*ESA 6th Advanced Training Course on Radar Polarimetry*

# INTRODUCTION TO TOMOGRAPHIC PHASE CALIBRATION


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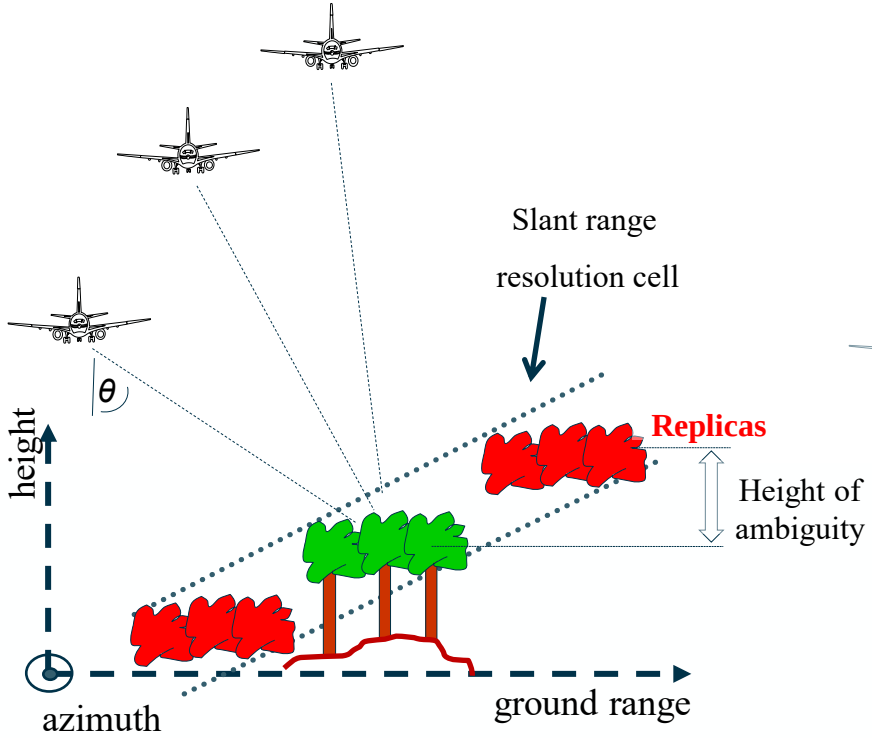
# Phase screens

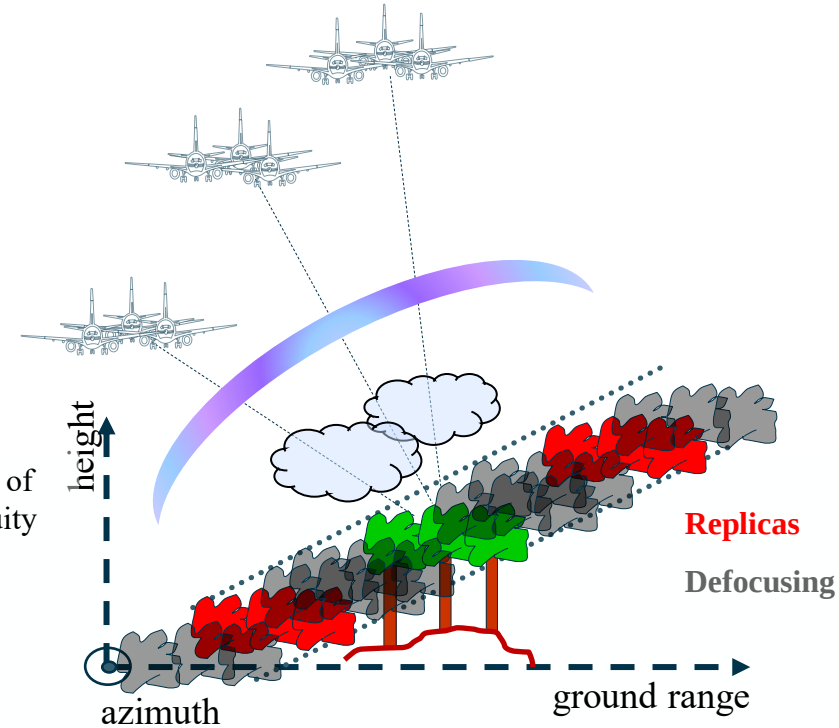
TomoSAR provides resolution in elevation by jointly focusing data from multiple baselines

- Resolution ⇔ total baseline span
- Vertical Ambiguity ⇔ baseline spacing

➢ Phase jitters result in signal defocusing

- Spaceborne: tropospheric and ionospheric phase screens
- Airborne: uncompensated platform motions ***on the order of a fraction of a wavelength***

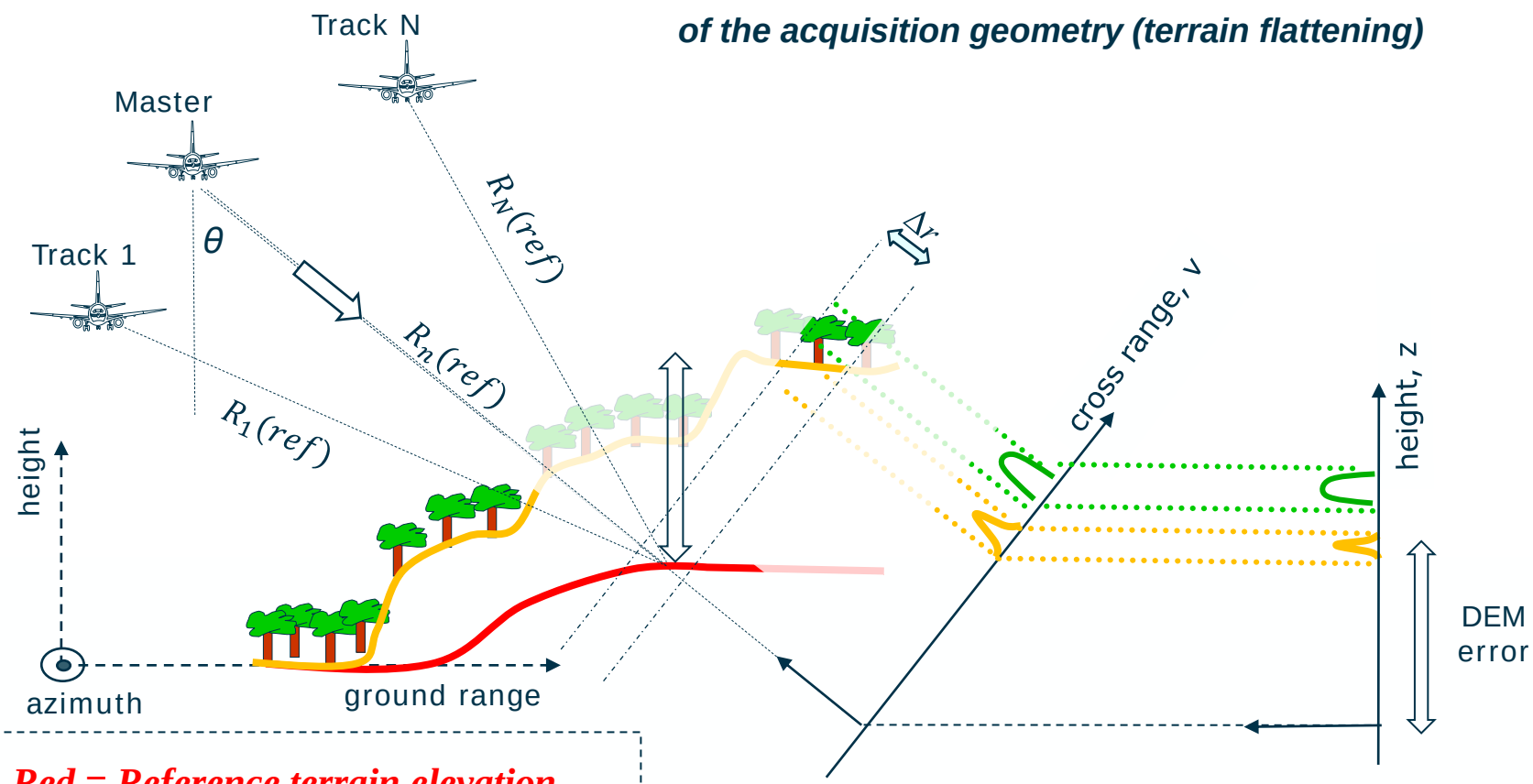


# Phase screens

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\} \cdot \int_C s(r,x,v) \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{r}v\right\} dv$$

