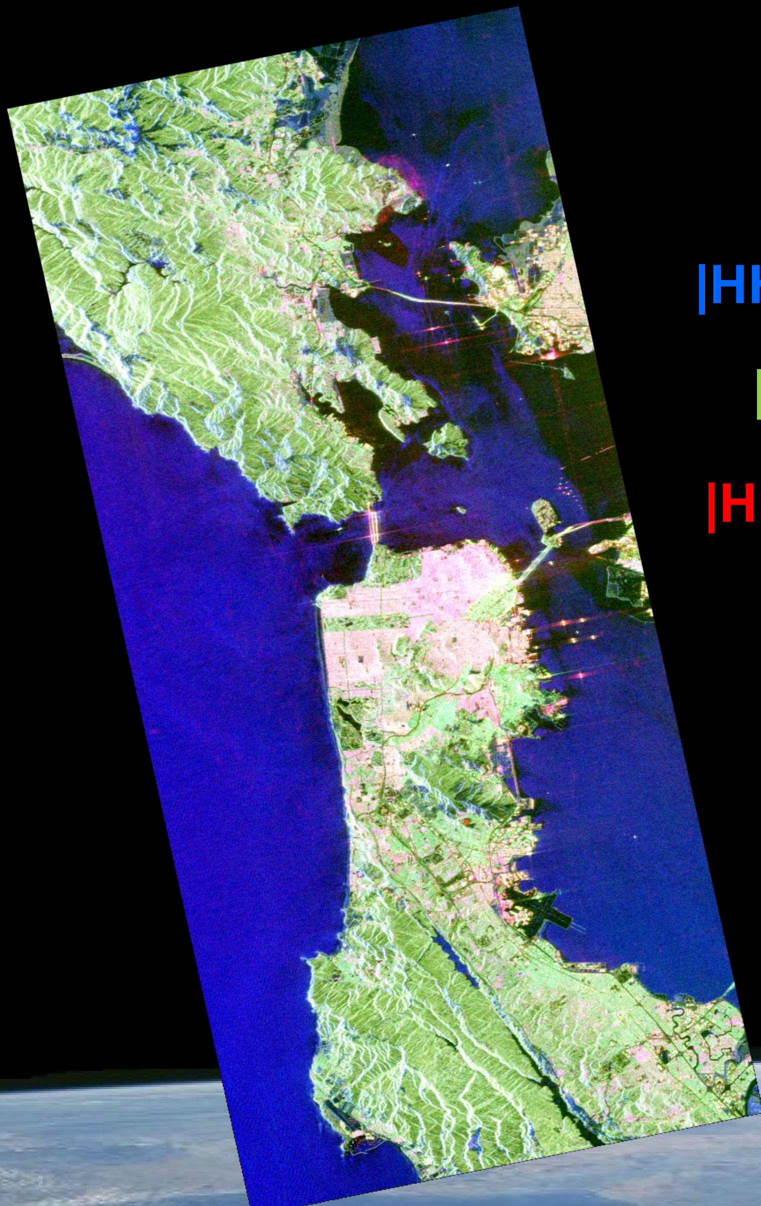
$|HH+VV|_{dB}$



$|HH-VV|_{dB}$

San Francisco Bay – (L-Band)

# Space-borne Sensors



$|HH+VV|_{dB}$

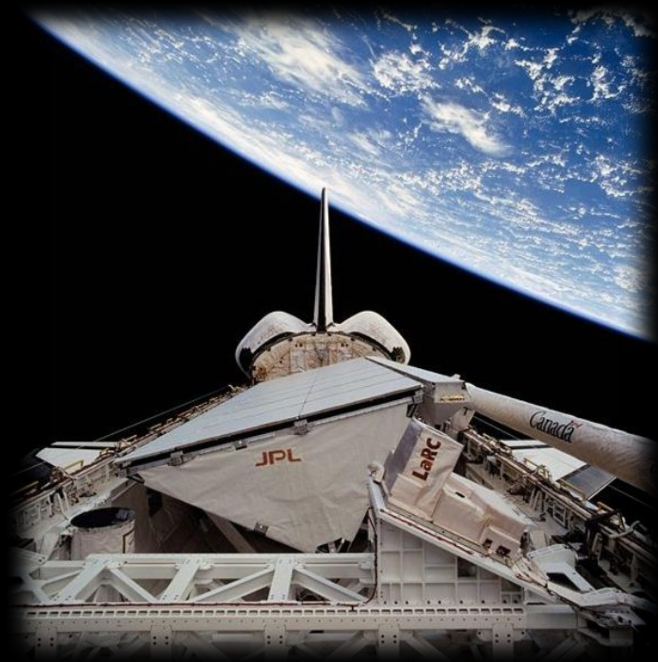
$|HV|_{dB}$

$|HH-VV|_{dB}$

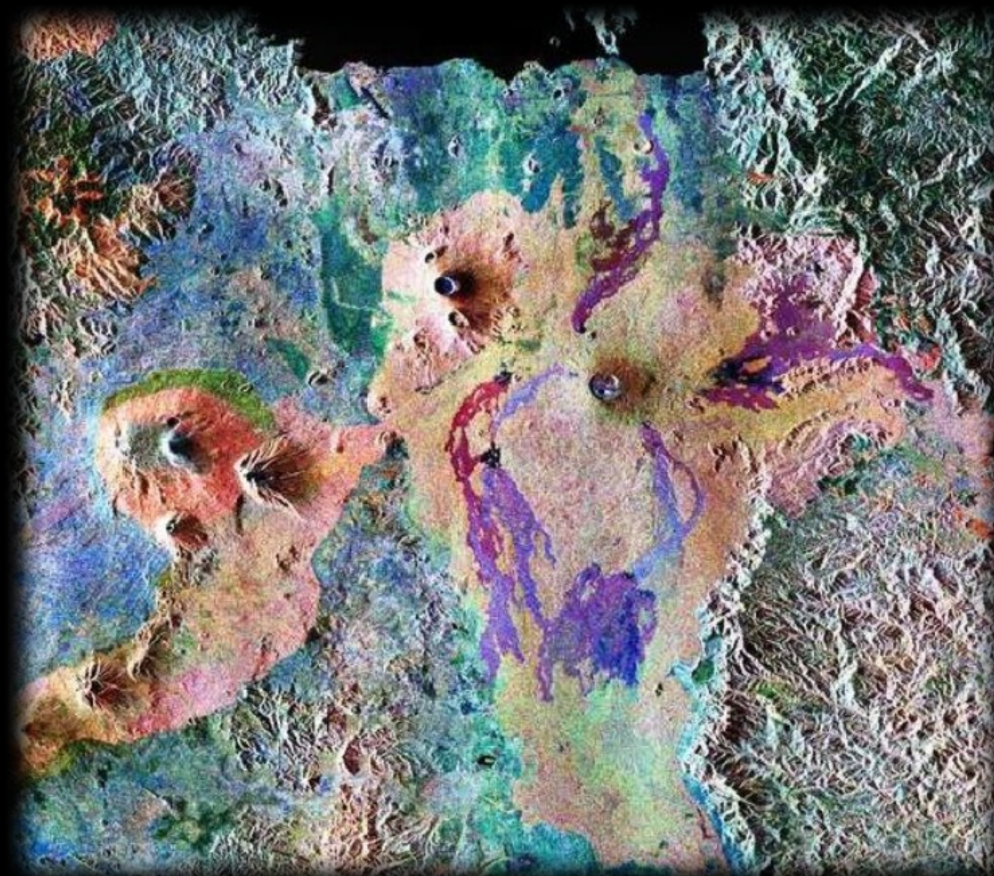

San Francisco Bay – (L-Band)

# Space-borne PolSAR Sensors

## SIR-C / X-SAR



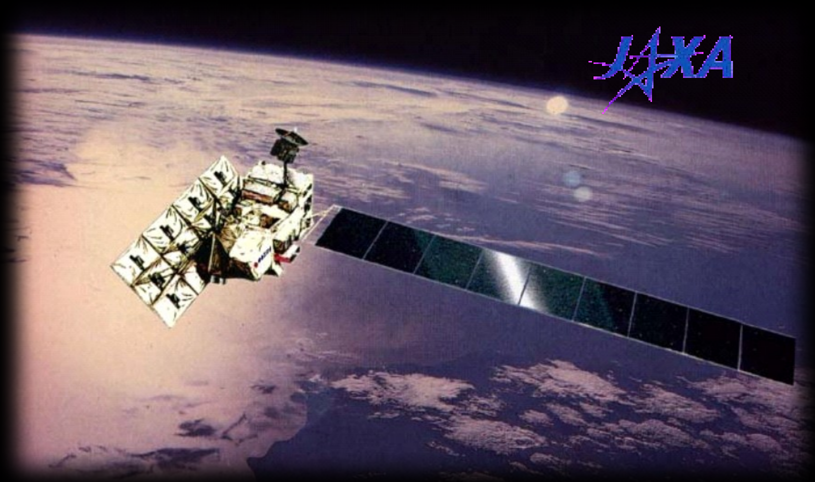

April 1994
L- and C-Band (Quad)
X-Band (Sngl)



Rwanda, Zaire, Uganda

# Space-borne PolSAR Sensors

## ALOS - PALSAR



January 2006

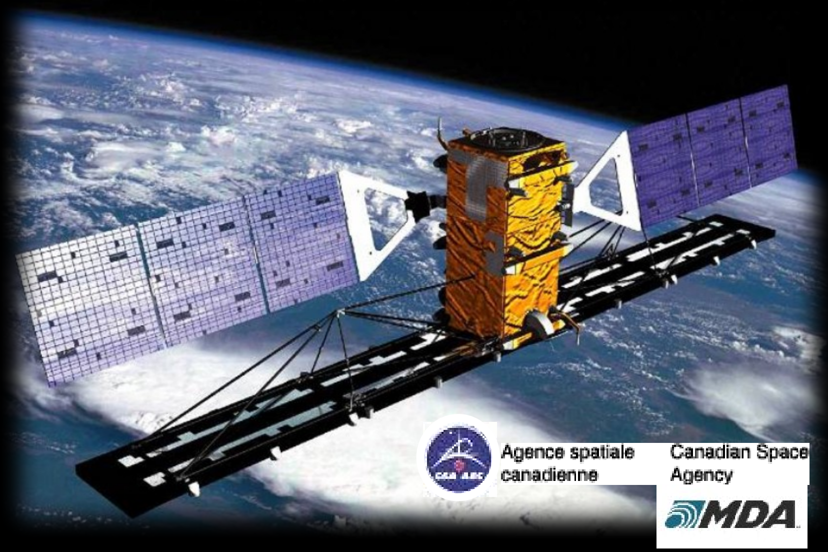
L-Band (Sngl / Twin / Quad)

ALOS : Advanced Land Observing Satellite

PALSAR : Phase Array L-Band SAR

# Space-borne PolSAR Sensors

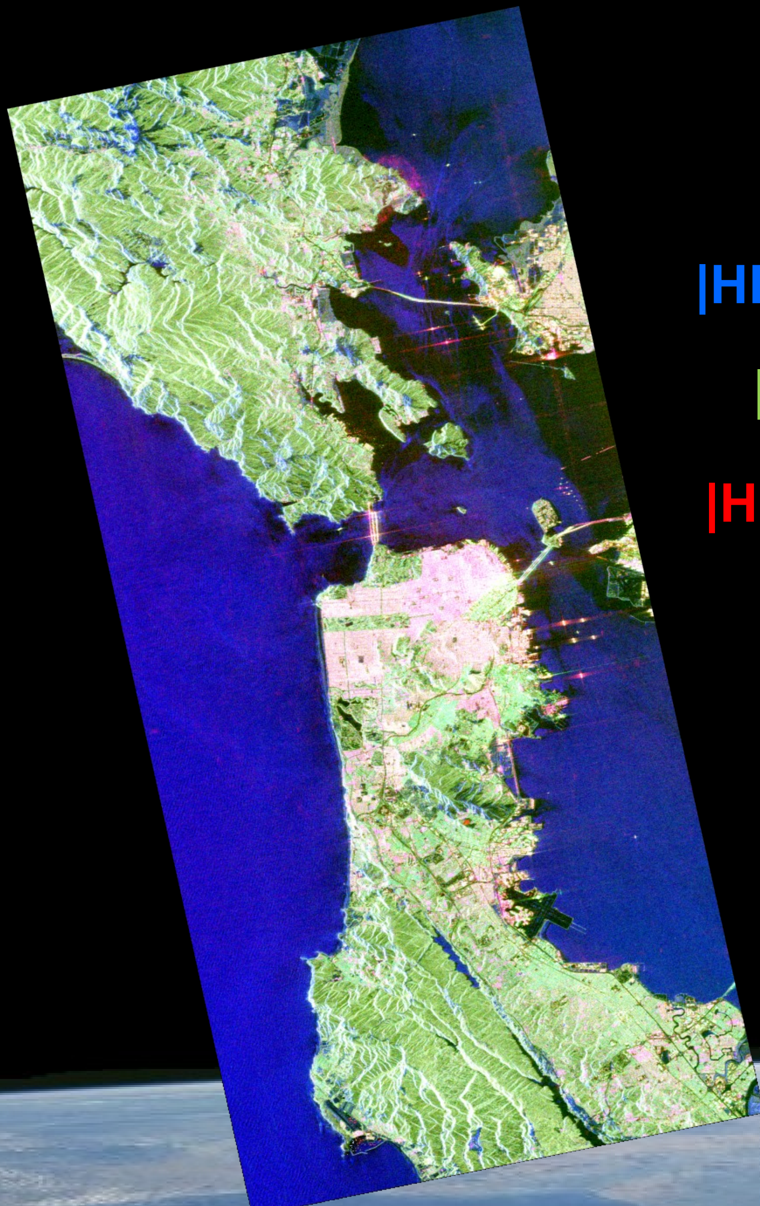
## RADARSAT - 2




**December 2007**

**C-Band (Quad)**

# Space-borne Sensors



$|HH+VV|_{dB}$

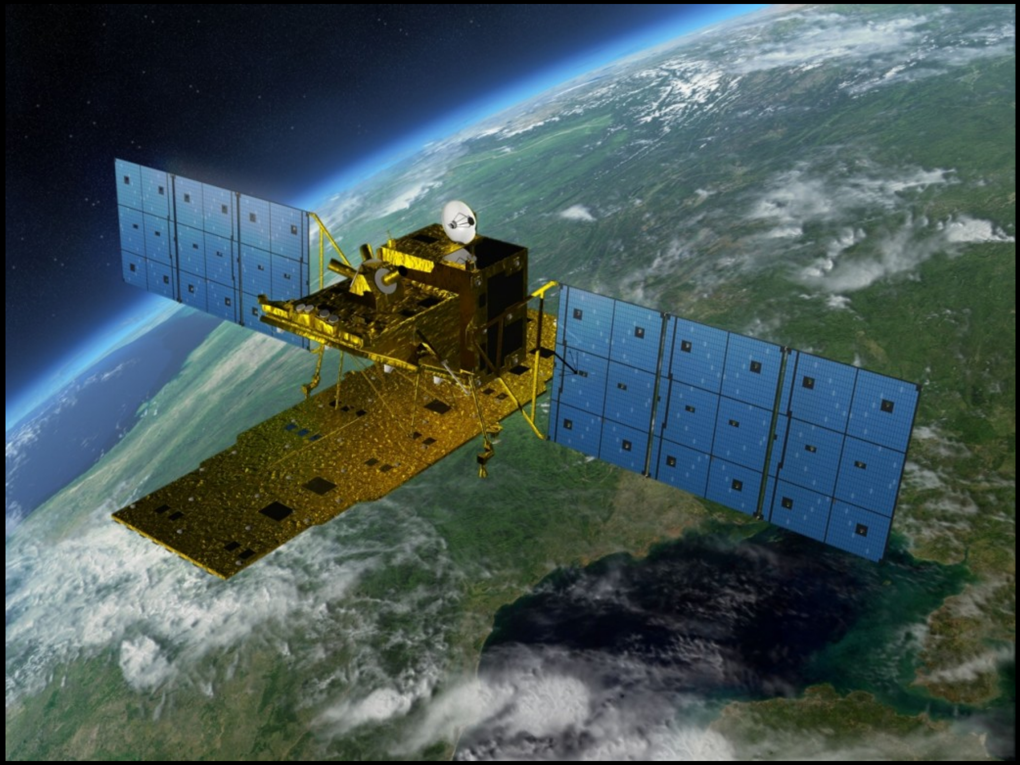
$|HV|_{dB}$

$|HH-VV|_{dB}$

San Francisco Bay – (L-Band and C-Band)

# Space-borne PolSAR Sensors

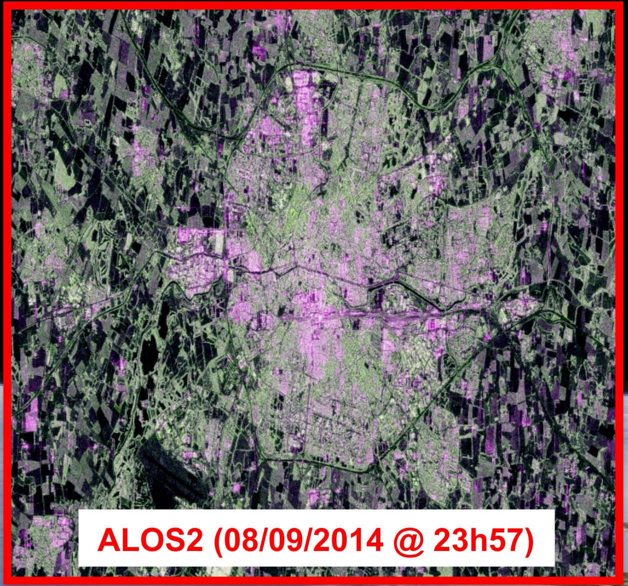
## ALOS - 2



**May 2014**

**L-Band (Quad)**

ALOS1 (30/04/2008 @ 22h34)



ALOS2 (08/09/2014 @ 23h57)

