# 7th Advanced Training Course on Radar Polarimetry

Toulouse, 2023

## *SAR BASICS & SAR TOMOGRAPHY THEORY*

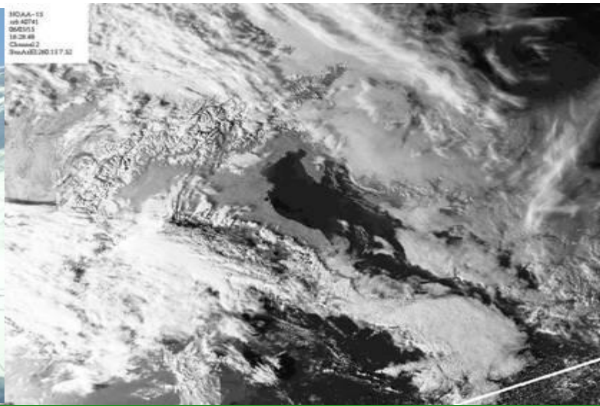
**Stefano Tebaldini**
Politecnico di Milano

# Radar Imaging

RADAR (***Radio Detection And Ranging***) is a technology to detect and study far off targets by transmitting EM pulses at radiofrequency and observing the backscattered echoes

***Some relevant features:***

1. **Active instrument: ⇔ *no need for external illumination source***
2. **Delay-based measurement ⇔ *target distance is obtained based on pulse two-way travel time***
3. **Microwaves penetrate through rain and clouds ⇔ *visibility in all weather conditions***
4. **Microwaves can penetrate into some natural media, like forests, snow, ice, sand ⇔ *sensitivity to the 3D structure of illuminated media***

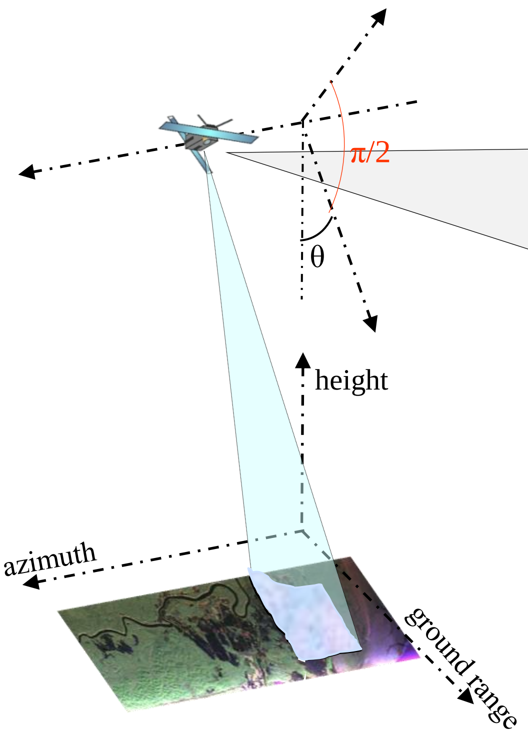


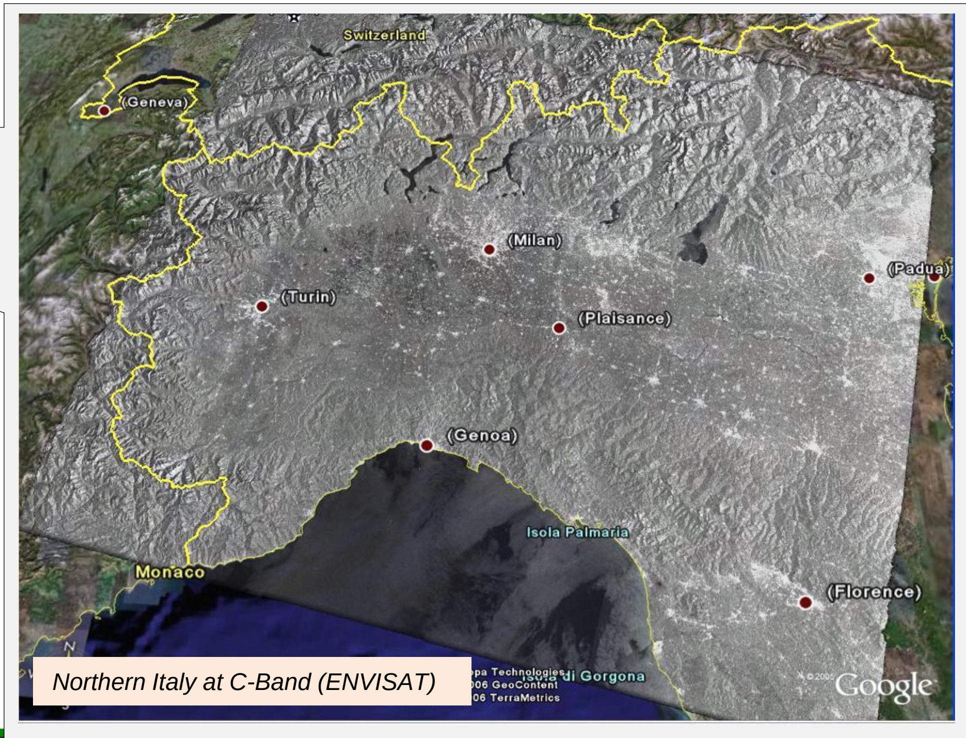
# SAR Imaging

SAR systems employ a RADAR sensor flown onboard a satellite platform to synthesize an antenna aperture as long as several kilometers

- Accurate measurement of Radar echoes backscattered from the targets as the system is flown along the satellite trajectory
- Image formation by Digital Processing techniques

⇒ The result is a high resolution **two-dimensional** map of the imaged scene



*Northern Italy at C-Band (ENVISAT)*

# SAR Imaging

*Key features:*

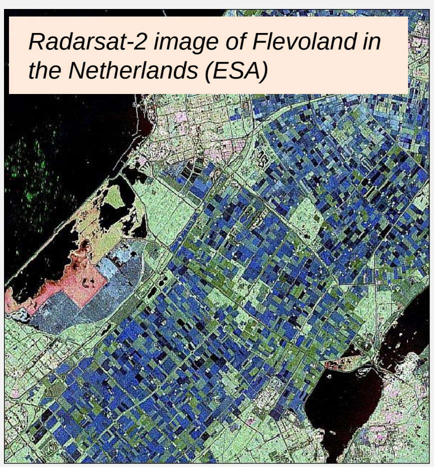
- Microwaves penetrate through rain and clouds ⇔ *visibility in all weather conditions*
- Aperture Synthesis⇔ *fine spatial resolution*
- Phase preserving ⇔ *millimeter accuracy about distance variations*

⇨ Spaceborne SARs provide *accurate* and *continuous* information about the Earth's surface and its evolution over time

***Topographic mapping***

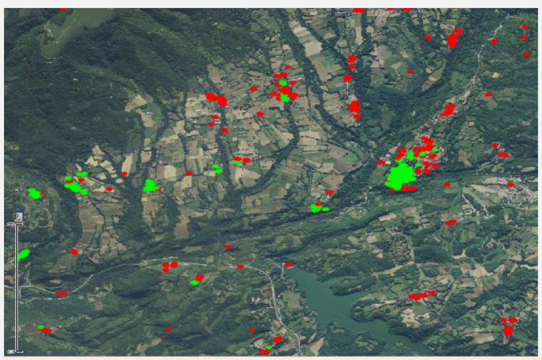
*DEM of Mount Etna, Sicily, derived from ERS-1 (ESA)*

***Land mapping***



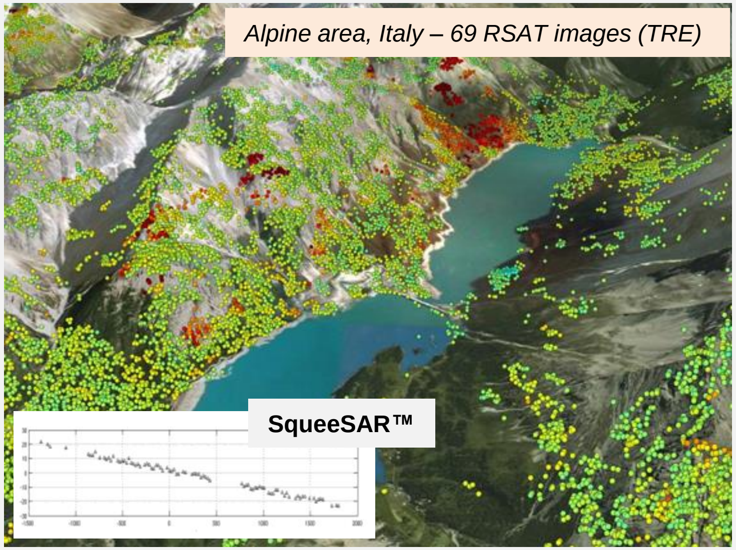
*Radarsat-2 image of Flevoland in the Netherlands (ESA)*

***Change detection***



*Post-earthquake change detection map in Amatrice, Italy, derived from Sentinel-1A (PoliMi)*

***Deformation monitoring***

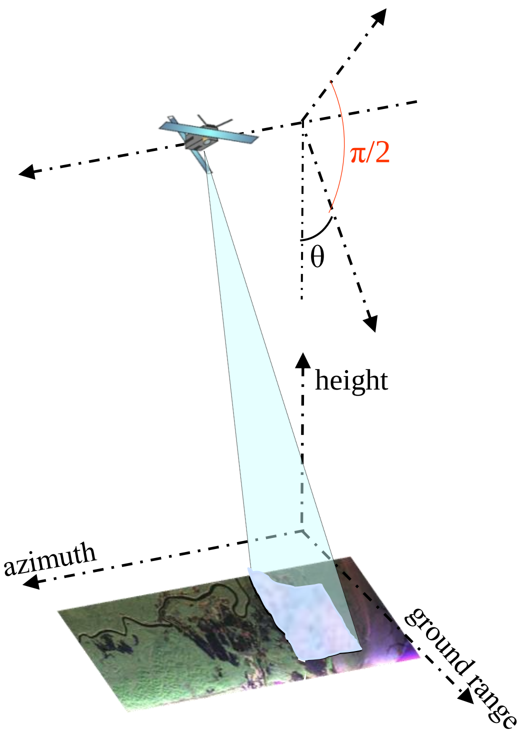


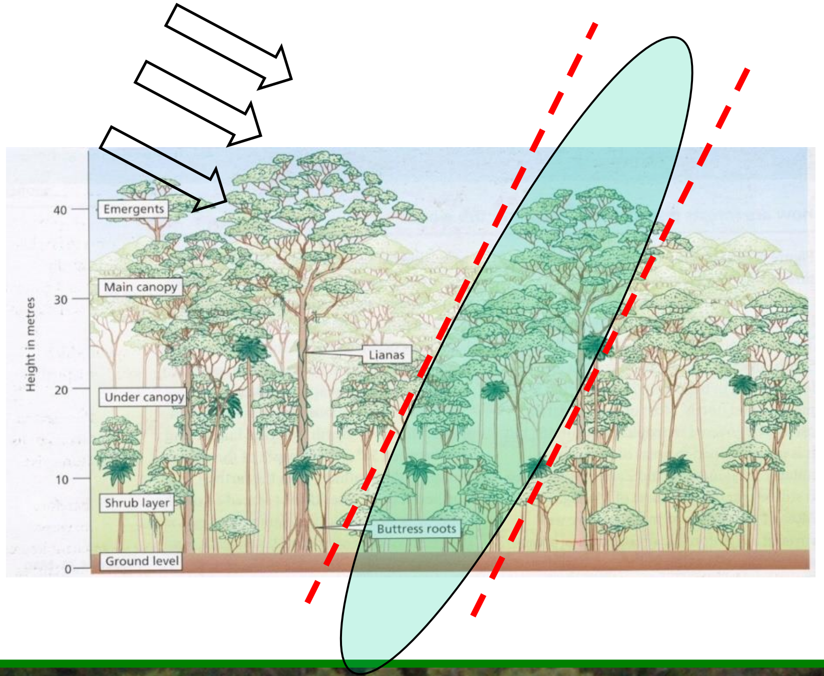

*Alpine area, Italy – 69 RSAT images (TRE)*

*Another key feature:*

- Microwaves **penetrate** into natural media, like forests, snow, ice, sand ⇔ *sensitivity to the three-dimensional structure of illuminated media*

⇒ A single pixel within a SAR image is actually a mixture of different scattering mechanisms distributed over height





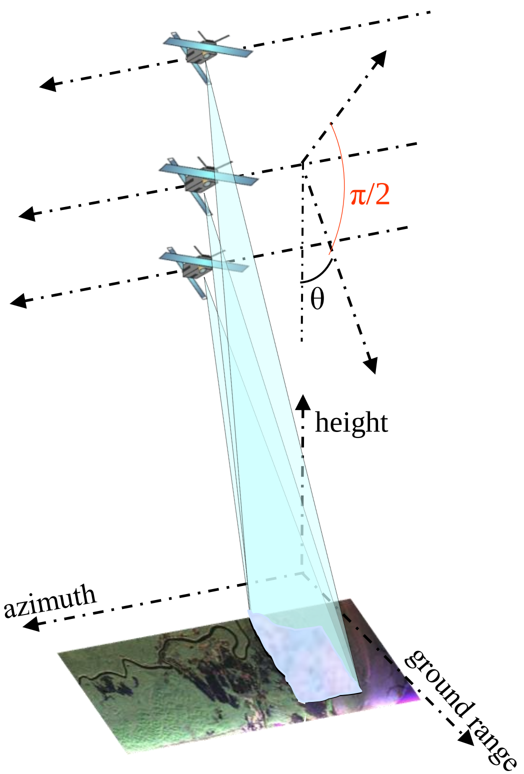
*Stefano Tebaldini*

# Tomographic SAR Imaging

TomoSAR systems employ a RADAR sensor flown along **multiple** trajectories

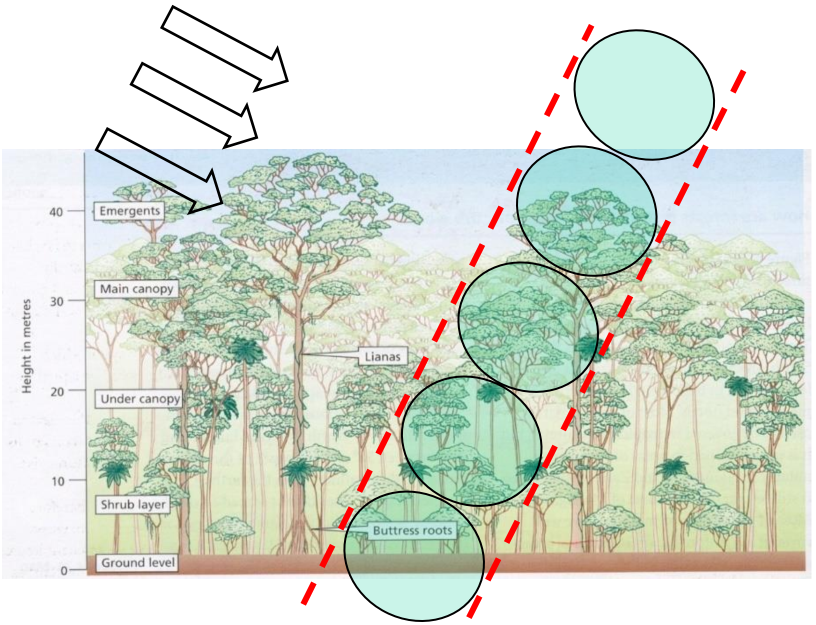
- Image formation by Digital Processing techniques

⇒ ***Three dimensional representation*** of Radar intensity at a given wavelength




***SAR produces pixels***

***TomoSAR produces voxels !!!***



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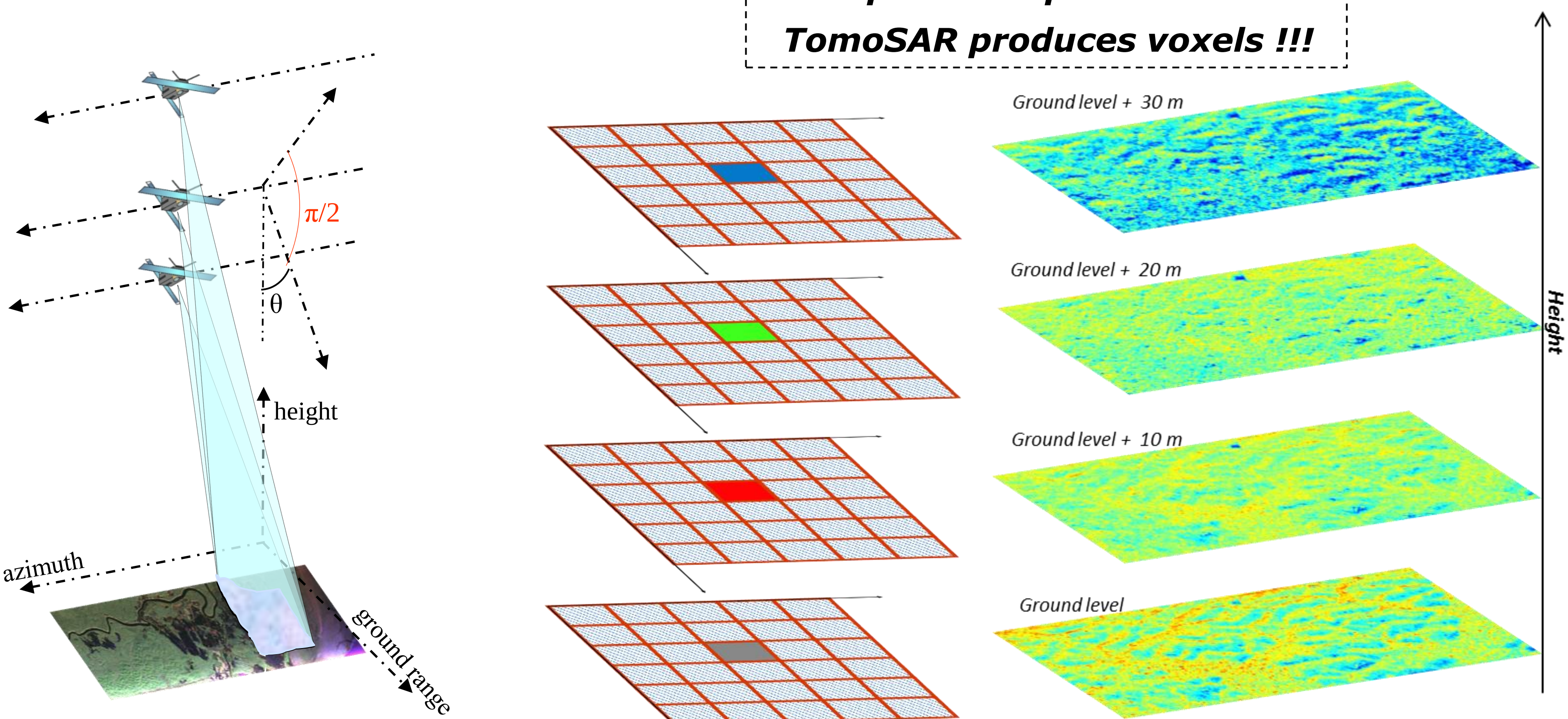
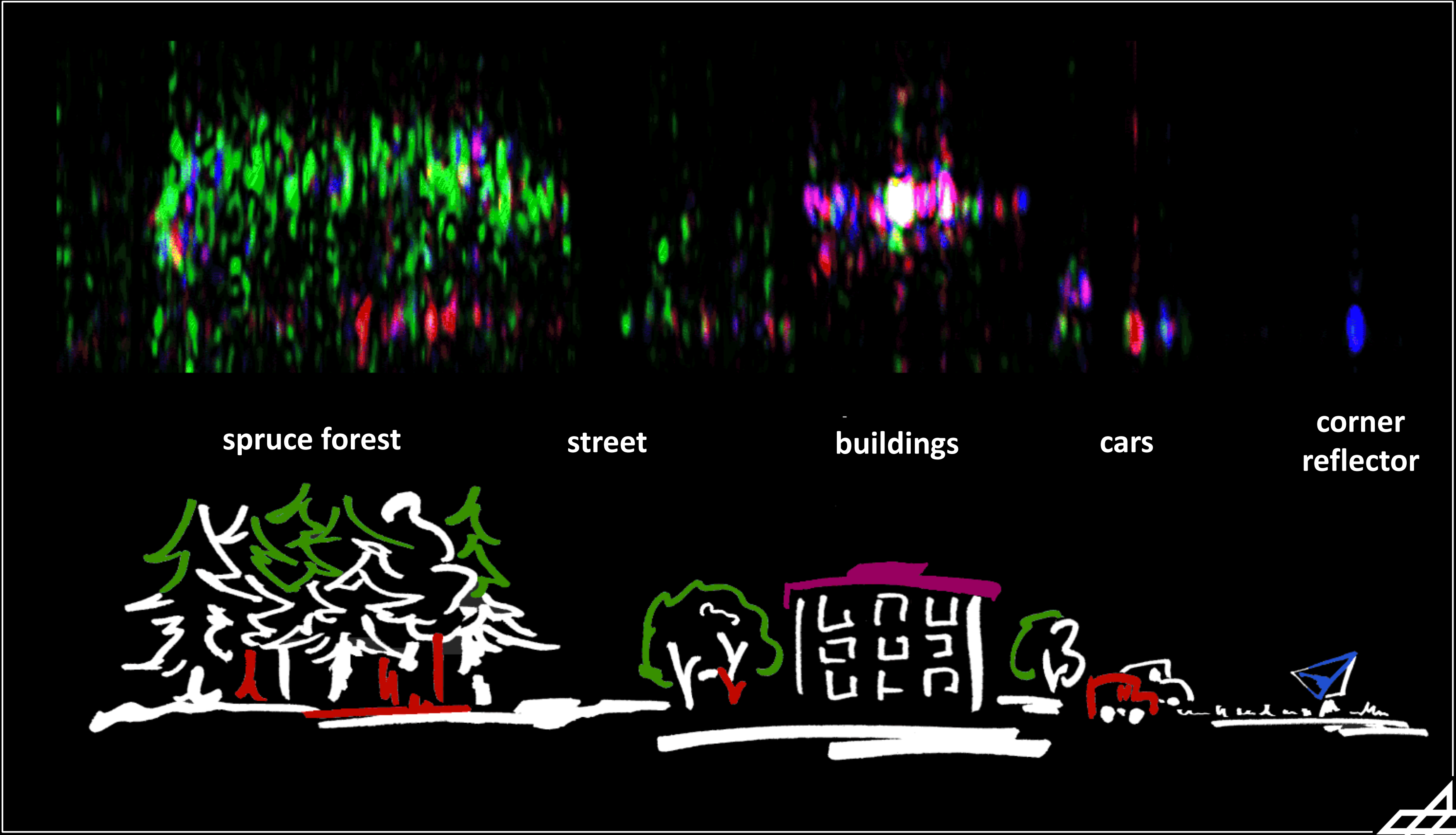


# *2000: First airborne demonstration* (Reigber & Moreira, TGRS)




Top: Polarimetric colour composite of tomographic slice in azimuth/height

HH+VV, HH-VV, 2*HV

Bottom: Schematic representation of the imaged slice

DLR

# Tomographic SAR Imaging

**TomoSAR is today an emerging remote sensing technology for imaging the interior structure of natural media from above by using electromagnetic (EM) waves**

**Timeline**

- Mid 90's: principle formulated (Knaell and Cardillo) & first experiment (Pasquali et al, Fortuny et al)
- 2000: First airborne demonstration (Reigber and Moreira, TGRS)
- 2007 – today: experimentation by Space Agencies (ESA, DLR, JPL) in the context of airborne and ground based campaigns, in view of future spaceborne applications on:
  - ✓ Forests
  - ✓ Agriculture
  - ✓ Ice sheets/glaciers
  - ✓ Snow
- 2024: launch date of the ESA P-Band Mission **BIOMASS** – global tomographic coverage of forested areas
- *near future* (?): spaceborne L-, C-, X-Band Tomography by future bistatic SAR systems

# TomoSAR & Forested areas

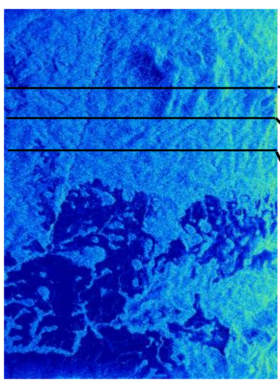
***Forest scenarios****: separation of backscatter from different heights within the vegetation*

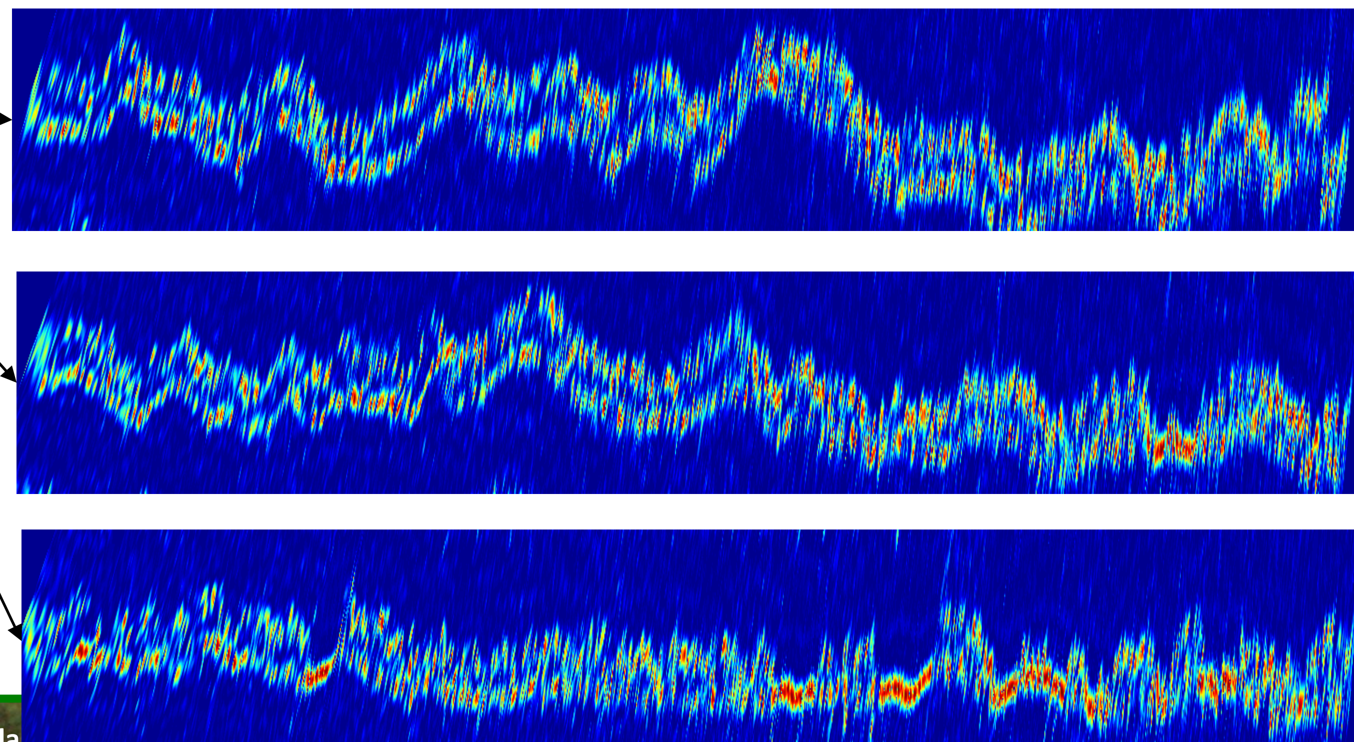
⇒ *Forest height*

⇒ *Sub-canopy terrain topography*

⇒ *Classification of forest structure*

⇒ *Improved forest biomass retrieval*





***Tomographic data from AfriSAR 2016 (ESA)***

*Site: Gabon*

*Acquisition by DLR & ONERA*

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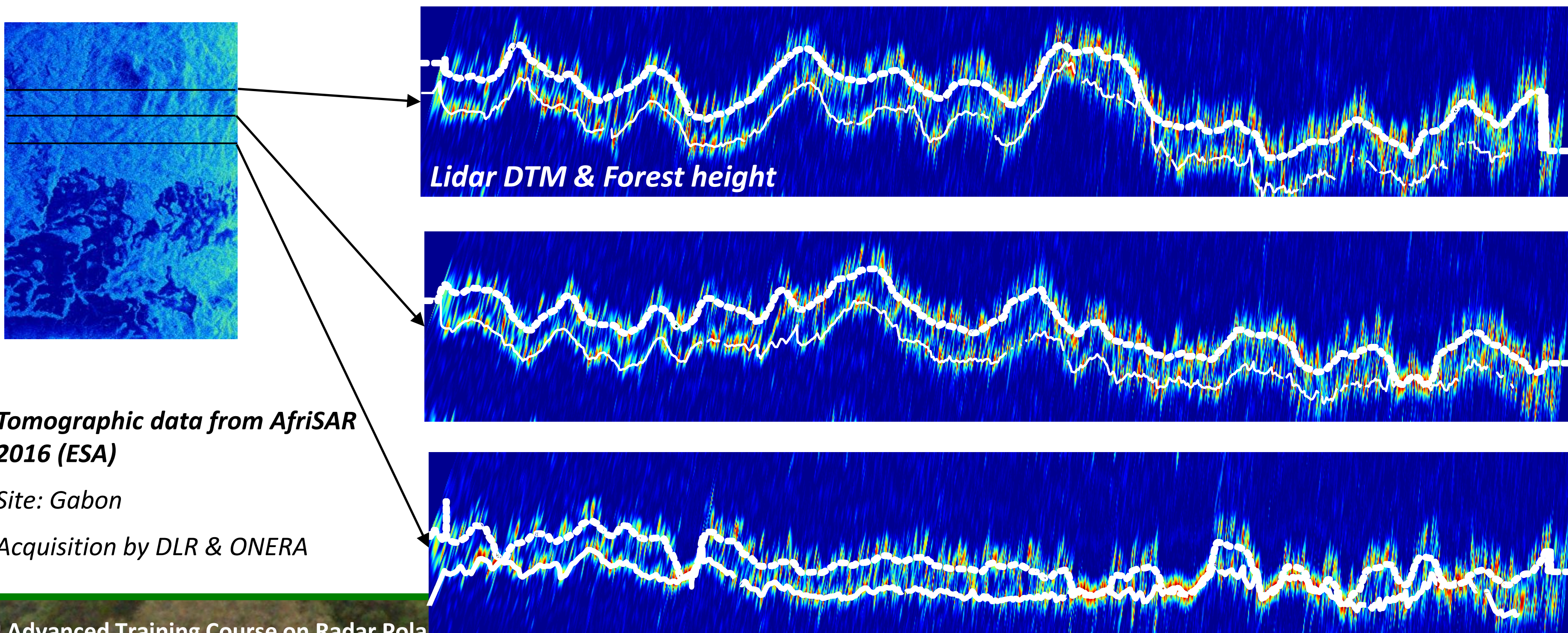


***Tomographic data from AfriSAR 2016 (ESA)***

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# TomoSAR & Forested areas

## *Forest height*

*Site: Lopé, Gabon*

*Data-set: AfriSAR (ESA)*

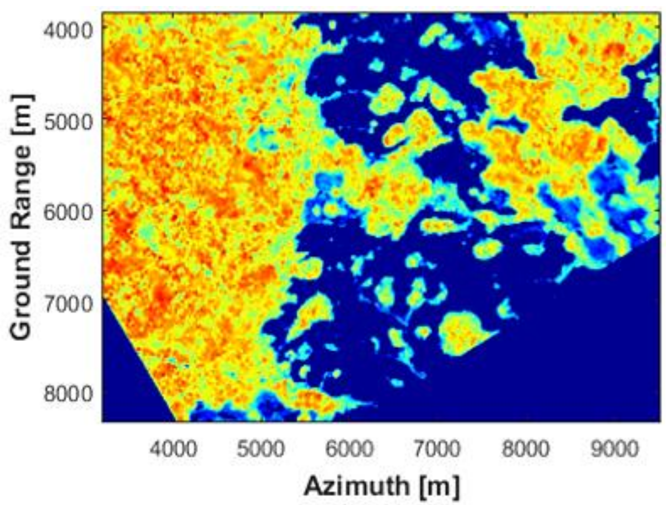
*Frequency: P-Band*

*$\sigma_{SAR\text{-}LIDAR} \approx 3$ m*

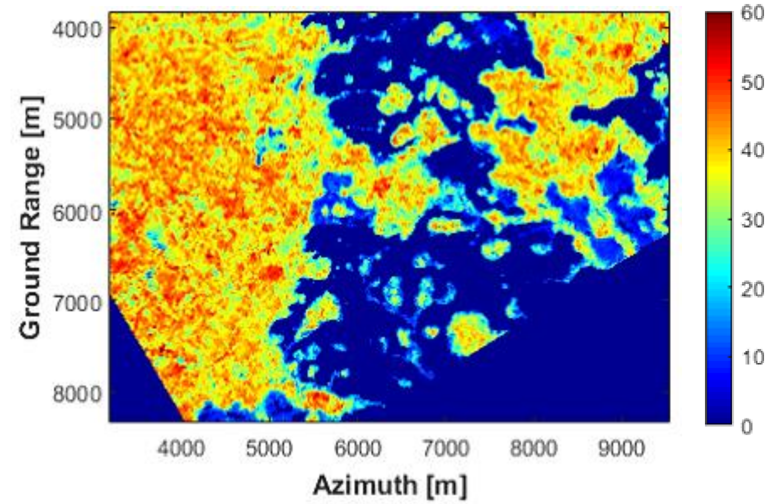
*@ 25 m*

***Yang et al., GRSL, 2020***

**Lidar Canopy height**




**TomoSAR Canopy height**




## *Sub-canopy terrain topography*

*Site: Mondah, Gabon*

*Data-set: AfriSAR (ESA)*

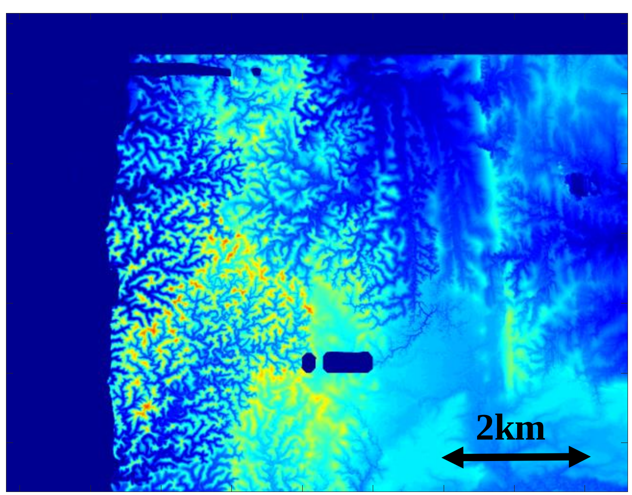
*Frequency: P-Band*

*$\sigma_{SAR\text{-}LIDAR} \approx 2.8$ m*

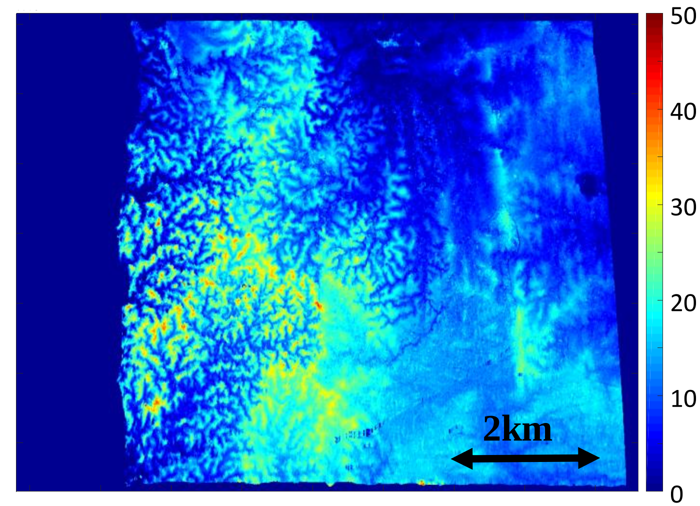
*@ 15 m*

***Mariotti et al., 2019***
***Pardini et al., 2018***
***Wasik et al., 2018***

**Lidar Terrain Model**

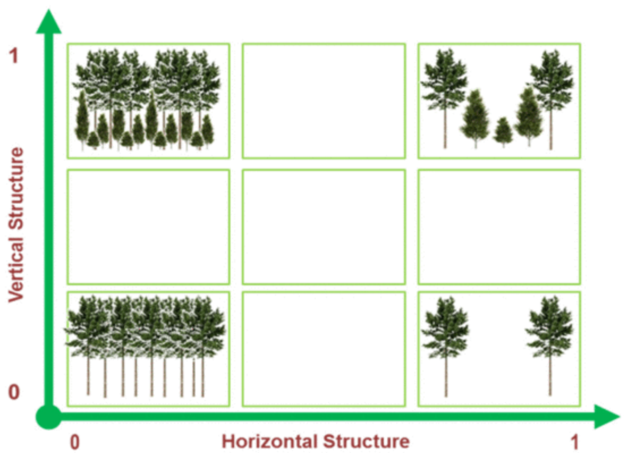


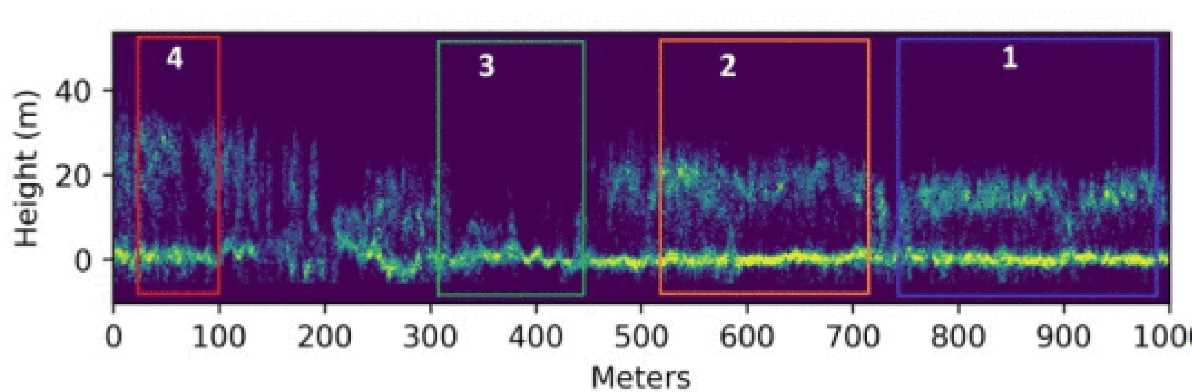

**TomoSAR Terrain Model**

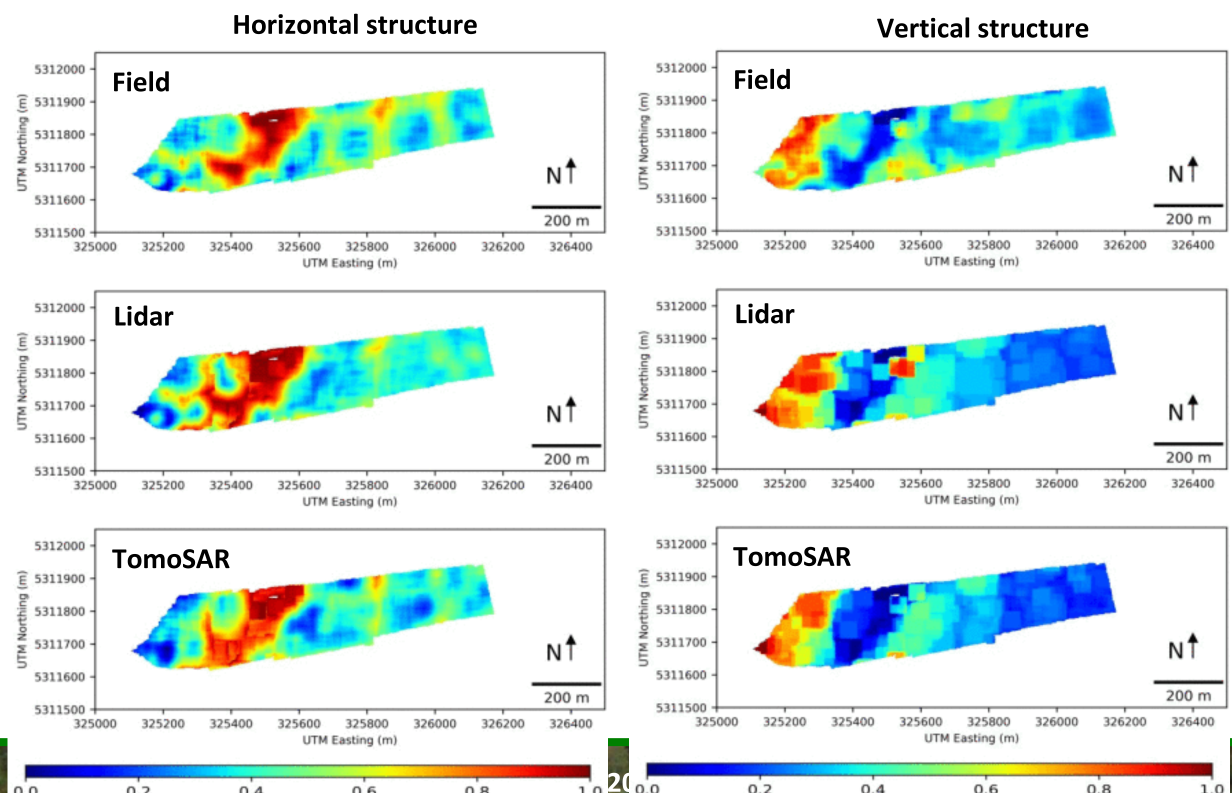


# TomoSAR & Forested areas

***Classification of forest structure***




*Site: Traunstein, Germany*

*Frequency: L-Band*

*Data-set by DLR*

***Tello et al., Journal of Selected Topics in Applied Earth Observations and Remote Sensing, 2018***

# TomoSAR & Forested areas

***Correlation between Radar intensity and Above Ground Biomass (AGB)***

- 2D SAR intensity is poorly correlated to AGB
- TomoSAR intensity at 0 m is poorly and negatively correlated to AGB
- TomoSAR intensity at main canopy height is highly correlated to AGB (≈ 50 Mg/ha per dB)

*Sites: Paracou, Nourages (French Guiana)*

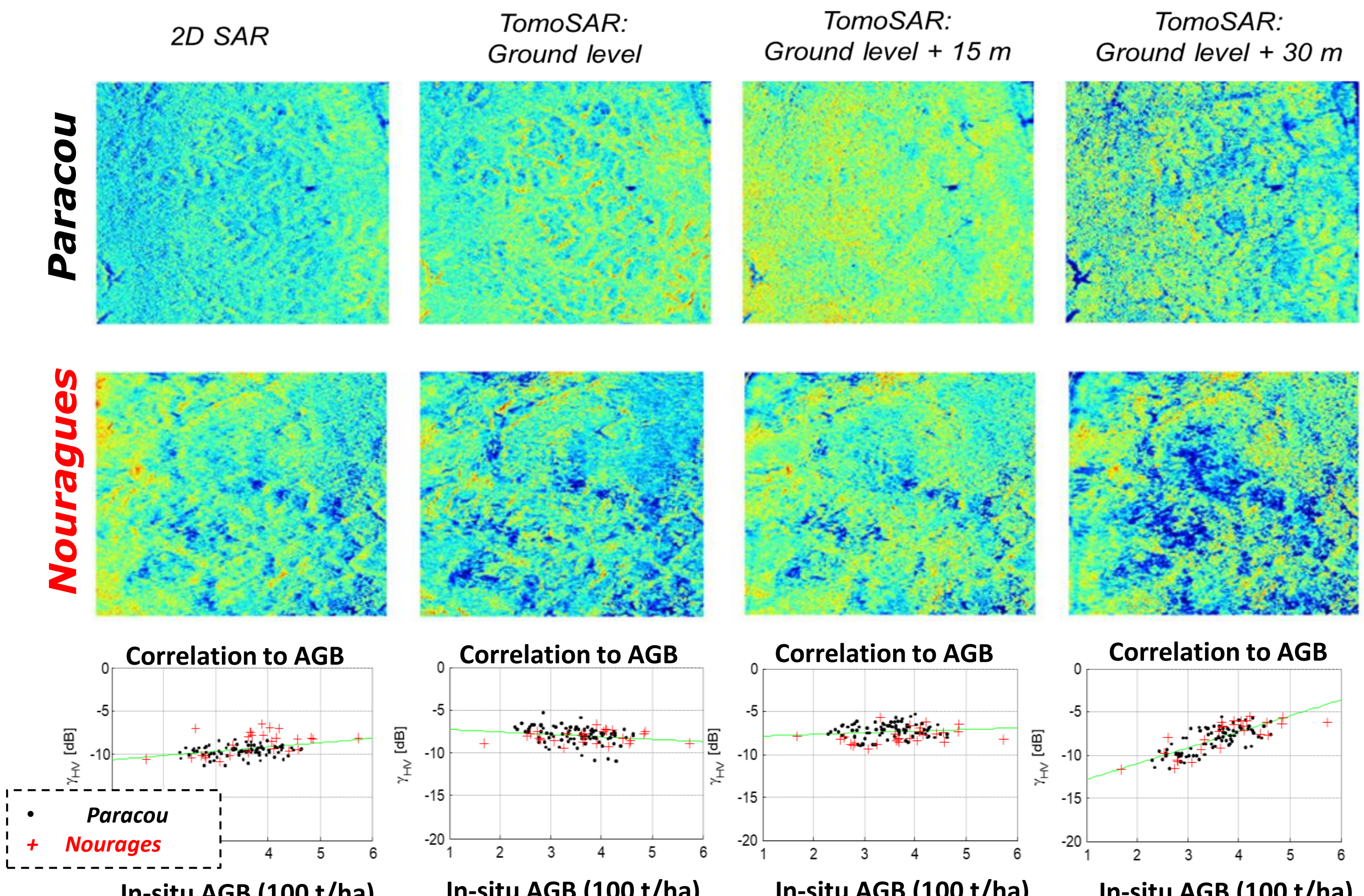
*Frequency: P-Band*

*Data-set: TropiSAR (ESA)*

*Data-set by ONERA*

***Ho Tong Minh et al., TGRS, 2014***

***Ho Tong Minh et al., Remote Sensing of Environment, 2016***



# TomoSAR & Forested areas

Results were confirmed for three African forest sites…..

