# → EARTH OBSERVATION SUMMER SCHOOL

## Earth System Monitoring & Modelling

30 July–10 August 2018 | ESA–ESRIN | Frascati (Rome) Italy

# Satellite gradiometry for geophysical research

Jörg Ebbing

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# Satellite gradiometry for geophysical research

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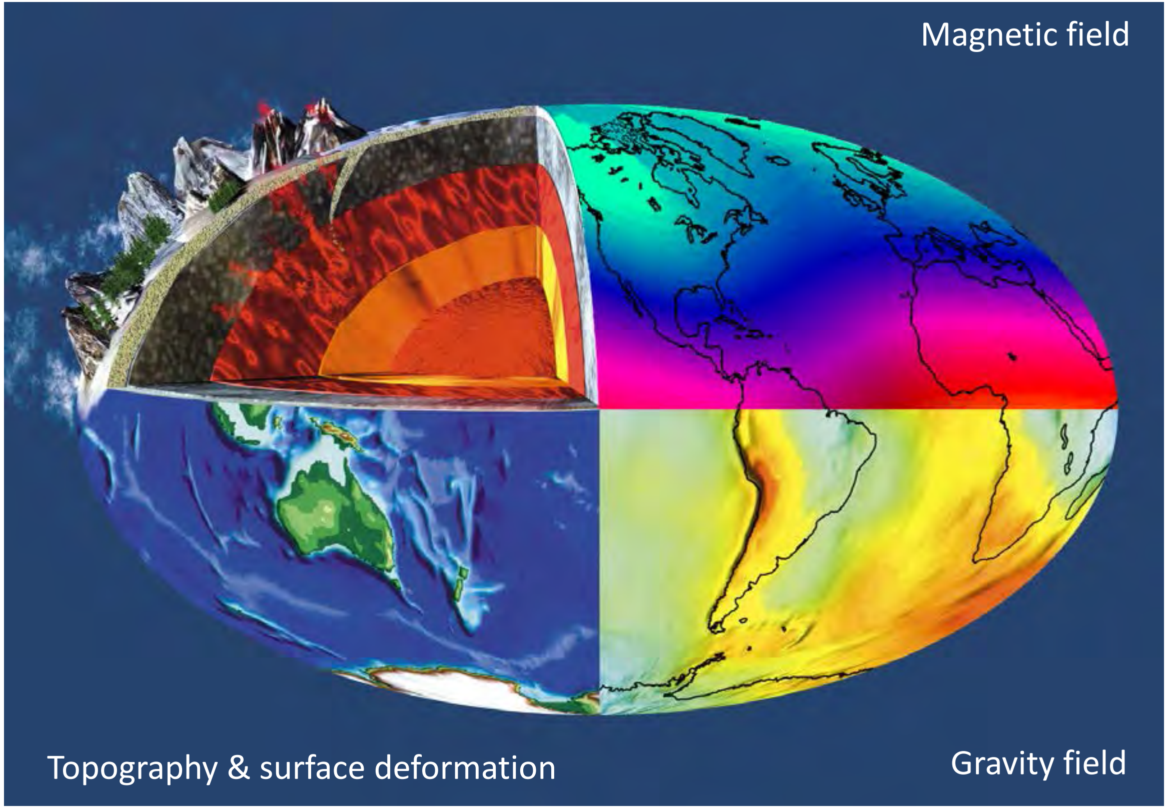
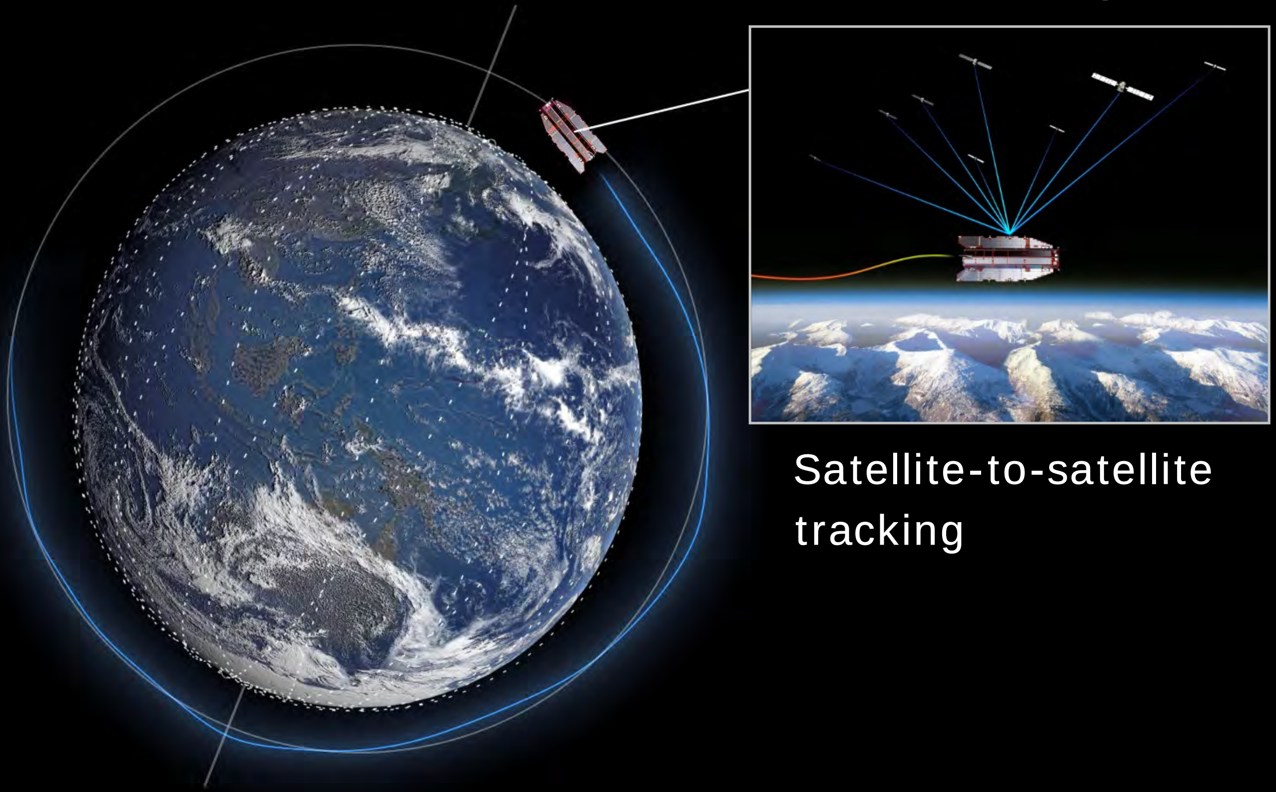
# Satellite data to study the dynamic (Solid) Earth