# Space-borne PolSAR Sensors

## Chang Zheng-4C - GaoFen-3 (GF-3)

### *Long March-4C - High Resolution-3*

**August 2016**

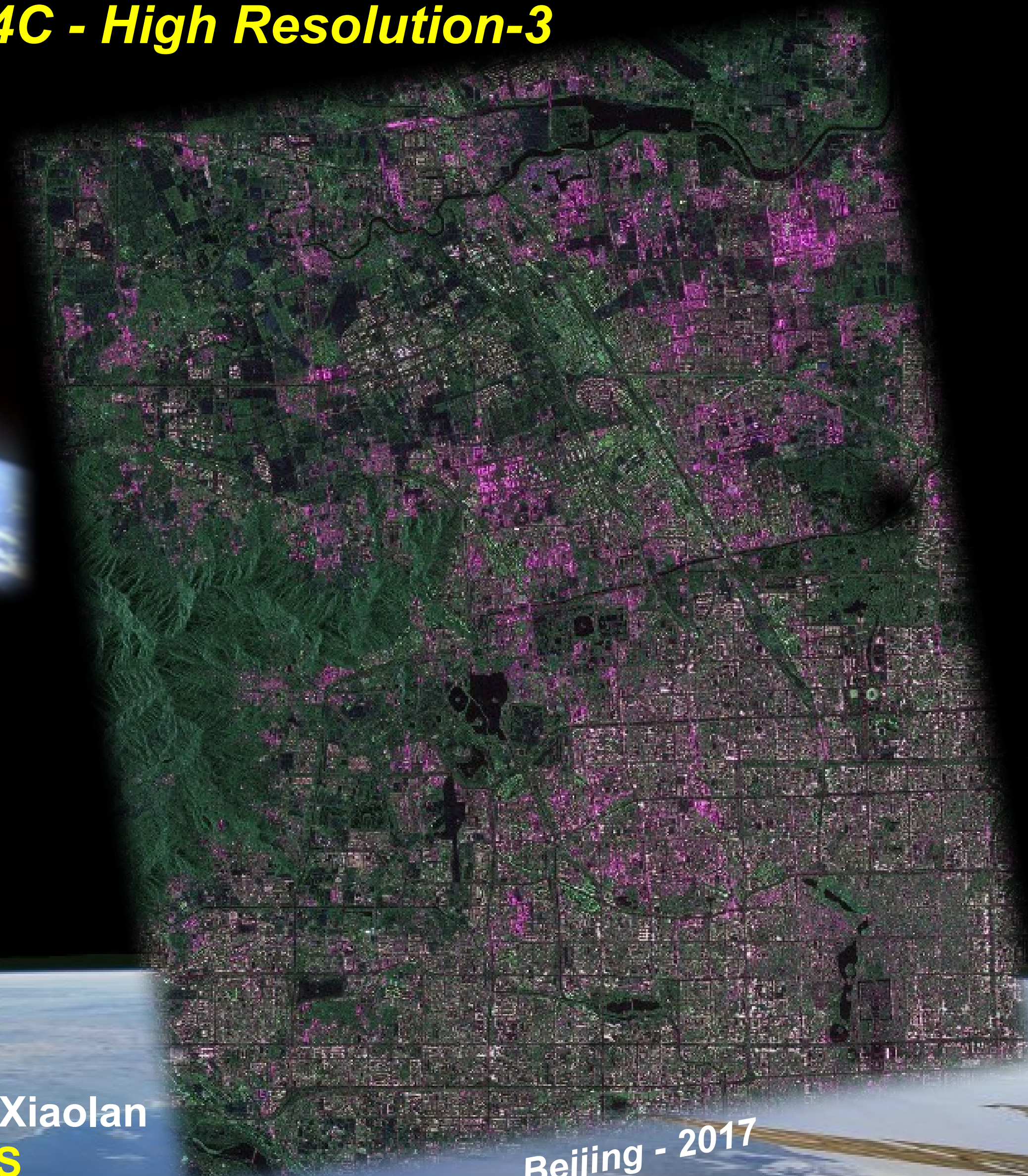
**C-Band (Quad)**

中国空间技术研究院
China Academy of Space Technology

IECAS

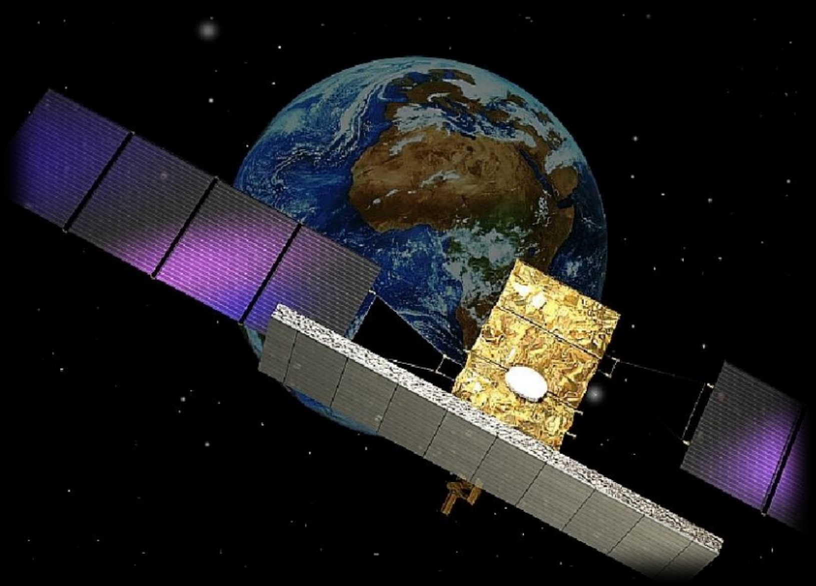
**Courtesy of Dr. Qiu Xiaolan**

**IECAS / GIPAS**



# Space-borne PolSAR Sensors

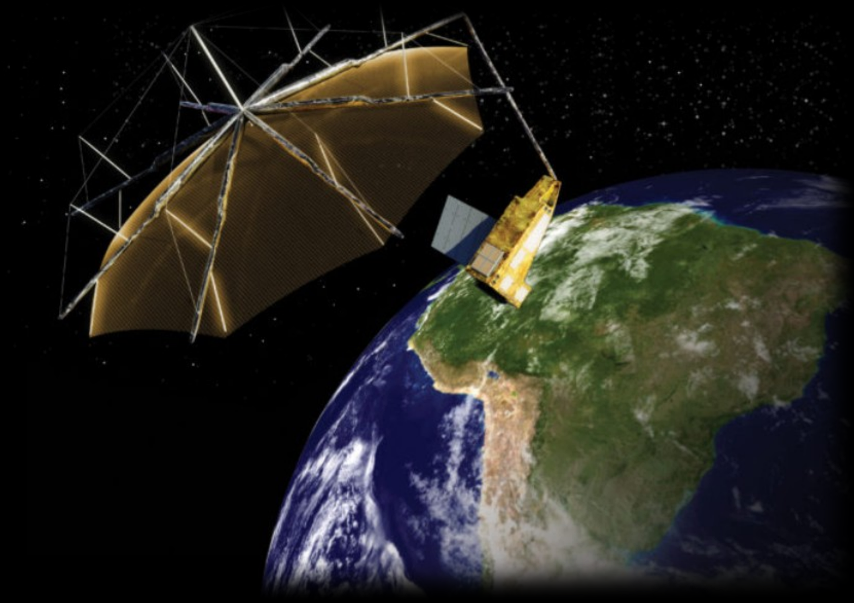
## COSMO - SkyMed - CSG



2A : 2018 2B : 2019

X-Band (Sngl / Dual / Quad Exp.)

## Earth Explorer - BIOMASS

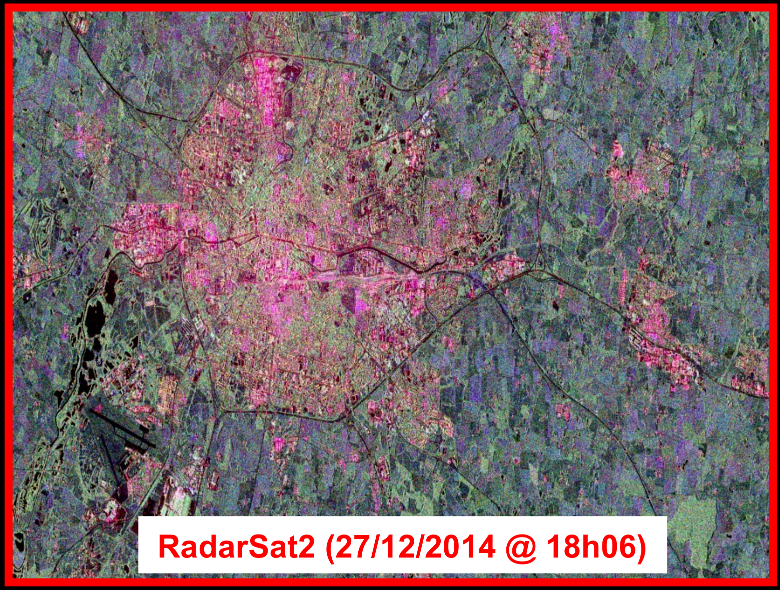


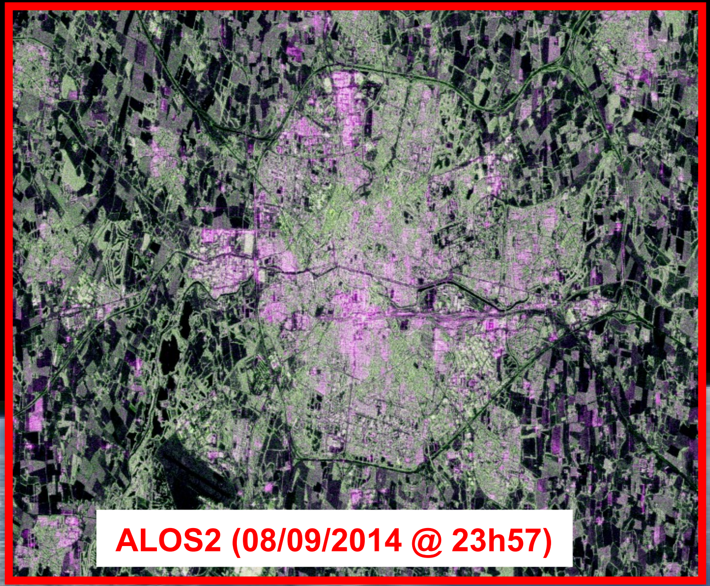
2021

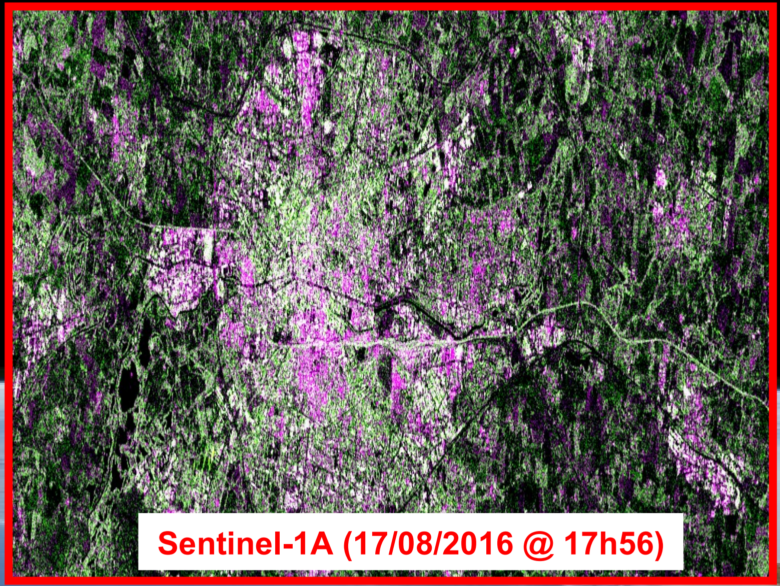
P-Band (Quad)

# Space-borne PolSAR Sensors

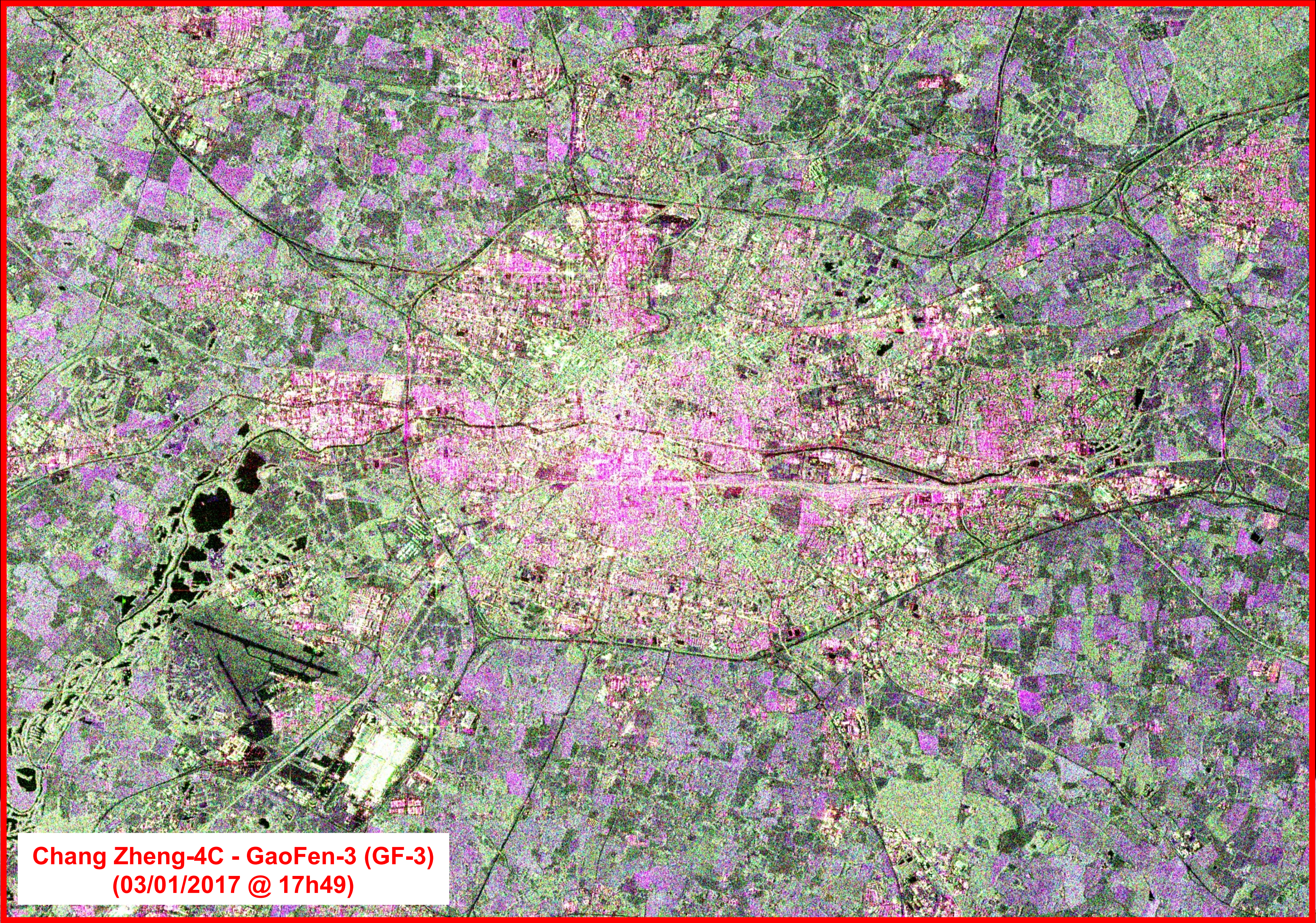
# Space-borne PolSAR Sensors

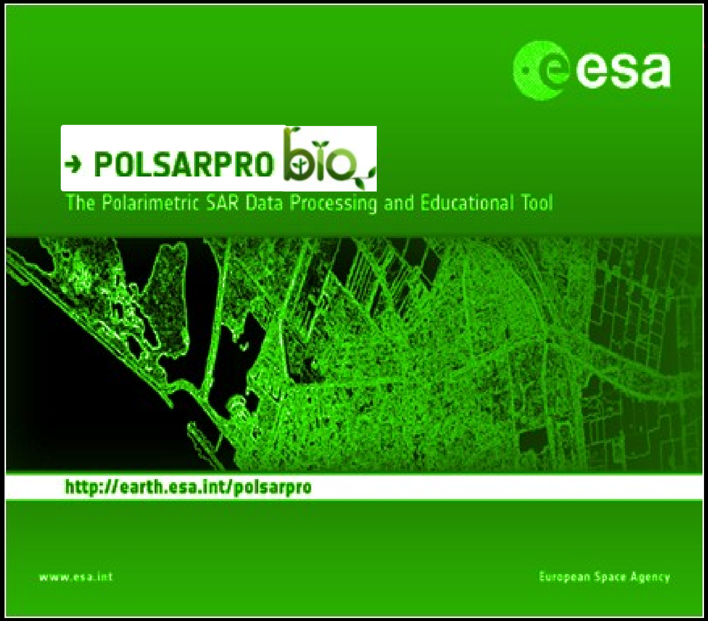



# Space-borne PolSAR Sensors



# ESA PolSARpro Toolbox




larimetric SAR data Processin
and educational tool

esa since 2003

- +3000 registered user
- +70 foreign countries

## International Collaborative Project

(Agencies, Research Centres, Universities)

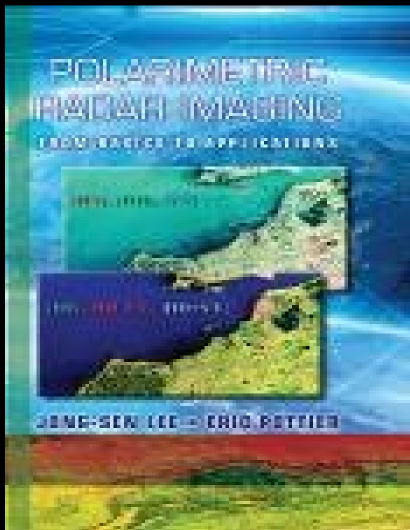
# ESA PolSARpro Toolbox


**Toolbox specifically designed to handle : Pol-SAR, Pol-InSAR , Pol-TomoSAR and Pol-TimeSAR data**

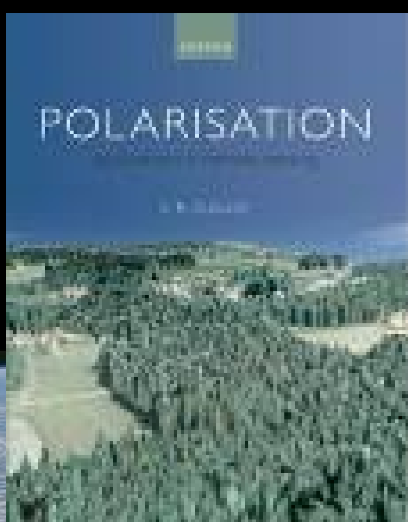
**Educational Software offering a tool for self-education in the field of Polarimetric SAR data processing and analysis**

**More than 1740 different Pol-SAR, Pol-InSAR, Pol-TomSAR, Pol-TimeSAR functionalities.**

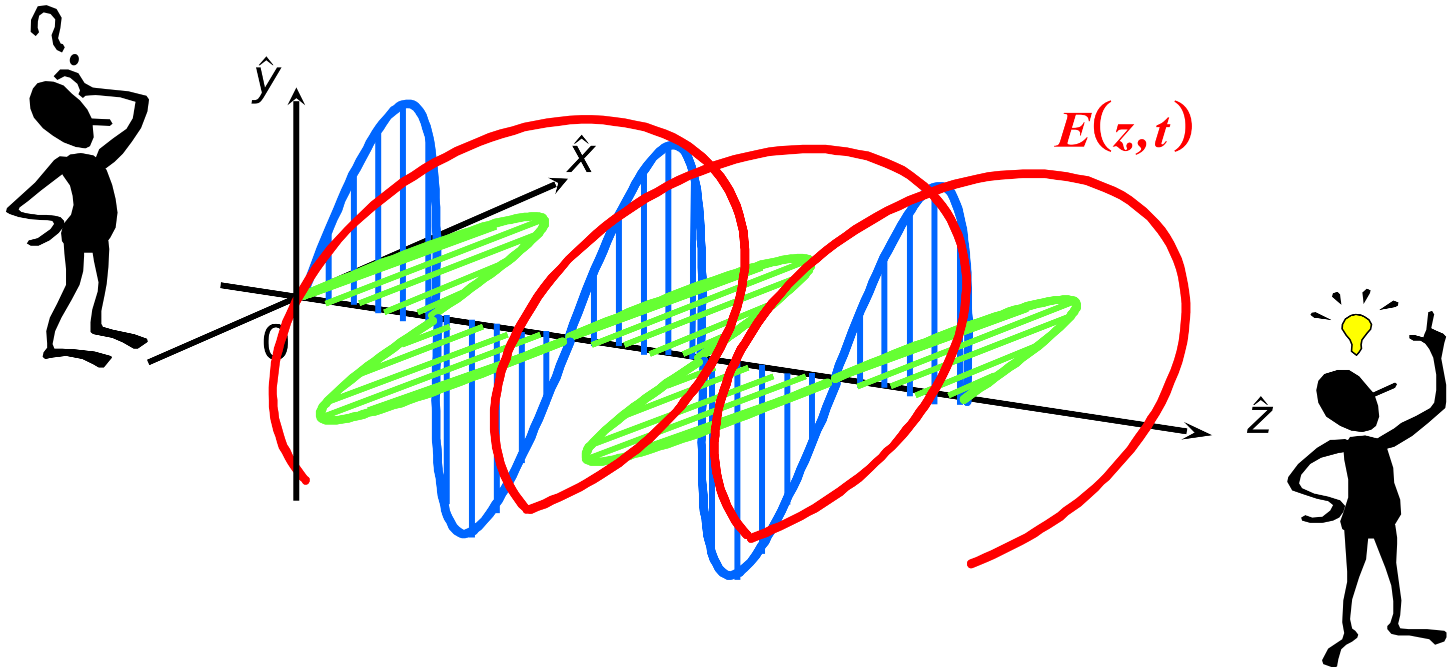
# Books On Polarimetric Radar
# SAR, Polarimetric Interferometry