*Sites: Paracou, Nourages (French Guiana)*

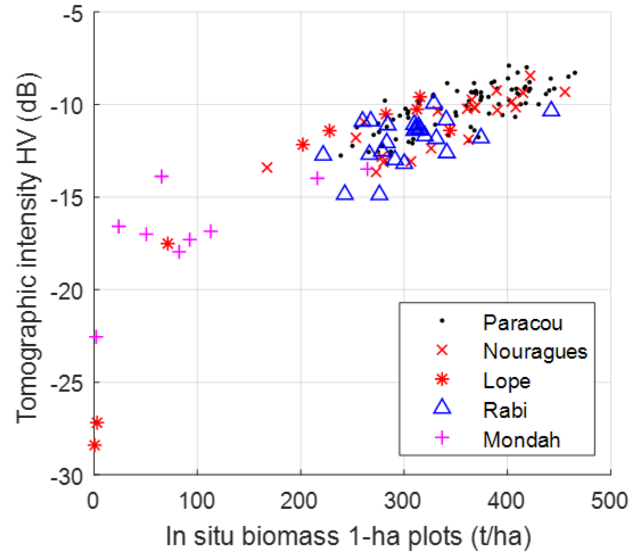
*Lopé, Rabi, Mondah (Gabon)*

*Frequency: P-Band*

*Data-sets: TropiSAR and AfriSAR (ESA)*

*Data-set by ONERA*

***Tebaldini et al., Geophysical Surveys, accepted***




…. and for a boreal site at L-Band

*Site: Krycklan (Sweden)*

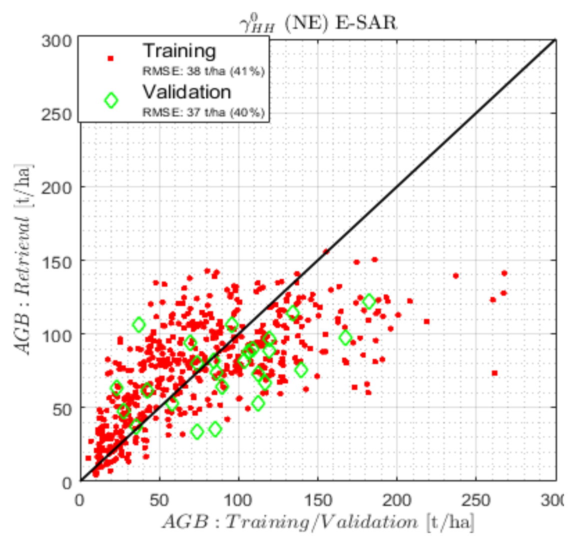
*Frequency: L-Band*

*Data-set: BioSAR 2 (ESA)*

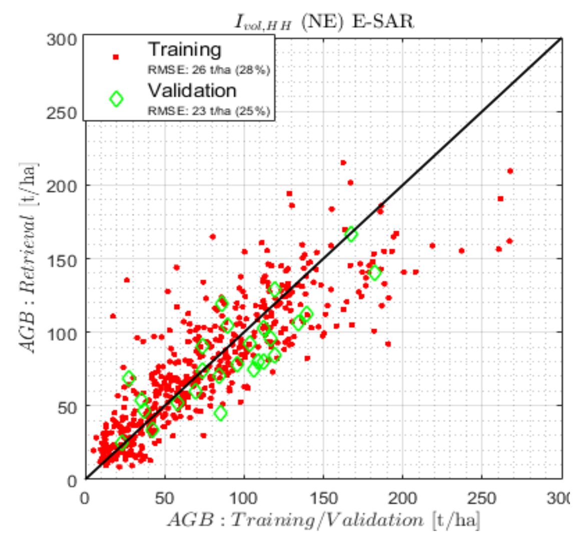
*Data-set by DLR*

***Blomberg et al., GRSL, 2018***

2D SAR




TomoSAR



# TomoSAR & Glaciers/ Ice sheets

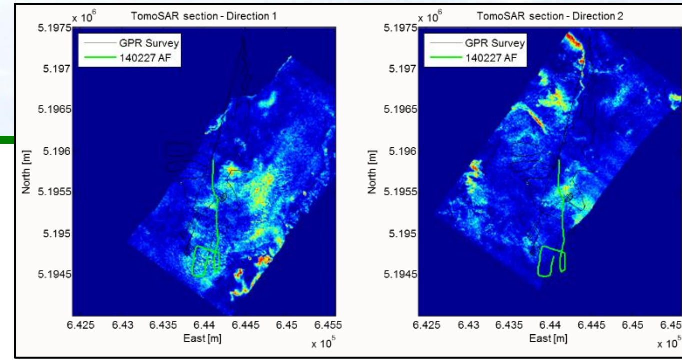
*Glaciers: inside view of the ice body*

⇒ *Bedrock detection below the ice surface*

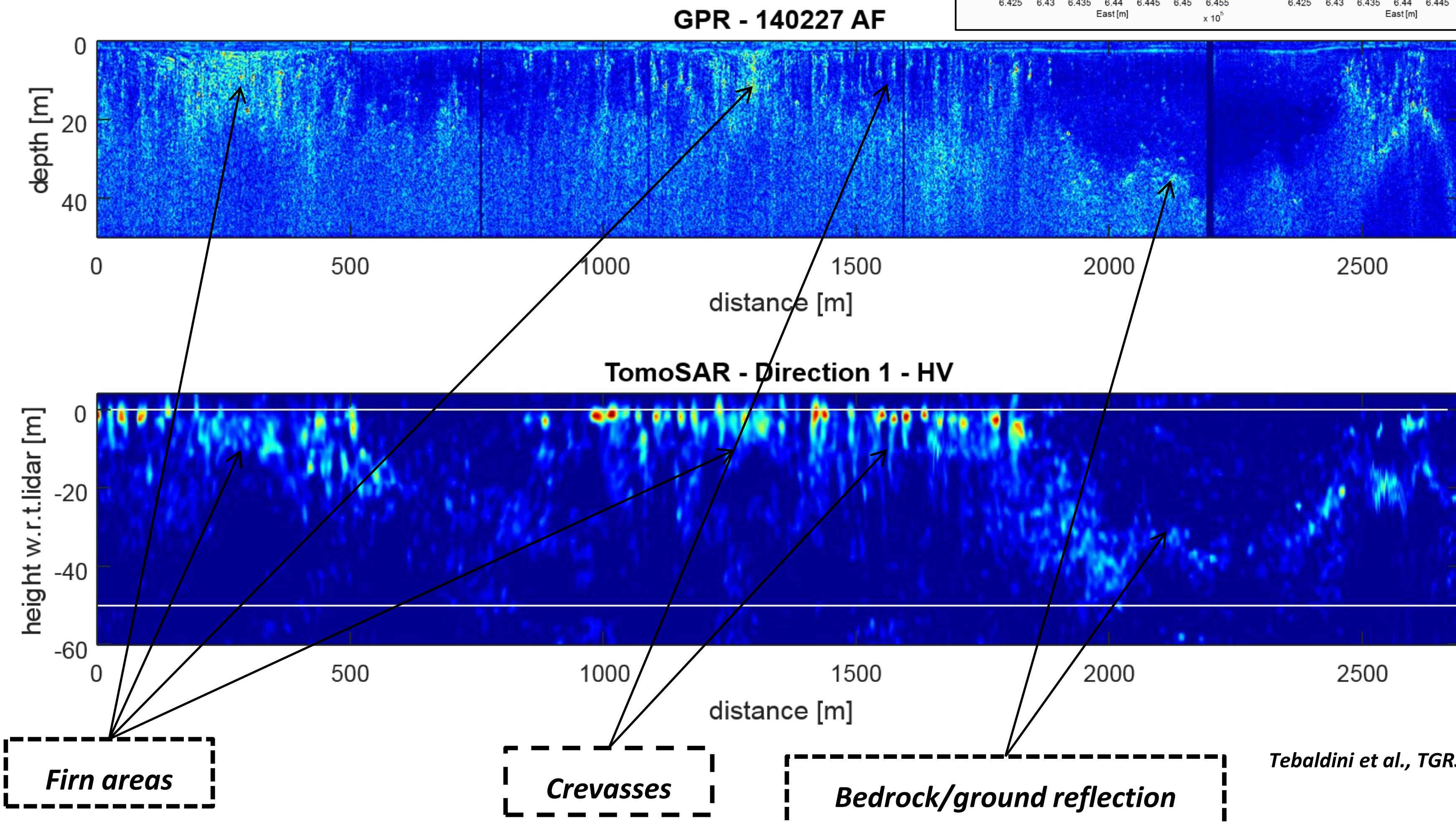
⇒ *Imaging of internal structures*

***Mittelbergferner,***
***Austrian Alps, March 2014***

# TomoSAR & Glaciers/ Ice sheets

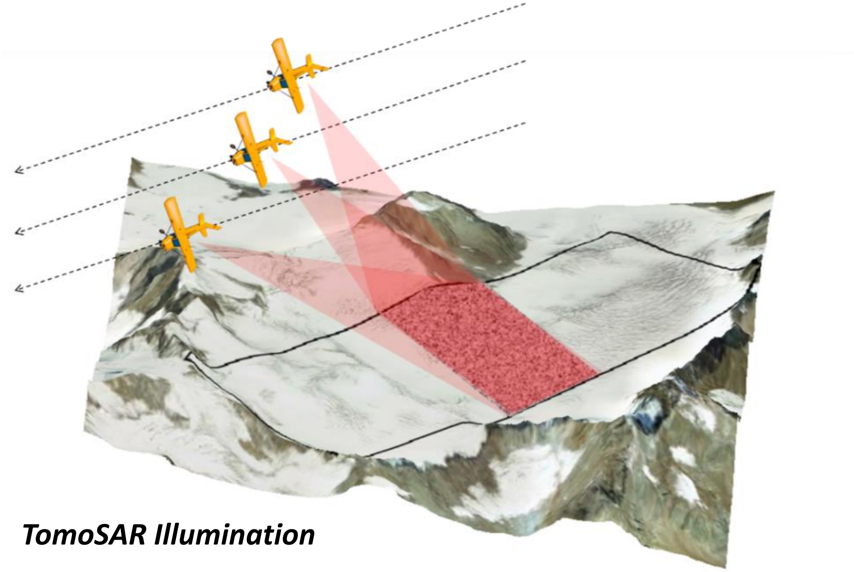



## *Comparison between 200 MHz Ground Penetrating Radar and L-Band TomoSAR*

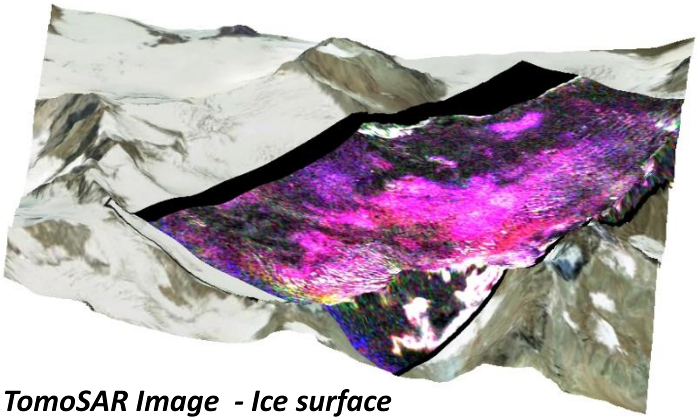



*Tebaldini et al., TGRS, 2016*

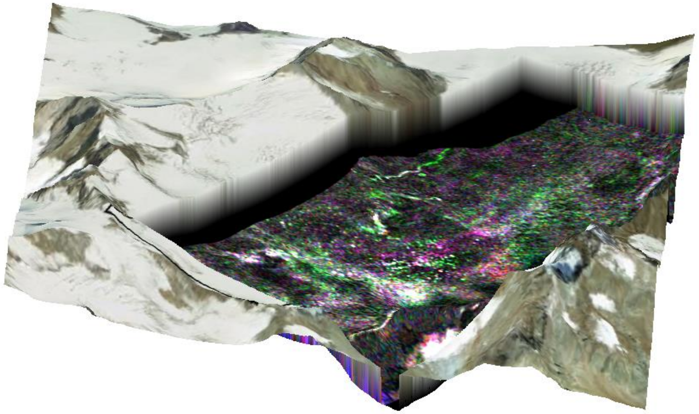
# TomoSAR & Glaciers/ Ice sheets



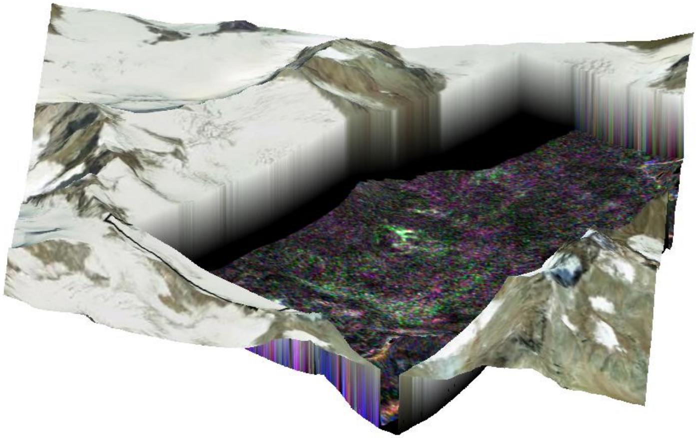
*TomoSAR Illumination*



*TomoSAR Image  - Ice surface*



*TomoSAR Image  - 25 m below the Ice surface*



*TomoSAR Image  - 50 m below the Ice surface*

# The Mittelbergferner @ L-Band

*Tebaldini et al., TGRS, 2016*

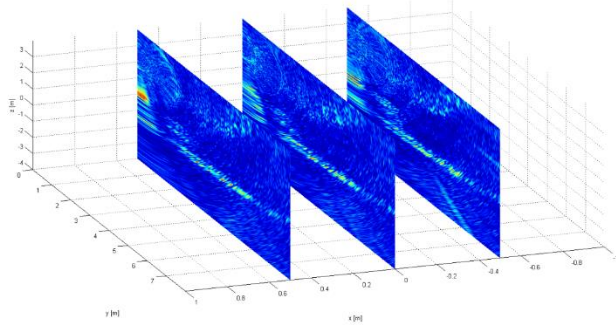
# TomoSAR & Snow

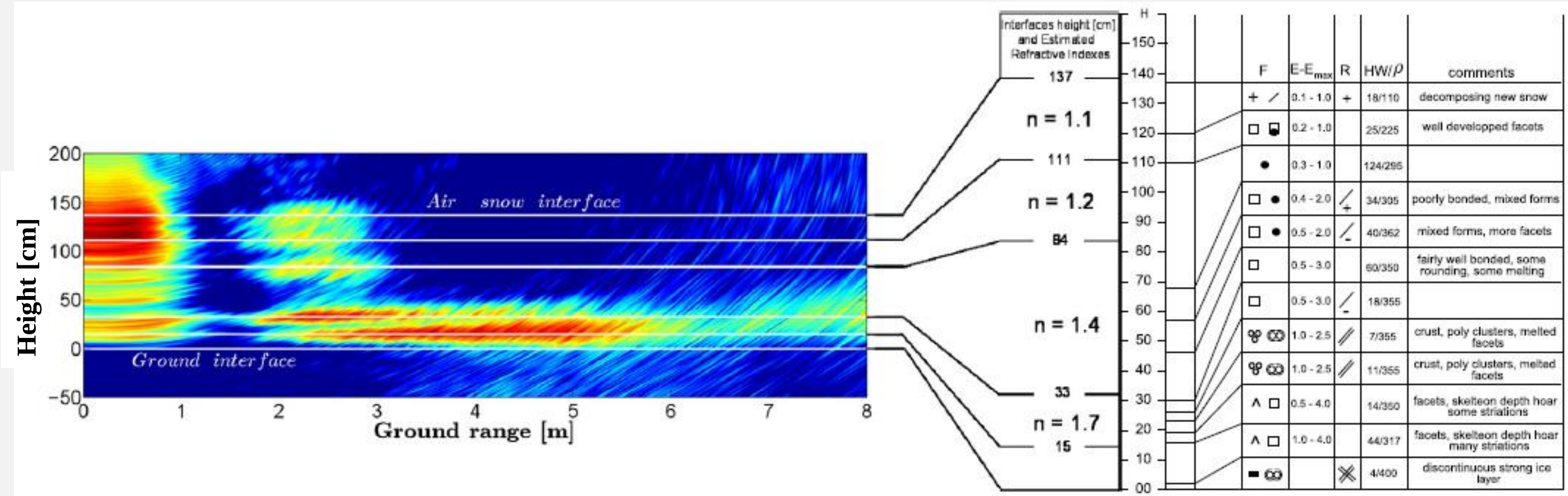
***Snow**: fine structure of snowpack layering*

⇒ *Total Snow depth*

⇒ *Refractive index*

⇒ *Internal layering*

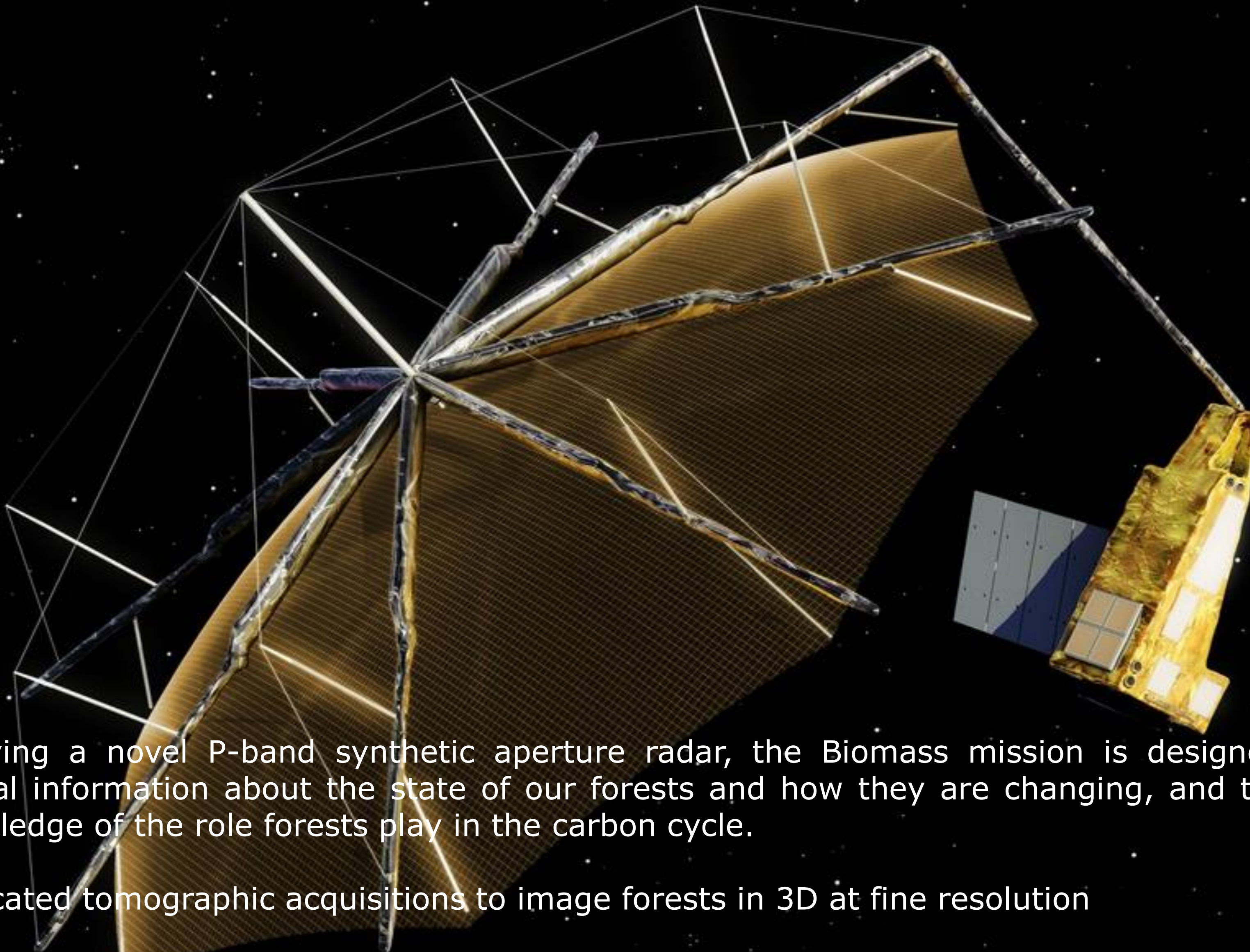





*Data from AlpSAR 2013 (Rennes 1, ESA)*

*Rekioua et al., Comptes Rendus Physique , 2017*

## *BIOMASS*



Carrying a novel P-band synthetic aperture radar, the Biomass mission is designed to deliver crucial information about the state of our forests and how they are changing, and to further our knowledge of the role forests play in the carbon cycle.

Dedicated tomographic acquisitions to image forests in 3D at fine resolution

To be launched in summer 2024
Site: Kourou, French Guiana
Rocket: Vega

# A step back…

RADAR (***Radio Detection And Ranging***) is a technology to detect and study far off targets by transmitting EM pulses at radiofrequency and observing the backscattered echoes

***Some relevant features:***

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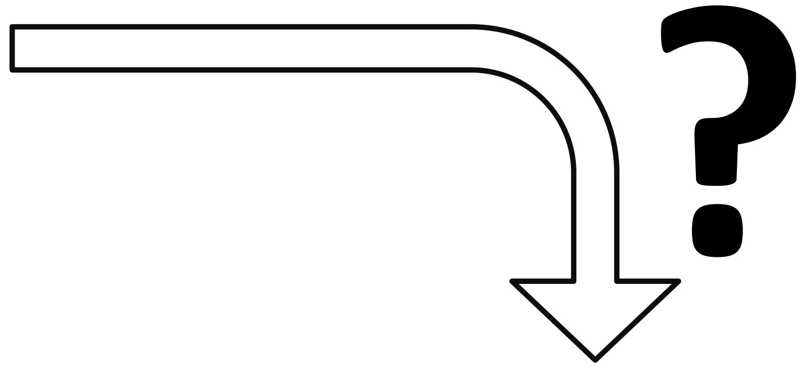

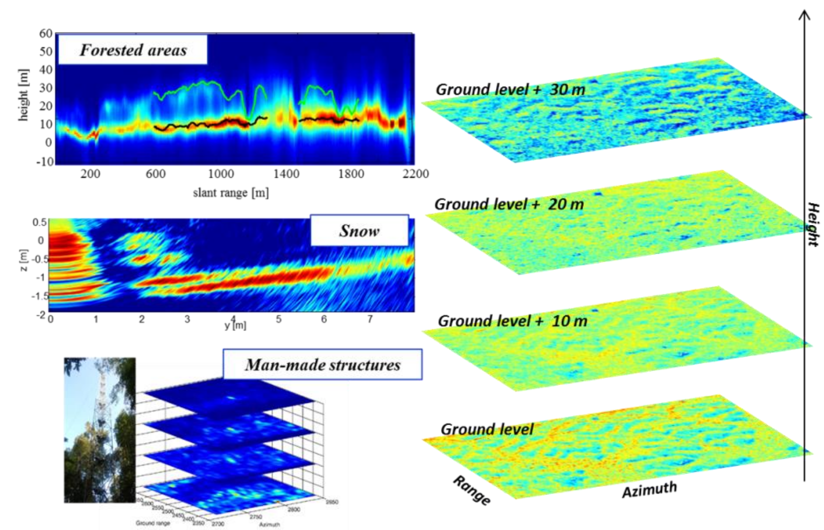

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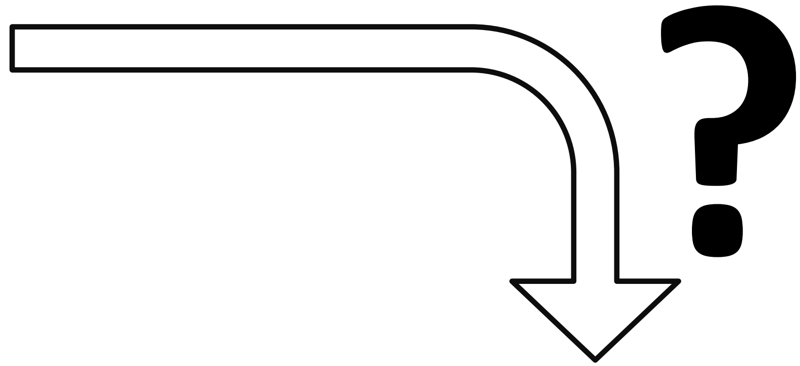
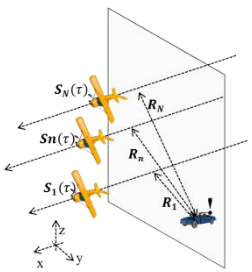
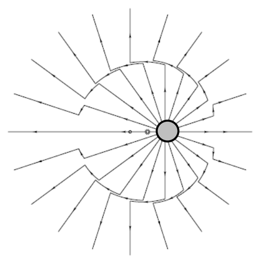


***Answer:***
***Yes (☺), with some efforts concerning***

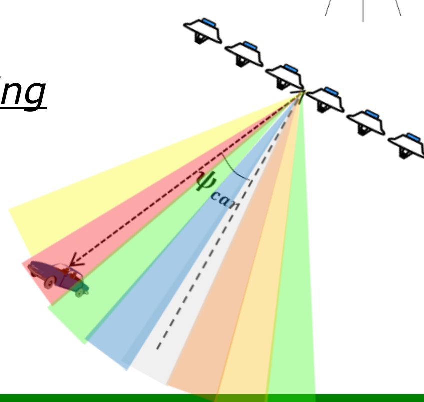


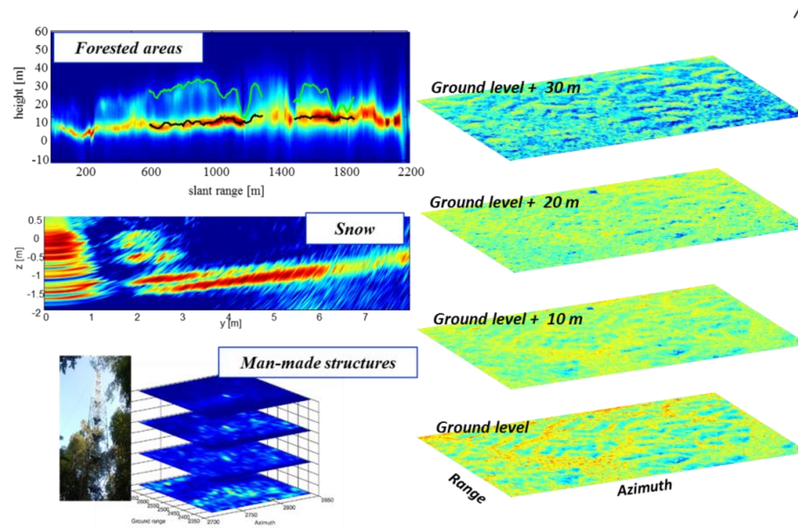
<u>*Geometry*</u>

<u>*Waves*</u>



<u>*Radar Processing*</u>



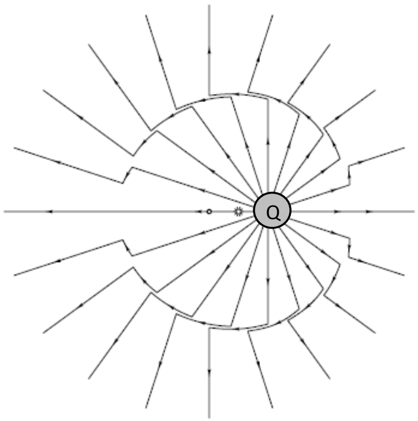


# *Waves*

# Must knows about waves (at least for today)

**Electromagnetic waves**

- EM wave = perturbation in the intensity of the static EM field
- Effect of the universal speed limit
- Triggered by charges in non-uniform motion
- Propagation velocity in free space: $c \approx 3 \cdot 10^8$ m/s



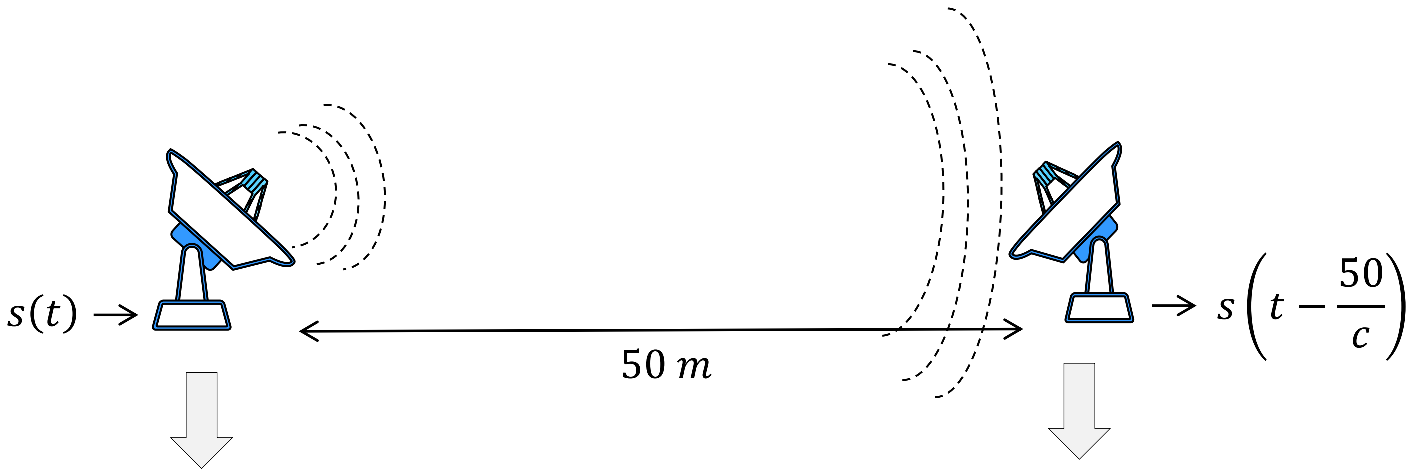

**Representation**

We can describe waves as signals that vary over **time** and **space** as:

$$s\left(t - \frac{z}{c}\right)$$

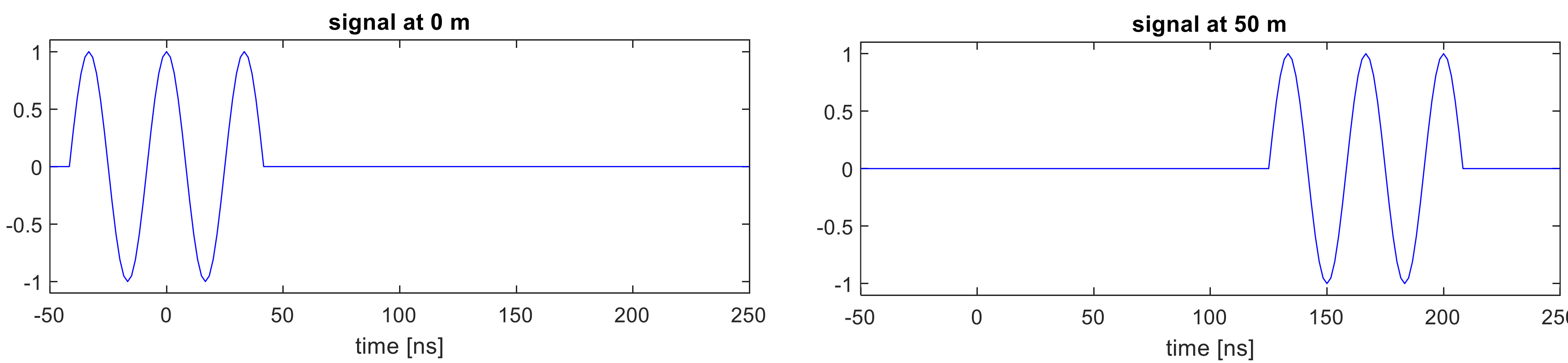
with $s(t)$ some waveform

When we express a signal as a function of time, we are implicitly assuming that the signal is measured at a **fixed point in space**



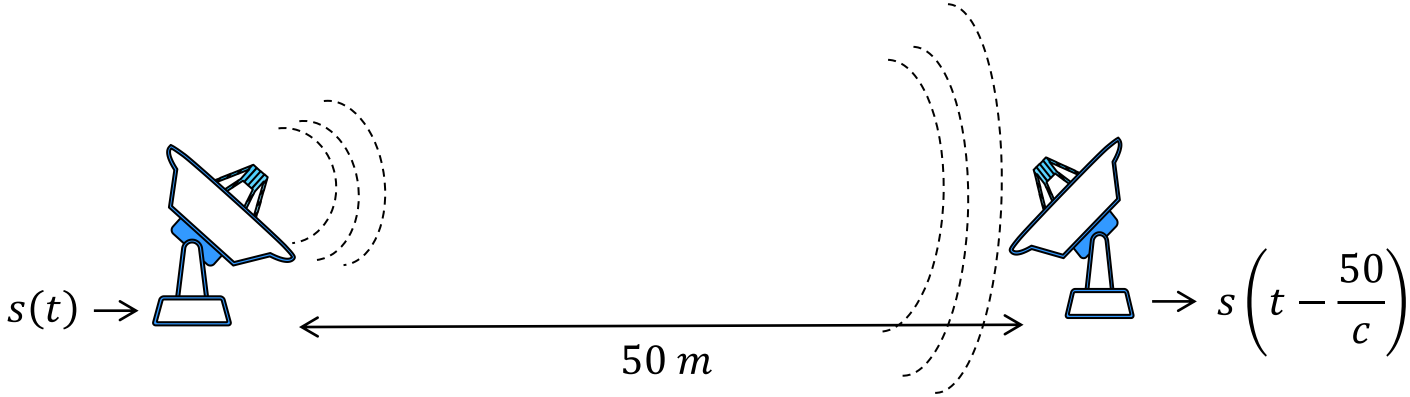

$s(t)$

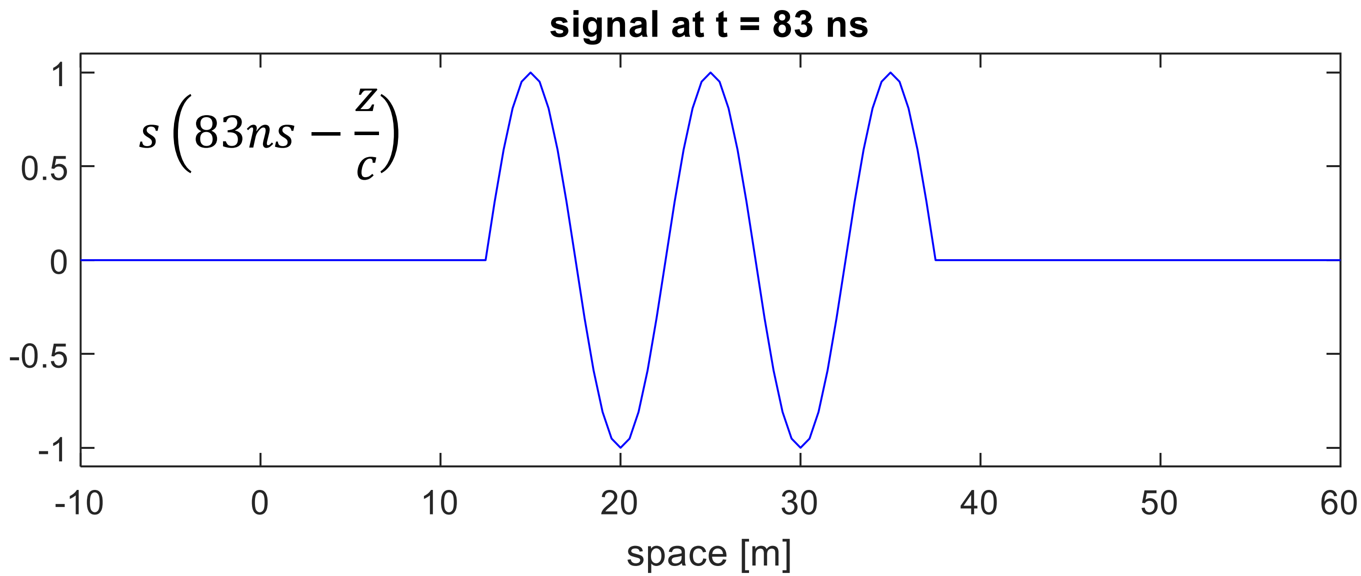
$s\left(t - \frac{50}{c}\right)$



# Must knows about waves (at least for today)

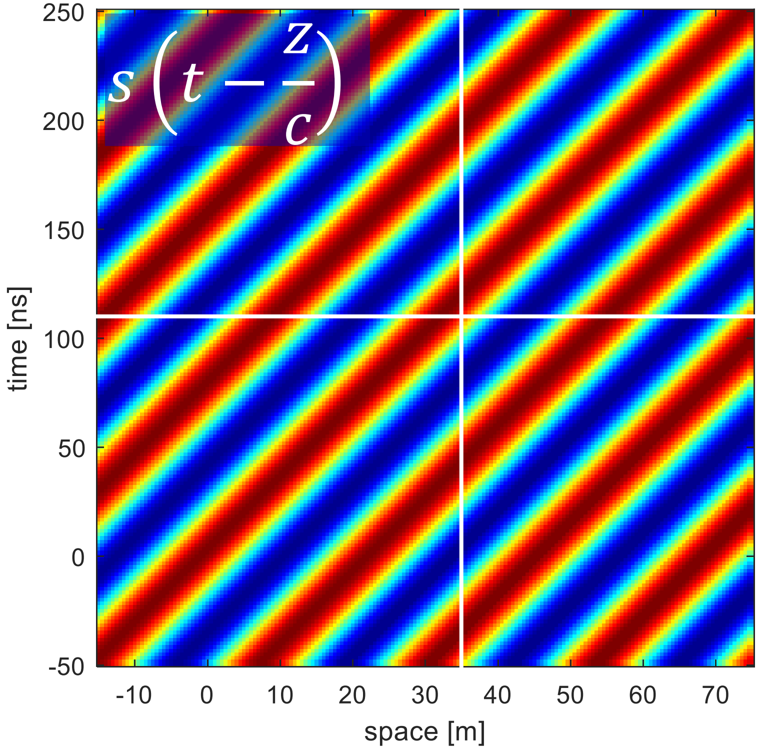
Equivalently, we can express a signal as a function of space by imaging that we can freeze the time at a **fixed instant** and take a snapshot of the signal distribution over space

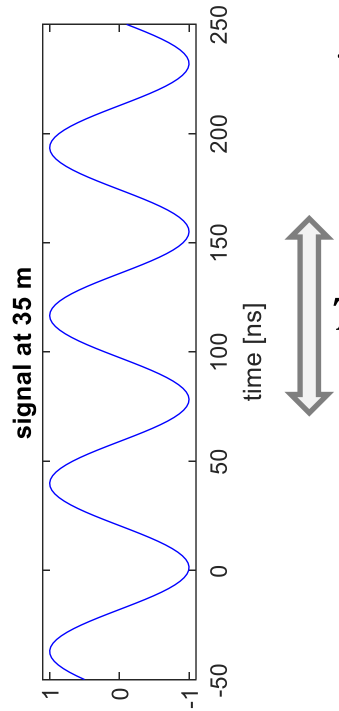


# Must knows about waves (at least for today)

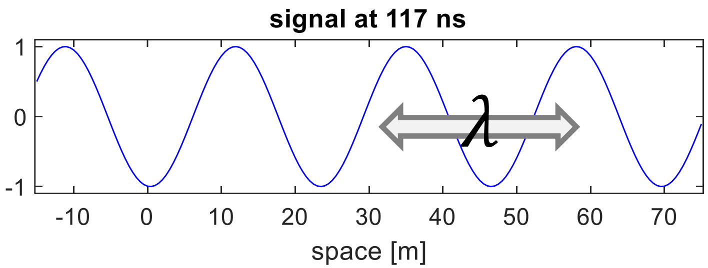
For the case of a monochromatic wave we have $s(t) = cos(2\pi f_o t)$




The temporal period is obtained as

$$T = \frac{1}{f_o}$$




The spatial period, a.k.a. ***the wavelength***, is obtained as

$$\lambda = \frac{c}{f_o}$$

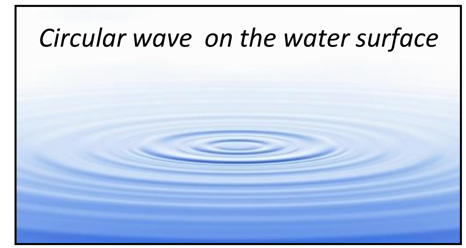
# *Geometric principles of target localization*

# Radio detection and ranging

RADAR (***Radio Detection And Ranging***) is a technology to detect and study far off targets by transmitting EM pulses at radiofrequency and observing the backscattered echoes

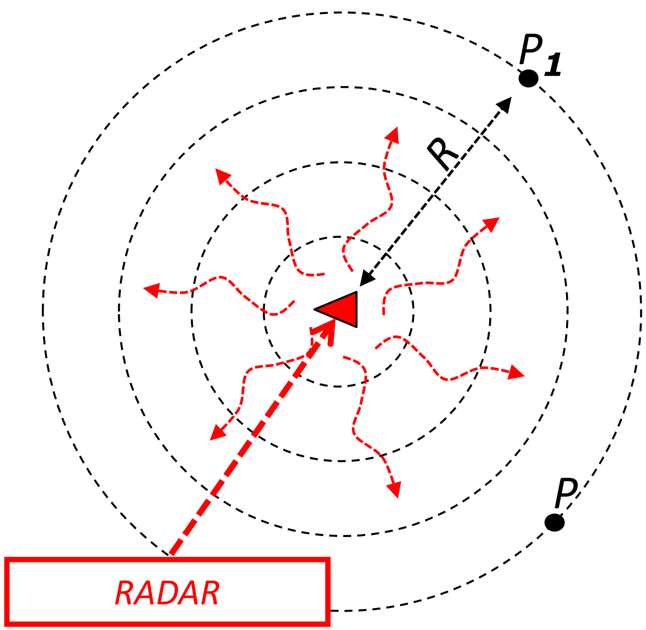
*Simplified description*

*I) The transmitted signal propagates away from the RADAR sensors in **all directions**$^{(*)}$ in the form of a **spherical wave***



*Circular wave on the water surface*

*$^{(*)}$Note: real antennas actually radiate over an angular sector, depending on size and wavelength*




**$s_{Tx}(t)$ = $s(t)$ = transmitted signal**

**$s_1(t)$ = signal received at point $P_1$**

$$s_1(t) = s\left(t - \frac{R}{c}\right)$$

$R$ = distance between $P_1$ and the RADAR

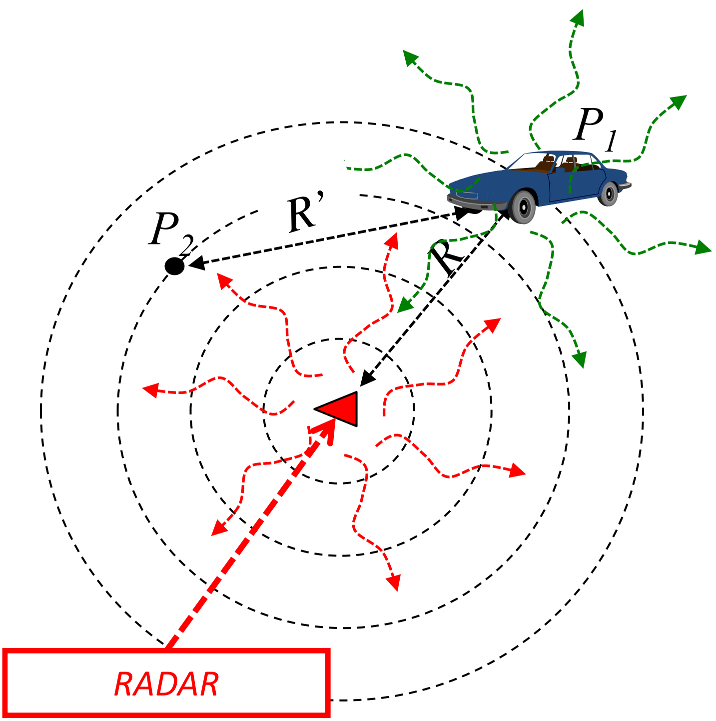
$c$ = speed of light

The received signal depends on distance only

$\Rightarrow$ Any antenna at distance **R** from the RADAR receives the same signal $s_1(t)$

*Simplified description*

*II) The signal interacts with surrounding objects (targets) ⇔ backscattered echoes*




As a first approximation the backscattered echo can be represented by imaging the target as a new source of spherical waves

**$s_1(t)$ = signal received at point $P_1$**

$$s_1(t) = s\left(t - \frac{R}{c}\right)$$

*R* = distance between $P_1$ and the RADAR

*c* = speed of light

**$s_2(t)$ = backscattered signal received at point $P_2$**

$$s_2(t) = A \cdot s_1\left(t - \frac{R'}{c}\right) = A \cdot s\left(t - \frac{R'+R}{c}\right)$$

*R' = distance from $P_1$ to $P_2$*

*A* = constant accounting for the interaction of the impinging signal with the target

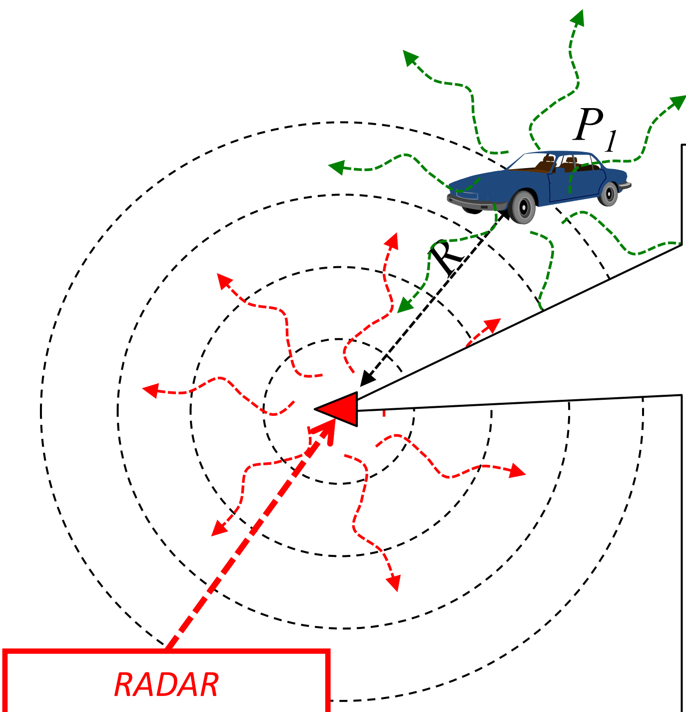
*Simplified description:*

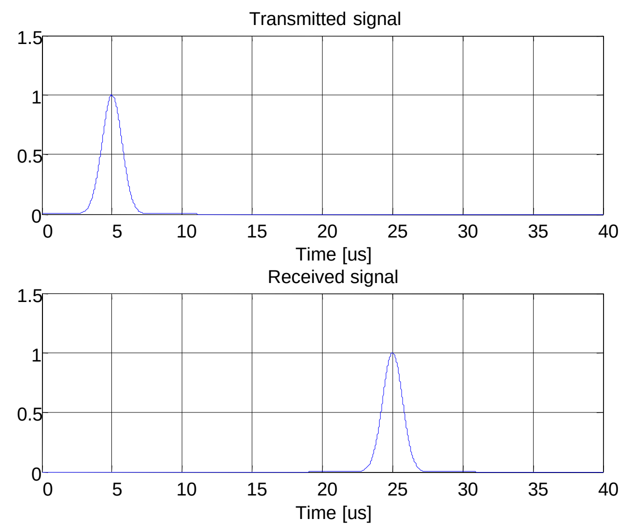
*III) The backscattered echo is received by the RADAR sensor*

**$s_{Rx}(t)$ = backscattered signal received by the RADAR**

$$s_{Rx}(t) = A \cdot s_1\left(t - \frac{R}{c}\right) = A \cdot s\left(t - \frac{2R}{c}\right)$$

⇨ *The distance of a target from the RADAR is known by measuring the pulse round-trip time*




*Example*

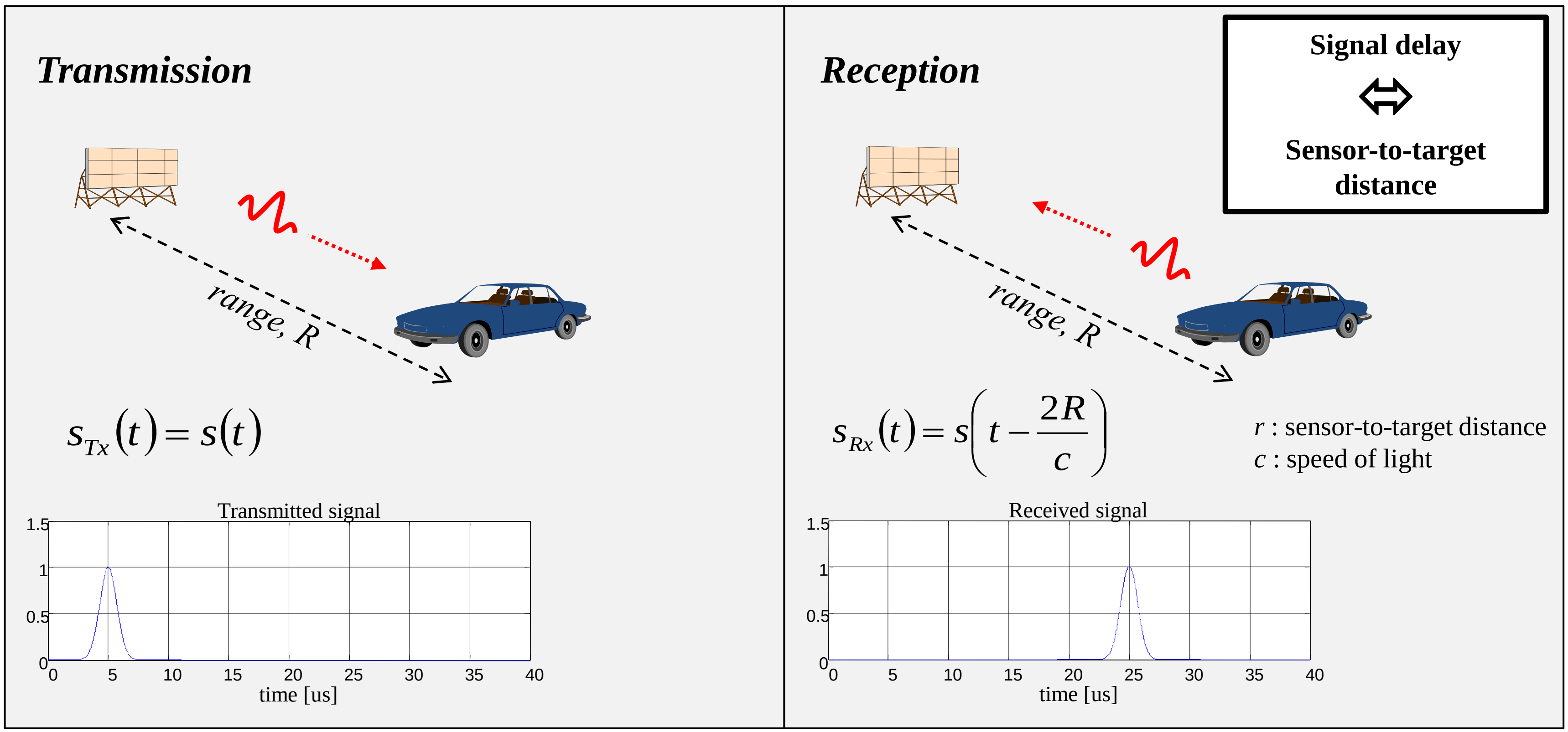
Measured delay: $\tau = 20$ *microseconds*

*Propagation velocity: c = 300000 Km/s*

$\Rightarrow$ **Range: R = $c \cdot \tau/2$ = 3 Km**

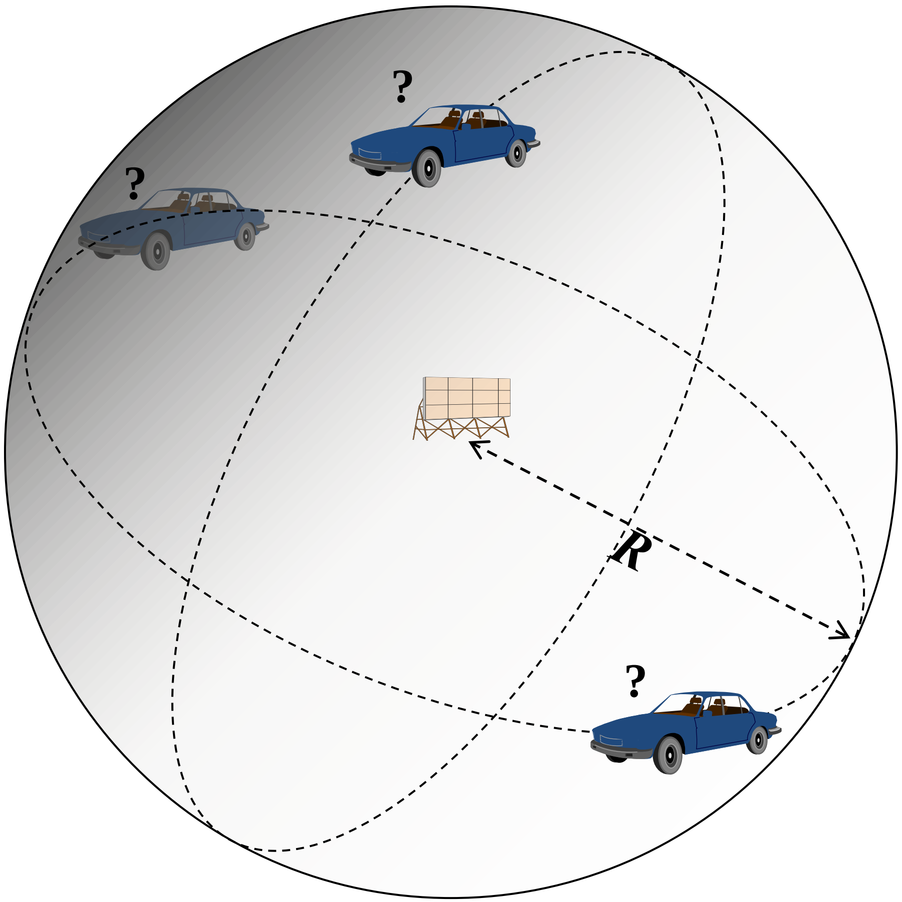
# Localization in 1D

*Delay measurement*



# Localization in 1D

*Delay measurement ⇔ Localization on the surface of a sphere*



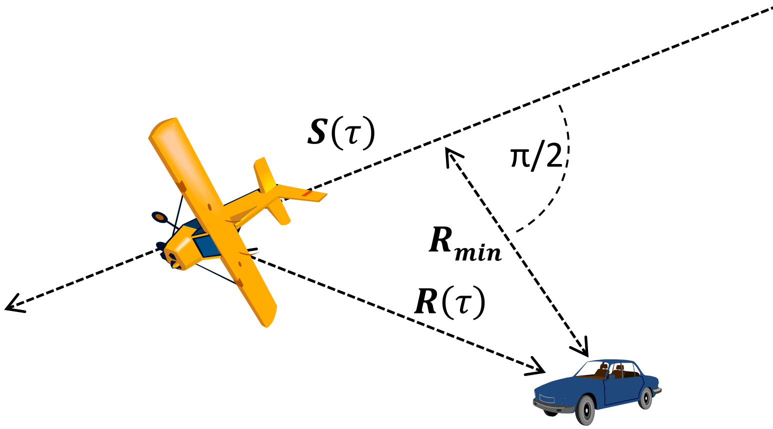

The target is bound to lie on a sphere

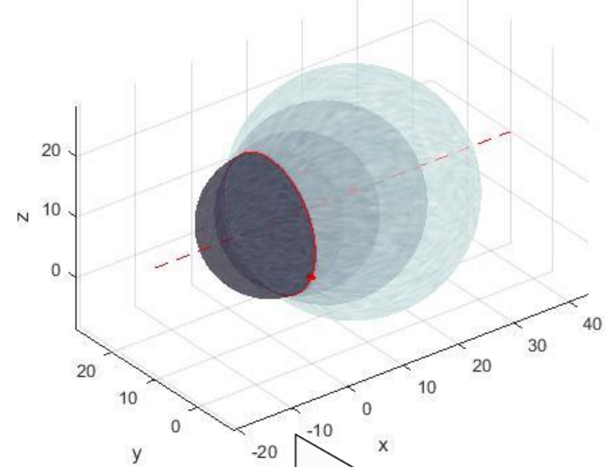
- Centered on the RADAR
- Of radius R

⇒ ***1D Localization***

# Localization in 2D (SAR)

Flying a RADAR along a straight line = measuring the distance from the target to each point on the line



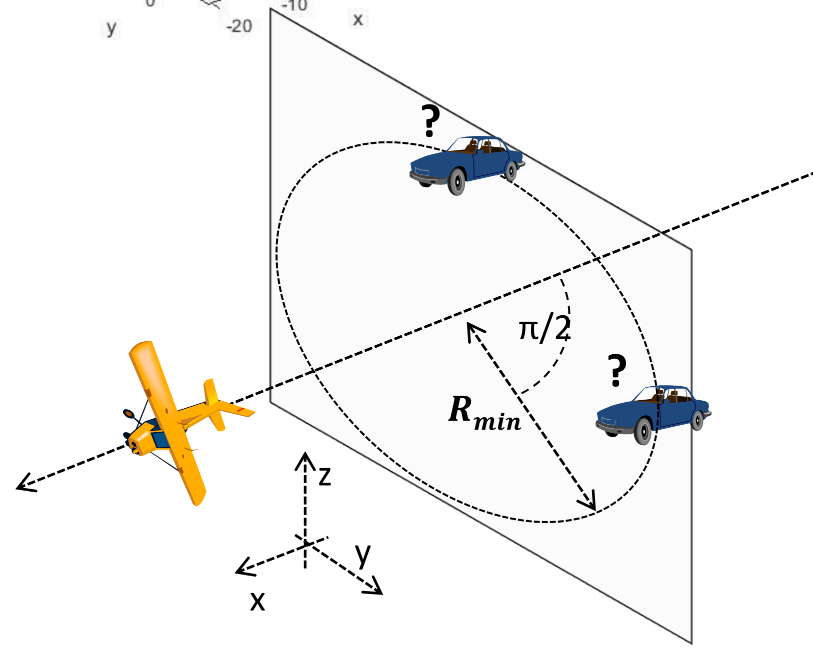

The target is bound to lie on the intersection of all the spheres:

- Centered in $S(\tau)$
- Of radius $R(\tau)$

$\Rightarrow$ The target is bound to lie on the circle:

- Centered on the trajectory
- Perpendicular to the trajectory (*yz* plane)
- Of radius $R_{min}$

$\Rightarrow$ 2D Localization



# Localization in 3D (TomoSAR)

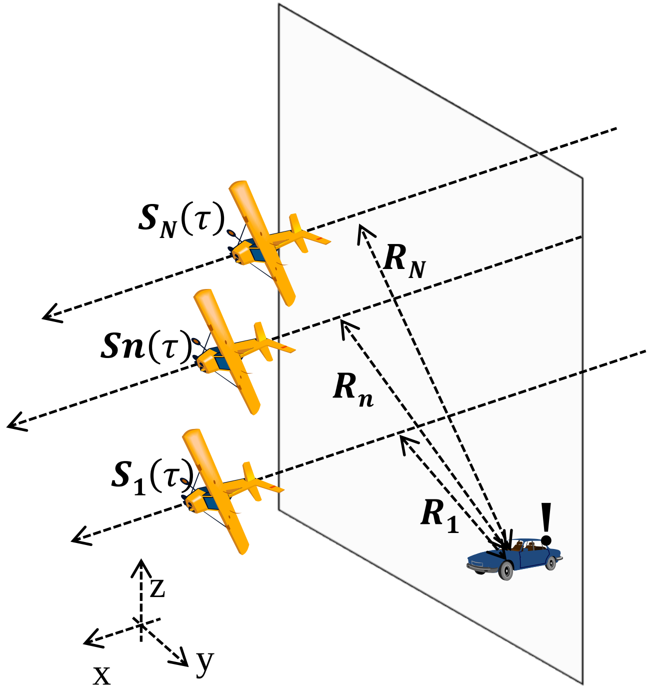
Flying a RADAR along multiple lines = measuring the distance from the target to multiple lines

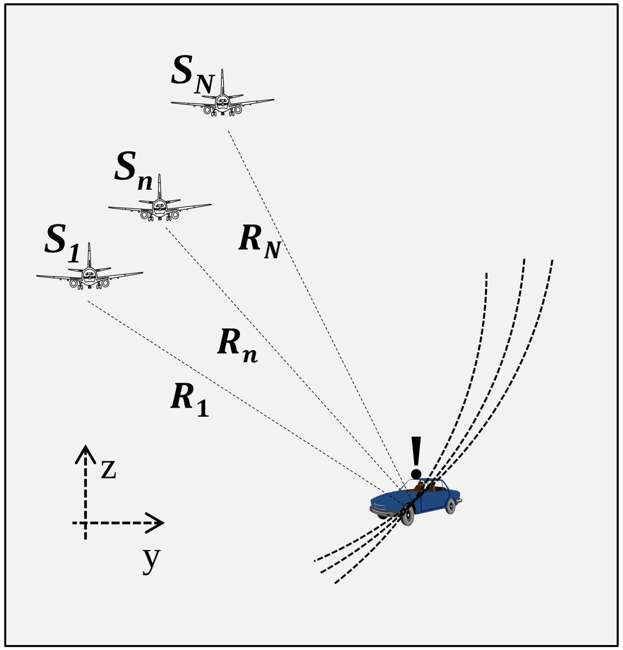
The target is bound to lie on the circles:

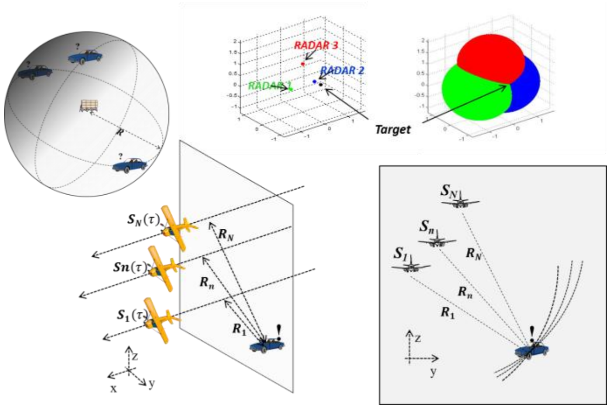
- Centered on each trajectory
- Perpendicular to the trajectory ,
- Of radius $\boldsymbol{R_1} \dots \boldsymbol{R_n} \dots . \boldsymbol{R_N}$

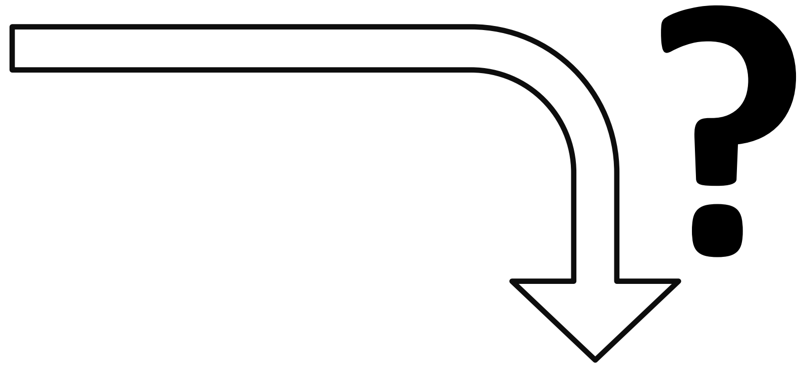
$\Rightarrow$ Only 1 solution in the 3D space !