***Phase offset to be removed based on knowledge of the acquisition geometry (terrain flattening)***




***Red = Reference terrain elevation***

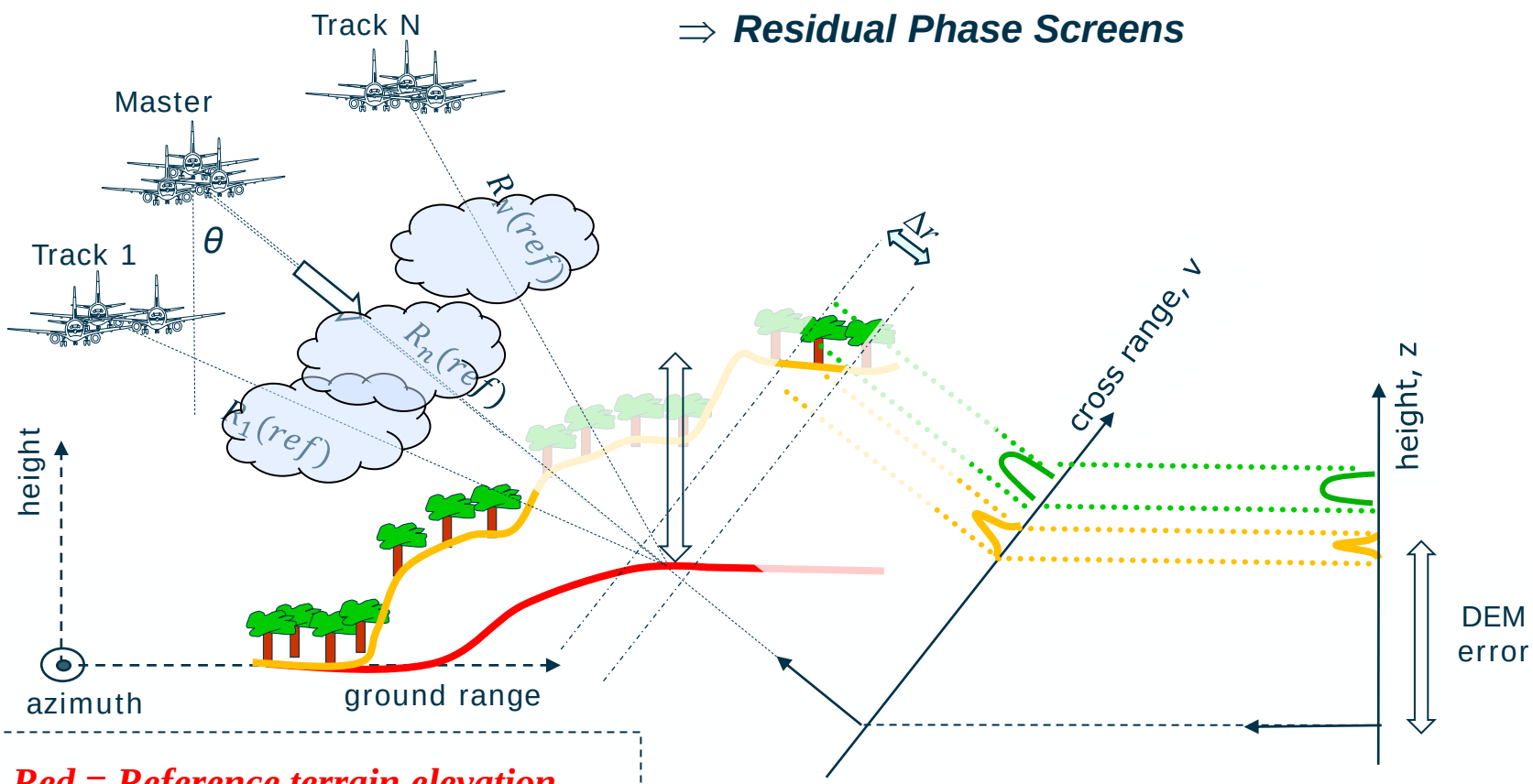
***Orange = True terrain elevation***

# Phase screens

$$I_n(r,x) = exp\left\{-j\frac{4\pi}{\lambda}R_n(ref)\right\} \cdot \int_C s(r,x,v) \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{r}v\right\} dv$$

***Phase offset are not removed correctly !***

$\Rightarrow$ ***Residual Phase Screens***




***Red = Reference terrain elevation***

***Orange = True terrain elevation***

# Phase screens

Each focused SLC SAR image is obtained as the Fourier Transform of the scene complex reflectivity projected along the cross-range coordinate, and **Phase rotated by an (often unknown) phase screen $\boldsymbol{\alpha_n(r,x)}$**

$$I_n(r,x) = exp\{-j\alpha_n(r,x)\} \int_C s(r,x,v) \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{r}v\right\} dv$$

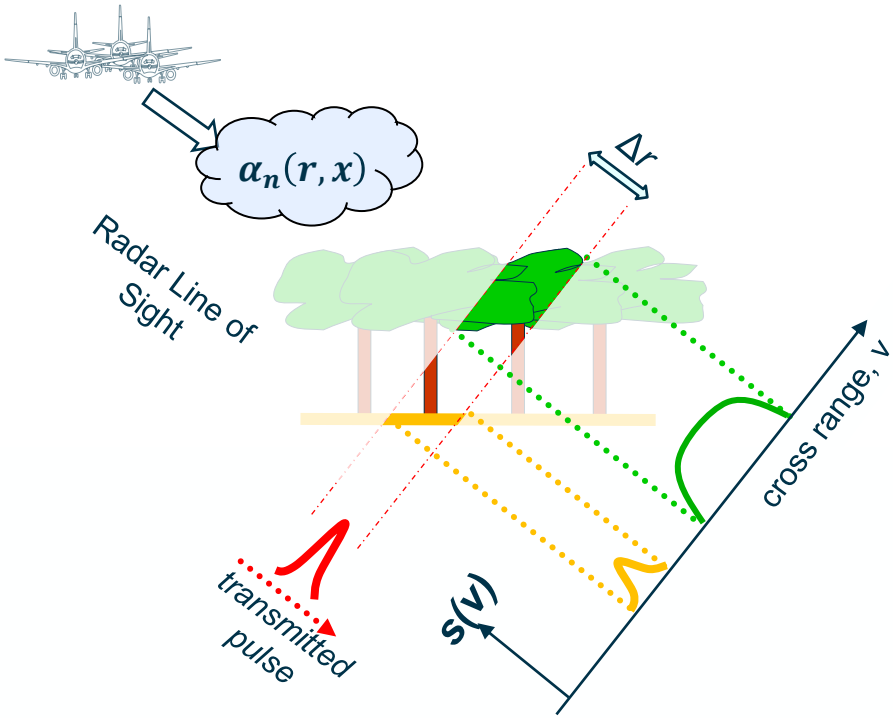
$I_n(r,x)$ : SLC pixel in the *n-th* image

$s(r,x,v)$: cross-range projection of target reflectivity within the SAR resolution cell at $(r,x)$

$b_n$ : normal baseline for the *n-th* image

$\lambda$ : carrier wavelength

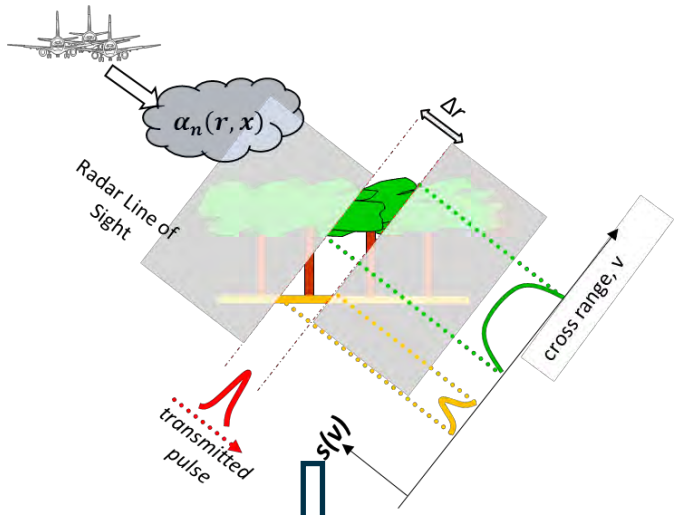
$\boldsymbol{\alpha_n(r,x)}$***: phase screen for the n-th pass***

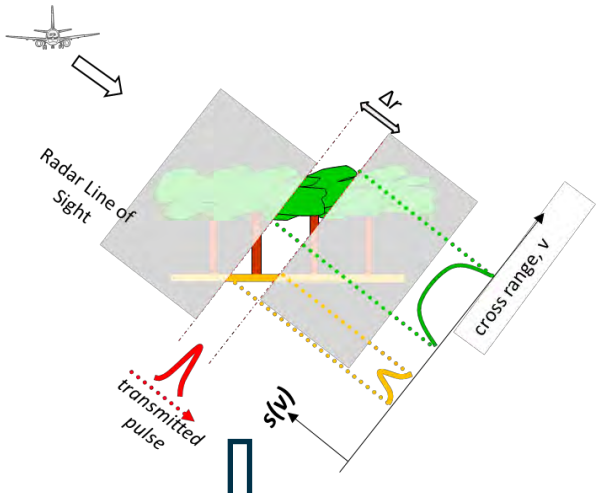


# Phase calibration

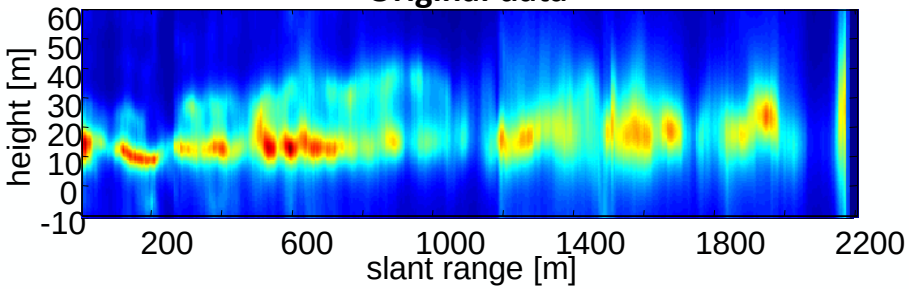
## *Phase calibration = phase screen removal*

$$I_n(r,x) = \cancel{exp\{-j\alpha_n(r,x)\}} \int_C s(r,x,v) \cdot exp\left\{-j\frac{4\pi}{\lambda}\frac{b_n}{r}v\right\} dv$$