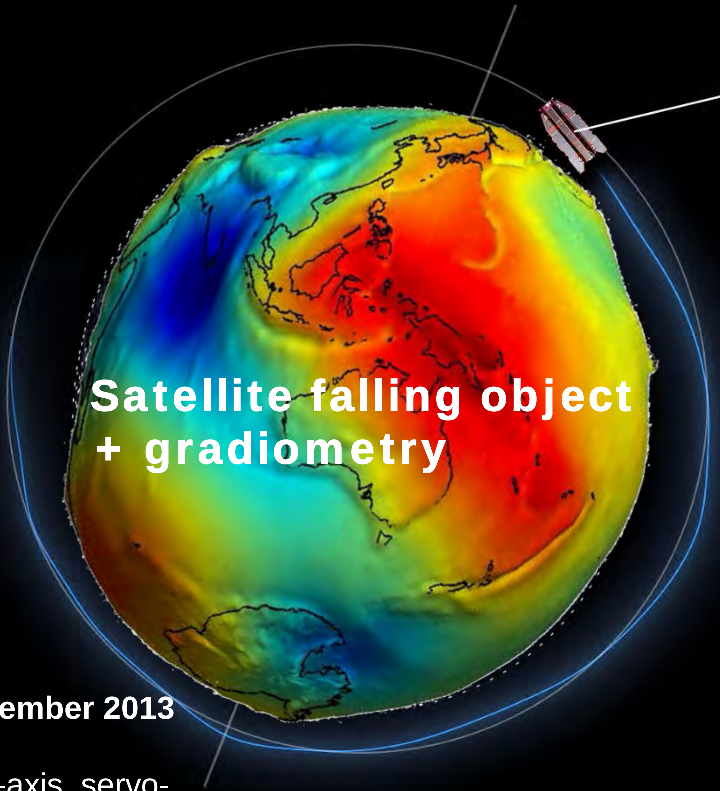


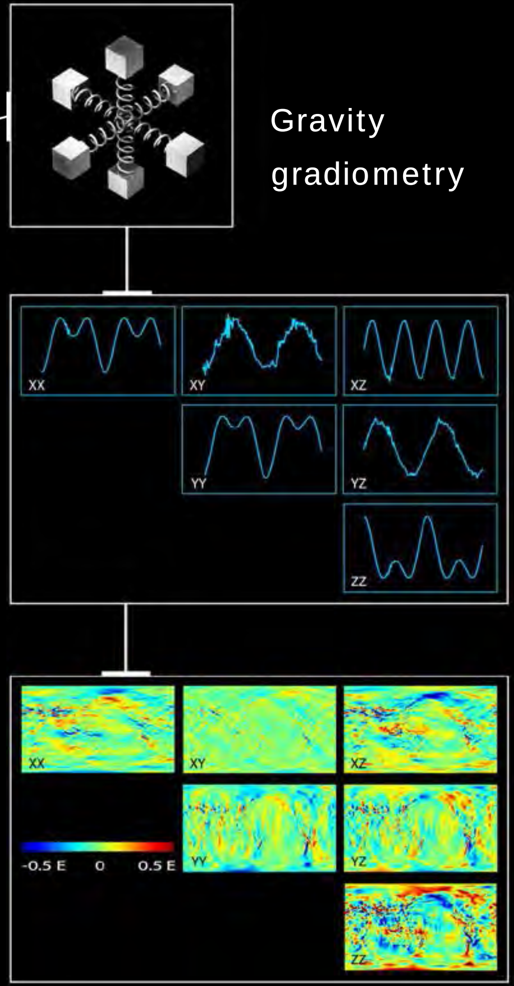

# Measurement concepts



Satellite-to-satellite tracking

# Measurement concepts




**Mission period**
**17 March 2009 – 11 November 2013**

Gradiometer; 3 pairs of 3-axis, servo-controlled, capacitive accelerometers (each pair separated by a distance of about 0.5 m).

0.5 gram

Super-tanker acceleration
due to attracting snowflake:

$0.000000000005\ m/s^2$

smallest acceleration measurable
in space by GOCE

1 000 000 metric tonnes

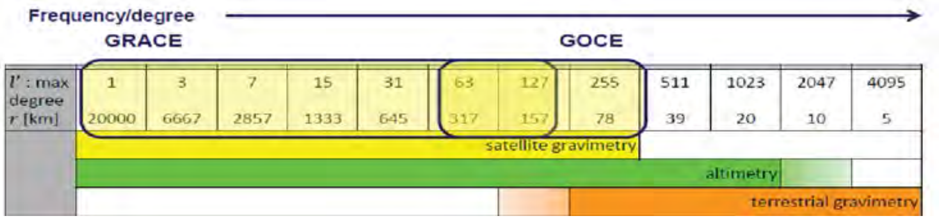
# GOCE gravity gradient data for lithospheric modeling and geophysical exploration

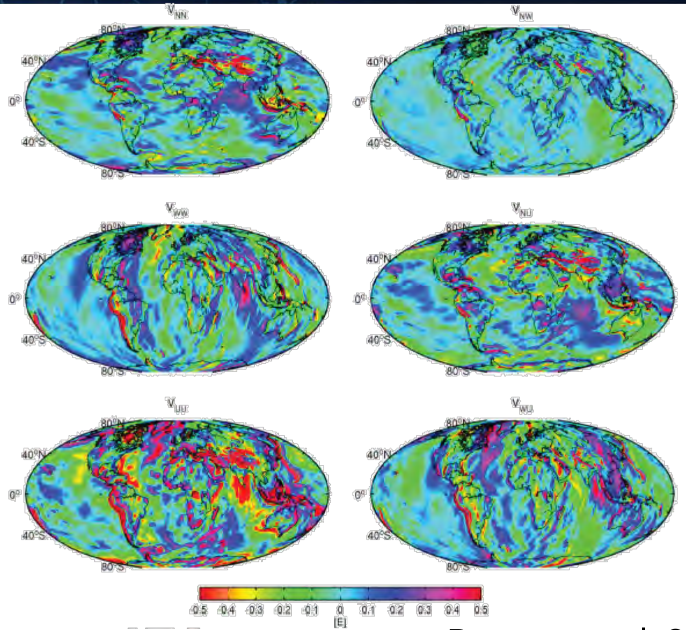
Data sets from the GOCE mission have two main advantages compared with earlier global gravity models:

(1) The GOCE gravity model has higher resolution in the transitional wavelength between earlier satellite and terrestrial gravity data. Only based on GOCE gravity data, it would be feasible to provide a gravity field with 80 km resolution

(2) The second and more revolutionary novelty is that GOCE measures gravity gradients.

⇒ How useful for geophysical research and exploration?

