ESA 6th Advanced Training Course on Radar Polarimetry

POLARIMETRIC SAR TOMOGRAPHY

Introduction

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-1 From InSAR

to PolinSAR

and PolTomoSAR

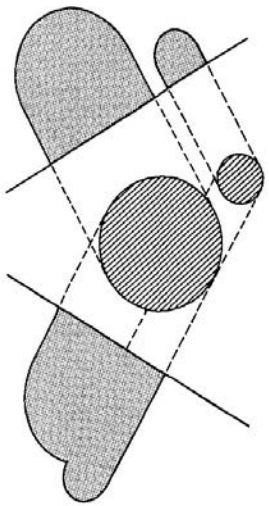
- 2 Lecture contents

Basic concept of tomographic imaging

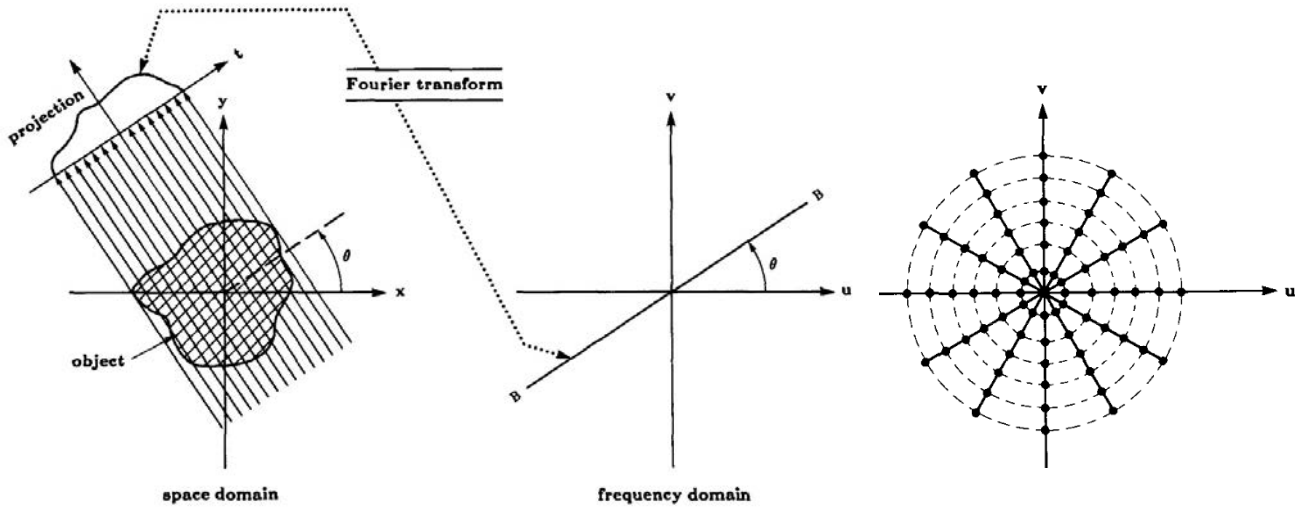
"Fundamentally, tomographic imaging deals with reconstructing an image from its projections"

A.C. Kak, M. Slaney, PCT, 1987

Projections of 2 cylinders



Fourier slice theorem in the non diffracting case

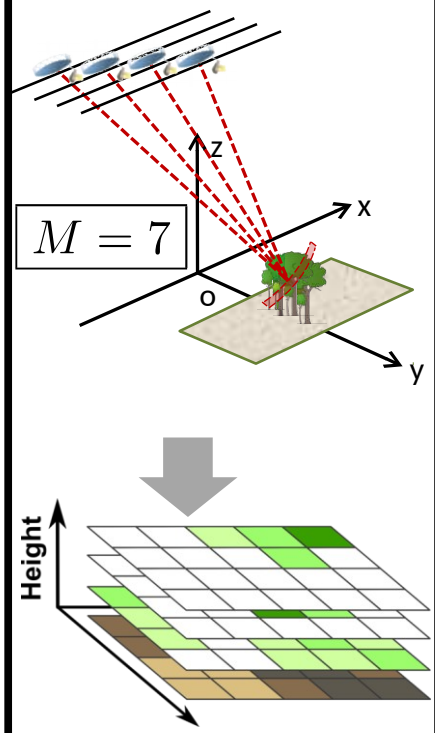


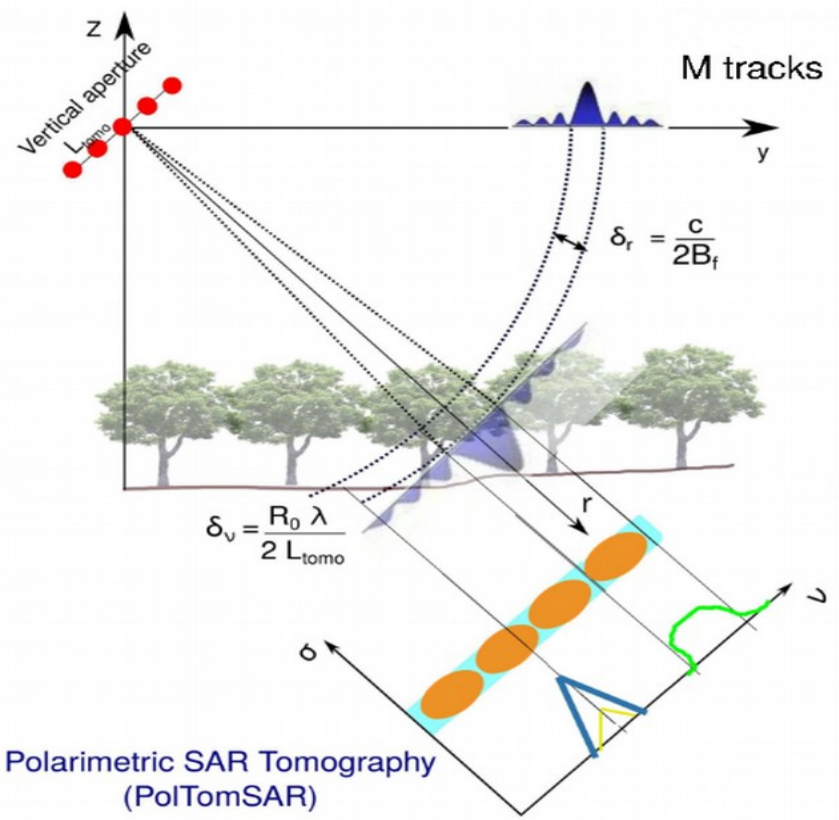
Synthetic Aperture Radar tomographic imaging

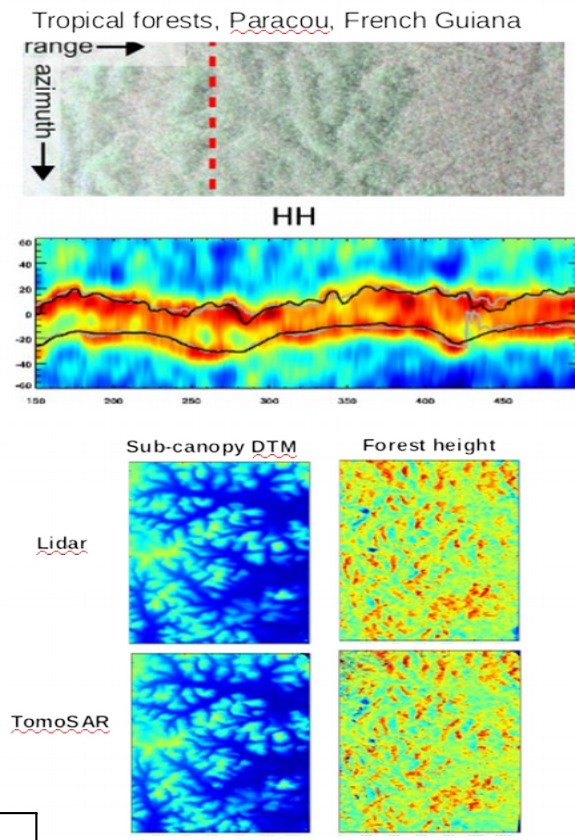
- Diffraction tomography
- Coherent processing
- Generalization of interferometric SAR processing using **a synthetic array**

Polarimetric SAR tomography

BIOMASS
PolTomoSAR mode





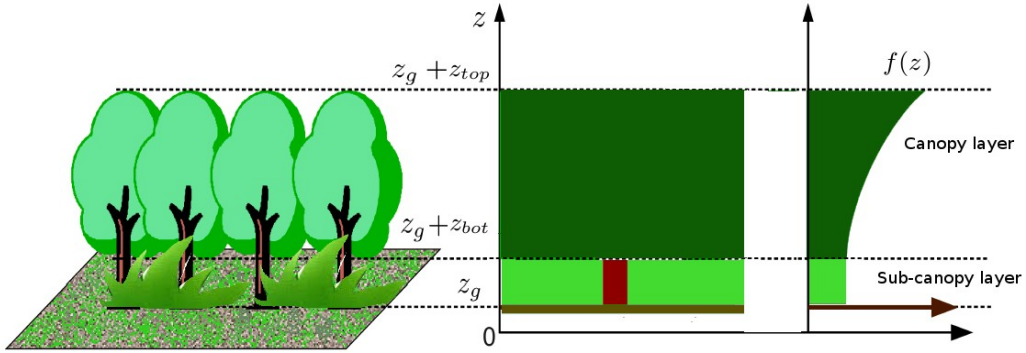


$$\delta z \propto \frac{1}{L_{tomo}} \approx \frac{z_{amb}}{M}$$

InSAR vertical decorrelation over volumes

Volumetric media inSAR response modeling

- Vertical reflectivity structure $\sigma_{v_e}(\vec{r}) = \sigma_{v_e}(z) = A_{v_e} f(z)$



- InSAR coherence $\gamma = \gamma_{th} \quad \gamma_{proc} \quad \gamma_{temp} \quad \gamma_{surf} \quad \gamma_z$

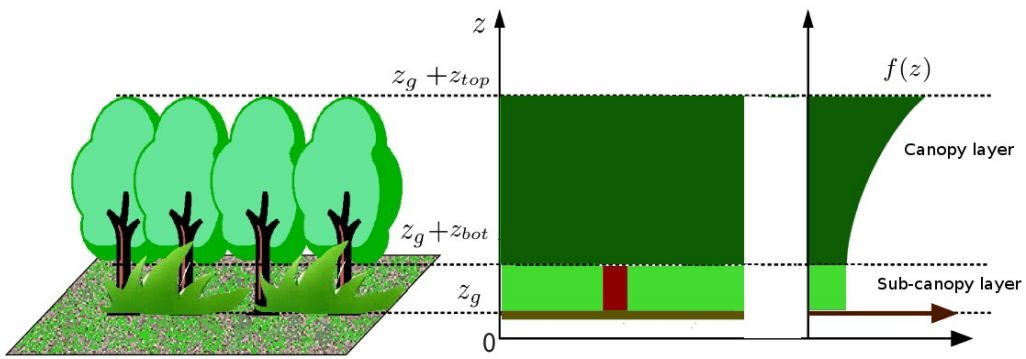
- Decorrelation due to vertical structure :

$$\boxed{\gamma_z = \frac{\int \sigma_{v_e}(z)\, \mathrm{e}^{jk_z z}\, \mathrm{d}z}{\int \sigma_{v_e}(z) \mathrm{d}z}} \qquad k_z = \frac{k_c B_\perp}{r \sin\theta}$$