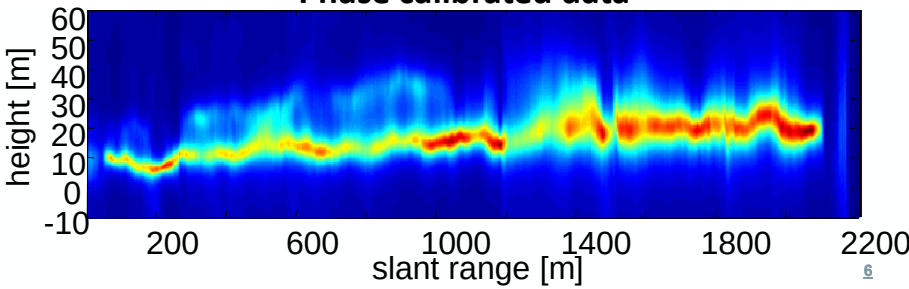



**Original data**




**Phase calibrated data**



# Phase screen impact on TomoSAR: a rough assessment

***Scenario: repeat pass tomographic survey***

$$S_n \longrightarrow \otimes \longrightarrow S_n exp(j\alpha_n) \cong S_n + \underbrace{jS_n\alpha_n}$$

$$\uparrow$$

$$exp(j\alpha_n)$$

***Equivalent noise term***

$$\boldsymbol{P_{noise} = P_{signal} \cdot \sigma_\alpha^2}$$

- $S_n$ = SLC image from the *n-th* pass
- $\alpha_n$ = phase screen for the *n-th* pass

# Phase screen impact on TomoSAR: a rough assessment

***Scenario: repeat pass tomographic survey***

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***Equivalent SNR*** $\approx \boldsymbol{\dfrac{1}{\sigma_\alpha^2}}$

- $S_n$ = SLC image from the *n-th* pass
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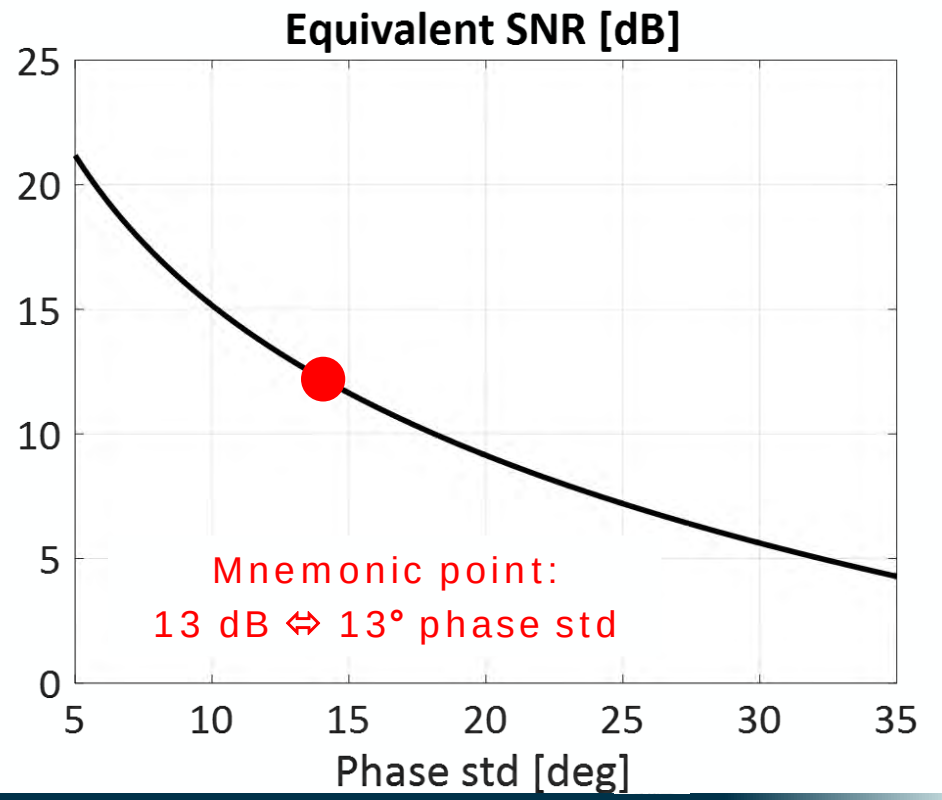


# Phase screen impact on TomoSAR: a rough assessment

***Scenario: repeat pass tomographic survey***

$$S_n \longrightarrow \otimes \longrightarrow S_n exp(j\alpha_n) \cong S_n + \underbrace{jS_n\alpha_n}$$

$$exp(j\alpha_n)$$

***Equivalent noise term***

$$\boldsymbol{P_{noise} = P_{signal} \cdot \sigma_\alpha^2}$$

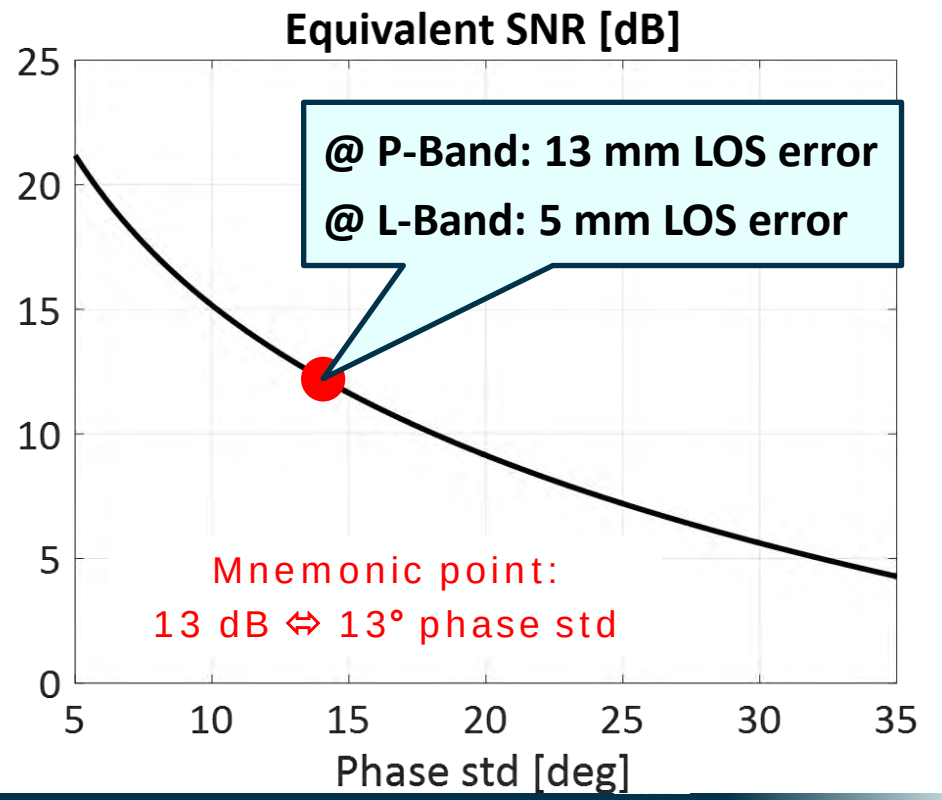
***Equivalent SNR*** $\approx \dfrac{\mathbf{1}}{\boldsymbol{\sigma_\alpha^2}}$

- $S_n$ = SLC image from the *n-th* pass
- $\alpha_n$ = phase screen for the *n-th* pass

**Conversion to antenna position error:**

$$\alpha_n = \frac{4\pi}{\lambda} dr_n$$

$dr_n$ = Line of Sight (LOS) error about antenna position in the *n-th* pass



# Phase screen impact on TomoSAR: a rough assessment

Impact on TomoSAR focusing depends on baseline distribution and processing algorithm

***Keep it simple rule***: we assume:

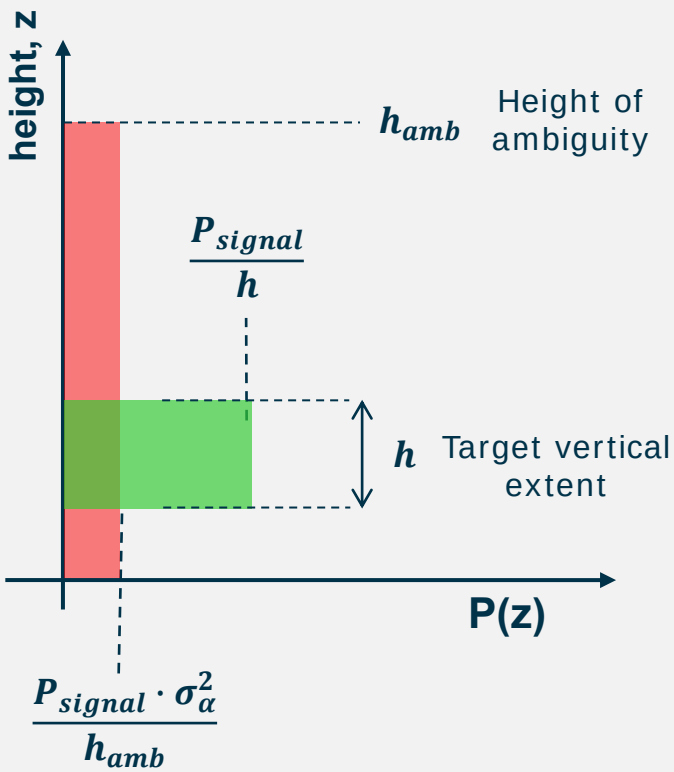
- Uniform baseline distribution ⇔ well defined height of ambiguity
- Linear processing ⇔ analytical assessment

*Tomogram Expected Value*

⇒ *The useful signal is focused in the height sector corresponding to the vertical extent of the target*