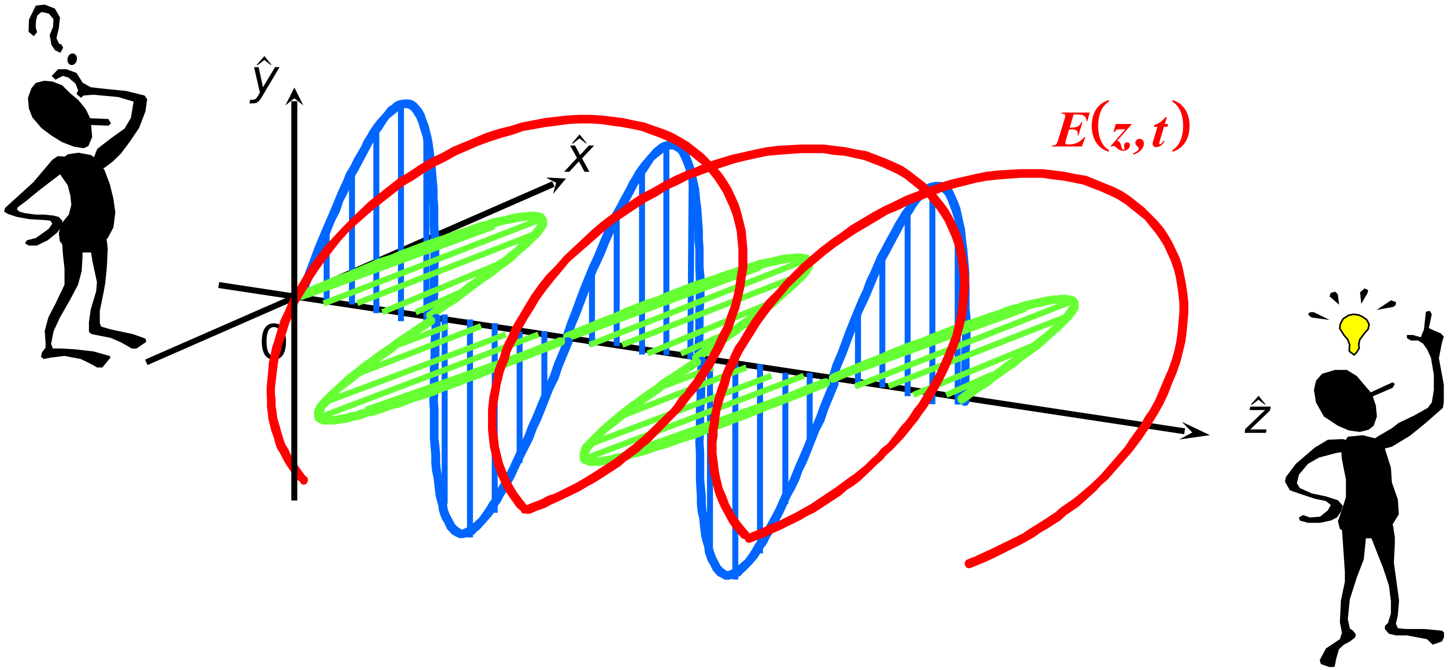

**Polarimetric Radar Imaging: From basics to applications**
***Jong-Sen LEE – Eric POTTIER***
CRC Press; 1st ed., February 2009, pp 422
ISBN: 978-1420054972




**Polarisation: Applications in Remote Sensing**
***Shane R. CLOUDE***
Oxford University Press, October 2009, pp 352
ISBN: 978-0199569731




# BASIC CONCEPTS




# WAVE POLARIMETRY

# PROPAGATION EQUATION

**REAL ELECTRIC FIELD VECTOR** $\vec{E}(z,t)$

## MAXWELL EQUATIONS

**MAXWELL – FARADAY EQUATION** $$\nabla \wedge \vec{E}(z,t) = -\frac{\partial \vec{B}(z,t)}{\partial t}$$

**MAXWELL – AMPERE EQUATION** $$\nabla \wedge \vec{H}(z,t) = \vec{J}_T(z,t)$$

**GAUSS THEOREM** $$\nabla \cdot \vec{D}(z,t) = \rho(z,t)$$

$$\nabla \cdot \vec{B}(z,t) = 0$$

# PROPAGATION EQUATION

$$\nabla \wedge \left(\nabla \wedge \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla \cdot \left(\nabla \vec{A}\right)$$

## PROPAGATION EQUATION

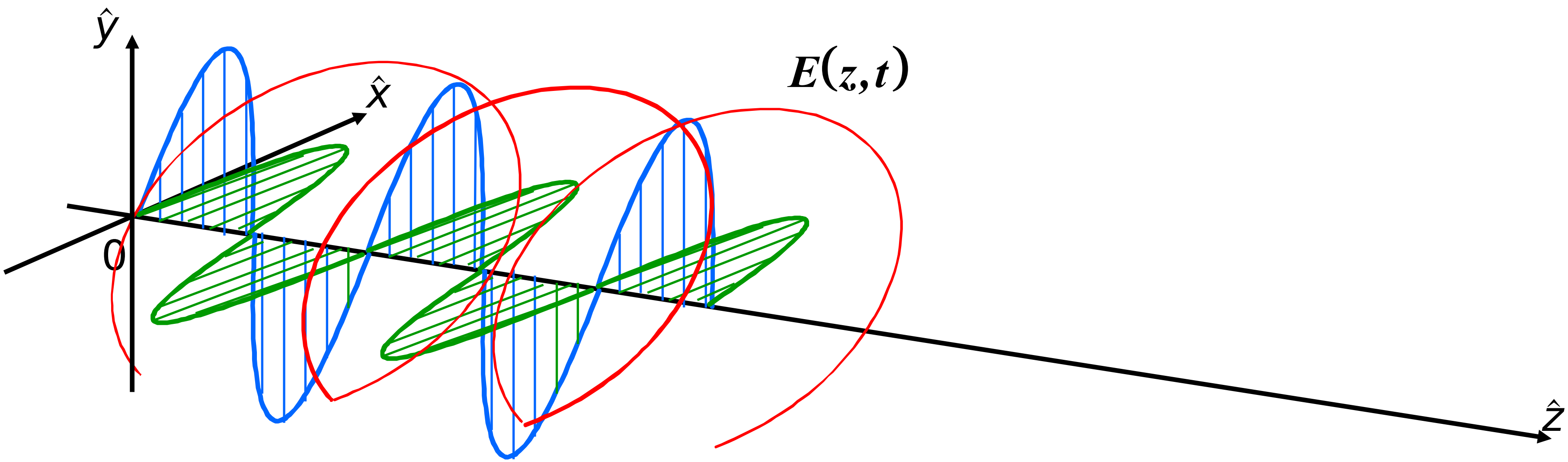
$$\nabla^2 \vec{E}(z,t) - \mu\varepsilon \frac{\partial^2 \vec{E}(z,t)}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}(z,t)}{\partial t} = -\frac{1}{\varepsilon} \frac{\partial \rho(z,t)}{\partial t}$$

## HELMHOLTZ PROPAGATION EQUATION

$$\nabla^2 \vec{E}(z,t) - \mu\varepsilon \frac{\partial^2 \vec{E}(z,t)}{\partial t^2} = 0$$

**Source Free, Linear, Homogeneous, Isotropic, Dielectric and lossless Medium**

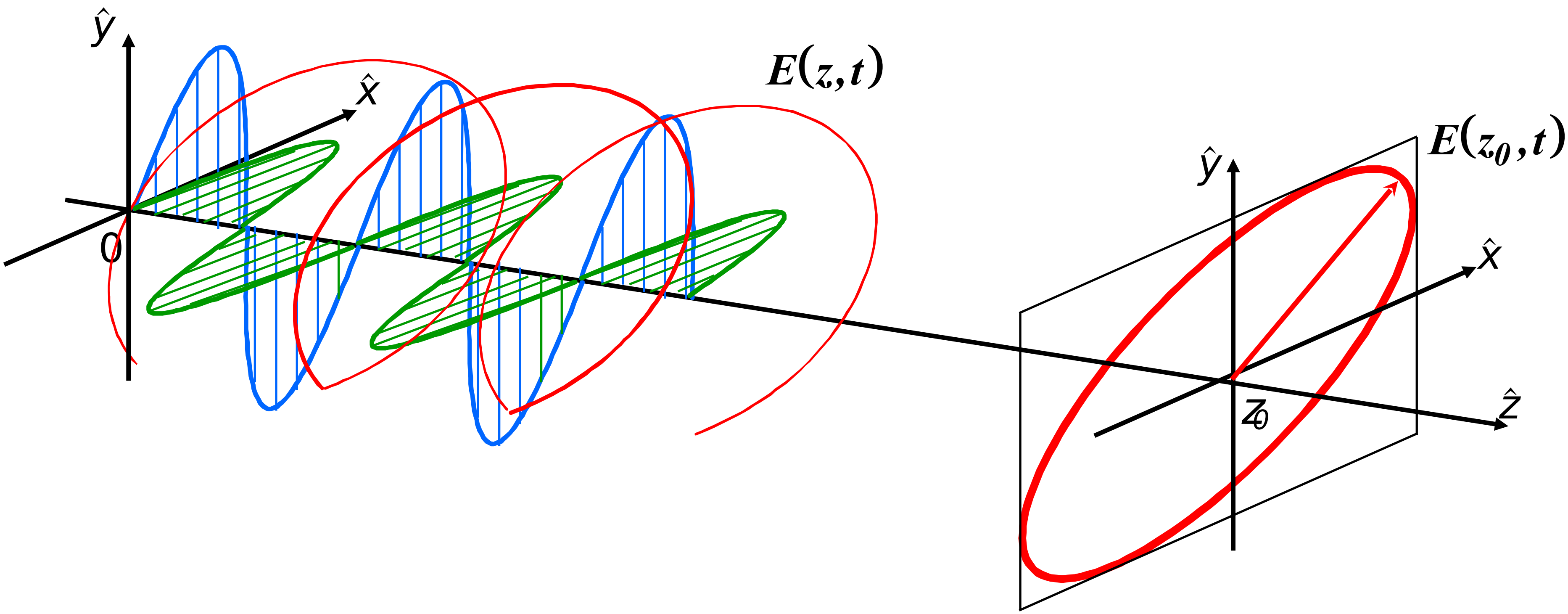
# POLARISATION ELLIPSE




**REAL ELECTRIC FIELD VECTOR**

$$\vec{E}(z,t) = \begin{Bmatrix} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{Bmatrix}$$

# POLARISATION ELLIPSE

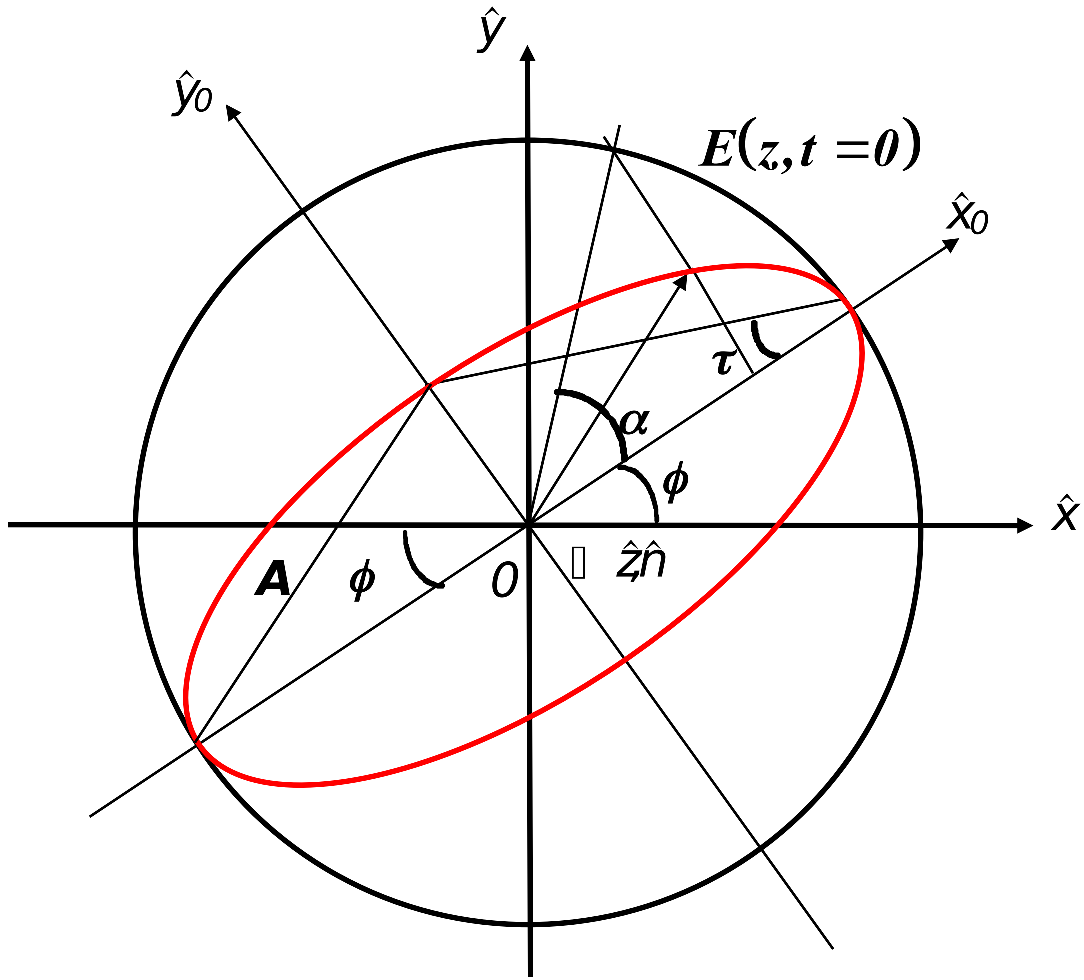



**THE REAL ELECTRIC FIELD VECTOR MOVES IN TIME ALONG AN ELLIPSE**

$$\left(\frac{E_x}{E_{0x}}\right)^2 - 2\frac{E_x E_y}{E_{0x} E_{0y}}\cos(\delta) + \left(\frac{E_y}{E_{0y}}\right)^2 = \sin^2(\delta)$$

**With:** $\delta = \delta_y - \delta_x$

# POLARISATION ELLIPSE




**A** : **WAVE AMPLITUDE**

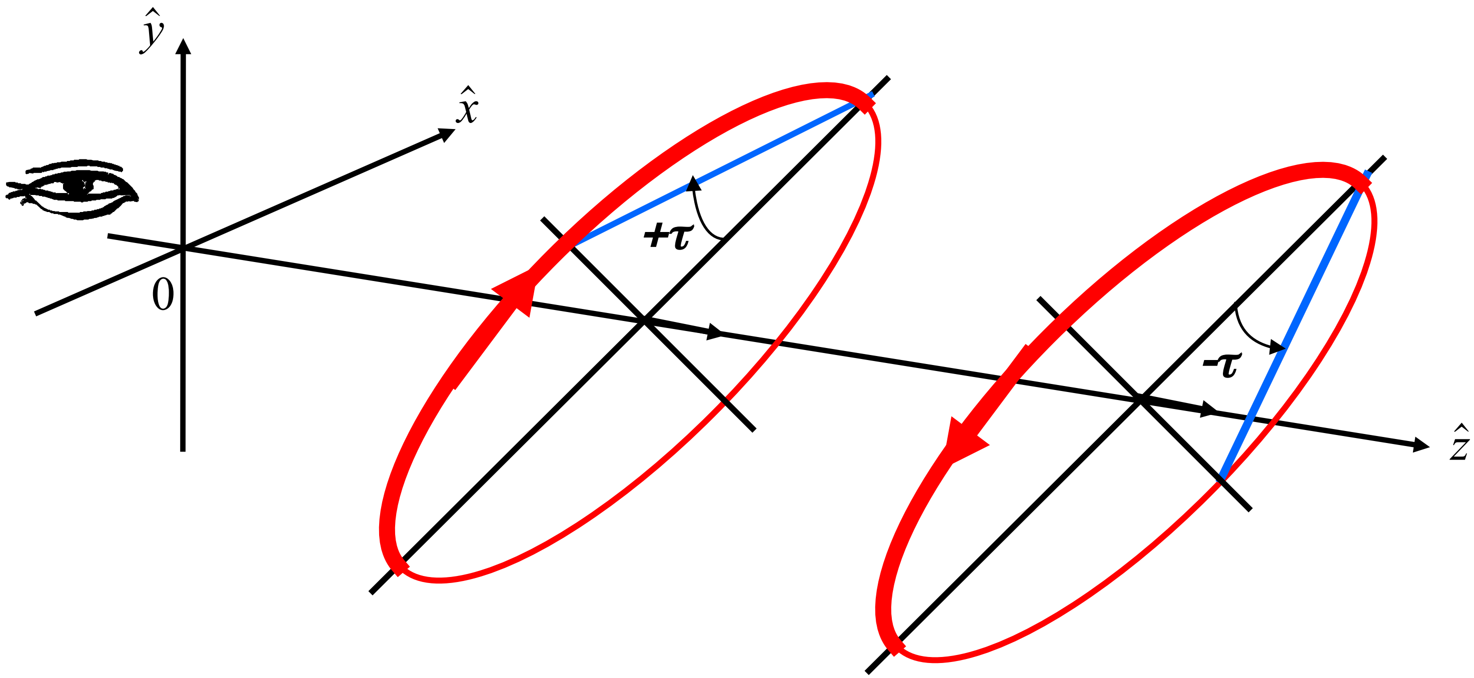
$\alpha$ : **ABSOLUTE PHASE**

$\phi$ : **ORIENTATION ANGLE** $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$

$\tau$ : **ELLIPTICITY ANGLE** $0 \leq \tau \leq \frac{\pi}{4}$

# POLARISATION HANDENESS

ROTATION SENSE: LOOKING INTO THE DIRECTION OF THE WAVE PROPAGATION




ANTI-CLOCKWISE ROTATION

LEFT HANDED POLARISATION

ELLIPTICITY ANGLE : $\tau > 0$

CLOCKWISE ROTATION

RIGHT HANDED POLARISATION

ELLIPTICITY ANGLE : $\tau < 0$

$$-\frac{\pi}{4} \leq \tau \leq \frac{\pi}{4}$$

# JONES VECTOR

**REAL ELECTRIC FIELD VECTOR**

$$\vec{E}(z,t) = \begin{cases} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$

**PHASOR = JONES VECTOR**

$$\underline{E} = \begin{bmatrix} E_x = E_{ox}e^{j\delta_x} \\ E_y = E_{oy}e^{j\delta_y} \end{bmatrix}$$

**With:** $\vec{E}(z,t) = \Re\left(\underline{E}\,e^{j(\omega t - kz)}\right)$

## GEOMETRICAL PARAMETERS

ABSOLUTE PHASE

$$\alpha = \delta_x$$

AMPLITUDE

$$A = \sqrt{E_{0x}^2 + E_{0y}^2}$$

ORIENTATION ANGLE

$$\tan 2\phi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta$$

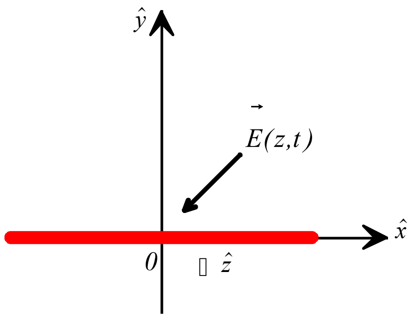
ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta$$