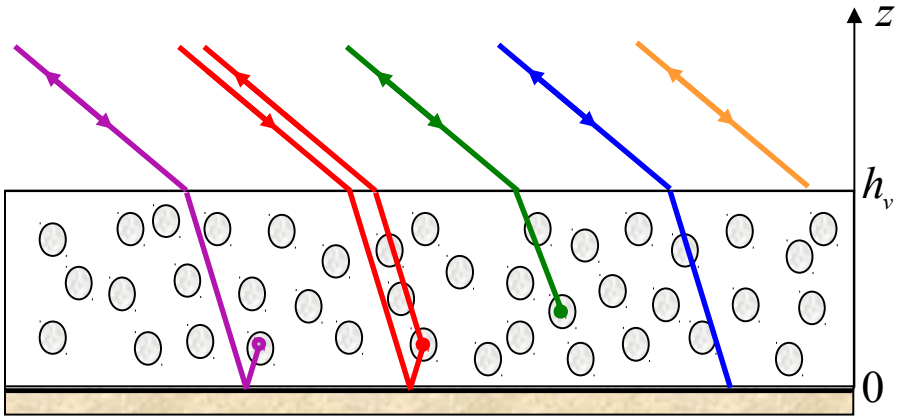
- Fourier transform-like **coherence-structure relationship**

$$\boxed{\gamma_z \xleftrightarrow{FT} \sigma_{v_e}(z)}$$

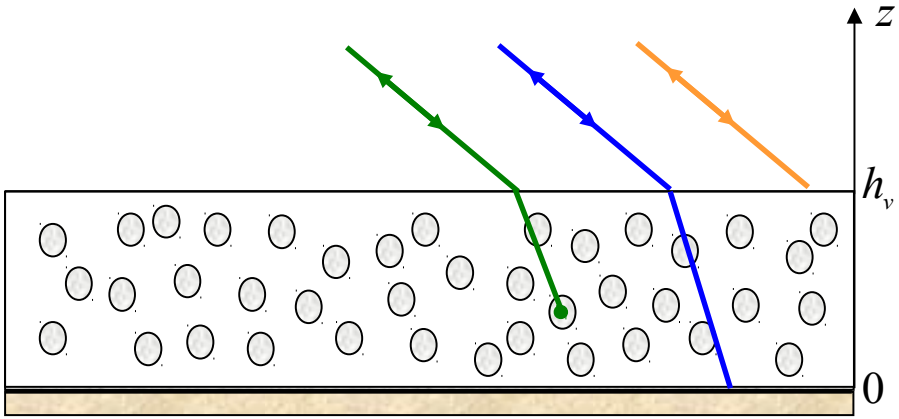
InSAR RVOG model



Modeling at order 1



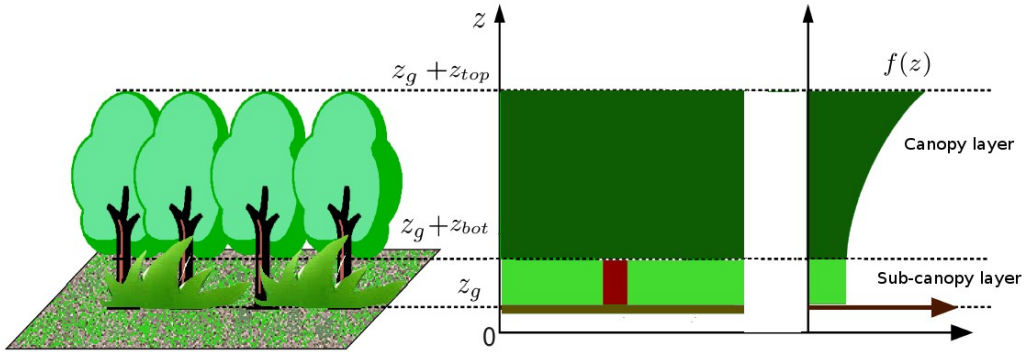
Parameter Estimation

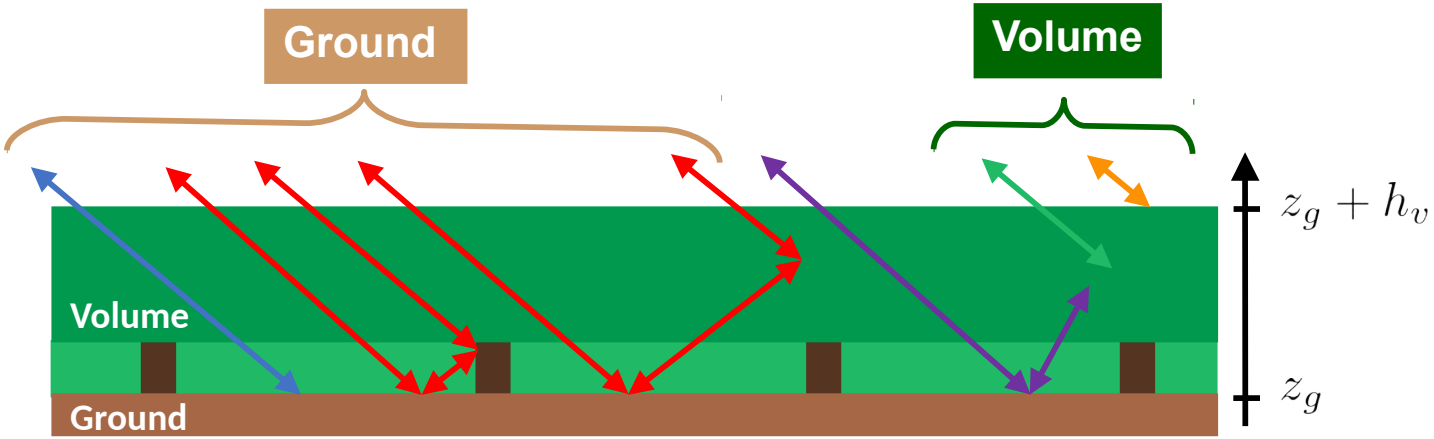


Parameter estimation often requires to simplify models

- omitting negligible terms
- merging contributions that cannot be discriminated (e.g. ground and double-bounce)

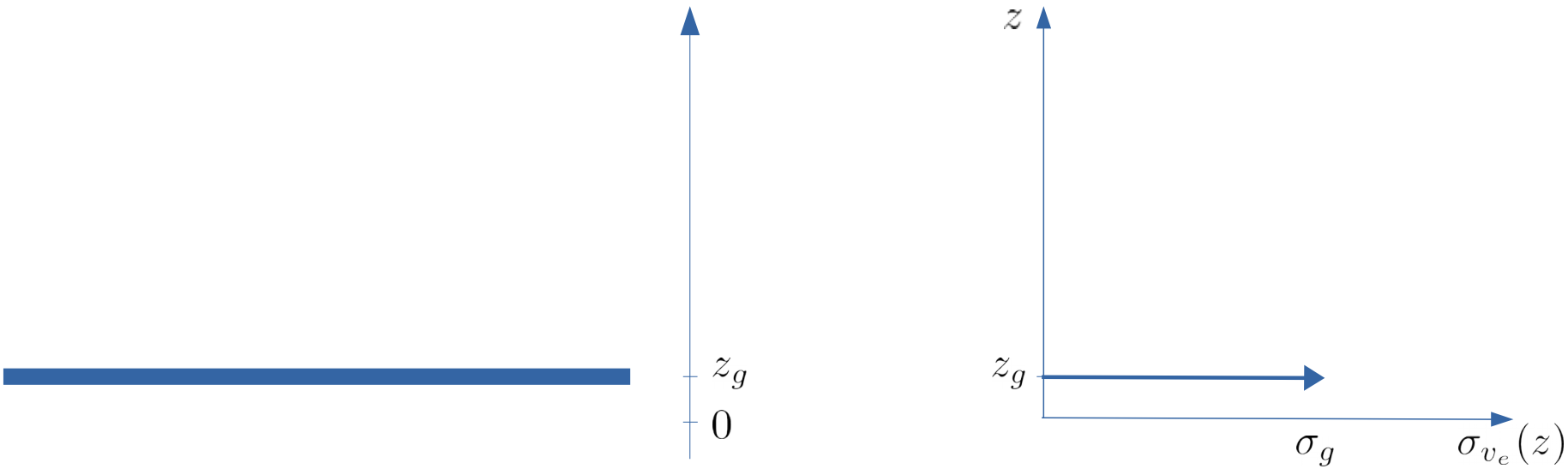
InSAR RVOG model





- 2 significant and uncorrelated mechanisms :
  - ⇨ volume + underlying ground
- low density medium
  - ⇨ no refraction

Ground only

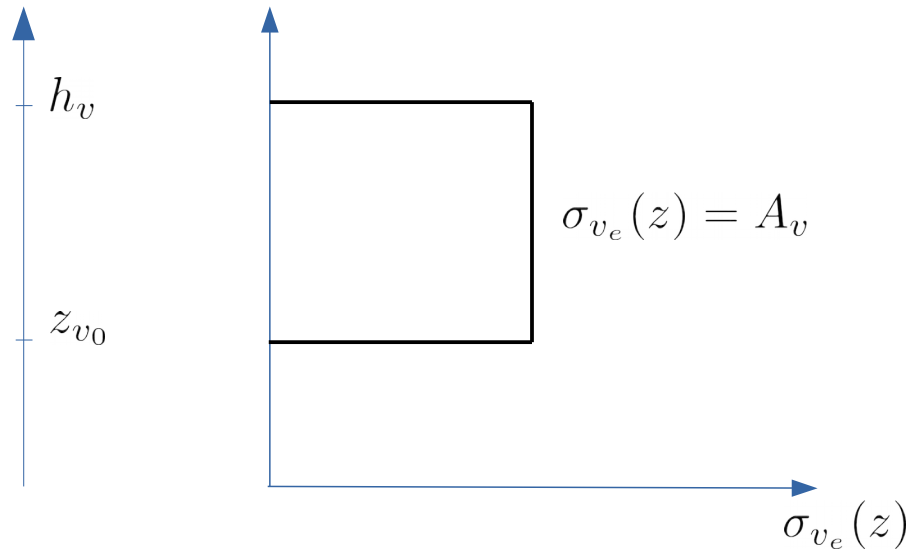


$$\gamma_z = \frac{\int \sigma_{v_e}(z)\, \mathrm{e}^{jk_z z}\, \mathrm{d}z}{\int \sigma_{v_e}(z) \mathrm{d}z}$$

$$\sigma_{v_e}(z) = \sigma_g \delta(z - z_g) \quad \Rightarrow \quad \gamma_z = \mathrm{e}^{jk_z z_g}$$

InSAR well adapted to topography estimation

Non attenuating random volume only



$$\gamma_z = \frac{\int \sigma_{v_e}(z)\, \mathrm{e}^{jk_z z}\, \mathrm{d}z}{\int \sigma_{v_e}(z) \mathrm{d}z}$$

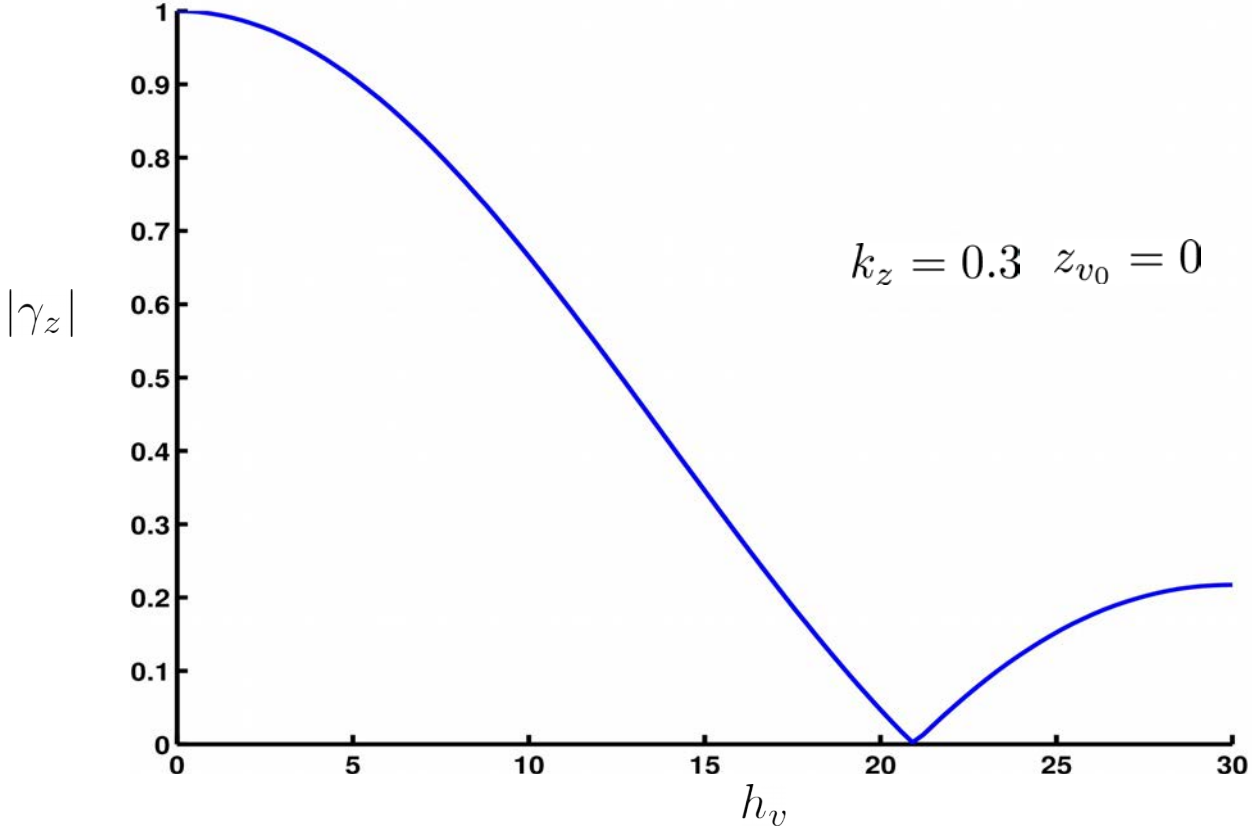
No underlying ground

Null extinction: $\sigma_{v_e}(z) = A_v$

$$\gamma_z = \frac{1}{d} \int_{z_{v_0}}^{h_v} \mathrm{e}^{jk_z z}\, \mathrm{d}z$$

$$\gamma_z = \mathrm{e}^{jk_z \frac{h_v + z_{v_0}}{2}}\, \mathrm{sinc}\left(\frac{k_z d}{2}\right)$$