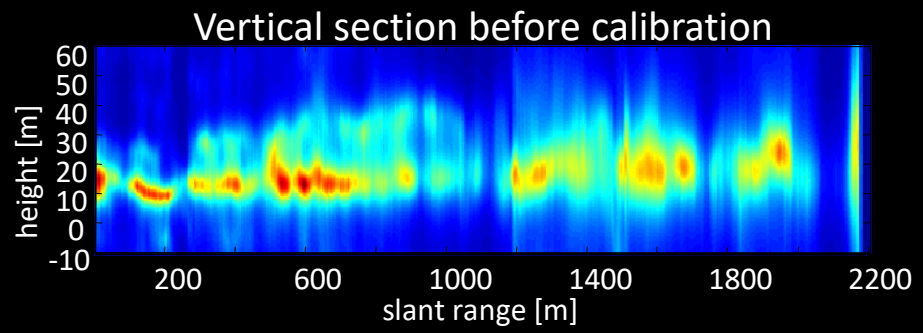
⇒ *Noise is spread all over the ambiguous height interval*

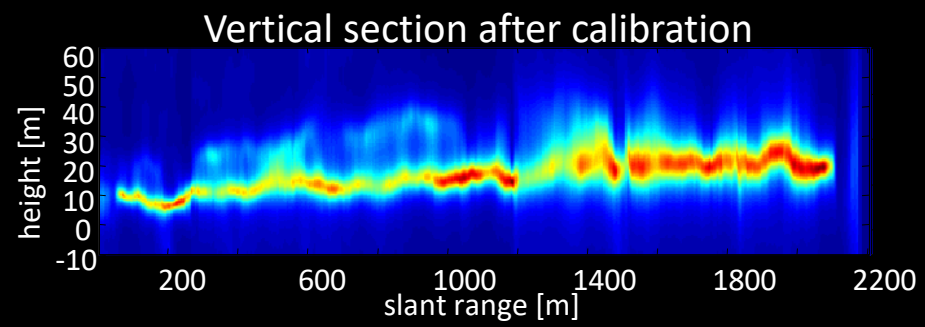
$$\Rightarrow \quad SNR_{Tomo} \approx \frac{1}{\sigma_\alpha^2} \frac{h_{amb}}{h}$$

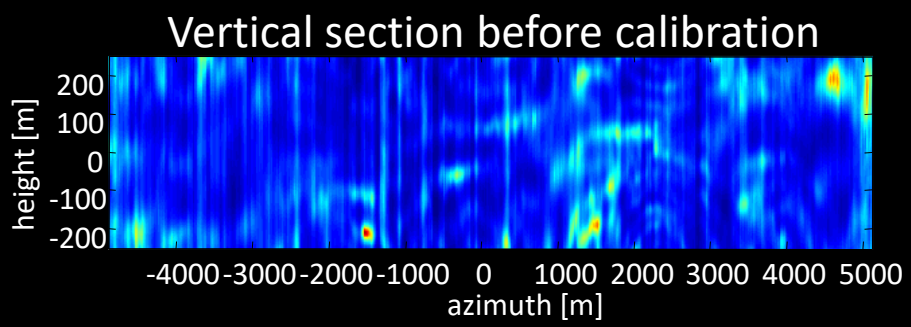


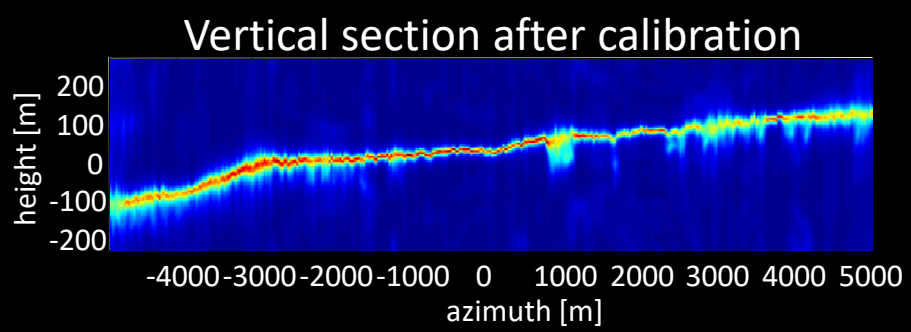
# *BIOSAR 2007 – DLR – P-Band*

| | |
|---|---|
| **Site** | **Remningstorp, Southern Sweden** |
| **Period** | Spring 2007 |
| **Scene** | Semi-boreal forest |
| **Topography** | Flat |
| **Carrier frequency** | P-Band |
| **Number of passes** | 9 |
| **Vertical resolution** | ≈ 10 m (near range) to ≈ 40 m (far range) |



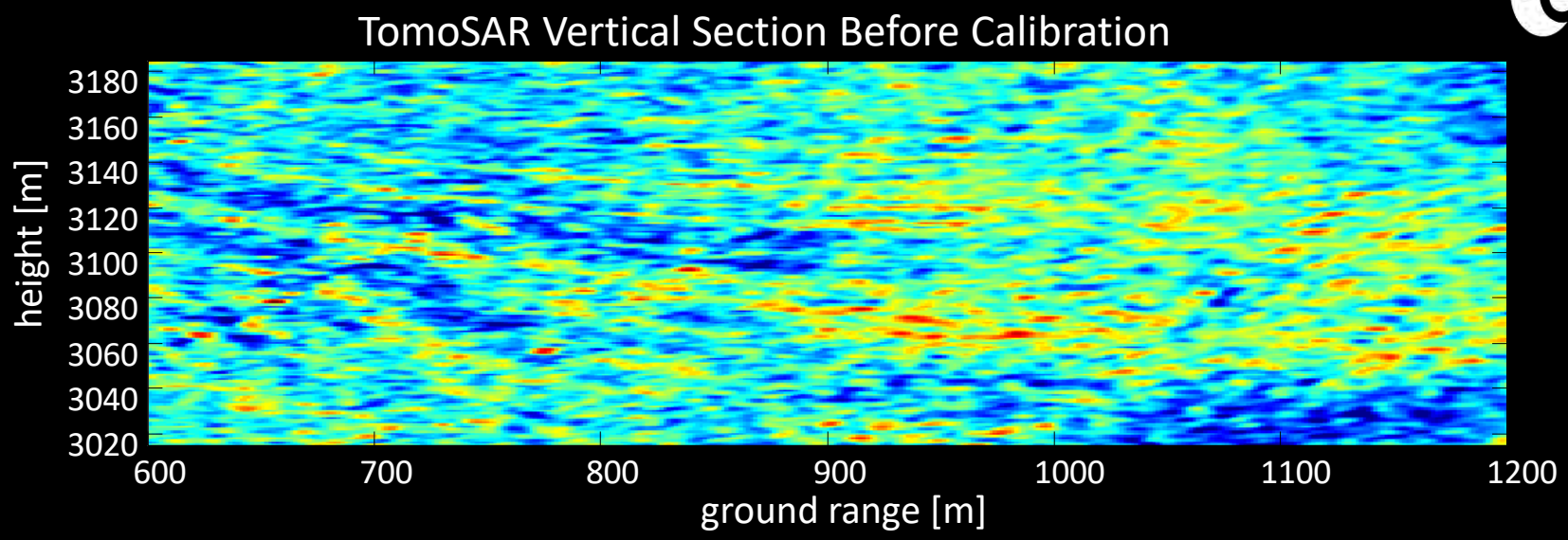



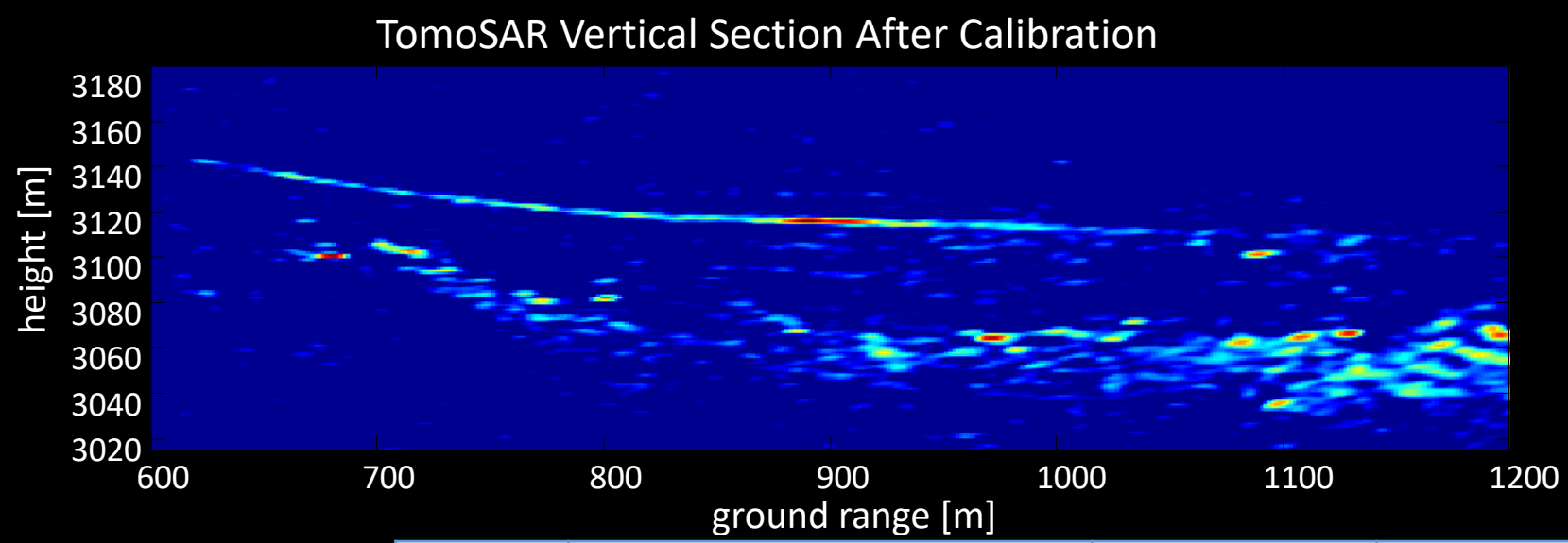




# *IceSAR 2012 – DTU – P-Band*

| | |
|---|---|
| **Site** | **K-transect, Southwest Greenland** |
| **Period** | May 2012 |
| **Scene** | Ice |
| **Topography** | Flat |
| **Carrier frequency** | P-Band |
| **Number of passes** | 8 |
| **Vertical resolution** | > 50 m |