$\Rightarrow$ 3D localization

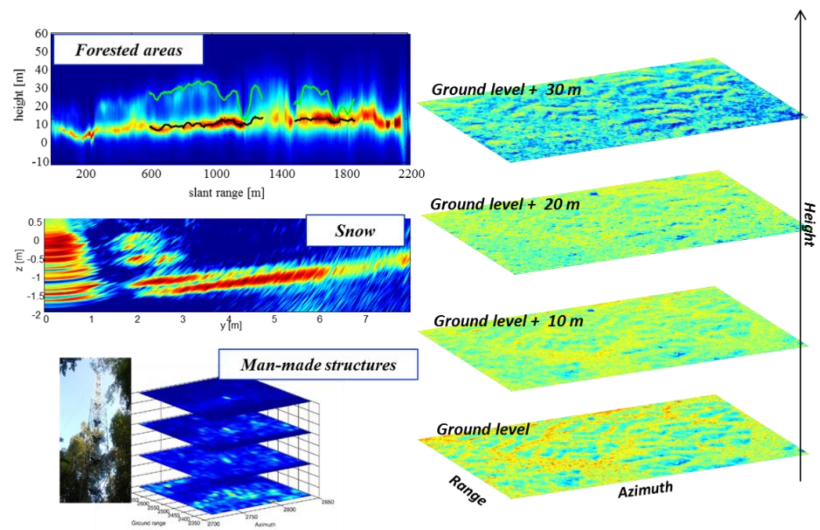


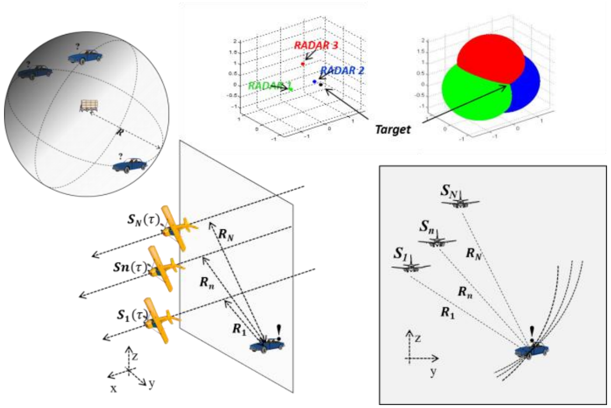



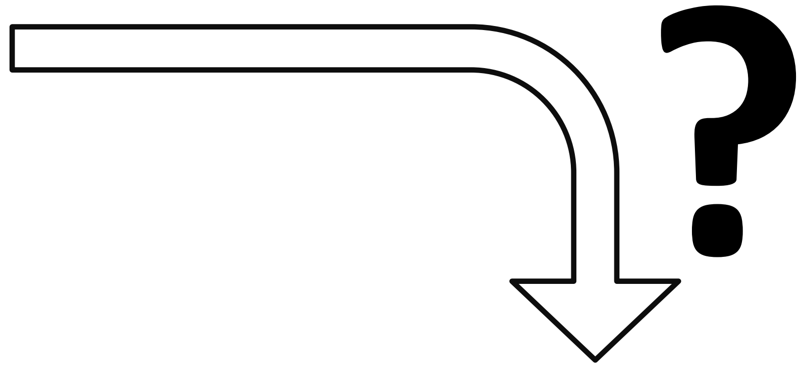


***Geometry unveils the principle why flying multiple trajectories results in the capability to localize a target in the 3D space***



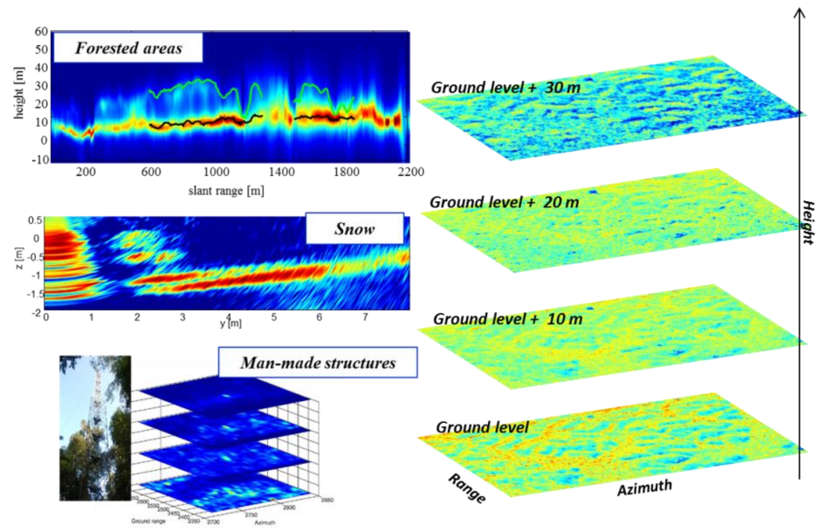


**_Geometry unveils the principle why flying multiple trajectories results in the capability to localize a target in the 3D space_**

**_Missing elements:_**

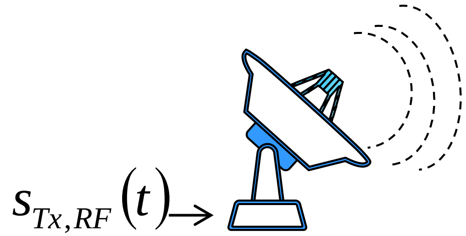
- **_Resolution_**
- **_What if there are many targets ?!!!_**



# *RADAR signals*

# Radar signals

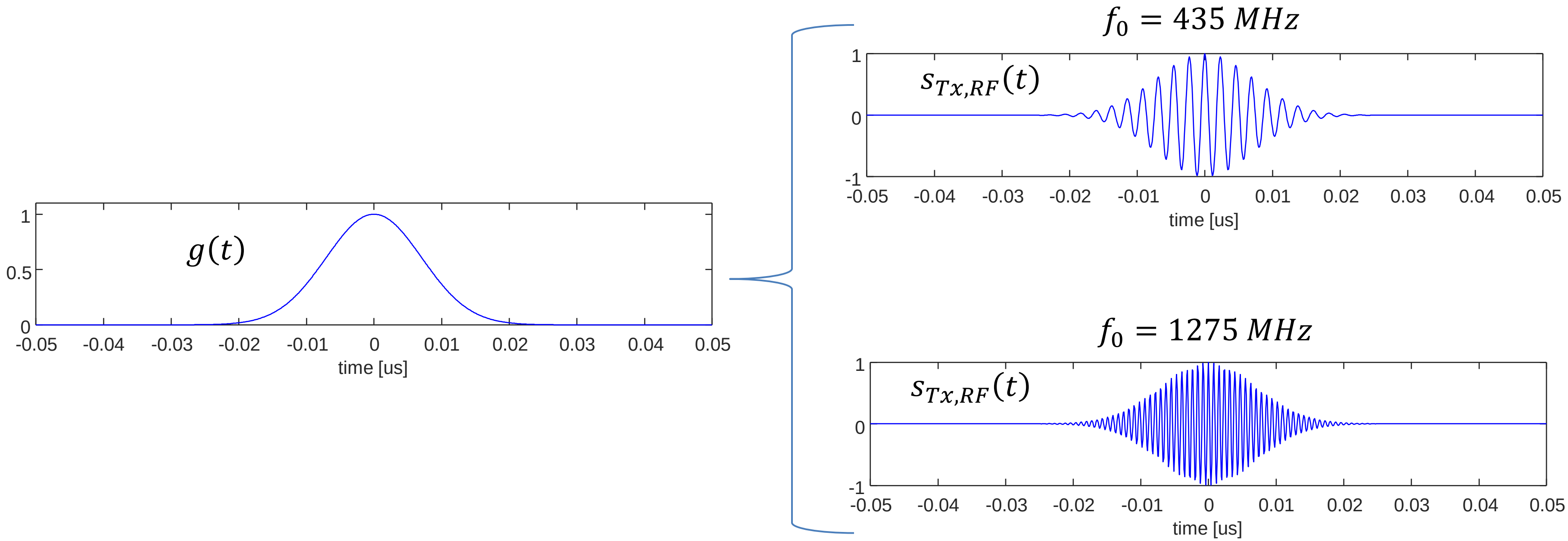
RADARs transmit and receive Radiofrequency (RF) pulses




$$s_{Tx,RF}(t) = g(t) \cdot cos(2\pi f_0 t)$$

*g(t) = short EM pulse*

*$f_0$ = carrier frequency*



# Radar signals

RADARs transmit and receive Radiofrequency (RF) pulses

$s_{Tx,RF}(t) \rightarrow$

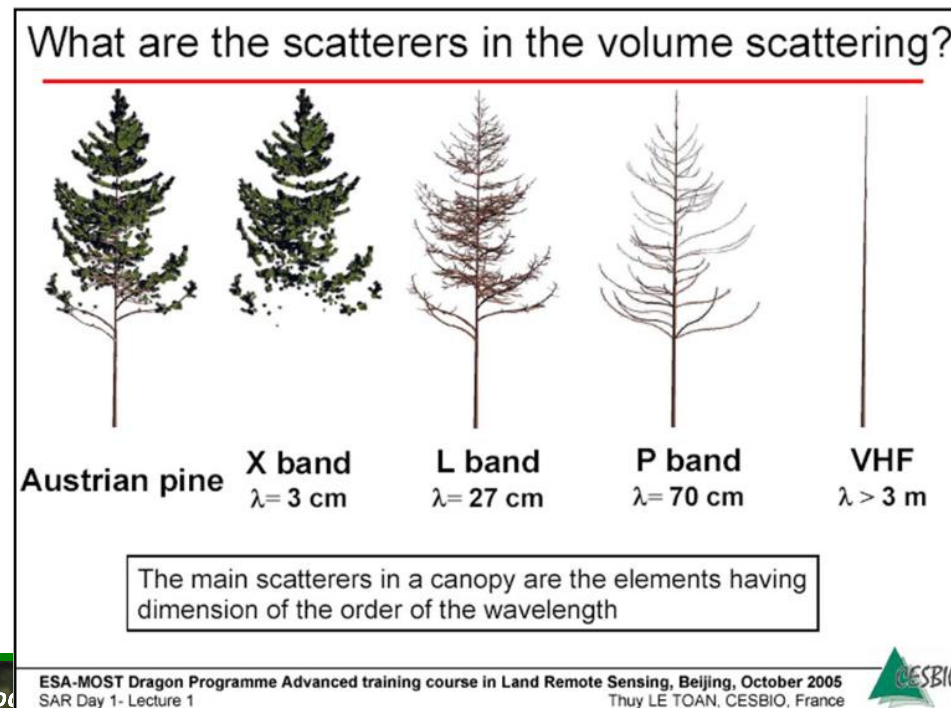
$$s_{Tx,RF}(t) = g(t) \cdot cos(2\pi f_0 t)$$

*g(t) = short EM pulse*

*$f_0$ = carrier frequency*

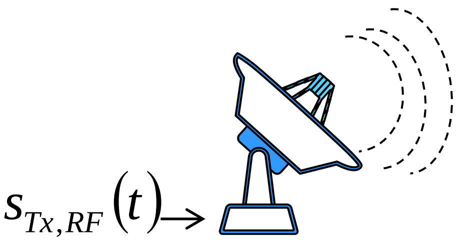
The carrier frequency (or wavelength $\lambda = \frac{c}{f_o}$) is perhaps the most important parameter in the design of a Radar sensor, as it determines:

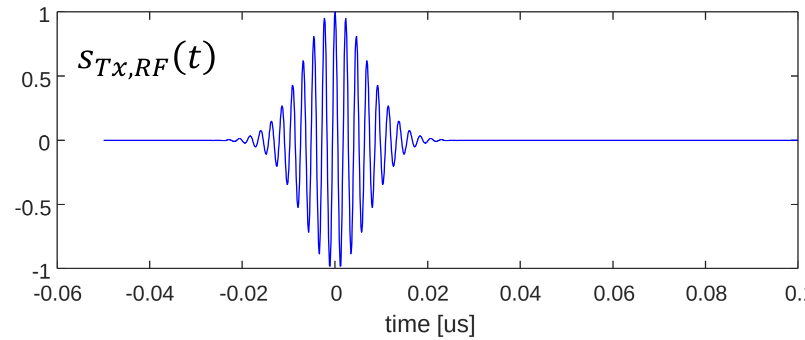
- The antenna to be used
- The RF hardware to be used
- **The features of the observed targets which the signal is sensitive to**

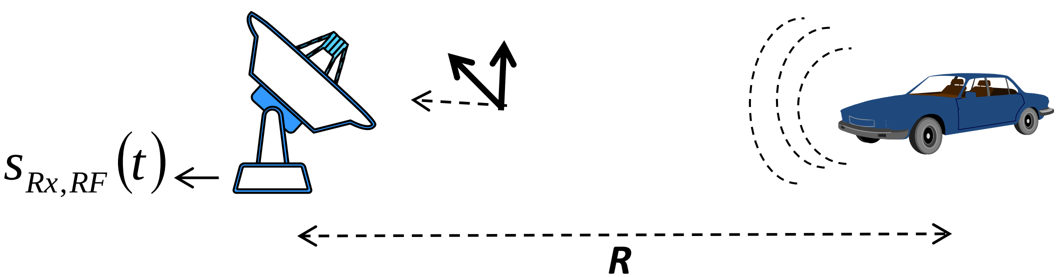


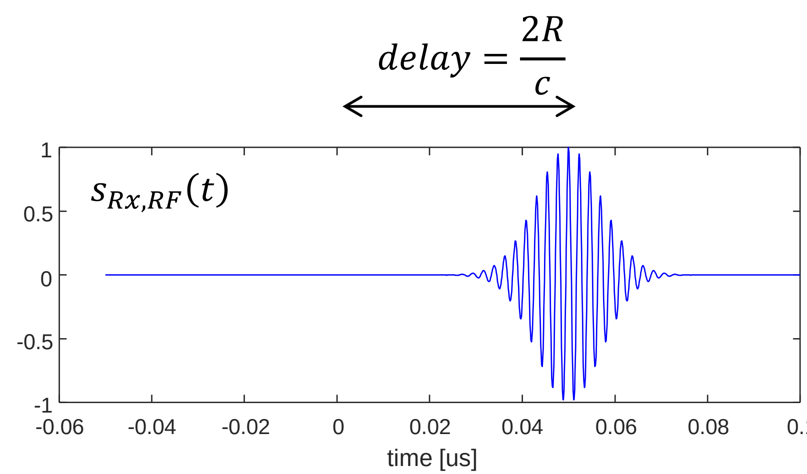
# Radar signals

On a mathematical ground, the signal backscattered by a target is represented as a delayed version of the transmitted signal




$$s_{Rx,RF}(t) = A \cdot s_{Tx,RF}(t-d)$$

$$d = \frac{2R}{c}$$

# Radar signals

Following basic trigonometry, the received signal is expressed as:

$$s_{Rx,RF}(t) = \underbrace{A \cdot g(t-d) \cdot cos(2\pi f_0 d)}_{\text{In phase component } I(t)} \cdot cos(2\pi f_0 t) - \underbrace{A \cdot g(t-d) \cdot sin(2\pi f_0 d)}_{\text{Quadrature component } -Q(t)} \cdot sin(2\pi f_0 t)$$

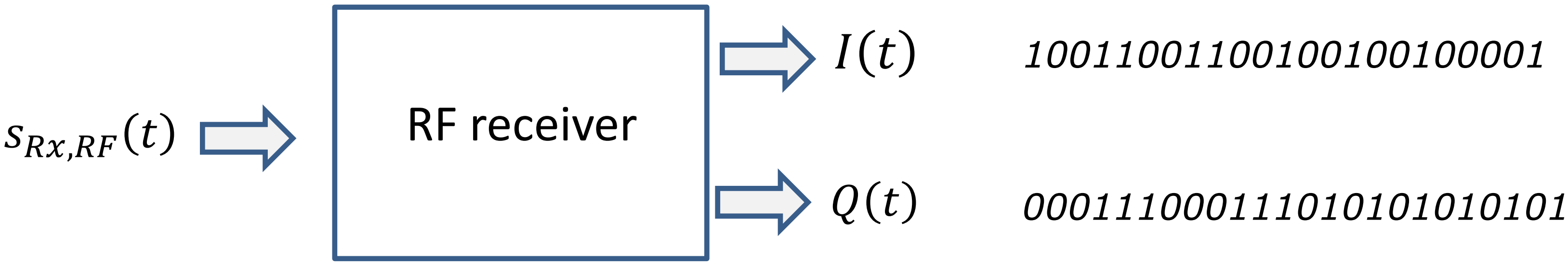
Where we define the in-phase and quadrature signals as:

- $I(t) = A \cdot g(t-d) \cdot cos(2\pi f_0 d)$
- $Q(t) = -A \cdot g(t-d) \cdot cos(2\pi f_0 d)$

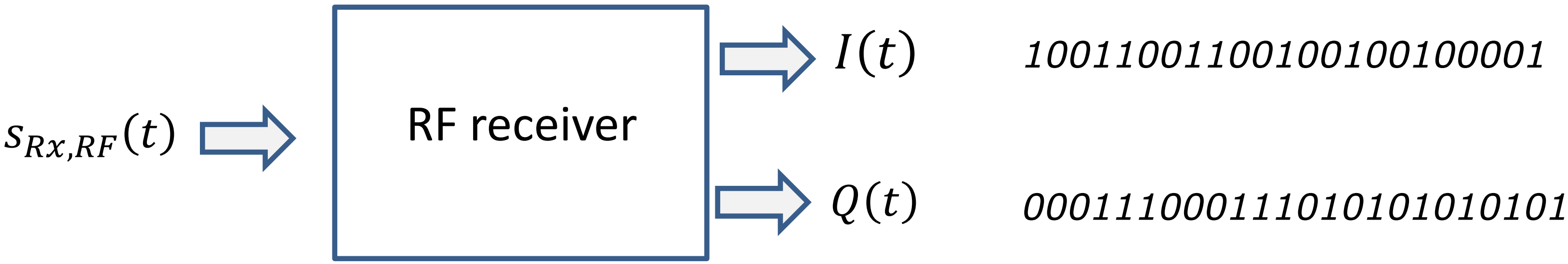
The **information** about the target is carried by the amplitude and delay parameters *A* and *d*, which are embedded in the in-phase and quadrature signals $I(t)$ and $Q(t)$
(we already know the value of the carrier, so no new info in it)

# Radar signals

The $I(t)$ and $Q(t)$ of RF signals are extracted by RF circuitry, stored as numerical signals…



The $I(t)$ and $Q(t)$ of RF signals are extracted by RF circuitry, stored as numerical signals…




… and represented as a single **complex** signal, simply referred to as the (complex envelope of the) received signal

$$s_{Rx}(t) = I(t) + jQ(t)$$

*j = imaginary unit*

The complex representation is ubiquitous in the study of all wave phenomena

One good reason why it is used: it allows to make large use of the properties of complex exponentials (easy!)

$\Rightarrow$ ***noticeable simplification!***

# Radar signals

Going back to the case of the received signal, we have:

$$s_{Rx,RF}(t) = \underbrace{A \cdot g(t-d) \cdot cos(2\pi f_0 d)}_{\text{In phase component } I(t)} \cdot cos(2\pi f_0 t) - \underbrace{A \cdot g(t-d) \cdot sin(2\pi f_0 d)}_{\text{Quadrature component } -Q(t)} \cdot sin(2\pi f_0 t)$$

$$s_{Rx}(t) = I(t) + jQ(t) = A \cdot g(t-d) \cdot e^{-j2\pi f_0 d}$$

# Radar signals

Finally, recalling that:

- The delay is obtained as $d = \frac{2R}{c}$
- The wavelength is obtained is $\lambda = \frac{c}{f_0}$

we obtain the usual expression of the received signal used in large part of the Radar literature:

$$s_{Rx}(t) = A \cdot g\left(t - \frac{2R}{c}\right) \cdot e^{-j\frac{4\pi}{\lambda}R}$$

# Radar signals

Finally, recalling that:

- The delay is obtained as $d = \frac{2R}{c}$
- The wavelength is obtained is $\lambda = \frac{c}{f_0}$

we obtain the usual expression of the received signal used in large part of the Radar literature:

$$s_{Rx}(t) = A \cdot g\left(t - \frac{2R}{c}\right) \cdot e^{-j\frac{4\pi}{\lambda}R}$$

***Amplitude***:
this term is related to the strength of the wave backscattered by the target

***Delayed pulse***:
this term allows for the determination of a target's distance from the Radar

***Phase***:
*this is where the magic starts…*

# *The frequency domain*

# Frequency domain

The signals presented in the last section were represented by drawing their variation over time, or by writing equations where the signal amplitude depends on the time like $g(t)$

This particular representation is referred to as ***time domain***

An alternative representation is built by representing a signal as a collection of **sinusoids** (either real or complex), i.e.:

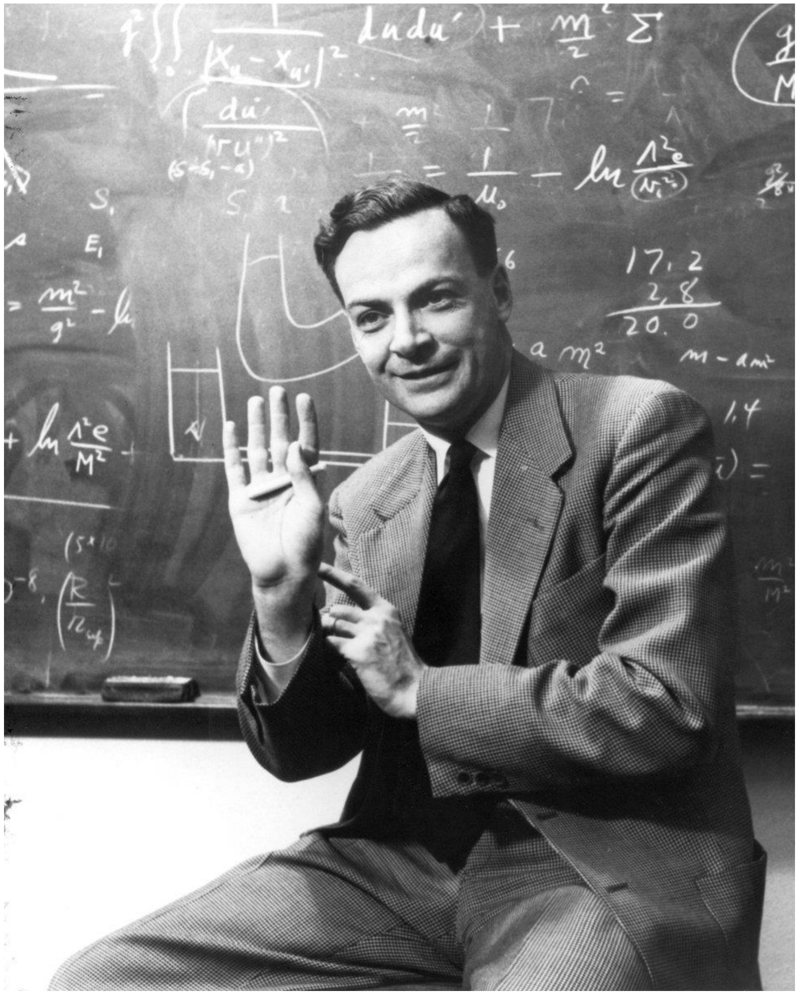
$$g(t) = G_1 e^{j2\pi f_1 t} + G_2 e^{j2\pi f_2 t} + G_3 e^{j2\pi f_3 t} + \cdots$$

we say that the signal $g(t)$ "contains" the sinusoids at frequency $f_1,\ f_2,\ f_3$… and the amplitudes $G_1$,$G_2$, $G_3$ represent the "strength" of each of those sinusoids.

**Question**: can we represent any signal in the frequency domain?

In other terms, can we always represent a signal as a collection of sinusoids?

We answer with the help of Richard Feynman:



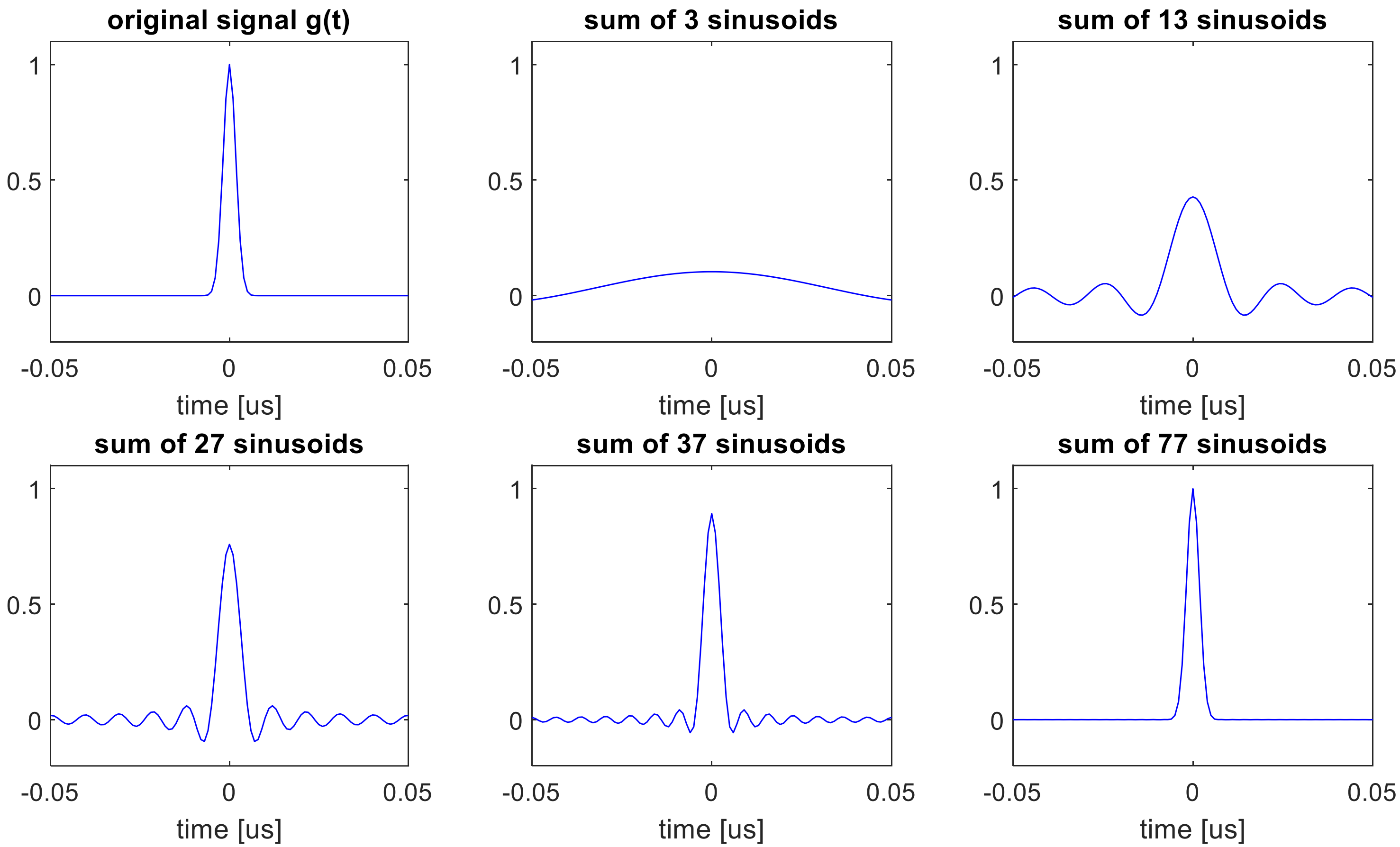
*In what circumstances can a curve be represented as a sum of a lot of cosines?*

*Answer:*

*In all ordinary circumstances, except for certain cases the mathematicians can dream up. Of course, the curve must have only one value at a given point, and it must not be a crazy curve which jumps an infinite number of times in an infinitesimal distance, or something like that. But aside from such restrictions any reasonable curve (one that a singer is going to be able to make by shaking her vocal cords) can always be compounded by adding cosine waves together.*

# Frequency domain

Practically, this means we can **always** represent a signal in terms of a sum of sinusoids, as long as we consider a sufficient number of sinusoids



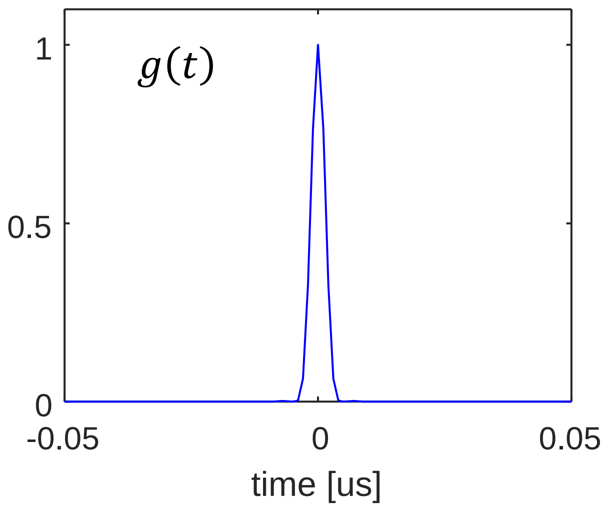
# Frequency domain

Practically, this means we can **always** represent a signal in terms of a sum of sinusoids, as long as we consider a sufficient number of sinusoids

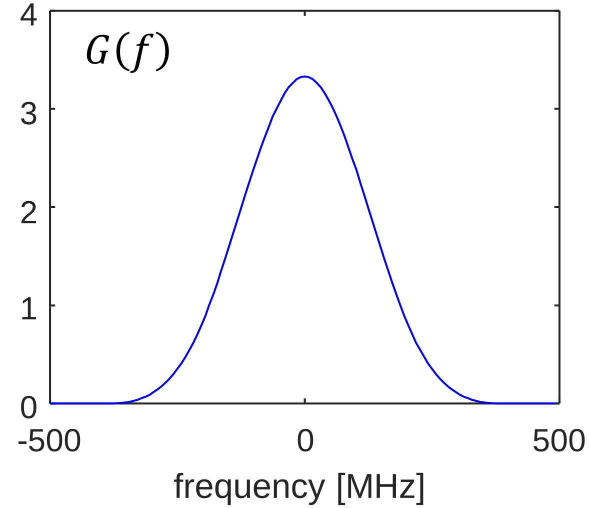
In this way, we can represent the signal $g$ by drawing (or writing) the series of the amplitudes as a function of the frequency of the associated sinusoid, i.e.: $G(f)$

$$g(t) = G_1 e^{j2\pi f_1 t} + G_2 e^{j2\pi f_2 t} + G_3 e^{j2\pi f_3 t} + \cdots$$

This particular representation is referred to as ***frequency domain***



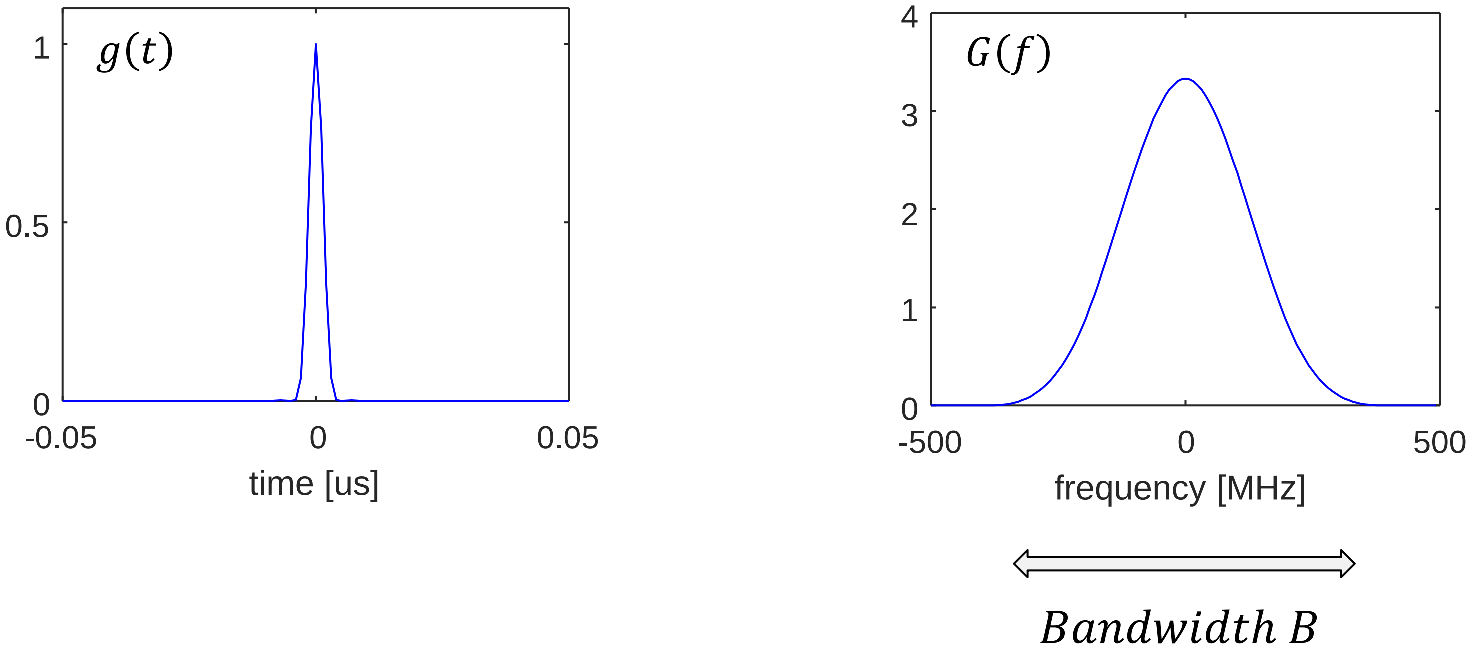

*Time domain representation*




*Frequency domain representation*

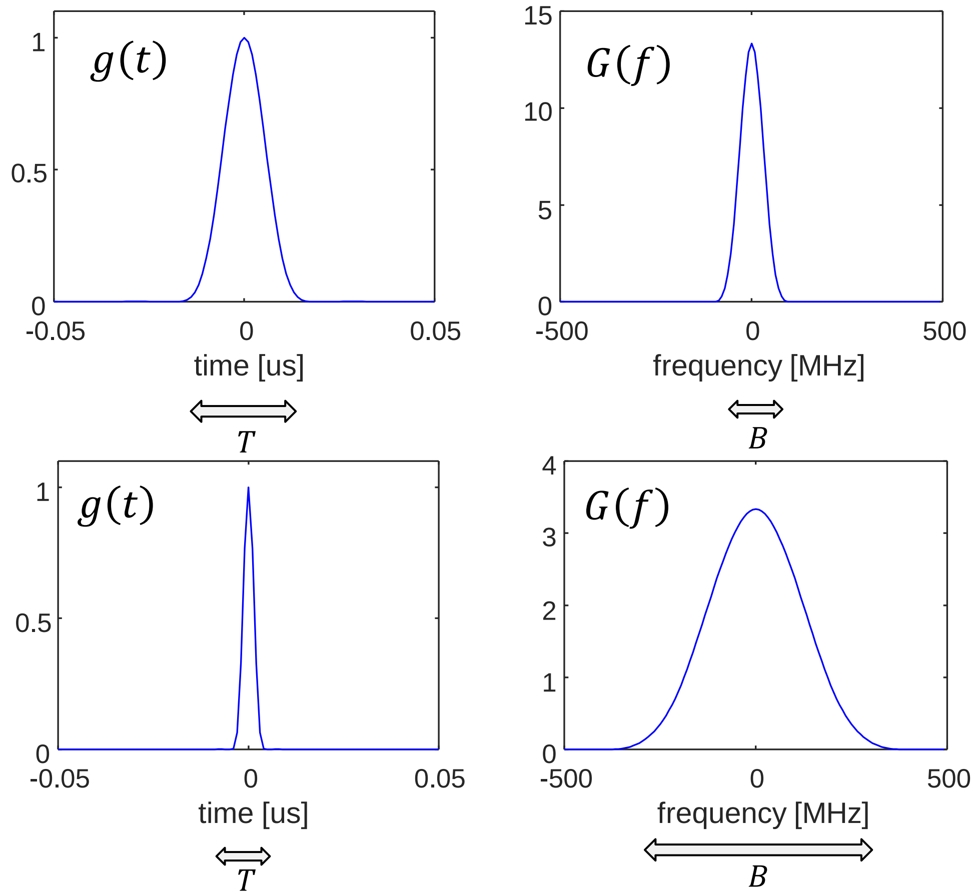
# Bandwidth

The **bandwidth** of a signal is defined as the "length" of the interval where we find the frequencies that are contained in it



For Radar pulses we can state the rule that (with some exceptions we will not discuss):

***signal bandwidth is inversely proportional to signal duration***




$$T \approx \frac{1}{B}$$

$$T \approx \frac{1}{B}$$

**Question**: how do we get to know which frequencies contribute to a signal? How do we compute their amplitudes $G(f)$?

# The Fourier Transform

Answer: we calculate the ***Fourier Transform*** of the signal

For a signal represented as a sequence of time samples in our computer, the Fourier Transform is expressed as:

$$G(f) = \sum_n g(t_n) \cdot e^{-j2\pi f t_n}$$

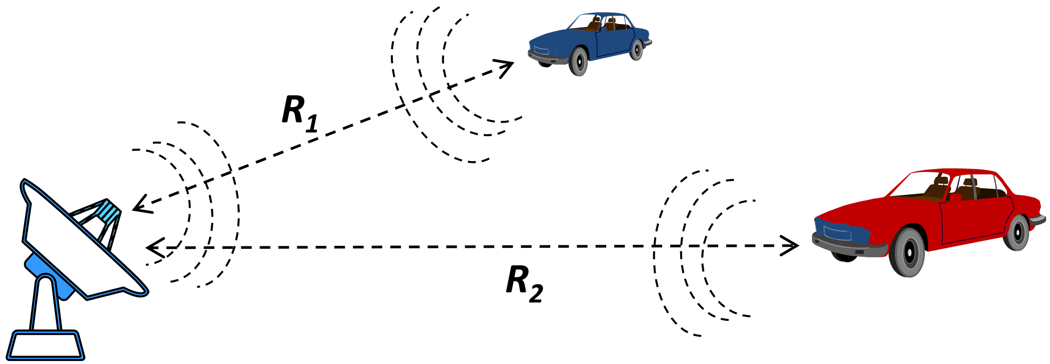
Which states a simple recipe:

- Choose (at will) a particular frequency $f$
- Take the original signal $g(t_n)$ and multiply its time samples times $e^{-j2\pi f t_n}$
- Sum over all samples
- Repeat for any frequency $f$ we want to evaluate

# *Range resolution*

# Range resolution

The concept of bandwidth leads us to directly to the important concept of ***range resolution***, intended as the capability to distinguish (resolve) two targets found at slightly different distances from the Radar

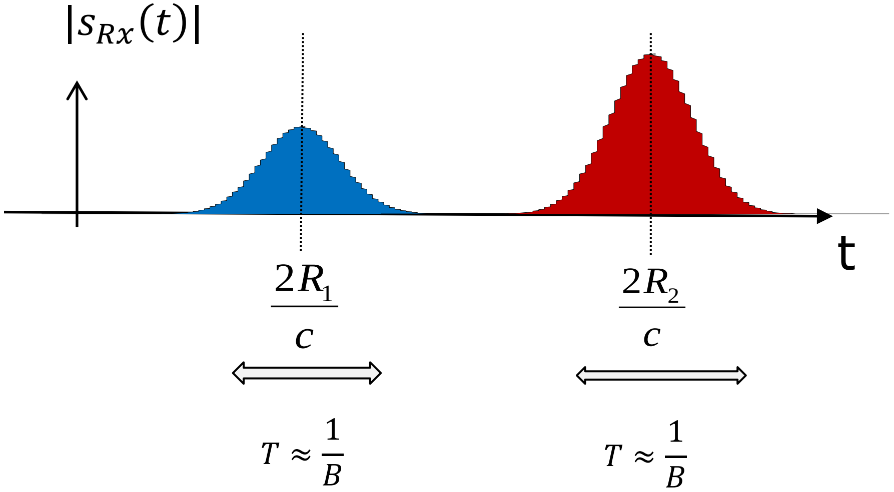



The received signal is now expressed as the sum of two signals associated with the two targets

$$s_{Rx}(t) = A_1 \cdot g\left(t - \frac{2R_1}{c}\right) \cdot e^{-j\frac{4\pi}{\lambda}R_1} + A_2 \cdot g\left(t - \frac{2R_2}{c}\right) \cdot e^{-j\frac{4\pi}{\lambda}R_2}$$

# Range resolution

If we plot signal magnitude, we obtain the following graph




$\Rightarrow$ We can tell there are two targets as long as the received signal exhibits **two distinct peaks**

This occurs upon the condition that:

$$\frac{2|R_2 - R_1|}{c} \geq T \quad \Longrightarrow \quad |R_2 - R_1| \geq \frac{c}{2B} = \Delta R$$

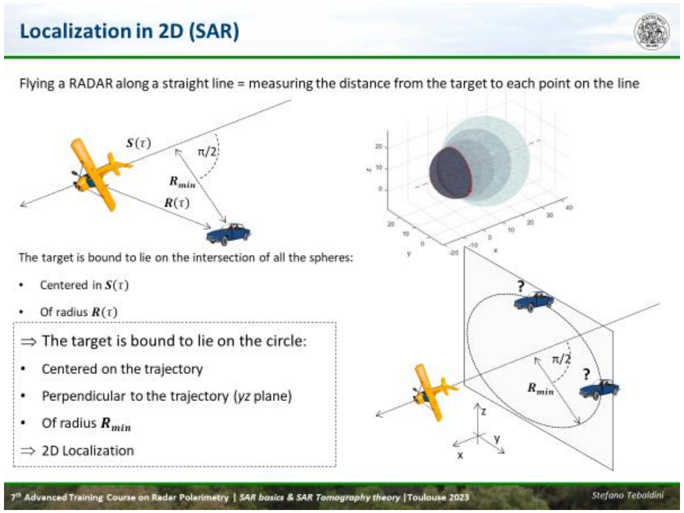
Where $\Delta R = \frac{c}{2B}$ is referred to as the **range resolution** of the Radar

# Range resolution

| B | ΔR | Typical SAR case |
|---|---|---|
| 6 MHz | 25 m | P-Band (≈ 400 MHz carrier) spaceborne SAR (due to ITU regulations) |
| 40 MHz | 3.75 m | L-Band (≈ 1300 MHz carrier) spaceborne SAR |
| 150 MHz – 500 MHz | 1 m – 0.3 m | Low frequency airborne SAR (hundreds of MHz to few GHz)<br><br>X-band (≈ 10 GHz carrier) spaceborne SAR |
| 1 GHz – 5 GHz | 0.15 m – 0.03 m | Higher frequency (≥ 10 GHz carrier) airborne SAR (some cases)<br><br>Higher frequency (≥ 70 GHz carrier) automotive Radar |

# *Angular resolution*

# Angular resolution




Moving a RADAR along a straight line = measuring the distance from the target to each point on the line

⇔ 2D Localization

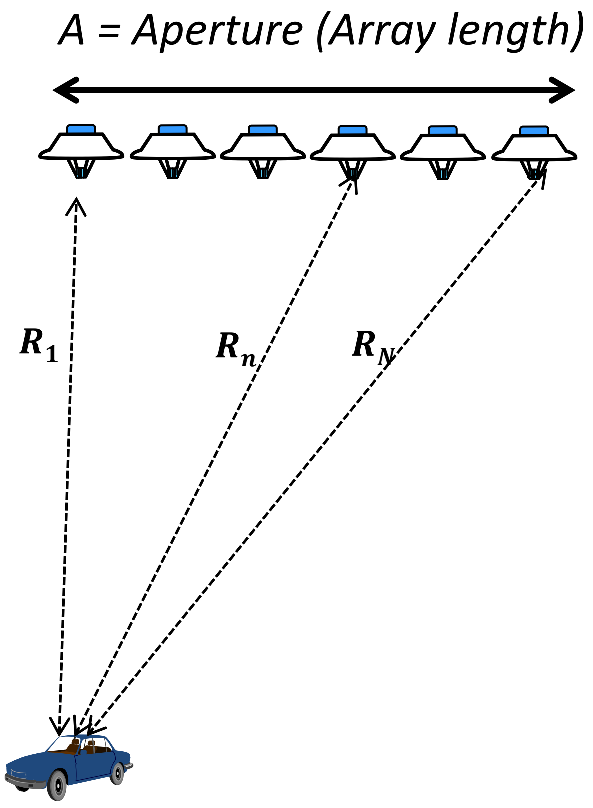
***How ?***

***Let's take a step back….***

# Angular resolution

Consider an array of *N* antennas, sequentially emitting a ***monochromatic wave***

*Note: Monochromatic wave* ⇔ *0 bandwidth* ⇔ $g(t) = 1$ ⇔ *no range resolution*




*Transmitted signal*

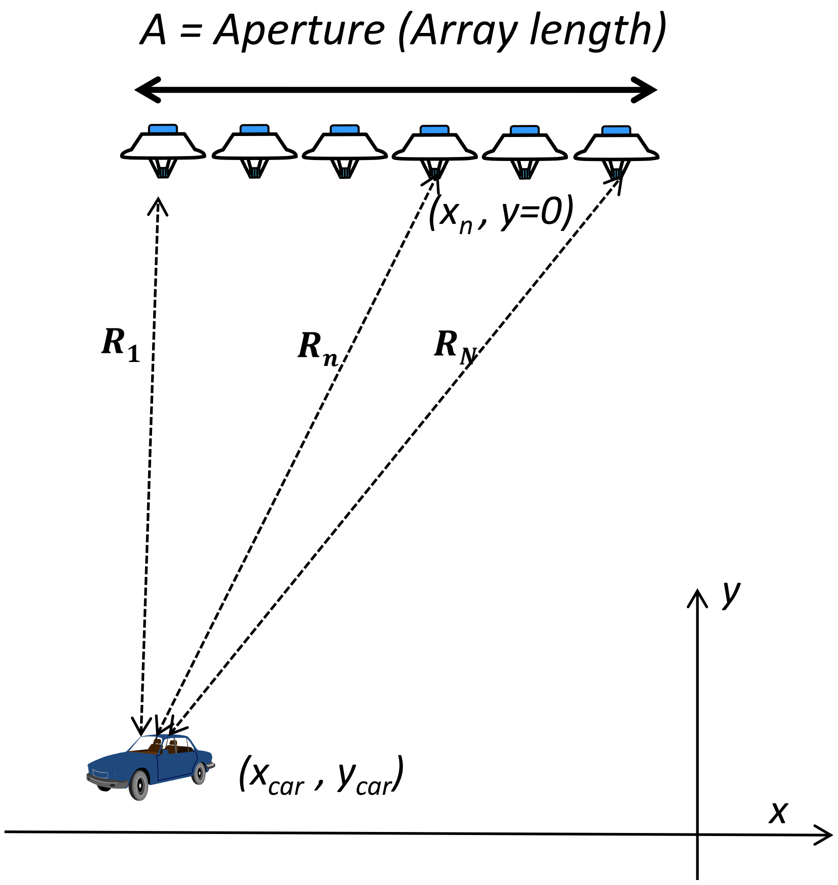
$$s_{Tx}(t) = exp(j2\pi f_0 t)$$

*Received signal*

$$s_{Rx}(n) = A_{car} \cdot e^{-j\frac{4\pi}{\lambda}R_n}$$

Consider an array of *N* antennas, sequentially emitting a ***monochromatic wave***

*Note: Monochromatic wave* ⇔ *0 bandwidth* ⇔ $g(t) = 1$ ⇔ *no range resolution*




*Received signal*

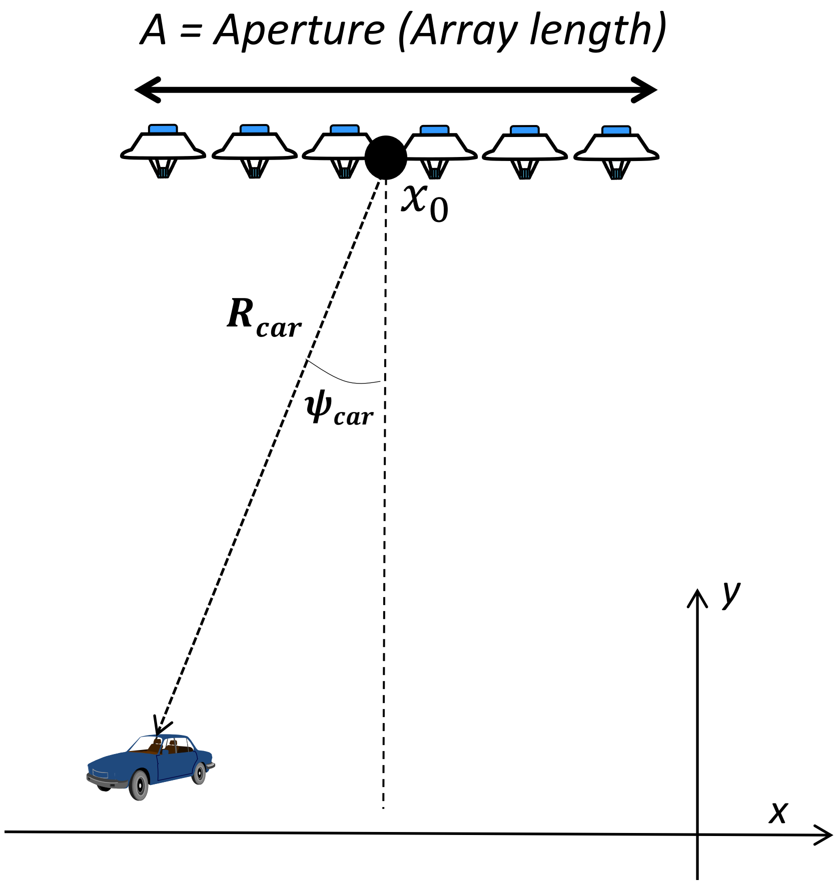
$$s_{Rx}(n) = A_{car} \cdot e^{-j\frac{4\pi}{\lambda}R_n}$$

$$R_n = \sqrt{(x_n - x_{car})^2 + (y_{car})^2}$$

# Angular resolution

Consider an array of *N* antennas, sequentially emitting a ***monochromatic wave***

*Note: Monochromatic wave ⇔ 0 bandwidth ⇔ $g(t) = 1$ ⇔ no range resolution*




*Received signal*

$$s_{Rx}(n) = A_{car} \cdot e^{-j\frac{4\pi}{\lambda}R_n}$$

$$R_n \cong R_{car} + sin(\psi_{car}) \cdot (x_n - x_0)$$

*Valid for* $R_{car} >> A$

# Angular resolution

Consider an array of *N* antennas, sequentially emitting a ***monochromatic wave***