InSAR RVOG analysis



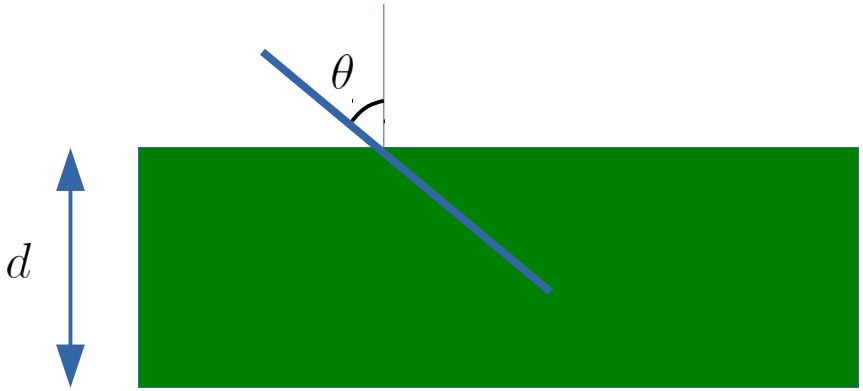
Volume center height

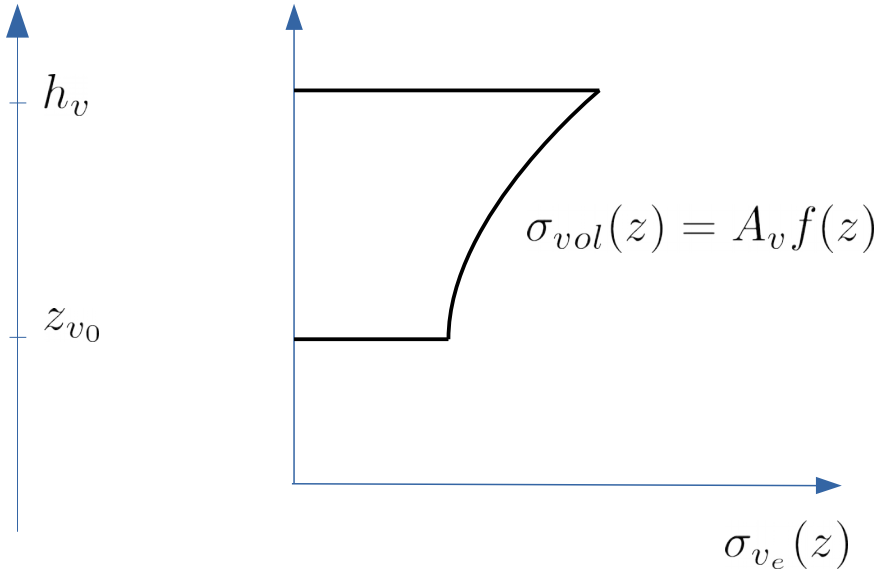
Volume width

$$\gamma_z = \mathrm{e}^{jk_z \frac{h_v + z_{v_0}}{2}} \operatorname{sinc}\left(\frac{k_z d}{2}\right)$$

InSAR well adapted to volume analysis under specific conditions

Attenuating random volume only



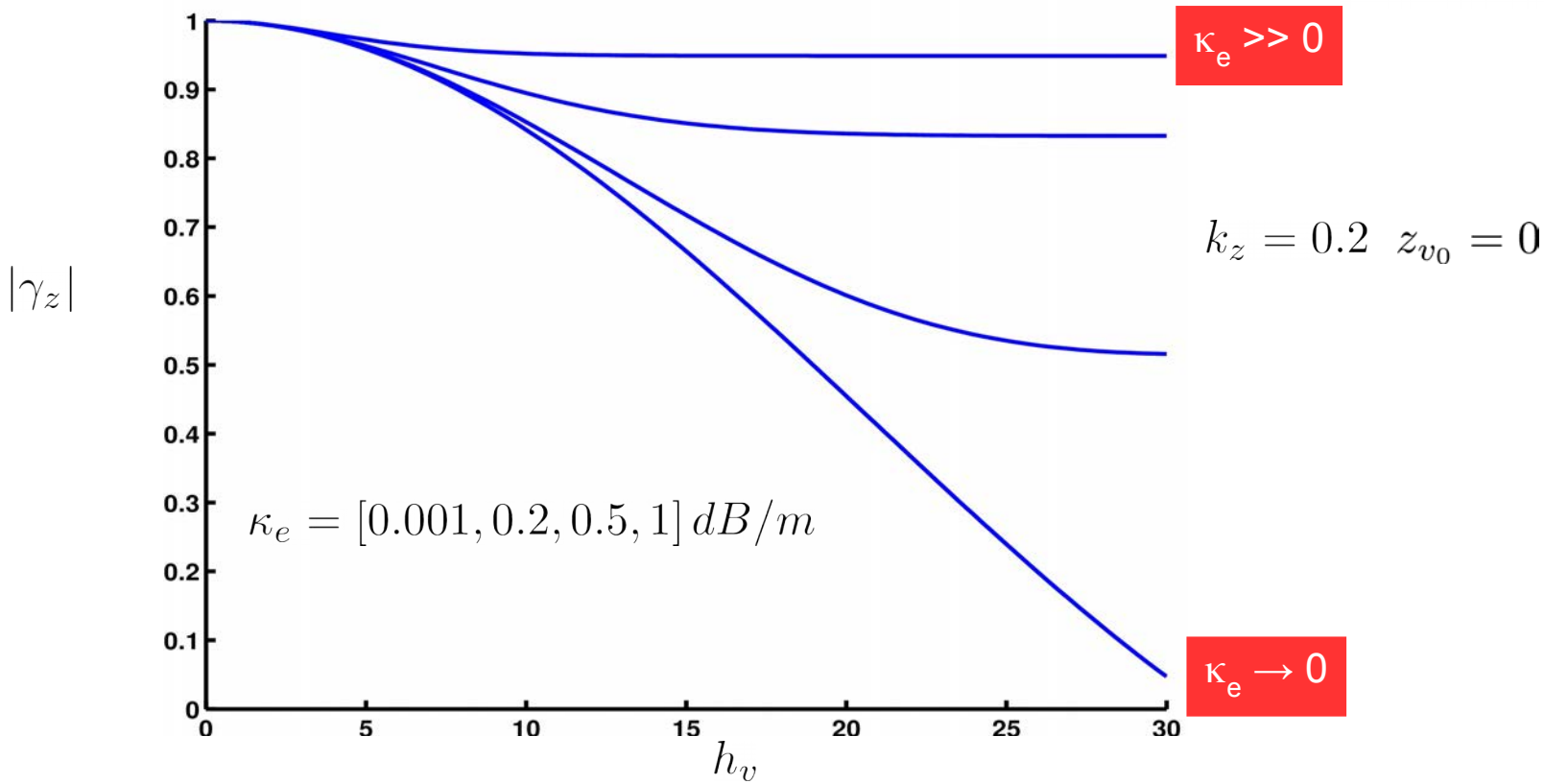


Linear differential extinction $\mathrm{d}I = -\kappa_e I \mathrm{d}s = -\frac{\kappa_e}{\cos\theta} I \mathrm{d}z$

Effective reflectivity density
constant extinction $\sigma_{vol}(z) = A_v \, \mathrm{e}^{-2\frac{\kappa_e}{\cos\theta}(h_v - z)} = A_v f(z)$

Backscattered volume intensity $I_v = \int_{z_{v_0}}^{h_v} \sigma_{vol}(z)\mathrm{d}z = \int_{z_{v_0}}^{h_v} A_v f(z)\mathrm{d}z$

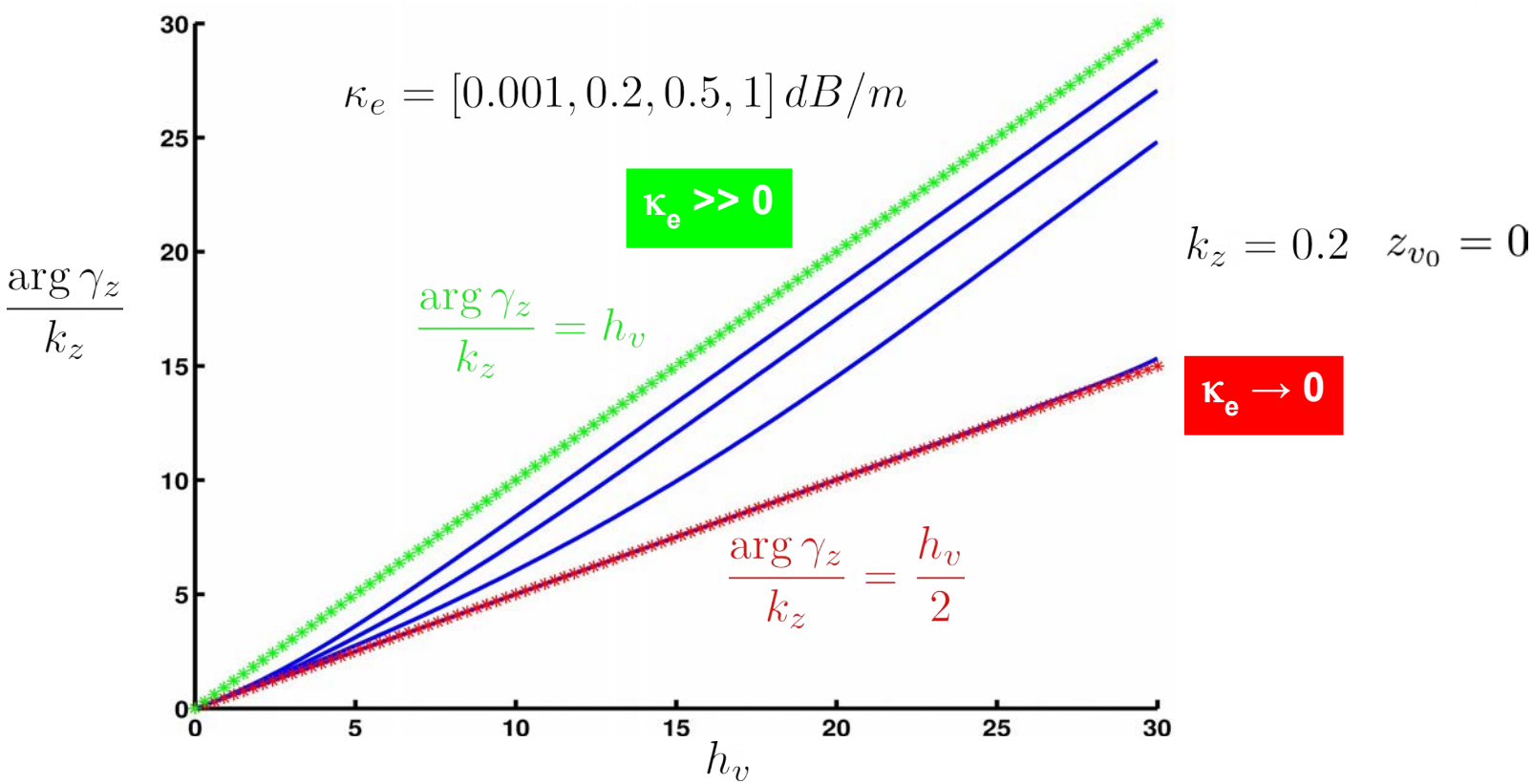
InSAR RVOG analysis



$$\gamma_{vol} = \gamma_z = e^{jk_z z_{v_0}} \frac{p}{p_1} \left(\frac{e^{p_1 d} - 1}{e^{pd} - 1} \right) \qquad p = \frac{2\kappa_e}{\cos\theta} \qquad p_1 = \frac{2\kappa_e}{\cos\theta} + jk_z$$

InSAR $|\gamma_z| \rightarrow \hat{h}_v$ ambiguous estimation

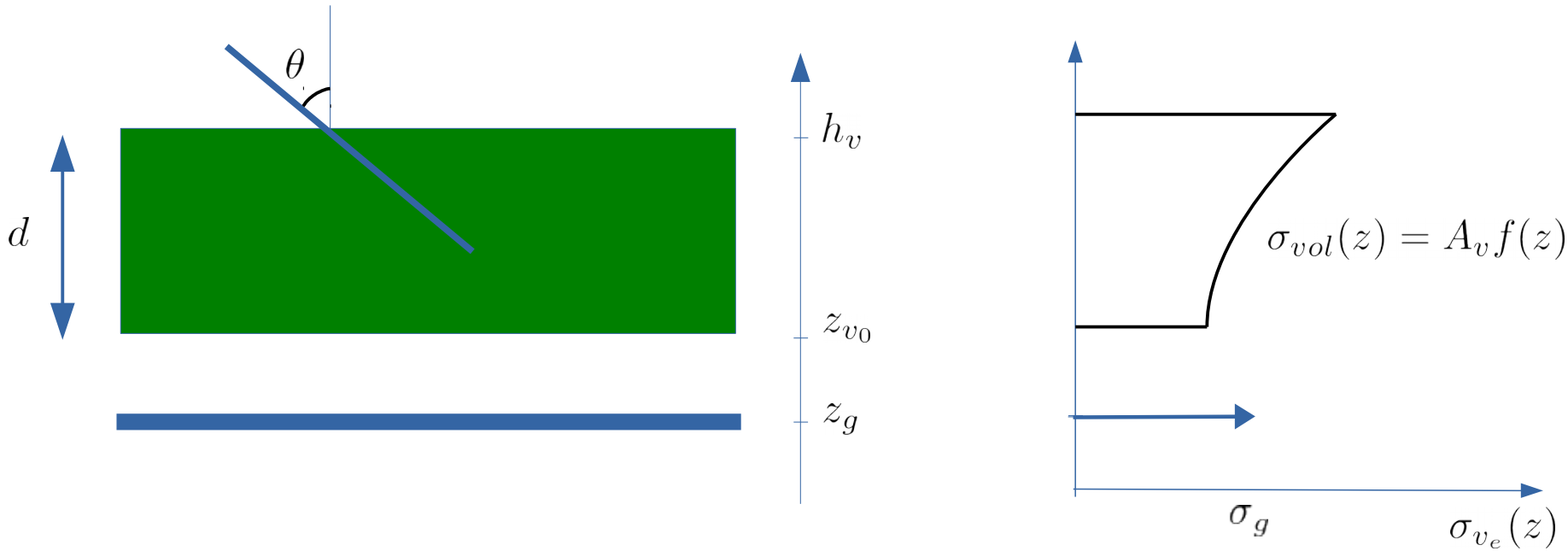
InSAR RVOG analysis



InSAR $\arg(\gamma_z) \rightarrow \hat{h}_v$ **ambiguous estimation**

Unambiguous solution for known $\sigma_{vol}(z)$ **shape :** $|\gamma_z|, \arg(\gamma_z) \rightarrow \hat{h}_v$