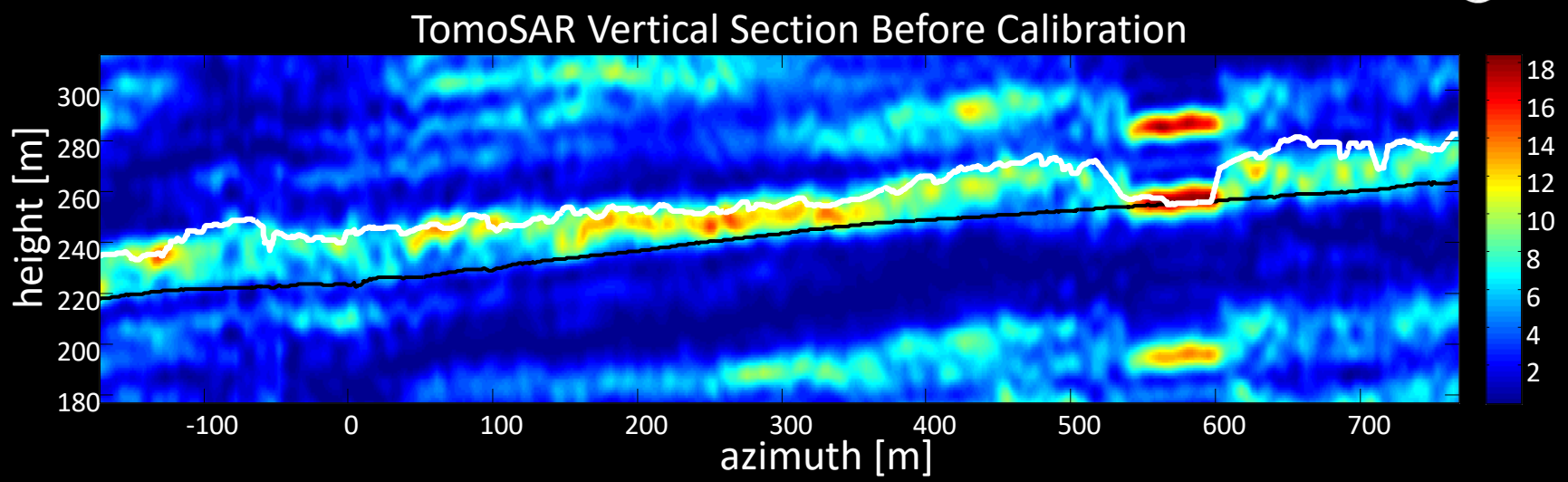
# AlpTomoSAR 2014 – MetaSensing – L-Band

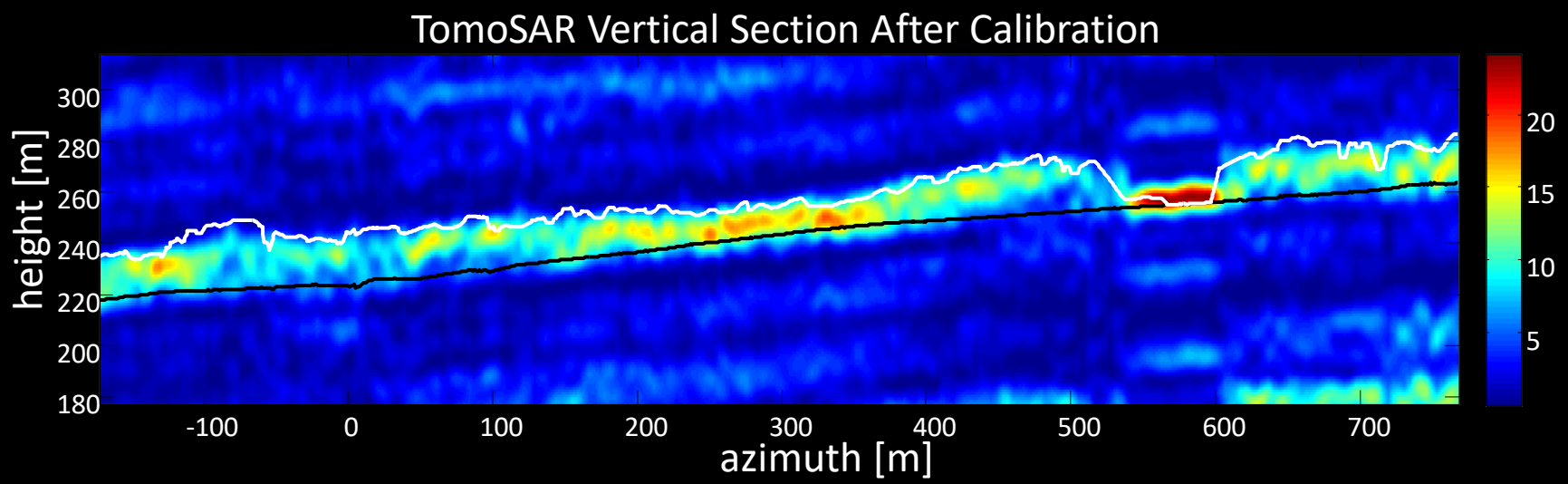



| | | | |
|---|---|---|---|
| **Site** | **Mittelbergferner, Austrian Alps** | **Topography** | **2500-3500 m a.s.l.** |
| **Period** | March 2014 | **Carrier frequency** | L-Band |
| **Scene** | Alpine Glacier | **Vertical resolution** | ≈ 2 m |