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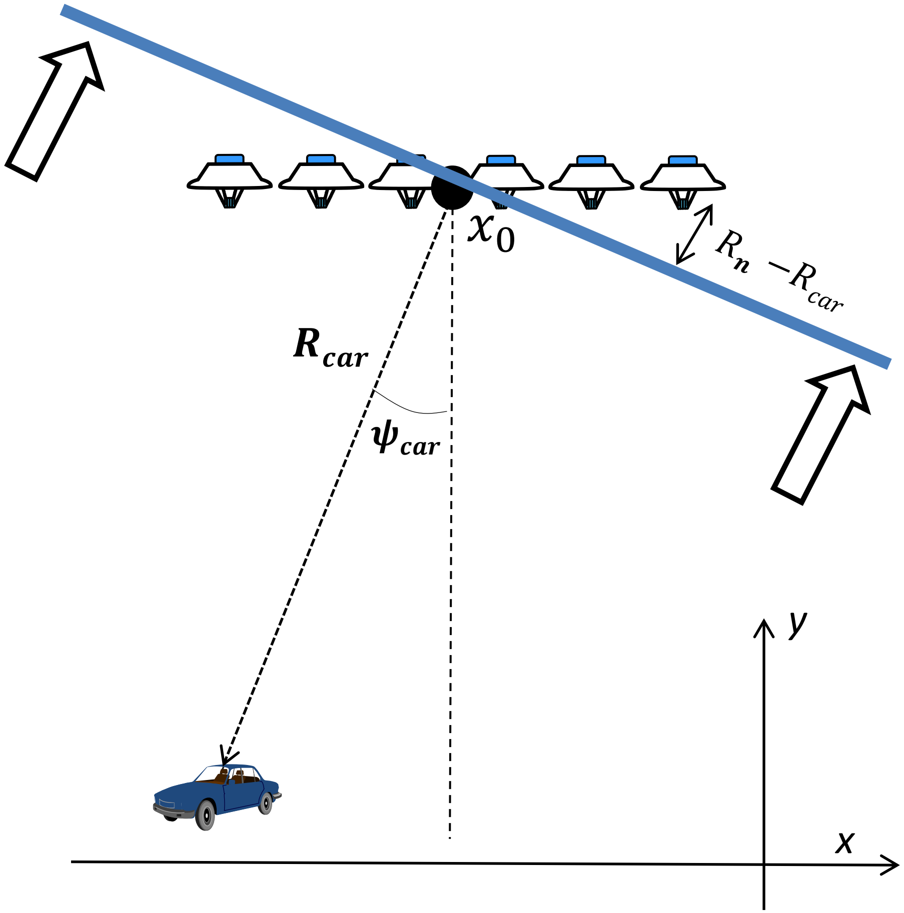


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*Valid for* $R_{car} >> A$

***Equivalent to a planar wavefront from the car to the antenna array***

# Angular resolution

***Plane wavefront approximation***

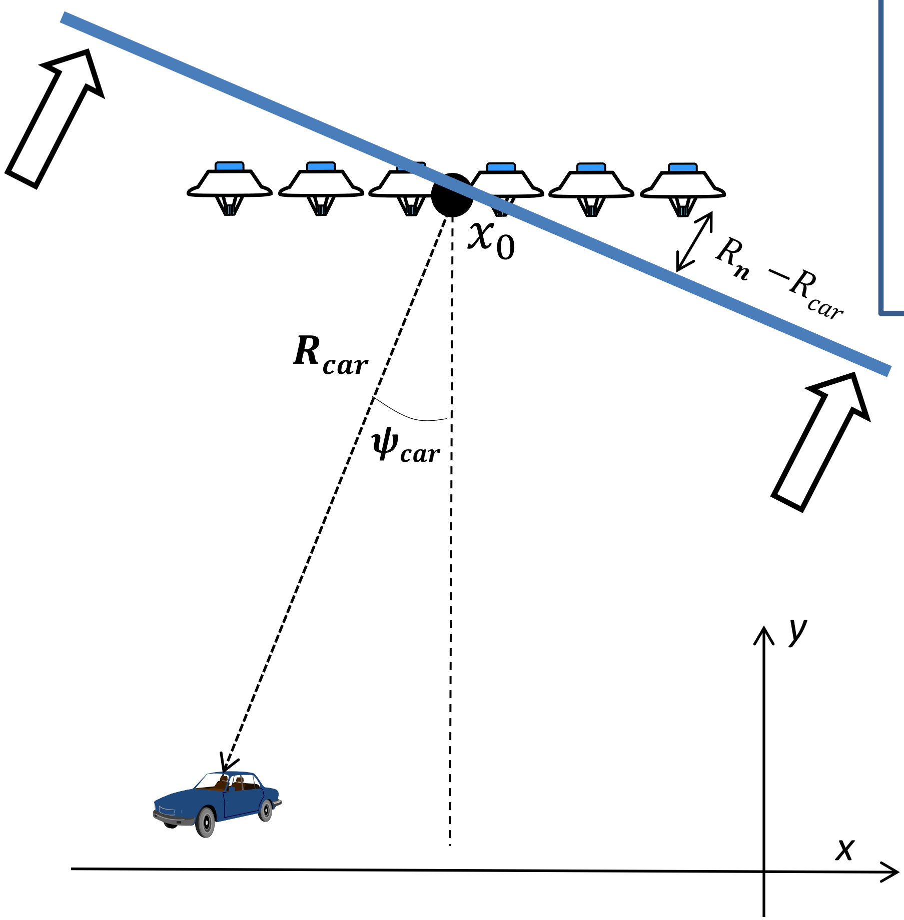
$$R_{\boldsymbol{n}} \cong R_{car} + sin(\psi_{car}) \cdot (x_n - x_0)$$

***Received signal***

$$s_{Rx}(n) \cong A_{car} e^{-j\frac{4\pi}{\lambda}R_{car}} \cdot e^{-j\frac{4\pi}{\lambda}sin(\psi_{car})\cdot x_n}$$

*Complex sinusoid with spatial frequency*

$$f_{car} = \frac{2}{\lambda} sin(\psi_{car})$$




*The Direction of Arrival (DoA) of the wavefront impinging on the array can be found by measuring the spatial frequency along the array*

# Angular resolution

***Plane wavefront approximation***

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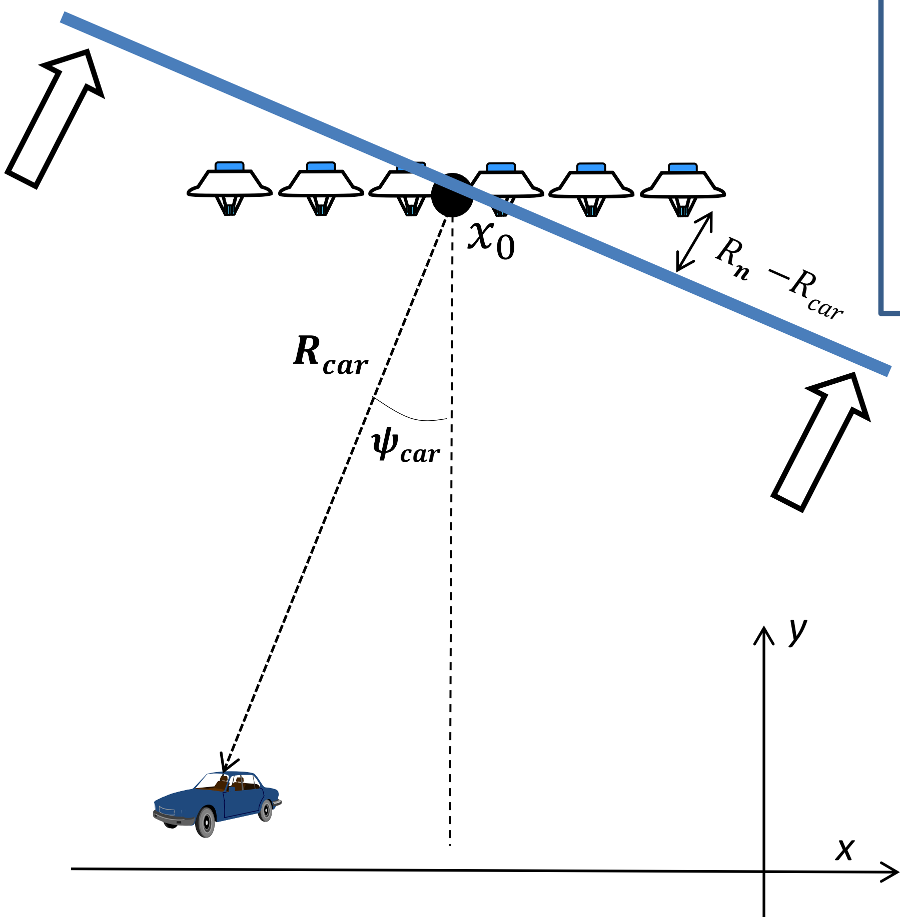


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*The Direction of Arrival (DoA) of the wavefront impinging on the array can be found by measuring the spatial frequency along the array*

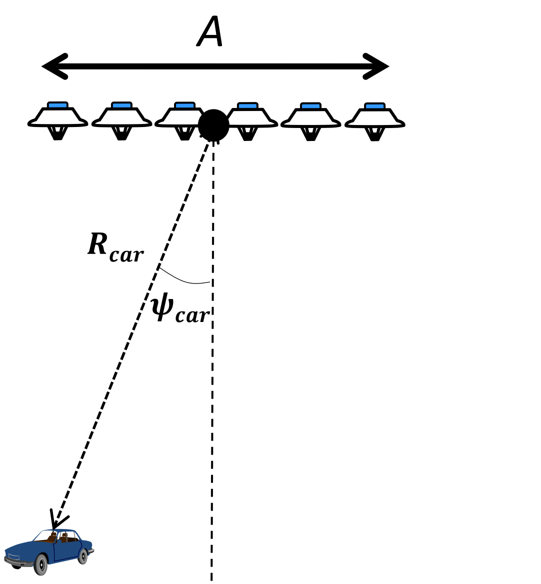
*We need a Fourier Transform!*

# Angular resolution

## Signal along the array

$$s_{Rx}(n) \cong A_{car} e^{-j\frac{4\pi}{\lambda}R_{car}} \cdot e^{-j2\pi f_{car} \cdot x_n}$$

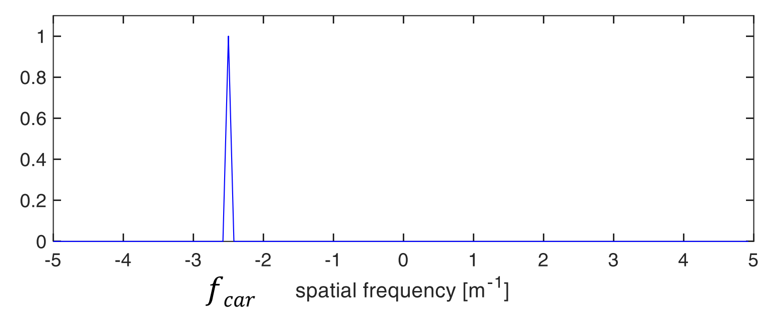
$$f_{car} = \frac{2}{\lambda} sin(\psi_{car})$$




## Fourier Transform

The signal to be transformed contains a single sinusoid at frequency $f_{car}$

$\Rightarrow$ We would expect its Fourier Transform to show a single peak at frequency $f_x = f_{car}$



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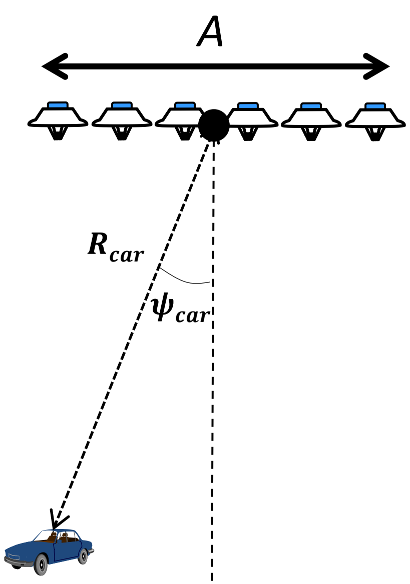
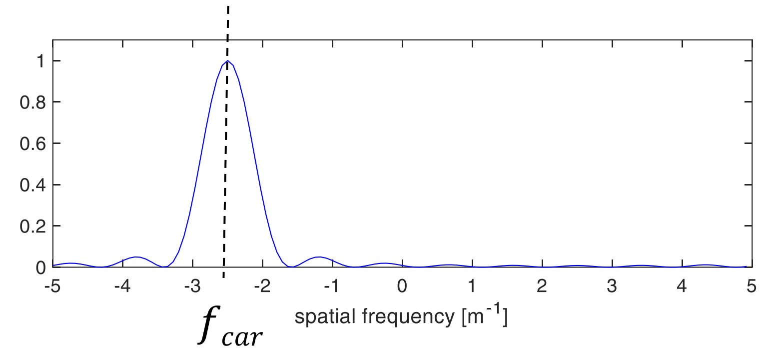



## Fourier Transform

The signal to be transformed contains a single sinusoid at frequency $f_{car}$

However, we find something *quite different*

- A peak is present at the right position ($f_x = f_{car}$)…
- … but is spread across an interval of frequencies



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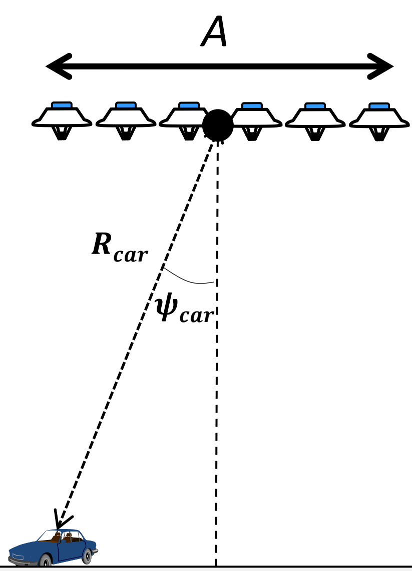


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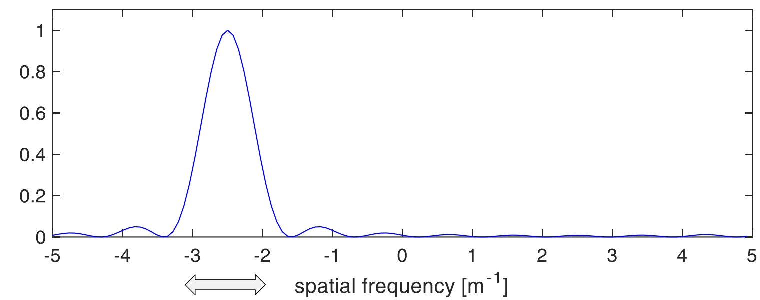



$$\Delta f \approx \frac{1}{A}$$

The reason for the spread is the inverse proportionality between signal duration and bandwidth

The signal along the array has a "*duration*" of $A$ meters hence its FT has a bandwidth $\Delta f \approx \frac{1}{A}$

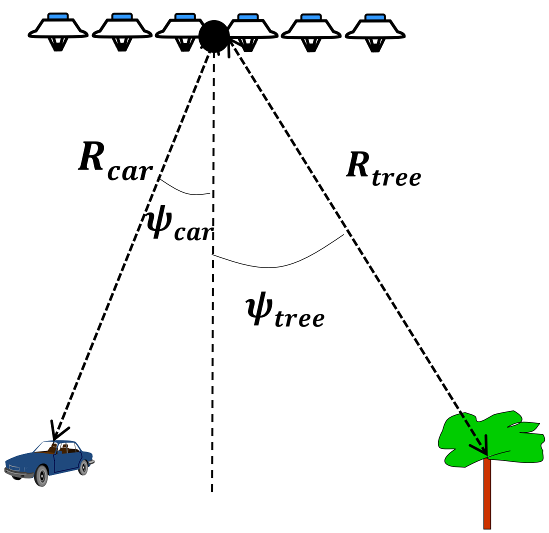
# Angular resolution

The link between array aperture and spatial bandwidth leads us directly to the important concept of ***angular resolution***, intended as the capability to distinguish (resolve) two targets found at slightly different angles w.r.t. the Radar

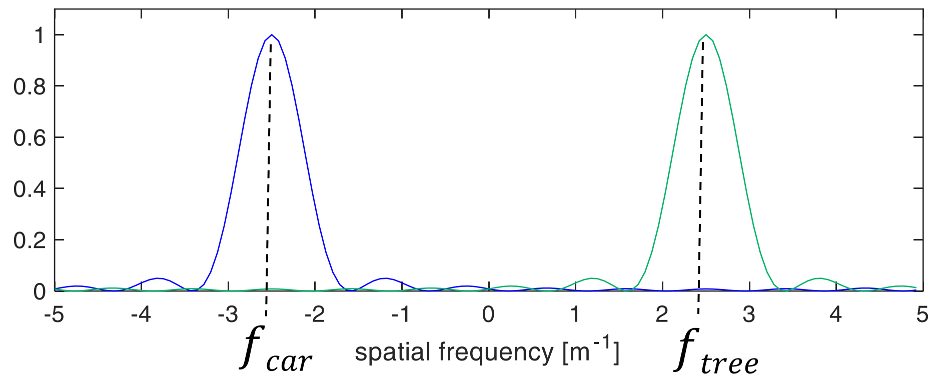
## Signal along the array

$$s_{Rx}(n)$$
$$\cong A_{car} e^{-j\frac{4\pi}{\lambda}R_{car}} \cdot e^{-j2\pi f_{car} \cdot x_n} + A_{tree} e^{-j\frac{4\pi}{\lambda}R_{tree}} \cdot e^{-j2\pi f_{tree} \cdot x_n}$$

$$f_{car} = \frac{2}{\lambda} sin(\psi_{car}) \qquad f_{tree} = \frac{2}{\lambda} sin(\psi_{tree})$$




## Fourier Transform



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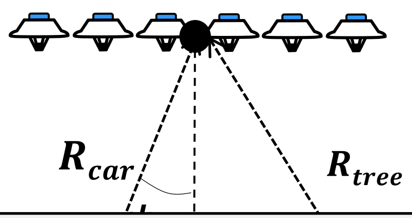
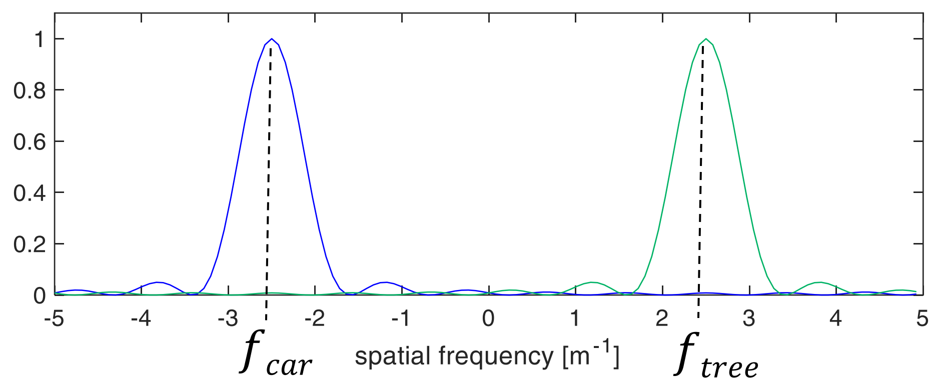



### Fourier Transform




$\Rightarrow$ We can tell there are two targets as long as the received signal exhibits **two distinct peaks**

This occurs upon the condition that:

$$|f_{car} - f_{tree}| \geq \Delta f \approx \frac{1}{A} \Longrightarrow |\psi_{car} - \psi_{tree}| \geq \Delta\psi \approx \frac{\lambda}{2A}$$

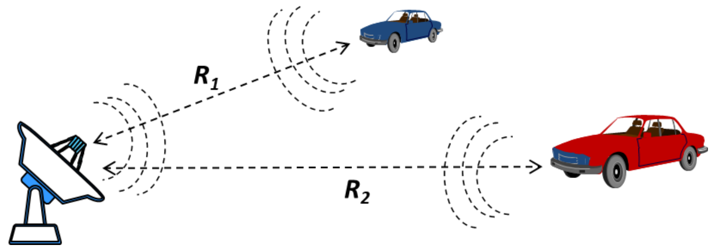
Where $\Delta\psi = \frac{\lambda}{2A}$ is referred to as the **angular resolution** of the array

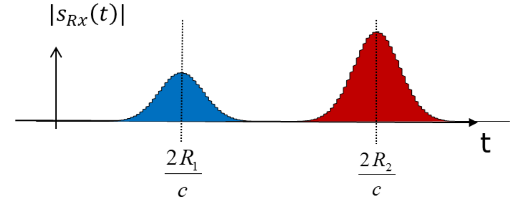
# 2D resolution

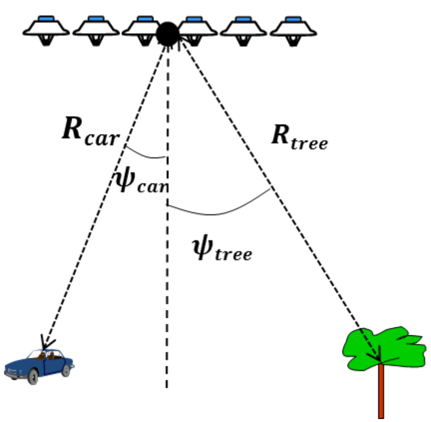
Bandwidth

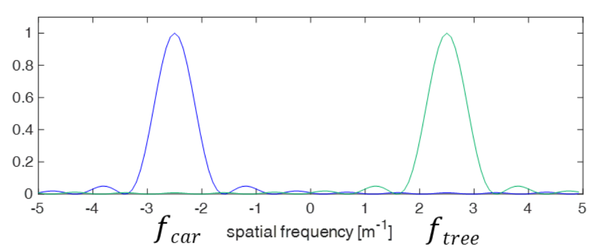
⇔

Range resolution



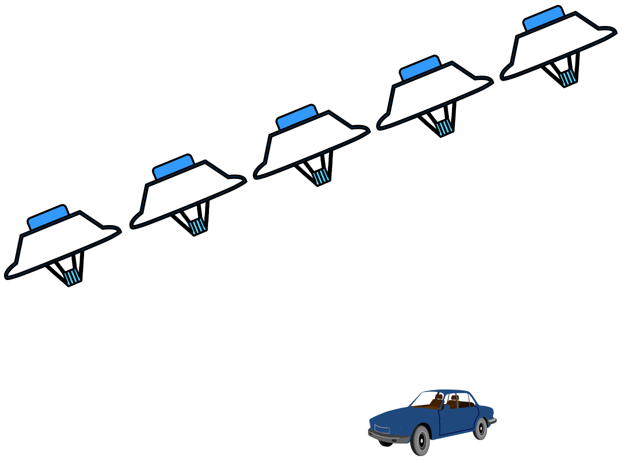

Antenna array emitting a monochromatic wave

⇔

Angular resolution

# 2D resolution

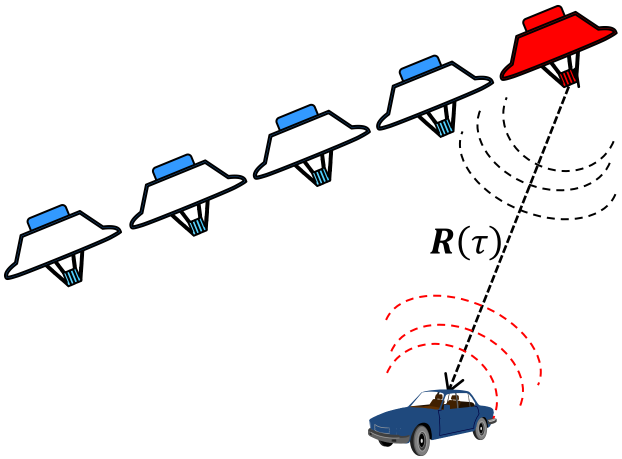
***Antenna array emitting RF pulses***

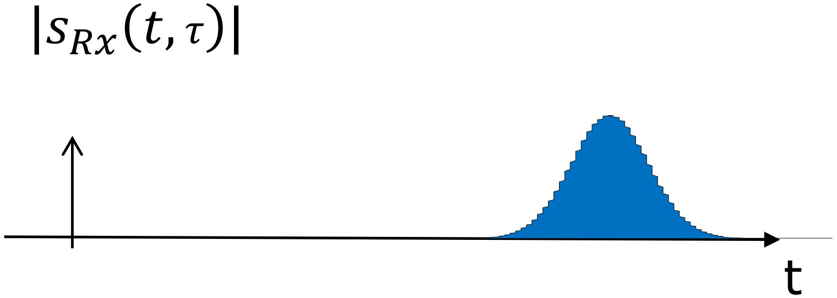


# 2D resolution

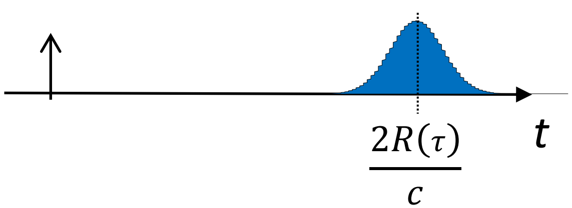
$\tau$ = flight time (or *slow time*, in jargon)

$t$ = time w.r.t. transmission (or *fast time*, in jargon)




$$s_{Rx}(t,\tau) = A_{car}\, g\left(t - \frac{2R(\tau)}{c}\right) \cdot e^{-j\frac{4\pi}{\lambda}R(\tau)}$$



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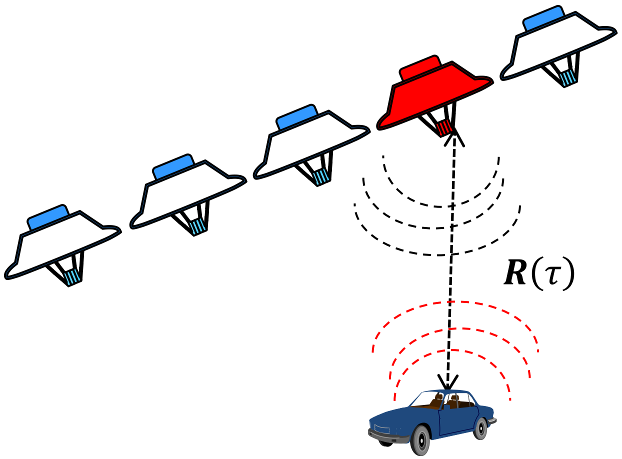


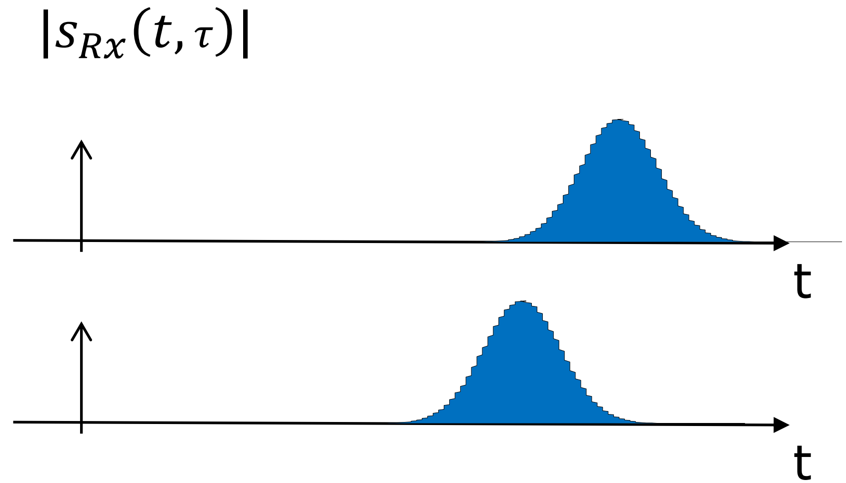


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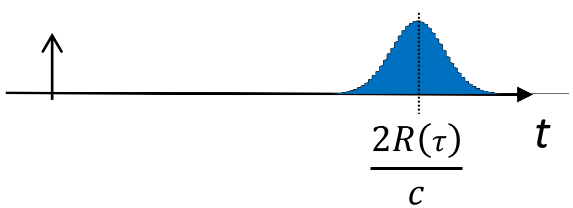

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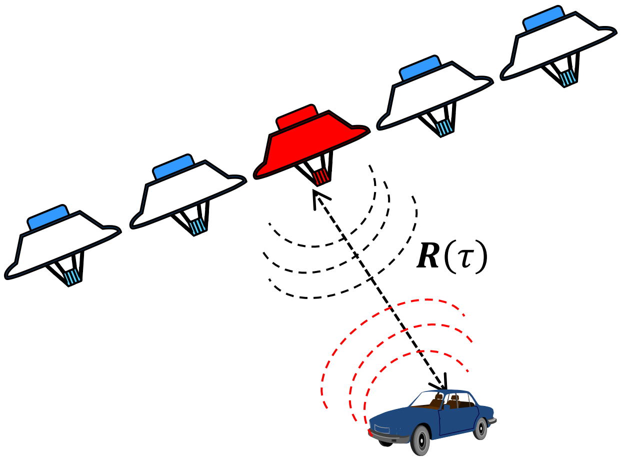


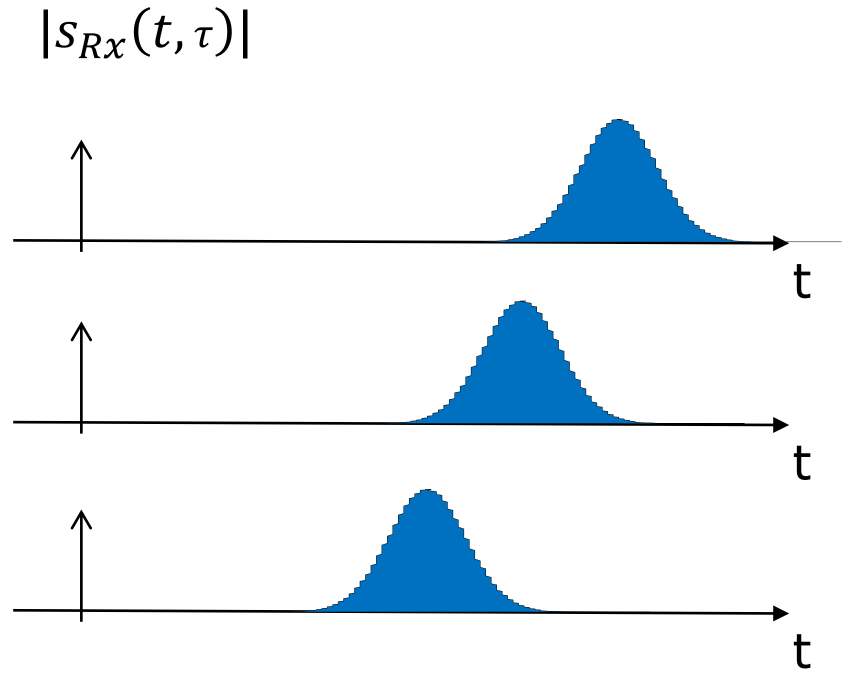


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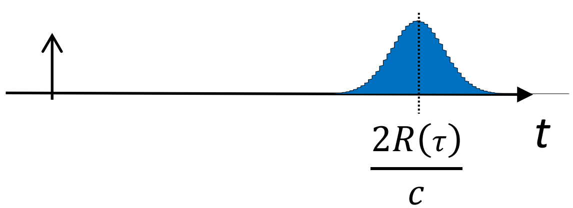

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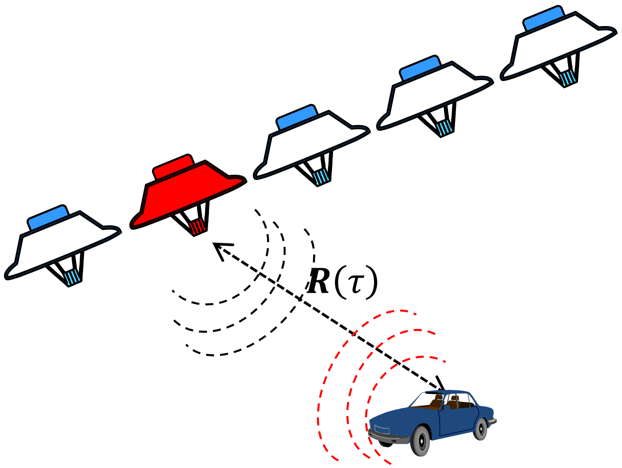


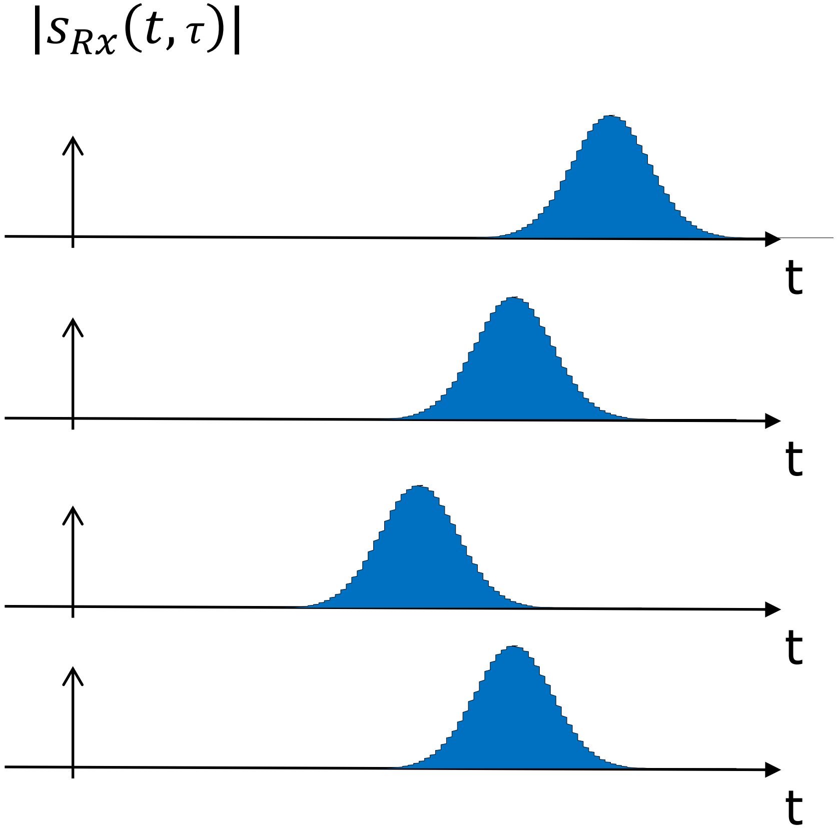


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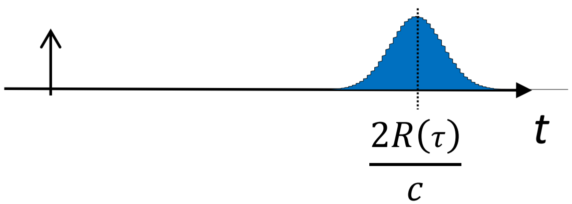

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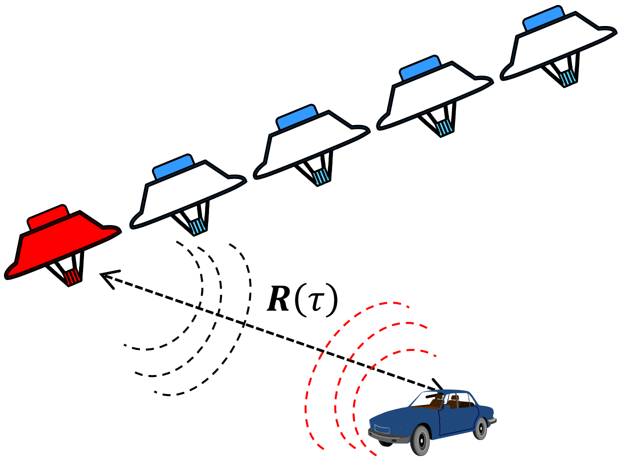
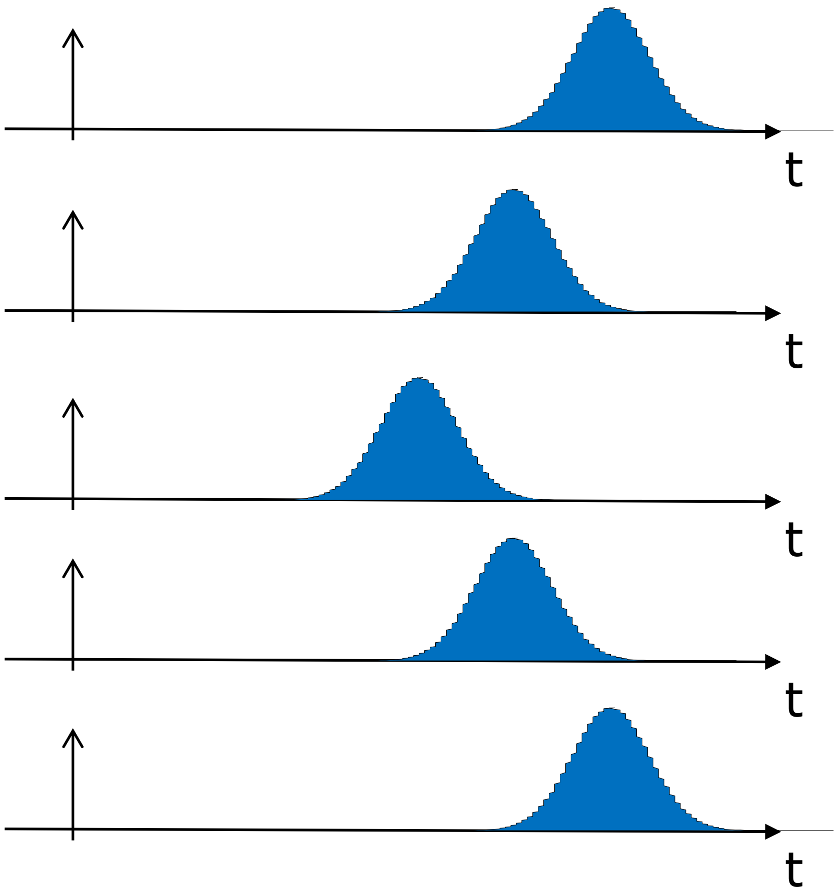



$|s_{Rx}(t,\tau)|$

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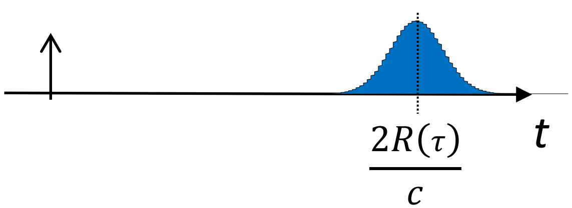

# 2D resolution

$\tau$ = flight time (or *slow time*, in jargon)

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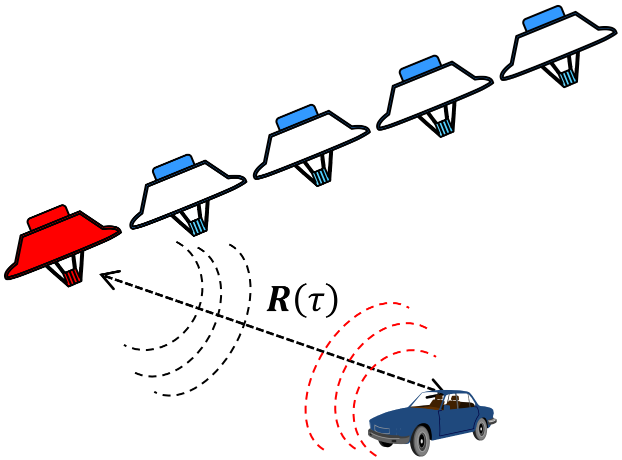
*The delay variation is referred to as **range migration***

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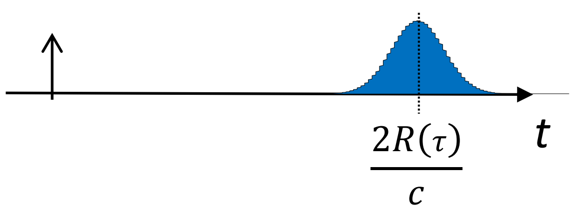
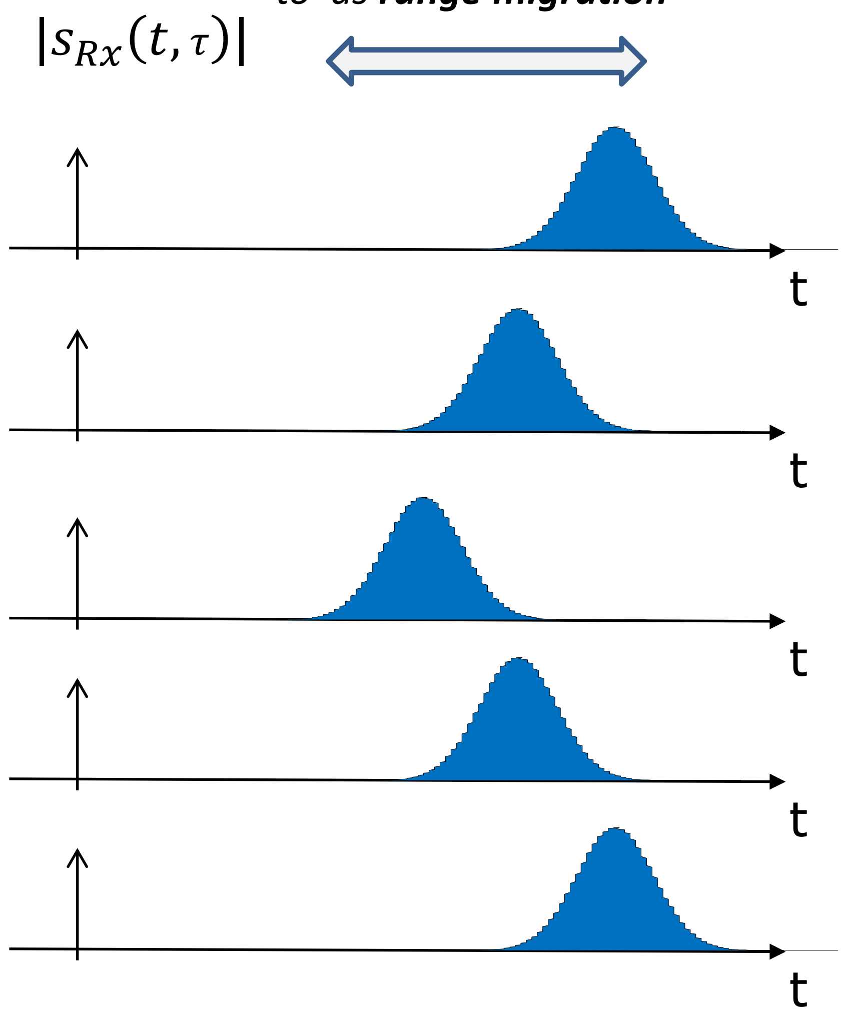
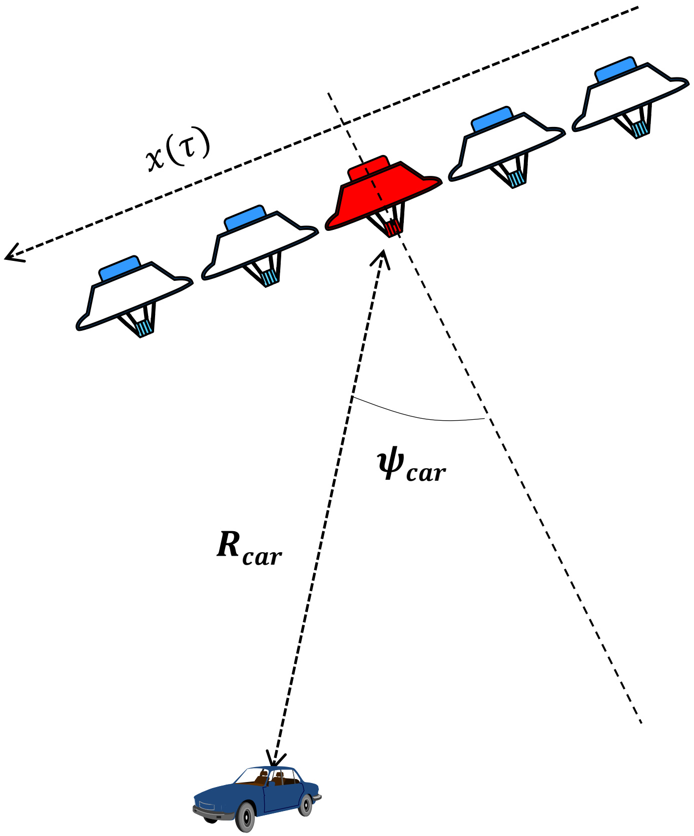


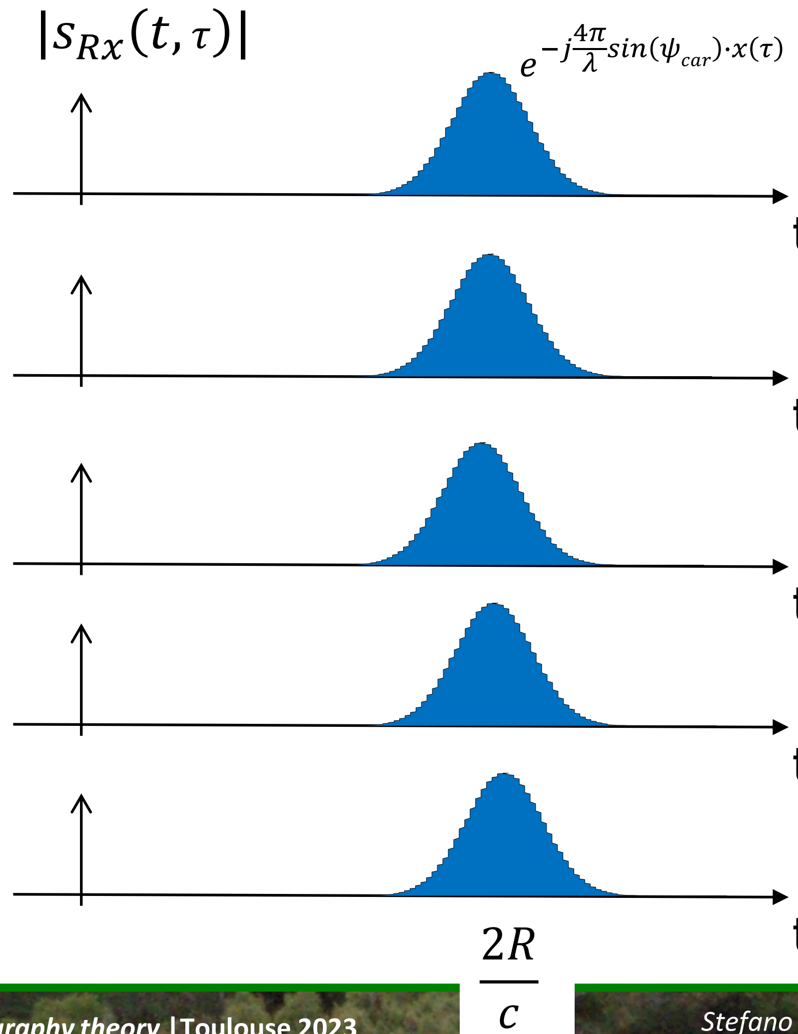



# 2D resolution

***Hp1: range migration is negligible*** ⇔ ***we can tell the range from the delay*** **(*along t*)**

***Hp2: plane wavefront approximation*** ⇔ ***we can tell the angular position from the frequency*** **(*along*** $\tau$**)**

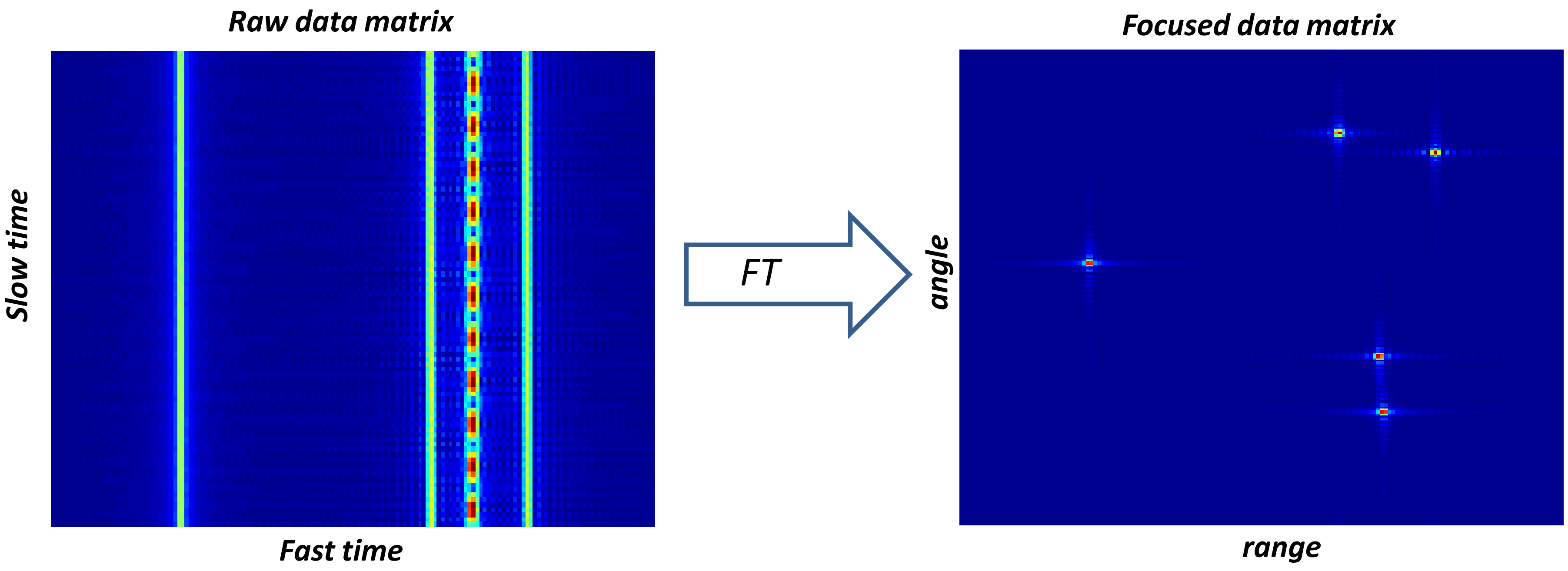


# 2D resolution

Practically, we compute a FT for any value of the fast time

$$S_{Rx}(R,\psi) = \sum_{\tau} s_{Rx}\left(t = \frac{2R}{c}, \tau\right) \cdot e^{-j\frac{4\pi}{\lambda}sin(\psi)x(\tau)}$$



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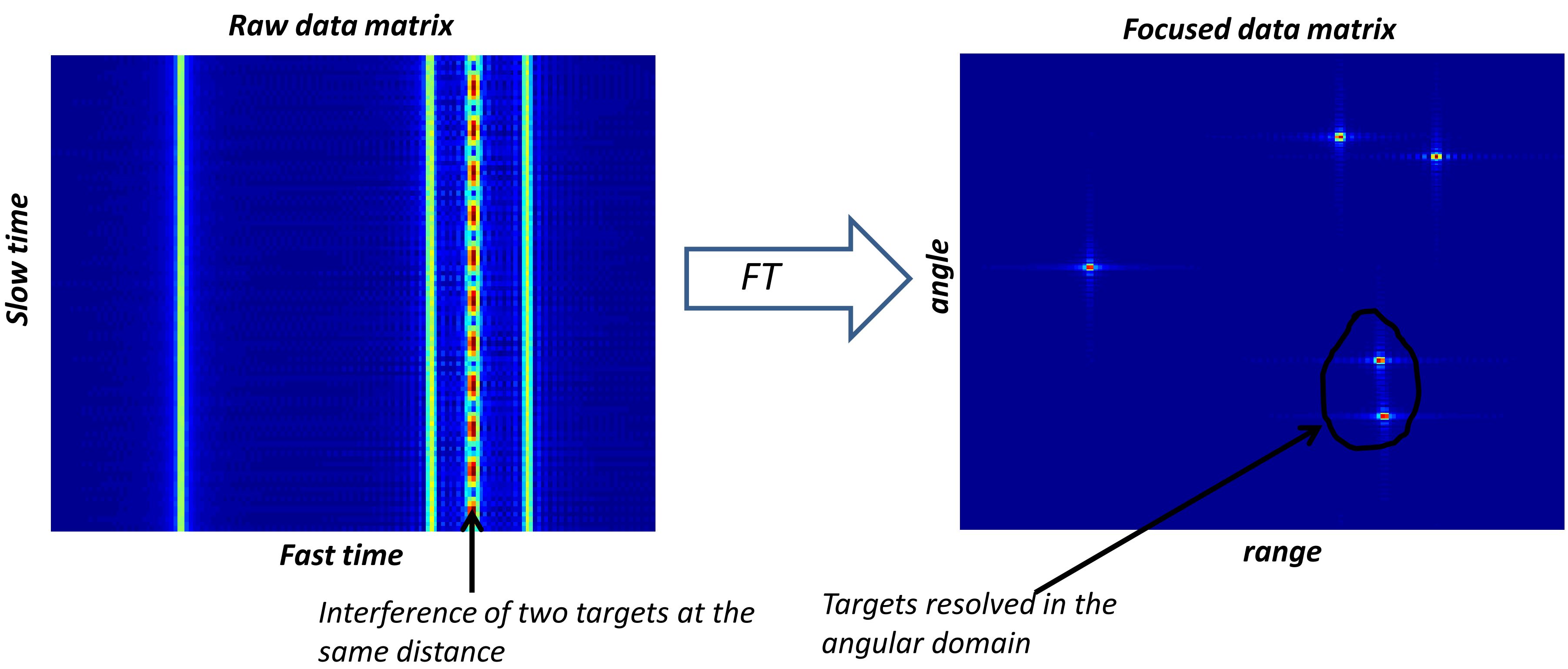
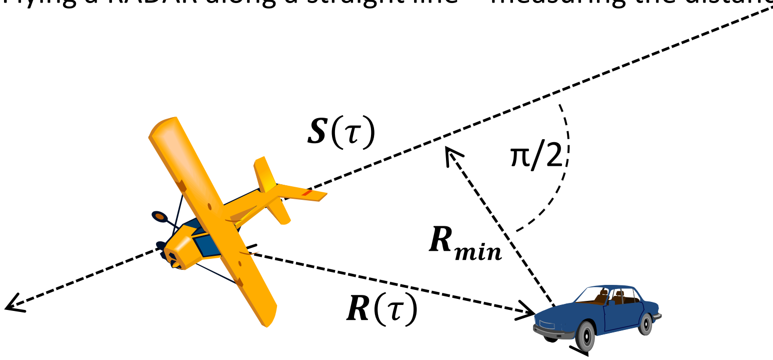


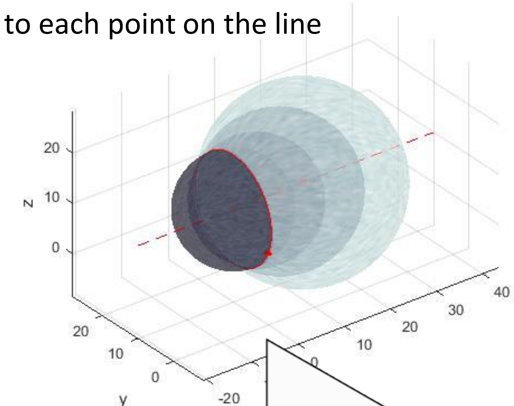
# *SAR Imaging*

# SAR imaging – geometrical interpretation

***Synthetic Aperture Radars (SAR) employ a moving RADAR sensor, flown onboard a satellite or an aircraft, in order to synthesize an antenna as long as several kilometers***

