→ EARTH OBSERVATION SUMMER SCHOOL

Earth System Monitoring & Modelling

30 July–10 August 2018 | ESA–ESRIN | Frascati (Rome) Italy

Joint inversion of satellite and other geophysical data

Jörg Ebbing

Joint inversion of satellite and other geophysical data

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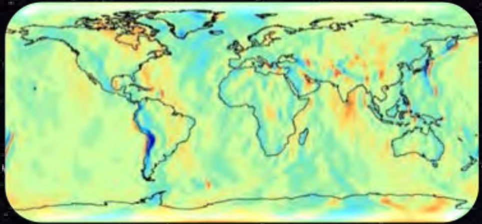
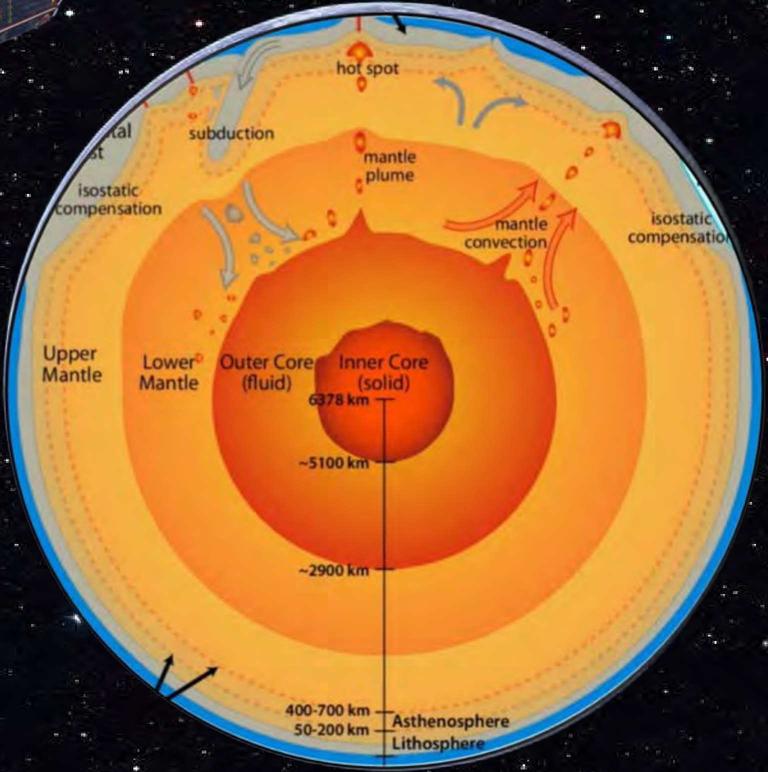


$$g \propto \frac{\rho V}{r^2}$$



Why invert gravity gradients?

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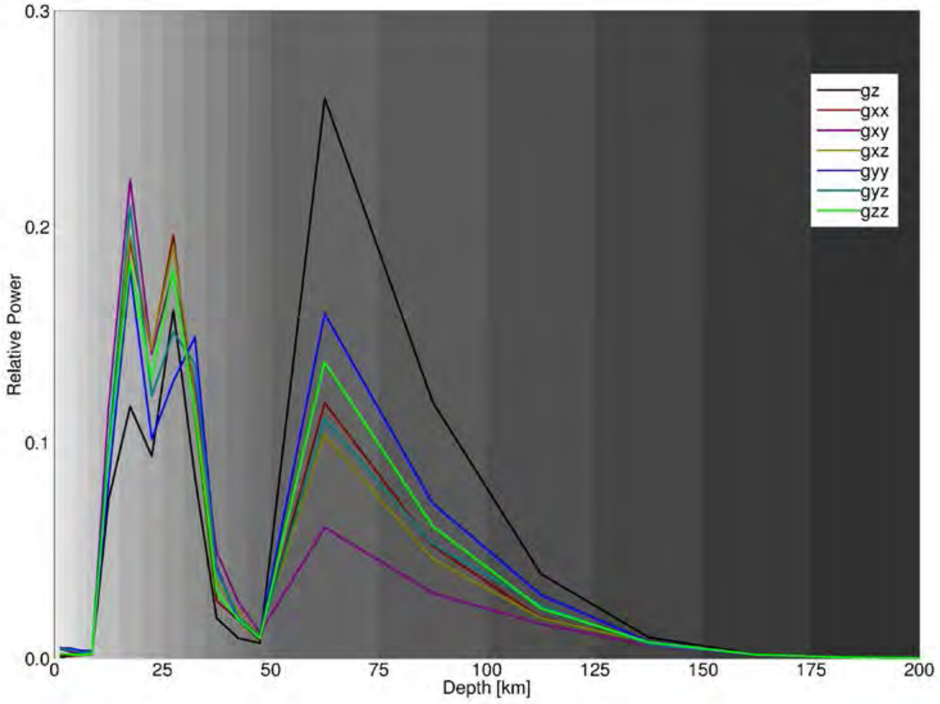

=> Different depth sensitivity of gradients can be exploited

Enhanced satellite gravity gradient imaging of Earth

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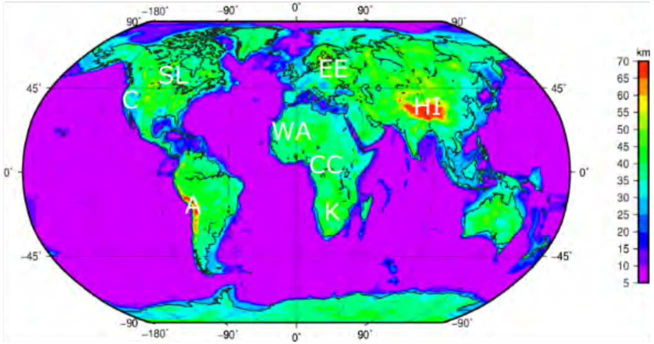
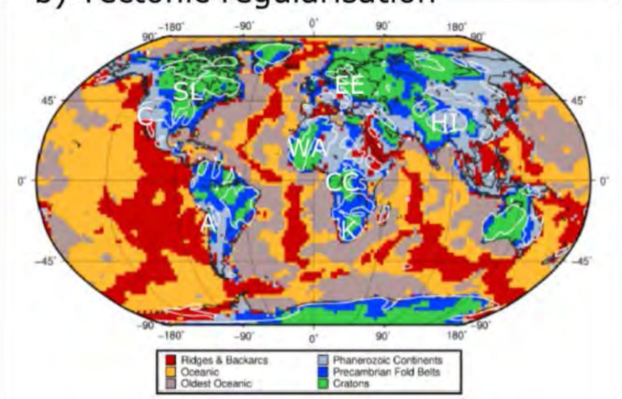
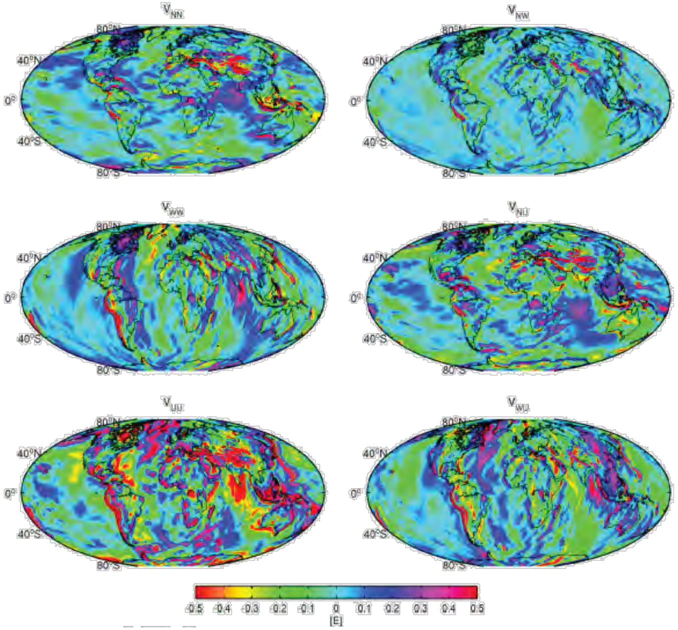
C) Crustal thickness (Crust1.0)



b) Tectonic regularisation





Enhanced satellite gravity gradient imaging of Earth

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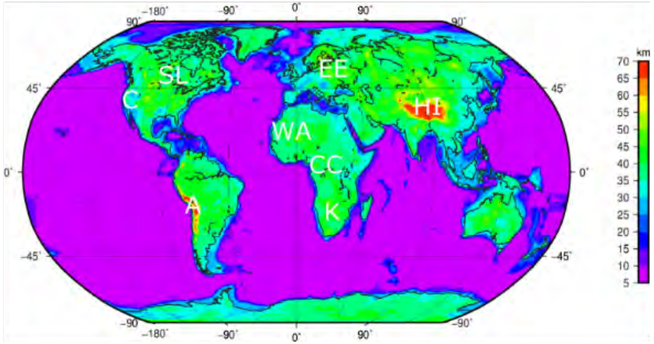
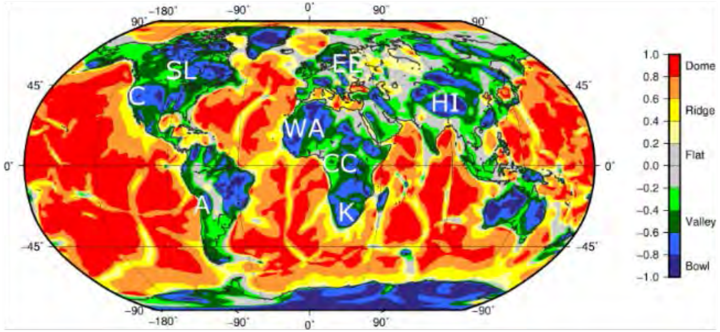
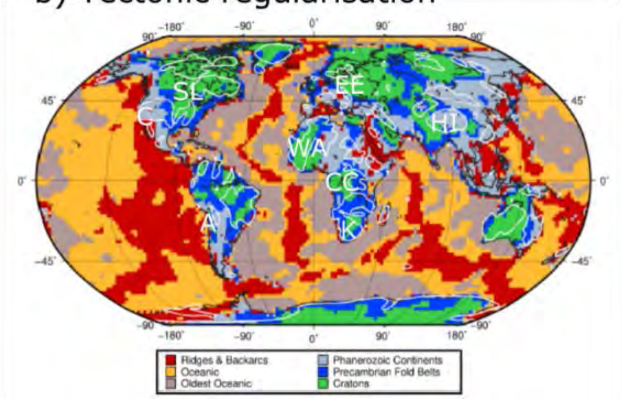
C) Crustal thickness (Crust1.0)



a) Shape Index



b) Tectonic regularisation



Shape index is a combination of minimum and maximum curvature:
indicates tectonic regimes with different crustal characteristics

$$S_i = \frac{2}{\pi}\arctan\left(\frac{G_{zz}}{\sqrt{(G_{xx}-G_{yy})^2+4{G_{xy}}^2}}\right)$$

Non-uniqueness in gravity inversion

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- Finding any density distribution that explains the gravity field is **trivially easy** (sort of)
- Example
  - Gravity measured at height h
  - Put surface density distribution $\sigma(x, y)$ at height 0
  - Use Fourier-Transform to get density

$$\sigma(k_x, k_y) = \frac{g(k_x, k_y)}{G} e^{\sqrt{k_x^2 + k_y^2}\, h}$$

  - This is basically filtering
- Meaningful density inversion requires **constraints**!