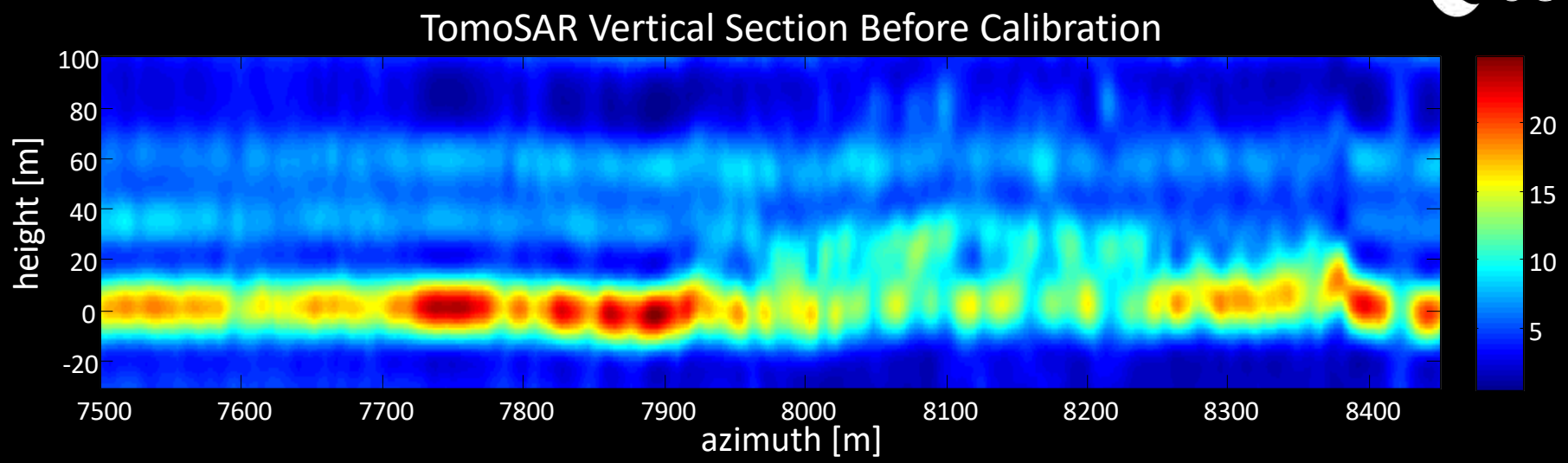
# BIOSAR 2008 – DLR – L-Band

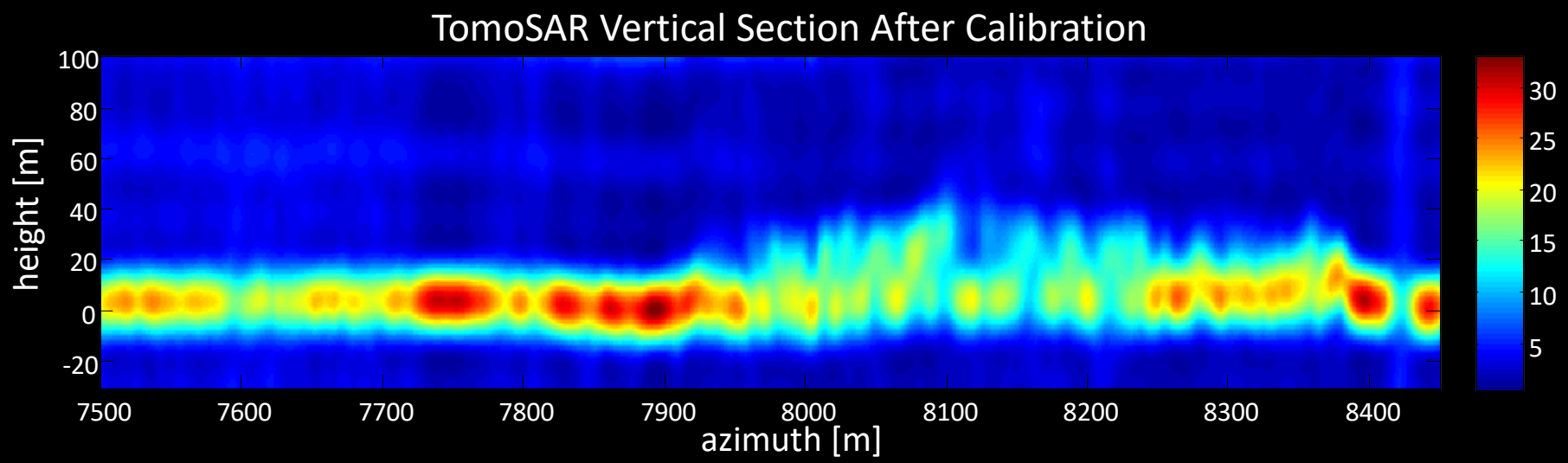



| | | | |
|---|---|---|---|
| **Site** | **Krycklan, Northern Sweden** | **Topography** | **Hilly** |
| **Period** | Fall 2008 | **Carrier frequency** | L-Band |
| **Scene** | Boreal forest | **Vertical resolution** | 6 m to 25 m |

# TropiSAR 2009 – ONERA – P-Band




| | | | |
|---|---|---|---|
| **Site** | **Paracou, French Guiana** | **Topography** | **Hilly** |
| **Period** | August 2009 | **Carrier frequency** | P-Band |
| **Scene** | Tropical forest | **Vertical resolution** | ≈15 m |

# Phase screen impact on TomoSAR

**TomoSAR provides resolution in elevation by jointly focusing data from multiple baselines**

- Resolution ⇔ total baseline span
- Vertical Ambiguity ⇔ baseline spacing

➢ **Phase screens result in signal defocusing**

- Spaceborne: tropospheric and ionospheric propagation
- ***Airborne: uncompensated platform motions on the order of a fraction of a wavelength***

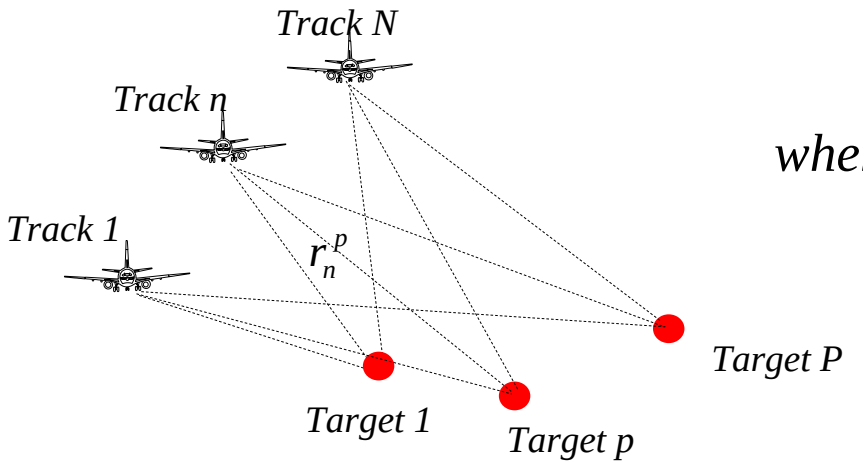
Most current navigational systems employed by airborne SARs do not provide the required sub-wavelength accuracy concerning the position of one flight with respect to another

⇨ **Need for a data-driven Phase Calibration procedure**

# Phase calibration of airborne data – *Double Localization*

*Problem statement:*

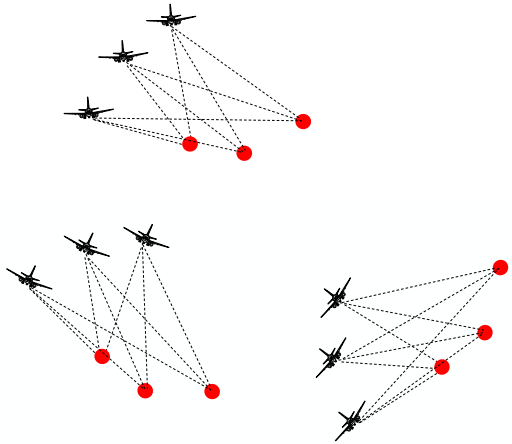
Find the position of *N* sensors and *P* target given the set of all sensor-to-target distances (for every along track location)




$$\text{where: } \hat{r}_n^p = r_{ref}^p + \frac{\lambda}{4\pi}\hat{\varphi}_n^p$$

- The distance set is invariant to translation and rotation
  $\Rightarrow$ The problem offers (at least) three degrees of freedom

  Convenient choices:
  - The position of one sensor can be fixed arbitrarily (i.e.: the Master position is known)
  - The elevation angle of one target can be fixed arbitrarily

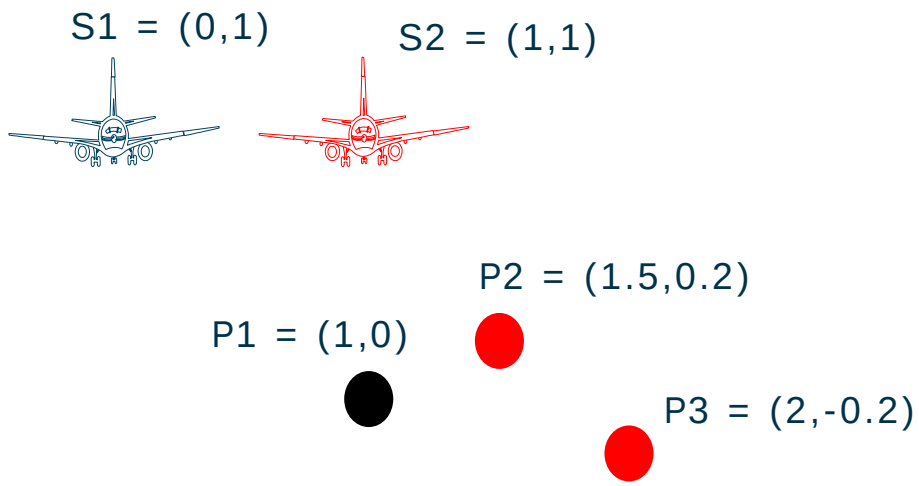


# Double Localization

*Problem statement:*

Find the position of *N* sensors and *P* target given the set of all sensor-to-target distances

## *Single-baseline case: N = 2*



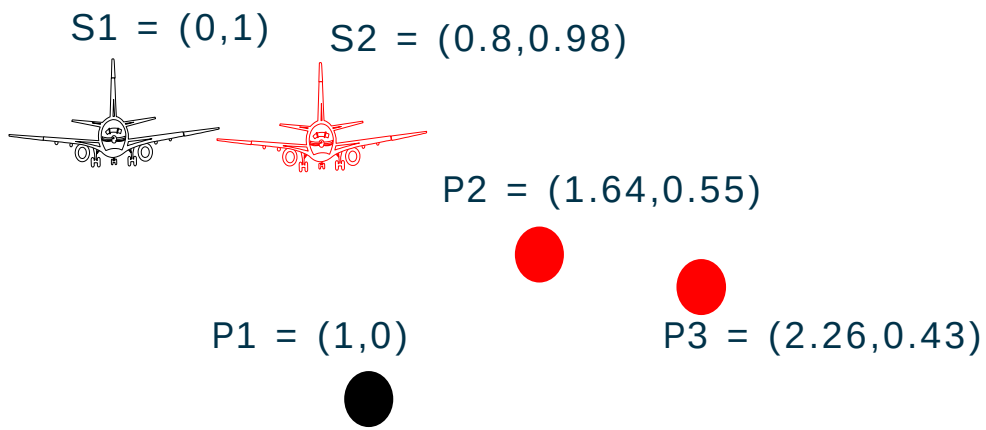

| Distance Set | P1 (ref) | P2 | P3 |
|---|---|---|---|
| S1 (ref) | 1.41 | 1.70 | 2.33 |
| S2 | 1.00 | 0.94 | 1.56 |

# Double Localization

*Problem statement:*

Find the position of *N* sensors and *P* target given the set of all sensor-to-target distances

## *Single-baseline case: N = 2*




| Distance Set | P1 (ref) | P2 | P3 |
|---|---|---|---|
| S1 (ref) | 1.41 | 1.70 | 2.33 |
| S2 | 1.00 | 0.94 | 1.56 |

*Two different single-baseline geometries may yield the same set of distances !*

- (Infinite) other single-baseline geometries exist that yield the same set of sensor-to-target distances