Attenuating random volume and ground



Backscattered volume intensity $I_v = \int_{z_{v_0}}^{h_v} \sigma_{vol}(z)\mathrm{d}z = \int_{z_{v_0}}^{h_v} A_v f(z)\mathrm{d}z$

Backscattered ground intensity $I_g = f(z_{v_0})\sigma_g = \mathrm{e}^{-2\frac{\kappa_e}{\cos\theta}d}\,\sigma_g$

InSAR RVOG analysis

- Coherence formulation $\sigma_{v_e}(z) = \sigma_{vol}(z) + \delta(z - z_g) I_g$

$$\gamma_z = \frac{\int \sigma_{v_e}(z)\, \mathrm{e}^{jk_z z}\, \mathrm{d}z}{\int \sigma_{v_e}(z) \mathrm{d}z} = \frac{\int \sigma_{vol}(z)\, \mathrm{e}^{jk_z z}\, \mathrm{d}z + I_g\, \mathrm{e}^{jk_z z_g}}{\int \sigma_{vol}(z) \mathrm{d}z + I_g}$$

$$\boxed{\gamma_z = \frac{\gamma_{vol} + m\, \mathrm{e}^{jk_z z_g}}{1 + m}}$$

- Ground to volume intensity ratio $m = \dfrac{I_g}{I_v} \in \mathbb{R}^+$
- Coherence interpretation

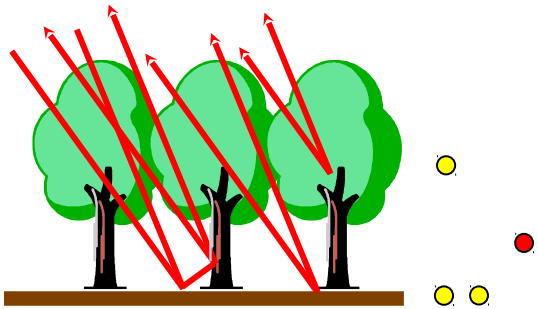
$$m \to 0 \Rightarrow \begin{cases} \arg \gamma_z & \approx \arg \gamma_{vol} \\ |\gamma_z| & \leq 1 \end{cases} \qquad m \to +\infty \Rightarrow \begin{cases} \arg \gamma_z & \approx \phi_g \\ |\gamma_z| & = 1 \end{cases}$$

$$0 < m < +\infty \Rightarrow ?$$

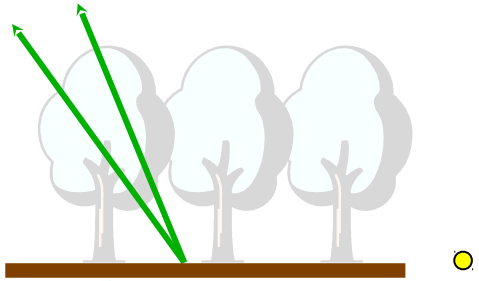
InSAR based RVOG analysis: under-determined problem

→ another source of diversity is needed : polarization ?

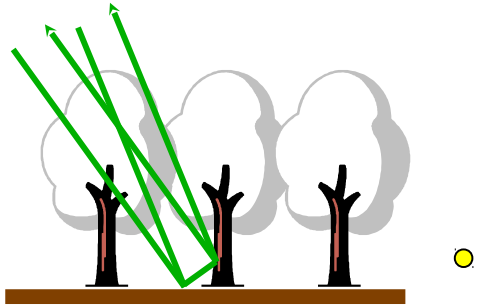
PolInSAR RVOG analysis



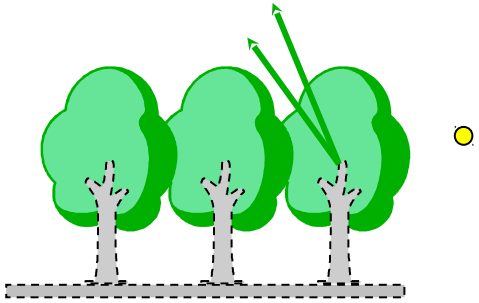
HH+VV



HH-VV

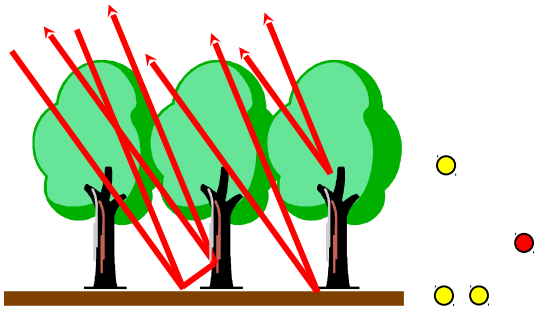


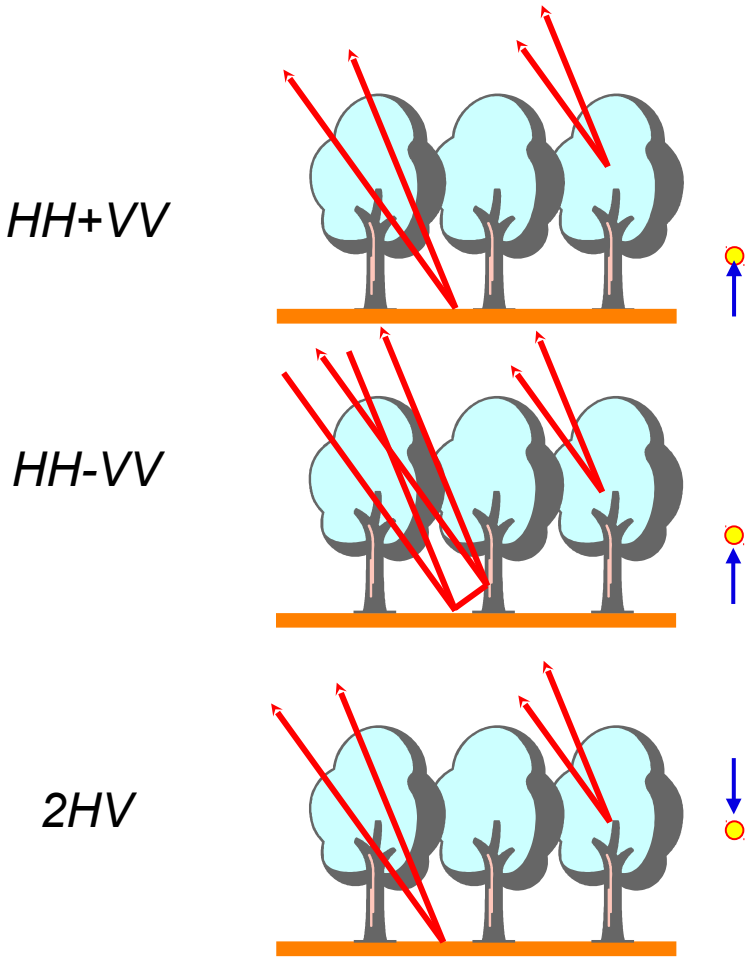
2HV



IN A PERFECT WORLD

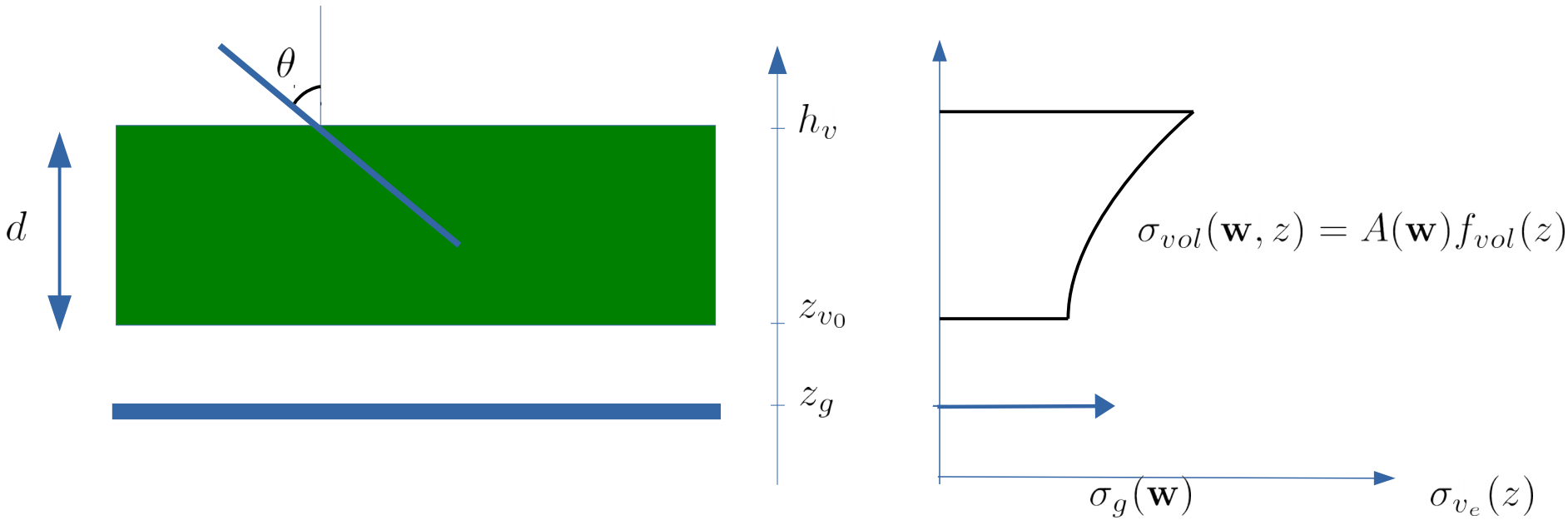
PolInSAR RVOG analysis





IN A REAL WORLD

Attenuating random volume and ground : polarimetric case

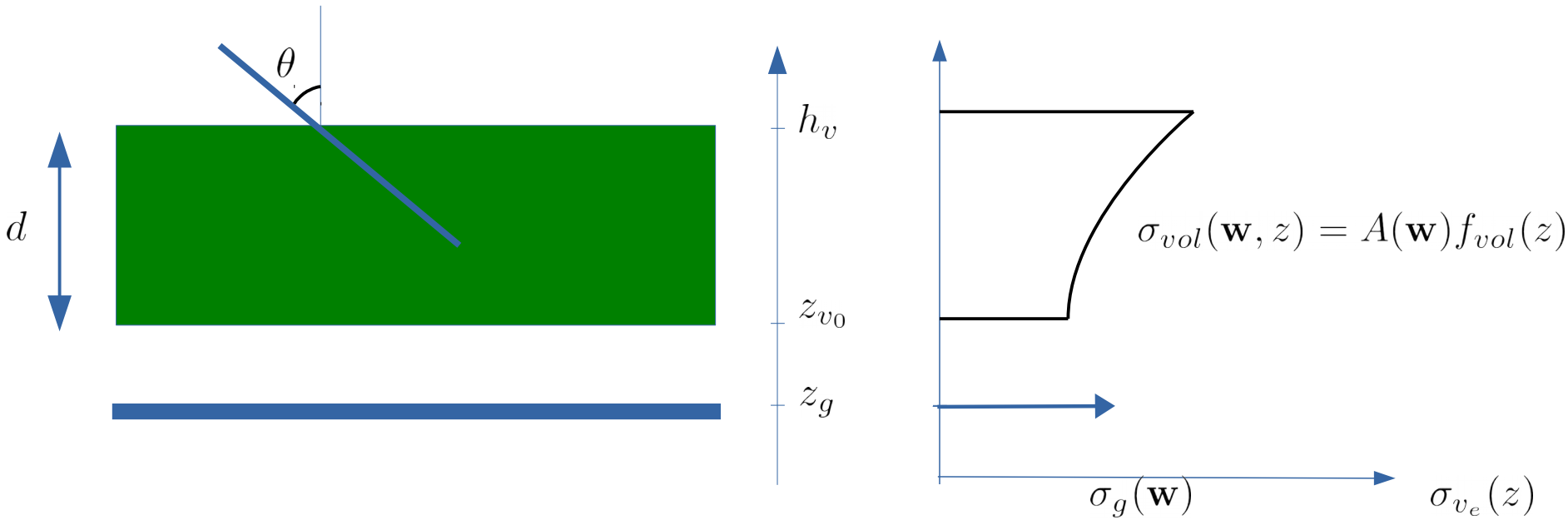


Unpolarized Linear differential extinction $\mathrm{d}I = -\kappa_e I \mathrm{d}s = -\frac{\kappa_e}{\cos\theta} I \mathrm{d}z$