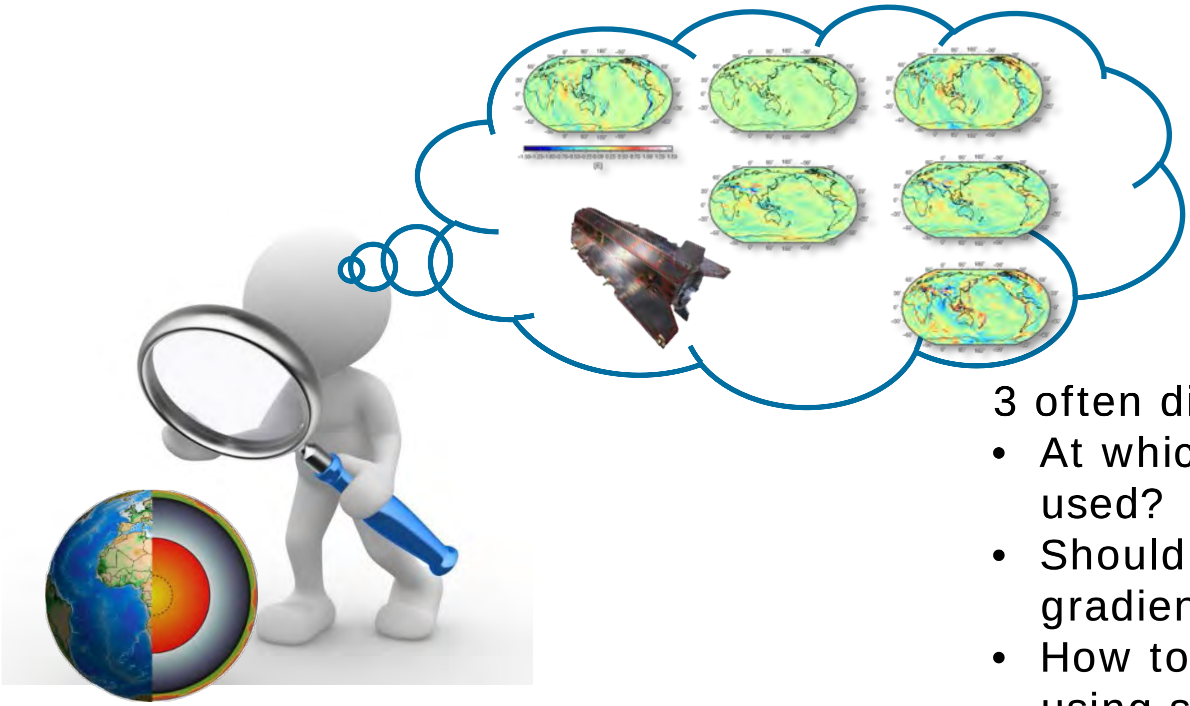



Bouman et al. 2016

Grids can be downloaded from http://eo-virtual-archive1.esa.int/GOCEGradients.html

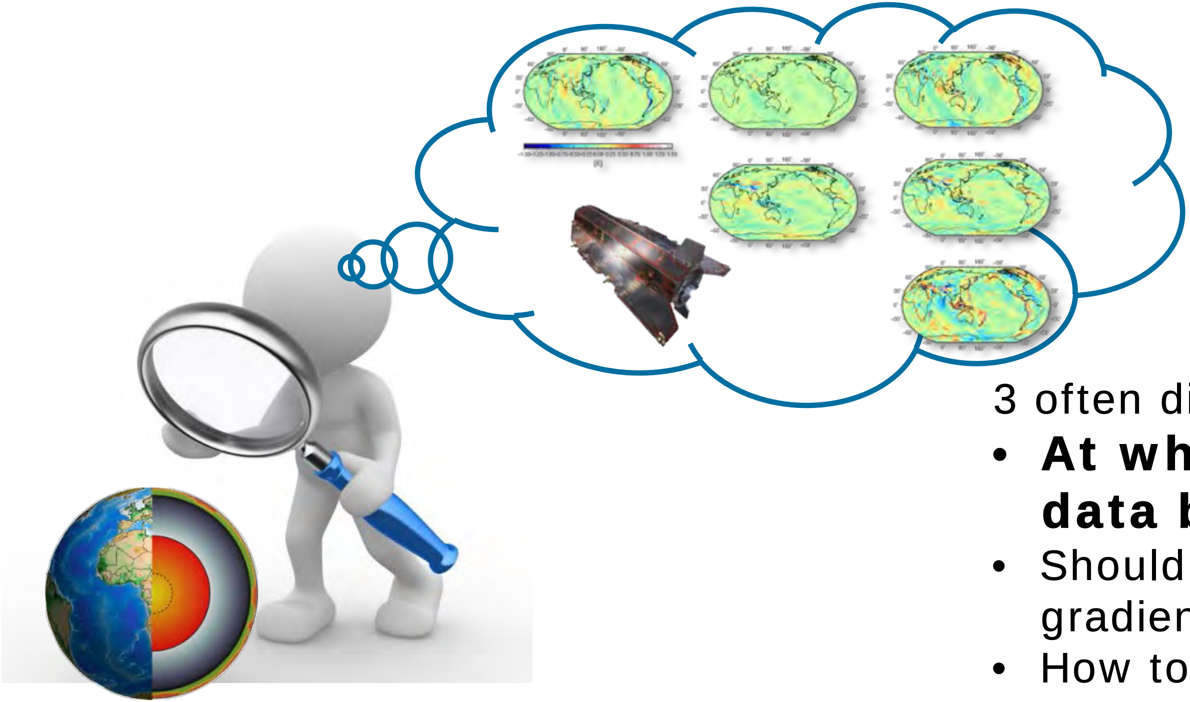
# Enhanced satellite gravity (gradient) imaging of Earth?



3 often discussed questions:

- At which altitude should the data be used?
- Should one use gravity or gravity gradients?
- How to represent the Earth when using satellite data?

# Enhanced satellite gravity (gradient) imaging of Earth?



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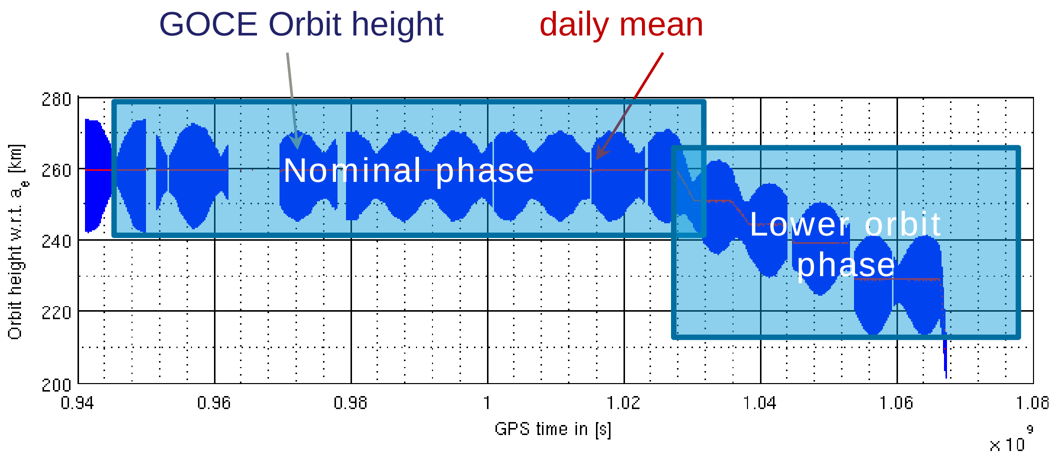
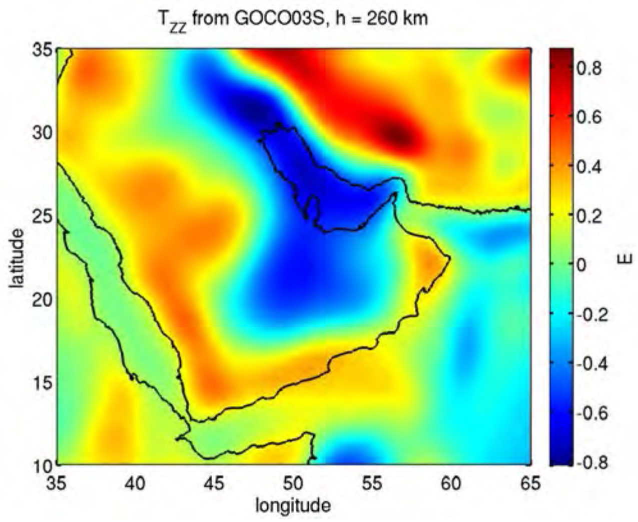
# GOCE orbit heights