Flying a RADAR along a straight line = measuring the distance from the target to each point on the line



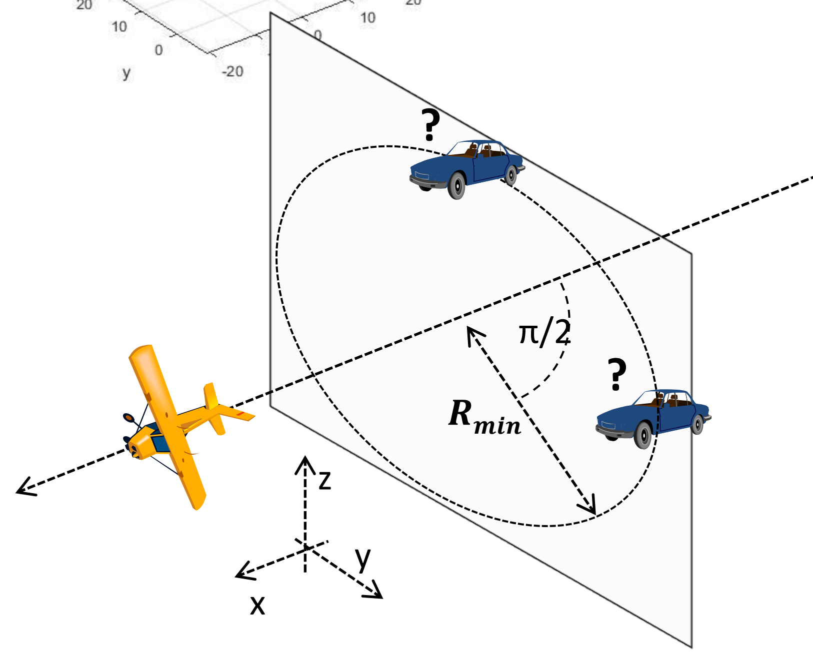

The target is bound to lie on the intersection of all the spheres:

- Centered in $S(\tau)$
- Of radius $R(\tau)$

⇒ The target is bound to lie on the circle:

- Centered on the trajectory
- Perpendicular to the trajectory (*yz* plane)
- Of radius $R_{min}$

⇒ 2D Localization



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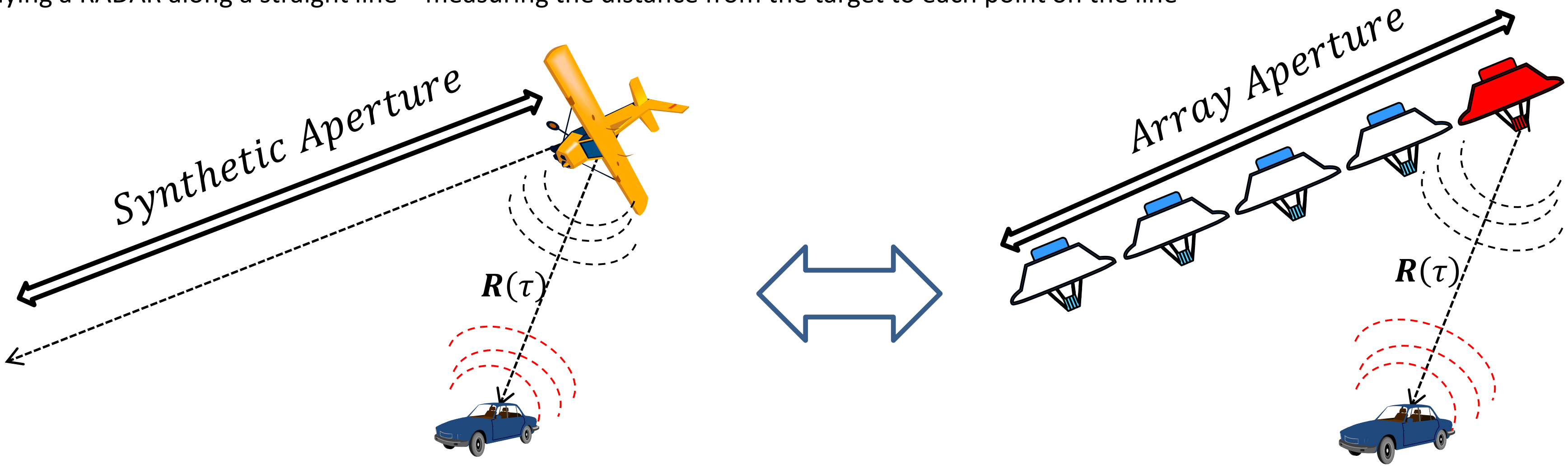
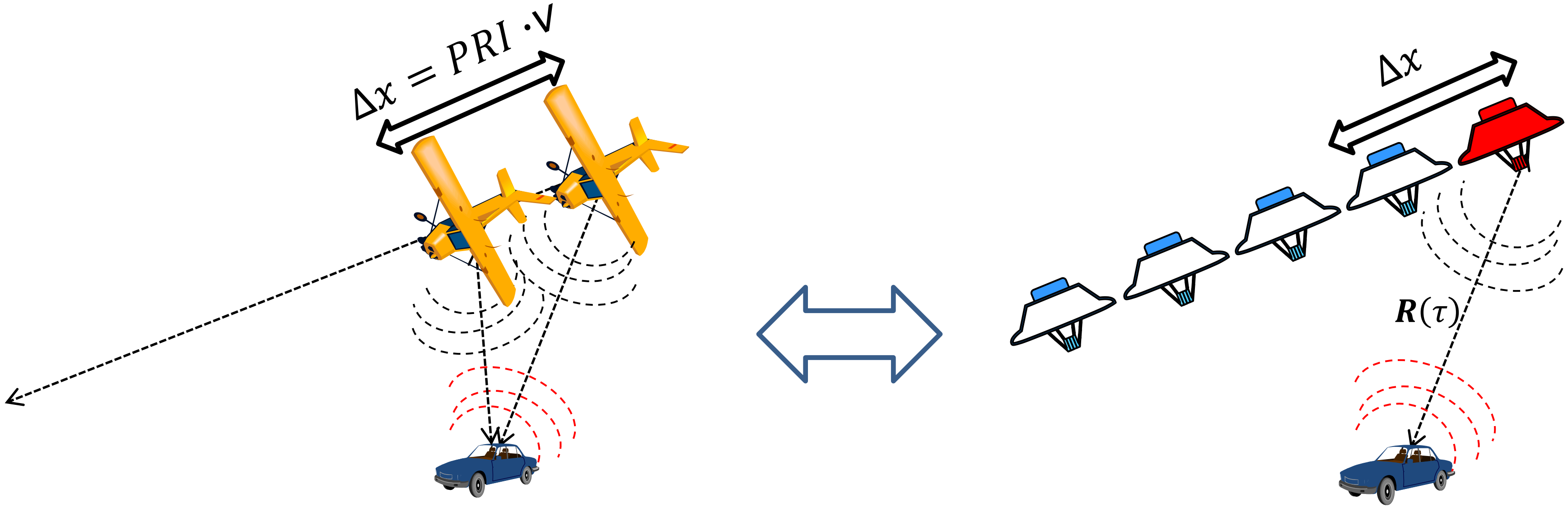


***Start-stop approximation:***

the platform is assumed to be completely still in air (or in space) during pulse transmission and reception




$\Rightarrow$ ***Equivalent to an antenna array !***

***PRI = Pulse Repetition Interval [s]***

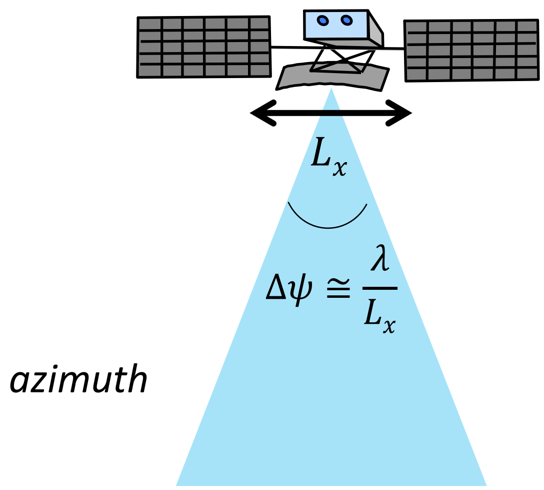
***PRF = Pulse Repetition Frequency [Hz]***

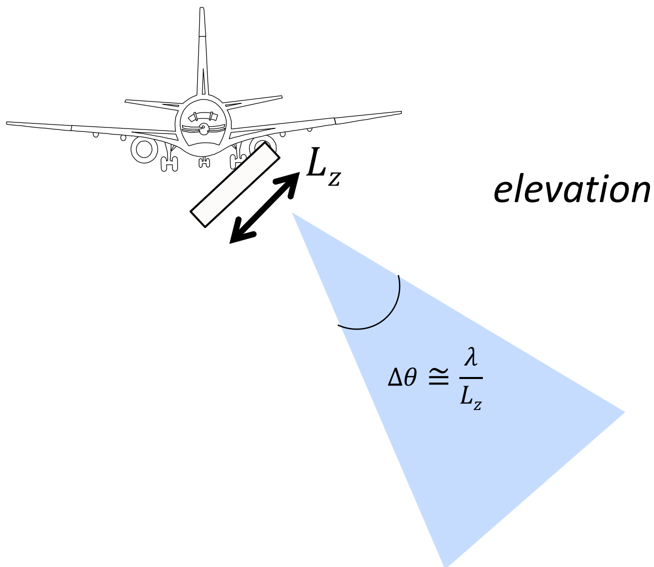
***v = Platform speed [m/s]***

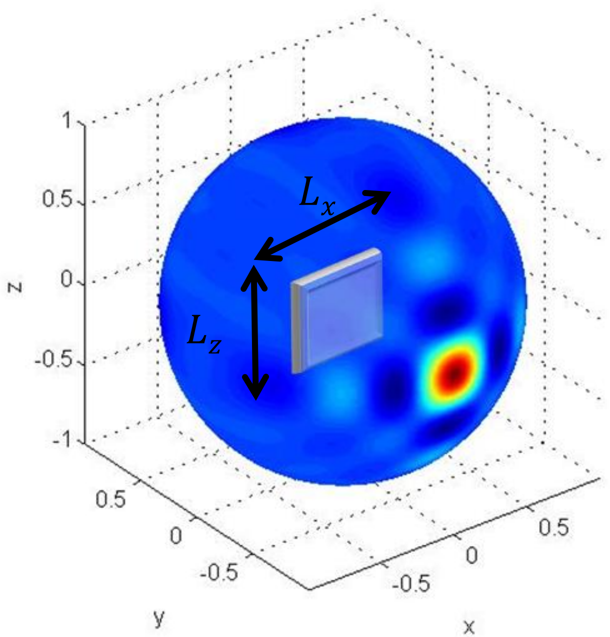
***How long is the synthetic aperture ?***

Target illumination is limited to an angular sector, depending on wavelength and antenna size

$$\Delta\psi \cong \frac{\lambda}{L_x} \qquad \Delta\theta \cong \frac{\lambda}{L_z}$$



# SAR imaging

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***Spaceborne SAR at C-Band***

*Lx = 10-15 m*

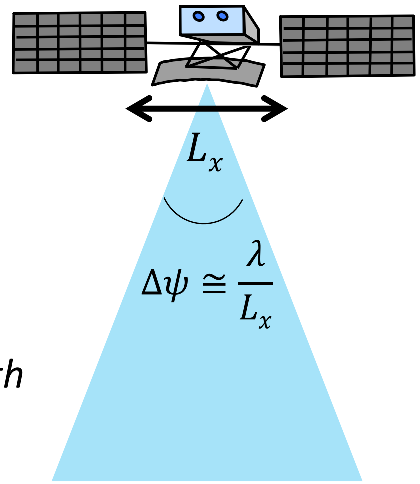
*λ = 5.6 cm*

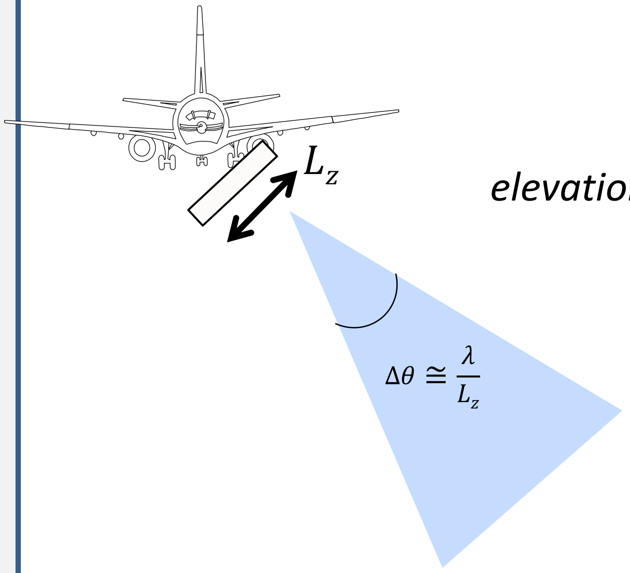
$\Delta\psi \cong 0.5\,°$


***Airborne SAR at P-Band***

*λ = 0.7 m*

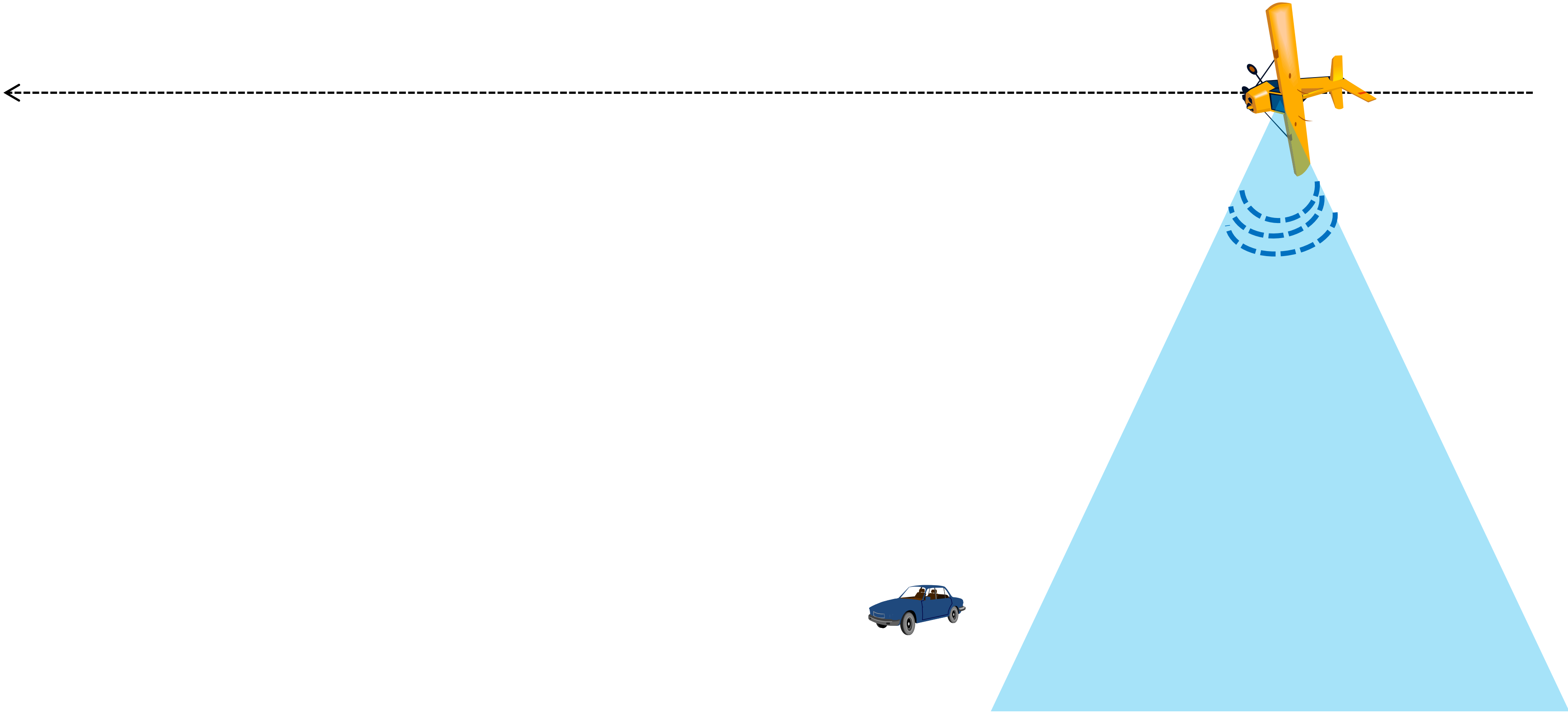
$\Delta\psi \cong 20\,°$



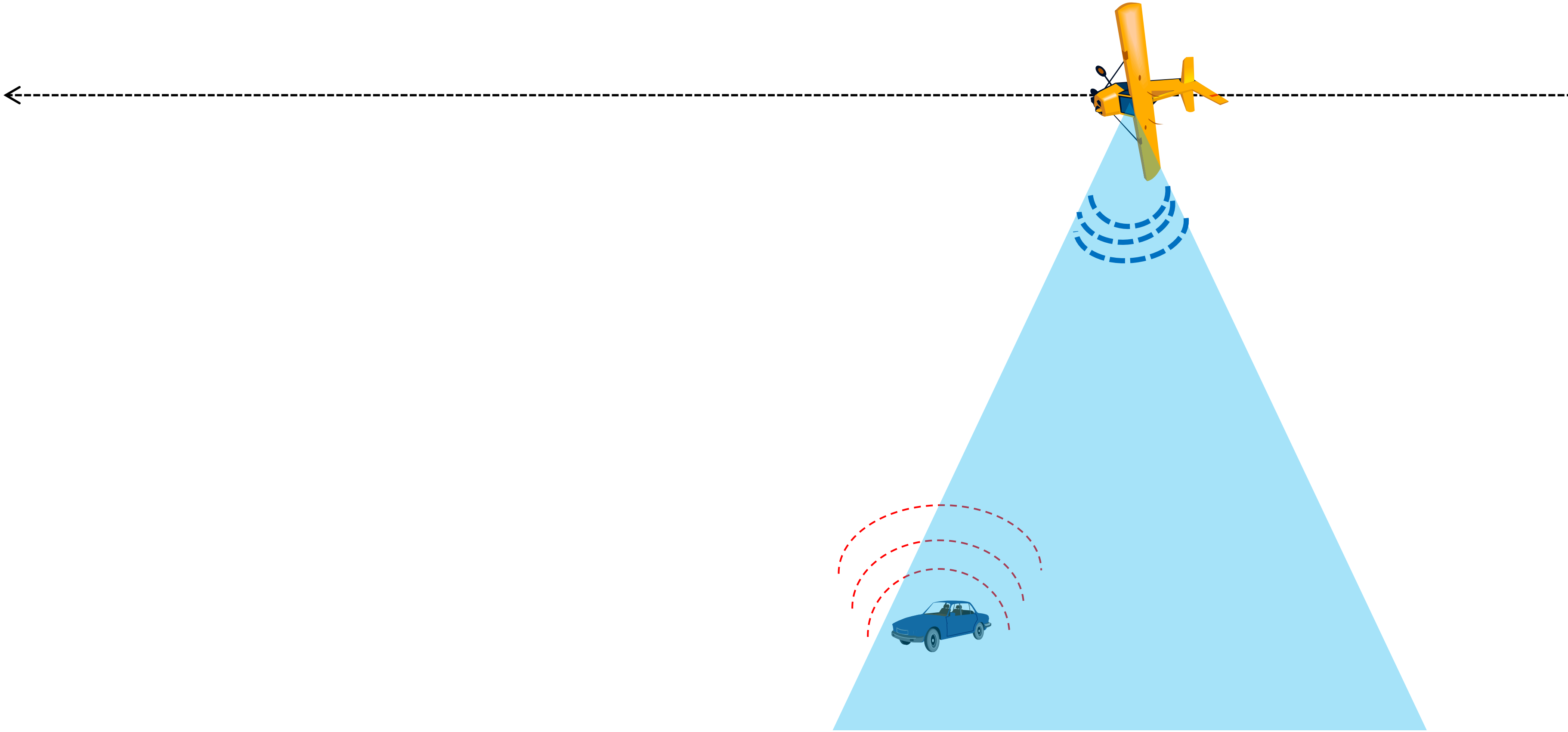
# SAR imaging

⇒ ***Targets are illuminated only a fraction of the time***



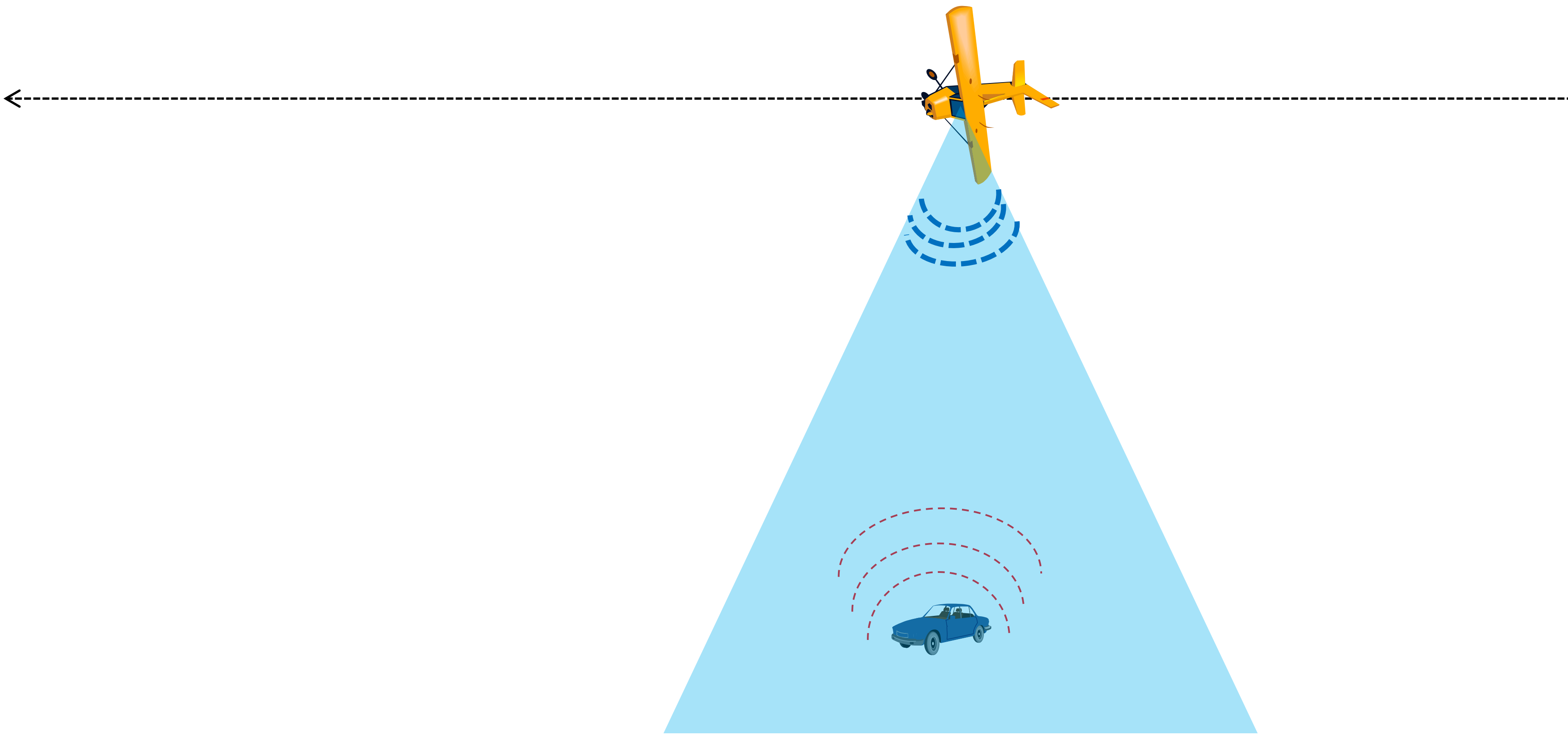
# SAR imaging

⇒ ***Targets are illuminated only a fraction of the time***



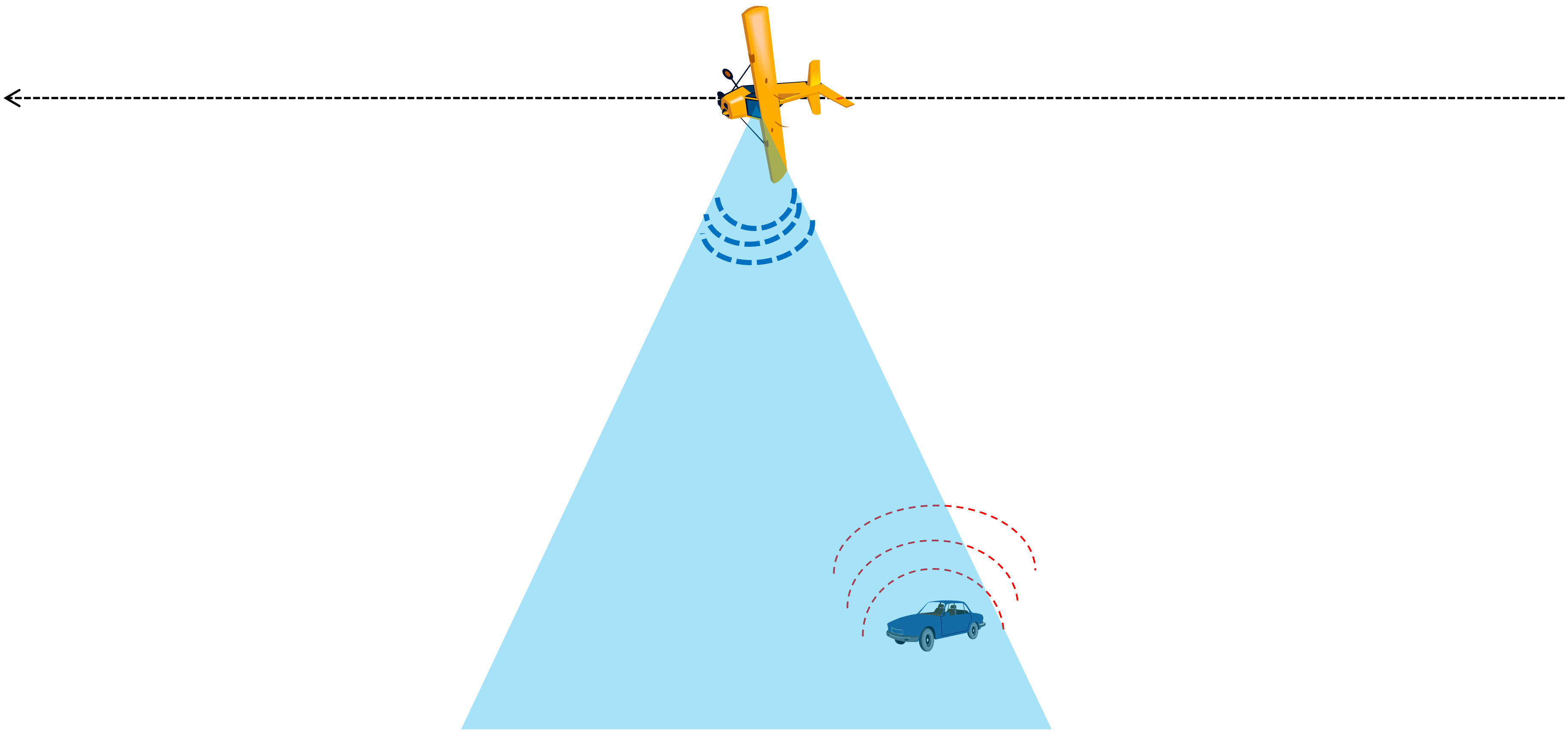
# SAR imaging

⇒ ***Targets are illuminated only a fraction of the time***



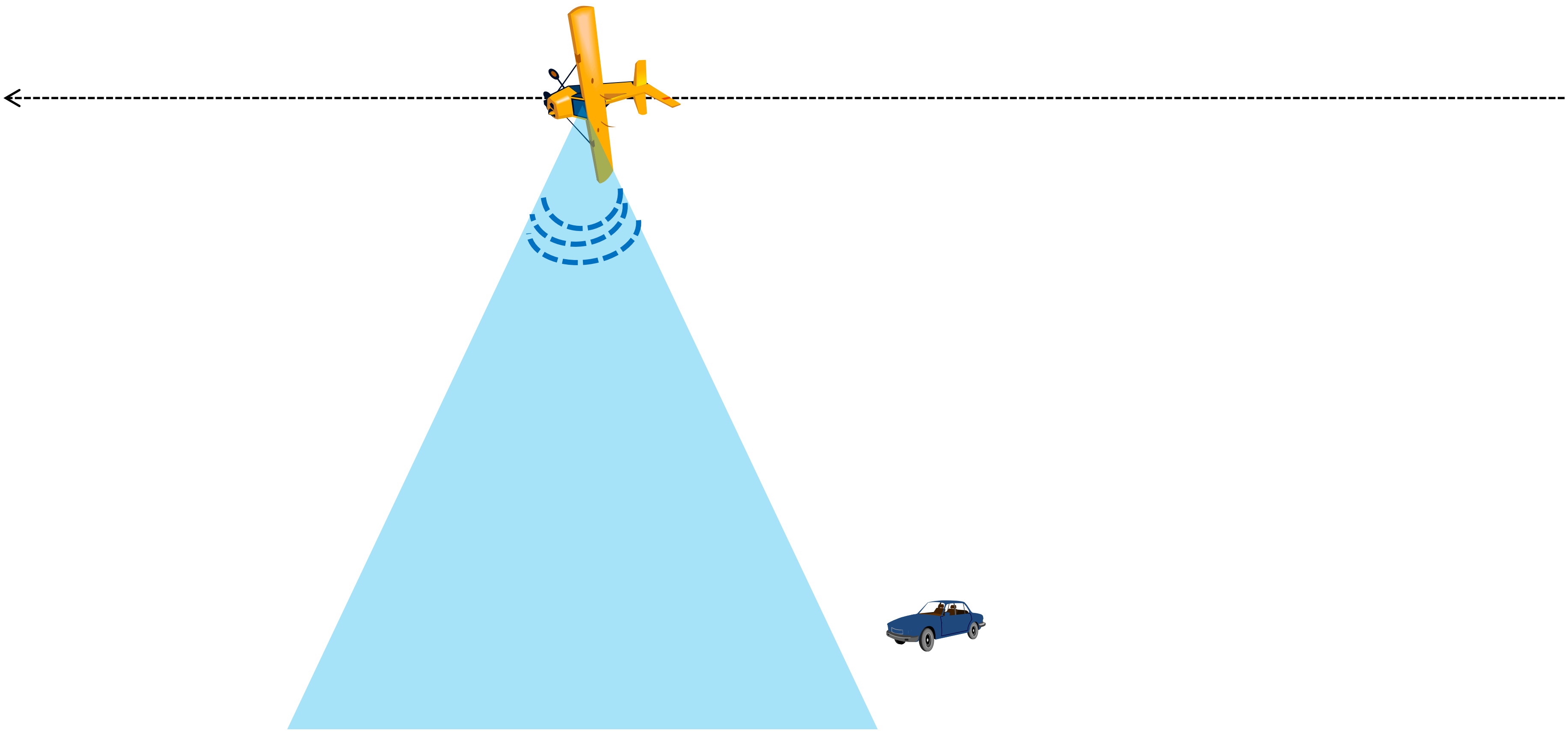
# SAR imaging

⇒ ***Targets are illuminated only a fraction of the time***



# SAR imaging

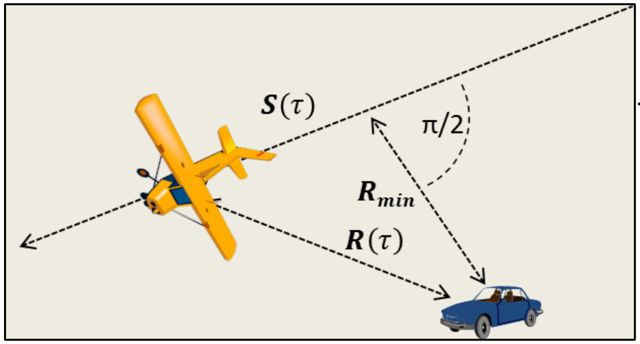
⇒ ***Targets are illuminated only a fraction of the time***

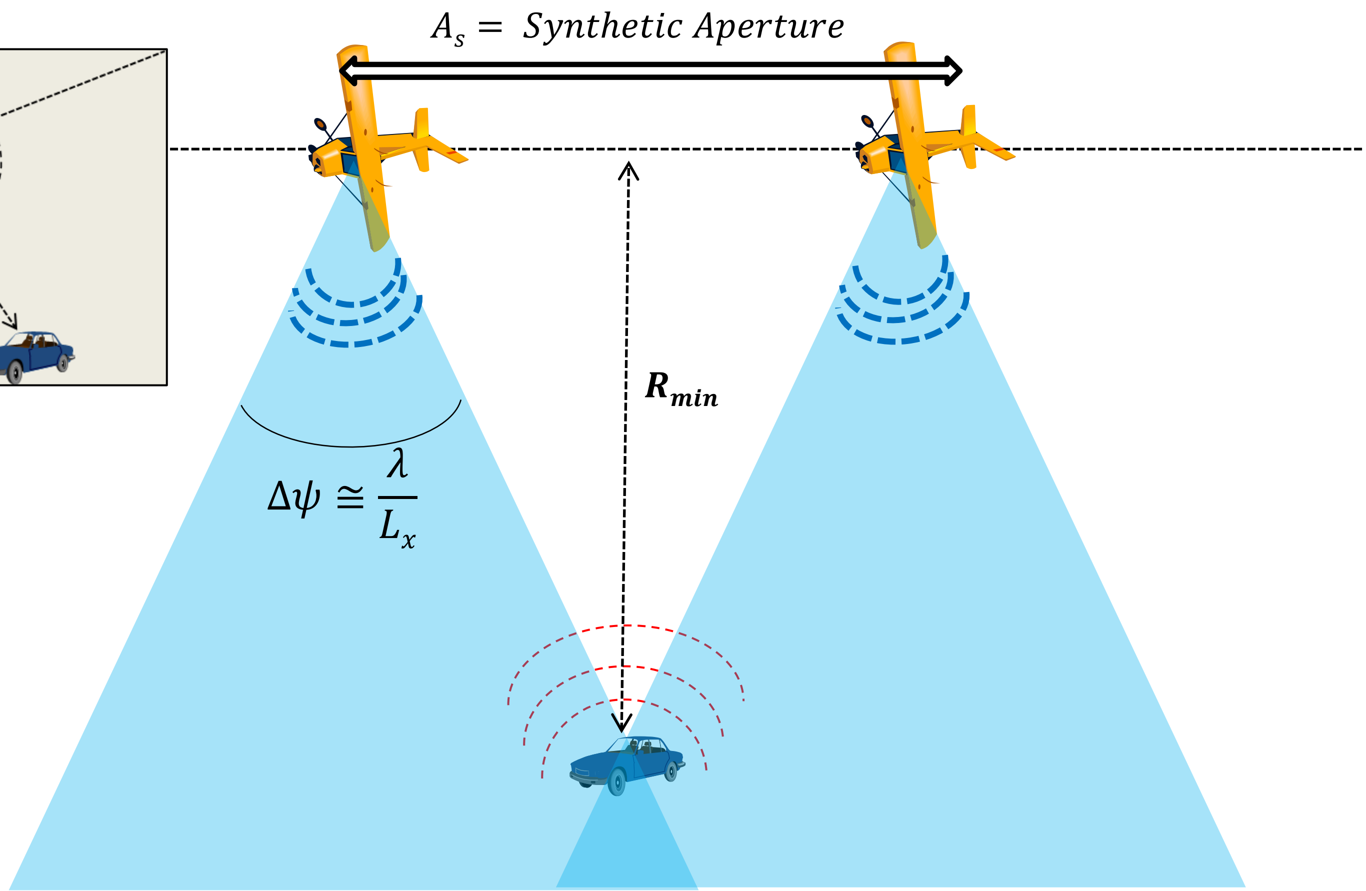


# SAR imaging

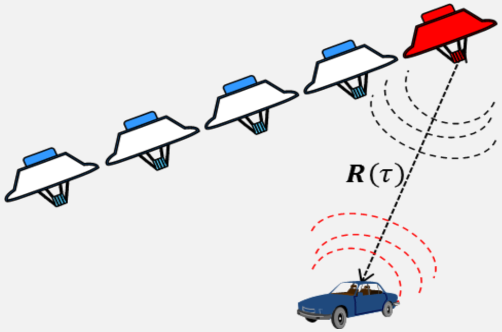
***Length of the synthetic aperture***

$$A_s \cong \Delta\psi \cdot \boldsymbol{R_{min}}$$



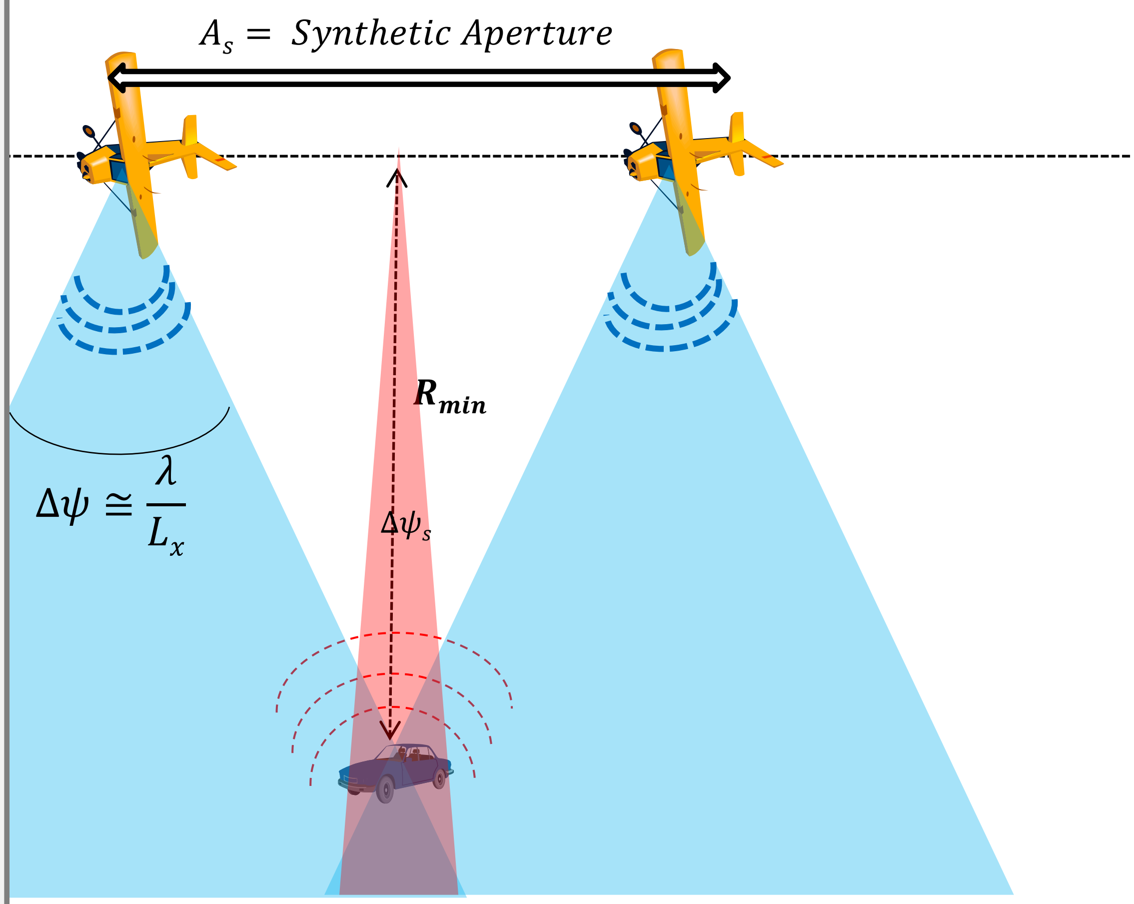



***Result from array theory***



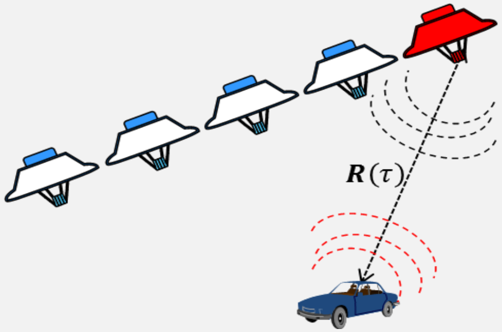

Aperture length translates to angular resolution according to

$$\Delta\psi_s \cong \frac{\lambda}{2A_s}$$



# SAR imaging

***Result from array theory***

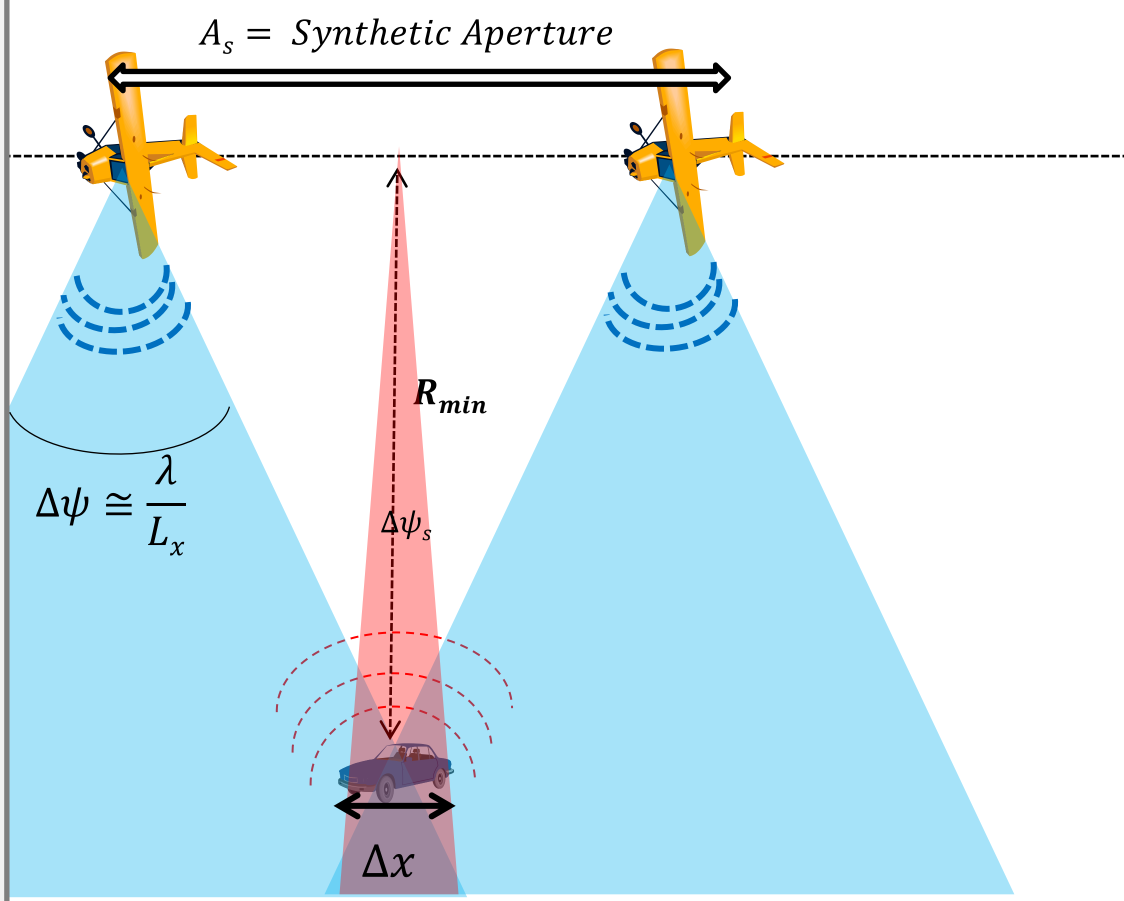



Aperture length translates to angular resolution according to

$$\Delta\psi_s \cong \frac{\lambda}{2A_s}$$

Angular resolution translates into horizontal resolution according to

$$\Delta x \cong \Delta\psi_s \cdot \boldsymbol{R}_{min}$$

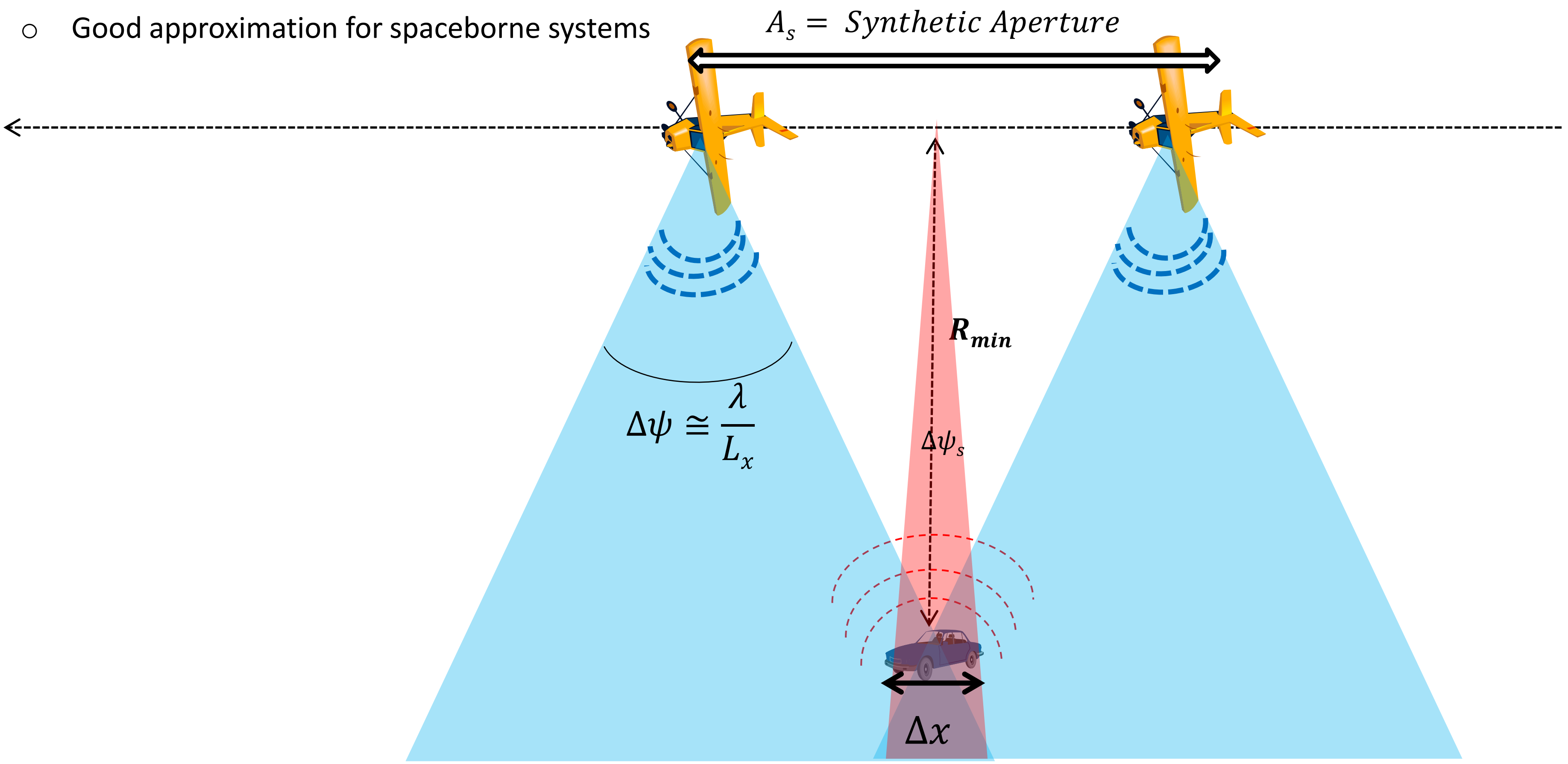


# SAR imaging

***Horizontal (along-track) resolution = half the antenna length***

$$\Delta x \cong \frac{L_x}{2}$$

- Independent on target's distance from the trajectory
- Good approximation for spaceborne systems

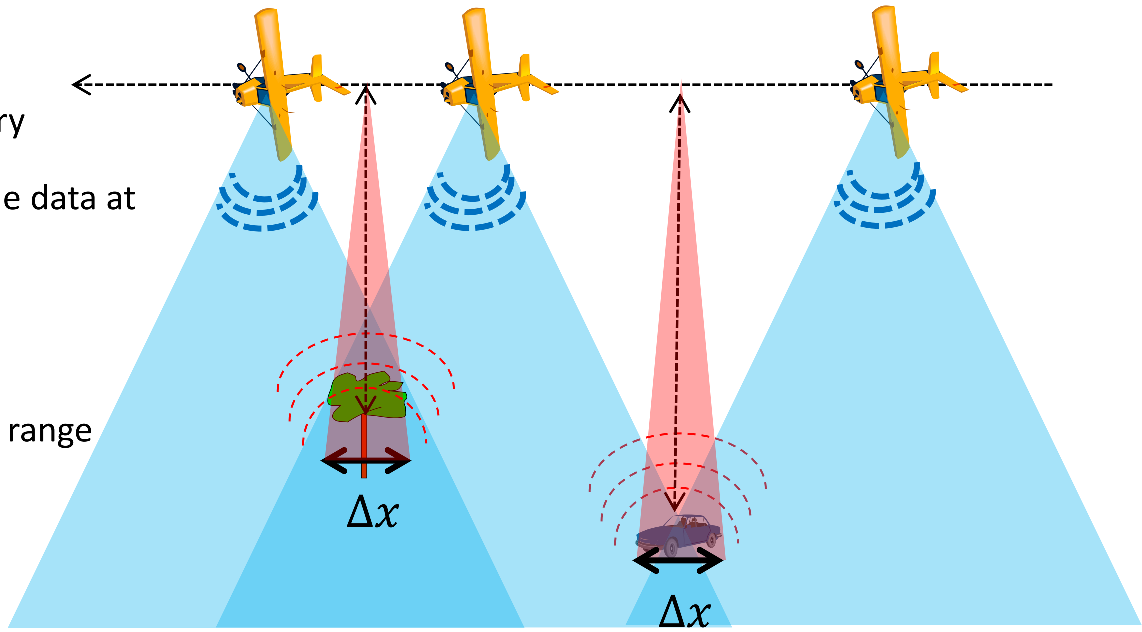


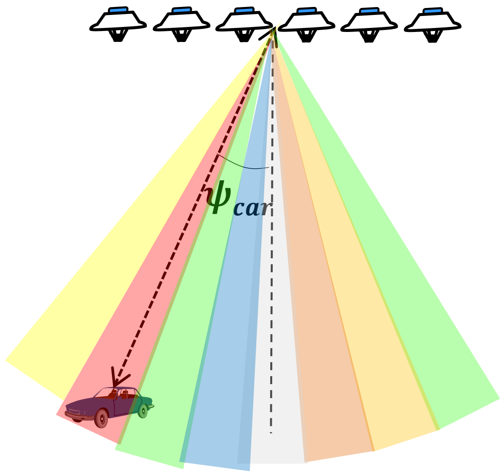
# SAR imaging

***Synthetic Aperture Radar***

- Synthetic Aperture sliding along the trajectory
- Angular resolution is obtained by focusing the data at a **fixed** angle

  Typical choice: ψ = 0 (Zero-Doppler)
- Synthetic aperture length depends on target range
- Same **horizontal resolution** for all targets



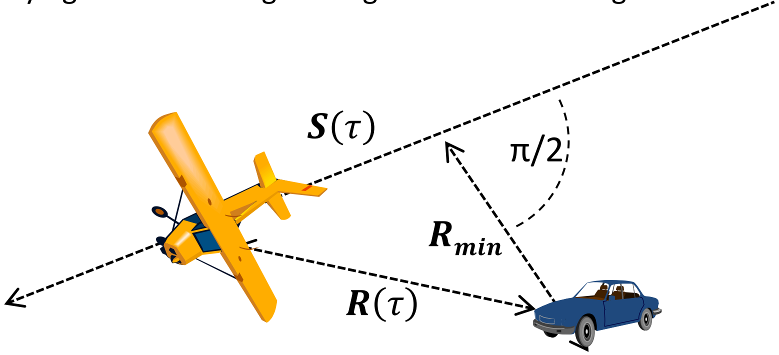

***Antenna array***

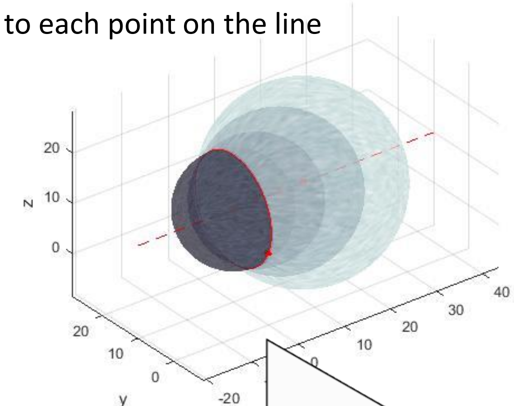
- Real aperture in a fixed position
- Angular resolution is obtained by focusing the data at **different** angles
- Same **angular resolution** for all targets

# SAR imaging – geometrical interpretation

***Synthetic Aperture Radars (SAR) employ a moving RADAR sensor, flown onboard a satellite or an aircraft, in order to synthesize an antenna as long as several kilometers***

Flying a RADAR along a straight line = measuring the distance from the target to each point on the line