Effective reflectivity density: $\sigma_{vol}(\mathbf{w}, z) = A(\mathbf{w}) f_{vol}(z) = \mathbf{w}^H \mathbf{T}_{vol} \mathbf{w} f_{vol}(z)$

Ground reflectivity $\sigma_g(\mathbf{w}, z) = \mathbf{w}^H \mathbf{T}_g \mathbf{w}\, \delta(z - z_g)$

Attenuating random volume and ground : polarimetric case



Unpolarized Linear differential extinction

$$\mathrm{d}I = -\kappa_e I \mathrm{d}s = -\frac{\kappa_e}{\cos\theta} I \mathrm{d}z$$

$$I_{vol}(\mathbf{w}, z) = \mathbf{w}^H \mathbf{T}_{vol} \mathbf{w} \int_{z_{v_0}}^{h_v} f_{vol}(z) \mathrm{d}z$$

$$I_g(\mathbf{w}, z) = f_{vol}(z_{v_0}) \mathbf{w}^H \mathbf{T}_g \mathbf{w} = \mathrm{e}^{-2\frac{ke}{\cos\theta} d} \mathbf{w}^H \mathbf{T}_g \mathbf{w}$$

Pol-InSAR RVOG analysis

Unpolarized volume coherence

$$\gamma_{vol}(\mathbf{w}) = \frac{\int A(\mathbf{w}) f(z)\, \mathrm{e}^{jk_z z}\, \mathrm{d}z}{\int A(\mathbf{w}) f(z) \mathrm{d}z} = \gamma_{vol}$$

- Polarized GVR $$m(\mathbf{w}) = \frac{I_g(\mathbf{w})}{I_{vol}(\mathbf{w})}$$

- PolinSAR coherence $$\gamma_z(\mathbf{w}) = \frac{\gamma_{vol} + m(\mathbf{w})\, \mathrm{e}^{jk_z z_g}}{1 + m(\mathbf{w})}$$

- Coherence interpretation : find a polarisation vector so that

Plausible

$$m \to 0 \Rightarrow \begin{cases} \arg \gamma_z & \approx \arg \gamma_{vol} \\ |\gamma_z| & \leq 1 \end{cases}$$

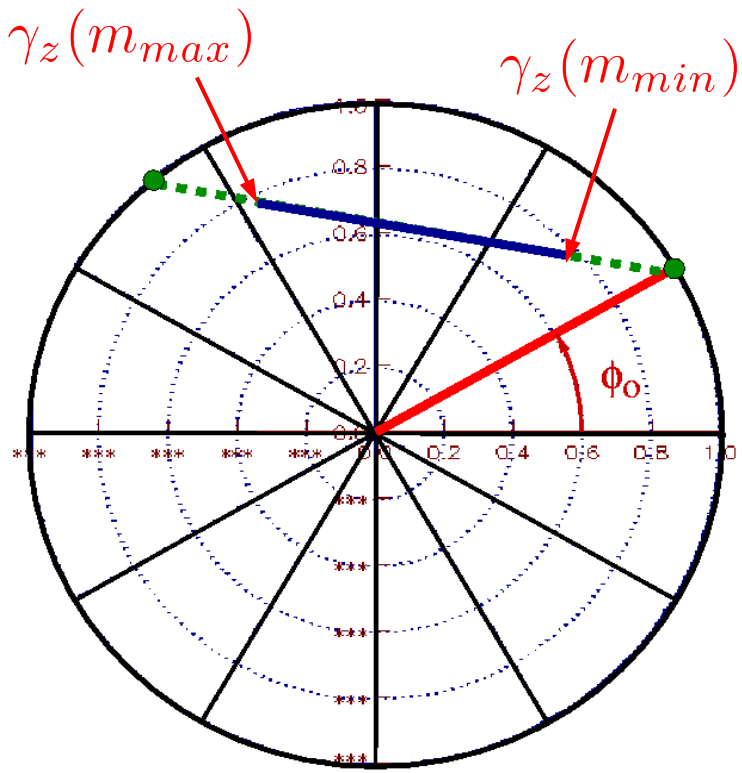
Unlikely

$$m \to +\infty \Rightarrow \begin{cases} \arg \gamma_z & \approx \phi_g \\ |\gamma_z| & = 1 \end{cases}$$

$$0 < m < +\infty \Rightarrow ?$$

RVOG COHERENCE MODEL : LINE MODEL

$$\gamma_z(\mathbf{w}) = \frac{\gamma_{vol} + m(\mathbf{w})\, e^{jk_z z_g}}{1 + m(\mathbf{w})}$$



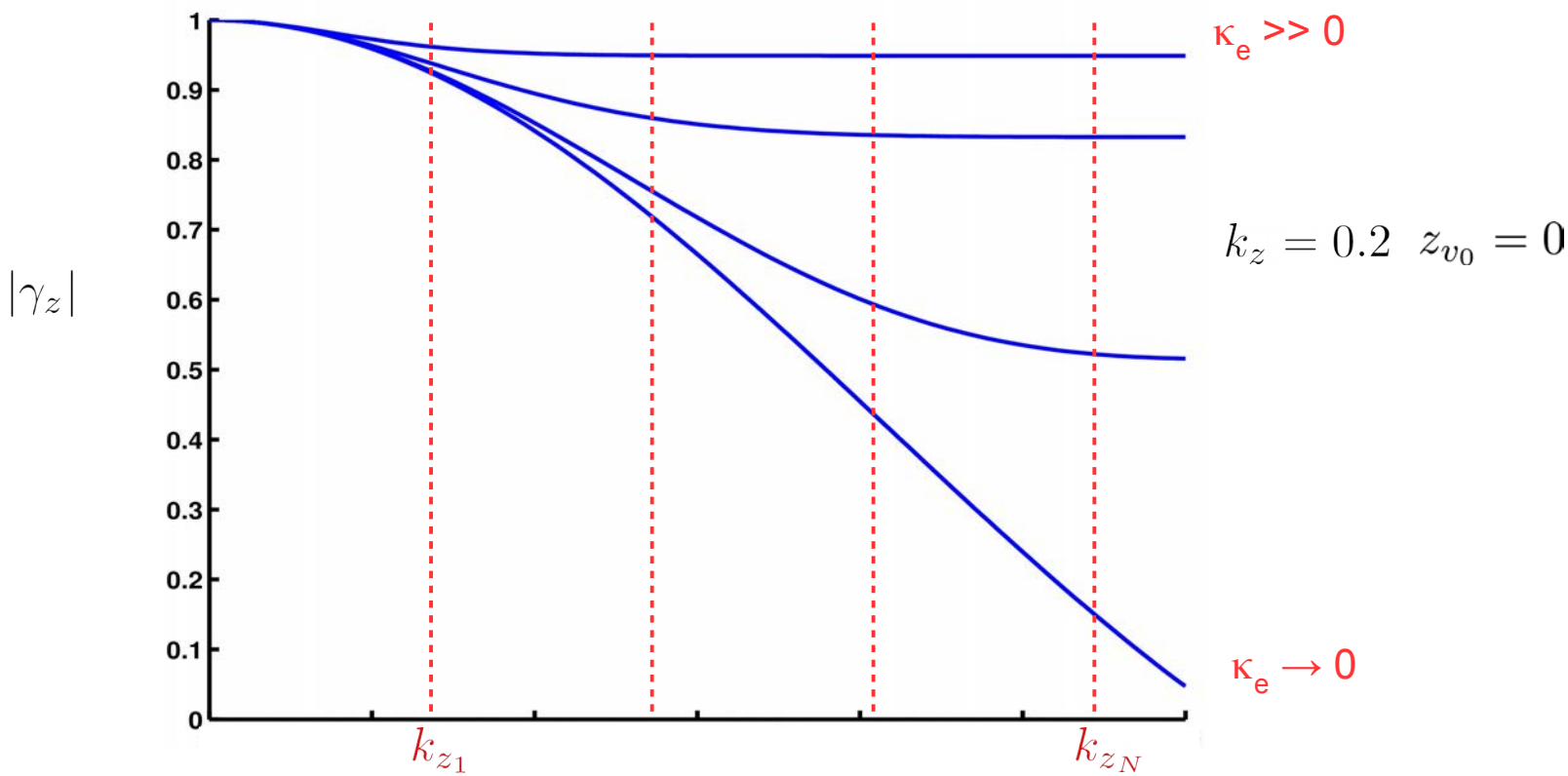
Equation of a straight line in the complex plane

Ground estimation through interpolation

PolinSAR RVOG solution :

- **some estimates remain ambiguous, requires phase diversity**
- **assumes a shape for volume extinction**

TomoSAR (MB-Pol-InSAR) RVOG analysis



Additional spatial diversity

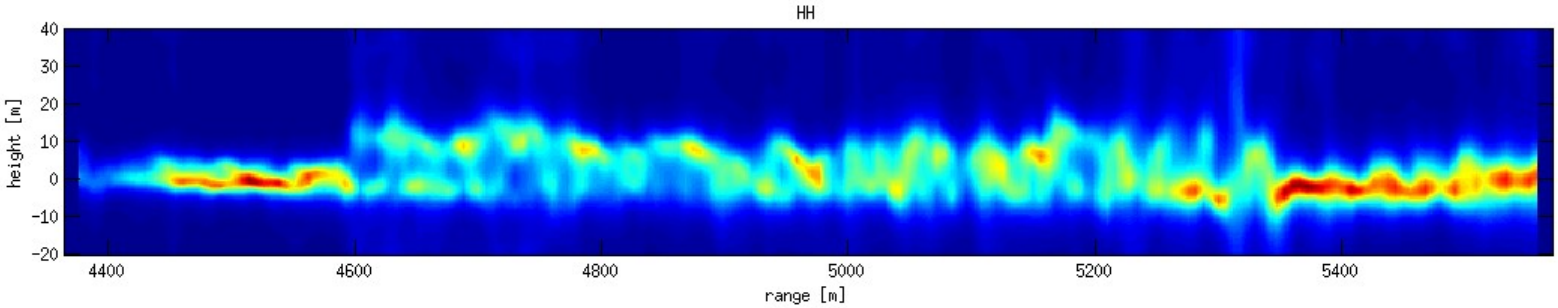
$\{\gamma_z(k_{z_n})\}_{n=1}^{N} \longrightarrow$

- Unambiguous height estimation for known f(z) shape
- Estimation of f(z) or **non-parametric analysis**
- PolTomoSAR (MB-Pol-InSAR) : $\{\gamma_z(k_{z_n}), \mathbf{w}\}_{n=1}^{N}$