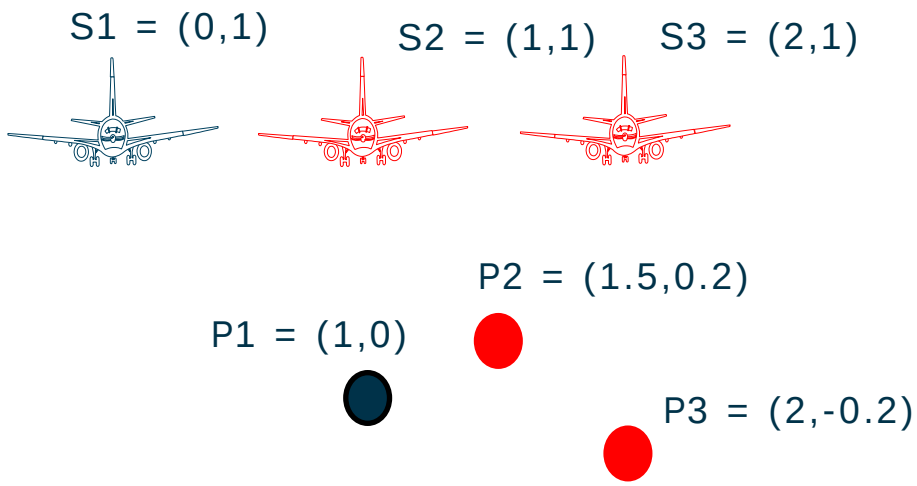
⇒ The problem is **not treatable** in the single-baseline case

# Double Localization

*Problem statement:*

Find the position of *N* sensors and *P* target given the set of all sensor-to-target distances

## *Multi-baseline case: N > 2*



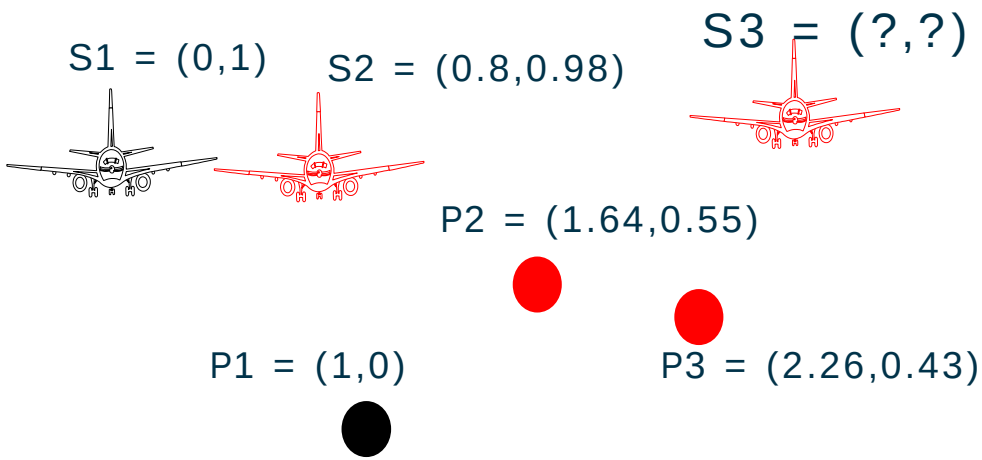

| Distance Set | P1 (ref) | P2 | P3 |
|---|---|---|---|
| S1 (ref) | 1.41 | 1.70 | 2.33 |
| S2 | 1.00 | 0.94 | 1.56 |
| S3 | 1.41 | 0.94 | 1.2 |

# Double Localization

*Problem statement:*

Find the position of *N* sensors and *P* target given the set of all sensor-to-target distances

## *Multi-baseline case: N > 2*

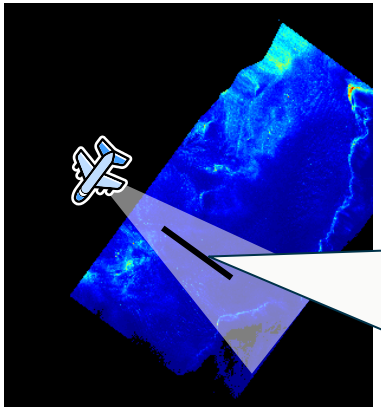


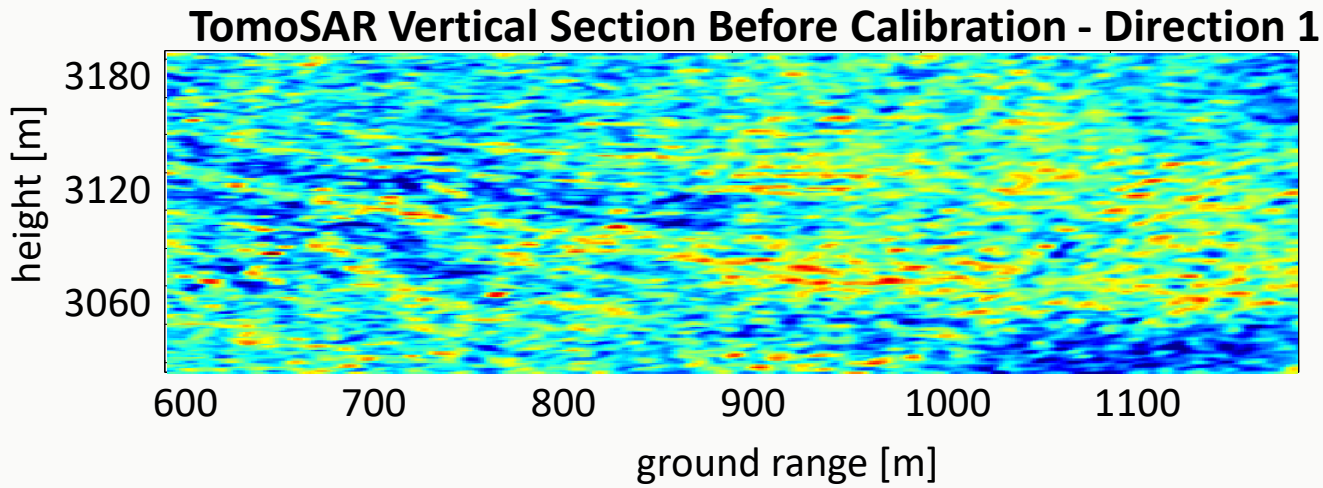

| Distance Set | P1 (ref) | P2 | P3 |
|---|---|---|---|
| S1 (ref) | 1.41 | 1.70 | 2.33 |
| S2 | 1.00 | 0.94 | 1.56 |
| S3 | XXX | XXX | XXX |

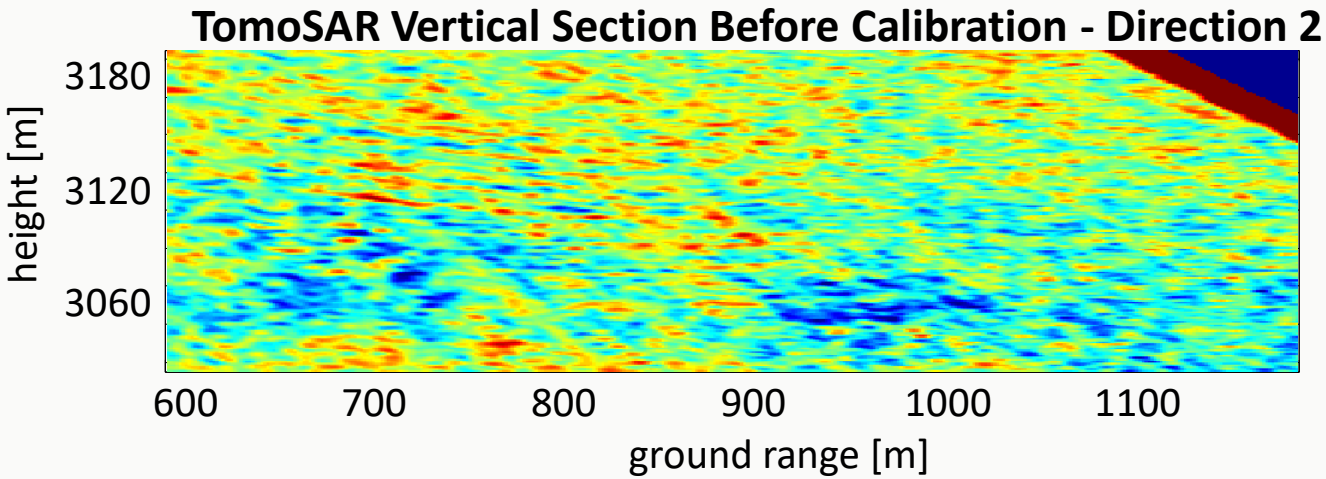
- No other multi-baseline geometry exists that yield the same set of sensor-to-target distances

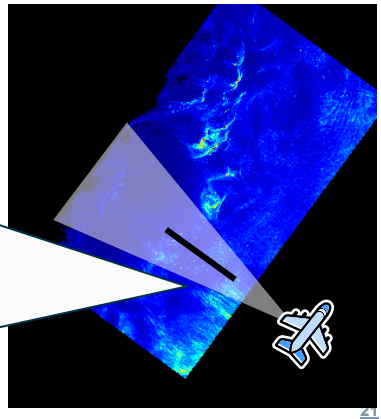
⇒ The problem becomes **treatable** in the multi-baseline case