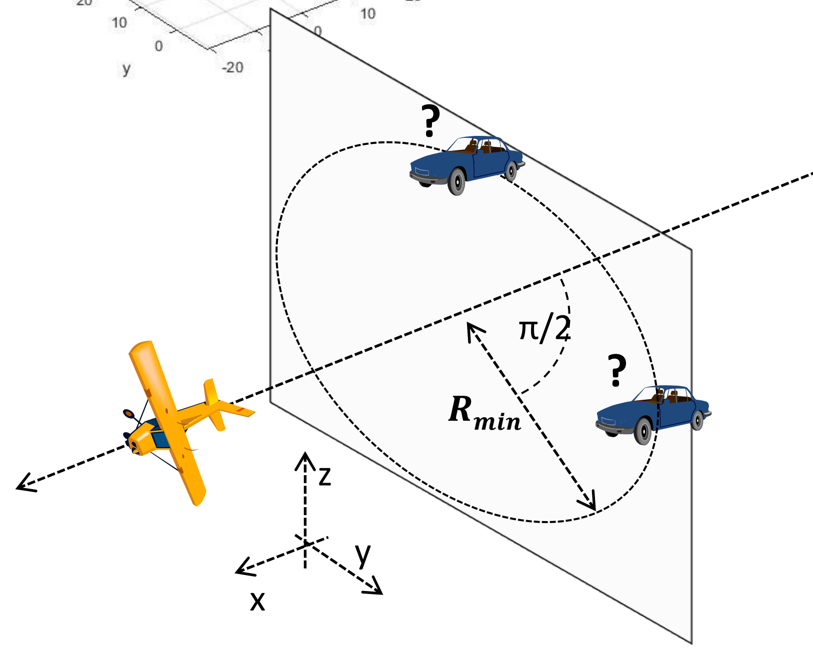

The target is bound to lie on the intersection of all the spheres:

- Centered in $S(\tau)$
- Of radius $R(\tau)$

$\Rightarrow$ The target is bound to lie on the circle:

- Centered on the trajectory
- Perpendicular to the trajectory (*yz* plane)
- Of radius $R_{min}$

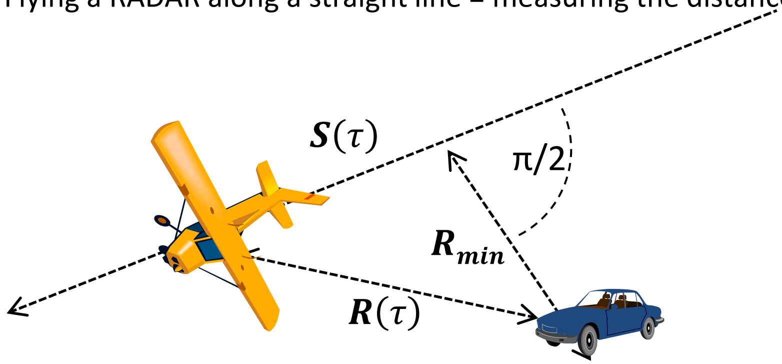
$\Rightarrow$ 2D Localization

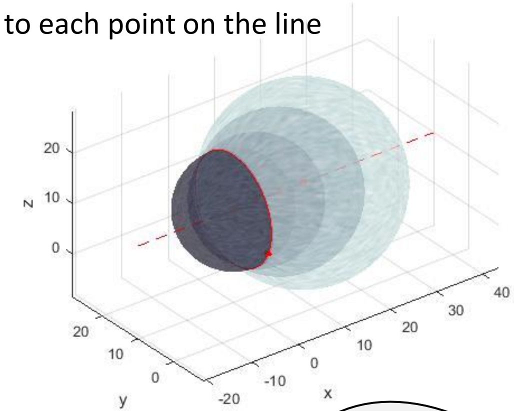


# SAR imaging – physical interpretation

***Synthetic Aperture Radars (SAR) employ a moving RADAR sensor, flown onboard a satellite or an aircraft, in order to synthesize an antenna as long as several kilometers***

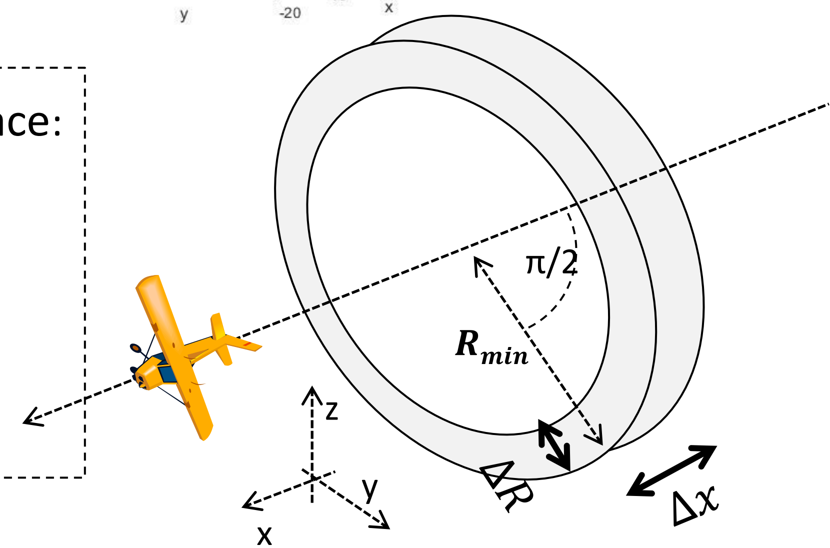
Flying a RADAR along a straight line = measuring the distance from the target to each point on the line



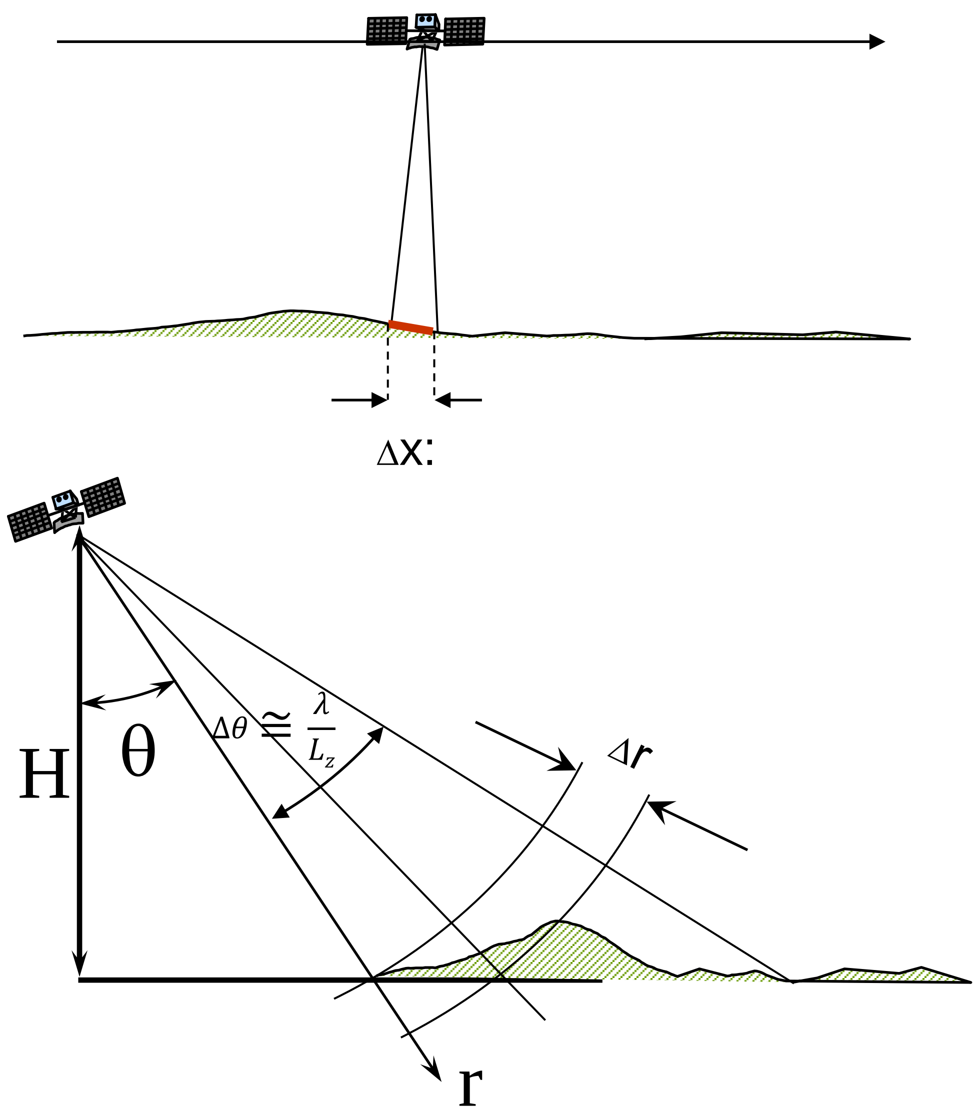



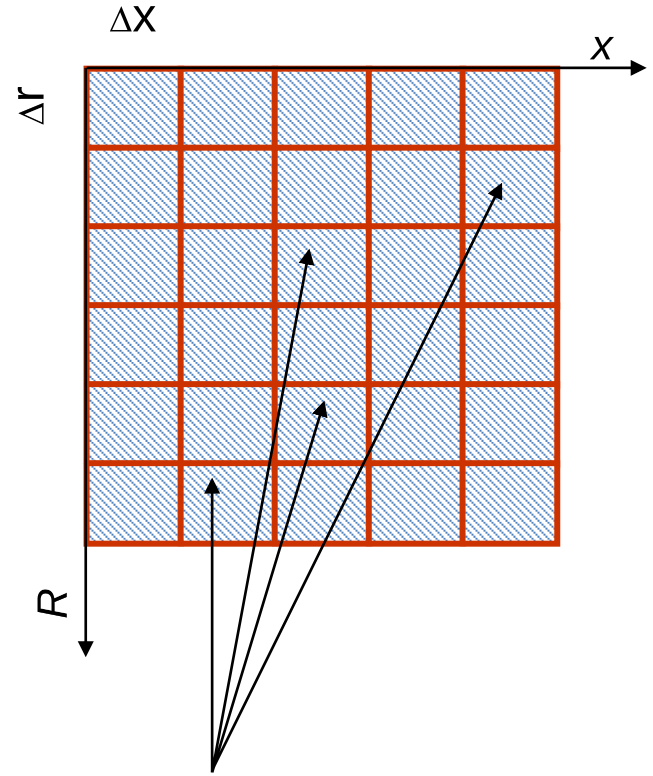

$\Rightarrow$ The target is bound to lie in the region of space:

- Centered on the trajectory
- Perpendicular to the trajectory (*yz* plane)
- Range thickness $\Delta R = \frac{c}{2B}$
- Along-track thickness $\Delta x = \frac{\lambda R}{2A_s} \cong \frac{L_x}{2}$



# SAR imaging



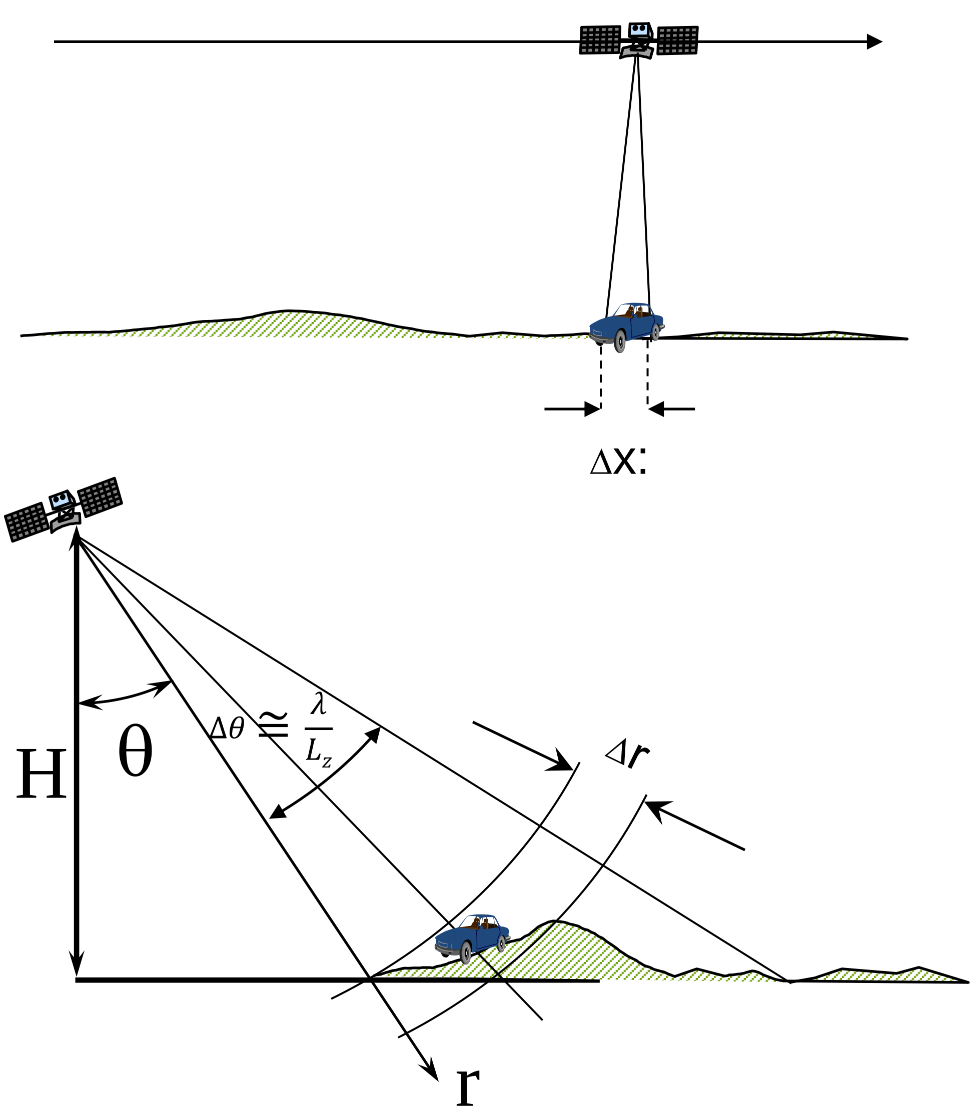




Each pixel is associated with a range/azimuth resolution cell

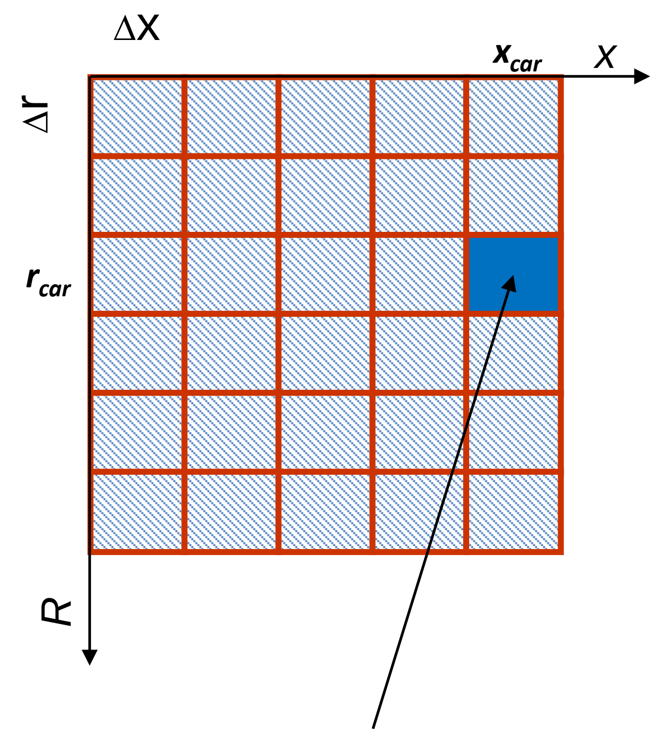
***SAR jargon***

- *R* = (slant) range
- *x* = azimuth

# SAR imaging




## *SAR image*

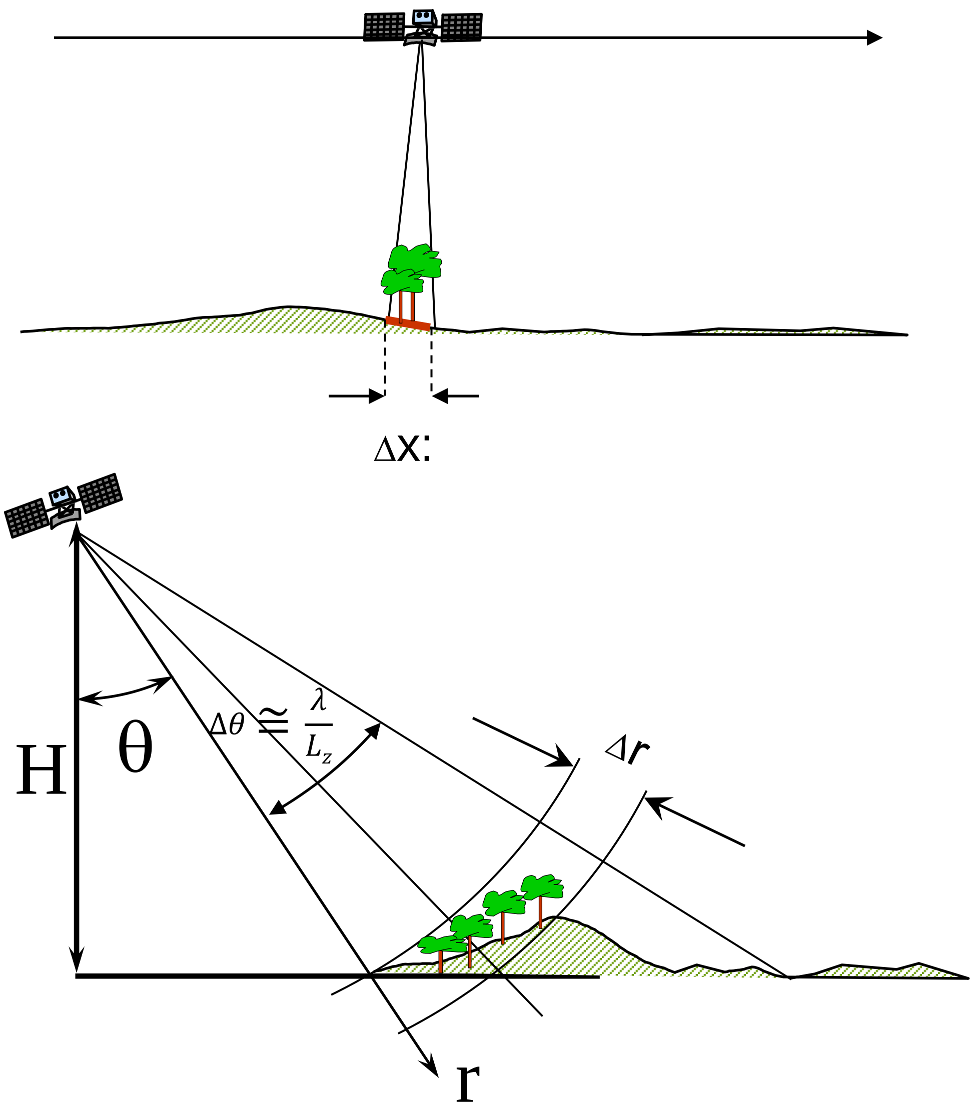



***This pixel value corresponds to the target at position $x_{car}$, $r_{car}$***

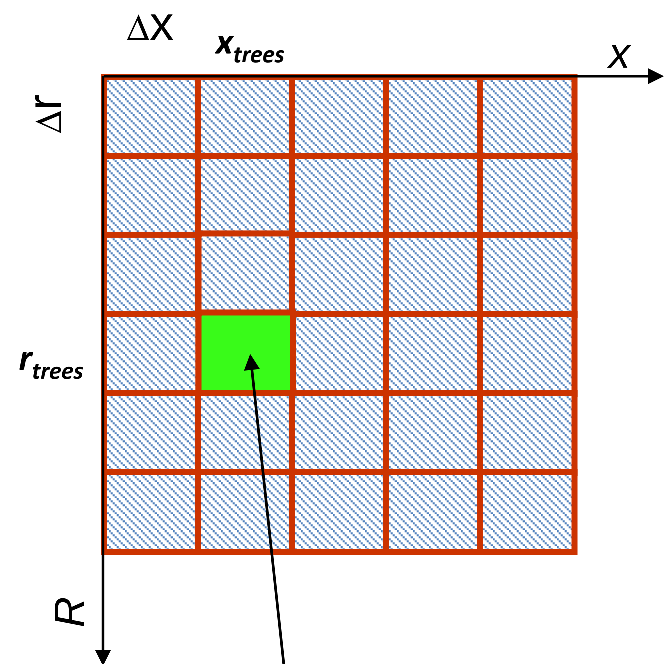
***SAR jargon***

- *R* = (slant) range
- *x* = azimuth

# SAR imaging




## *SAR image*




***This pixel value is arises from the interference of all trees within the resolution cell***

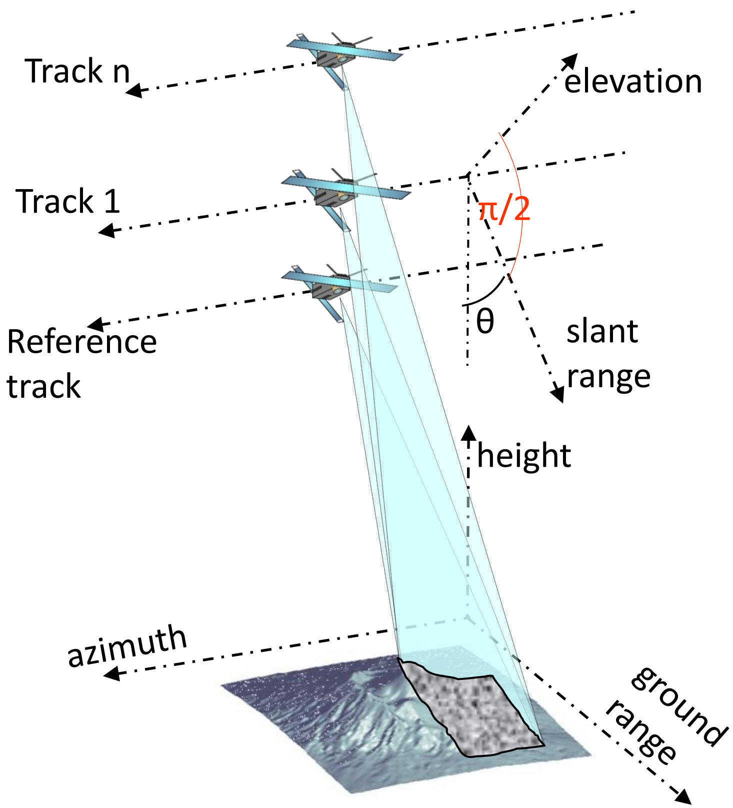
***SAR jargon***

- *R* = (slant) range
- *x* = azimuth

# *TomoSAR Imaging*

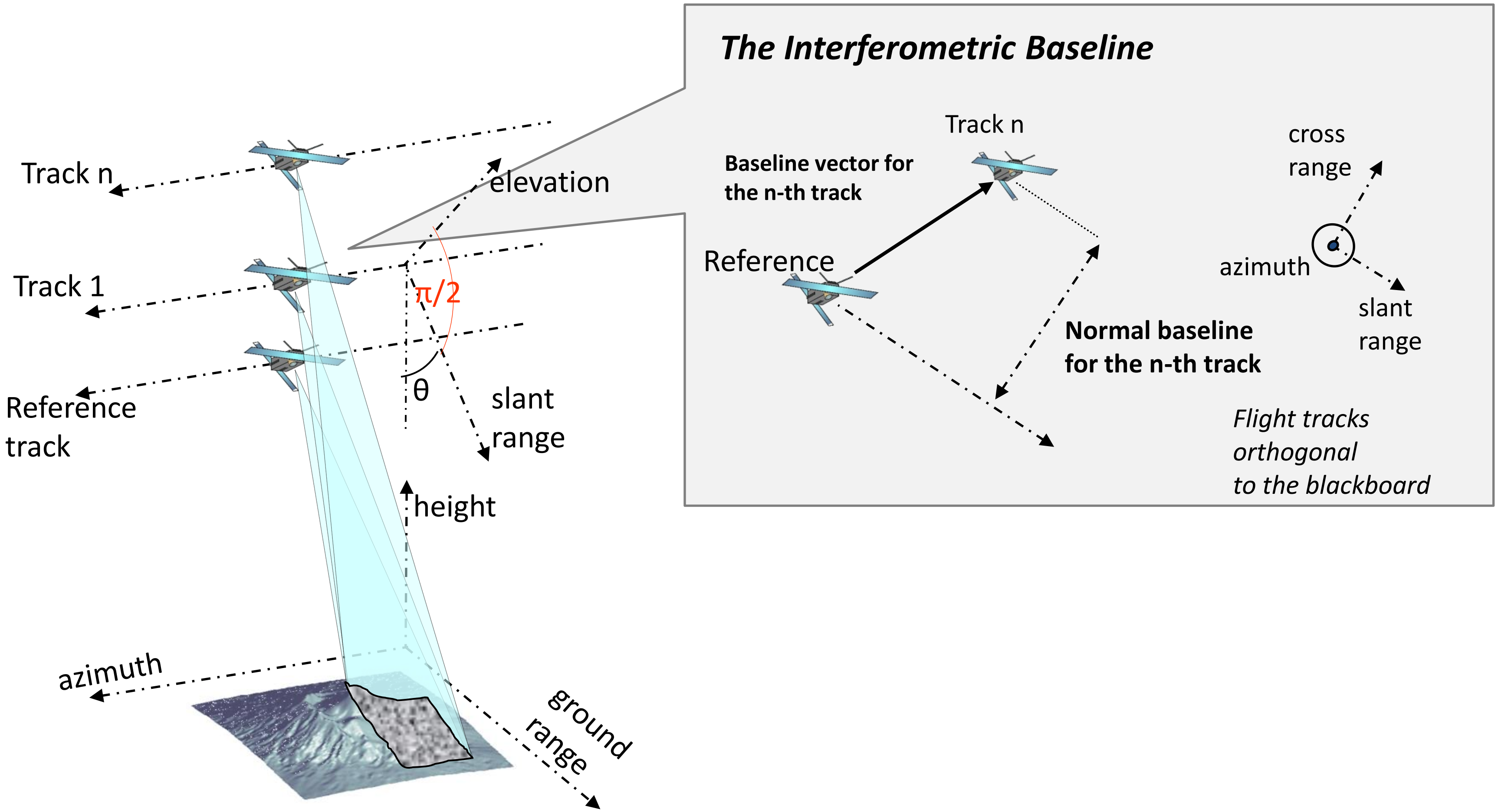
# TomoSAR Imaging

Multiple baselines ⇔ Illumination from multiple points of view



# TomoSAR Imaging

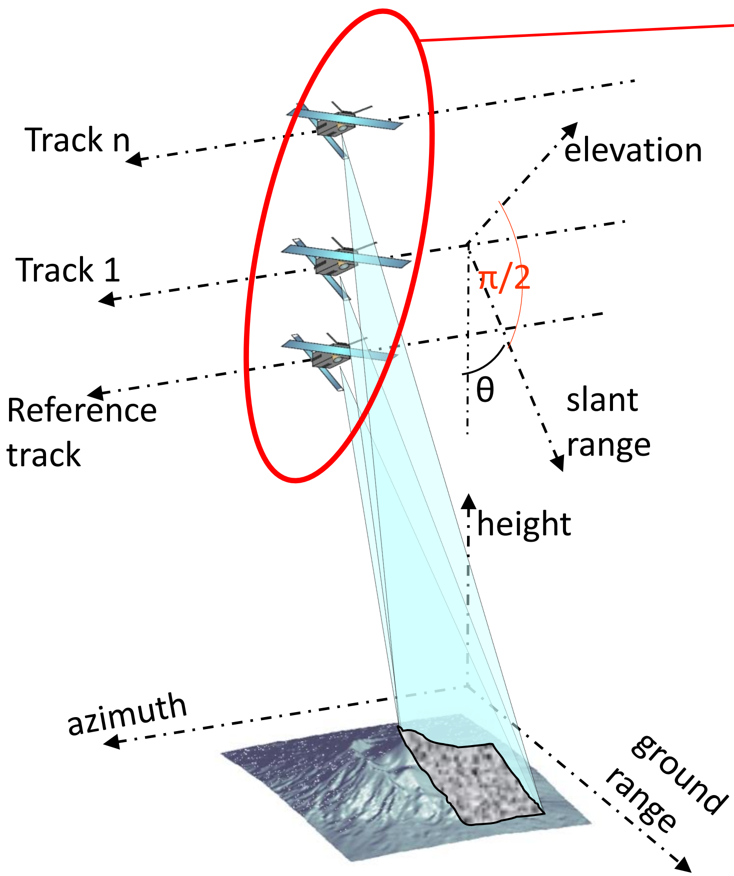
Multiple baselines ⇔ Illumination from multiple points of view

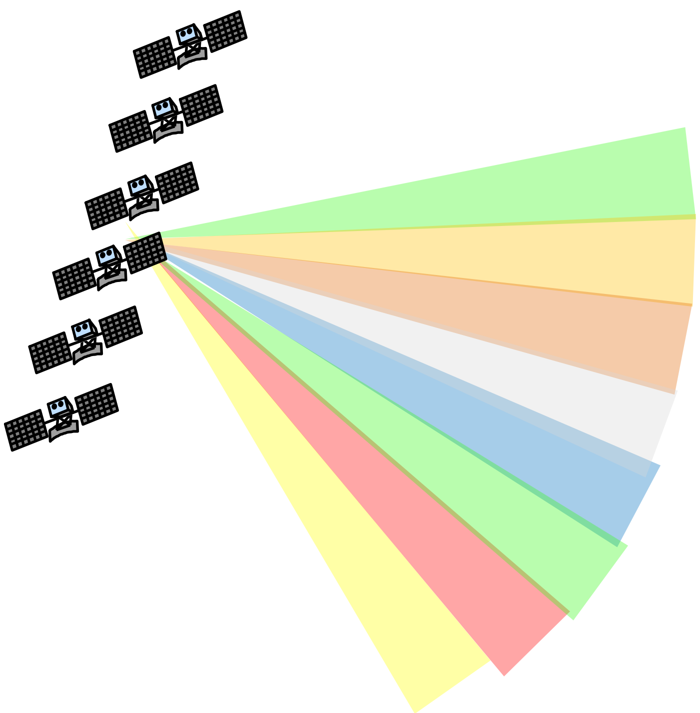


# TomoSAR Imaging

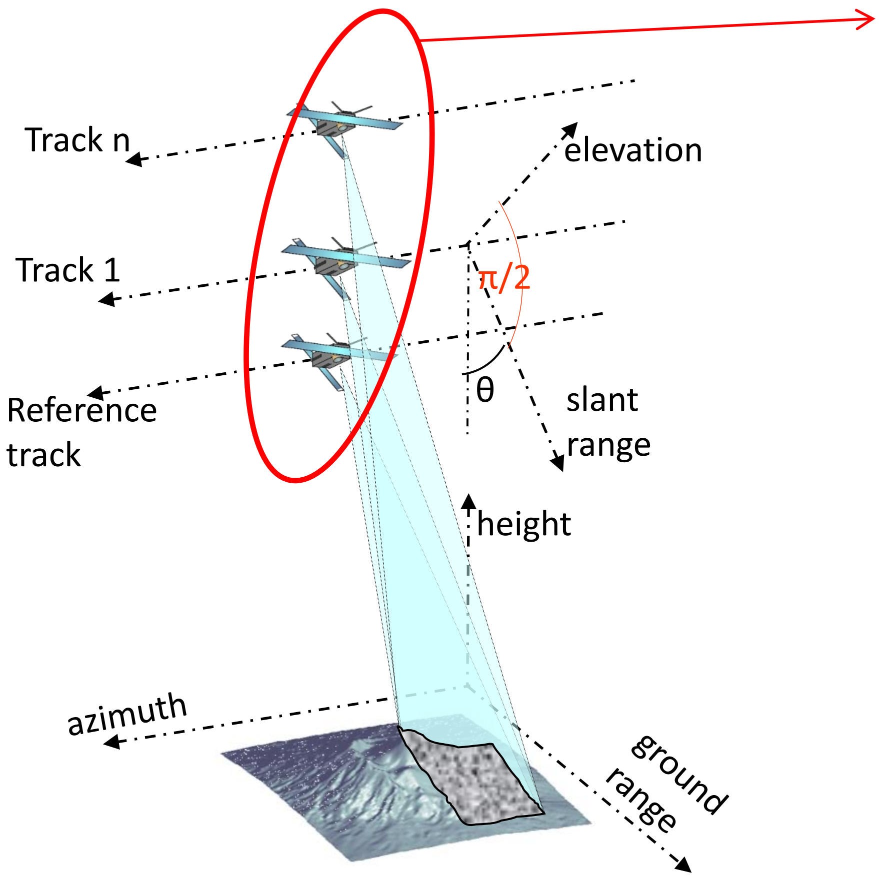
Multiple baselines ⇔ Illumination from multiple points of view

***An antenna array is formed at each azimuth position***



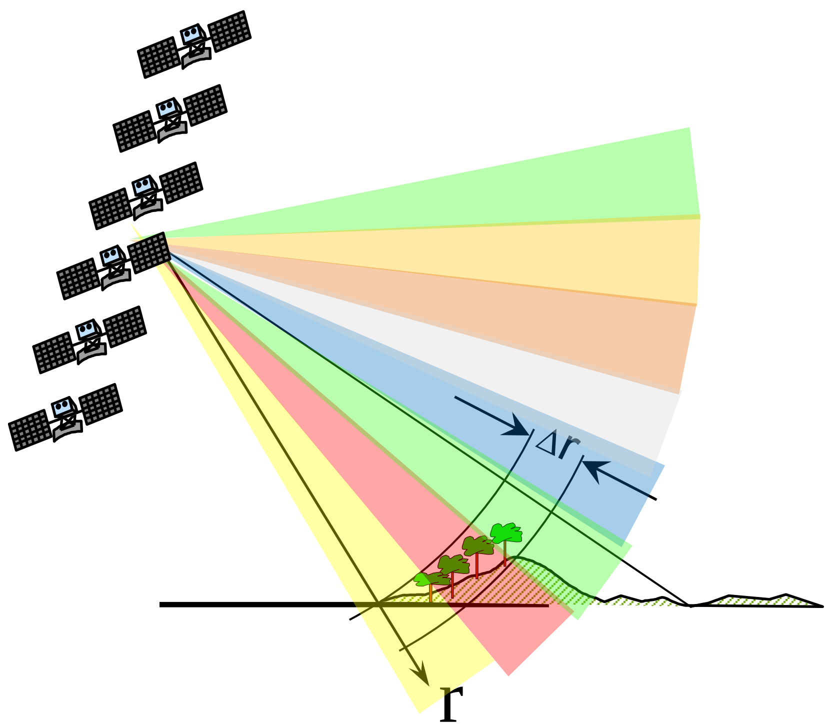



Multiple baselines ⬄ Illumination from multiple points of view

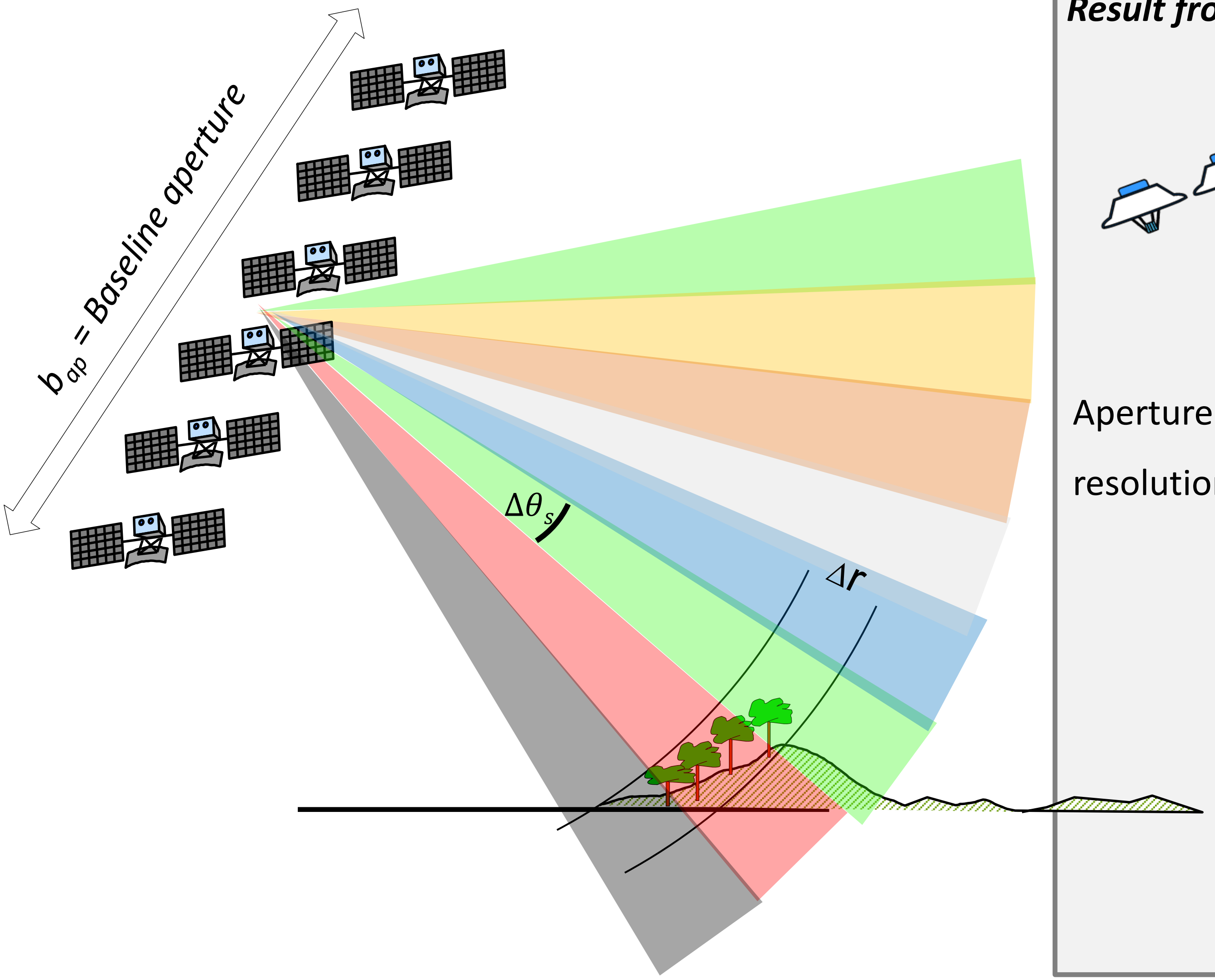



***An antenna array is formed at each azimuth position***

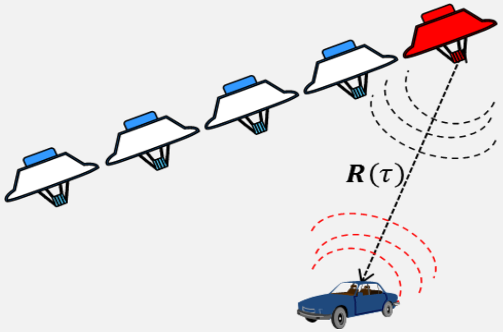
***⬄ Resolution of targets at different elevation within each SAR range/azimuth resolution cell***



# TomoSAR Resolution



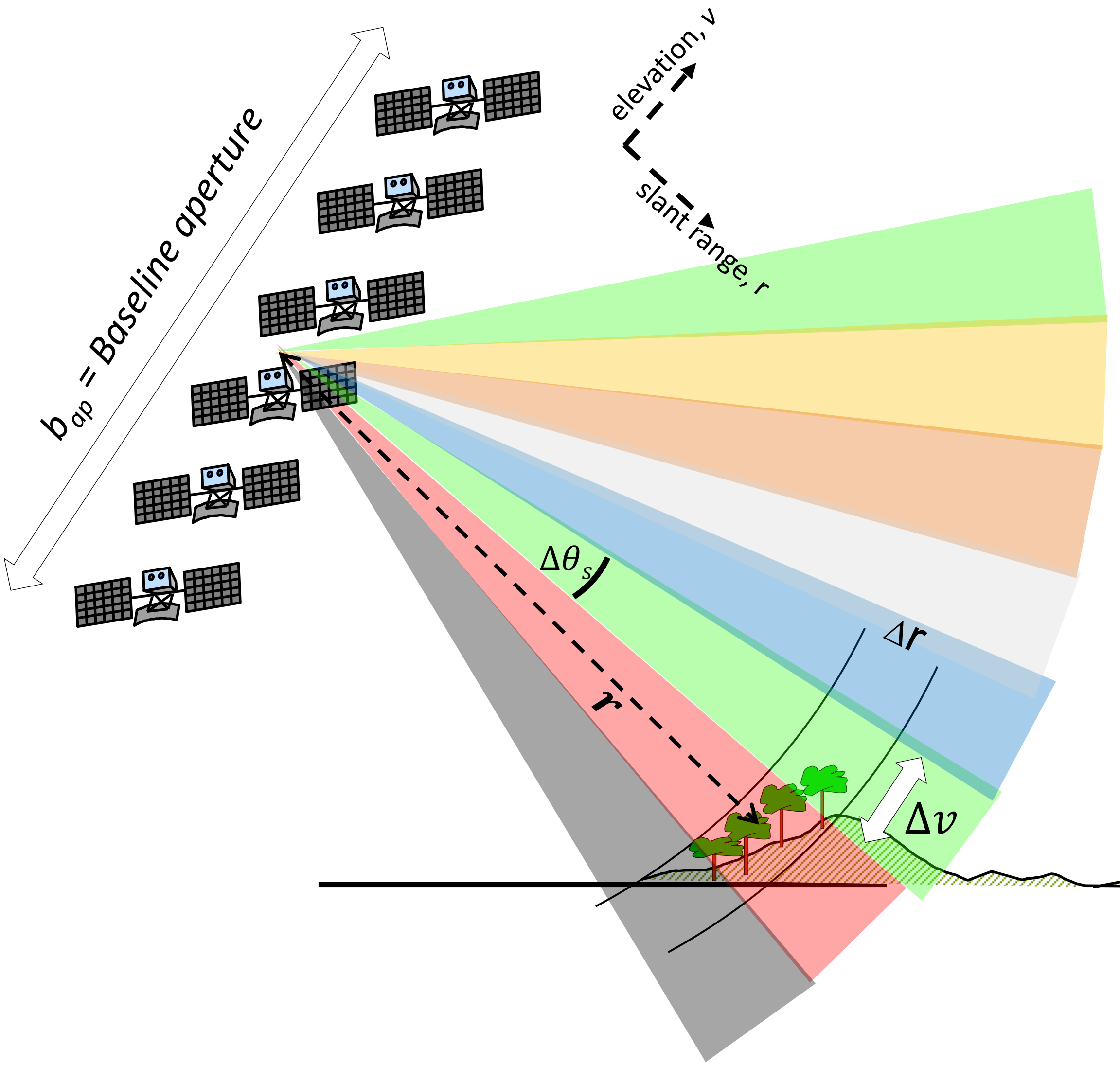

***Result from array theory***



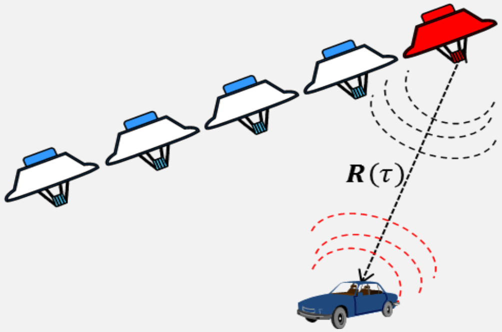

Aperture length translates to angular resolution according to

$$\Delta\theta_s \cong \frac{\lambda}{2b_{ap}}$$

# TomoSAR Resolution




***Result from array theory***



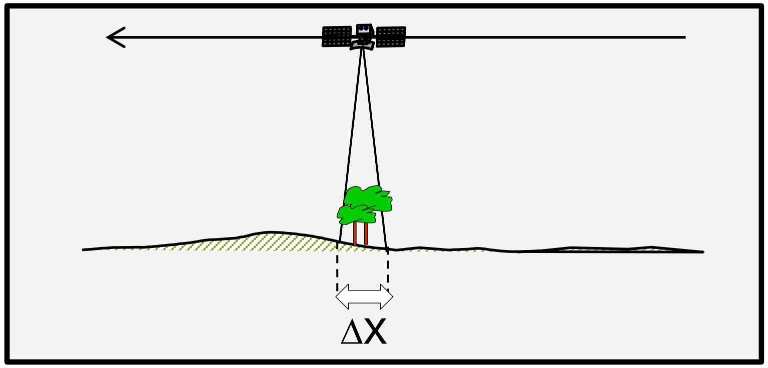

Aperture length translates to angular resolution according to

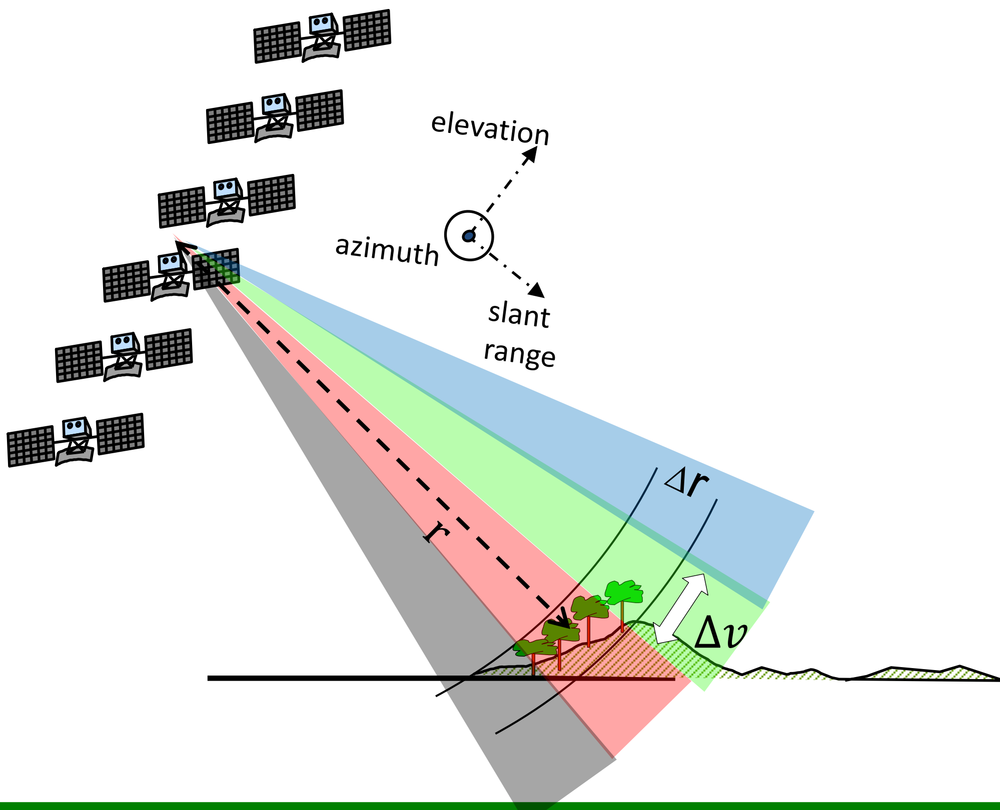
$$\Delta\theta_s \cong \frac{\lambda}{2b_{ap}}$$

Angular resolution translates into **elevation** resolution according to

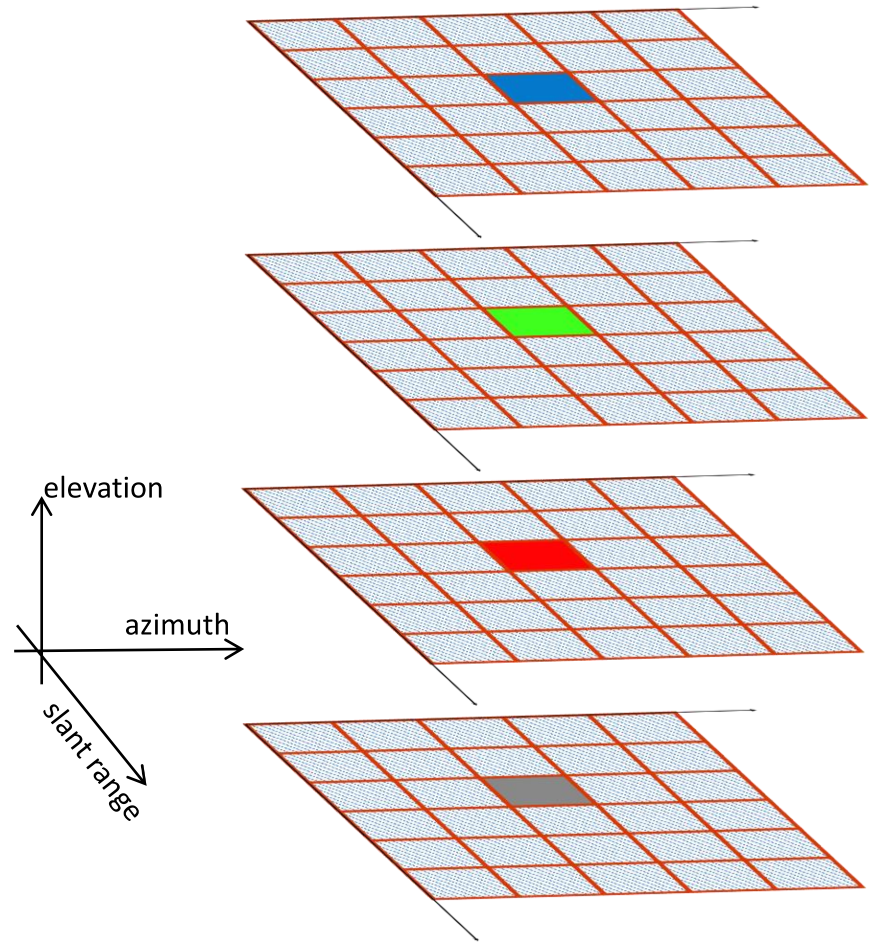
$$\Delta v \cong \Delta\theta_s \cdot r$$

# TomoSAR Resolution Cell



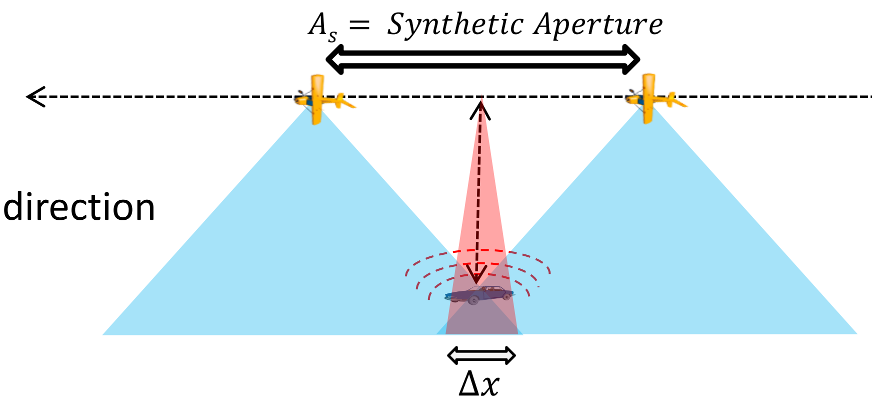

***TomoSAR cube***
Each voxel is associated with a range/azimuth/elevation resolution cell

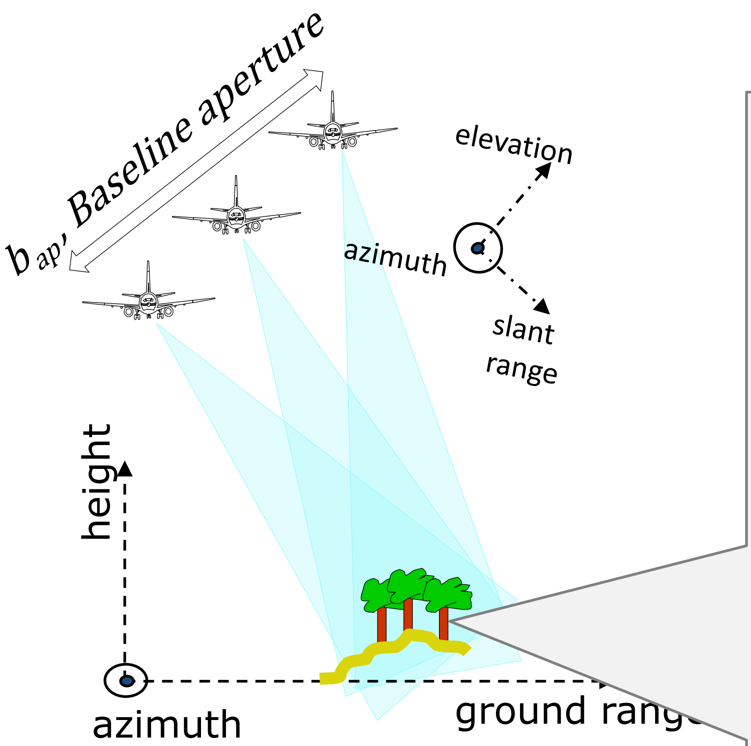


# TomoSAR Resolution Cell

Resolution is determined by

- Pulse bandwidth along the slant range direction
- Along-track synthetic aperture length in the azimuth direction
- Baseline aperture in the elevation direction



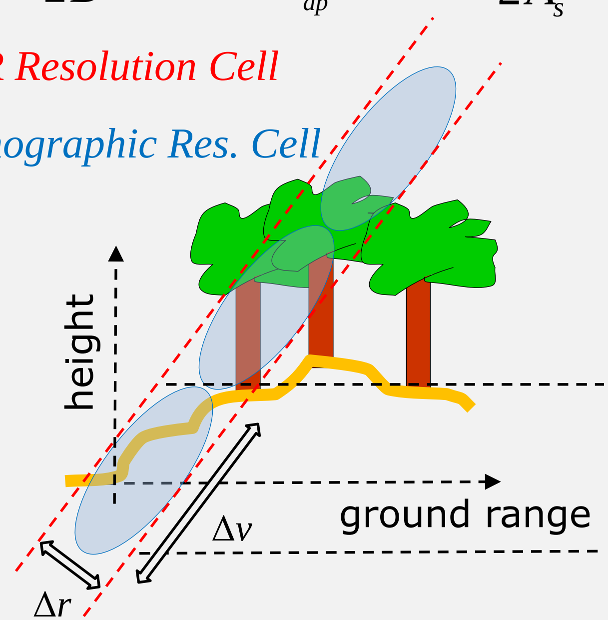




$$\Delta r = \frac{c}{2B} \qquad \Delta v = \frac{\lambda r}{2b_{ap}} \qquad \Delta x = \frac{\lambda r}{2A_s}$$