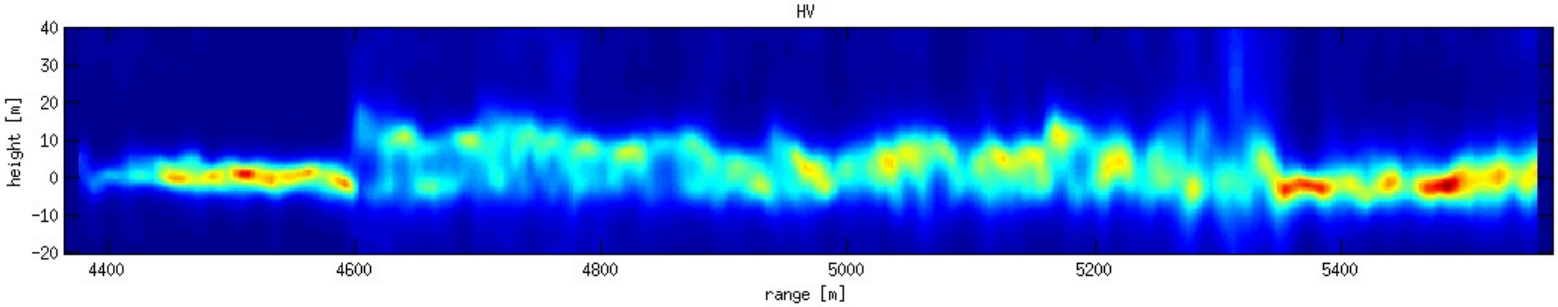
InSAR phases, polarization & TomoSAR

L-band BIOSAR2, Capon tomograms

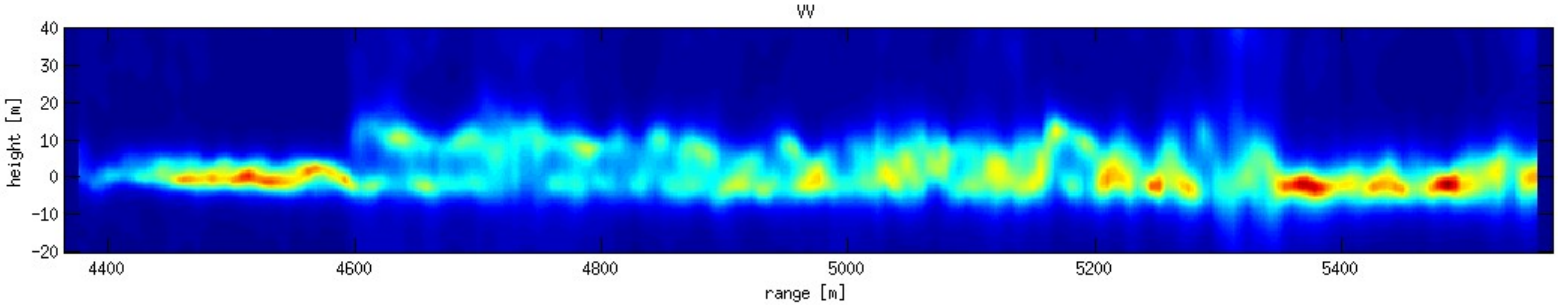
HH



HV



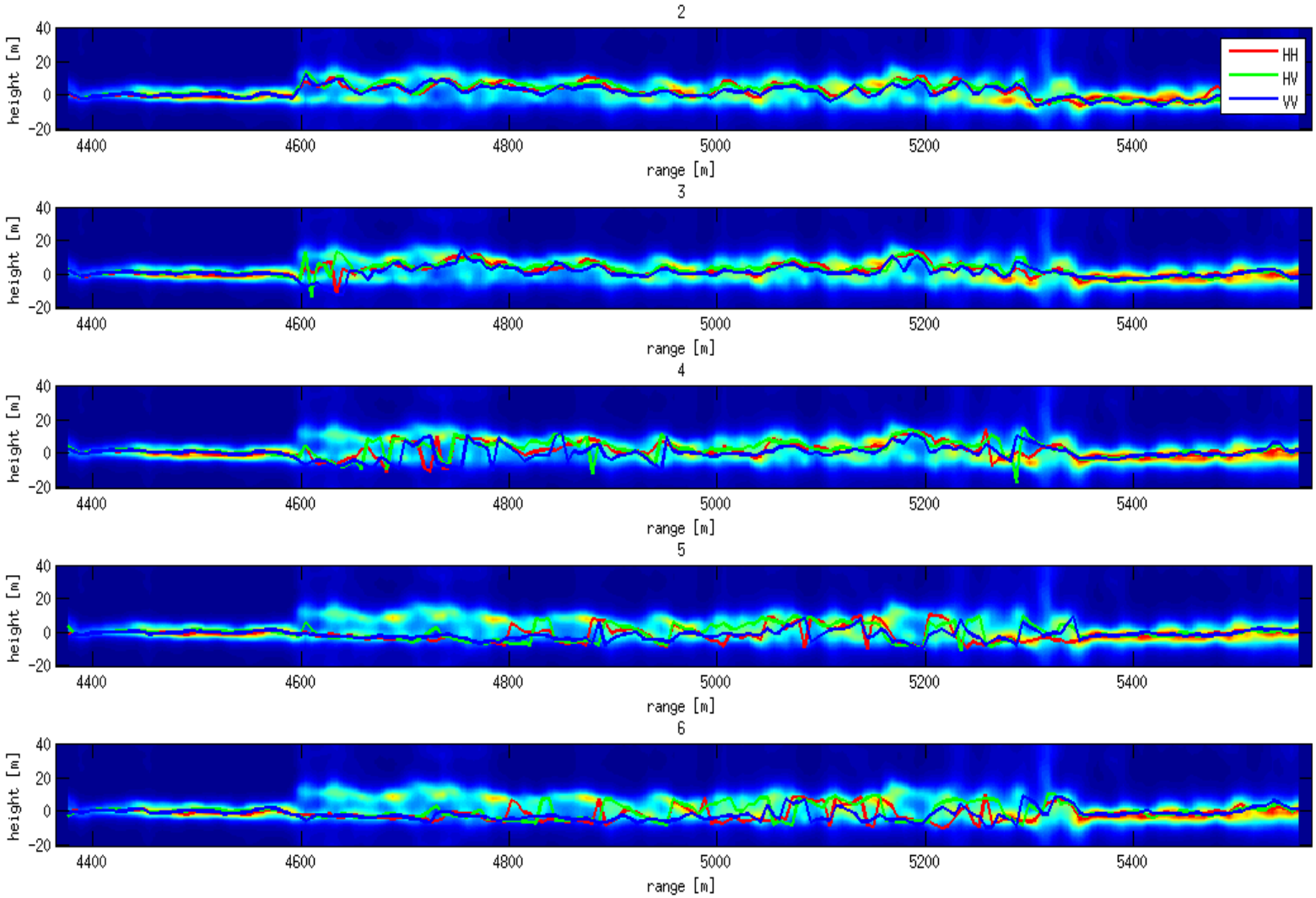
VV



Polarimetric diversity

POL-InSAR phase center heights

$h(\phi_{12})$



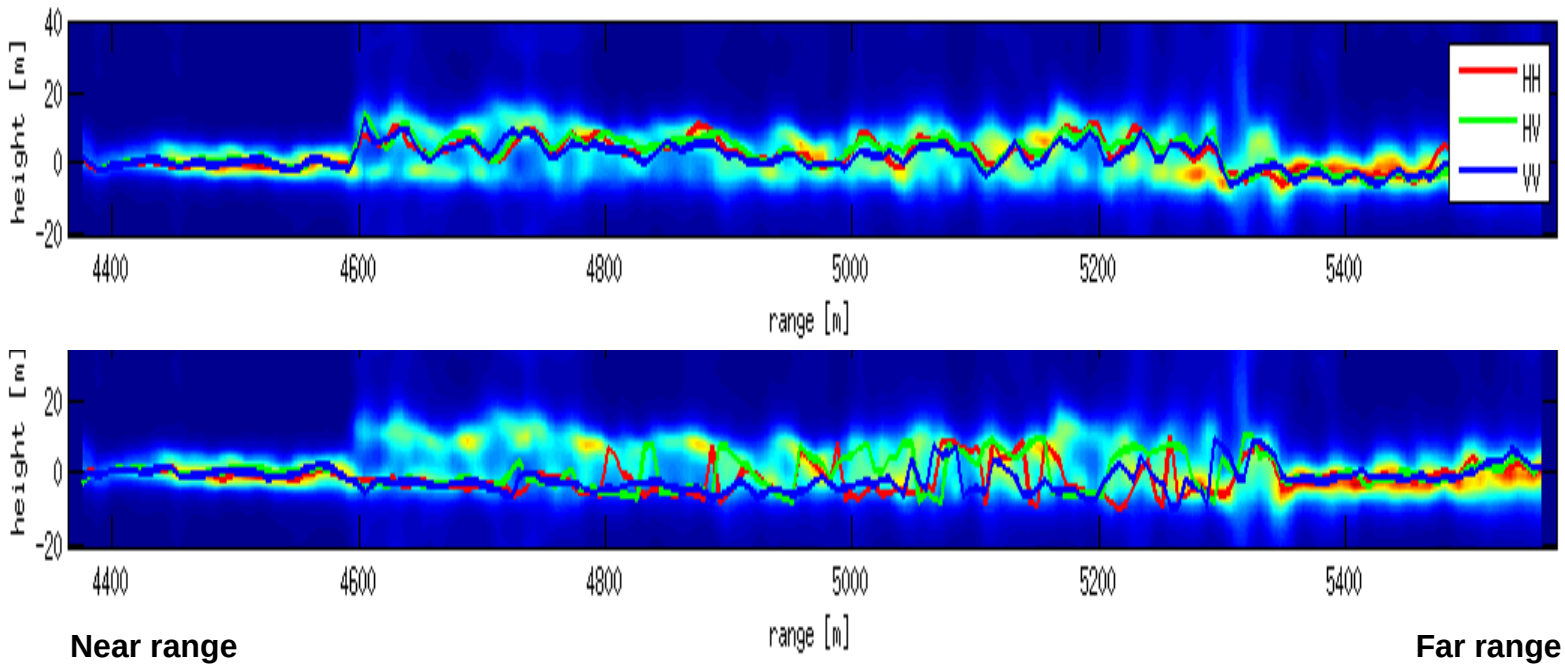
$h(\phi_{16})$

Polarimetric diversity

POL-InSAR phase center heights

$h(\phi_{12})$

$h(\phi_{16})$



Single-baseline PolinSAR:

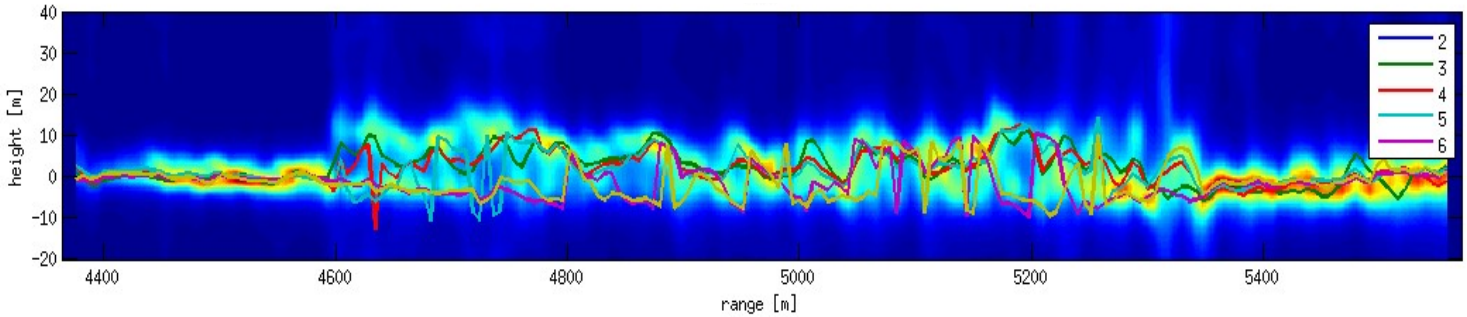
- Phase Center height diversity not always guaranteed
- Requires **specific k_z** (baseline) values: **adequate volume decorrelation**

InSAR phases, polarization & TomoSAR

Spatial diversity

MB-InSAR phase center heights

HH



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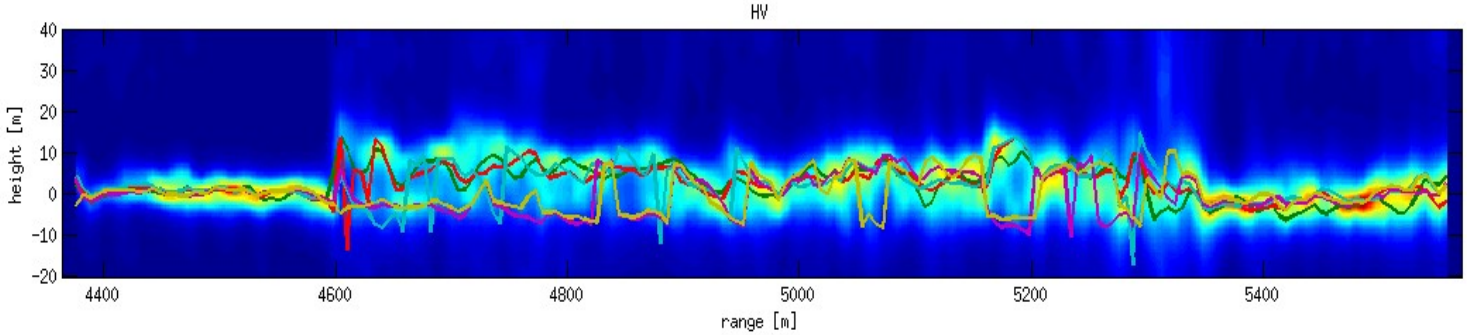
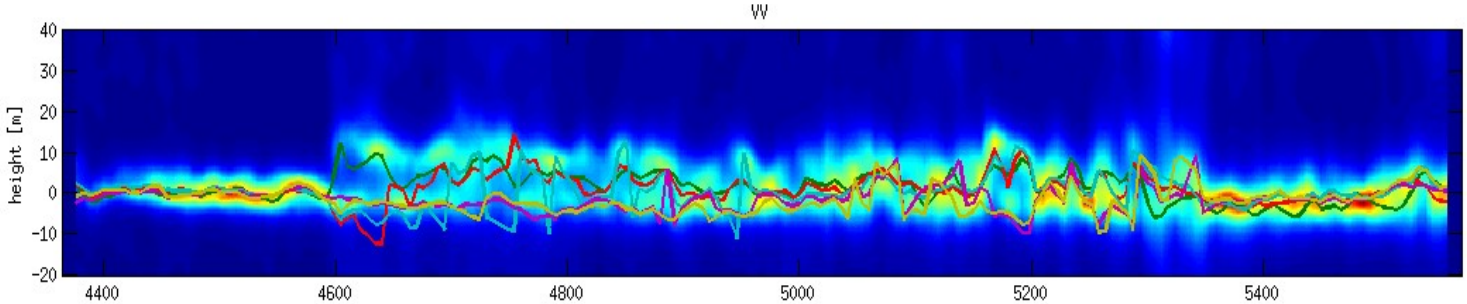