POLARISATION HANDENESS: $Sign(\tau)$

# JONES VECTOR

## HORIZONTAL POLARISATION STATE

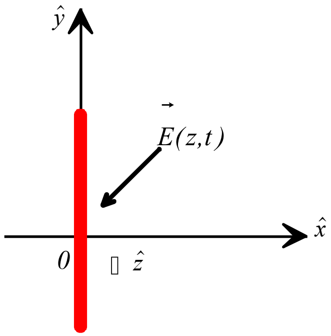



$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\phi = 0$$

$$\tau = 0$$

## VERTICAL POLARISATION STATE

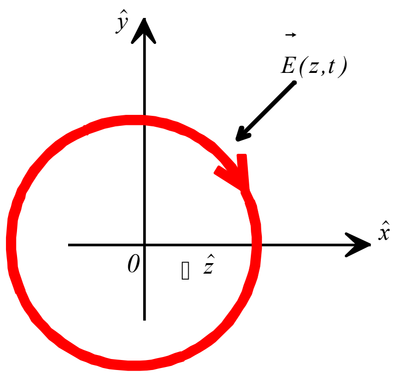



$$\underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\phi = \frac{\pi}{2}$$

$$\tau = 0$$

## LEFT CIRCULAR POLARISATION STATE

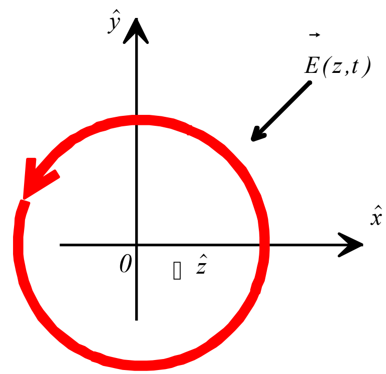



$$\underline{LC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}$$

$$-\frac{\pi}{2} \le \phi \le +\frac{\pi}{2}$$

$$\tau = +\frac{\pi}{4}$$

## RIGHT CIRCULAR POLARISATION STATE

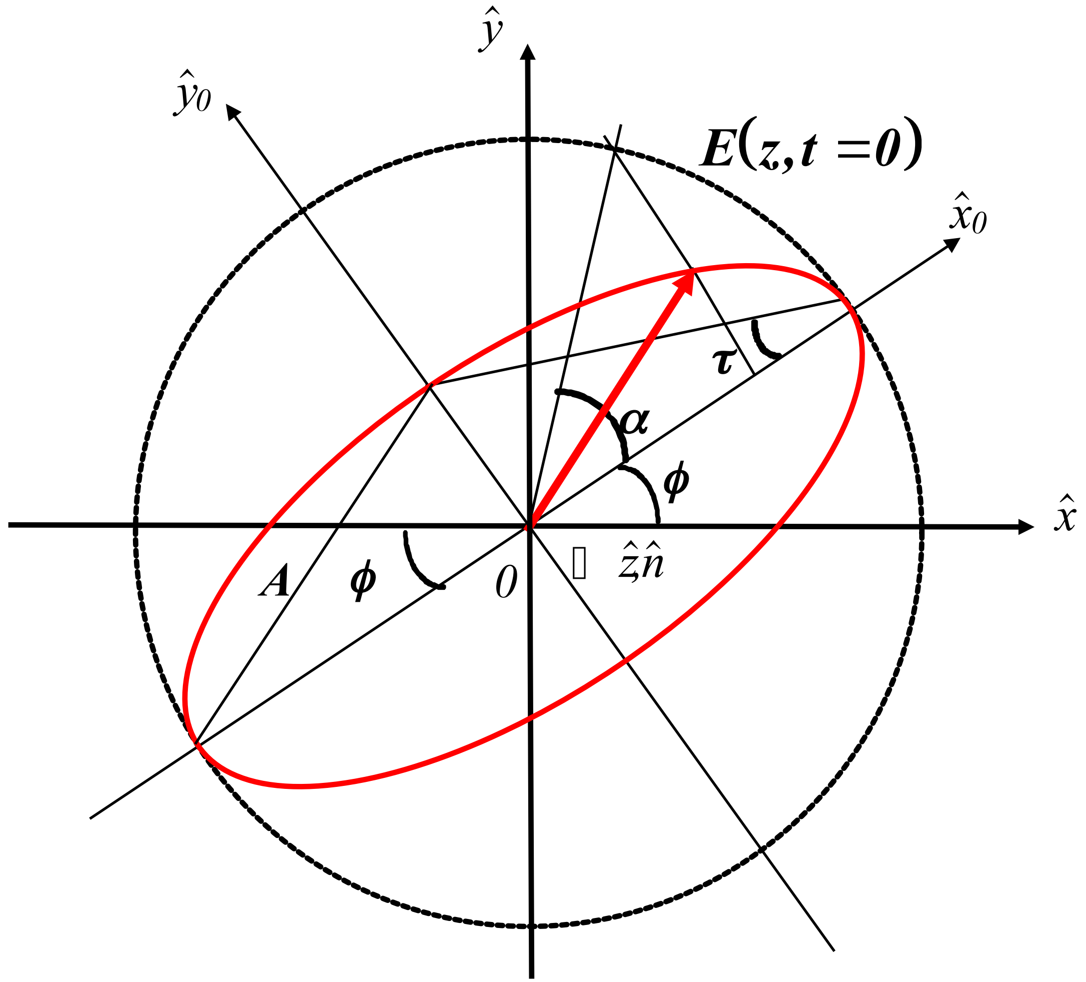



$$\underline{RC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

$$-\frac{\pi}{2} \le \phi \le +\frac{\pi}{2}$$

$$\tau = -\frac{\pi}{4}$$

# JONES VECTOR




$$\underline{E} = A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} \hat{u}_x$$

# ORTHOGONAL JONES VECTOR

## JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{ox} e^{j\delta_x} \\ E_{oy} e^{j\delta_y} \end{bmatrix}$$

$$= A \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} \hat{u}_x$$

↓

## POLARISATION ALGEBRA

*NORM OF A JONES VECTOR* $\left\| \underline{E} \right\| = \sqrt{E_{0x}^2 + E_{0y}^2}$

*SCALAR PRODUCT* $\langle \underline{A}, \underline{B} \rangle = \underline{A}^{T*} \underline{B}$

*ORTHOGONALITY* $\langle \underline{A}, \underline{A}_\perp \rangle = 0$

# ELLIPTICAL BASIS TRANSFORMATION

## JONES VECTOR

$$\underline{E} = A\begin{bmatrix}\cos(\varphi) & -\sin(\varphi)\\ \sin(\varphi) & \cos(\varphi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}\hat{u}_x$$

## ORTHOGONAL JONES VECTOR

$$\underline{E} = A\begin{bmatrix}\cos(\varphi) & -\sin(\varphi)\\ \sin(\varphi) & \cos(\varphi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}\hat{u}_y$$

$$\downarrow$$

$$\left[\underline{E}, \underline{E}\right] = A\begin{bmatrix}\cos(\varphi) & -\sin(\varphi)\\ \sin(\varphi) & \cos(\varphi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}\left[\hat{u}_x, \hat{u}_y\right]$$

$$\downarrow$$

## ELLIPTICAL BASIS TRANSFORMATION

# ELLIPTICAL BASIS TRANSFORMATION

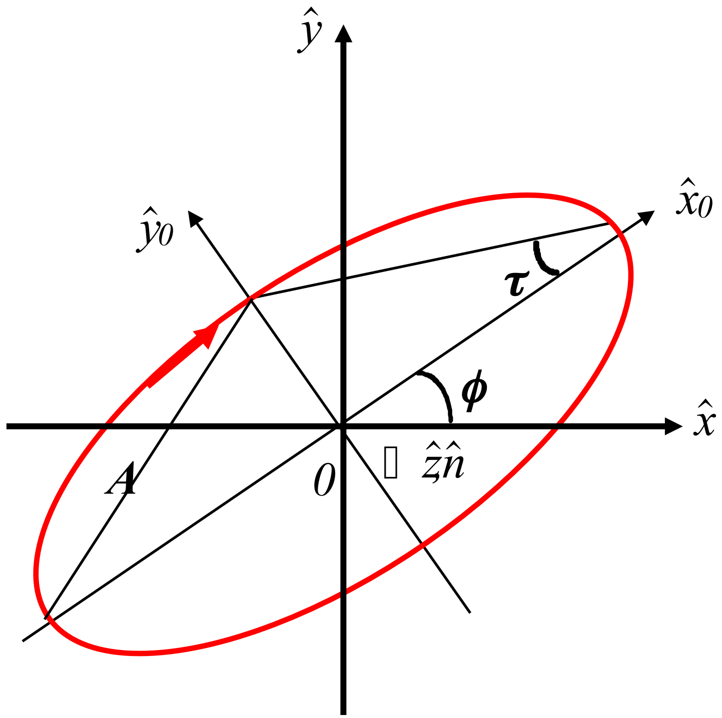
## SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

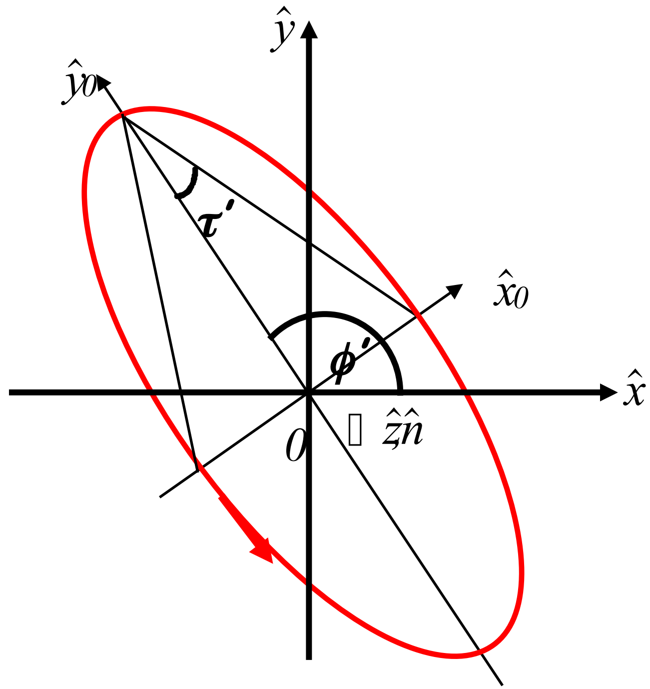
$$[U(\varphi,\tau,\alpha)] = \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

## ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$[U_{(A,A)\mapsto(B,B)}] = [U(\varphi,\tau,\alpha)]^{-1}$$
$$= \begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix}$$

# ORTHOGONAL JONES VECTOR

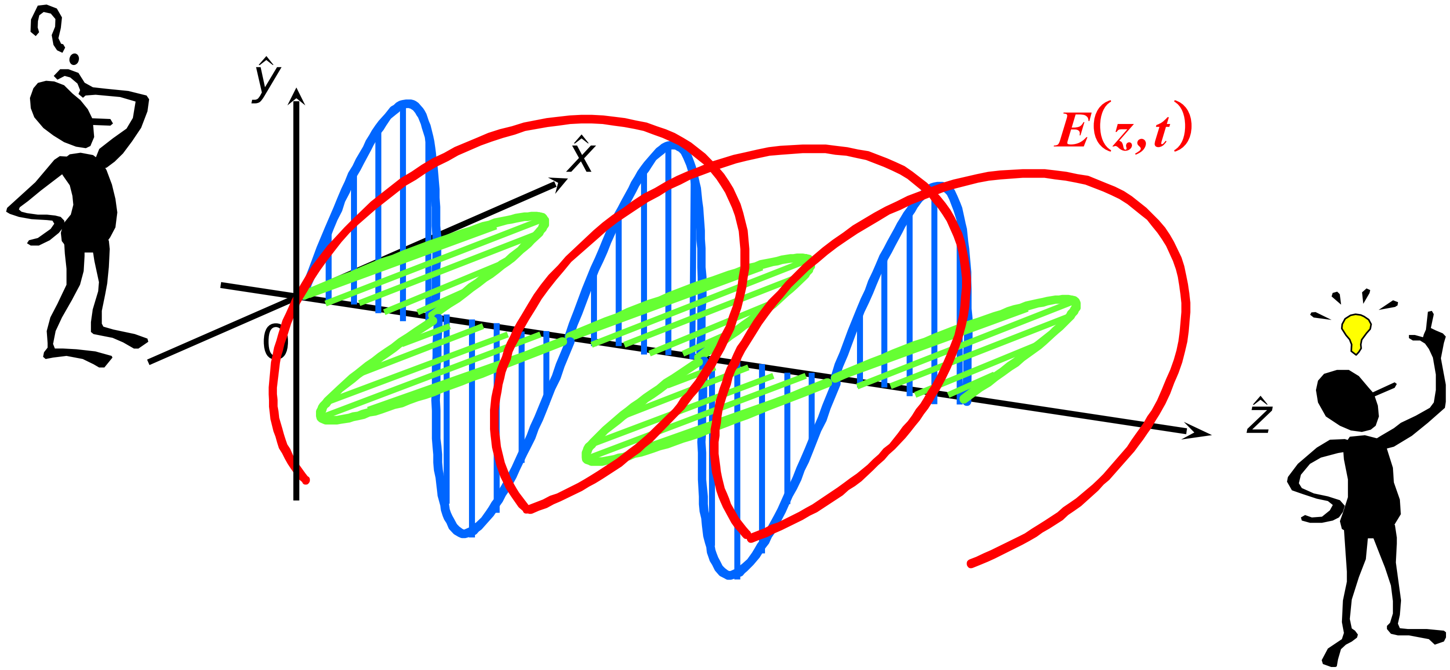



## ORTHOGONALITY CONDITIONS

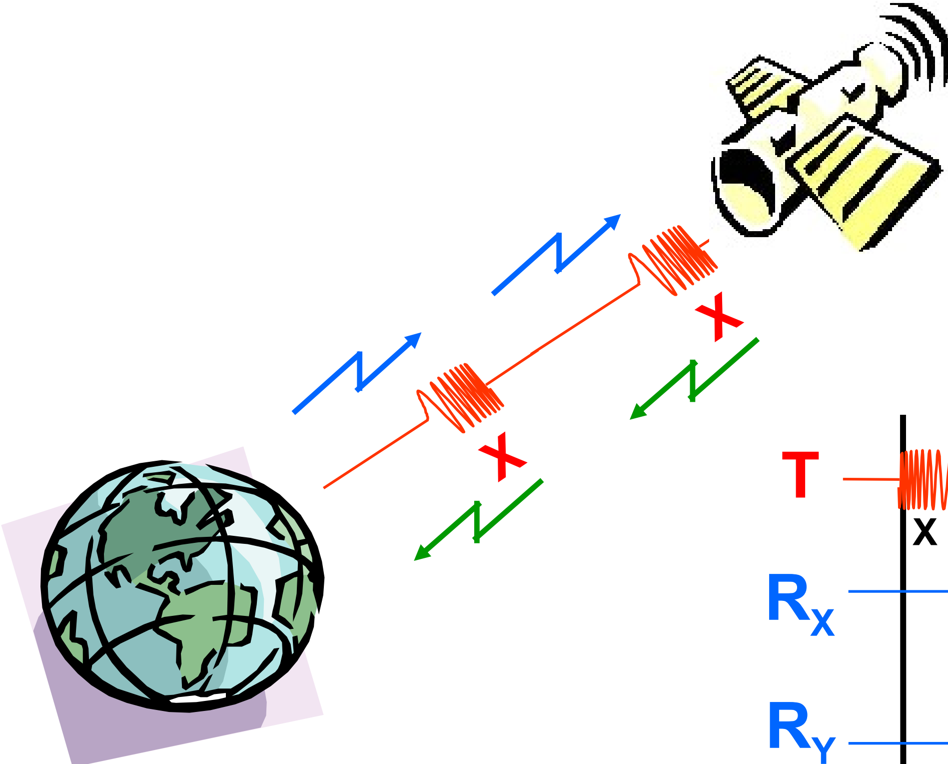
$$(\varphi,\tau) \mapsto \begin{Bmatrix} \varphi' = \varphi + \frac{\pi}{2} \\ \tau' = -\tau \end{Bmatrix}$$