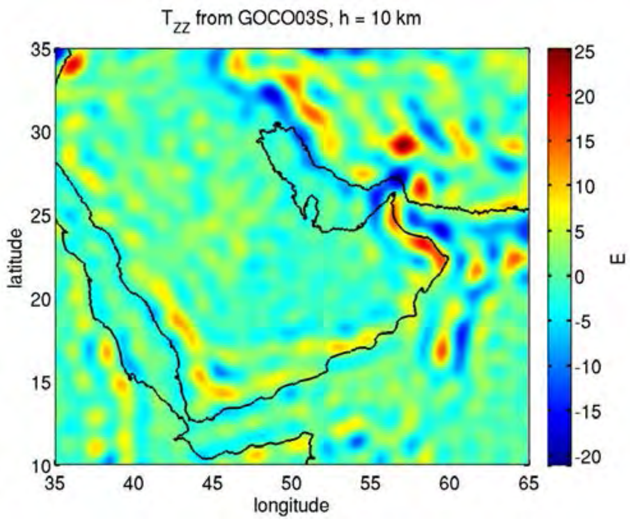



- Four times orbit lowering
- 30 km Orbit height lowering accounts for approx. 30% signal increase (d/o > 200)
- Nominal phase had perigee height of 255 km
- Lower orbit phase had perigee height of 225 km

# GOCE data @ satellite altitude / Earth's surface

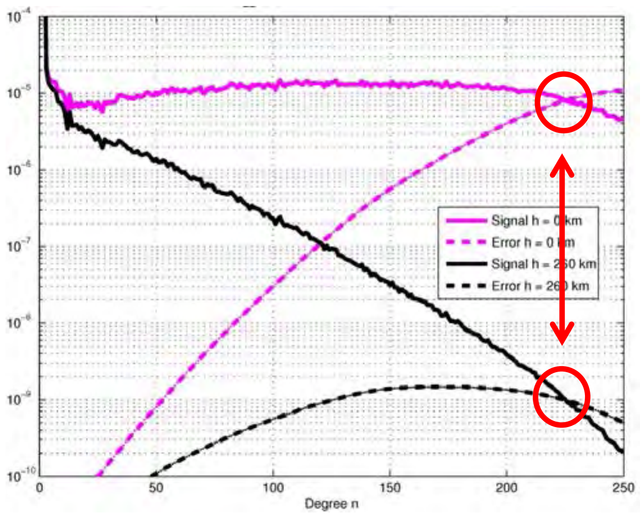
- Signal @ satellite altitude is smooth
- Downward continuation enhances signal power & details



# GOCE data @ satellite altitude / Earth’s surface

$V_{ZZ}$ degree RMS h = 0 & 260 km



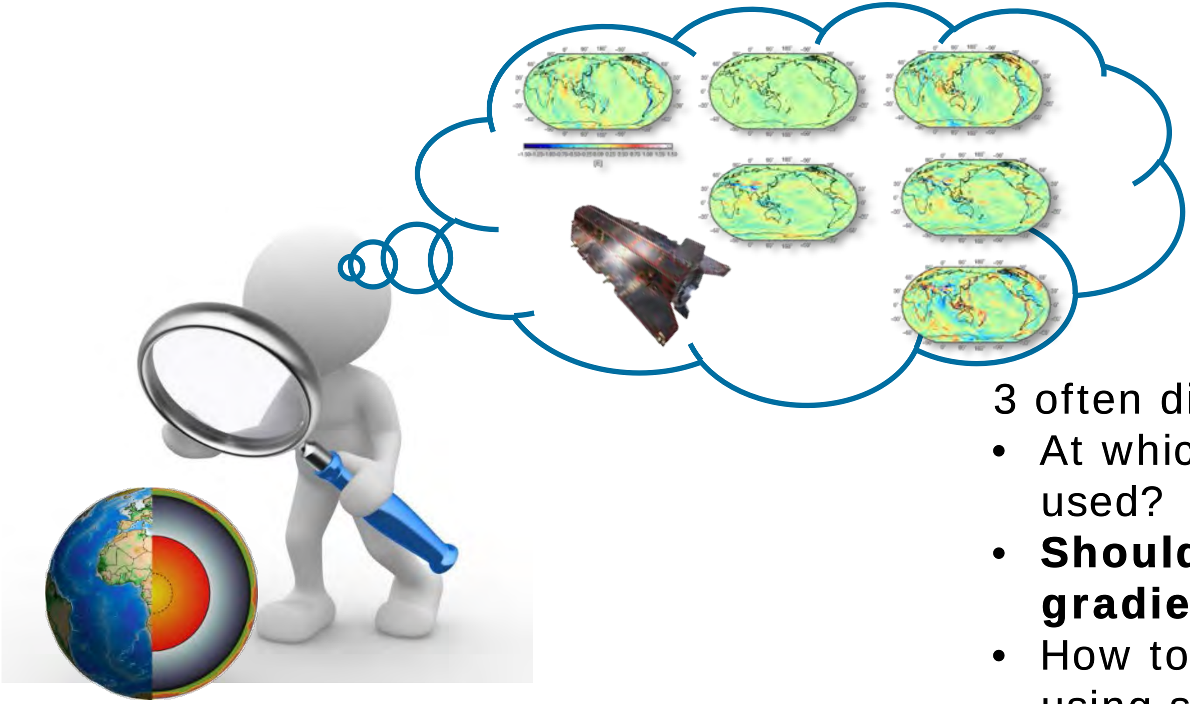

GOCO03S $V_{ZZ}$ signal & error, L = 225

| Saudi Arabia | Height | | |
|---|---|---|---|
| | 0 km | 10 km | 260 km |
| Signal RMS | 5.1 E | 4.1 E | 0.3 E |
| Model error | 1.3 E | 0.9 E | 0.4 mE |

- Downward continuation also amplifies noise
- Effective resolution of data does not change
- Omission error becomes much larger (mainly high frequency topography)

For model inversion it is probably best to use data close to their original point of acquisition.

# Enhanced satellite gravity (gradient) imaging of Earth?



3 often discussed questions:

- At which altitude should the data be used?
- **Should one use gravity or gravity gradients?**
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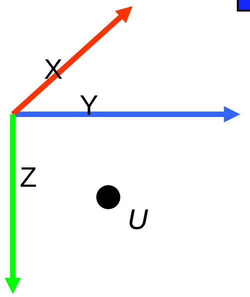
# Scalar potential, vector acceleration, tensor gradient



**scalar field potential**

*U(x,y,z)*



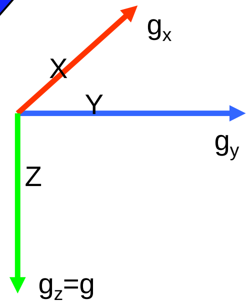

spatial derivative

**vector field acceleration**

$$\frac{\partial U}{\partial x} = g_x$$

$$\frac{\partial U}{\partial y} = g_y$$

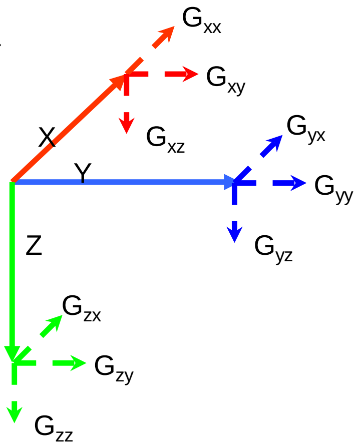
$$\frac{\partial U}{\partial z} = g_z$$




spatial derivative

**tensor field gradient**

$$\begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix}$$




People also use U or T or G to denote the gravity and gradient components.
Coordinate systems might be (X, Y, Z) or (N, W, U) as shown in subscripts.