Global crustal model Crust1.0

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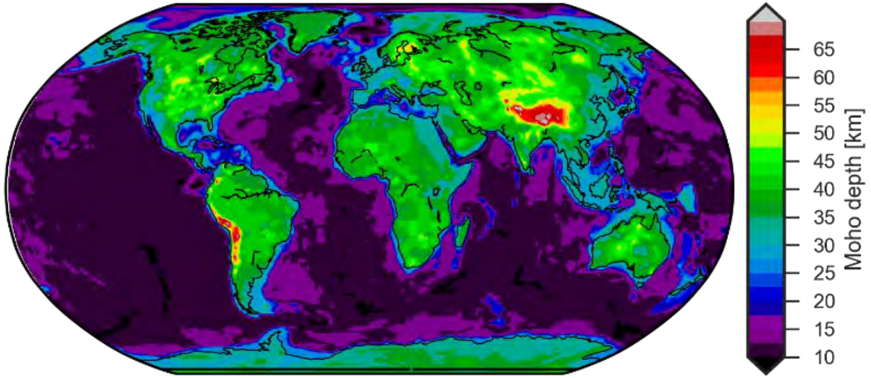
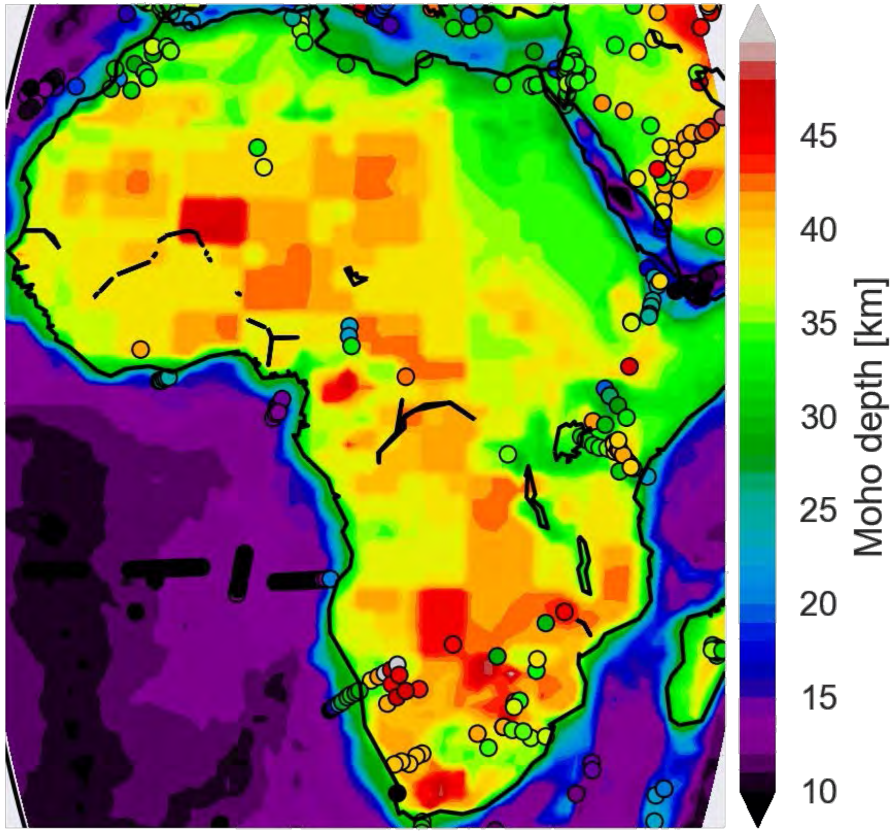


Moho depth from Crust1.0
(Laske et al. 2013)

- De-facto standard for global crustal models
- Gives crustal layers and seismic velocities
- Based on results from active seismics combined using **expert knowledge**
- Uses predefined geological domains

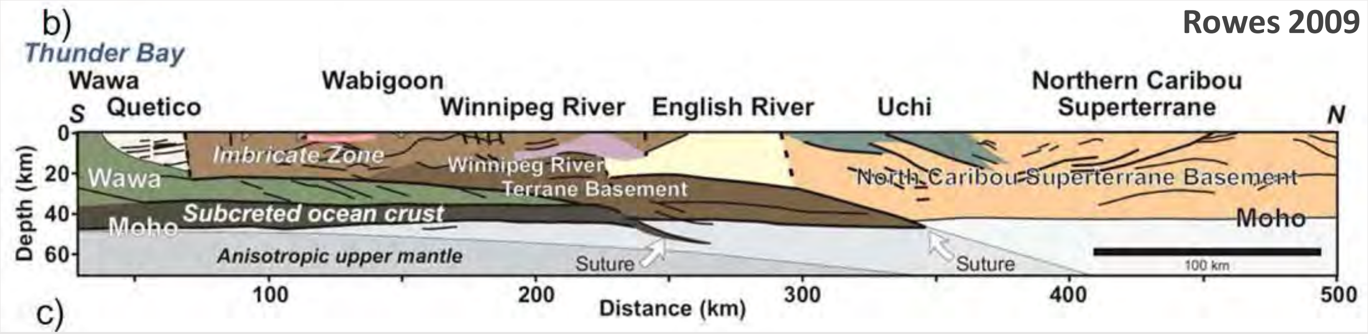


Active source seismology (continental scale)

Hydraulic vibrations

Chemical Explosives [1]

Peaceful nuclear Explosions [2]



Continental scale
Refraction and reflection profiles

Global crustal models are based on compilations of **published interpretations**!

Global crustal model Crust1.0

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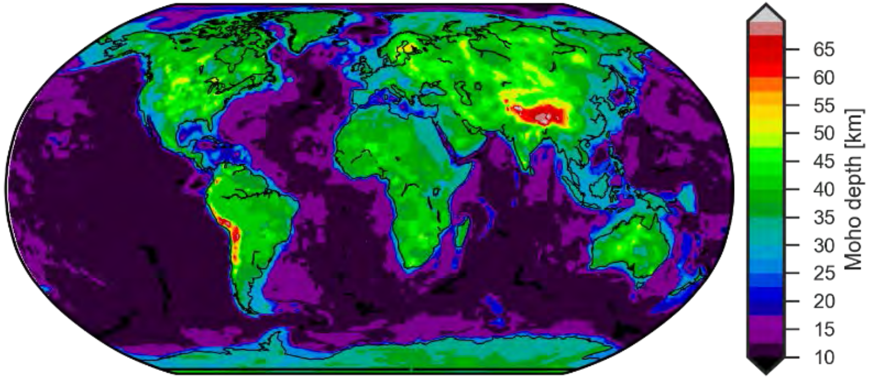
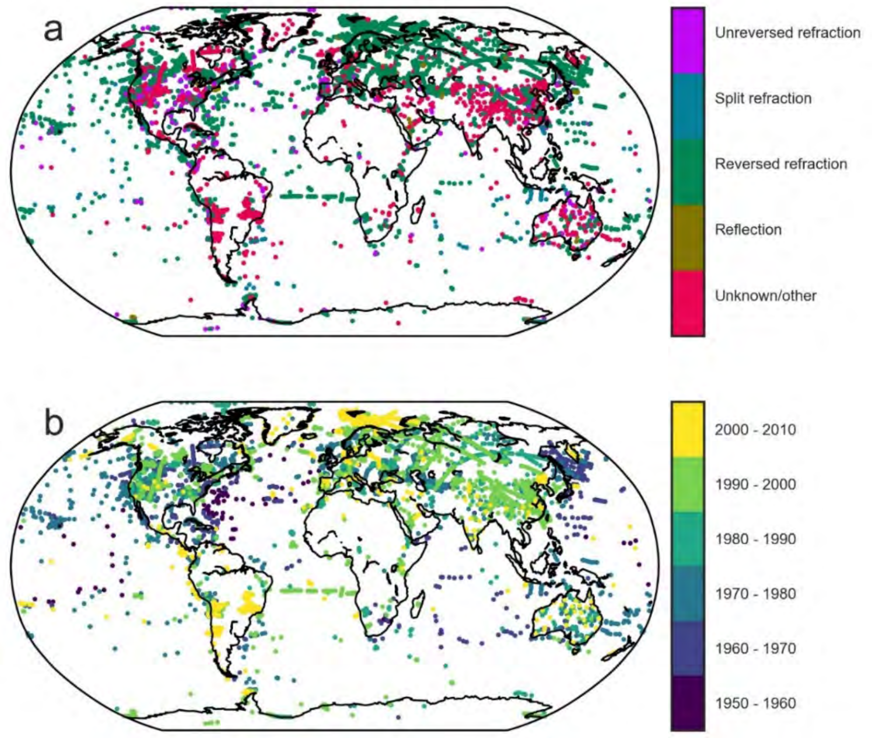


Moho depth from Crust1.0
(Laske et al. 2013)

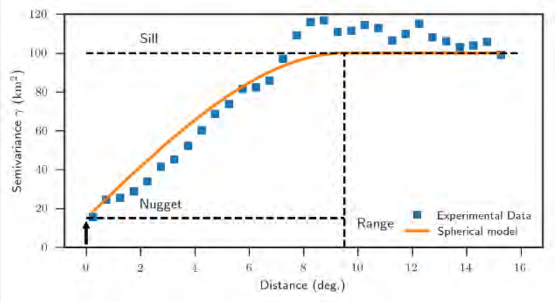
Global coverage of active source seismology



- De-facto standard for global crustal models
- Gives crustal layers and seismic velocities
- Based on results from active seismics combined using **expert knowledge**
- Uses predefined geological domains

Kriging interpolation

Semivariance:
Mean squared difference as a function of separation



- Range: 9.5° (approx. 1000 km)
- Nugget: 15 km²
- Sill: 100 km²

Meaning of parameters

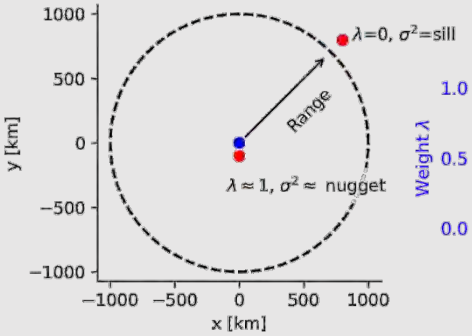
- **Nugget**: Small-scale + measurement errors
- **Range**: Correlation distance
- **Sill**: Scale of variability

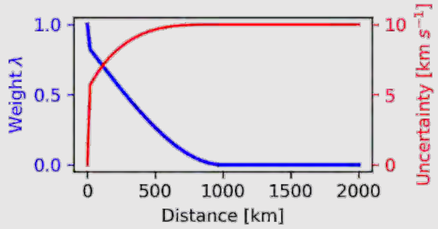
Kriging equation:

$$\hat{Z}^* = \mu + \sum_i \lambda_i (Z_i - \mu)$$

- Known values $Z_i = Z(x_i)$
- Unknown value $Z^* = Z(x^*)$
- Mean value μ
- Weights λ_i depend on point distances and semivariogram