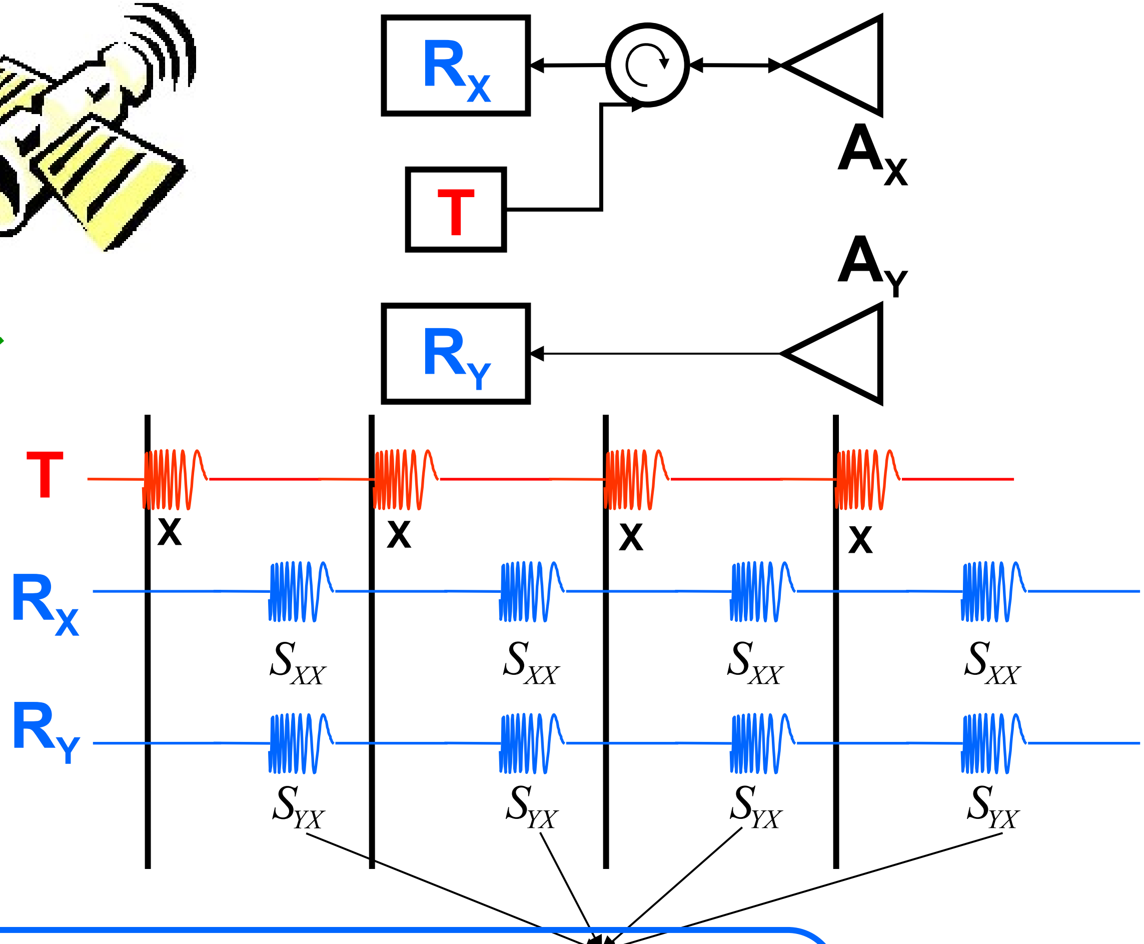
➡ **CHANGE OF POLARISATION HANDENESS**




# SCATTERING POLARIMETRY

# WAVE POLARIMETRY



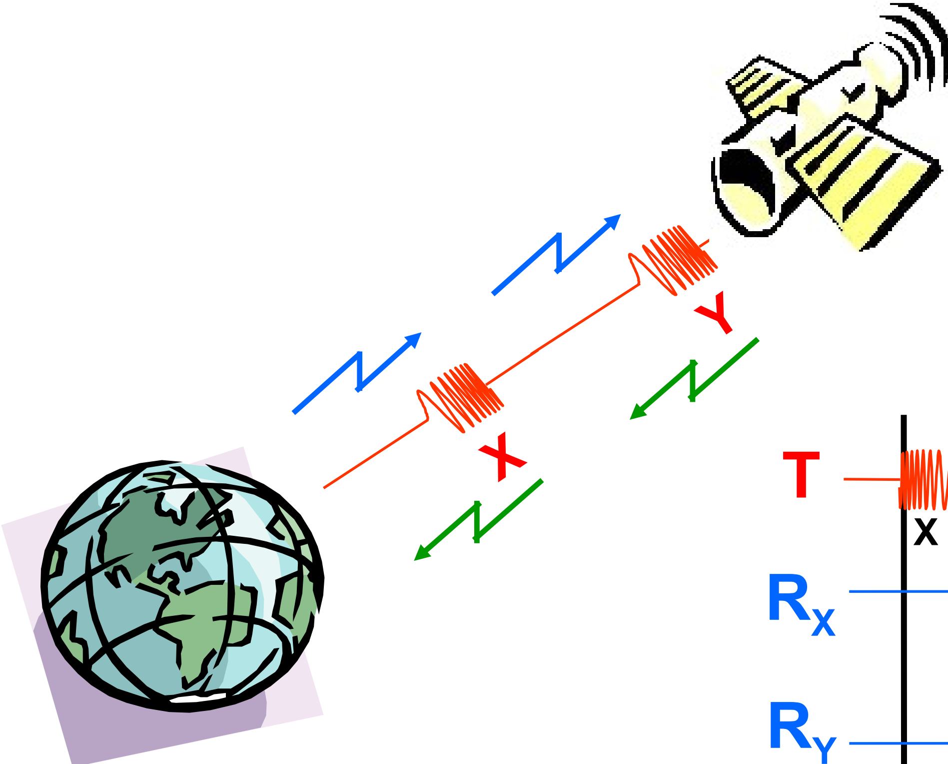

**TRANSMITTER:** X

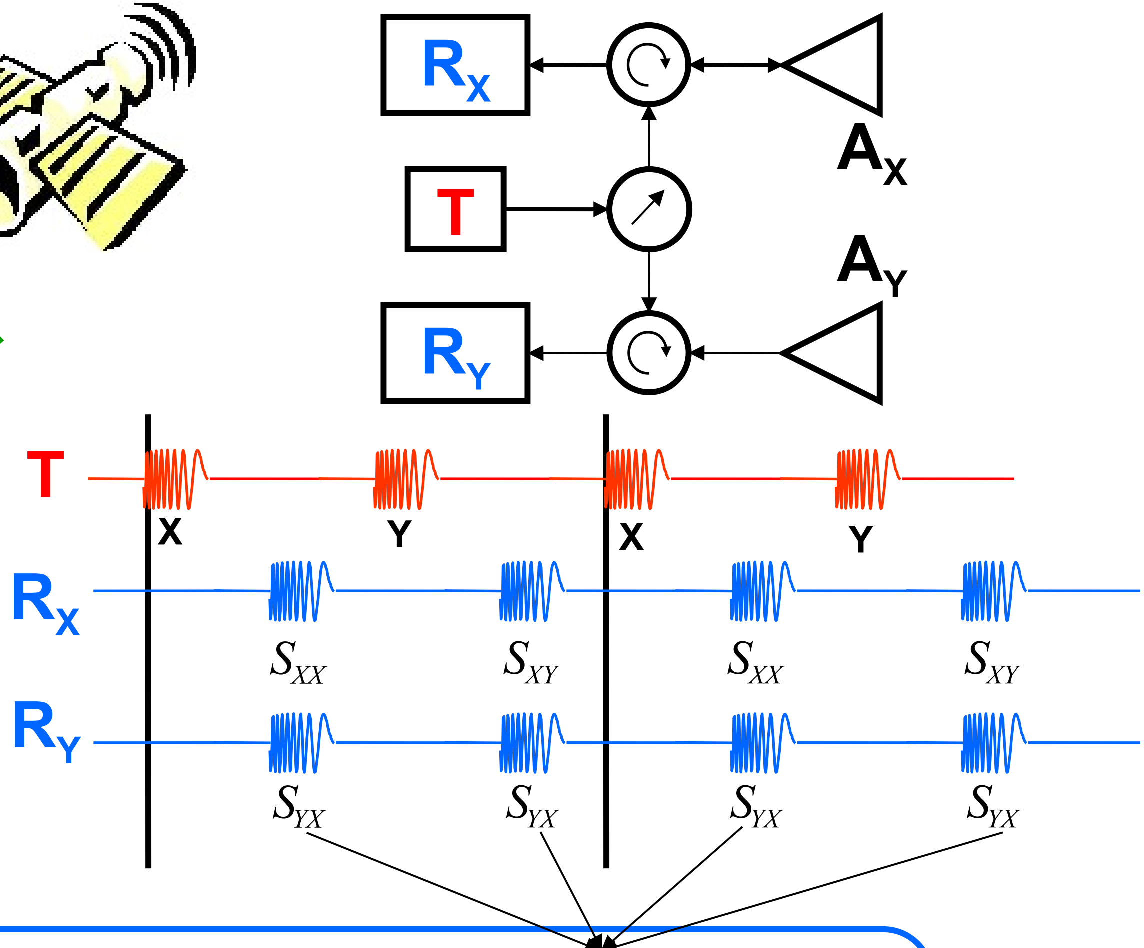
**RECEIVERS:** X & Y

JONES VECTORS $\left\{ \underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix} \right\}$

WAVE POLARIMETRY

# SCATTERING POLARIMETRY




**TRANSMITTER:** X & Y

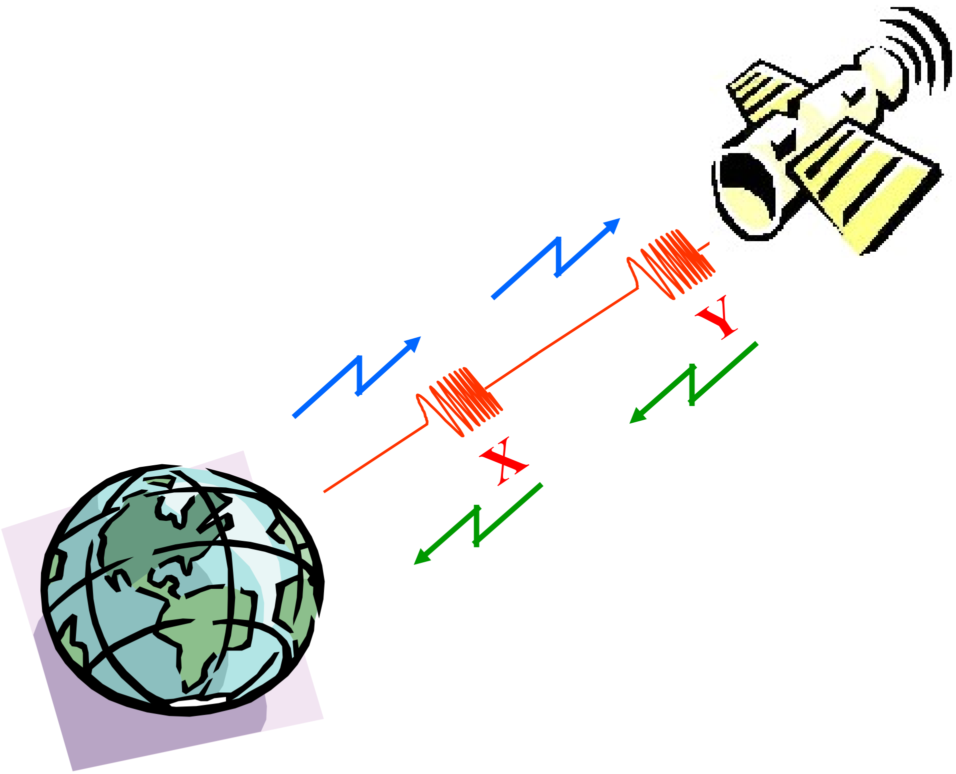
**RECEIVERS:** X & Y

**SINCLAIR MATRICES**

$$\left\{ [S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \right\}$$

**SCATTERING POLARIMETRY**

# POLARIMETRIC DESCRIPTORS




**TRANSMITTER:** X & Y
**RECEIVERS:** X & Y

## THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

- [S] SINCLAIR Matrix
- k, Ω Target Vectors
- [K] KENNAUGH Matrix
- [T] Coherency Matrix
- [C] Covariance Matrix

# BACKSCATTERING MATRIX

$$[S] = \frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix} = \frac{e^{jkr}}{r}\begin{bmatrix} |S_{XX}|e^{j\varphi_{XX}} & |S_{XY}|e^{j\varphi_{XY}} \\ |S_{XY}|e^{j\varphi_{XY}} & |S_{YY}|e^{j\varphi_{YY}} \end{bmatrix}$$

**ABSOLUTE BACKSCATTERING MATRIX**

$$[S] = \frac{e^{jkr}e^{j\phi_{XX}}}{r}\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} \\ |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} & |S_{YY}|e^{j(\varphi_{YY}-\varphi_{XX})} \end{bmatrix}$$

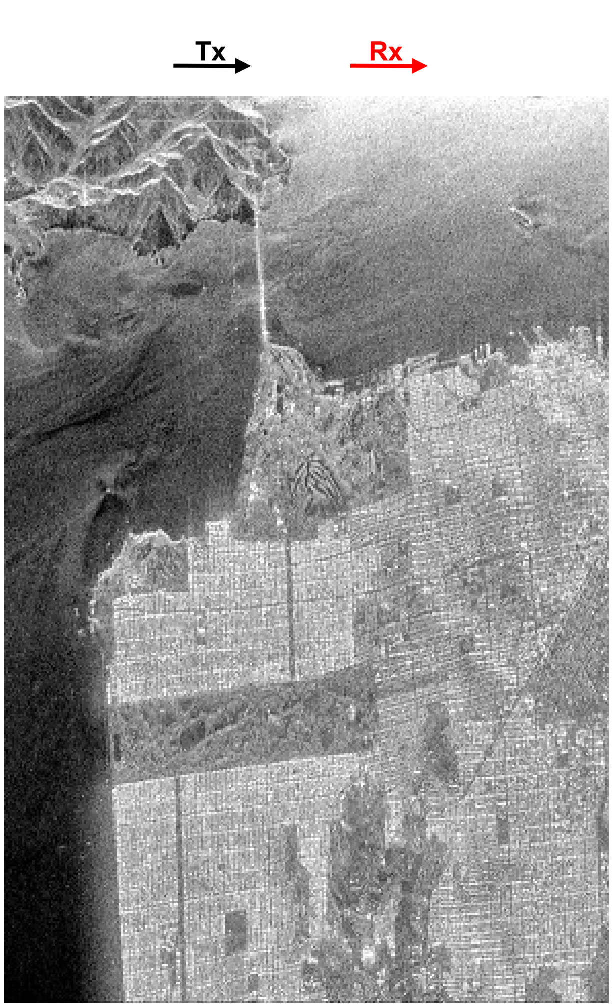
**Absolute Phase Factor**

**RELATIVE BACKSCATTERING MATRIX**

**Five Parameters: 3 Amplitudes and 2 Phases**

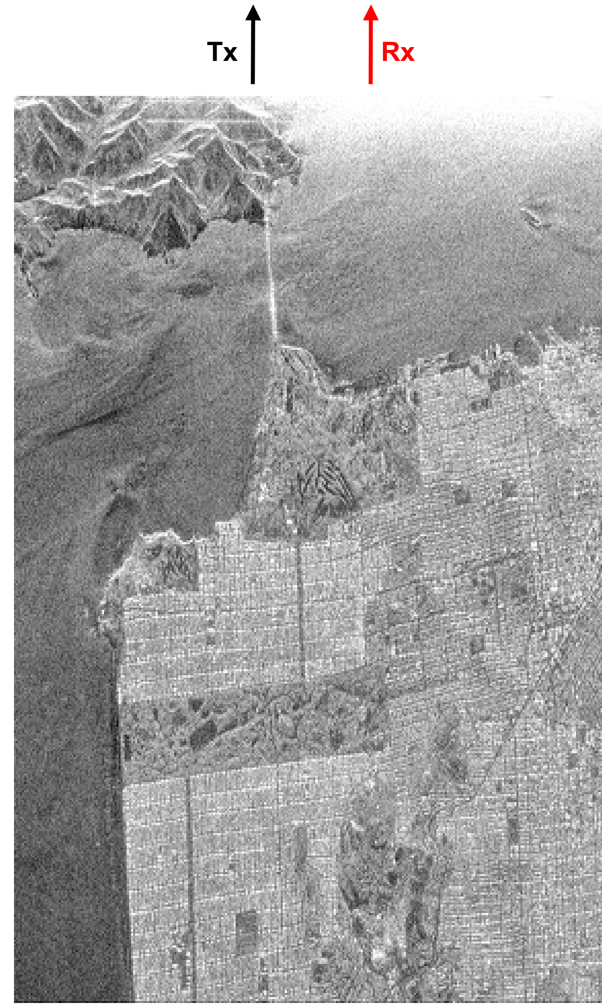
**SCATTERER POLARIMETRIC DIMENSION = 5**

# SCATTERING POLARIMETRY




$|HH|_{dB}$


$|HV|_{dB}$




$|VV|_{dB}$

-30dB -15dB 0dB

# SCATTERING POLARIMETRY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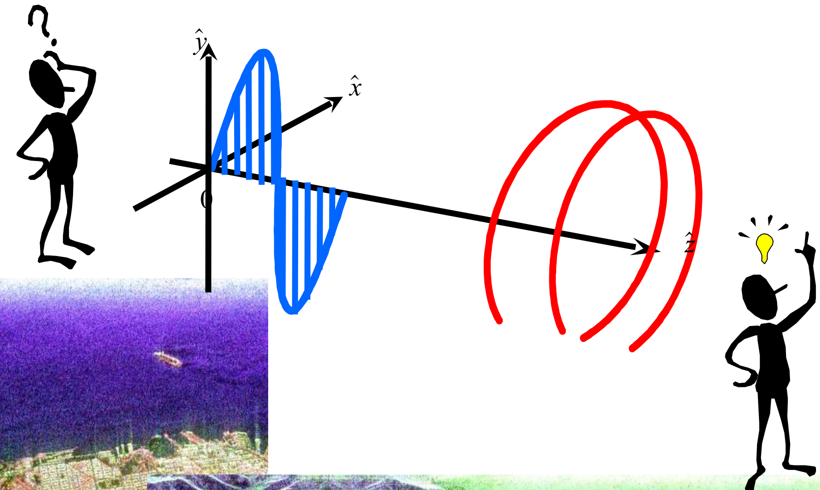
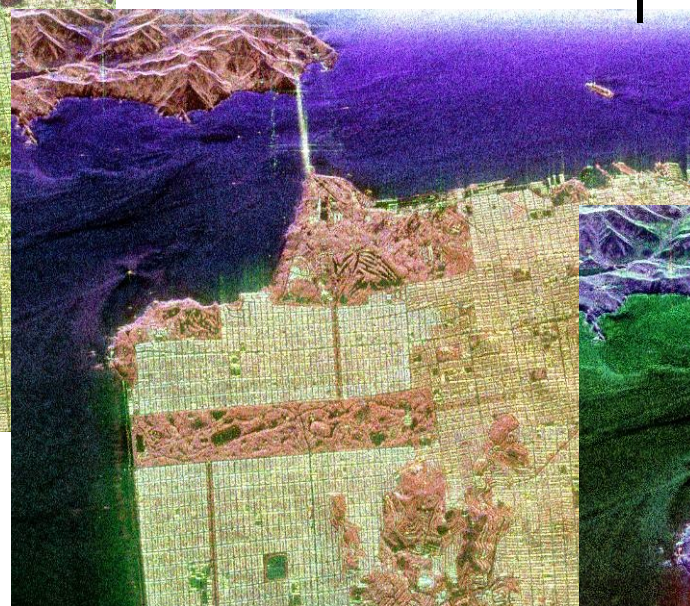
**Sinclair Color Coding**

**|HH| |HV| |VV|**

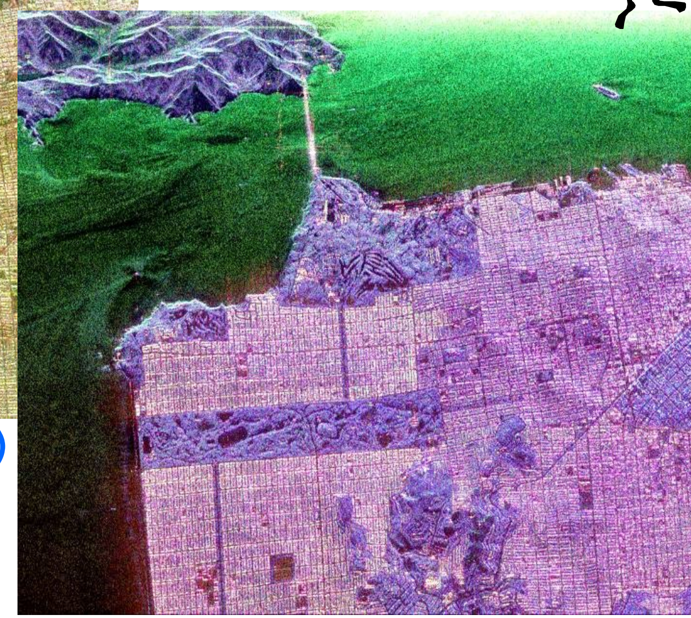
# ELLIPTICAL BASIS TRANSFORMATION



Pauli Color Coding (H,V)



Pauli Color Coding (+45,-45)



Pauli Color Coding (L,R)

Ernst LÜNEBURG
(PIERS95 - Pasadena)

# ELLIPTICAL BASIS TRANSFORMATION

$$\left[S_{(B,B)}\right] = \left[U_{(A,A)\mapsto(B,B)}\right]^T \left[S_{(A,A)}\right] \left[U_{(A,A)\mapsto(B,B)}\right]$$

**CON-SIMILARITY TRANSFORMATION**

$$\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

**SU(2) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX**

$$\left[U_{(A,A)\mapsto(B,B)}\right] \qquad \left[U(\varphi,\tau,\alpha)\right]^{-1}$$

$$\begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix}$$

$$\left[U_2(-\alpha)\right] \qquad \left[U_2(-\tau)\right] \qquad \left[U_2(-\varphi)\right]$$