# Gravity gradient is the derivative of gravity

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**Gradient: change in gravity components along three axial directions.**

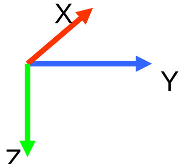
$$G_{xx} = \frac{\partial}{\partial x} g_x = \frac{\partial^2}{\partial x \partial x} U$$

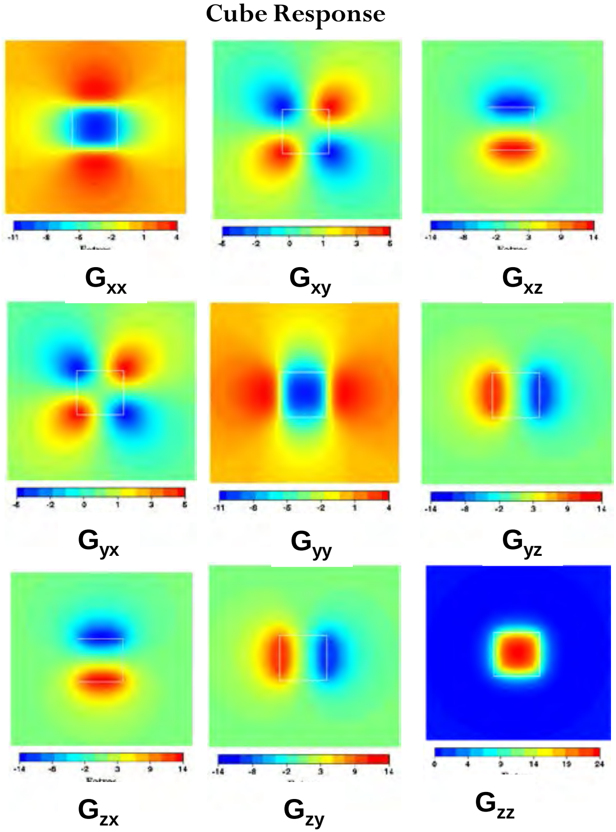
$$G_{xy} = \frac{\partial}{\partial y} g_x = \frac{\partial^2}{\partial x \partial y} U$$

**… and so on**

**Derivative**

| | x | y | z |
|---|---|---|---|
| **Component** x | $G_{xx}$ | $G_{xy}$ | $G_{xz}$ |
| y | $G_{yx}$ | $G_{yy}$ | $G_{yz}$ |
| z | $G_{zx}$ | $G_{zy}$ | $G_{zz}$ |



# Symmetry

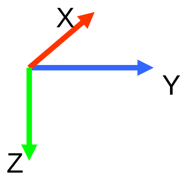


- The order of derivative does not matter:

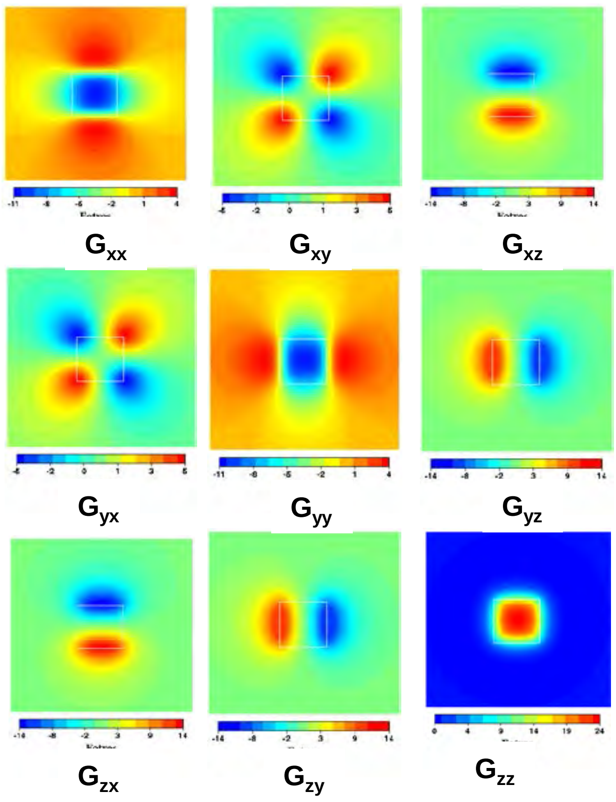
$$G_{xy} = \frac{\partial^2 U}{\partial y \partial x} = \frac{\partial^2 U}{\partial x \partial y} = G_{yx}$$

$$G_{xz} = G_{zx}$$

$$G_{yz} = G_{zy}$$




Cube Response



# Laplace Equation → Trace of the tensor sums to zero

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- Laplace Equation is valid in source free space:

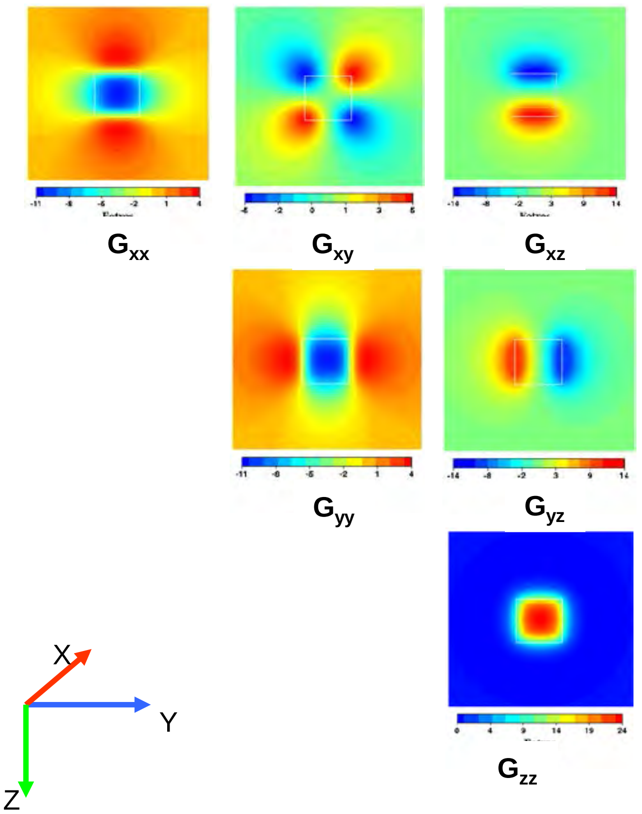
$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$$

$$\Updownarrow$$

$$G_{xx} + G_{yy} + G_{zz} = 0$$

- We can measure 6 gravity gradients, but only 5 are independent

Cube Response



# Gravity vs. gradients

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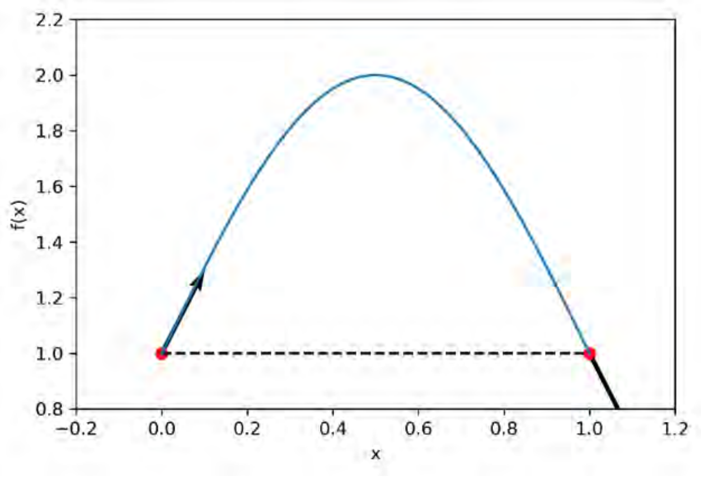
„The gravity gradients contain no additional information because they are all derivatives of the potential“

???