Example: One known point

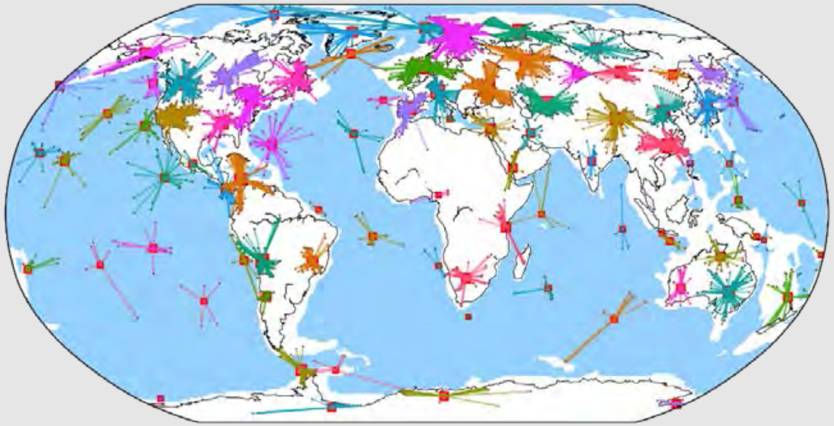




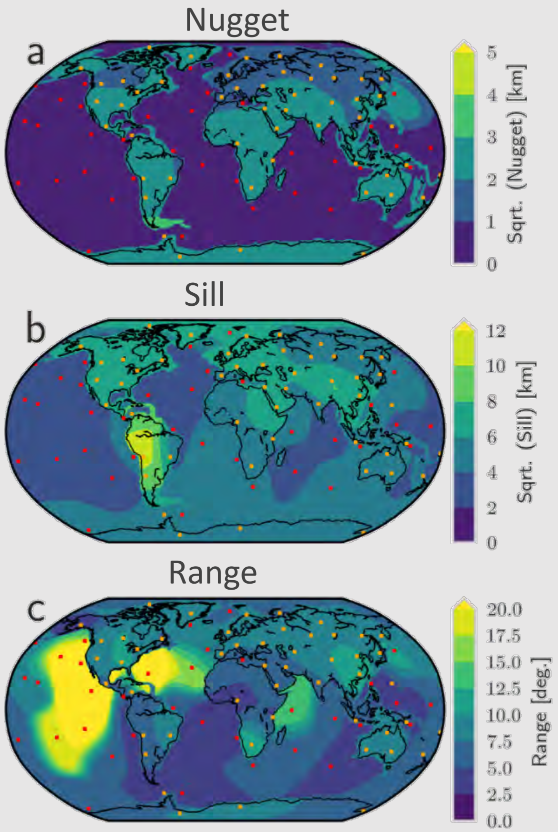
Global scale kriging – technical challenges

One semivariogram for entire Earth insufficient!

- Separate oceanic and continental domains
- Determine semivariograms for point clusters

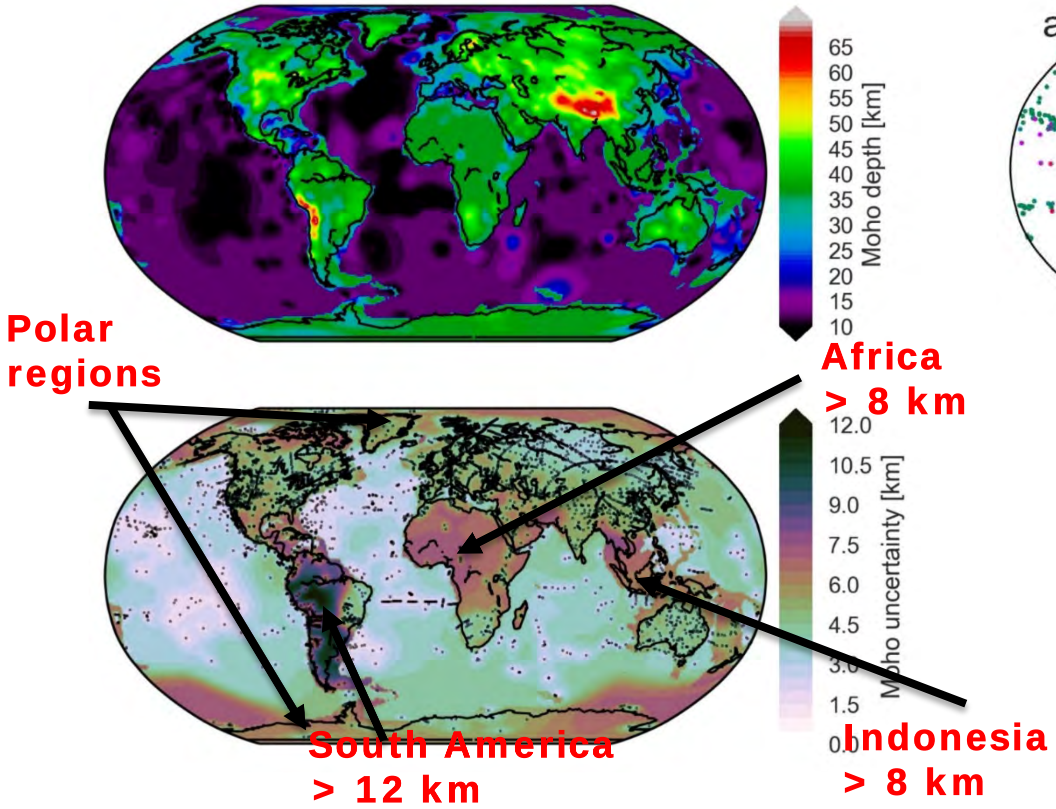


Result of Agglomerative Clustering

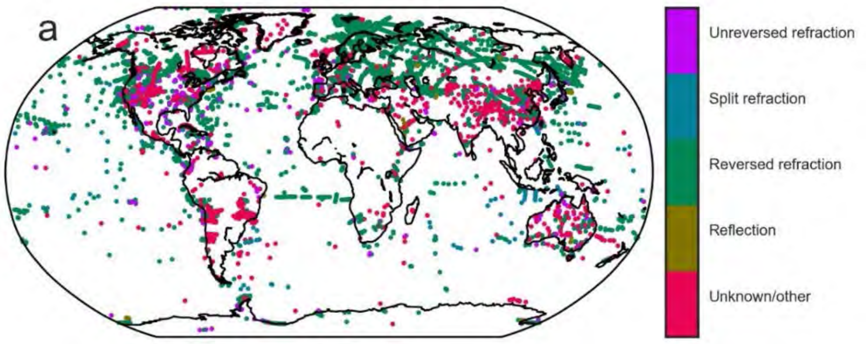


Global crustal model from interpolation

Moho depth from Interpolation



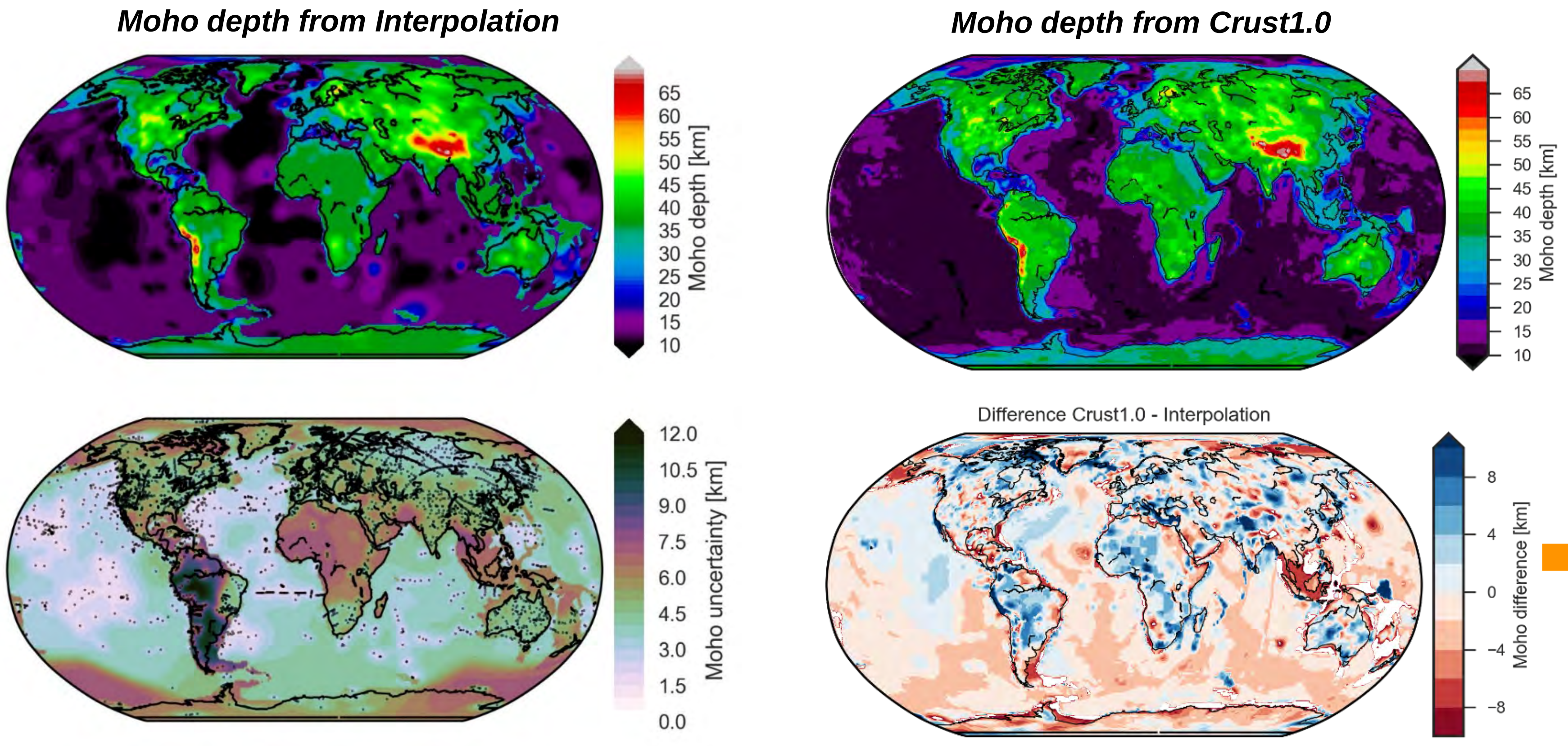
Global coverage of active source seismology



- Alternative by interpolation of constraints using only seismic information (velocity, depth)
- **No expert knowledge** required
- Provides uncertainties based on data quality/coverage

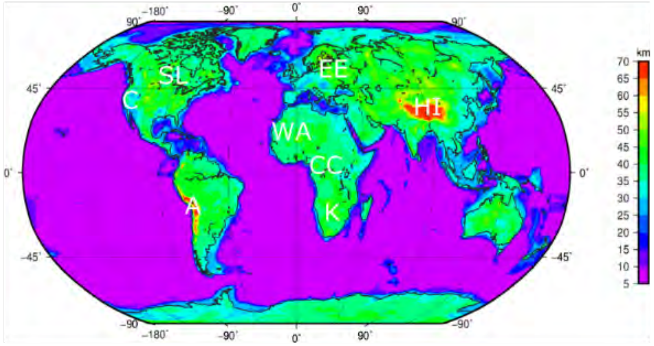
Median relative accuracy +/- 20 %

Comparison of global crustal models



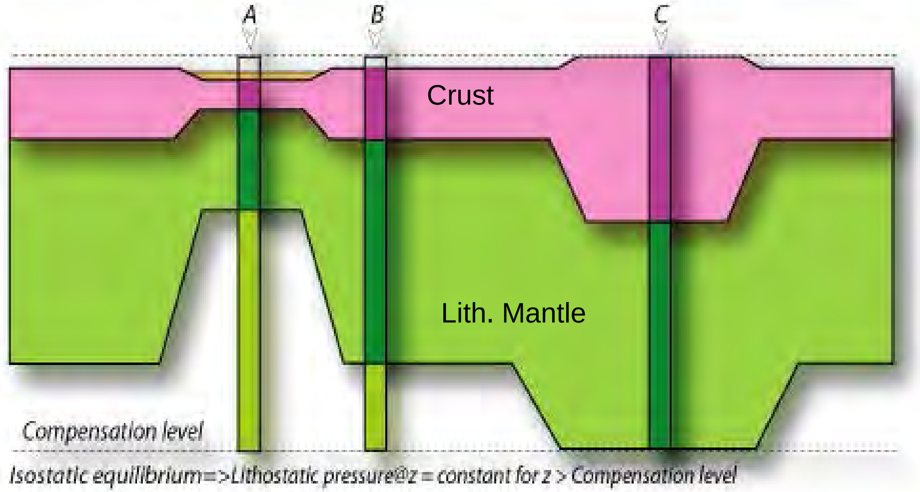
Enhanced satellite gravity gradient imaging of Earth