VV



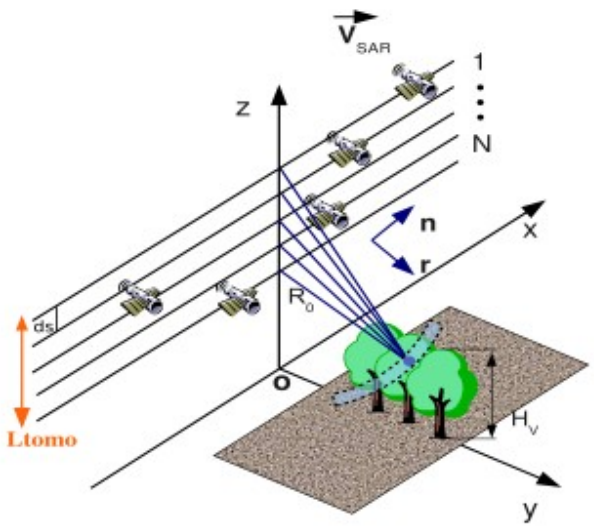
Tomography

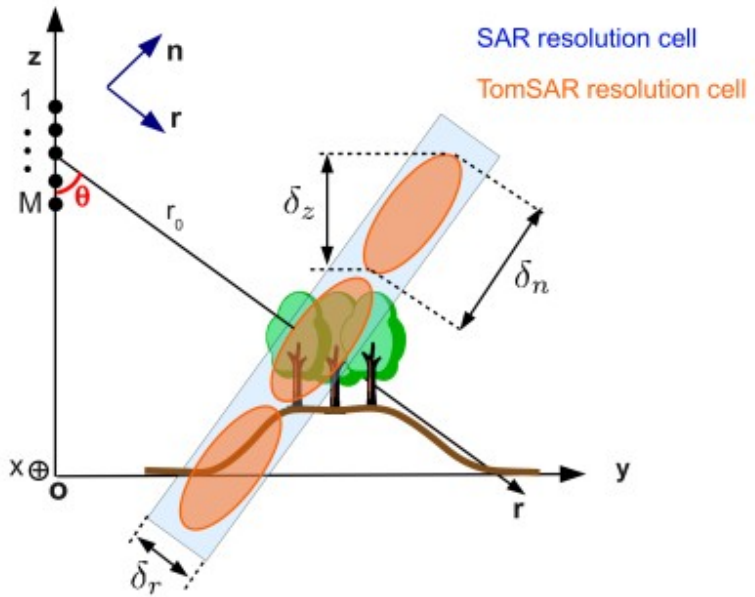
Phase Center height diversity through baseline agility

Multibaseline InSAR (MB-InSAR) tomography

Several mixed scatterers → many across-track positions

Acquisition geometry





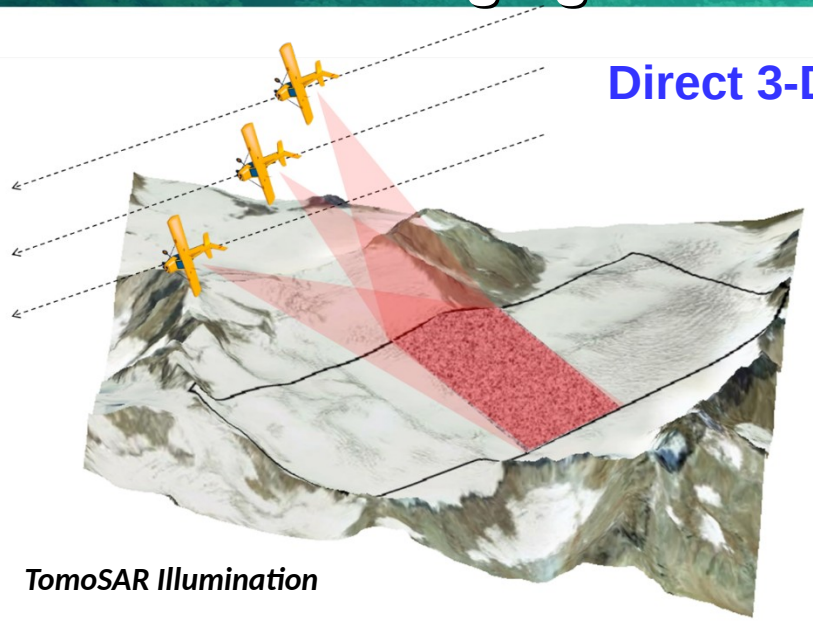
Processing options

- Direct 3-D imaging: coherent combination of M SAR acquistions

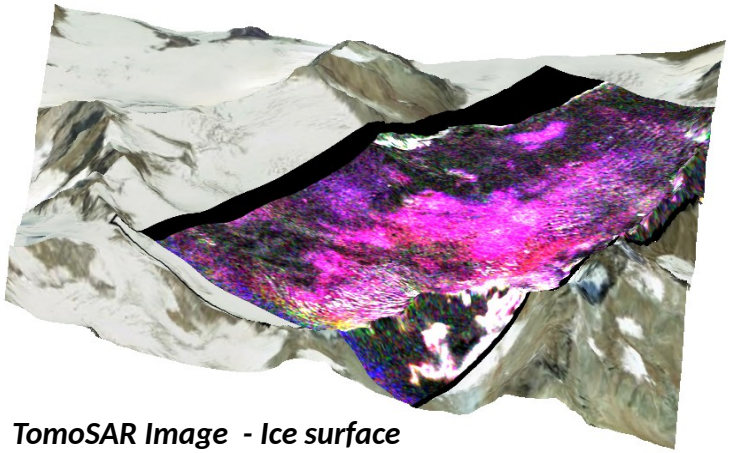
- M x 2-D focusing & coherent processing of M-InSAR quantities

Polarimetric SAR imaging of 3-D scenes

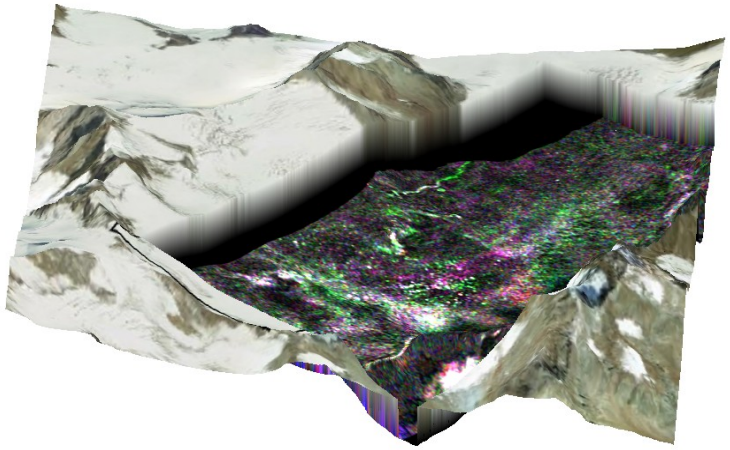
Direct 3-D imaging of an Alpine glacier at L band



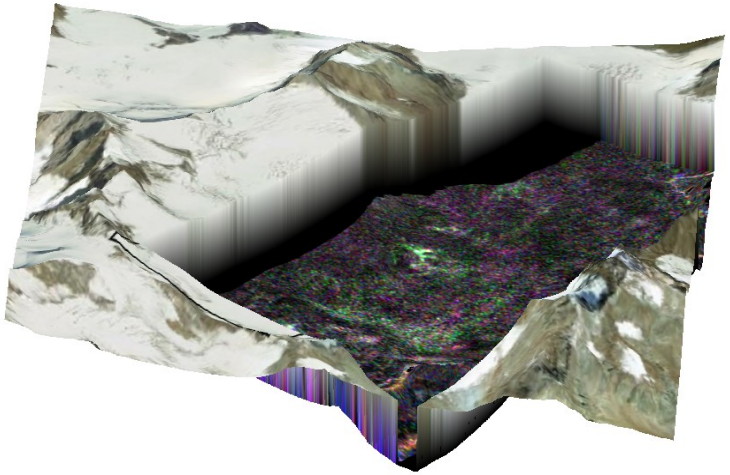
TomoSAR Illumination



TomoSAR Image - Ice surface



TomoSAR Image - 25 m below the Ice surface

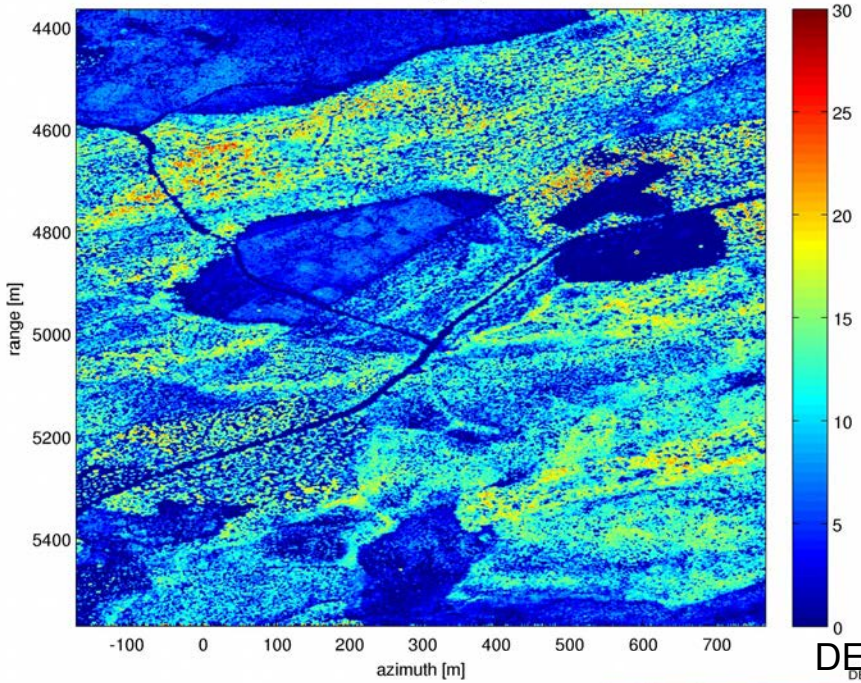


TomoSAR Image - 50 m below the Ice surface

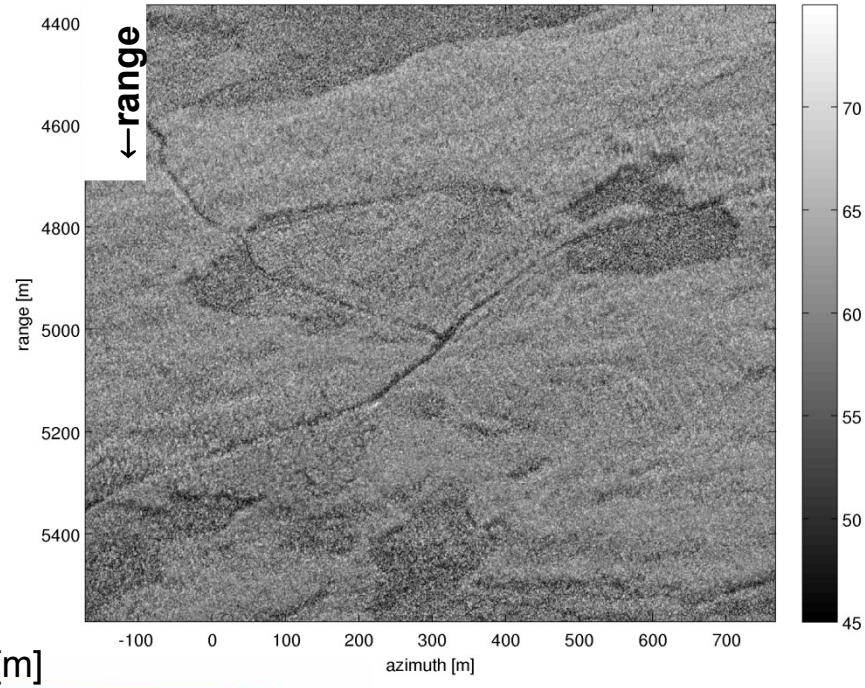
Polarimetric SAR imaging of 3-D scenes

BIOSAR II, Boreal forest, L band

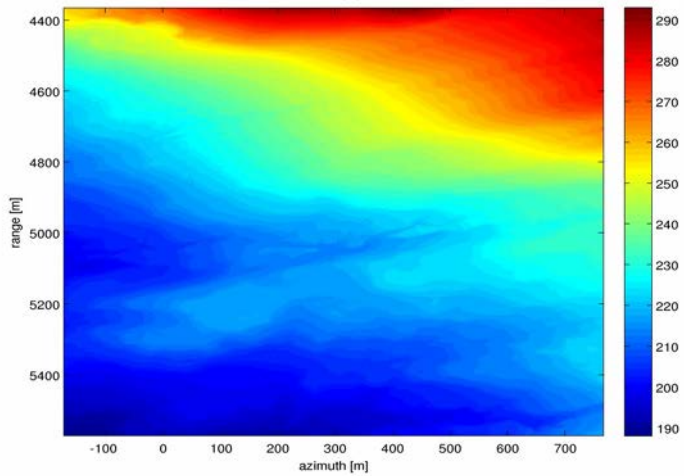
Forest height [m]