# ELLIPTICAL BASIS TRANSFORMATION


© Google Earth

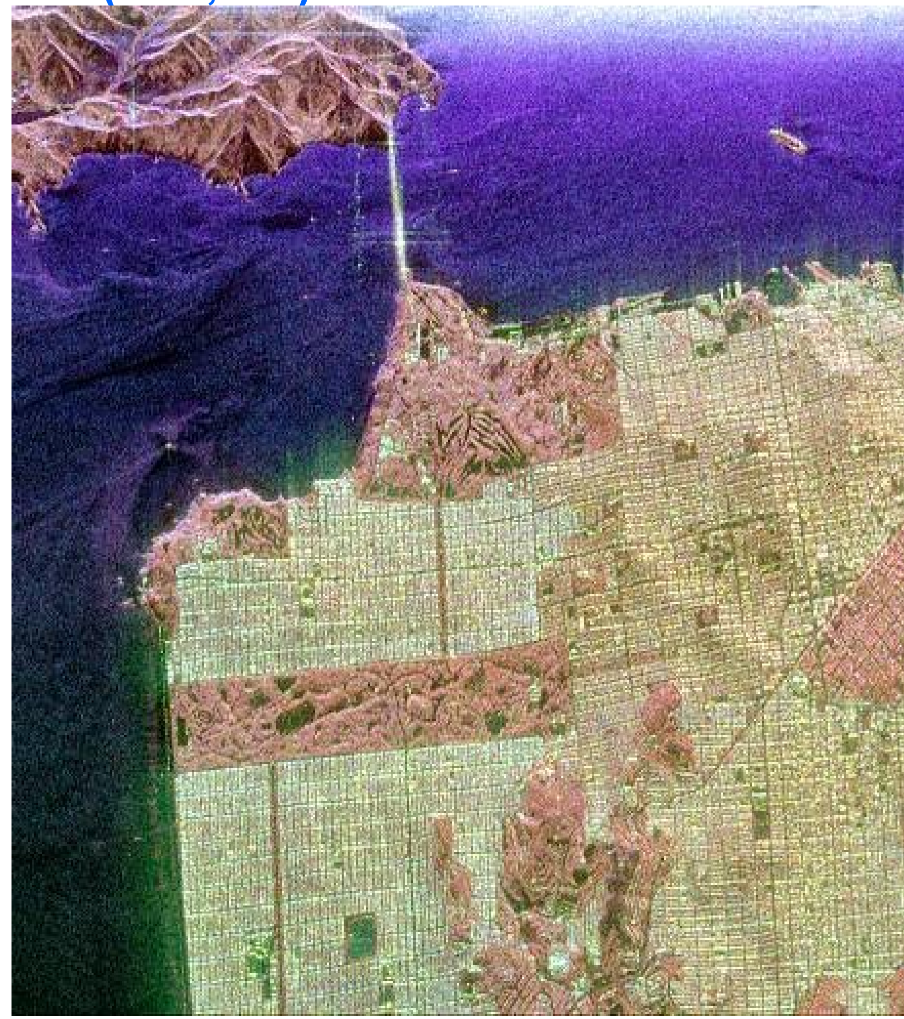
(H,V) POLARISATION BASIS

|HH+VV| |HV | |HH-VV|

# ELLIPTICAL BASIS TRANSFORMATION


© Google Earth

## (+45°,-45°) POLARISATION BASIS



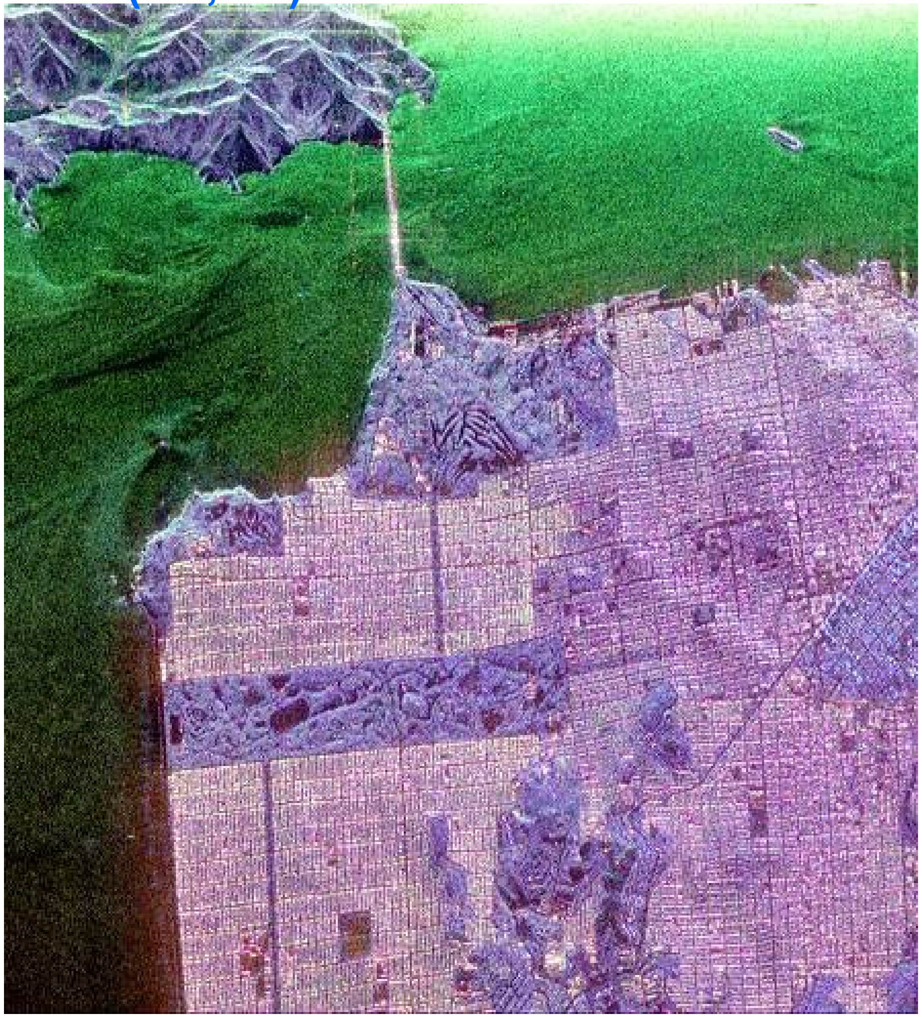
|AA+BB| |AB | |AA-BB|

With: A=Linear +45°, B=Linear –45°

# ELLIPTICAL BASIS TRANSFORMATION

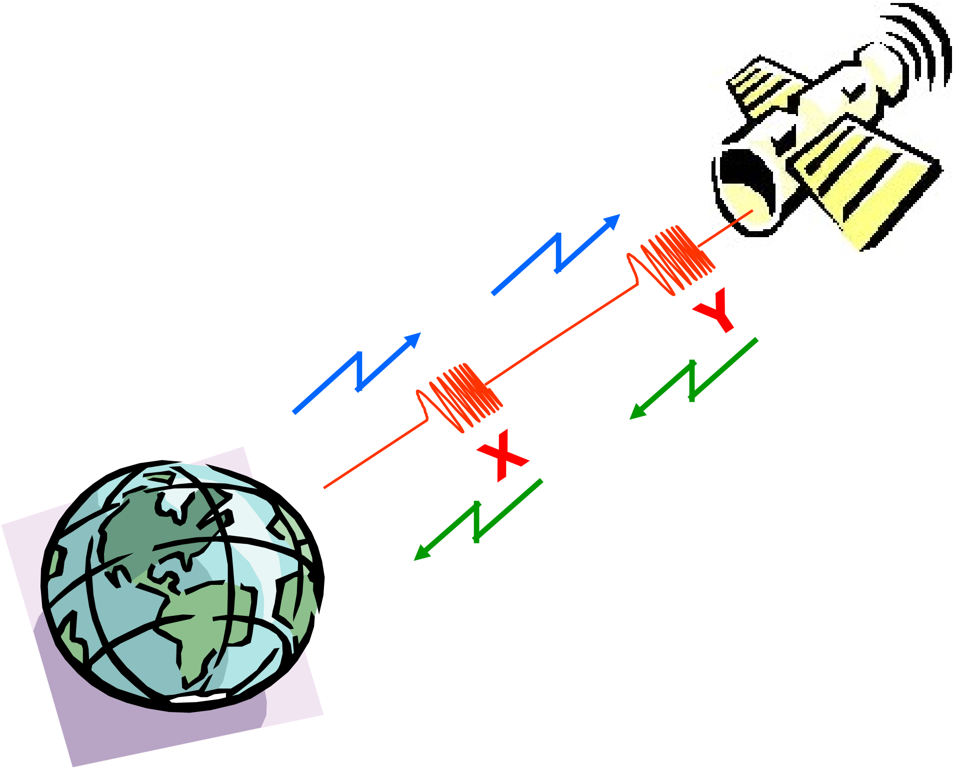

© Google Earth

## (LC,RC) POLARISATION BASIS



|LL+RR| |LR | |LL-RR|

# POLARIMETRIC DESCRIPTORS




TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S] SINCLAIR Matrix
k, Ω Target Vectors
[K] KENNAUGH Matrix
[T] Coherency Matrix
[C] Covariance Matrix

# TARGET VECTORS

## SCATTERING VECTOR TRANSFORMATIONS

**Pauli Scattering Vector:**

$$\underline{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ 2S_{XY} \end{bmatrix}$$

**Lexicographic Scattering Vector:**

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ \sqrt{2}S_{XY} \\ S_{YY} \end{bmatrix}$$

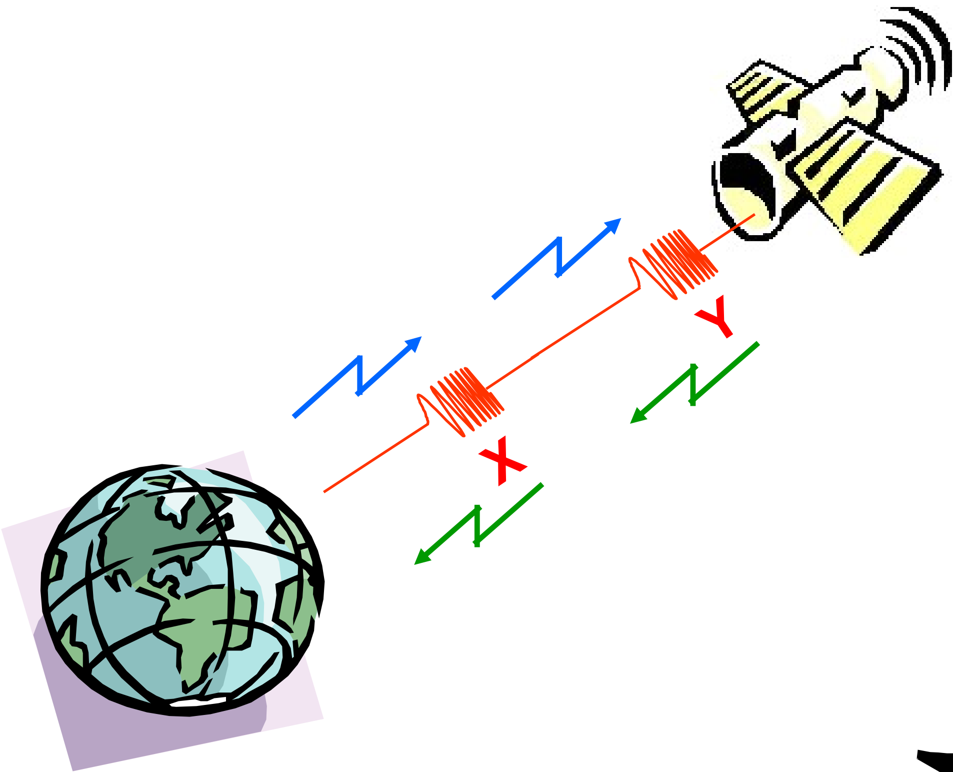
**UNITARY TRANSFORMATION**

$$\underline{k} = [D_3]\underline{\Omega} \quad and \quad \underline{\Omega} = [D_3]^{-1}\underline{k} = [D_3]^T\underline{k}$$

WHERE $[D_3]$ IS A SU(3) MATRIX
IN ORDER TO PRESERVE THE NORM
OF THE SCATTERING VECTOR

$$[D_3] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \end{bmatrix}$$

# POLARIMETRIC DESCRIPTORS




**TRANSMITTER:** X & Y
**RECEIVERS:** X & Y

**THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS**

| | |
|---|---|
| [S] | SINCLAIR Matrix |
| k, Ω | Target Vectors |
| [K] | KENNAUGH Matrix |
| **[T]** | **Coherency Matrix** |
| [C] | Covariance Matrix |

**STATISTICAL DESCRIPTION**

**PARTIAL SCATTERING POLARIMETRY**

## MONOSTATIC CASE

PAULI SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}} \left[ S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY} \right]^T$$

COHERENCY MATRIX [T]

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

HERMITIAN MATRIX - RANK 1

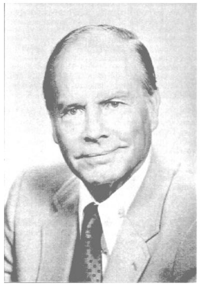
A0, B0+B, B0-B : HUYNEN TARGET GENERATORS

# HUYNEN PARAMETERS

## PHYSICAL INTERPRETATION

**MAN-MADE TARGET DECOMPOSITION**
**IDENTIFICATION and ANALYSIS**

« *PHENOMENOLOGICAL THEORY OF RADAR TARGETS* » (1970)



**A0 :** GENERATOR OF TARGET SYMMETRY

**B0+B :** GENERATOR OF TARGET NON-SYMMETRY

**B0-B :** GENERATOR OF TARGET IRREGULARITY

**C :** GENERATOR OF TARGET GLOBAL SHAPE (LINEAR)

**D :** GENERATOR OF TARGET LOCAL SHAPE (CURVATURE)

**E :** GENERATOR OF TARGET LOCAL TWIST (TORSION)

**F :** GENERATOR OF TARGET GLOBAL TWIST (HELICITY)

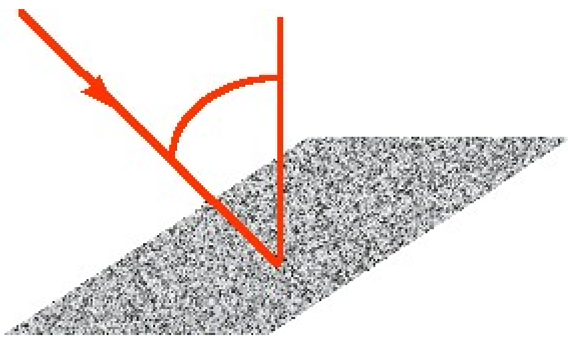
**G :** GENERATOR OF TARGET LOCAL COUPLING (GLUE)

**H :** GENERATOR OF TARGET GLOBAL COUPLING (ORIENTATION)

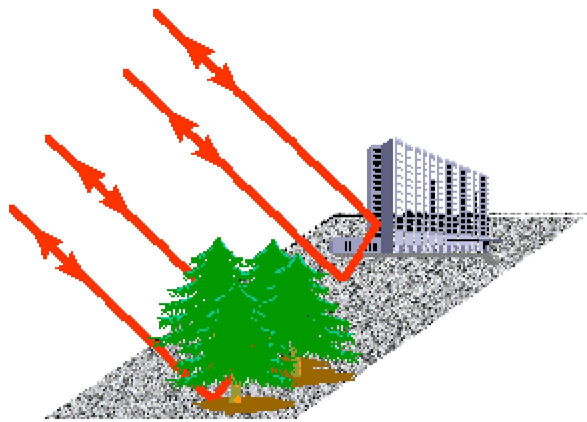
# TARGET GENERATORS

## PHYSICAL INTERPRETATION

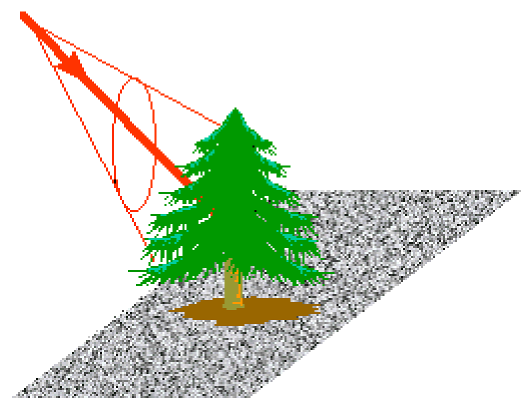
**SINGLE BOUNCE SCATTERING (ROUGH SURFACE)**



**DOUBLE BOUNCE SCATTERING**



**VOLUME SCATTERING**

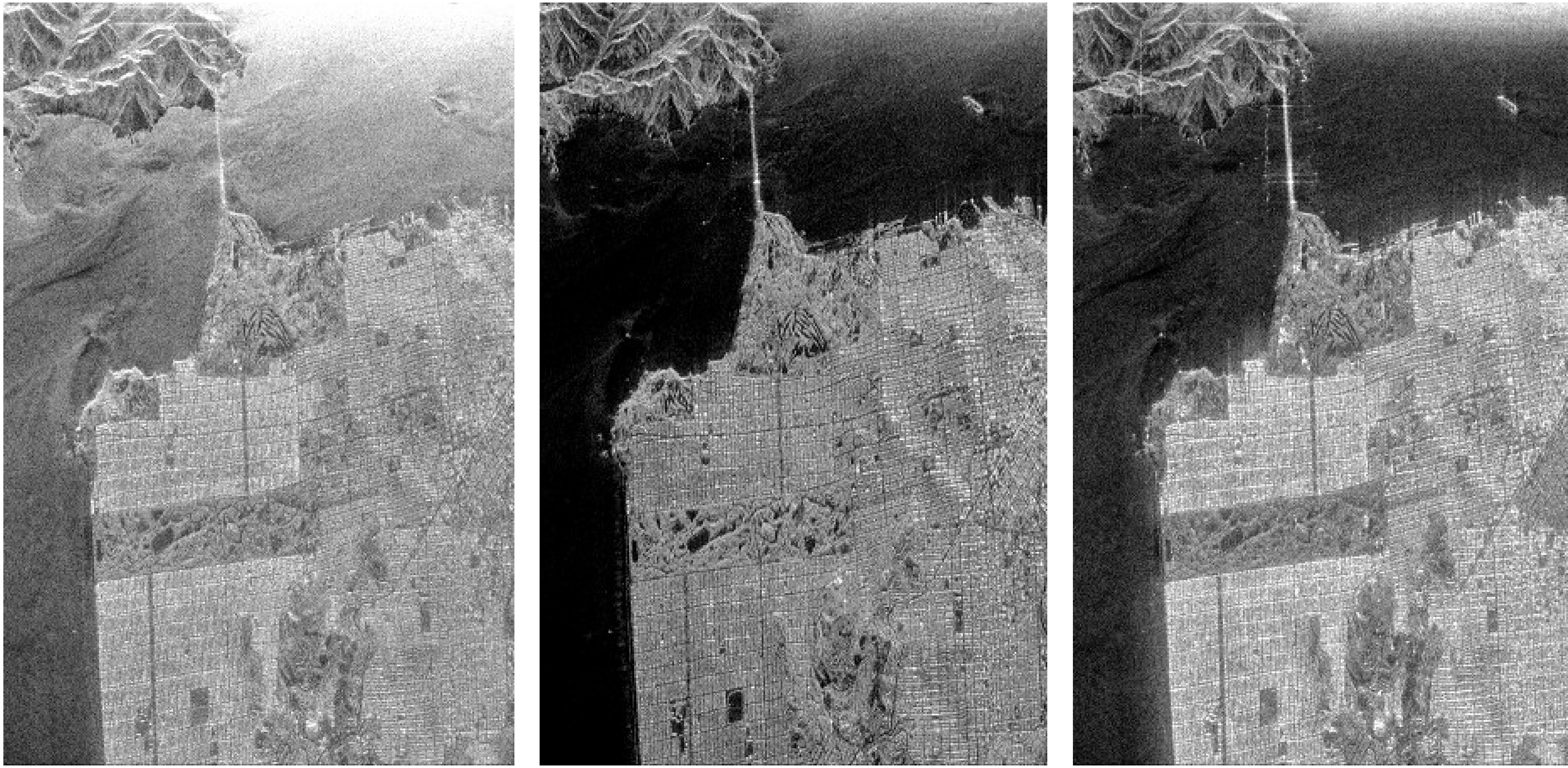


$$T_{11} = 2A_0 = \left|S_{XX} + S_{YY}\right|^2$$

$$T_{33} = B_0 - B = 2\left|S_{XY}\right|^2$$

$$T_{22} = B_0 + B = \left|S_{XX} - S_{YY}\right|^2$$

# TARGET GENERATORS



$|HH+VV|_{dB}$ $|HV|_{dB}$ $|HH-VV|_{dB}$

-30dB -15dB 0dB

# TARGET GENERATORS


© Google Earth

## (H,V) POLARISATION BASIS

|HH+VV| |HV | |HH-VV|

# ELLIPTICAL BASIS TRANSFORMATION

## SPECIAL UNITARY SU(2) GROUP

$$[U_2]=\begin{bmatrix}\cos(\varphi) & -\sin(\varphi)\\ \sin(\varphi) & \cos(\varphi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{-j\alpha} & 0\\ 0 & e^{j\alpha}\end{bmatrix}$$