C) Crustal thickness

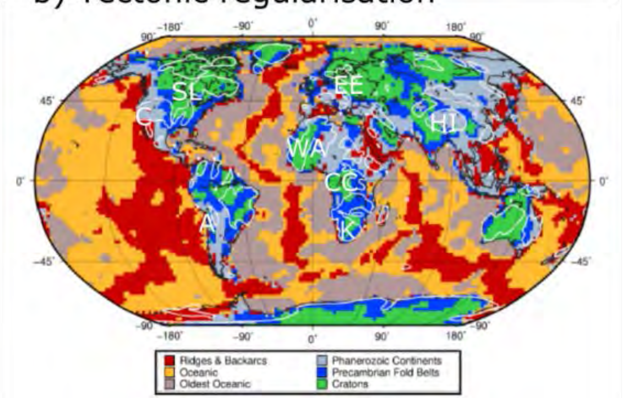


Moho =>

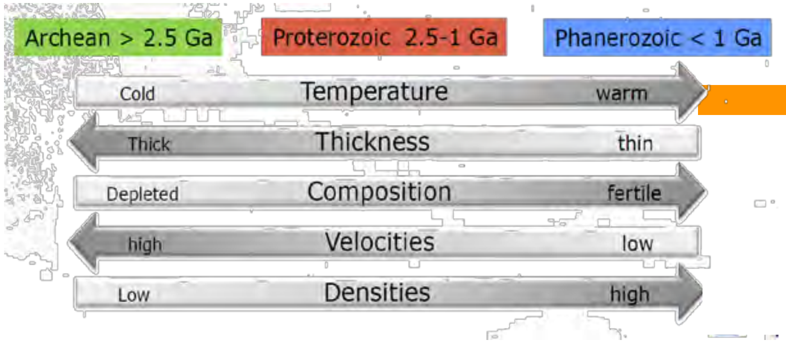
Base lithosphere =>



b) Tectonic regularisation



Tectonic regularisation relies on clustering seismic data related to age groups

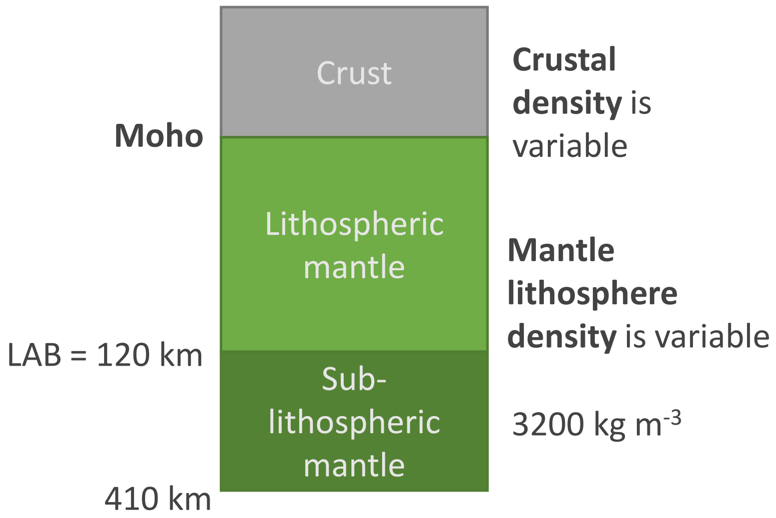


Invert satellite gravity gradient for lithospheric structure
-> 3 competing requirements

1. Fit the satellite gravity gradient data

2. Compatible with global crustal model

3. Compatible with isostatic balance



Possible issues

- Non-uniqueness/trade-offs
- Unclear data accuracy

Use Bayesian approach with Monte Carlo Markov Chain

Inversion strategy

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Bayesian inversion using **hierarchical Monte-Carlo-Markov-Chain** approach

Probabilistic inversion

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$$\boxed{\boldsymbol{d} = \boldsymbol{G}(\boldsymbol{m}) + \boldsymbol{\epsilon}}$$ (forward problem)

- $\boldsymbol{m}$: Moho, Crustal density, Mantle density
- $\boldsymbol{d}$: Observed Gravity and Topography data
- $\boldsymbol{G}$: Forward operator
- $\boldsymbol{\epsilon}$: Stochastic misfit

$$\boxed{P(\boldsymbol{m}|\boldsymbol{d}) = \frac{P(\boldsymbol{d}|\boldsymbol{m})P(\boldsymbol{m})}{P(\boldsymbol{d})} \propto P(\boldsymbol{d}|\boldsymbol{m})P(\boldsymbol{m})}$$ (Bayes theorem)

$P(\boldsymbol{m})$: **prior probability** (based on seismic constraints)

$P(\boldsymbol{d}|\boldsymbol{m})$: **Likelihood (data fit)**

Bayesian inversion

- Put all model parameters (Moho, LAB, crustal density, mantle density) at all grid cells in a vector $\boldsymbol{m}$
- Put all observed data in a vector $\boldsymbol{d}$
- Each model is assigned a **prior probability** (based on e.g. seismic constraints) $P(\boldsymbol{m})$
- Each model is assigned a **likelihood** based on how good it fits the data $P(\boldsymbol{d}|\boldsymbol{m})$
- Bayes theorem gives posterior probability:

$$\boxed{P(\boldsymbol{m}|\boldsymbol{d}) = \frac{P(\boldsymbol{d}|\boldsymbol{m})P(\boldsymbol{m})}{P(\boldsymbol{d})} \propto P(\boldsymbol{d}|\boldsymbol{m})P(\boldsymbol{m})}$$

Bayes theorem

Question: How good (likely) is a model

$$\boxed{P(\boldsymbol{m}|\boldsymbol{d}) \propto P(\boldsymbol{d}|\boldsymbol{m})P(\boldsymbol{m})}$$

Data fit

Prior information

The likelihood function

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$$\boxed{\boldsymbol{d} = \boldsymbol{G}(\boldsymbol{m}) + \boldsymbol{\epsilon}}$$

Random term $\boldsymbol{\epsilon}$ might reflect

- measurement errors
- forward modelling errors
- unmodelled contributions

Typical assumptions for $\boldsymbol{\epsilon}$

Proposed improvements

Independent Gaussian → Correlated Gaussian (gravity gradients)

Assumed variance (often ad-hoc) → Variable variance (Hierarchical MCMC)

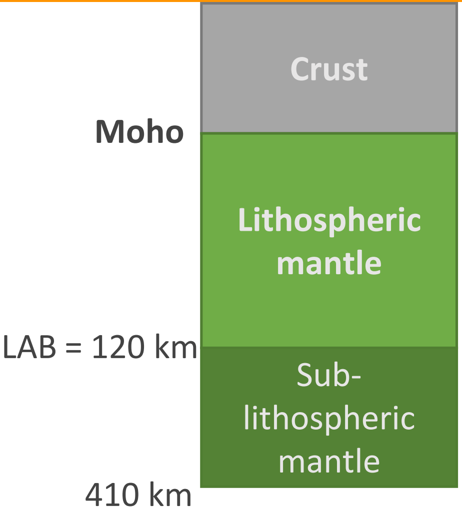
$$\boxed{P(\boldsymbol{m},\boldsymbol{\sigma}|\boldsymbol{d}) \propto P(\boldsymbol{d}|\boldsymbol{m},\boldsymbol{\sigma})P(\boldsymbol{m})P(\boldsymbol{\sigma})}$$

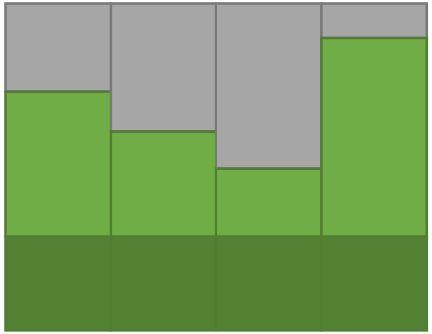
Probabilistic inversion

$$\boxed{\boldsymbol{d} = \boldsymbol{G}(\boldsymbol{m}) + \boldsymbol{\epsilon}}$$

$$P(\boldsymbol{m}, \boldsymbol{\sigma} | \boldsymbol{d}) \propto P(\boldsymbol{d} | \boldsymbol{m}, \boldsymbol{\sigma}) P(\boldsymbol{m}) P(\boldsymbol{\sigma})$$

Use MCMC (Metropolis-Hastings algorithm) to find ensemble of solutions $\boldsymbol{m}_i$



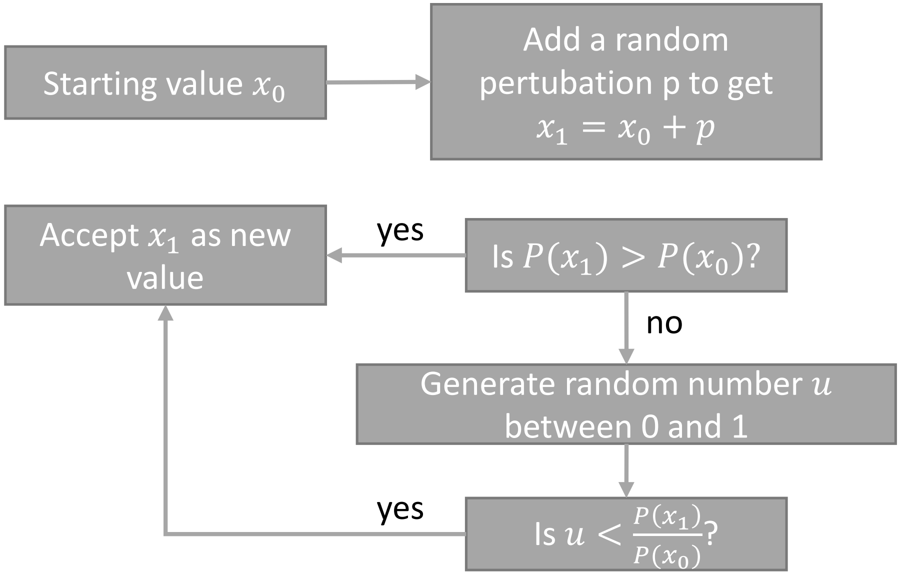


Each model: N columns

Metropolis-Hastings

Goal:
Generate samples from a probability distribution P(x)

In our case P(***x***) is P(***m*|*d***)



Result:
The sequence of $x_0, x_1, \ldots,$ are representative samples of $P(x)$!