HH intensity [dB]



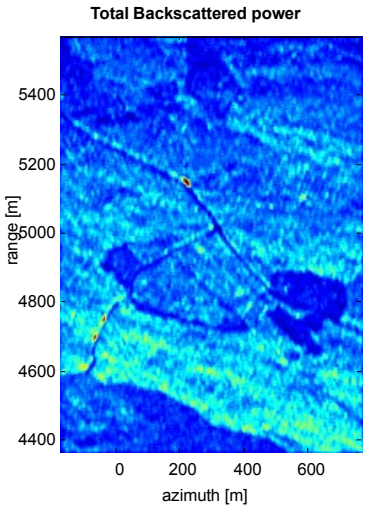
DEM [m]

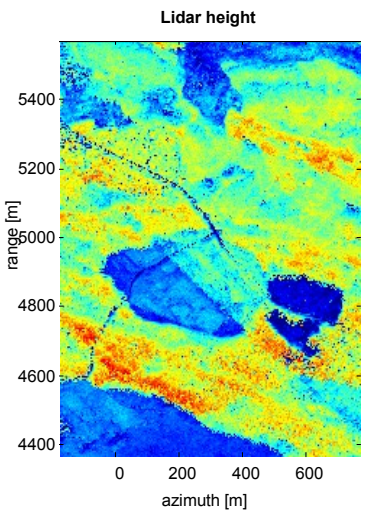


Polarimetric SAR imaging of 3-D scenes

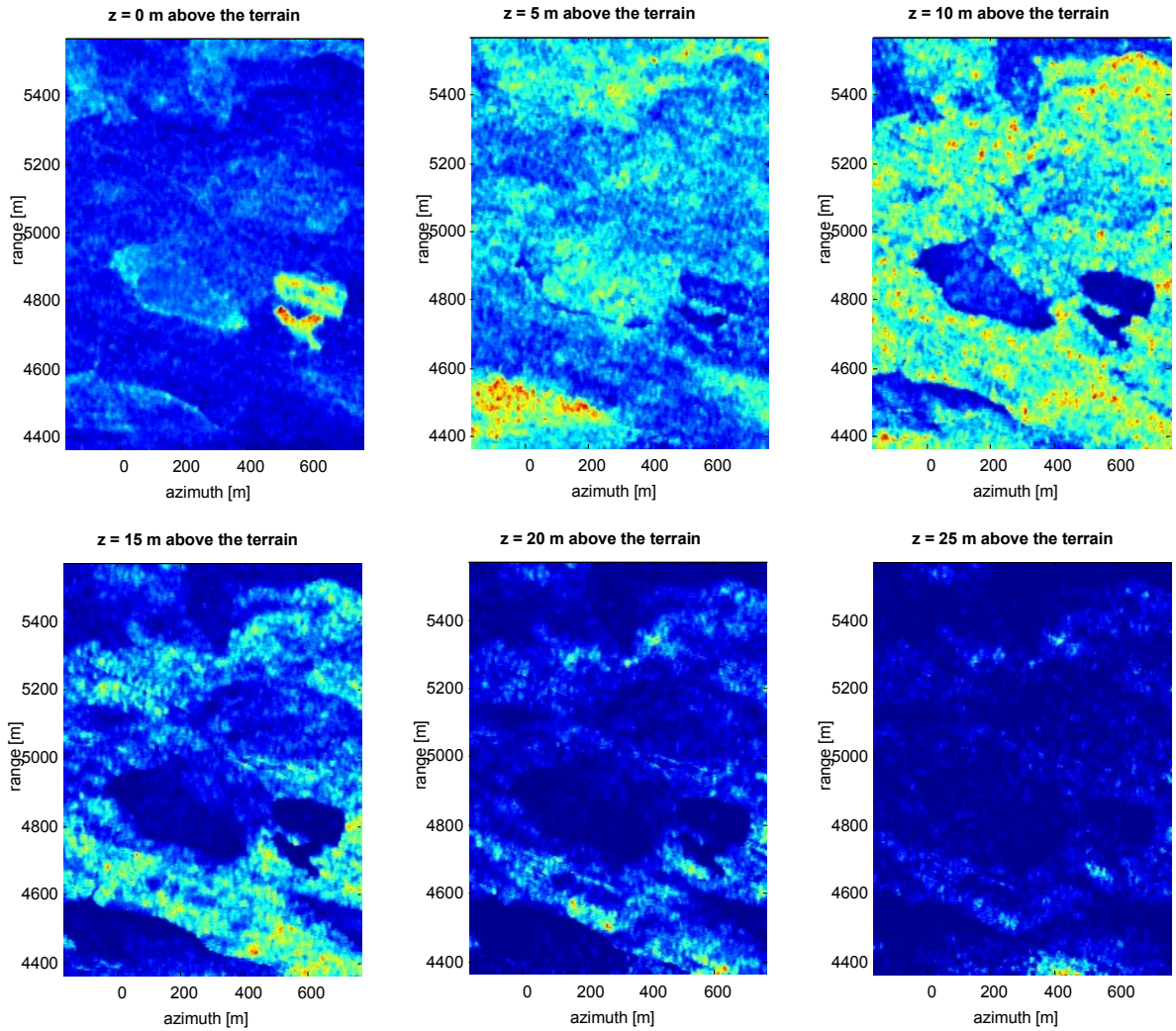
M x 2-D imaging of a Boreal forest at L band

SAR: 2D Imaging





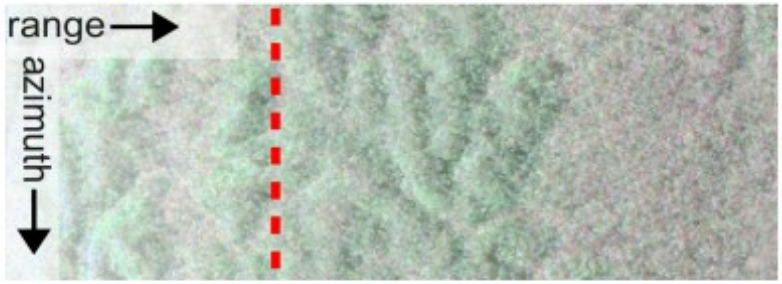
TomoSAR: 3D Imaging



- Power distribution in height direction
- Full-resolution CAPON

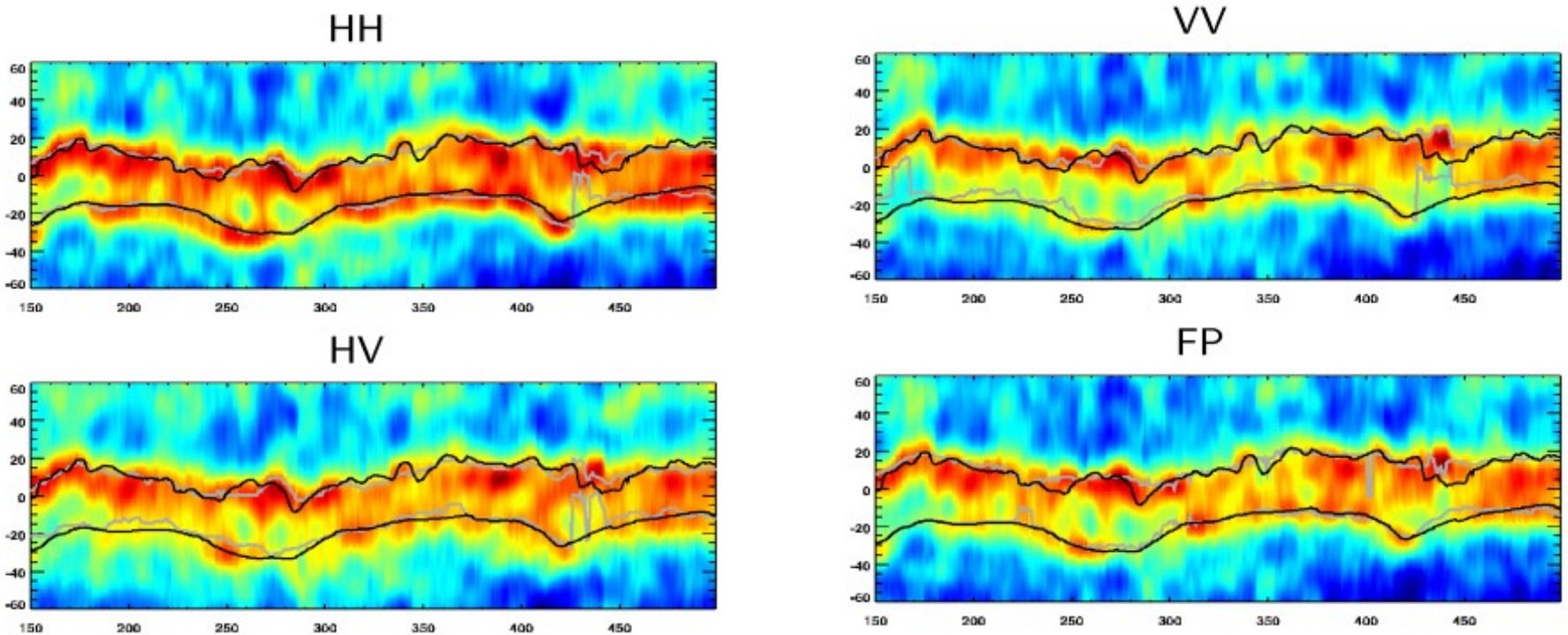
Tree height and ground topography estimation

Hybrid spectral approach



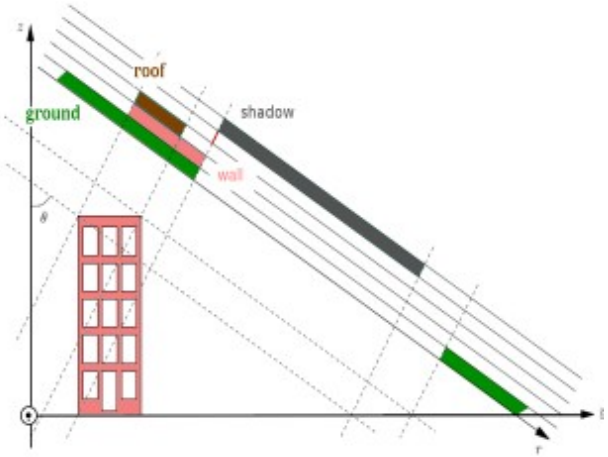
- Estimated profiles match LiDAR
- HH profiles : similar to FP case

LiDAR — TomSAR —



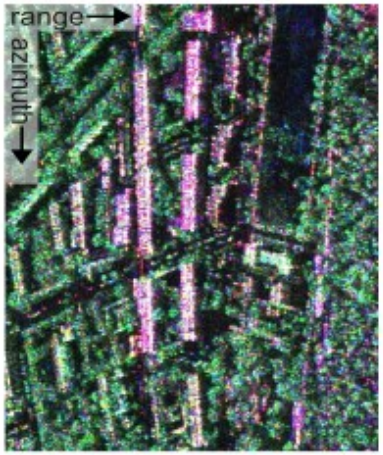
Polarimetric SAR imaging of 3-D scenes

Urban areas



Mixture of several contributions in the elevation direction

Pauli-coded SAR image



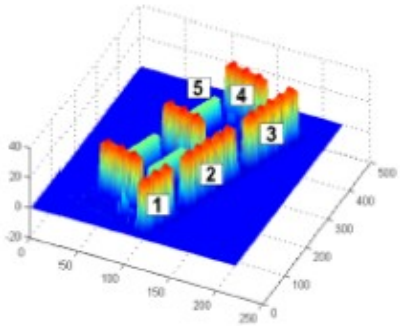
Optical image

Building reconstruction

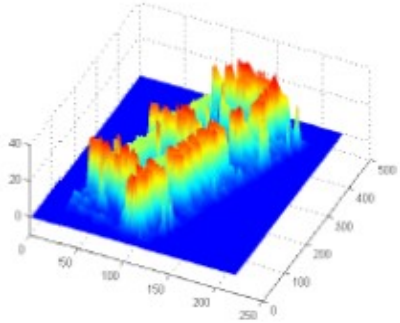
Google map

Bing map

LiDAR



Estimated by FP-NSF
($N_s = 2$)



Averaged z[m]	B1	B2	B3	B4	B5
LIDAR	30.0	30.2	30.1	30.8	16.3
Estimated	27.5	27.8	27.5	27.3	16.1

Course outline

1- Introduction

2- Principles of 3-D SAR imaging

3- Spectral analysis techniques for PolTomoSAR focusing

4- Campaign & results SARSIM toolbox

5- Practical