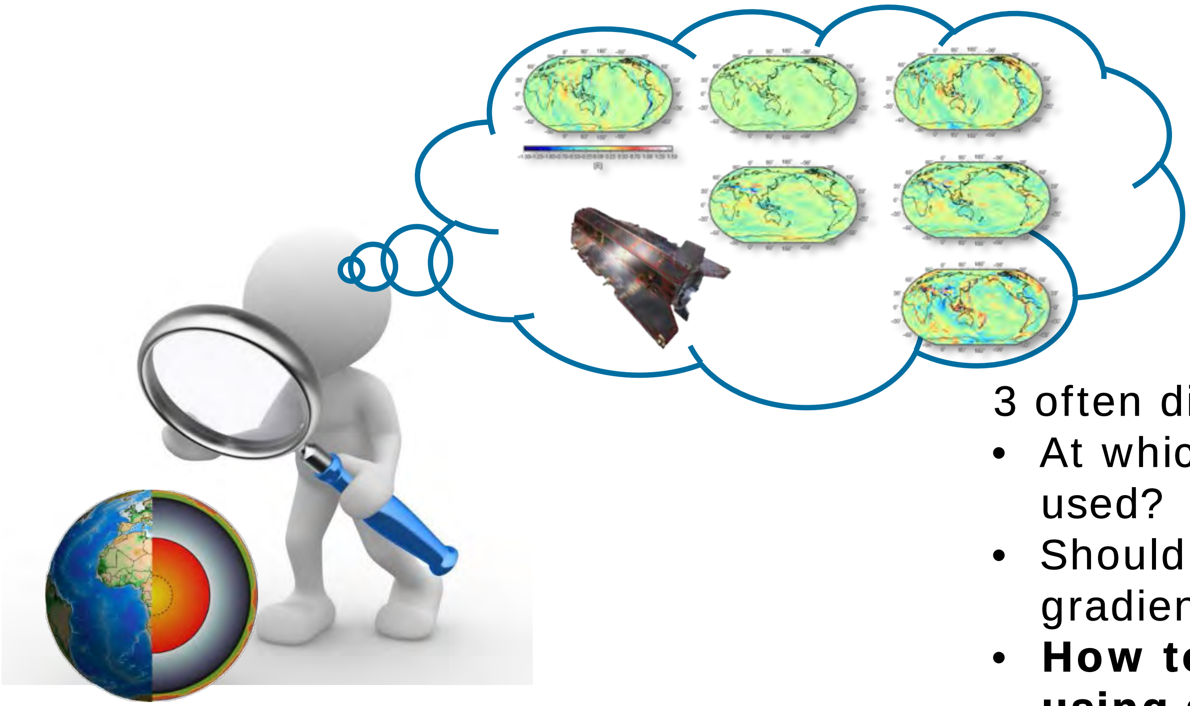
„The gravity gradients might contain additional information ***even though*** they are all derivatives of the potential“

„If $f(x_i)$ is known at $N$ discrete locations $x_i$, then no new information can be gained from knowing the derivative $f'(x_i)$.“

Counter example: Interpolation



# Enhanced satellite gravity (gradient) imaging of Earth?



3 often discussed questions:

- At which altitude should the data be used?
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- **How to represent the Earth when using satellite data?**

# GOCE satellite gradients for lithospheric modelling: the well-known NE Atlantic area

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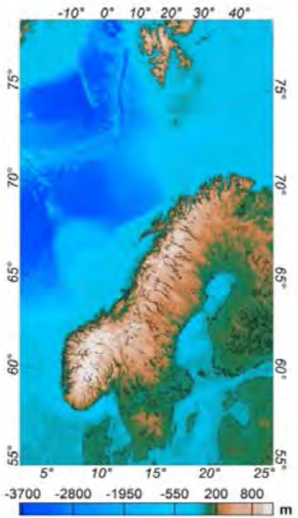
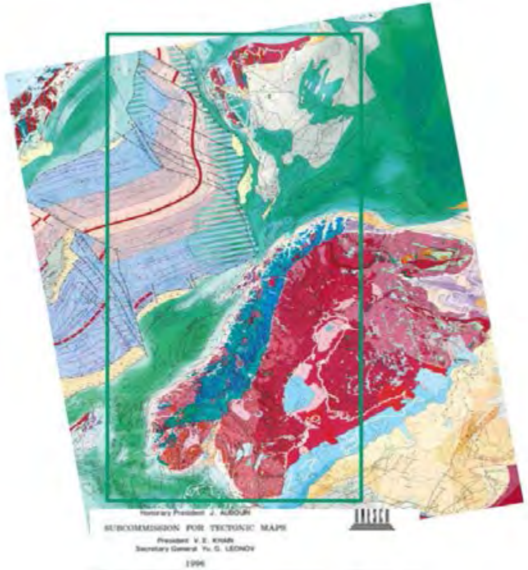
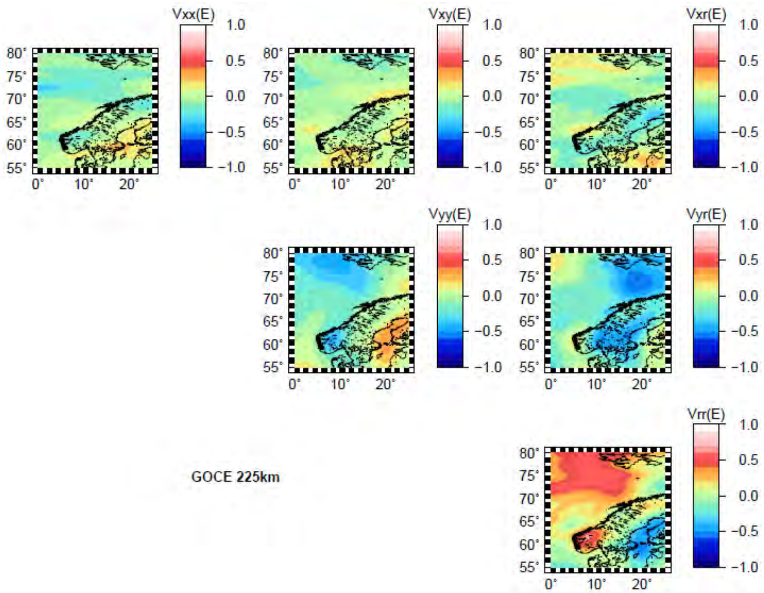