Test area: Mid-Atlantic

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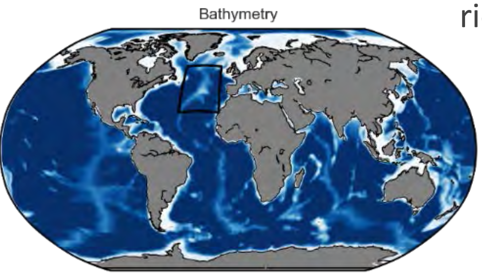
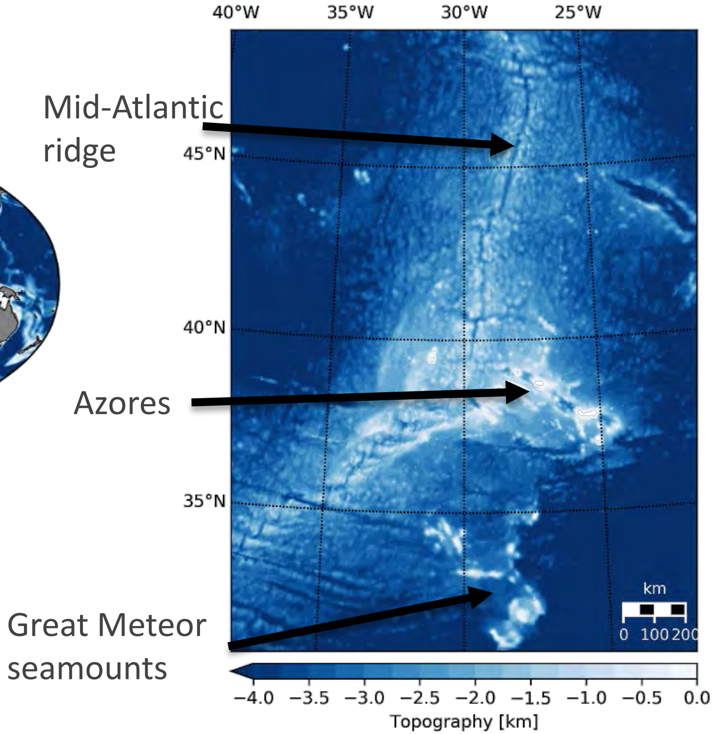
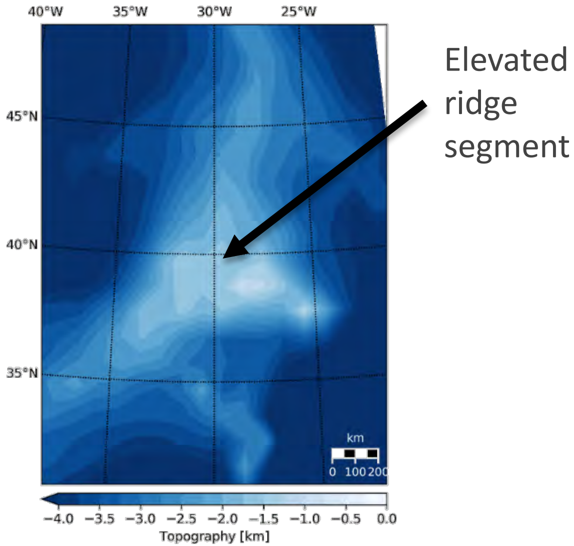




Topography from ETOPO1
(Amante and Eakins 2009)



Downsampled to 1° resolution

Why Azores?
- **Cooling trend in oceanic lithosphere**
- **Signature of hotspots**

Input data for inversion

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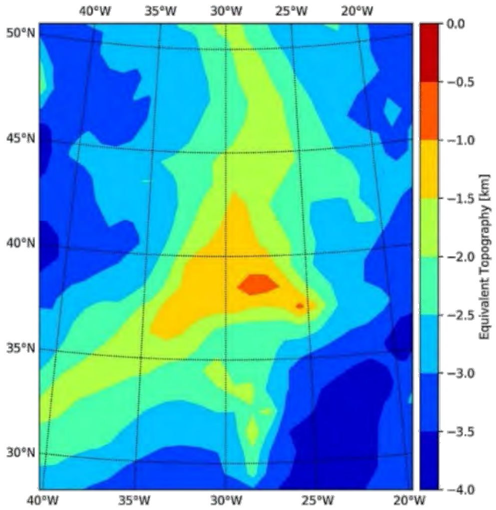
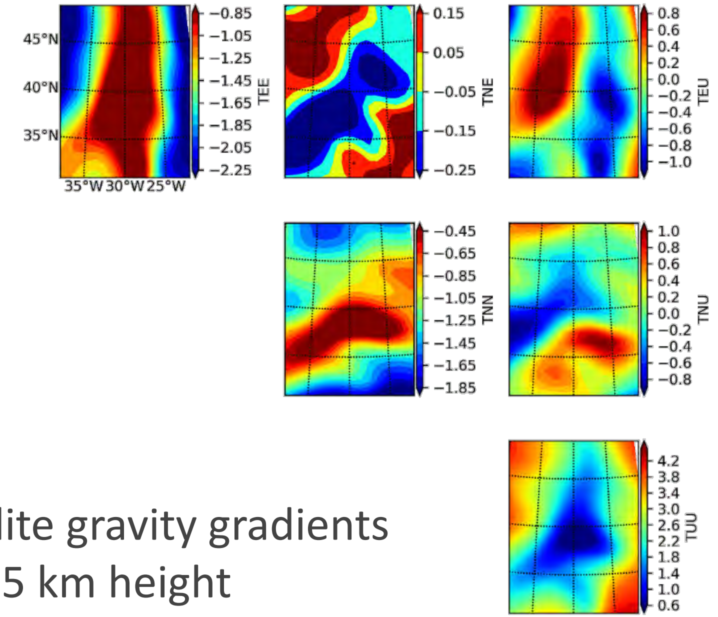
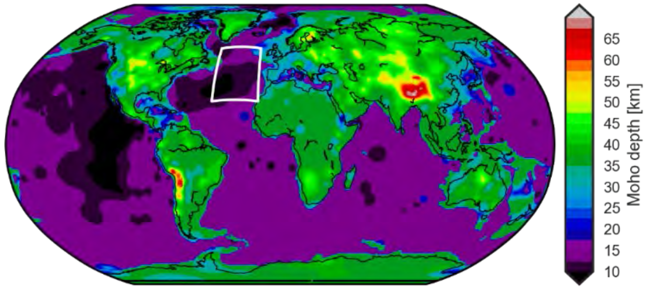


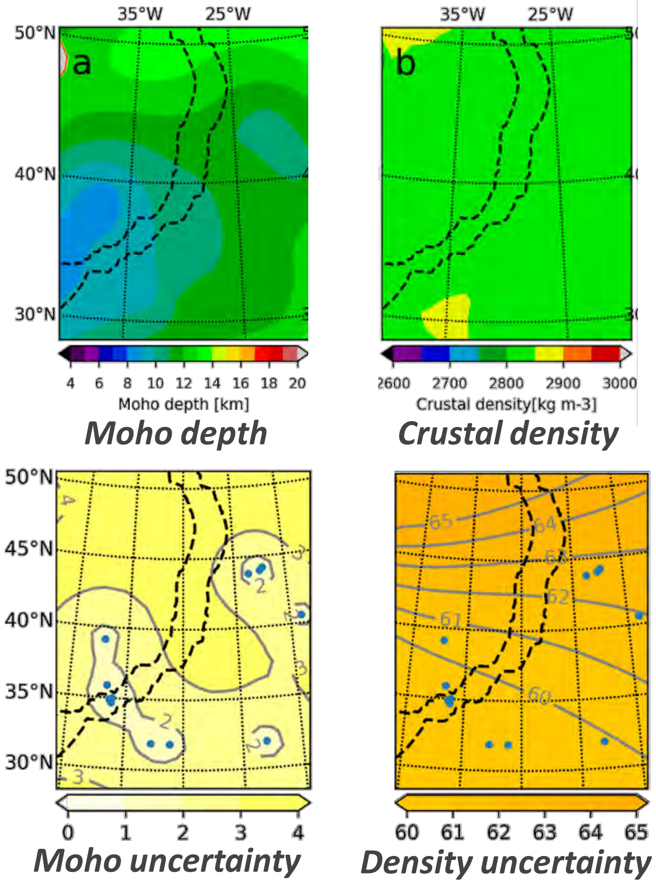
Equivalent topography
(Sediments corr.)



Satellite gravity gradients
@ 225 km height
(Topo+Sediments corr.)
„Bouguer anomaly“

Seismic constraints





Prediction based on inital data

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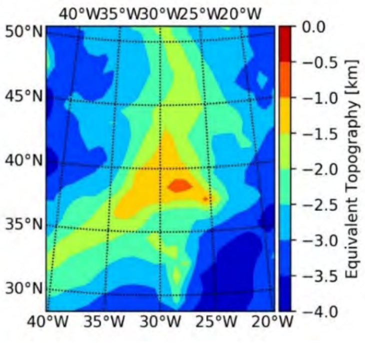
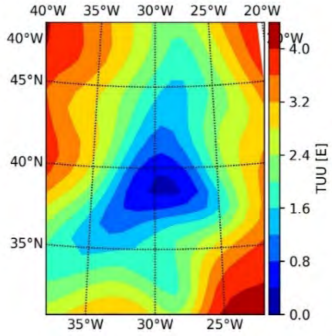
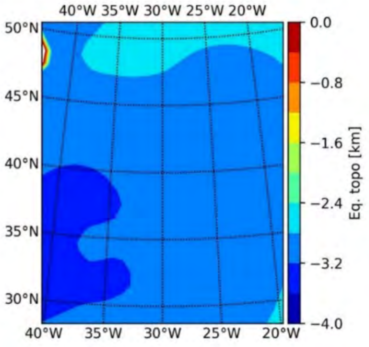


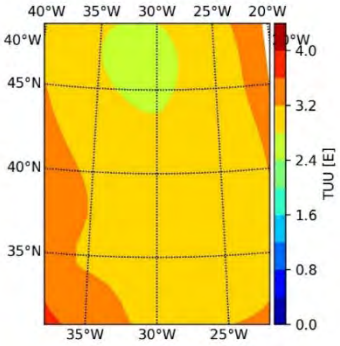
Observed Topography



Observed vertical gravity gradient



Predicted Topography



Predicted vertical gravity gradient

Inversion after 2 million iterations

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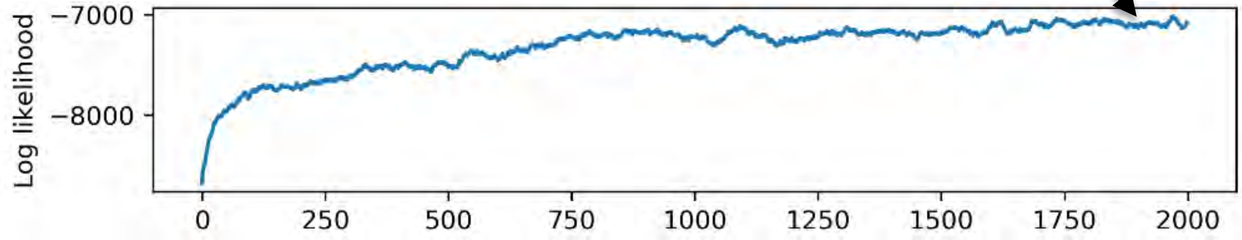
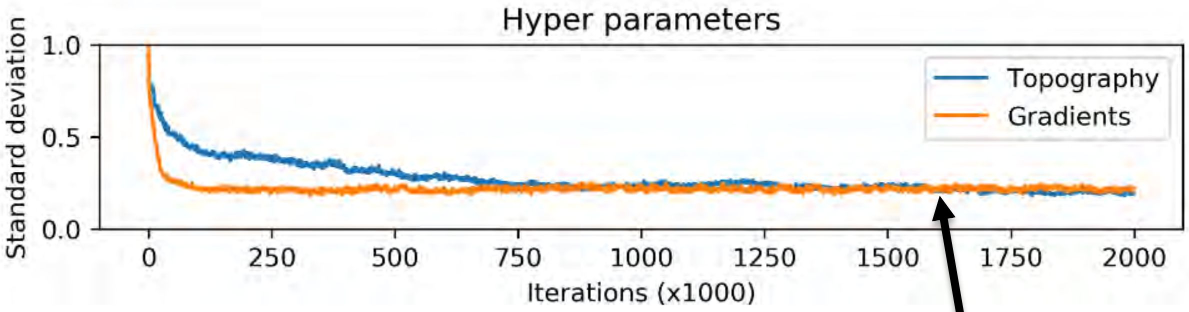
Convergence

$$P(\boldsymbol{m}, \sigma|\boldsymbol{d}) \propto P(\boldsymbol{d}|\boldsymbol{m}, \boldsymbol{\sigma})P(\boldsymbol{m})P(\boldsymbol{\sigma})$$