$[U_2(\varphi)]$ $[U_2(\tau)]$ $[U_2(\alpha)]$

## SPECIAL UNITARY SU(3) GROUP

$$\begin{bmatrix}1 & 0 & 0\\ 0 & \cos(2\varphi) & \sin(2\varphi)\\ 0 & -\sin(2\varphi) & \cos(2\varphi)\end{bmatrix}\begin{bmatrix}\cos(2\tau) & 0 & j\sin(2\tau)\\ 0 & 1 & 0\\ j\sin(2\tau) & 0 & \cos(2\tau)\end{bmatrix}\begin{bmatrix}\cos(2\alpha) & -j\sin(2\alpha) & 0\\ -j\sin(2\alpha) & \cos(2\alpha) & 0\\ 0 & 0 & 1\end{bmatrix}$$

**$[U_3(2\phi)]$** **$[U_3(2\tau)]$** **$[U_3(2\alpha)]$**

# ELLIPTICAL BASIS TRANSFORMATION

## SINCLAIR MATRIX

$$E^{s}_{(A,A)} = \left[S_{(A,A)}\right] E^{i}_{(A,A)} \qquad E^{s}_{(B,B)} = \left[S_{(B,B)}\right] E^{i}_{(B,B)}$$

$$\left[S_{(B,B)}\right] = \left[U_{(A,A)\mapsto(B,B)}\right]^{T} \left[S_{(A,A)}\right] \left[U_{(A,A)\mapsto(B,B)}\right]$$

**CON-SIMILARITY TRANSFORMATION**

## COHERENCY MATRIX

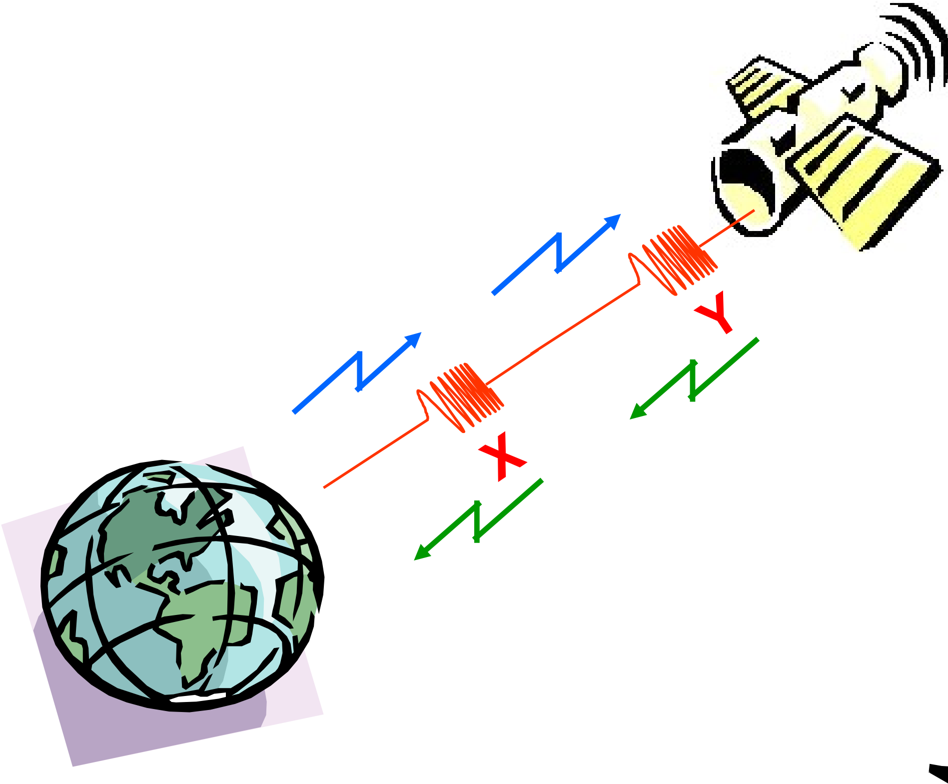
$$\left[T_{(B,B)}\right] = \left[U_{3(A,A)\mapsto(B,B)}\right] \left[T_{(A,A)}\right] \left[U_{3(A,A)\mapsto(B,B)}\right]^{-1}$$

**SIMILARITY TRANSFORMATION**

$$\left[U_{3(A,A)\mapsto(B,B)}\right]$$

**U(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX**

# POLARIMETRIC DESCRIPTORS




**TRANSMITTER:** X & Y
**RECEIVERS:** X & Y

**THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS**

| | |
|---|---|
| [S] | SINCLAIR Matrix |
| k, Ω | Target Vectors |
| [K] | KENNAUGH Matrix |
| [T] | Coherency Matrix |
| [C] | Covariance Matrix |

**STATISTICAL DESCRIPTION**

**PARTIAL SCATTERING POLARIMETRY**

# COVARIANCE MATRIX

## MONOSTATIC CASE

### LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \left[ S_{XX} \quad \sqrt{2} S_{XY} \quad S_{YY} \right]^T$$

↓

### COVARIANCE MATRIX [C]

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX} S_{XX}^* & \sqrt{2} S_{XX} S_{XY}^* & S_{XX} S_{YY}^* \\ \sqrt{2} S_{XY} S_{XX}^* & 2 S_{XY} S_{XY}^* & \sqrt{2} S_{XY} S_{YY}^* \\ S_{YY} S_{XX}^* & \sqrt{2} S_{YY} S_{XY}^* & S_{YY} S_{YY}^* \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

# COVARIANCE-COHERENCY MATRICES

**COHERENCY MATRIX** | **COVARIANCE MATRIX**

$$[T] = \underline{k} \cdot \underline{k}^{*T}$$

$$\underline{k} = \lfloor D_{3or4} \rfloor \underline{\Omega}$$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T}$$

**UNITARY TRANSFORMATION**

$$[T] = [D_{3or4}][C][D_{3or4}]^{T*}$$

**[*T*] and [*C*] HAVE THE SAME EIGENVALUES**

Both contain the same information about Polarimetric Scattering Amplitudes, Phase Angles and Correlations

**[*T*]** is closer related to Physical and Geometrical Properties of the Scattering Process, and thus allows a better and direct physical interpretation

**[*C*]** is directly related to the system measurables

**[*T*]** is directly related to the Kennaugh matrix and the Huynen parameters

# POLARIMETRIC DESCRIPTORS

## SINCLAIR MATRIX

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

EQUIVALENCE ?

## SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

↓

## COHERENCY MATRIX [*T*]

$$[T] = \underline{k} \cdot \underline{k}^{*T}$$

## SCATTERING VECTOR $\underline{\Omega}$

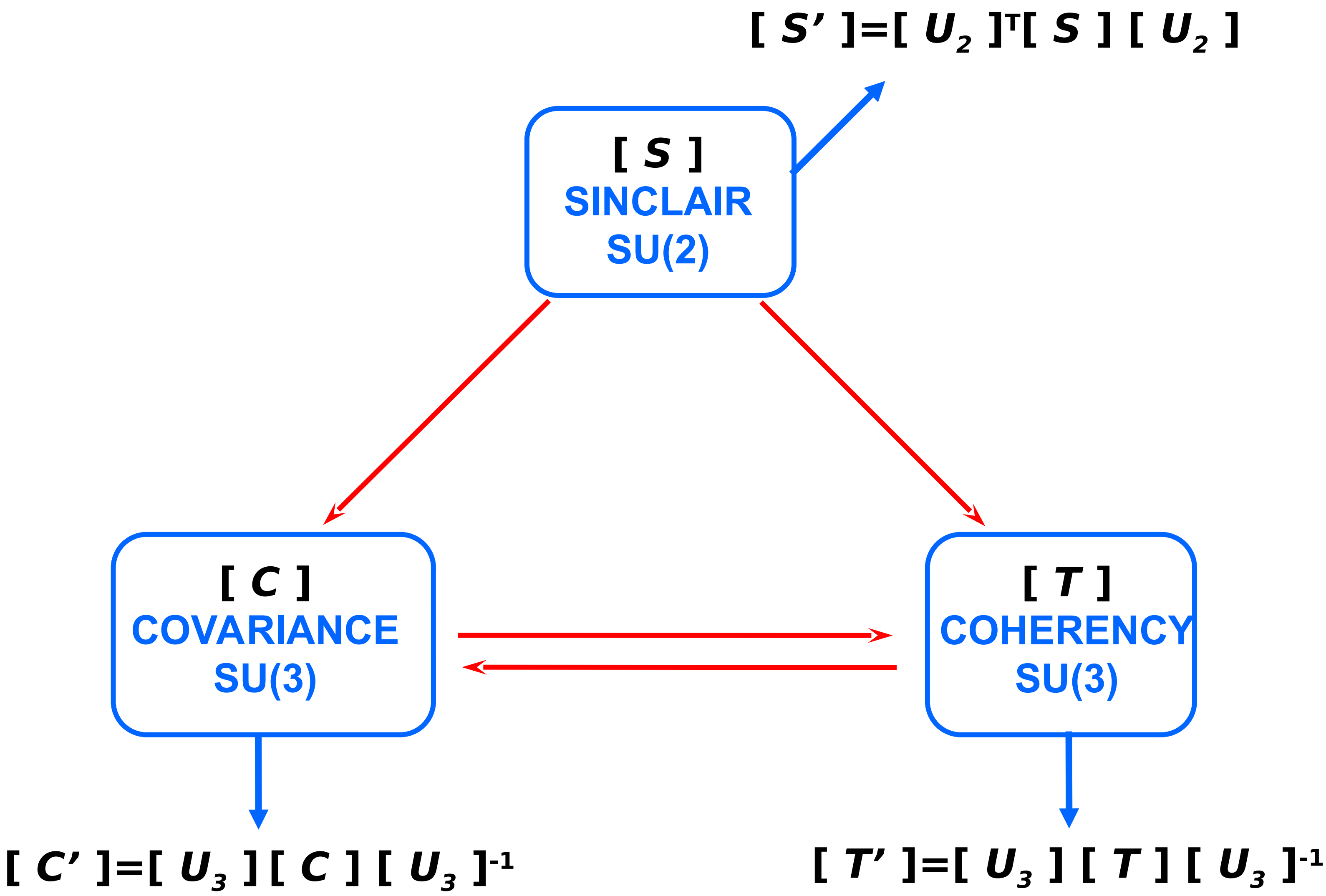
$$\underline{\Omega} = \left[S_{XX} \quad \sqrt{2}S_{XY} \quad S_{YY}\right]^T$$

↓

## COVARIANCE MATRIX [*C*]

$$[C] = \underline{\Omega}\,\underline{\Omega}^{T*}$$

# POLARIMETRIC DESCRIPTORS



# ELLIPTICAL BASIS TRANSFORMATION

## SPECIAL UNITARY SU(2) GROUP

$$\begin{bmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

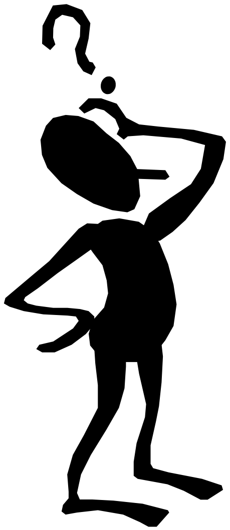
$$[U_2(\varphi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

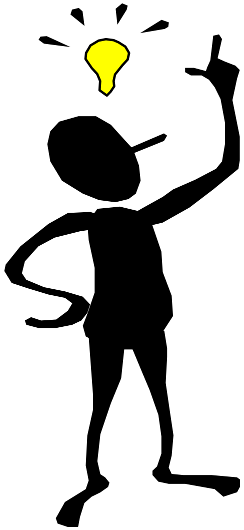
## SPECIAL UNITARY SU(3) GROUP (T Matrix)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\varphi) & \sin(2\varphi) \\ 0 & -\sin(2\varphi) & \cos(2\varphi) \end{bmatrix} \begin{bmatrix} \cos(2\tau) & 0 & j\sin(2\tau) \\ 0 & 1 & 0 \\ j\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} \cos(2\alpha) & -j\sin(2\alpha) & 0 \\ -j\sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[U_3(2\phi)] \qquad [U_3(2\tau)] \qquad [U_3(2\alpha)]$$

# TARGET EQUATIONS



## POLARIMETRIC GOLDEN NUMBER

## POLARIMETRIC TARGET DIMENSION

# TARGET EQUATIONS

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

**5 DEGREES OF FREEDOM**

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\phi_{XY-XX}, \phi_{YY-XX}$$

**TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »**

**=**

**5**

**KENNAUGH MATRIX [*K*]**

**COHERENCY MATRIX [*T*]**

**9 HUYNEN REAL PARAMETERS**

***(A0, B0, B, C, D, E, F, G, H)***

**COVARIANCE MATRIX [*C*]**

**9 REAL PARAMETERS**

**|*XX*|, |XY|, |*YY*|,**

**Re(*XXXY**), Im(*XXXY**)**

**Re(*XXYY**), Im(*XXYY**)**

**Re(*XYYY**), Im(*XYYY**)**

**9 - 5 = 4 TARGET EQUATIONS**

# TARGET EQUATIONS

## PURE TARGET – MONOSTATIC CASE

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

**3x3 HERMITIAN MATRIX - RANK 1**

↓

**9 PRINCIPAL MINORS = 0**

$$2A_0(B_0 + B) - C^2 - D^2 = 0 \qquad 2A_0(B_0 - B) - G^2 - H^2 = 0$$

$$-2A_0E + CH - DG = 0 \qquad B_0^2 - B^2 - E^2 - F^2 = 0$$

$$C(B_0 - B) - EH - GF = 0 \qquad -D(B_0 - B) + FH - GE = 0$$

$$2A_0F - CG - DH = 0 \qquad -G(B_0 + B) + FC - ED = 0$$

$$H(B_0 + B) - CE - DF = 0$$

# TARGET EQUATIONS

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

**5 DEGREES OF FREEDOM**

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\phi_{XY-XX}, \phi_{YY-XX}$$

**COHERENCY MATRIX [*T*]**

**9 HUYNEN REAL PARAMETERS**

**(*A0, B0, B, C, D, E, F, G, H*)**

**TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »**

**=**

**5**

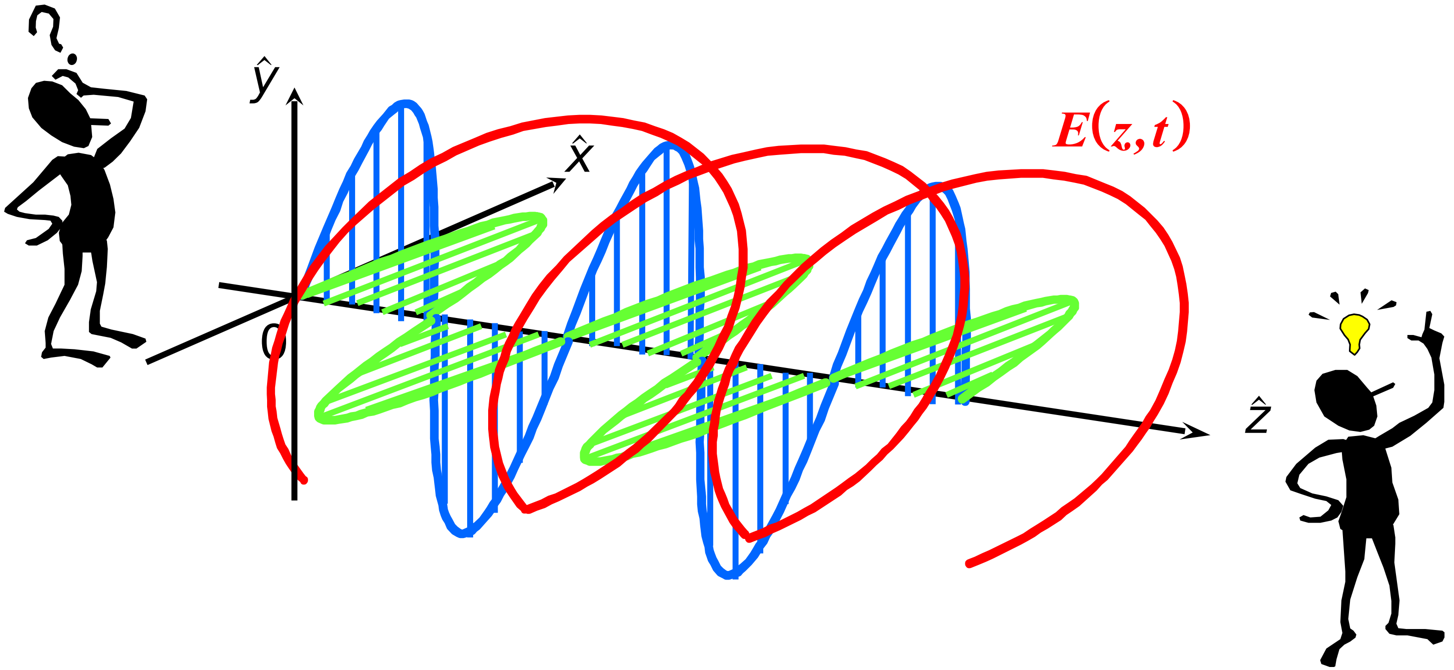
**9 - 5 = 4 TARGET EQUATIONS**

$$2A_0(B_0 + B) = C^2 + D^2$$

$$2A_0(B_0 - B) = G^2 + H^2$$

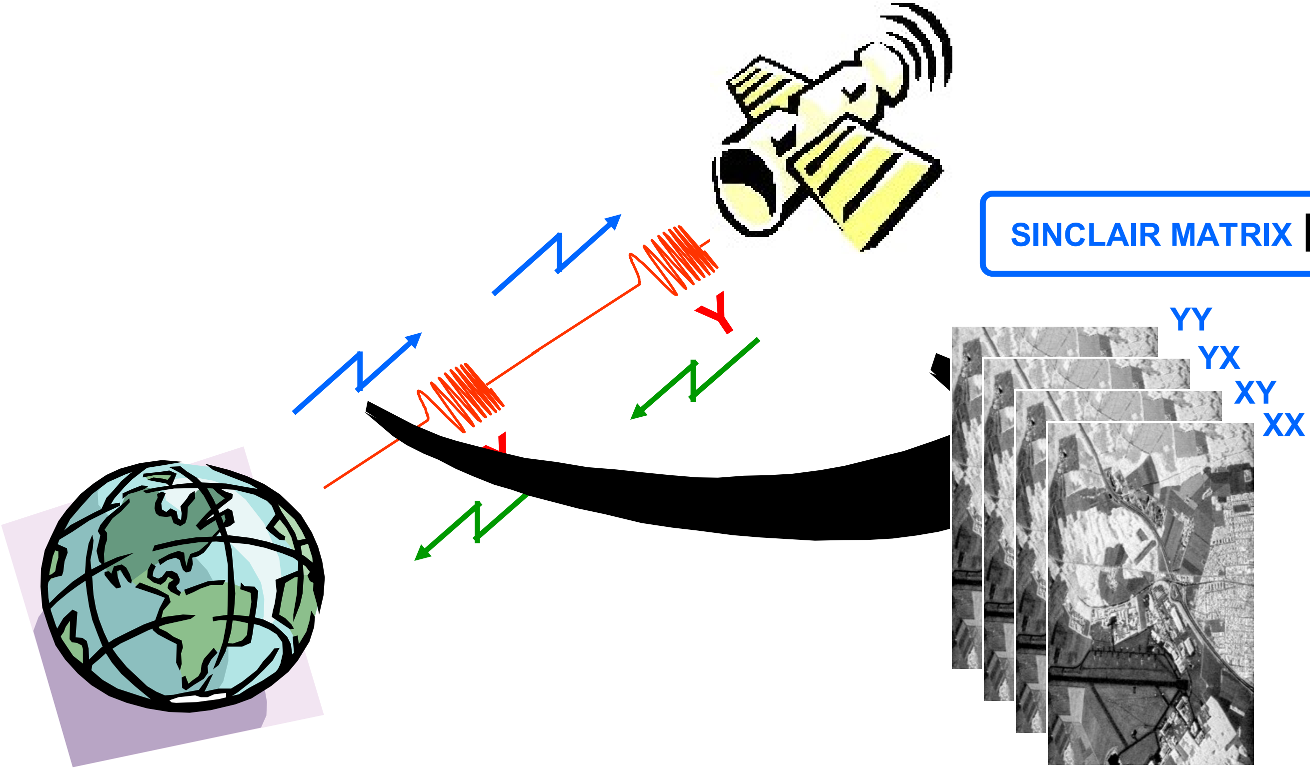
$$2A_0E = CH - DG$$

$$2A_0F = CG + DH$$




# POLARIMETRIC REMOTE SENSING

# SCATTERING POLARIMETRY




**SINCLAIR MATRIX [S]**

**THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS**

| | |
|---|---|
| [S] | SINCLAIR Matrix |
| k, Ω | Target Vectors |
| [K] | KENNAUGH Matrix |
| [T] | Coherency Matrix |
| [C] | Covariance Matrix |