*B: pulse bandwidth*
*λ: carrier wavelength*

*SAR Resolution Cell*

*Tomographic Res. Cell*




For most systems:

$$\Delta v >> \Delta r, \Delta x$$

$$\Delta z \cong \Delta v \cdot \sin(\theta)$$

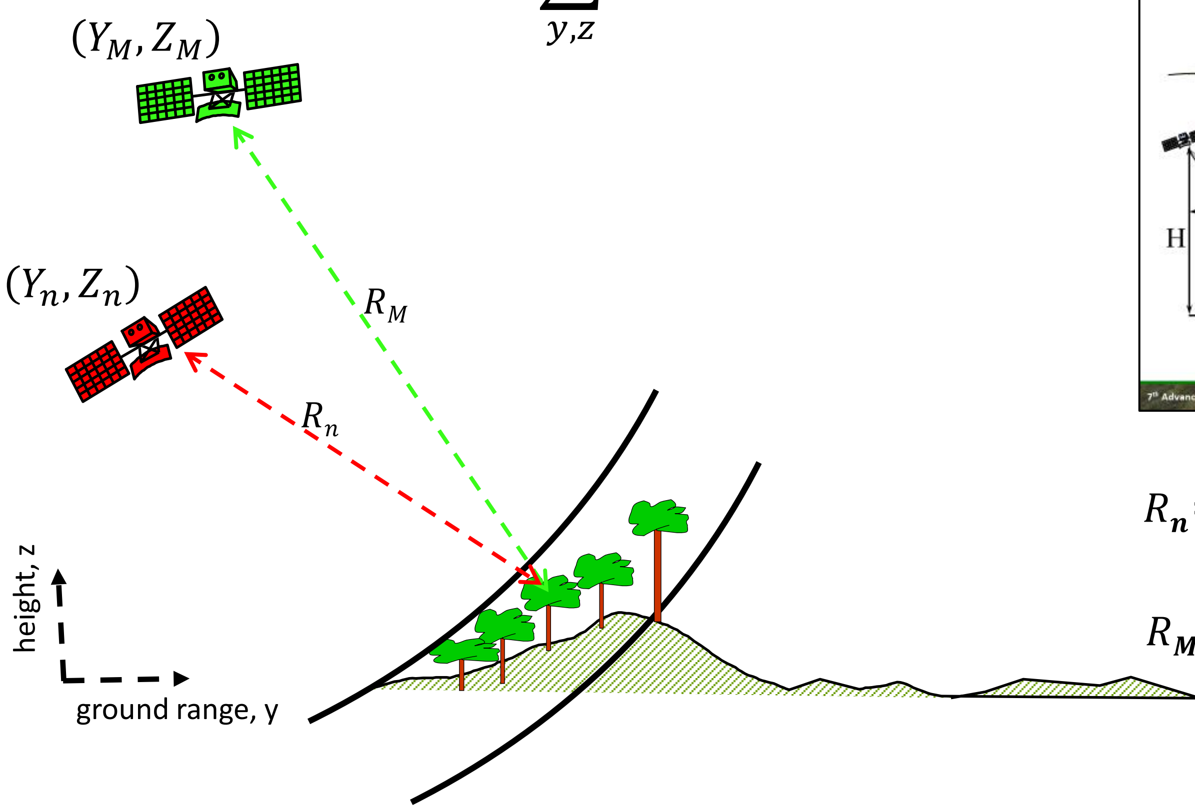
# *TomoSAR Processing*

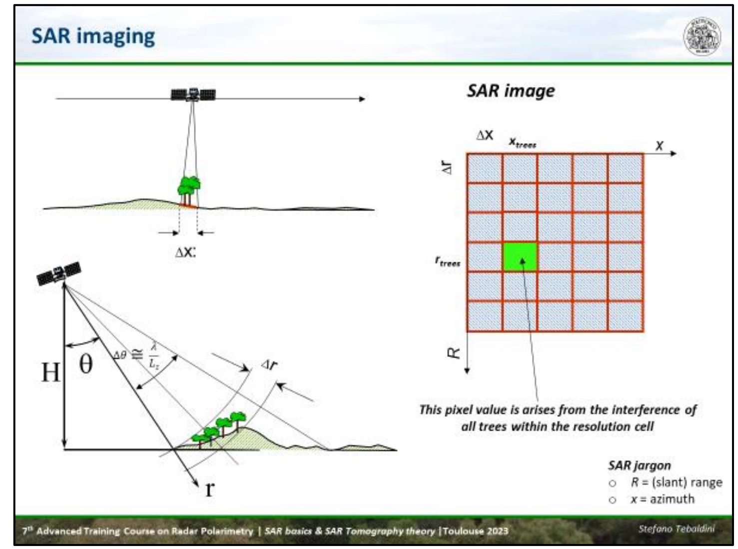
# SAR pixel – multiple baseline model

***SAR pixel = Sum of all elementary scatterer at different elevations within the same range/azimuth resolution cell***

- Each elementary scatterer is phase-rotated according to its distance from the Radar

$$I_n(r,x) = \sum_{y,z} A(y,z) \cdot e^{-j\frac{4\pi}{\lambda}R_n(y,z)}$$



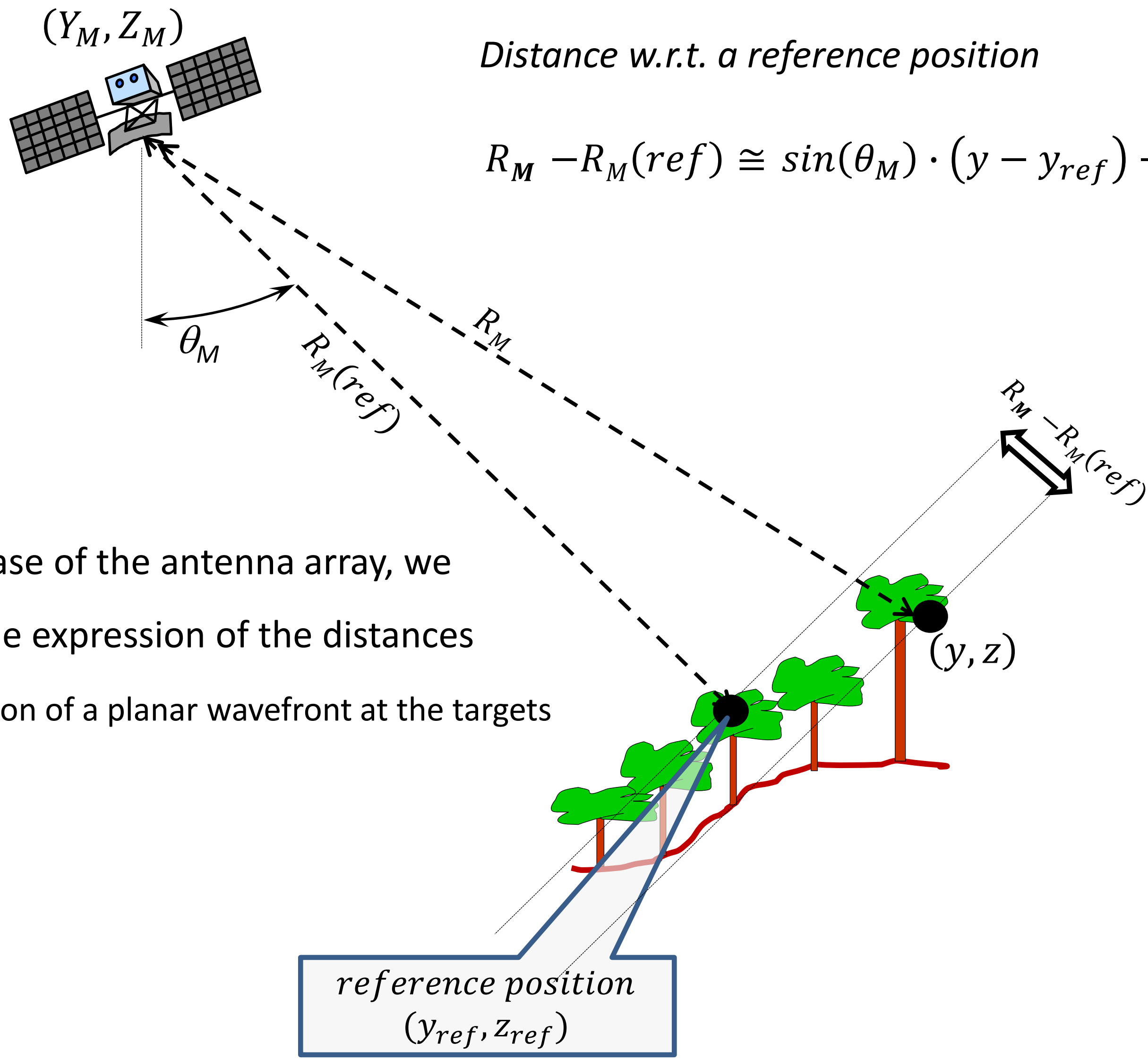

$$R_n = \sqrt{(Y_n - y)^2 + (Z_n - z)^2}$$

$$R_M = \sqrt{(Y_M - y)^2 + (Z_M - z)^2}$$

# SAR pixel – multiple baseline model

*Distance w.r.t. a reference position*

$$R_{\boldsymbol{M}} - R_M(ref) \cong sin(\theta_M) \cdot (y - y_{ref}) - cos(\theta_M) \cdot (z - z_{ref})$$




As in the case of the antenna array, we linearize the expression of the distances

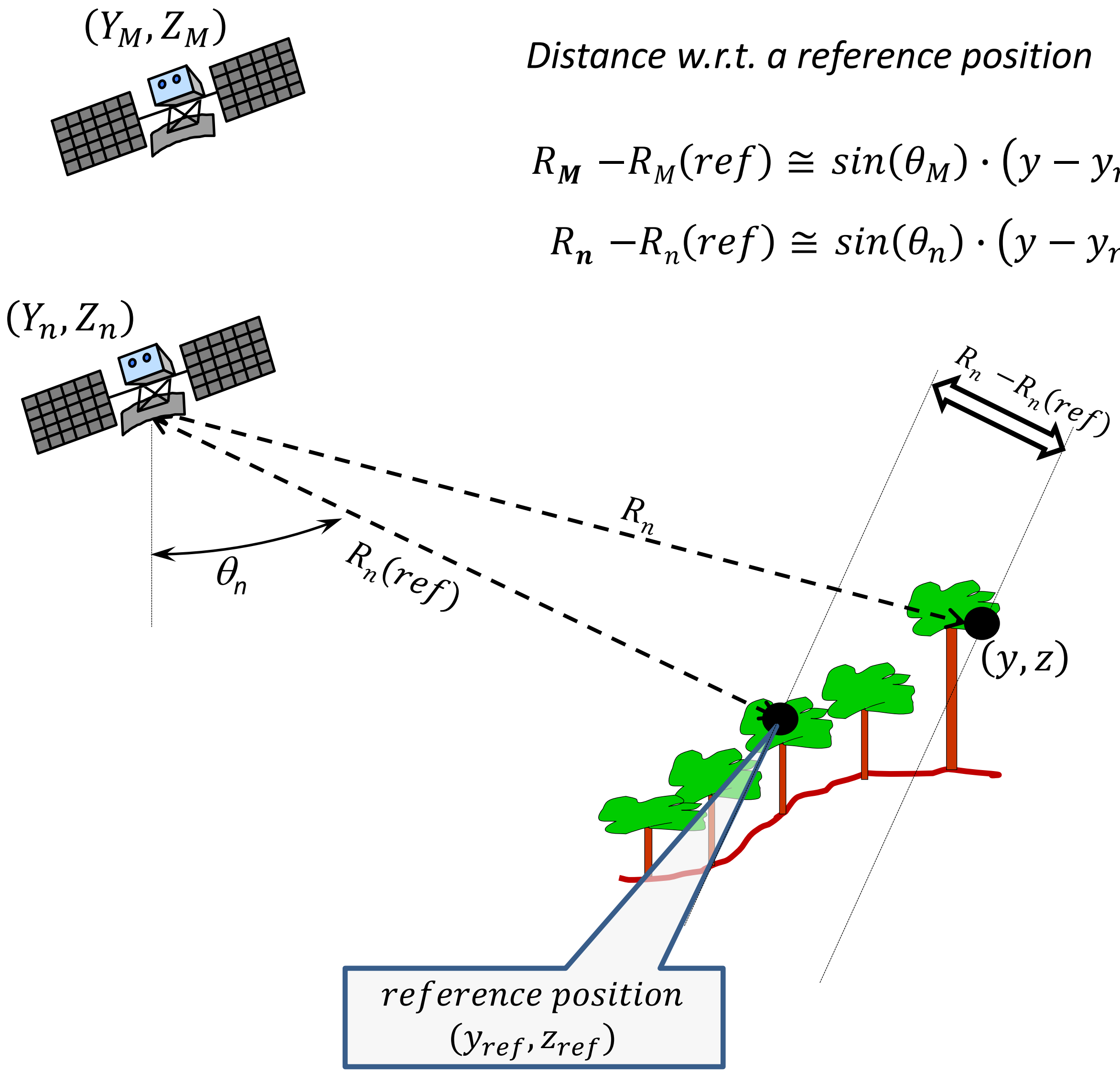
⇔ Assumption of a planar wavefront at the targets

# SAR pixel – multiple baseline model

Distance w.r.t. a reference position

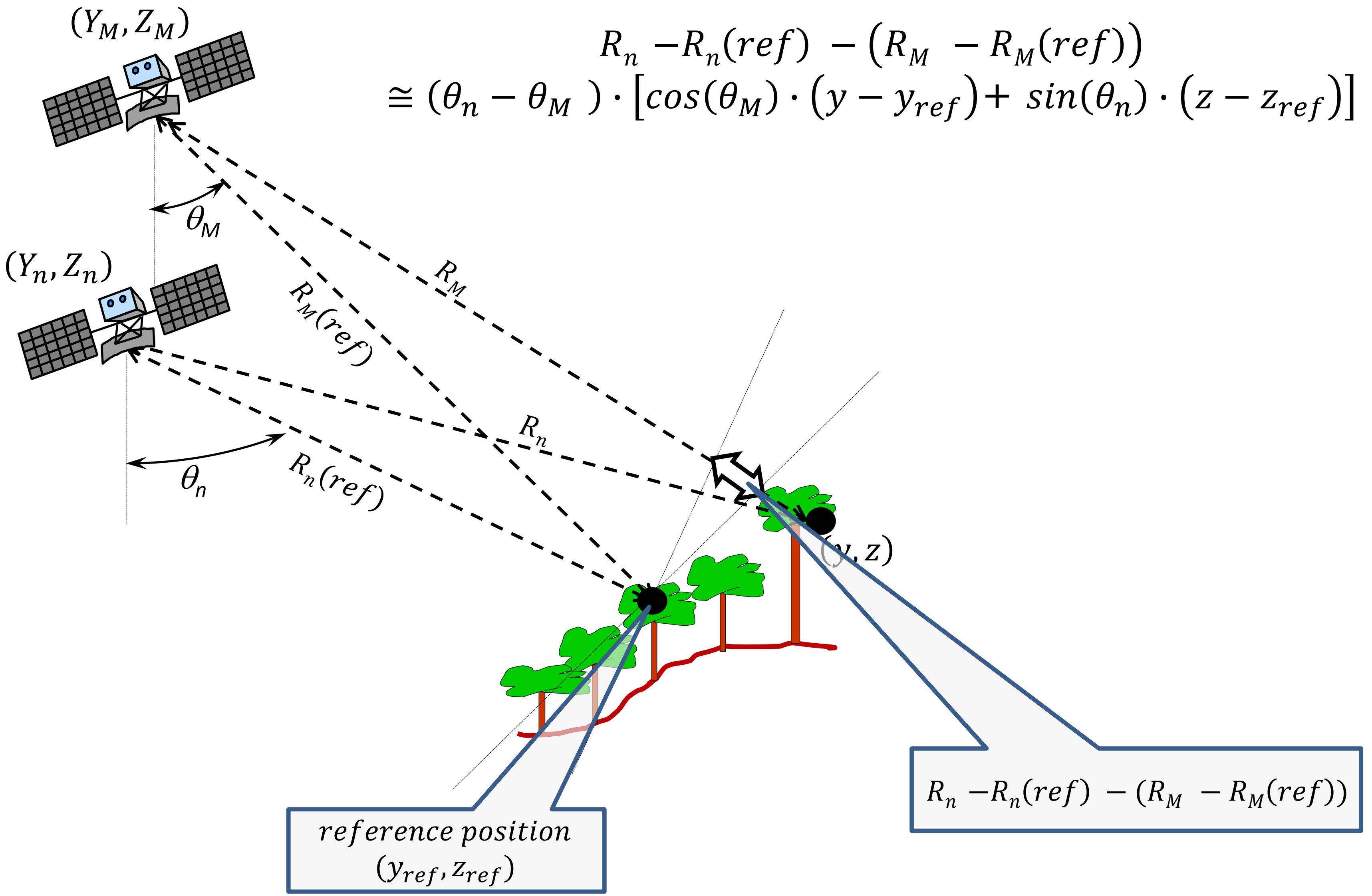
$$R_{\boldsymbol{M}} - R_M(ref) \cong sin(\theta_M) \cdot (y - y_{ref}) - cos(\theta_M) \cdot (z - z_{ref})$$

$$R_{\boldsymbol{n}} - R_n(ref) \cong sin(\theta_n) \cdot (y - y_{ref}) - cos(\theta_n) \cdot (z - z_{ref})$$



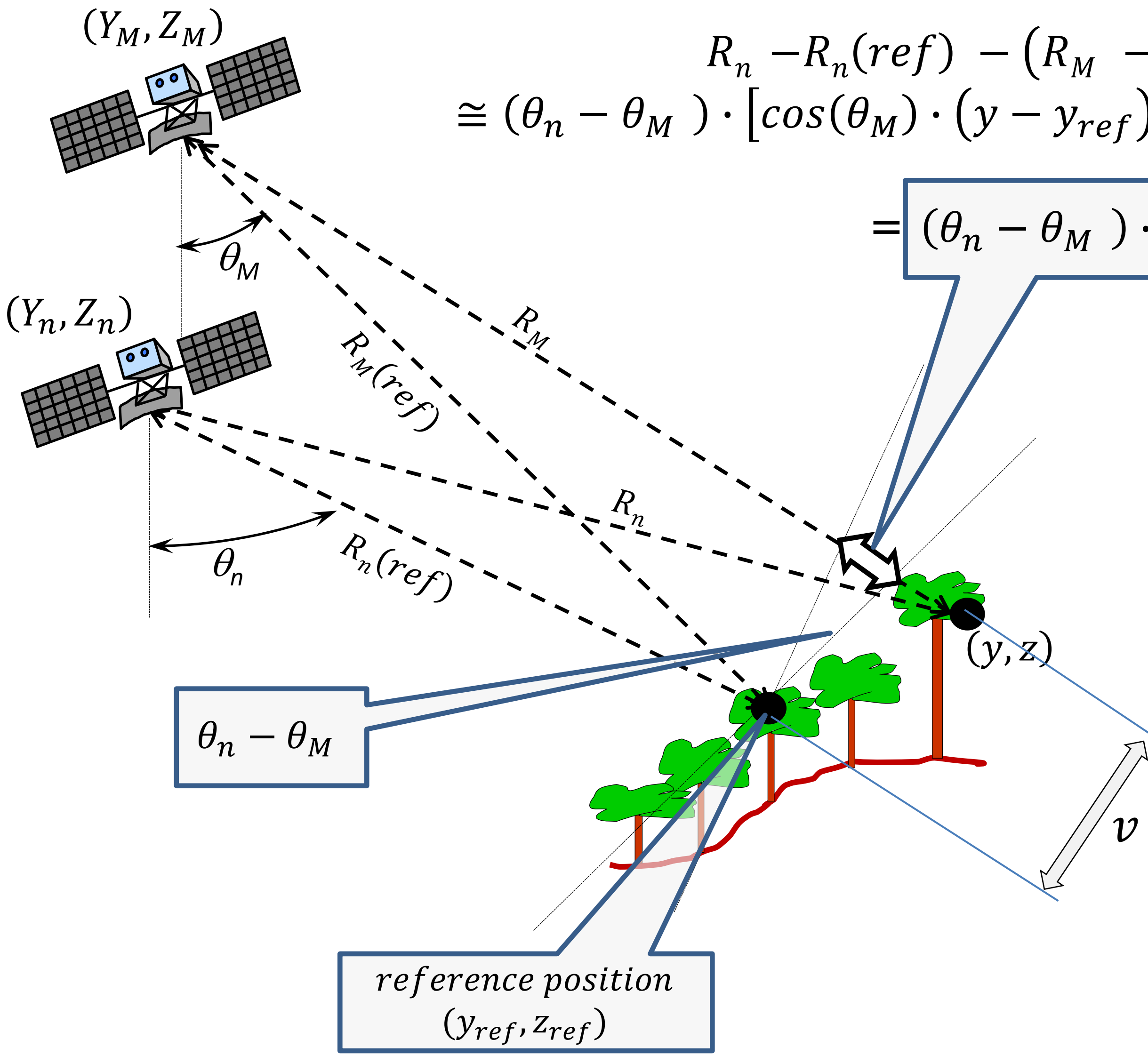
# SAR pixel – multiple baseline model

$$R_n - R_n(ref) - (R_M - R_M(ref))$$
$$\cong (\theta_n - \theta_M) \cdot [\cos(\theta_M) \cdot (y - y_{ref}) + \sin(\theta_n) \cdot (z - z_{ref})]$$



# SAR pixel – multiple baseline model

$$R_n - R_n(ref) - \left(R_M - R_M(ref)\right)$$
$$\cong (\theta_n - \theta_M)\cdot\left[\cos(\theta_M)\cdot(y - y_{ref}) + \sin(\theta_n)\cdot(z - z_{ref})\right]$$



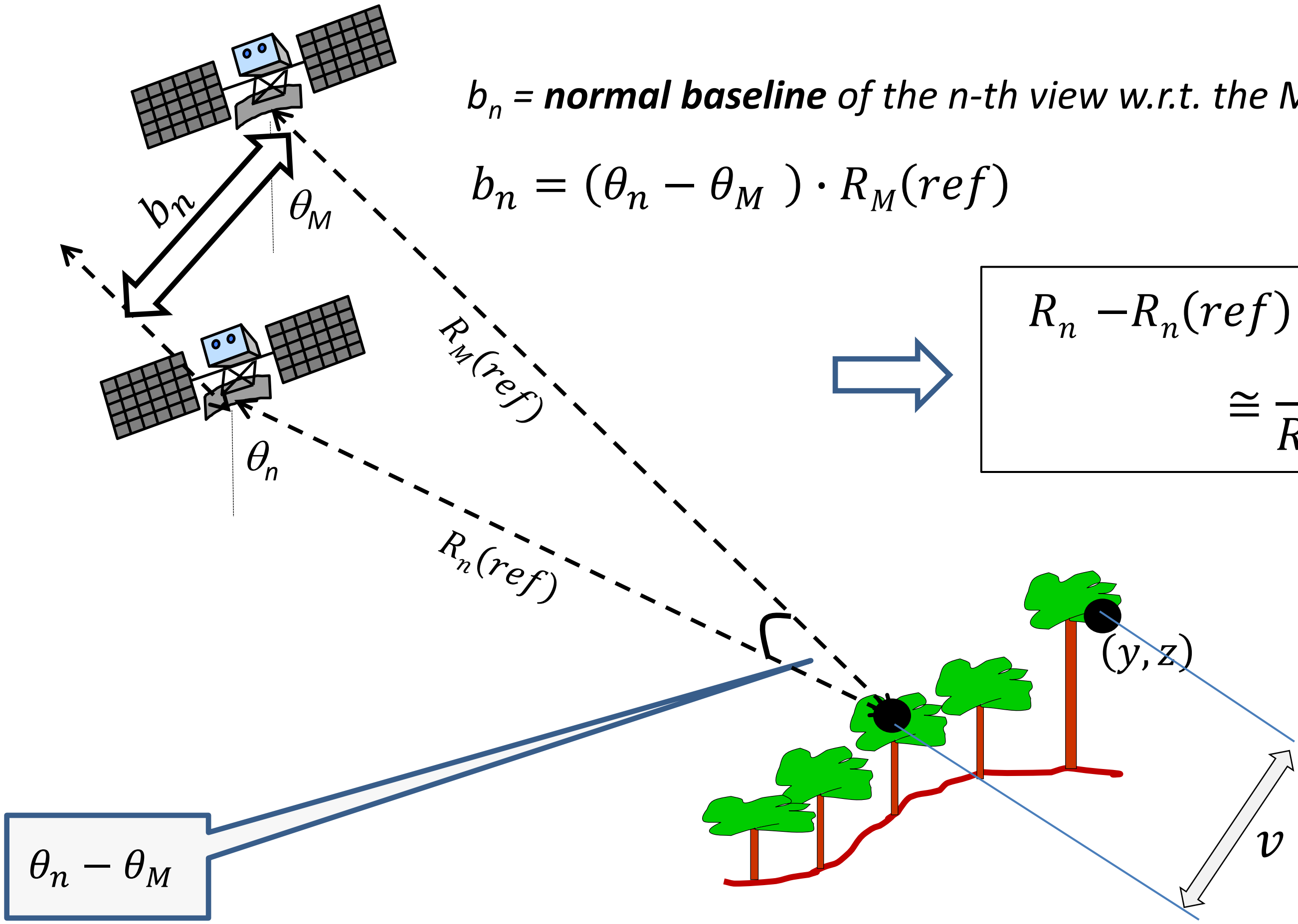

$v$ is the displacement of the target at $(y, z)$ w.r.t. to $(y_{ref}, z_{ref})$ in the direction of elevation

# SAR pixel – multiple baseline model

$b_n$ = ***normal baseline*** *of the n-th view w.r.t. the Master view*

$$b_n = (\theta_n - \theta_M) \cdot R_M(ref)$$

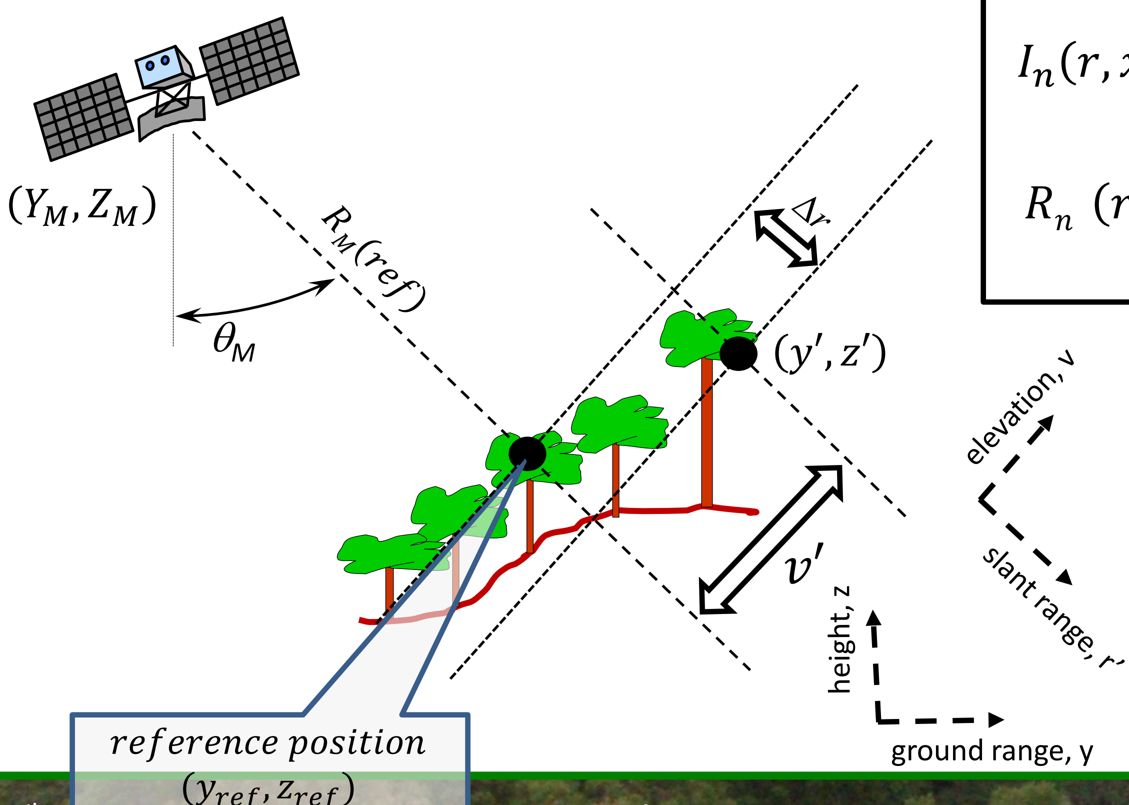
$$R_n - R_n(ref) - \left(R_M - R_M(ref)\right) \cong \frac{b_n}{R_M(ref)} \cdot v$$



# *SAR pixel – multiple baseline model*

The approximations above allow restating the SAR model in a new Cartesian coordinate system defined by *slant range, elevation with respect to a reference point and a reference orbit*

- The reference position is typically taken as the *(x,y,z)* position of the SAR pixel when projected onto a given Digital Terrain Model
- The choice of the reference orbit is largely arbitrary




$$I_n(r,x) = \sum_{r',v} A(r',v) \cdot e^{-j\frac{4\pi}{\lambda}R_n(r',v)}$$

$$R_n\ (r',v) \cong R_n(ref)\ + r' + \frac{b_n}{R_M(ref)} \cdot v$$

# SAR pixel – multiple baseline model

⇩

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\}\cdot\sum_{r',v}A(r',v)exp\left\{-j\frac{4\pi}{\lambda}r'\right\}\cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

# SAR pixel – multiple baseline model

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\}\cdot\sum_{r',v}A(r',v)exp\left\{-j\frac{4\pi}{\lambda}r'\right\}\cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

*Common terms for all baselines*

# SAR pixel – multiple baseline model

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\} \cdot \sum_{r',v} A(r',v)exp\left\{-j\frac{4\pi}{\lambda}r'\right\} \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

*Common terms for all baselines*

*Summing over r' we get*

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\} \cdot \sum_{v} s(v) \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

# SAR pixel – multiple baseline model

⇩

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\} \cdot \sum_{r',v} A(r',v)exp\left\{-j\frac{4\pi}{\lambda}r'\right\} \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

*Common terms for all baselines*

*Summing over r' we get*

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\} \cdot \sum_{v} s(v) \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

*Phase offset to be removed based on knowledge*

*of the acquisition geometry (terrain flattening)*

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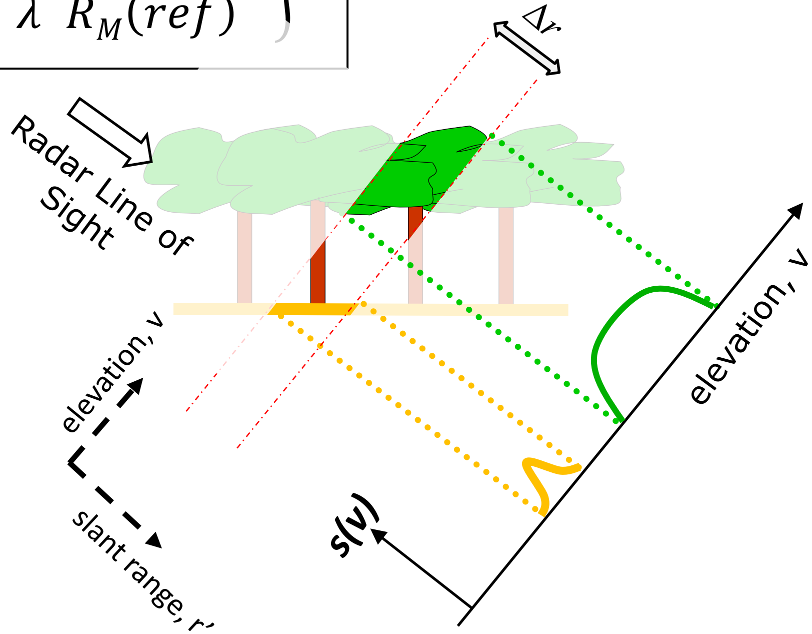
*Phase offset to be removed based on knowledge of the acquisition geometry (terrain flattening)*

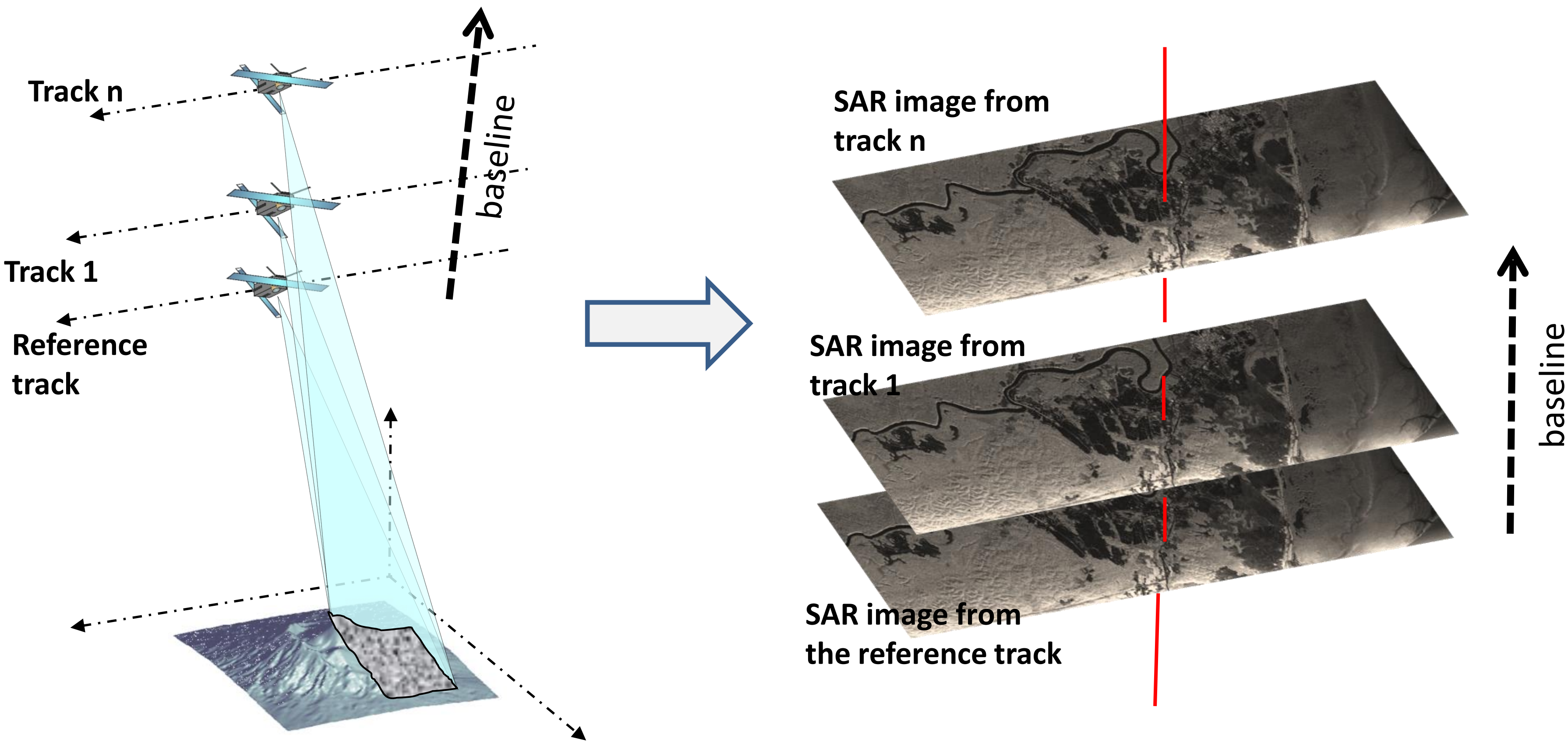
***s(v) = projection of the scatterers along elevation***



# *TomoSAR forward model*

The SAR pixel in the *n-th* image can finally be expressed in a simple form as follows:

$$I_n(r,x) = \sum_v s(v) \cdot exp\{-j2\pi f_v b_n\} \quad with\ f_v = \frac{2}{\lambda}\frac{v}{R_M(ref)}$$



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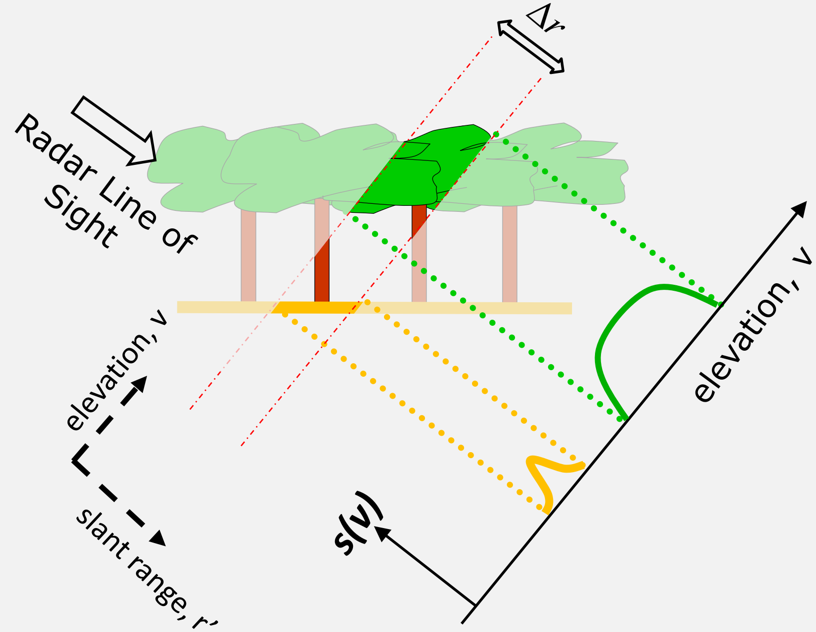
The signal obtained by taking the pixels at the same $(r,x)$ location in a stack of SAR images is contributed by a sum of complex sinusoids

- The frequencies of the sinusoids correspond to the elevations $v$ at which the targets are found
- The complex amplitude of the sinusoids are obtained by projecting the scatterers within the SAR resolution cell along elevation



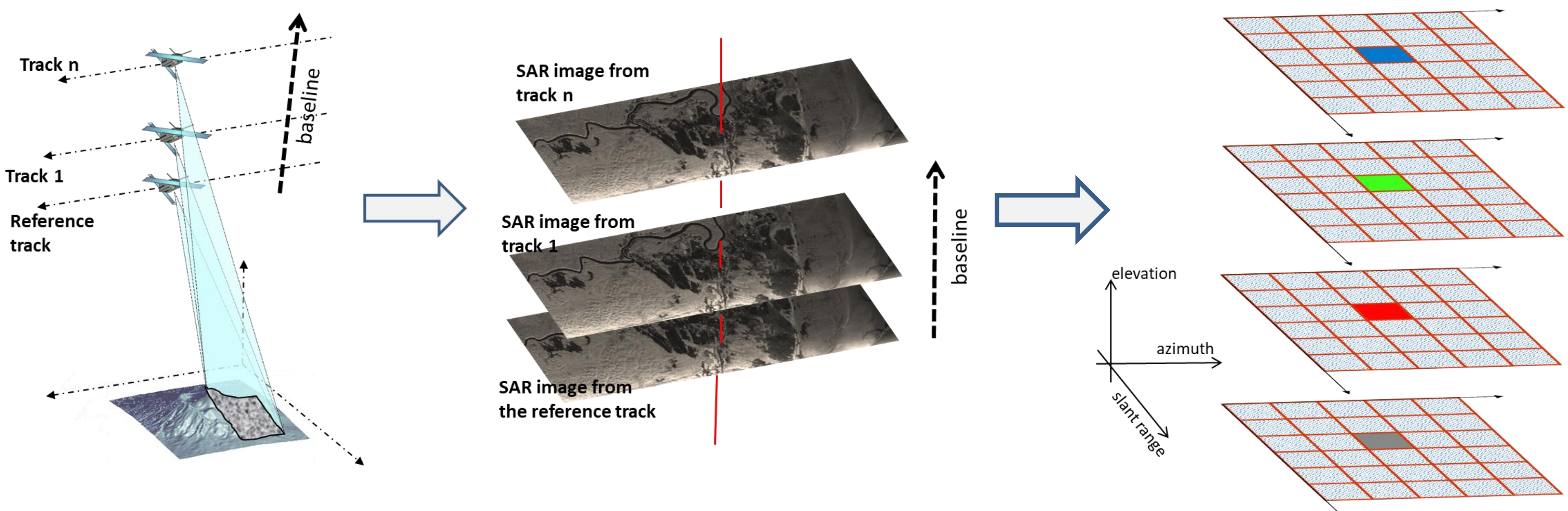

# *TomoSAR focusing algorithm*

Tomographic focusing consists in retrieving the amplitudes $s(v)$ from the signal $I_n(r,x)$

$\Rightarrow$ As always, this is done by computing a Fourier Transform

$$T(r,x,v) = \sum_n I_n(r,x) \cdot exp\left\{j\frac{2}{\lambda}\frac{v}{R_M(ref)}\right\}$$

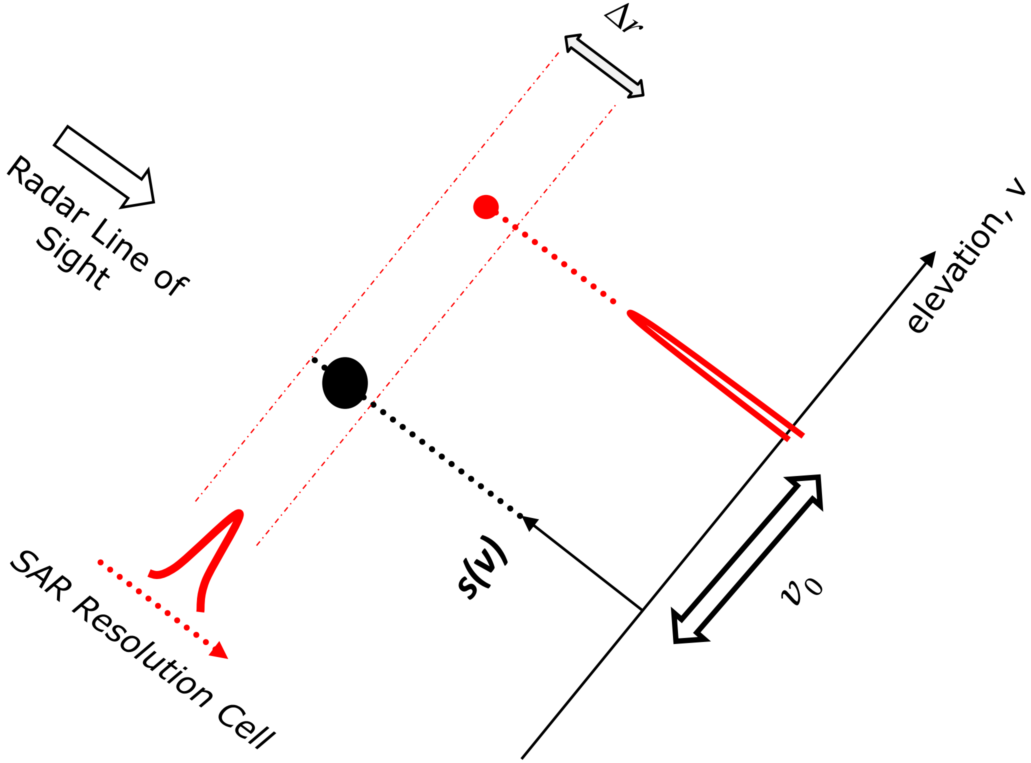
***Acquisition*** ***Stack of SAR images*** ***Tomographic voxels***



# TomoSAR – examples

*Case 1: a single point target*

$$I_n(r,x) = s_0 \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v_0\right\}$$




***s(v) = projection of the scatterers along elevation***

● = reference position

# *TomoSAR – examples*

*Case 1: a single point target*

$$I_n(r,x) = s_0 \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v_0\right\}$$

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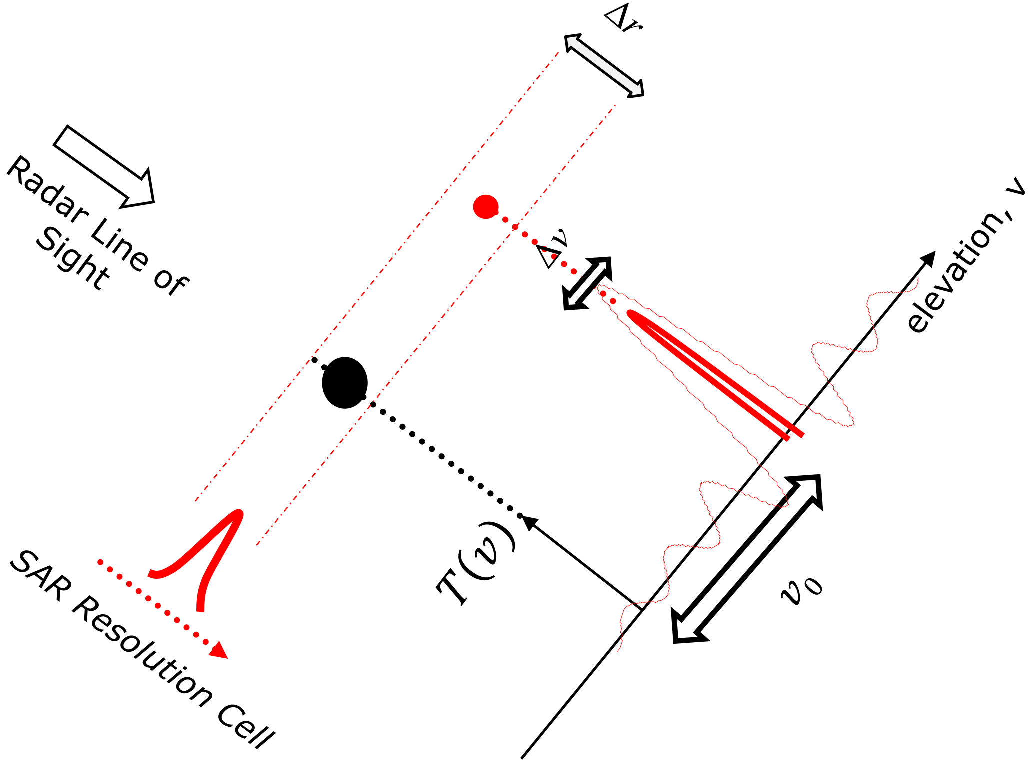



***T(v) = reconstruction by SAR Tomography***

*Elevation resolution is*

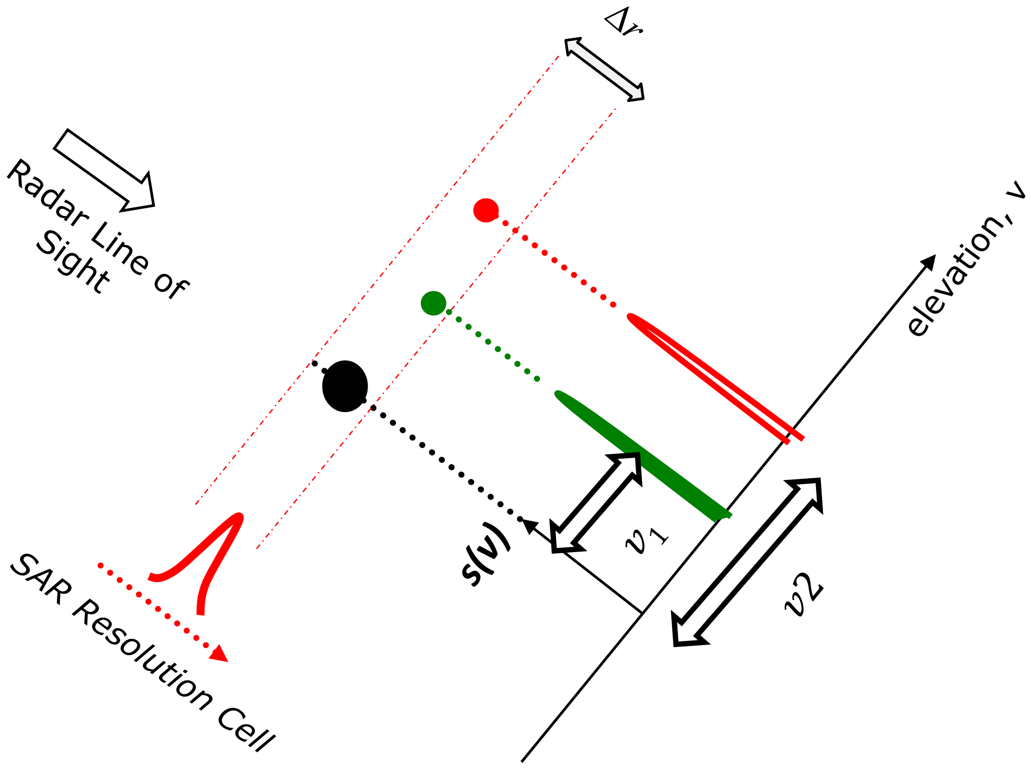
$$\Delta v = \frac{\lambda R_M(ref)}{2b_{ap}}$$

● = reference position

# TomoSAR – examples

*Case 2: two point targets*

$$I_n(r,x) = \sum_{p=1}^{2} s_p \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v_p\right\}$$




***s(v) = projection of the scatterers along elevation***

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# TomoSAR – examples

*Case 2: two point targets*

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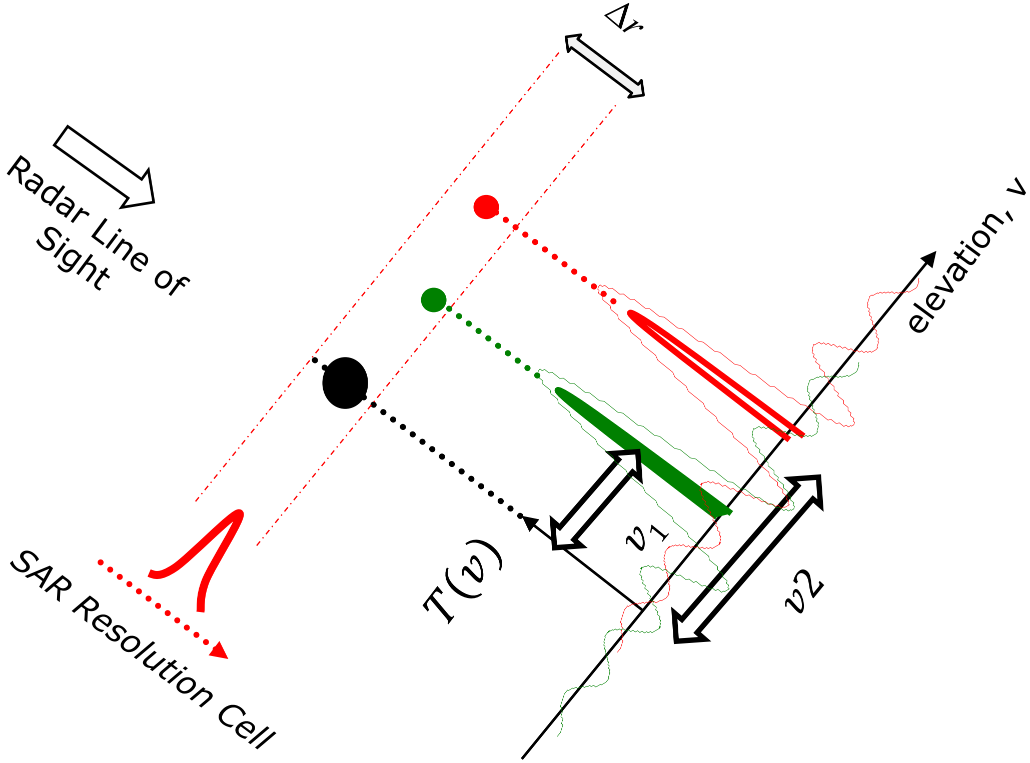


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***T(v) = reconstruction by SAR Tomography***

*Elevation resolution is*

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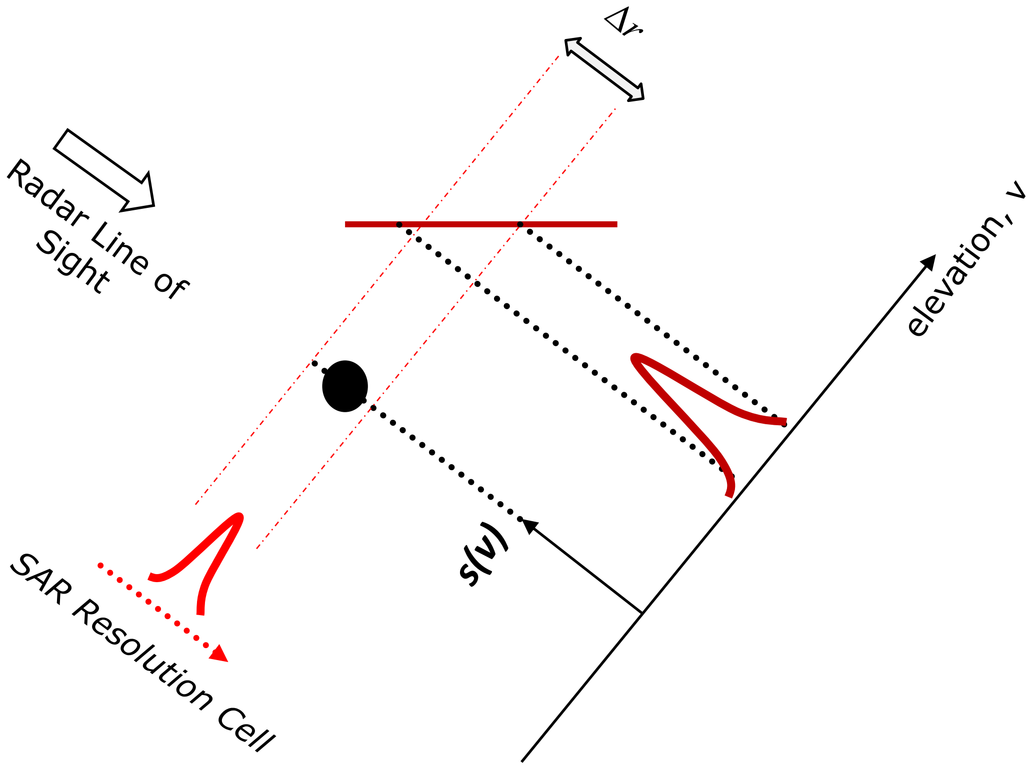
# TomoSAR – examples