Converges to ~0.2 (km or E)

Inversion results

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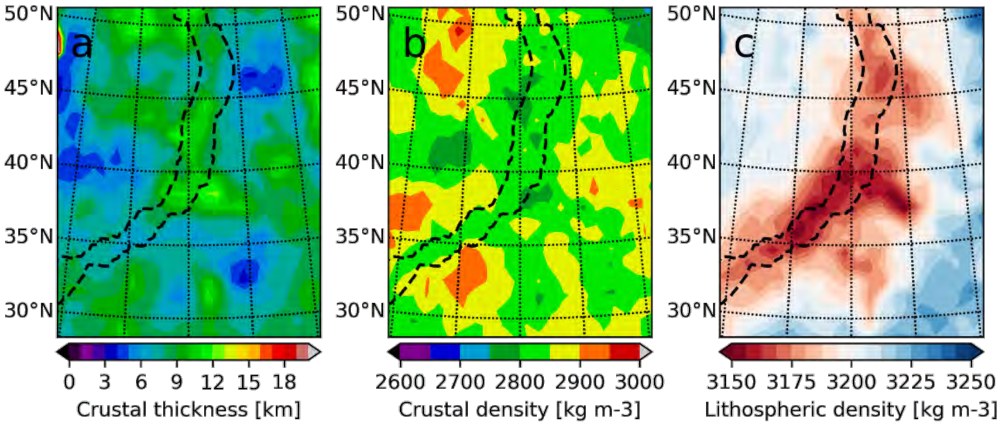
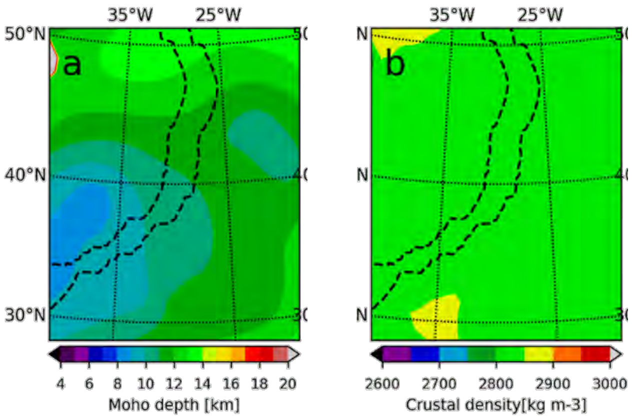
Inversion result
Mean of last
500,000 models

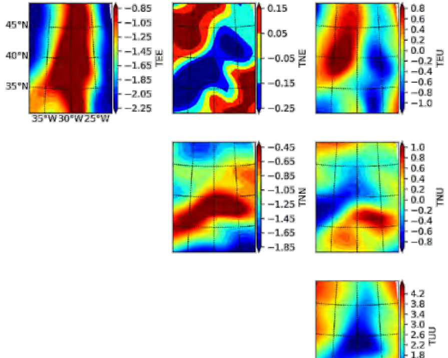


Starting values

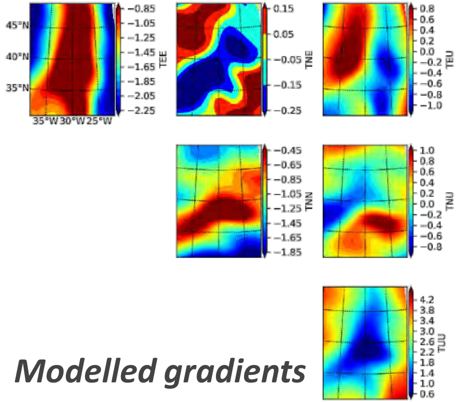


Data residuals (RMS):
G_{uu}: 0.06 E
Topography: 100 m

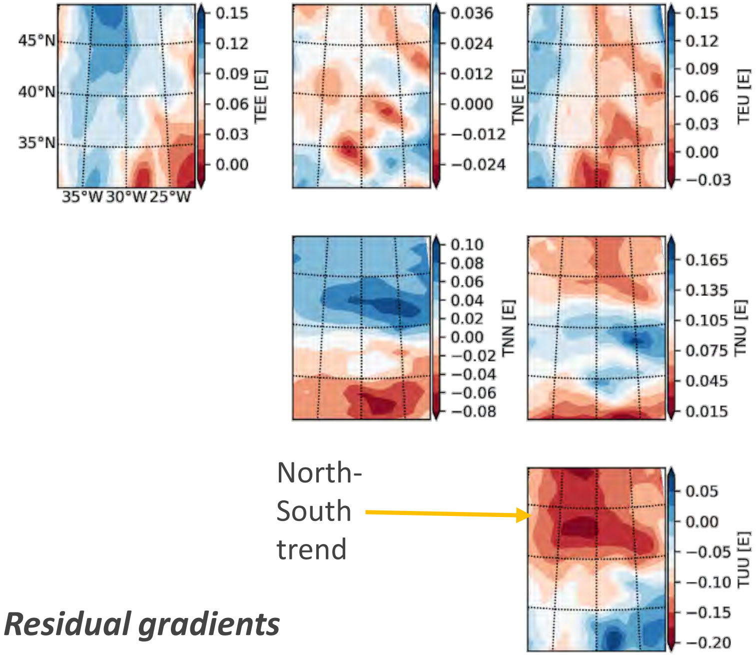
Gradient data fit (mean model!)



Observed gradients



Modelled gradients



Residual gradients

Component	EE	NE	EU	NN	NU	UU
St. Dev.	0.03 E	0.01 E	0.03 E	0.04 E	0.03 E	0.06 E

Topography and gradient data fit (mean model!)

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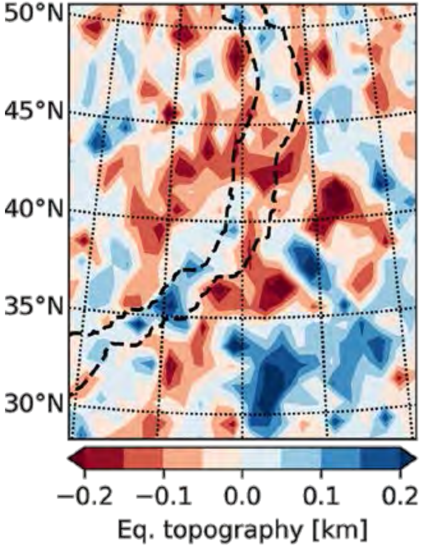
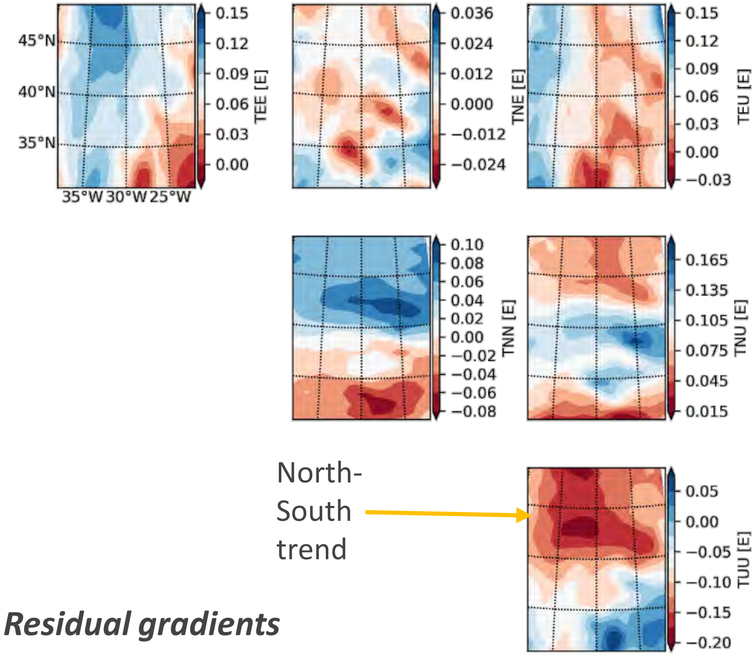


Residual
St. Dev: 0.09 km



Residual gradients

Component	EE	NE	EU	NN	NU	UU
St. Dev.	0.03 E	0.01 E	0.03 E	0.04 E	0.03 E	0.06 E

Spatial characteristics of gravity gradients

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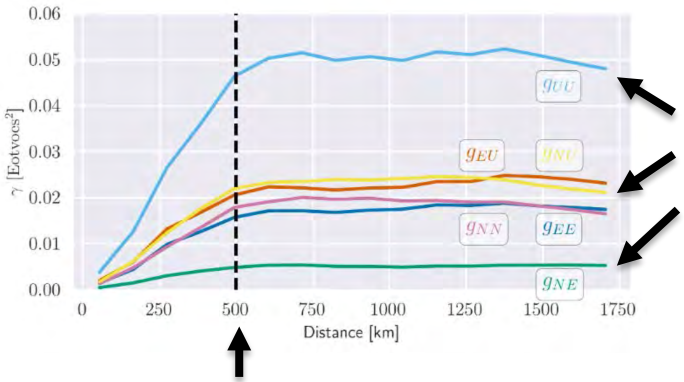
Semivariogram analysis of satellite free-air gravity gradients

$\gamma(h)$ = Average squared difference between points at a separation h



Plateau value corresponds to variance
-> Agrees with theoretical expectation

Distance where saturation reached = correlation distance

New misfit function

- Use analytical covariance function

$$C(h) = C_0 \exp(-\frac{h}{\rho})$$

- Determine covariance matrix between all pairs of point

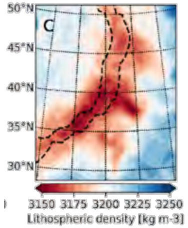
$$\Sigma_{ij} = C(h_{ij})$$

- Use multivariate normal distribution as misfit function

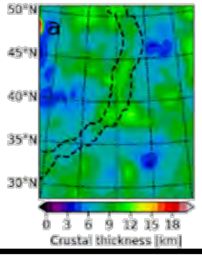
$$p(m|d) \propto |\Sigma|^{-\frac{1}{2}} e^{-\frac{1}{2}(d-G(m))^T \Sigma^{-1}(d-G(m))}$$