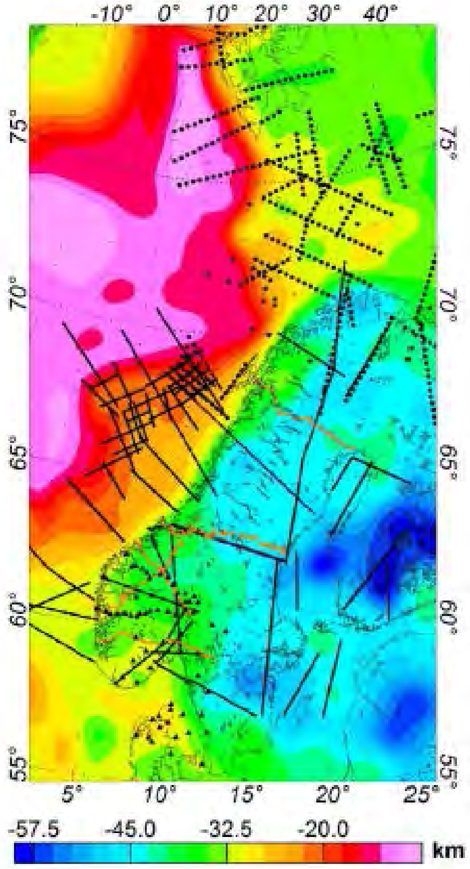



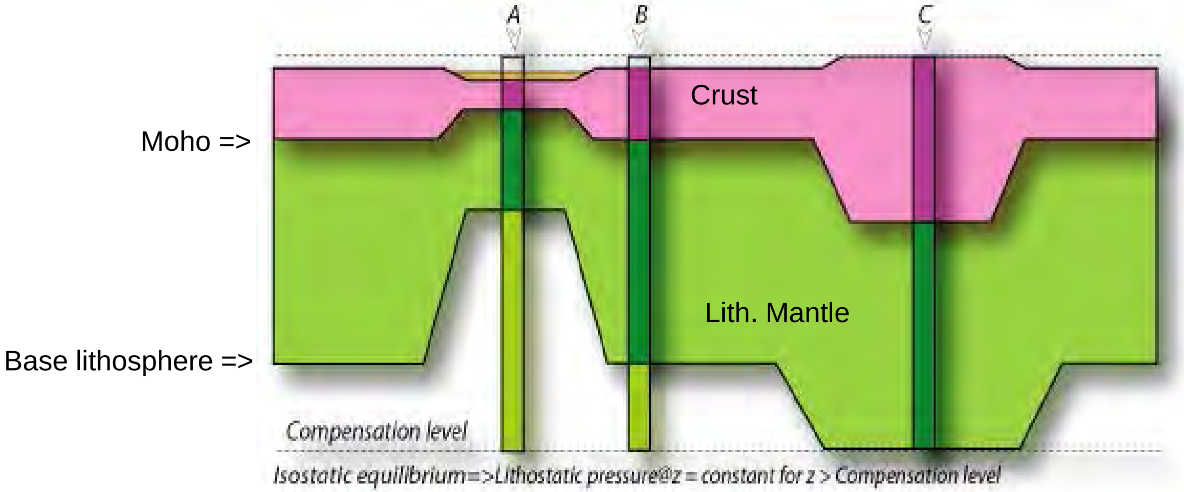


http://goce4interior.dgfi.badw.de/

# Importance of Moho depth and lithosphere






***Main density contrast at Moho, not base lithosphere***

# Flat Earth vs. Spherical calculations

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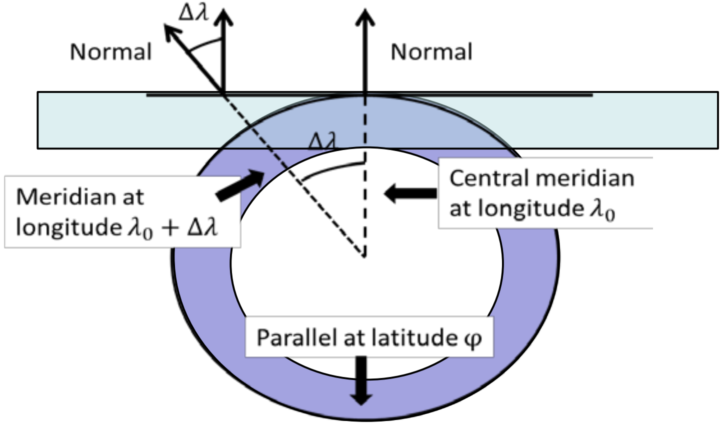



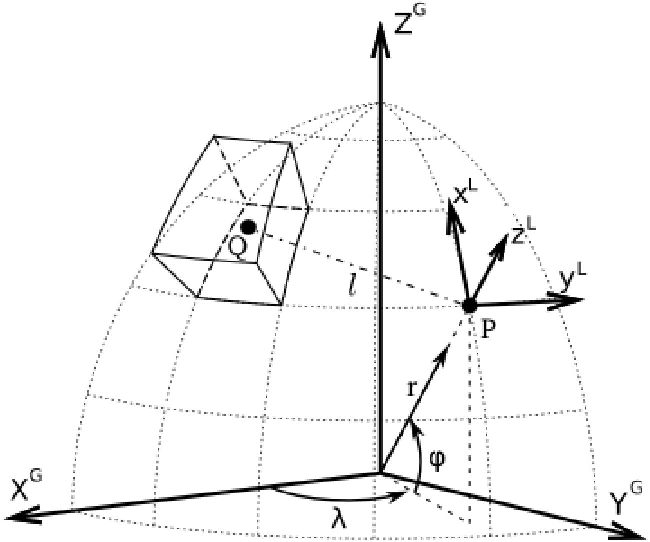


**Tesseroids v1.1**

http://leouieda.github.io/tesseroids/

# Forward modelled effect of Moho depth

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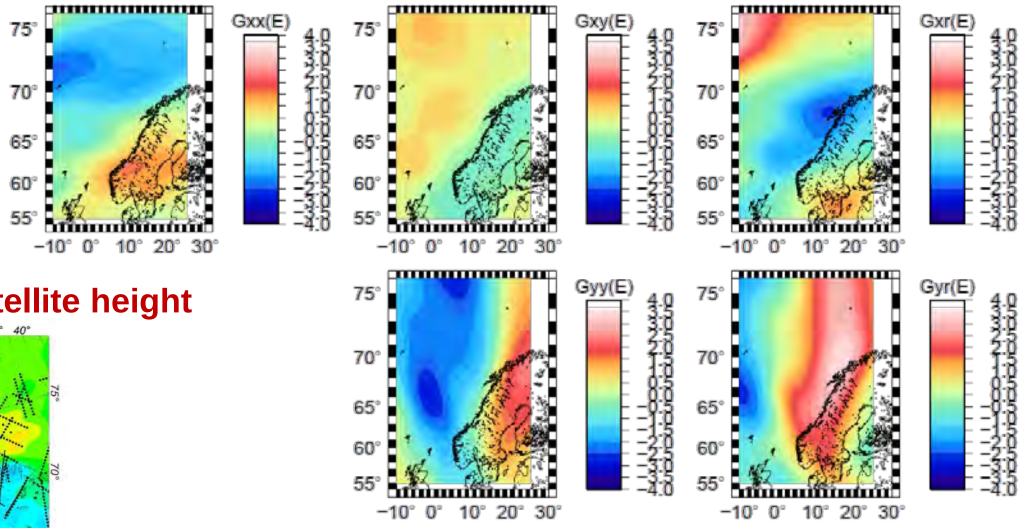
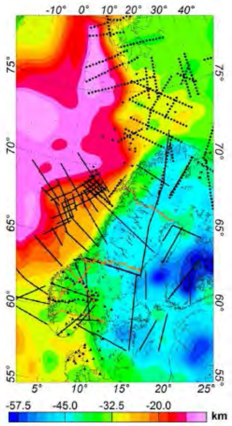
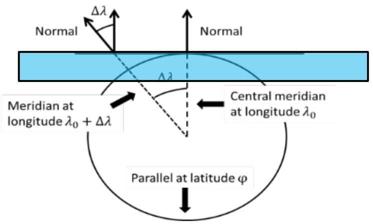
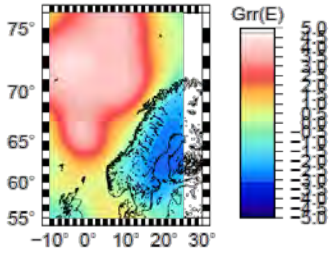