*Case 3: terrain*




● = reference position

***s(v) = projection of the scatterers along elevation***

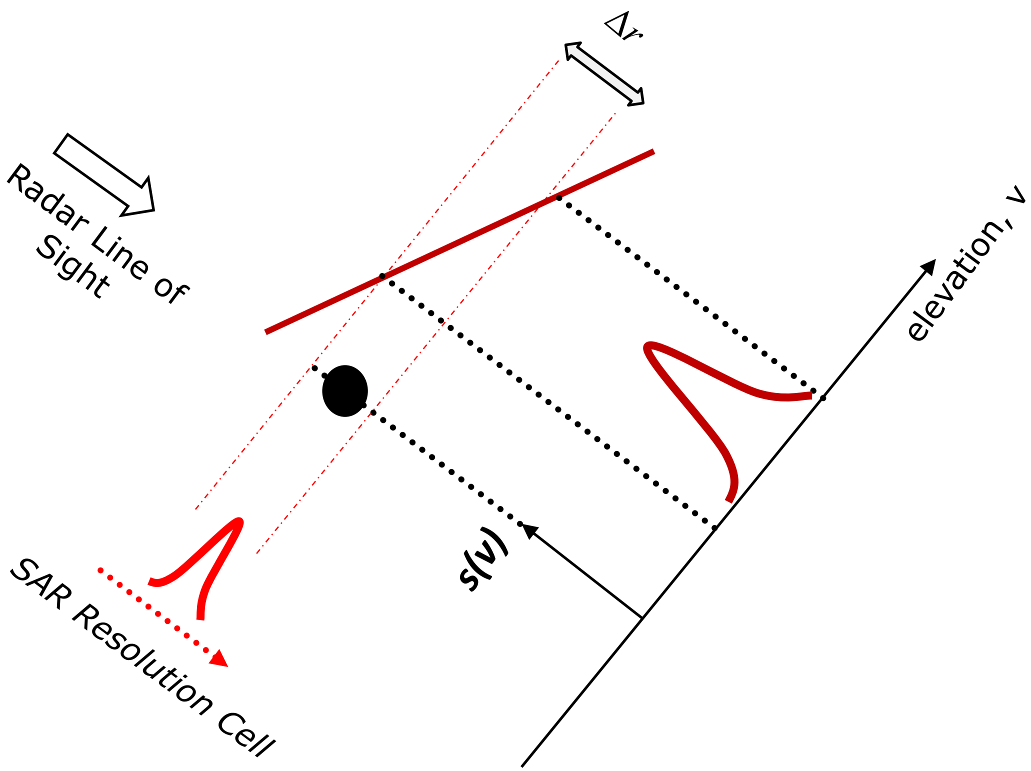
*Terrain = extended target*

⇔ *It does not project into a peak*

⇔ *Spread along elevation*

# TomoSAR – examples

*Case 3: terrain*




● = reference position

***s(v) = projection of the scatterers along elevation***

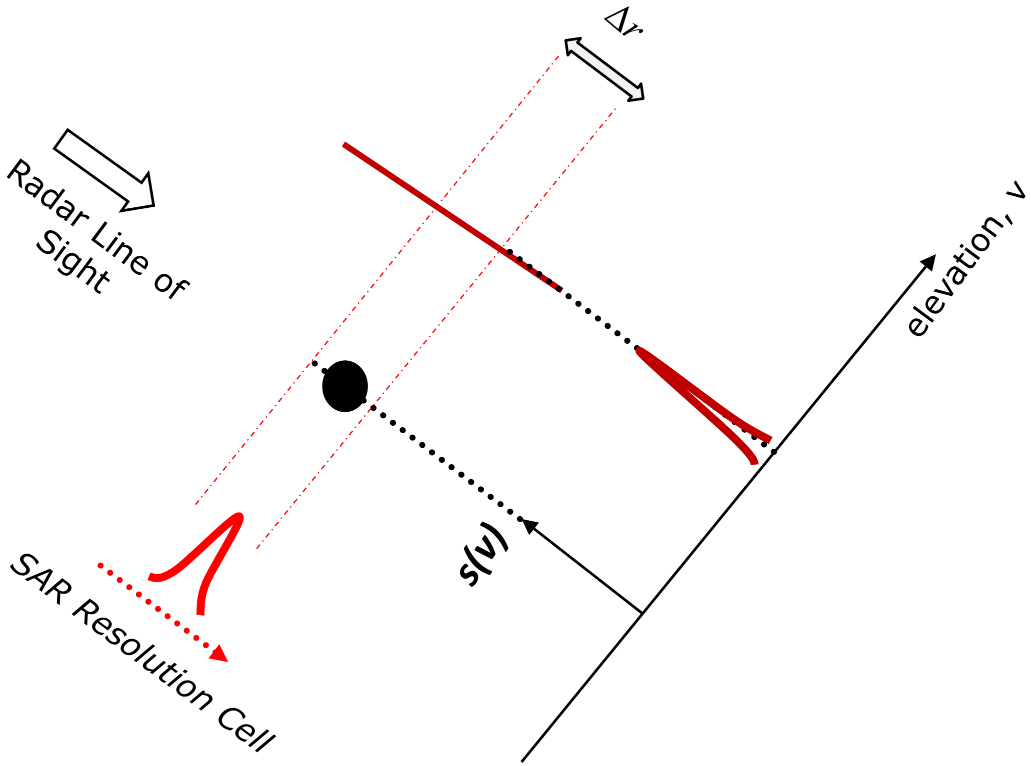
*Terrain = extended target*

⇔ *It does not project into a peak*

⇔ *Spread along elevation (depending on terrain slope)*

# TomoSAR – examples

*Case 3: terrain*




● = reference position

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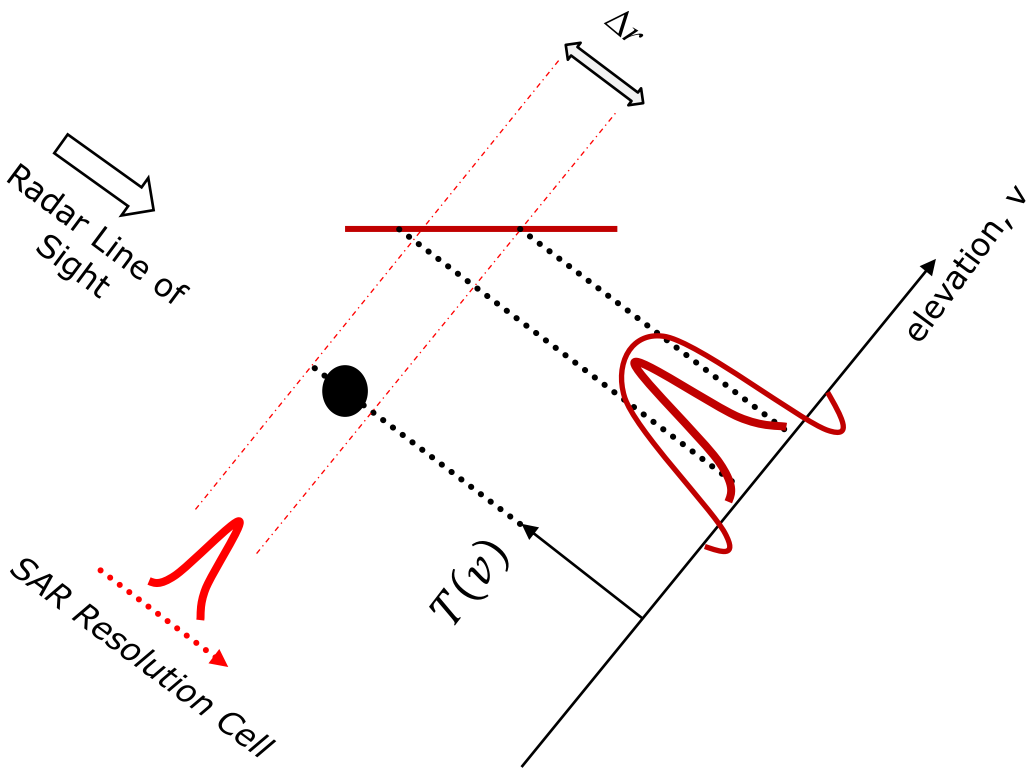
# TomoSAR – examples

*Case 3: terrain*

$$T(r,x,v) = \sum_n I_n(r,x) \cdot exp\left\{j\frac{2}{\lambda}\frac{v}{R_M(ref)}\right\}$$




***T(v) = reconstruction by SAR Tomography***

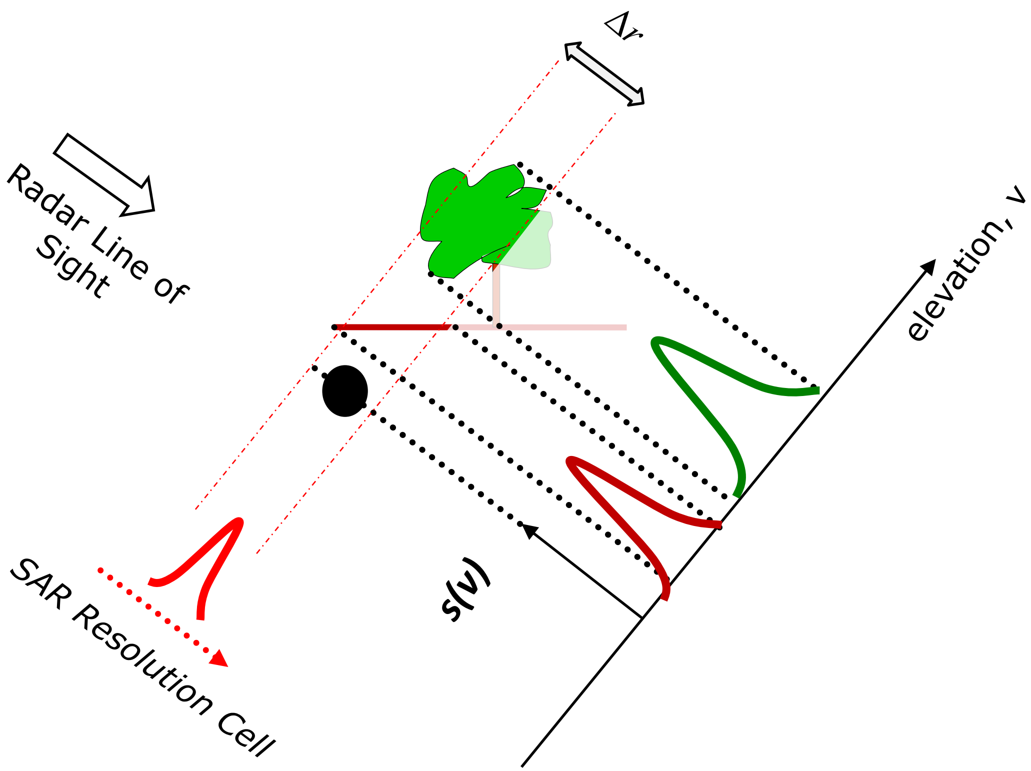
*Elevation resolution is*

$$\Delta v = \frac{\lambda R_M(ref)}{2b_{ap}}$$

● = reference position

# TomoSAR – examples

*Case 4: terrain + forest*



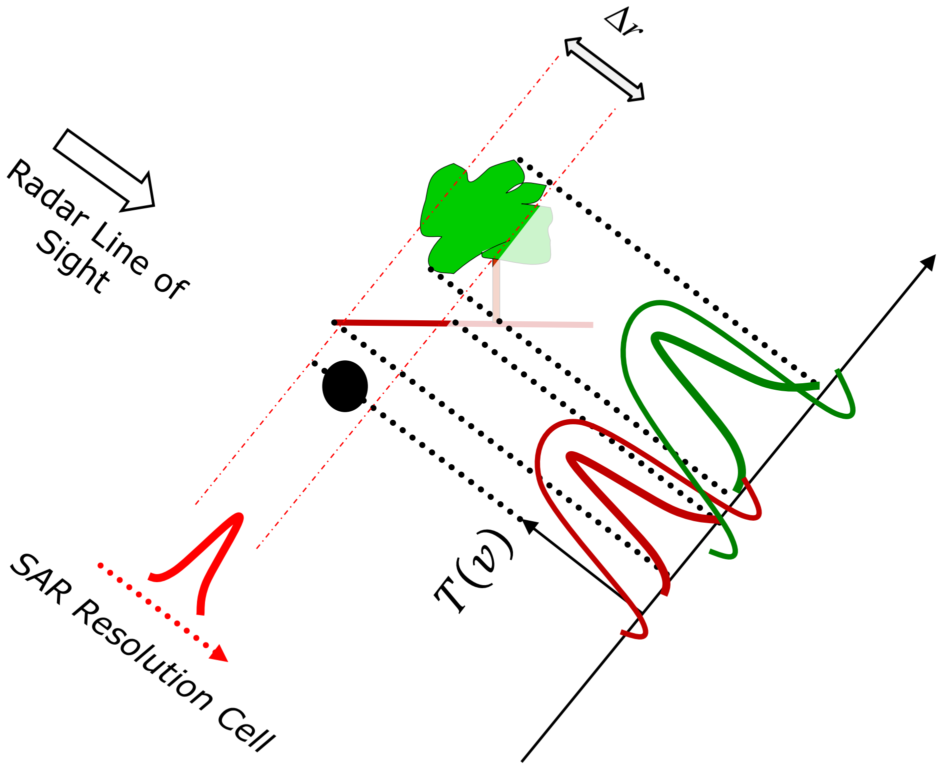

***s(v) = projection of the scatterers along elevation***

● = reference position

# TomoSAR – examples

*Case 4: terrain + forest*

$$T(r,x,v) = \sum_n I_n(r,x) \cdot exp\left\{j\frac{2}{\lambda}\frac{v}{R_M(ref)}\right\}$$




***T(v) = reconstruction by SAR Tomography***

*Elevation resolution is*

$$\Delta v = \frac{\lambda R_M(ref)}{2b_{ap}}$$

● = reference position

# TomoSAR inversion w.r.t. height

TomoSAR forward model

$$I_n(r,x) = \sum_v s(v) \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v\right\}$$

$I_n(r,x)$ : SLC pixel in the *n-th* image

$s$(r,x,$v$): projection of the scatterers along elevation

$b_n$ : normal baseline for the *n-th* image

$\lambda$ : carrier wavelength

Change of variable from cross range to height

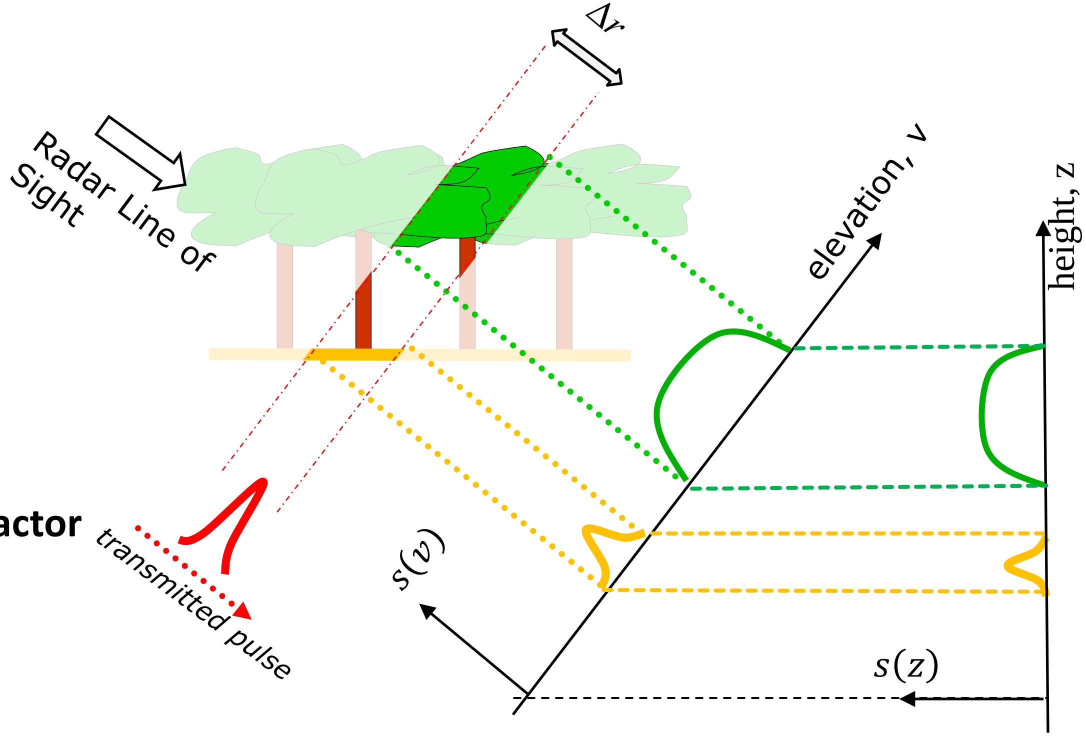
$$z = v \cdot \sin\theta$$

⇩

$$I_n(r,x) = \sum_z s(z) \cdot exp\{-jk_z(n)z\}$$

$k_z$ is usually referred to as **interferometric wavenumber** or **phase to height conversion factor**

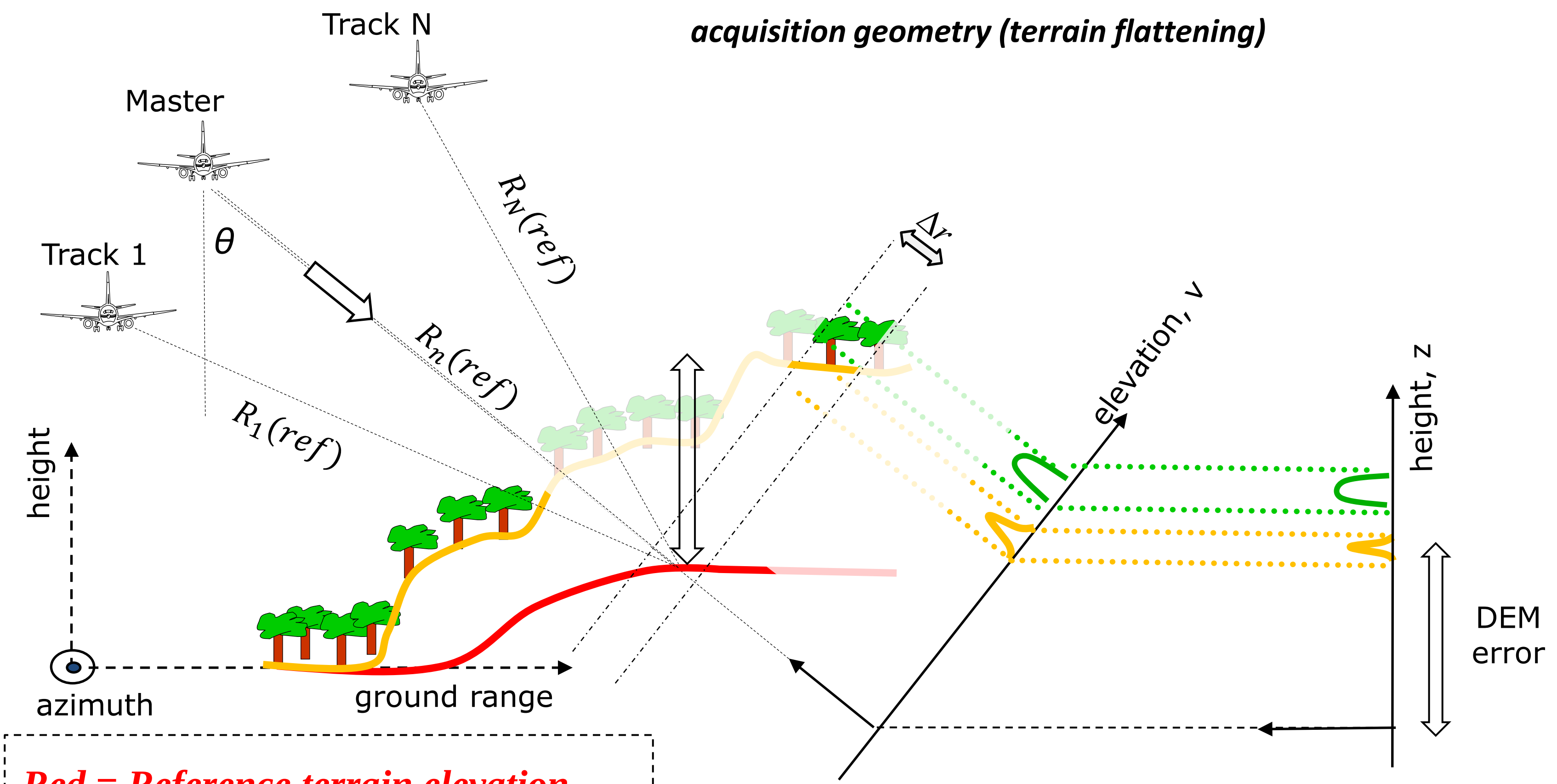
$$k_z(n) = \frac{4\pi}{\lambda R_M(ref)}\frac{b_n}{\sin\theta}$$



# Terrain flattening

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\}\cdot\sum_z s(z)\cdot exp\{-jk_z(n)z\}$$

***Phase offset to be removed based on knowledge of the acquisition geometry (terrain flattening)***



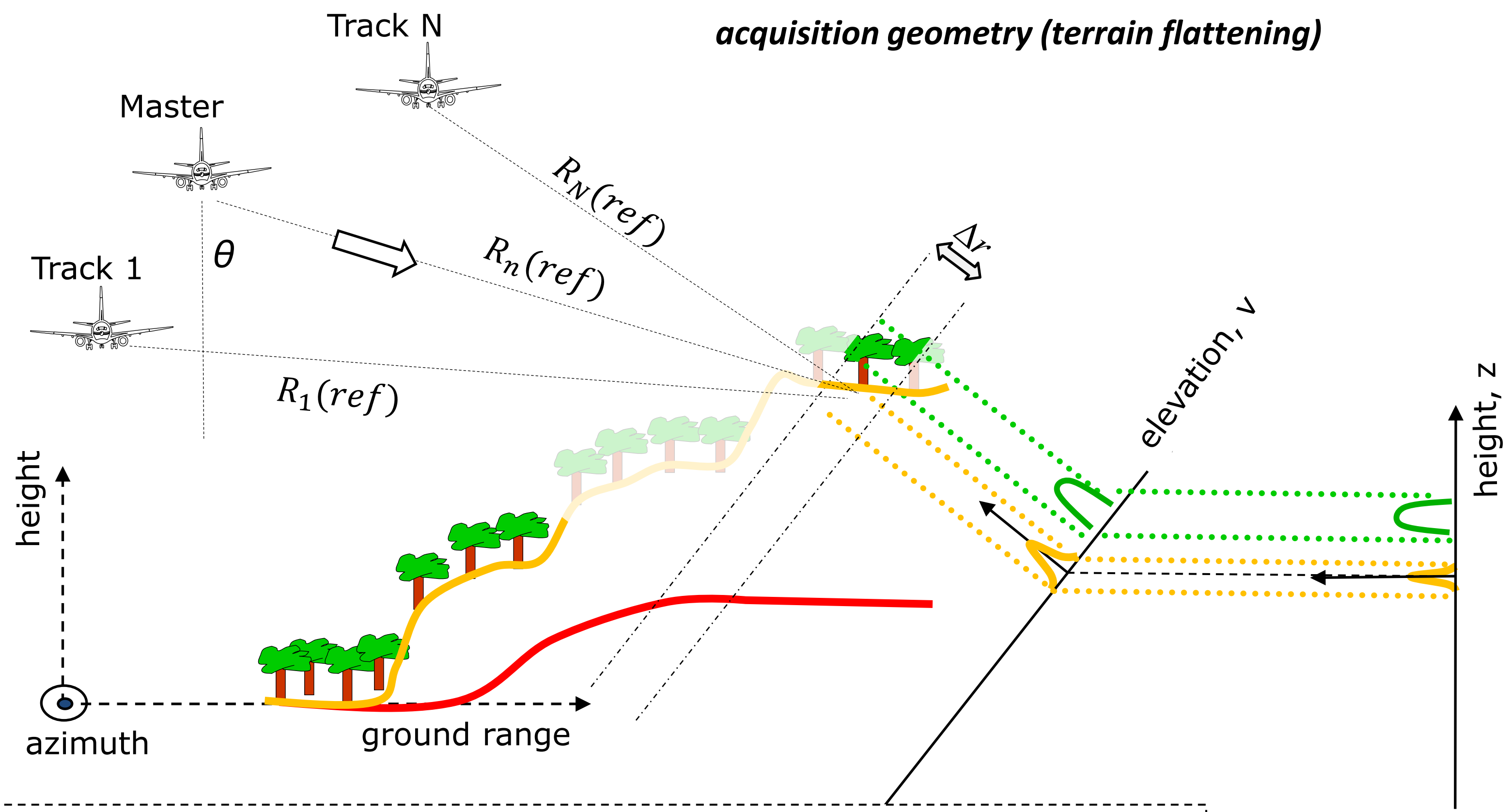

*Red = Reference terrain elevation*

*Orange = True terrain elevation*

# Terrain flattening

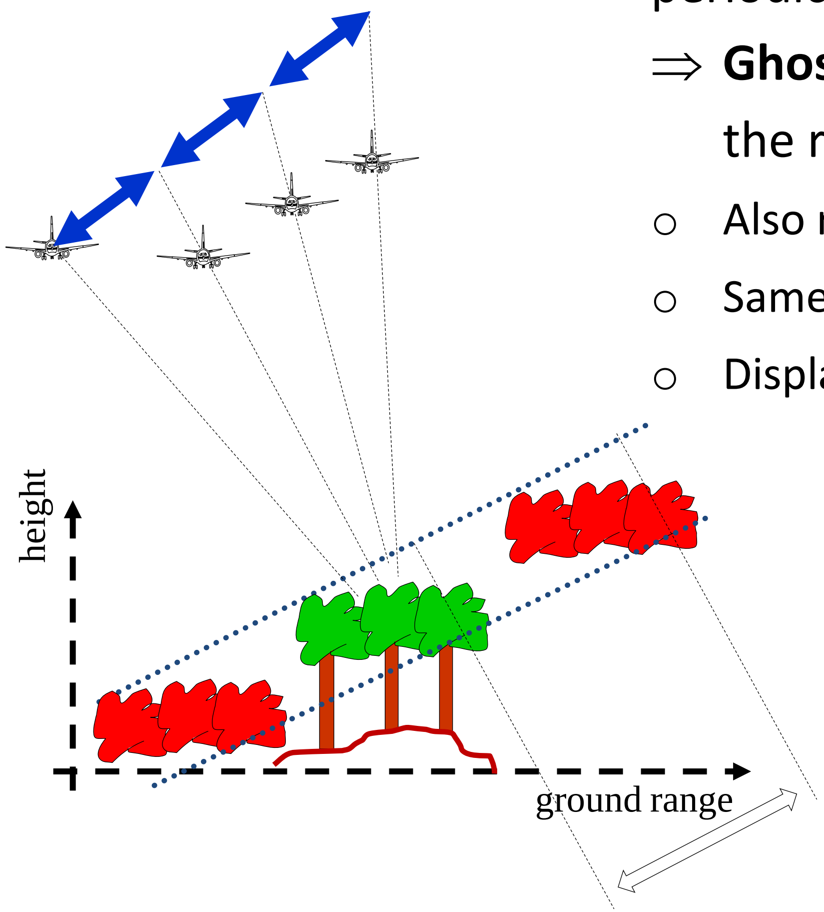
$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\} \cdot \sum_z s(z) \cdot exp\{-jk_z(n)z\}$$

***Phase offset to be removed based on knowledge of the acquisition geometry (terrain flattening)***




Orange = Reference terrain elevation = True terrain elevation

# Ambiguity

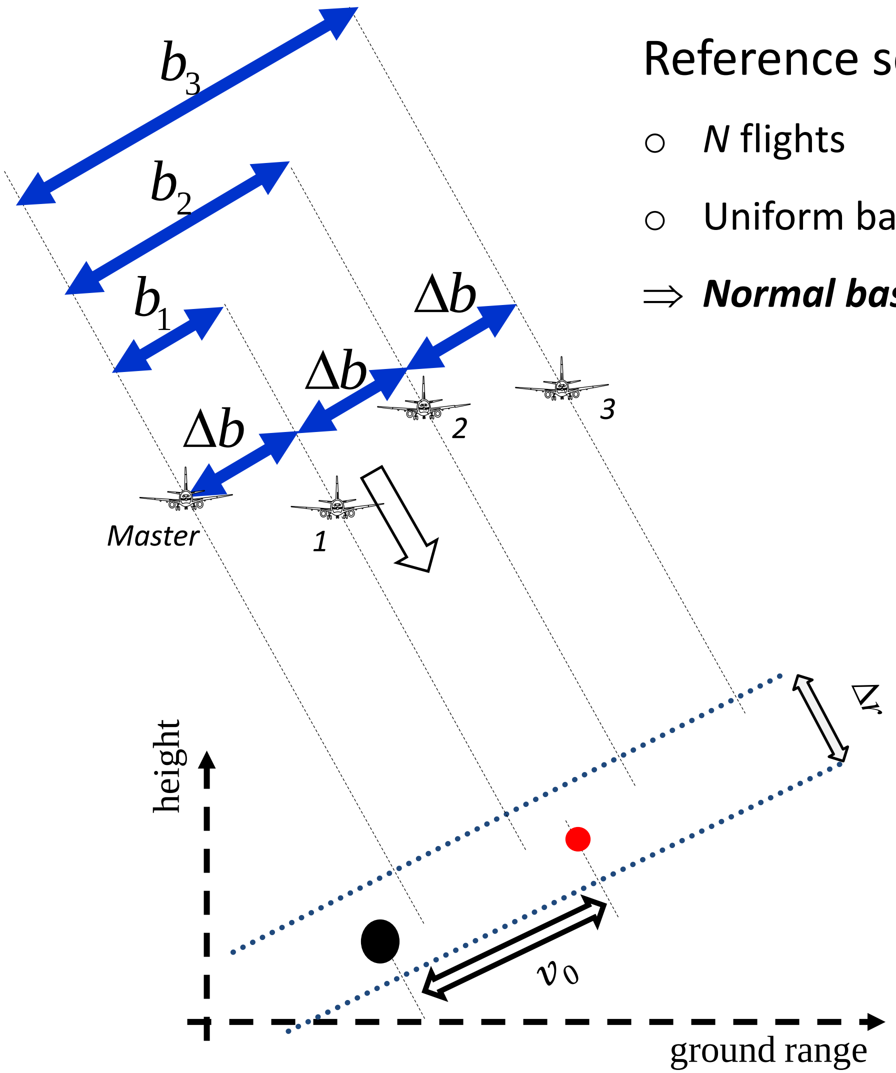



***Warning!*** The algorithm we have just seen produces periodic results

⇒ **Ghost** targets appearing at known position w.r.t. the real one

- Also referred to as ambiguous targets, or replicas
- Same range as the real target
- Displaced along elevation

# Ambiguity




● = reference position

## Reference scenario

- *N* flights
- Uniform baseline spacing *Δb*

⇒ ***Normal baseline for the n-th flight:*** $b_n = n \, \Delta b$

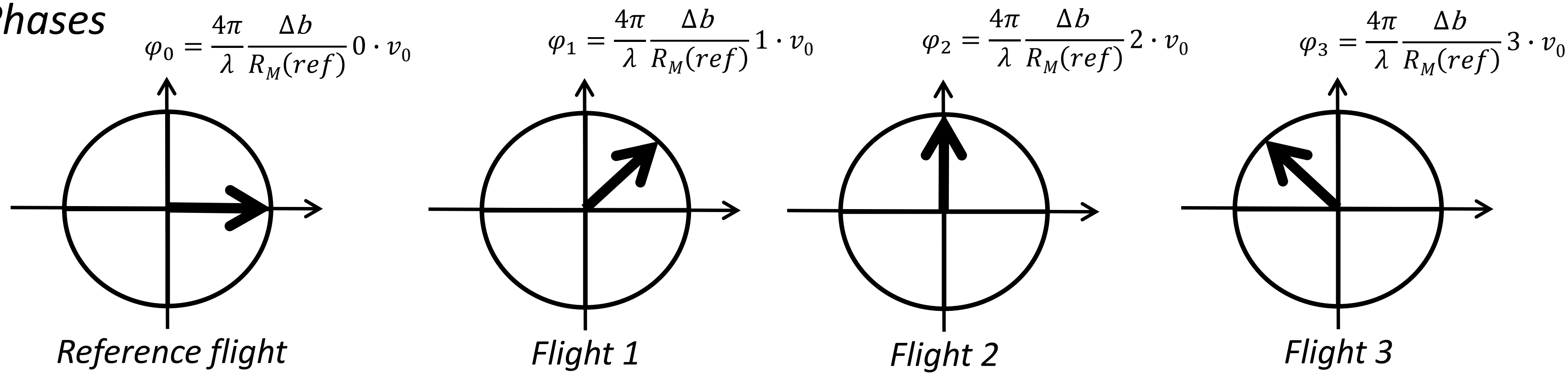
$$I_n(r,x) = s_0 \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v_0\right\}$$

$$= s_0 \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}n \cdot v_0\right\}$$

# Ambiguity

$$I_n(r,x) = s_0 \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}n \cdot v_0\right\}$$

*Phases*




Let's now consider another elevation $v_a$ such that $v_a = \frac{\lambda R_M(ref)}{2\Delta b}$

Then $\frac{4\pi}{\lambda}\frac{\Delta b}{r}n \cdot (v_0 + v_a) = \frac{4\pi}{\lambda}\frac{\Delta b}{r}n \cdot v_0 + 2\pi n \equiv \frac{4\pi}{\lambda}\frac{\Delta b}{r}n \cdot v_0$

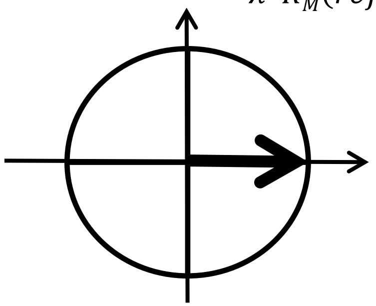
$\Rightarrow$ The two elevations $v_0$ and $v_a$ produce the same phase in all SAR images

# Ambiguity

$$I_n(r,x) = s_0 \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}n \cdot v_0\right\}$$

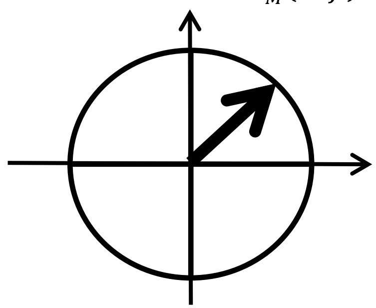
*Phases*

$$\varphi_0 = \frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}0 \cdot v_0$$



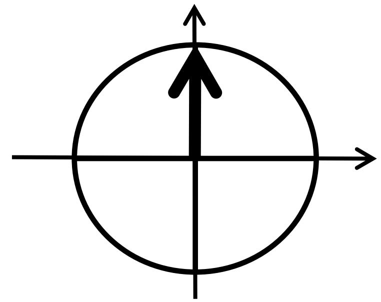
*Reference flight*

$$\varphi_1 = \frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}1 \cdot v_0$$



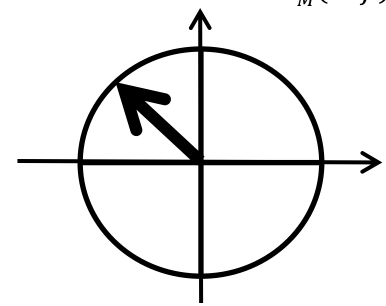
*Flight 1*

$$\varphi_2 = \frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}2 \cdot v_0$$



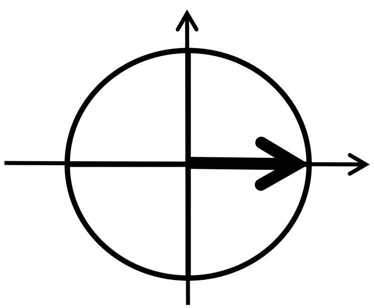
*Flight 2*

$$\varphi_3 = \frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}3 \cdot v_0$$



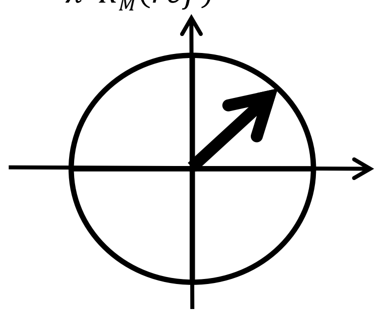
*Flight 3*

$$\varphi_0 = \frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}0 \cdot (v_0 + va)$$



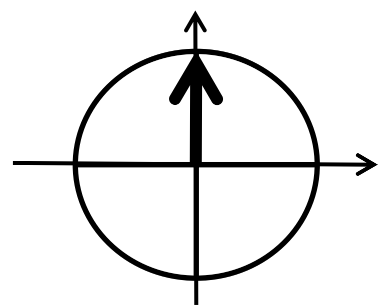
*Master*

$$\varphi_1 = \frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}1 \cdot (v_0 + va)$$



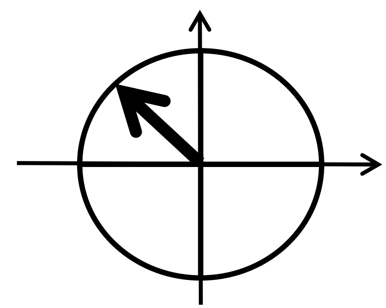
*Flight 1*

$$\varphi_2 = \frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}2 \cdot (v_0 + va)$$



*Flight 2*

$$\varphi_3 = \frac{4\pi}{\lambda}\frac{\Delta b}{R_M(ref)}3 \cdot (v_0 + va)$$

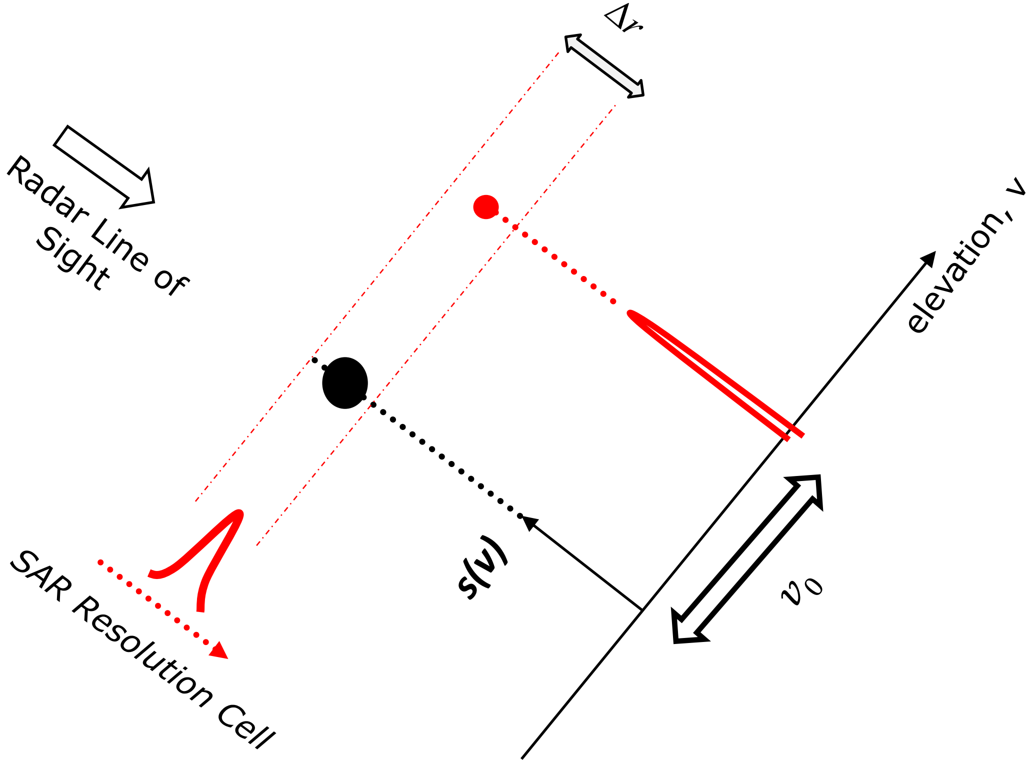


*Flight 3*

# TomoSAR – examples (II)

*Case 1: a single point target*

$$I_n(r,x) = s_0 \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v_0\right\}$$




***s(v) = projection of the scatterers along elevation***

● = reference position

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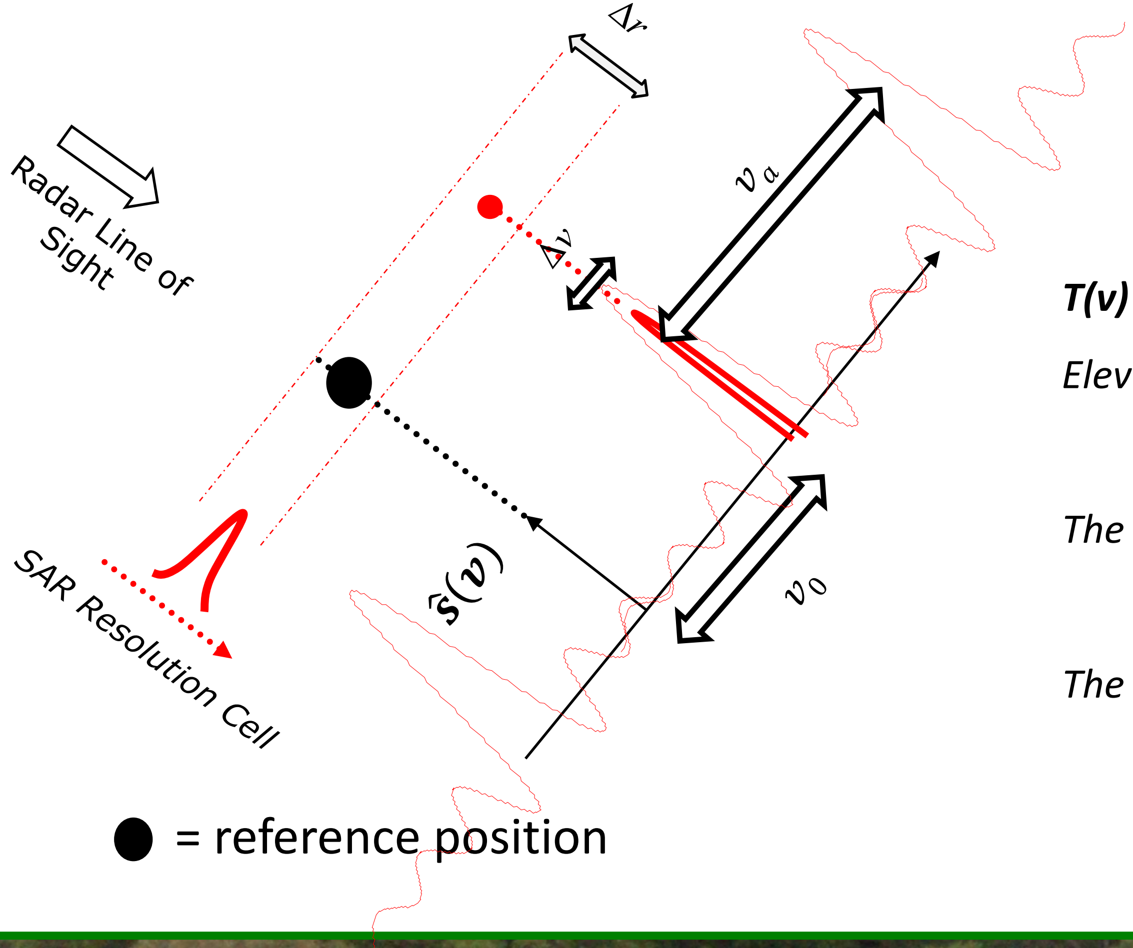
$$I_n(r,x) = s_0 \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{R_M(ref)}v_0\right\}$$

$$T(r,x,v) = \sum_n I_n(r,x) \cdot exp\left\{j\frac{2}{\lambda}\frac{v}{R_M(ref)}\right\}$$




***T(v) = reconstruction by SAR Tomography***

*Elevation resolution is*

$$\Delta v = \frac{\lambda R_M(ref)}{2b_{ap}}$$

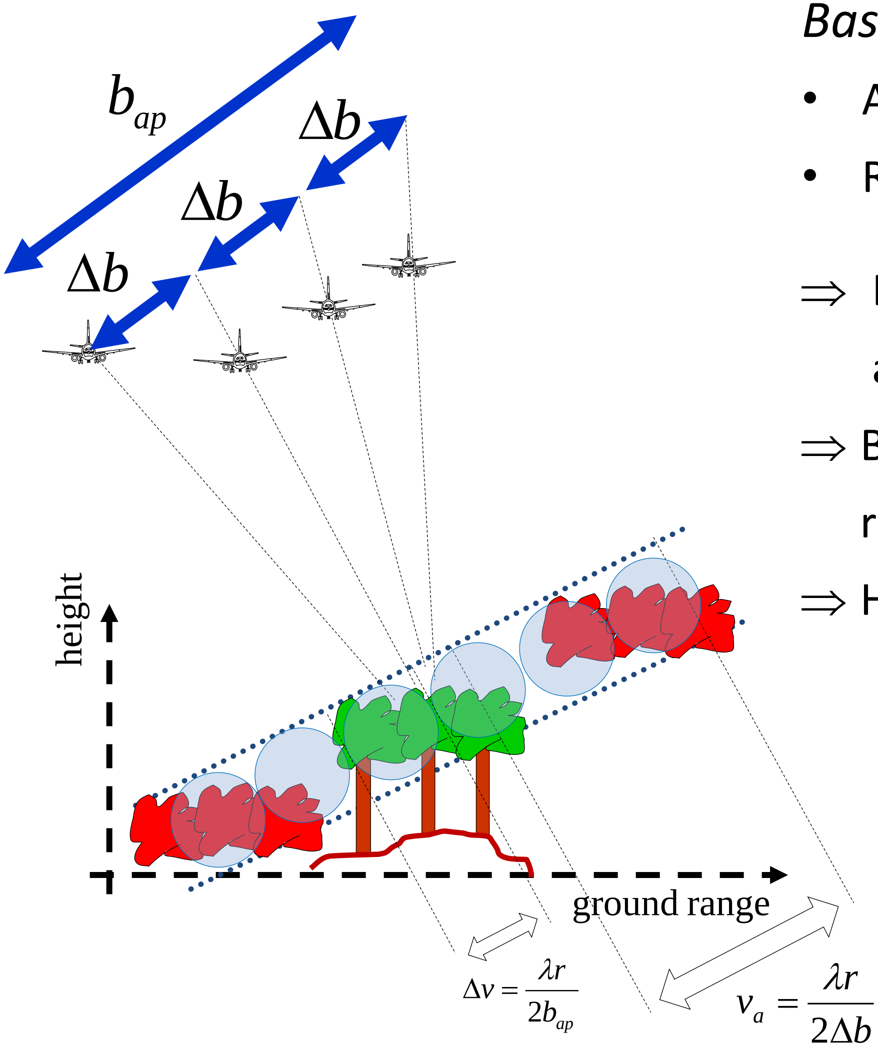
*The elevation of ambiguity is*

$$v_a = \frac{\lambda R_M(ref)}{2\Delta b}$$

*The corresponding height of ambiguity is*

$$z_a = \frac{\lambda R_M(ref)\sin\theta}{2\Delta b}$$

# Baseline design tips




*Baseline design tips*

- Ambiguity ⇔ baseline spacing
- Resolution ⇔ baseline aperture

⇒ Baseline spacing: small enough to ensure that ambiguous targets stay away from the real ones

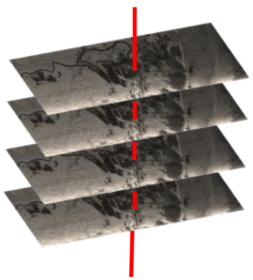
⇒ Baseline aperture: large enough to meet resolution requirement

⇒ How many passes ? $N \geq \frac{b_{ap}}{\Delta b} = \frac{v_a}{\Delta v}$

# Advanced TomoSAR

Current paradigm for forested areas: **retrieve the *vertical distribution of backscattered power* based on the observed *InSAR coherences***

***SLC data***



***Complex data vector (Nx1)***

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix}$$

⇨

***Coherence estimation (spatial multilooking)***

$$\gamma_{nm} = \frac{\langle d_n d_m^{\,*} \rangle_L}{\sqrt{\langle |d_n|^2 \rangle_L \langle |d_m|^2 \rangle_L}}$$

⇨

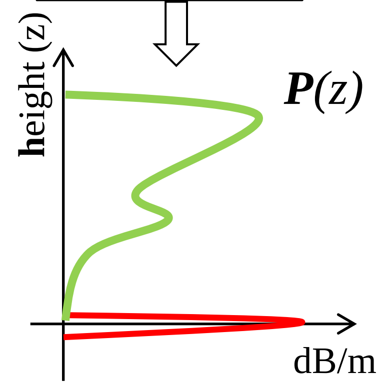
***Coherence matrix (NxN)***

$$\mathbf{R} = \begin{bmatrix} 1 & \gamma_{12} & \cdots & \gamma_{1N} \\ \gamma_{21} & 1 & & \\ \vdots & & \ddots & \vdots \\ \gamma_{N1} & & \cdots & 1 \end{bmatrix}$$

⇩

***TomoSAR processing***

⇩




## *Why ?*

- Equivalent to DFT if inversion is carried out using linear methods
- **Non-linear methods can be used to achieve:**
  - **Super-resolution** (super = finer than the limit from baseline aperture)
  - **Rejection of ambiguous targets** (using irregular baseline spacing)

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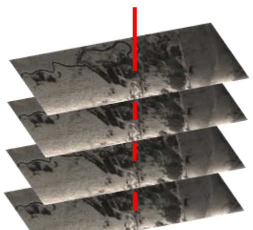

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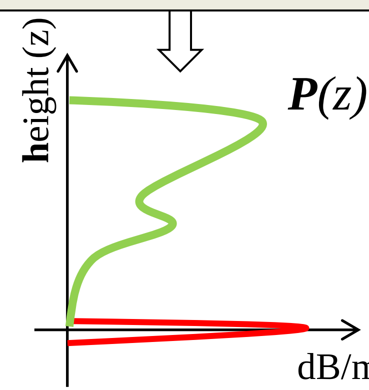
***Coherence matrix (NxN)***

$$\mathbf{R} = \begin{bmatrix} 1 & \gamma_{12} & \cdots & \gamma_{1N} \\ \gamma_{21} & 1 & & \\ \vdots & & \ddots & \vdots \\ \gamma_{N1} & & \cdots & 1 \end{bmatrix}$$

***W***

# **Later on today**

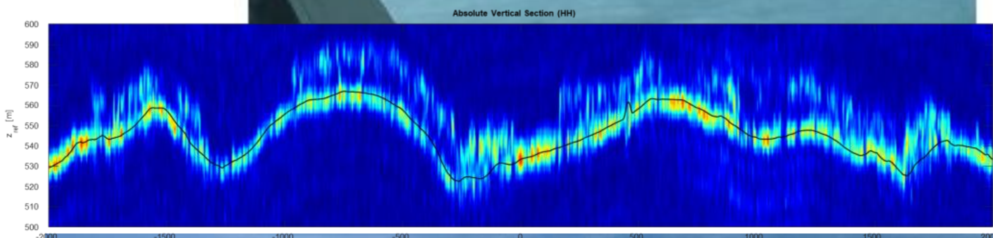
- Equivalent to DFT if inversion is carried out using linear methods
- **Non-linear methods can be used to achieve:**
  - **Super-resolution** (super = finer than the limit from baseline aperture)
  - **Rejection of ambiguous targets** (using irregular baseline spacing)



# *TomoSense*

The airborne campaign took place in 2020/21 at the Kermeter site in the Eifel National Park in North-Rine Westphalia in Germany. The campaign includes:

- Bistatic airborne SAR surveys at L- and C-Band collected by flying two aircraft in close formation, with one following the other at a nominal distance of approximately 20/30 m.
- The flights were programmed in synergy with the P-Band campaign BelSAR-P.
- In-situ collection of relevant forest parameters at approximately 80 plots.
- Collection of TLS data at a scale of 1 ha at 10 plots.
- Installation of 5 m trihedral reflectors for P-Band calibration




# TomoSense

The TomoSense data-set is intended to serve as an important basis for studies on microwave scattering from forested areas in the context of future studies on Earth Observation missions.

The data-set includes:

- Calibrated SAR images and tomographic cubes at different levels of processing
- ALS-derived maps of forest height and AGB
- Forest census
- TLS profiles.

Complex SAR images are already finely coregistered, phase calibrated, and ground steered, in such a way as to enable future researchers to directly implement any kind of interferometric or tomographic processing without having to deal with the subtleties of airborne SAR data.

In addition to that, the data-base comprises tomographic cubes representing forest scattering in 3D both in Radar and geographical coordinates, which are intended for use by non-Radar experts.

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The data-set includes:

- C
- A
- F
- T

**The whole data-set (Radar+Lidar) is public and free to use for scientific purposes**

**Just contact me at**

**[stefano.tebaldini@polimi.it](mailto:stefano.tebaldini@polimi.it)**

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