Capabilities and limitations

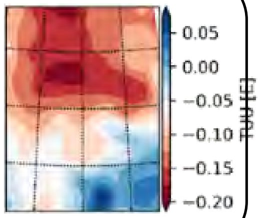
+ Expected lithospheric signal has been resolved



- Limited resolution in crust



+ Reasonable data fit

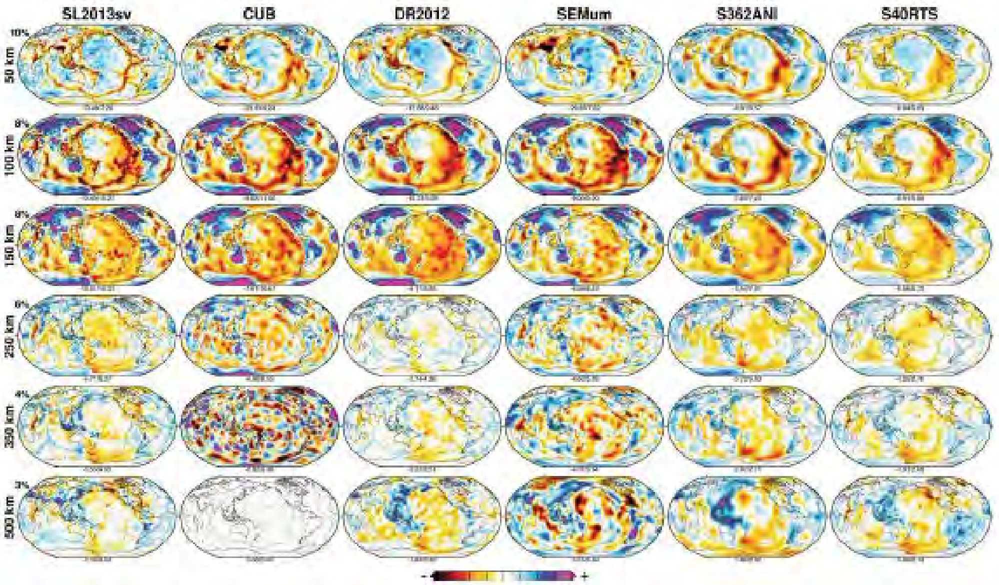


- Numerically inefficient

+ More observations are easy to include

+ More prior information is easy to add

Application to global inversion



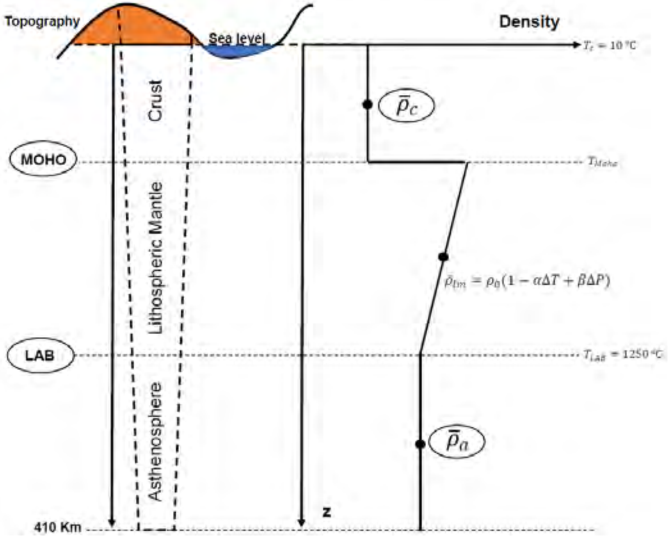
Joint inversion of satellite data and other geophysical information can provide a global reference/background model for applications from geodynamics to exploration

Can be used to evaluate seismic tomography and possibly reconcile observations

Initial Model

LITHO 1.0

The LITHO1.0 model is a 1° tessellated model of the crust and uppermost mantle of the earth, extending into the upper mantle to include the lithospheric lid and underlying asthenosphere.



Inversion set-up

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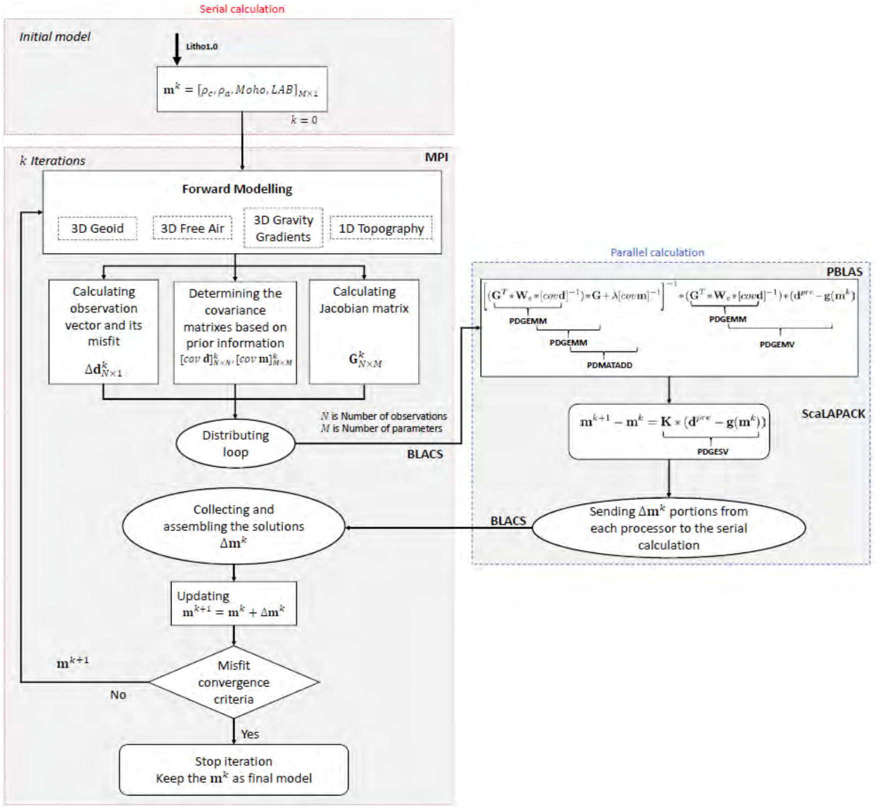
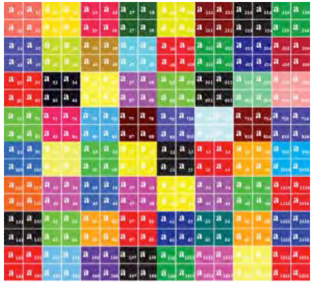
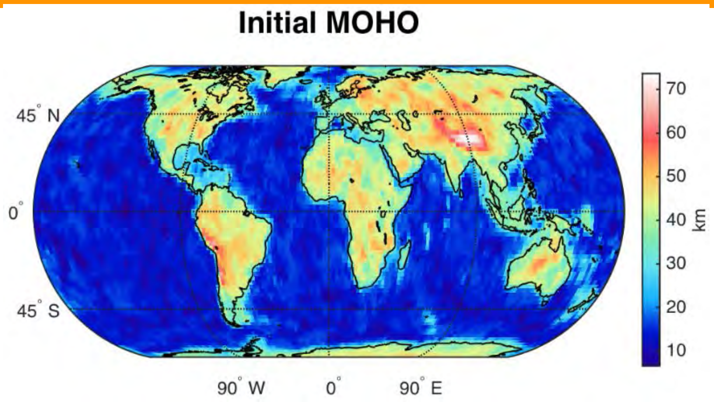
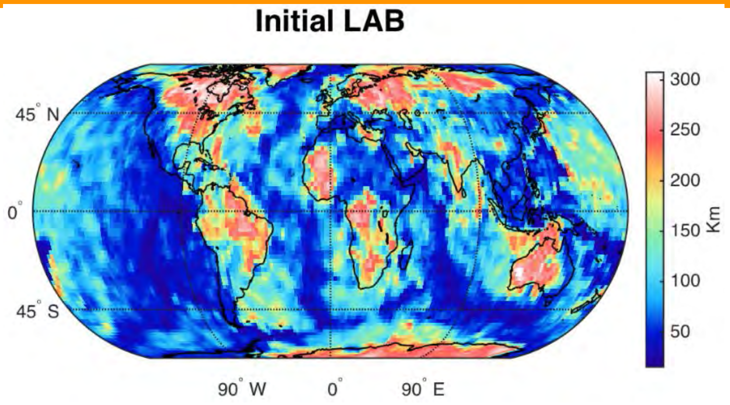
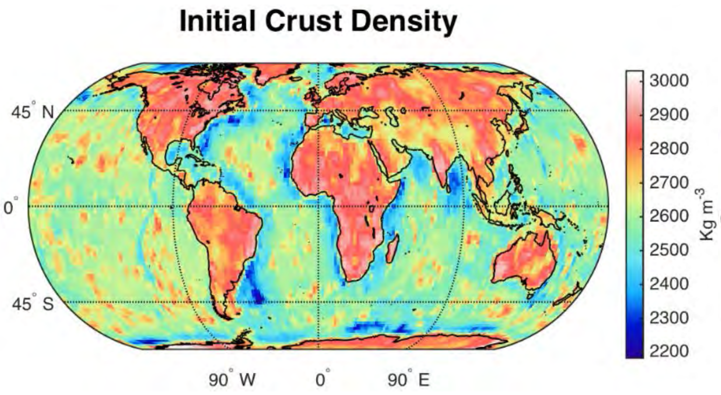