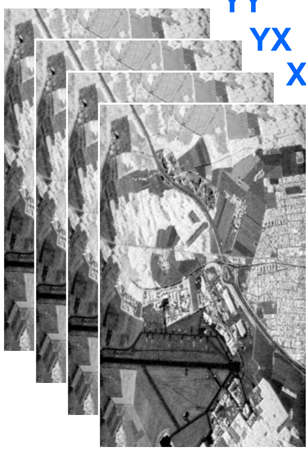
**TRANSMITTER: X & Y**
**RECEIVERS: X & Y**

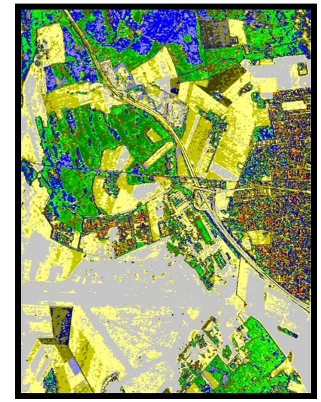
**POLARIMETRIC REMOTE SENSING**

# POLARIMETRIC REMOTE SENSING

**POL-SAR PROCESSING**
PHENOMENOLOGIC
QUALITATIVE ANALYSIS

YY
YX
XY
XX

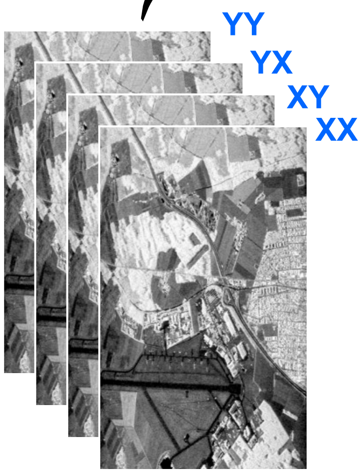


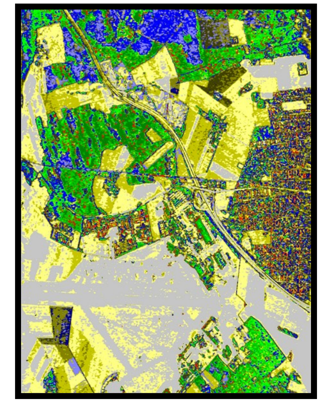


**POLARIMETRIC SPECKLE FILTERING** → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION** MONO/DUAL CHANNELS

# POLARIMETRIC REMOTE SENSING

**POL-SAR PROCESSING**
**PHENOMENOLOGIC**
**QUALITATIVE ANALYSIS**



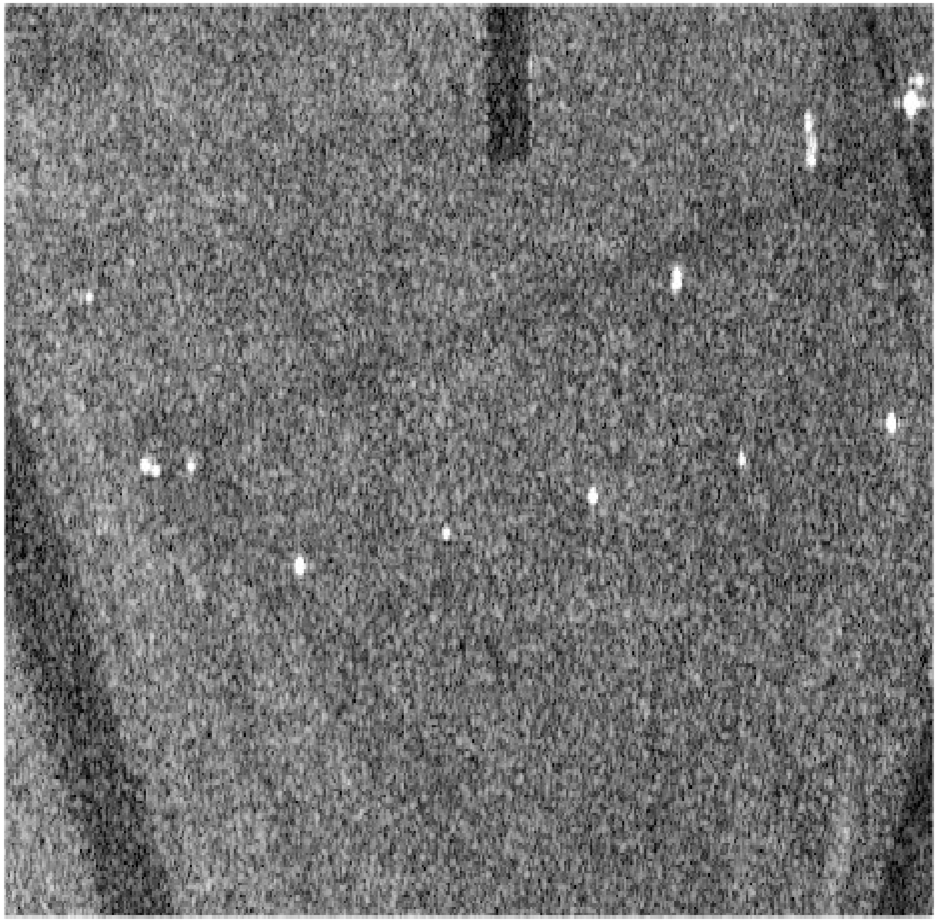



**POLARIMETRIC SPECKLE FILTERING**

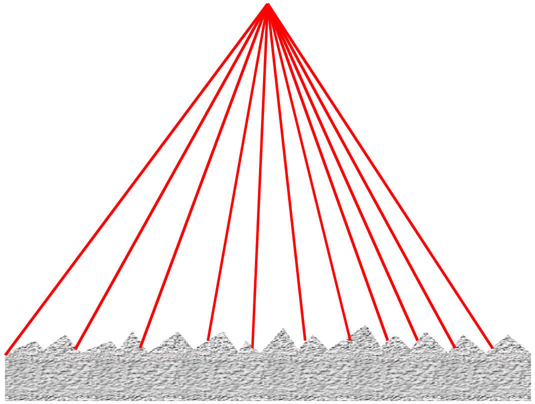
**POLARIMETRIC TARGET DECOMPOSITION**

**POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

# SPECKLE PHENOMENON



OBSERVATION POINT



SURFACE ROUGHNESS
WAVELENGTH

SCATTERING FROM DISTRIBUTED SCATTERERS

⬇

COHERENT INTERFERENCES OF WAVES SCATTERED FROM MANY RANDOMLY DISTRIBUTED ELEMENTARY SCATTERERS INSIDE THE RESOLUTION CELL

⬇

GRANULAR NOISE

⬇

SPECKLE PHENOMENON

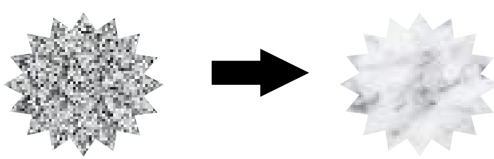
# SPECKLE FILTERING

SPECKLE PHENOMENON

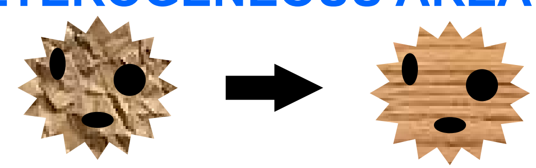
DISTORTION OF THE INTERPRETATION

SPECKLE FILTERING

HOMOGENEOUS AREA



HETEROGENEOUS AREA



SPECKLE REDUCTION
(RADIOMETRIC RESOLUTION)

DETAILS PRESERVATION
(SPATIAL RESOLUTION)

# POLSAR SPECKLE FILTERING

- Preserving polarimetric properties
  - Filter all elements equally like multi-look Processing
  - Select pixels with the same scattering property
- Introduce no cross-talk
  - Filter each element separately but equally
- Reduce speckle while preserving image quality

J.S. Lee, M.R. Grunes and G. De Grandi, "Polarimetric SAR Speckle Filtering and Its Impact on Terrain Classification" *IEEE TGRS*, September 1999

## POLARIMETRIC VECTORIAL SPECKLE FILTER

$$\langle [T] \rangle = \frac{1}{N} \sum_{i=1}^{N} \underline{k}_i \underline{k}_i^{*T}$$

$$\left[\hat{T}\right] = E\left([T]\right) - k\left[E\left([T]\right) - [T]\right] \rightarrow \left[\hat{T}\right]$$

**SPAN IMAGE**

$$S_S = \langle T_{11} \rangle + \langle T_{22} \rangle + \langle T_{33} \rangle$$

**LINEAR SCALAR LEE FILTER**

$$\hat{S} = E(S_S) - k\left[E(S_S) - S_S\right]$$

$$k = \frac{var(S)}{var(S_S)} = \frac{CV_{S_S}^2 - \sigma_v^2}{CV_{S_S}^2 \left[1 + \sigma_v^2\right]}$$

J.S. LEE

**Homogeneous Areas**

$$var(S) \approx 0 \;\Rightarrow\; k = 0 \;\Rightarrow\; \hat{S} = E(S_S)$$

**Highly Inhomogeneous Areas**

$$var(S) \mapsto var(S_s) \;\Rightarrow\; k = 1 \;\Rightarrow\; \hat{S} = S_S$$

## REFINED FILTER

# SPECKLE FILTERING

$$[T] = \underline{k}\,\underline{k}^{*T}$$

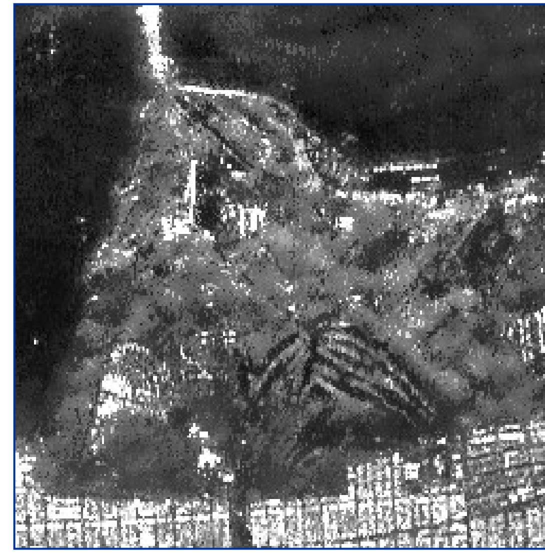
SPECKLE FILTER

**AVERAGING DATA**

**SECOND ORDER STATISTICS**

**COHERENCY MATRICES**

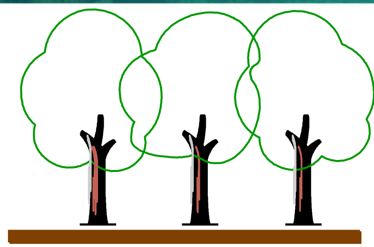
**SMOOTHING AVERAGING**



$$\langle [T] \rangle = \frac{1}{N} \sum_{i=1}^{N} \underline{k}_i \underline{k}_i^{*T}$$

**CONCEPT OF THE DISTRIBUTED TARGET**

# TARGET DECOMPOSITIONS



**PURE TARGET**

**POLARIMETRIC DISTRIBUTED TARGET « *DIMENSION* » = 5**

**COHERENCY MATRIX [*T*]**

**9 REAL DEPENDANT HUYNEN PARAMETERS (A0,B0,B,C,D,E,F,G,H)**

**9 - 5 = 4 TARGET EQUATIONS**

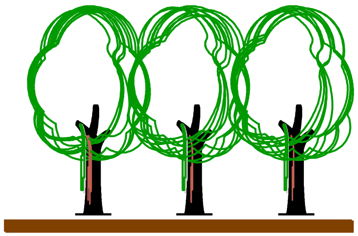
$$2A_0(B_0+B) = C^2 + D^2$$

$$2A_0(B_0-B) = G^2 + H^2$$

$$2A_0E = CH - DG$$

$$2A_0F = CG + DH$$

# TARGET DECOMPOSITIONS



**DISTRIBUTED TARGET**

**POLARIMETRIC DISTRIBUTED TARGET « *DIMENSION* » = 9**

**COHERENCY MATRIX <[*T*]>**

**9 REAL INDEPENDANT HUYNEN PARAMETERS**

**(<A0>,<B0>,<B>,<C>,<D>,<E>,<F>,<G>,<H>)**

## 9 TARGET INEQUATIONS

$$2\langle A_0\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle^2+\langle D\rangle^2$$

$$2\langle A_0\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle G\rangle^2+\langle H\rangle^2$$

$$2\langle A_0\rangle\langle E\rangle\geq\langle C\rangle\langle H\rangle-\langle D\rangle\langle G\rangle$$

$$2\langle A_0\rangle\langle F\rangle\geq\langle C\rangle\langle G\rangle+\langle D\rangle\langle H\rangle$$

$$\langle B_0\rangle^2\geq\langle B\rangle^2+\langle E\rangle^2+\langle F\rangle^2$$

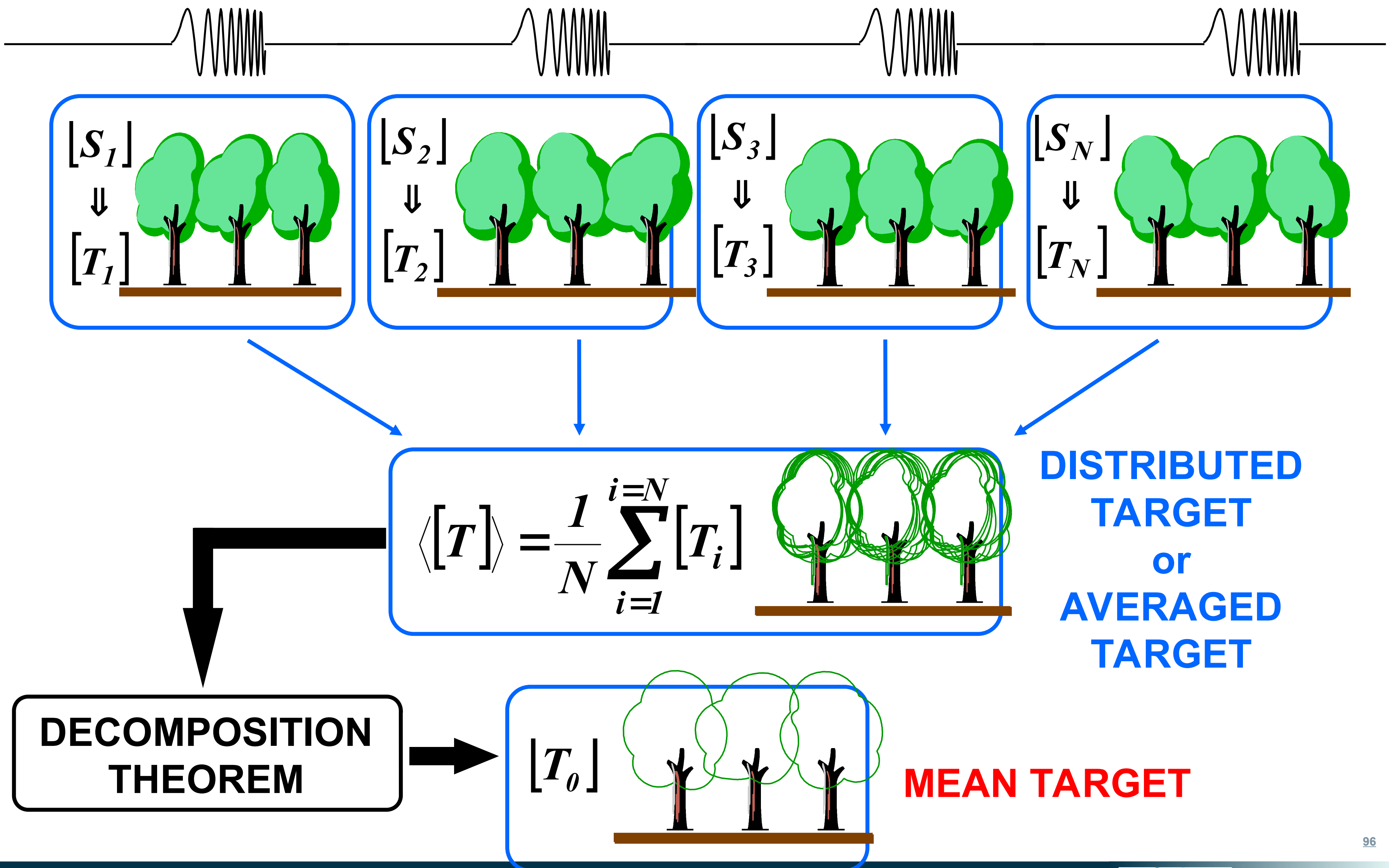
$$\langle H\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle\langle E\rangle+\langle D\rangle\langle F\rangle$$

$$\langle G\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle\langle F\rangle-\langle D\rangle\langle E\rangle$$

$$\langle C\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle H\rangle\langle E\rangle+\langle F\rangle\langle G\rangle$$

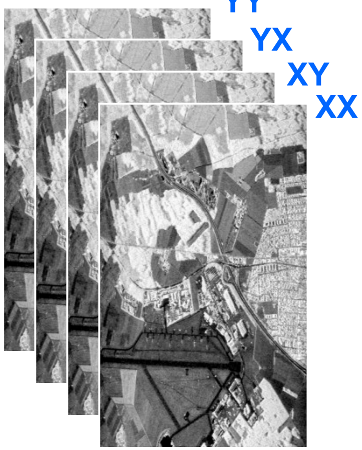
$$\langle D\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle F\rangle\langle H\rangle-\langle G\rangle\langle E\rangle$$

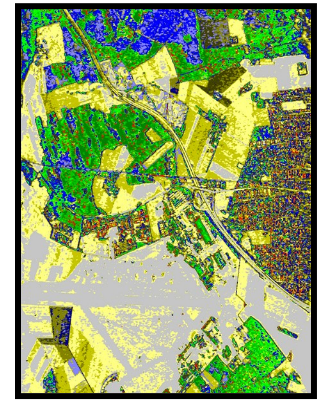
# TARGET DECOMPOSITIONS



# POLARIMETRIC REMOTE SENSING

**POL-SAR PROCESSING**
**PHENOMENOLOGIC**
**QUALITATIVE ANALYSIS**



**POLARIMETRIC SPECKLE FILTERING** → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

# TARGET DECOMPOSITIONS

[*S* ]

COHERENT DECOMPOSITION

E. KROGAGER (1990)

W.L. CAMERON (1990)

[*K* ]

TARGET DICHOTOMY

J.R. HUYNEN (1970)

R.M. BARNES (1988)

[*T* ]

[*C* ]

AZIMUTHAL SYMMETRY

EIGENVECTORS BASED DECOMPOSITION

MODEL BASED DECOMPOSITION

S.R. CLOUDE (1985)

W.A. HOLM (1988)

A.J. FREEMAN – S.L. DURDEN (1992)
Y. YAMAGUSHI (2005 - 2012), AN (2010)

EIGENVECTORS / EIGENVALUES ANALYSIS
&
MODEL BASED DECOMPOSITION

J.J. VAN ZYL (1992-2008), M. ARII (2010)
TSVM (R. TOUZI – 2007)

EIGENVECTORS / EIGENVALUES ANALYSIS
ENTROPY / ANISOTROPY

S.R. CLOUDE - E. POTTIER (1996-1997)