# Results from AlpTomoSAR

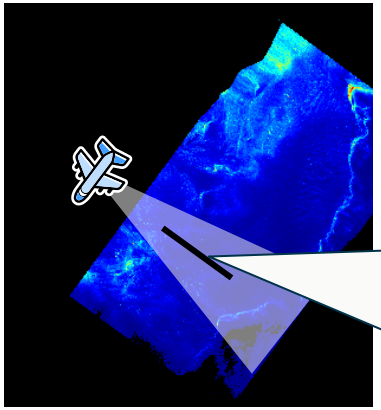


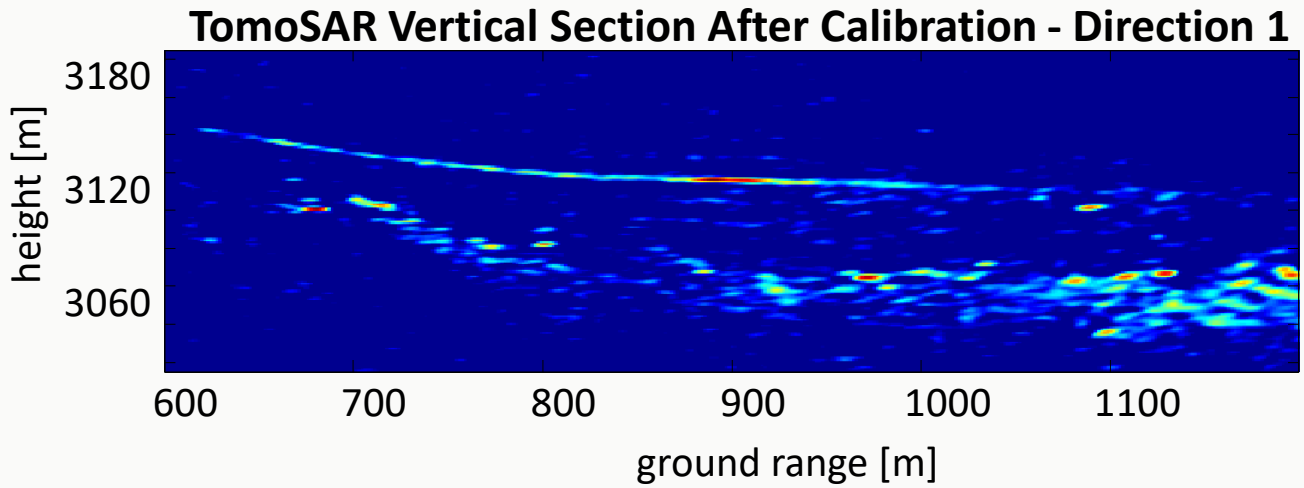


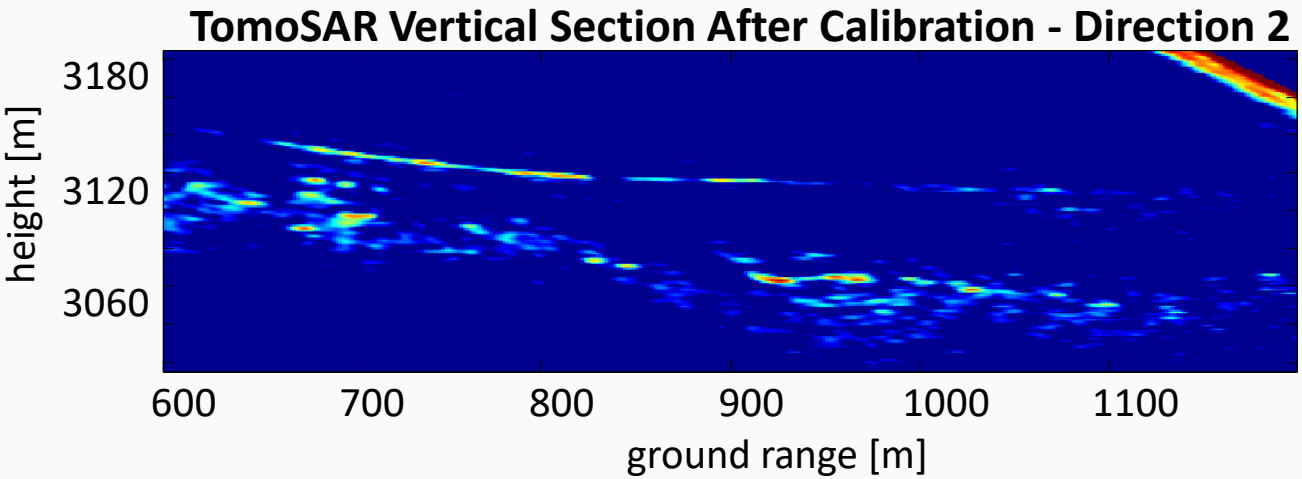



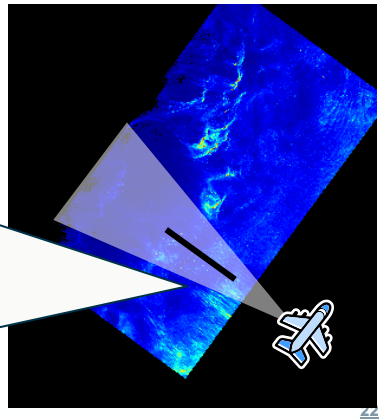
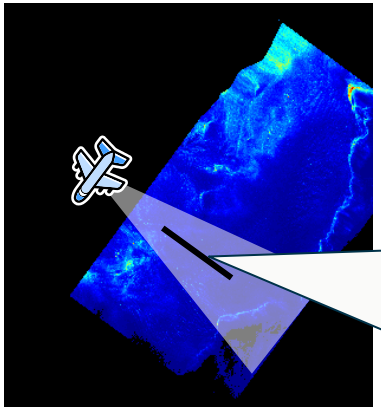
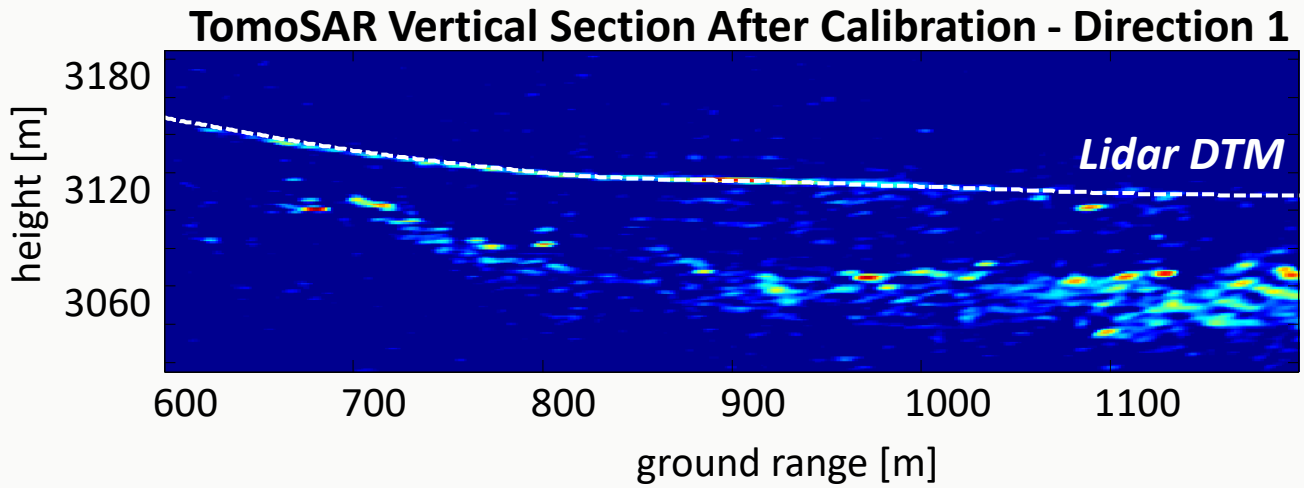
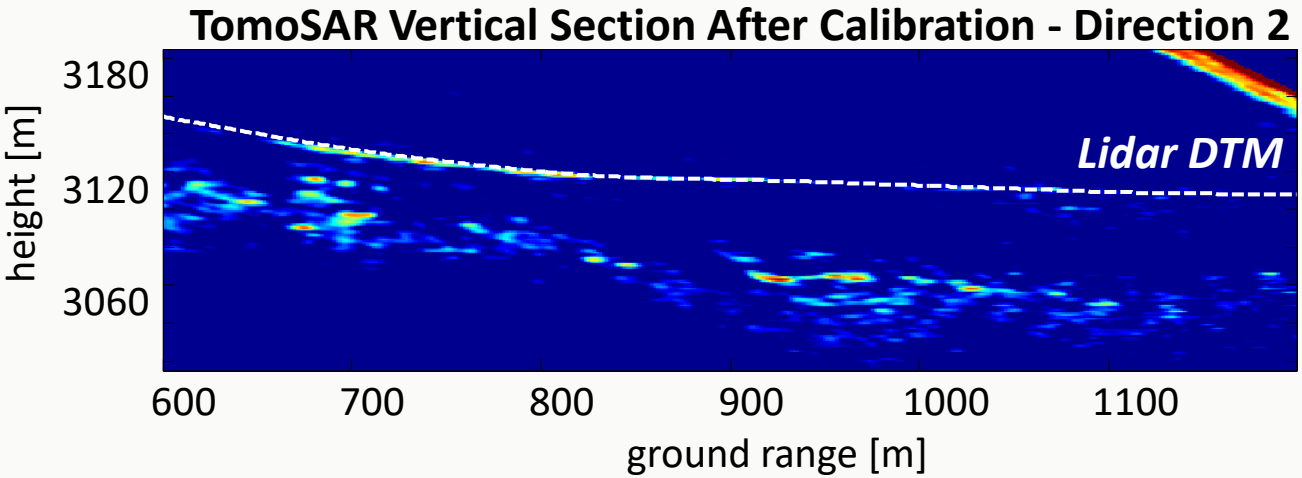
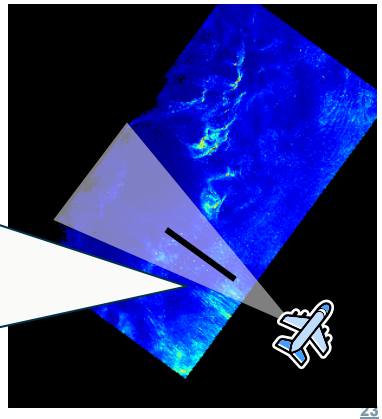



# Results from AlpTomoSAR



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