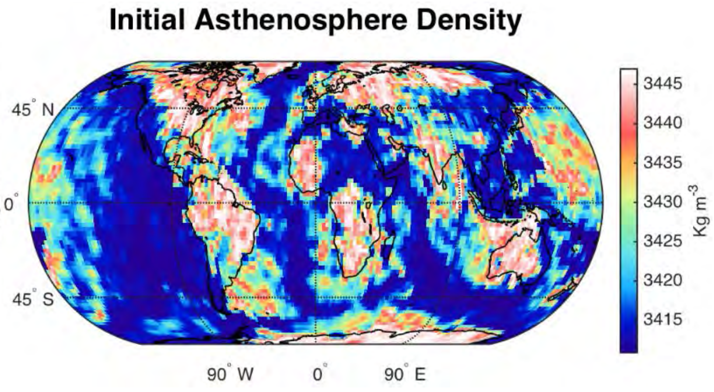


Initial Model









Fitting of Observations

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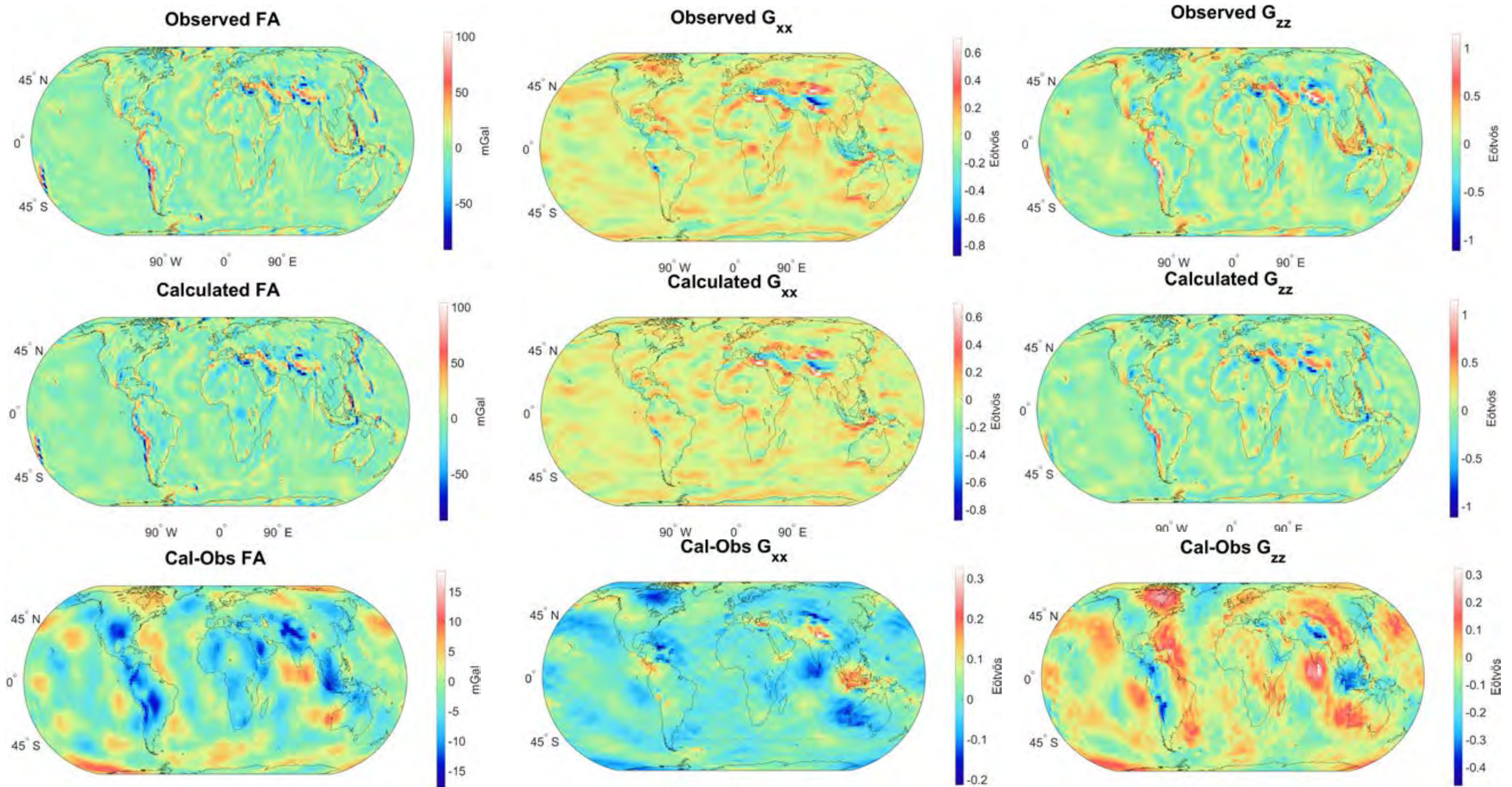

Estimating of Parameters

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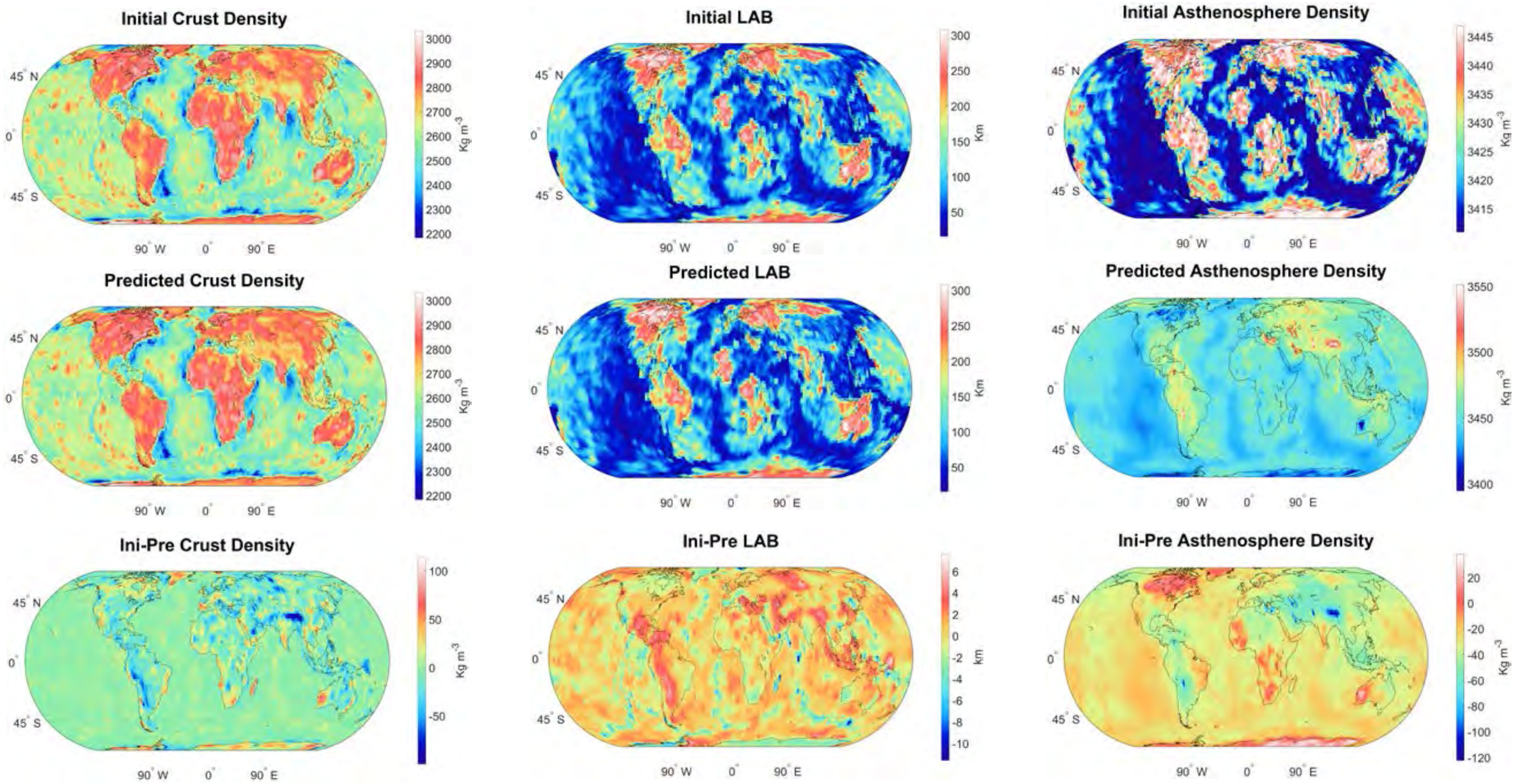
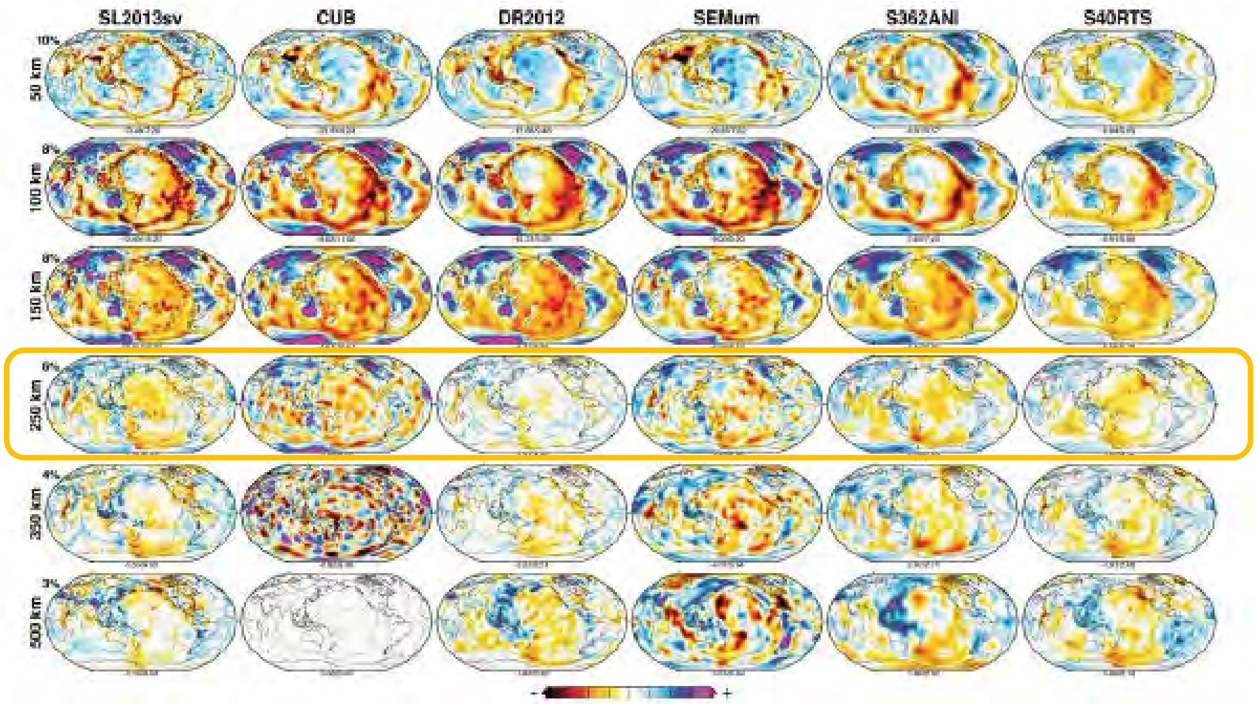
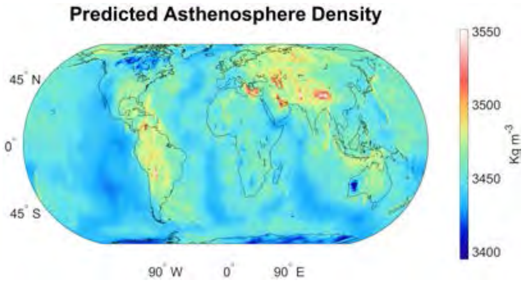


Application to global inversion





Similarities to tomography models -> comparison can provide temperature and composition

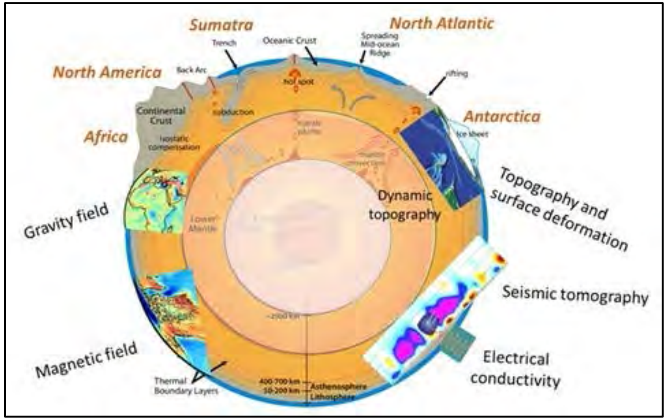
Summary

Satellite gravity data provide complementary information

Probabilistic inversion can exploit large suite of models
- Constraints can be implemented, e.g. seismic depths, data inter-dependencies
- But uncertainties should be included in inversion

Inversion results in density structure of the crust and upper mantle -> reflects temperature

Comparison of gravity inversion and seismic tomography can provide temperature and composition on a-> Application to Antarctica



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