**Satellite height**



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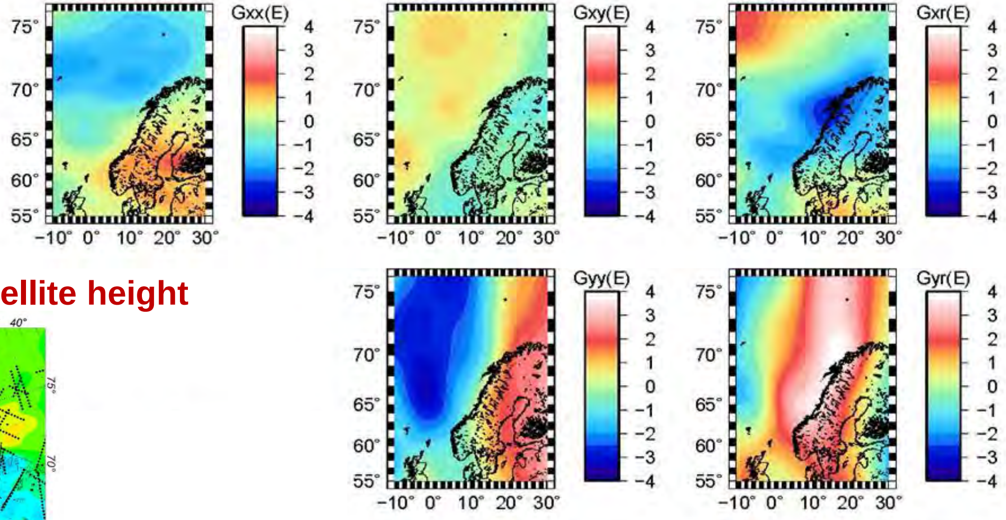


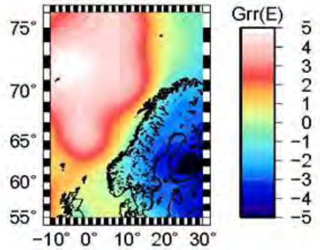


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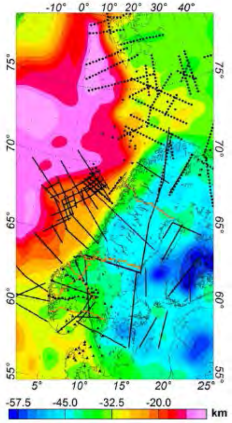
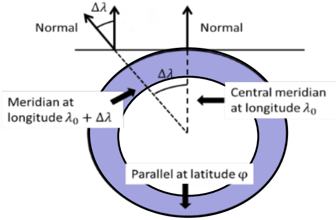
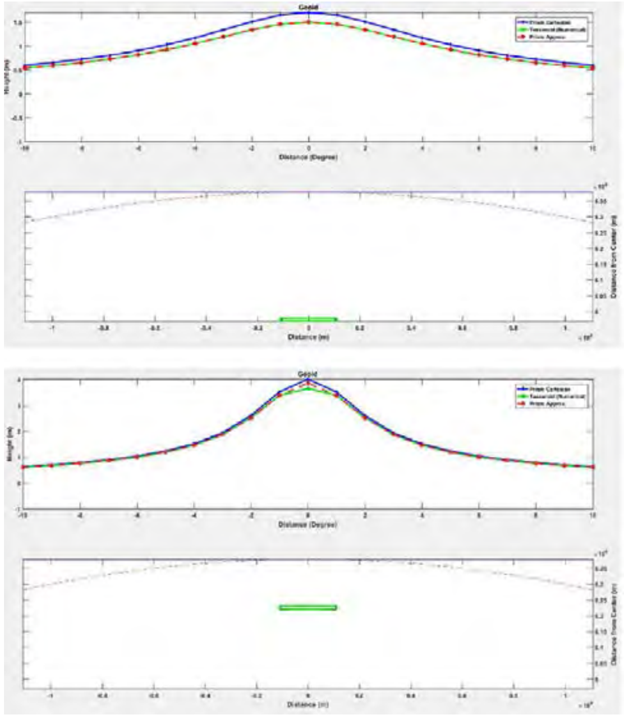


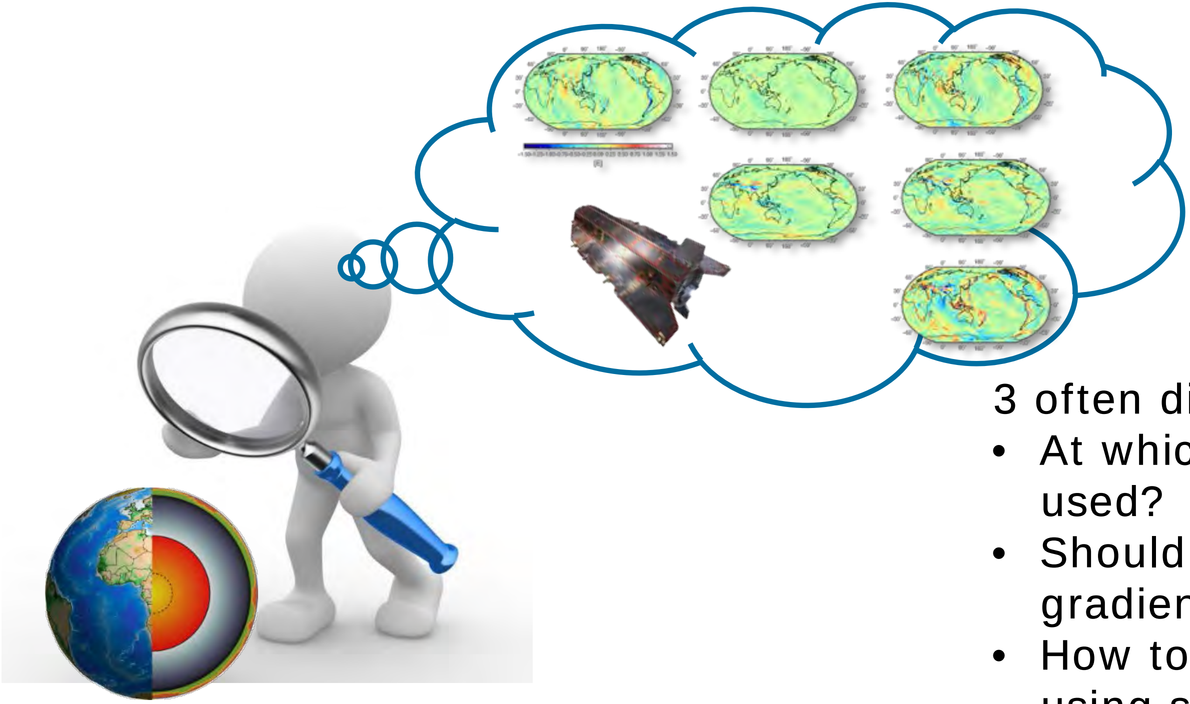
# Spherical vs. planar Earth






Distance matters:
The further away the measurement point is from the source, the more the lateral extension of sources has to be considered.

# Enhanced satellite gravity (gradient) imaging of Earth?



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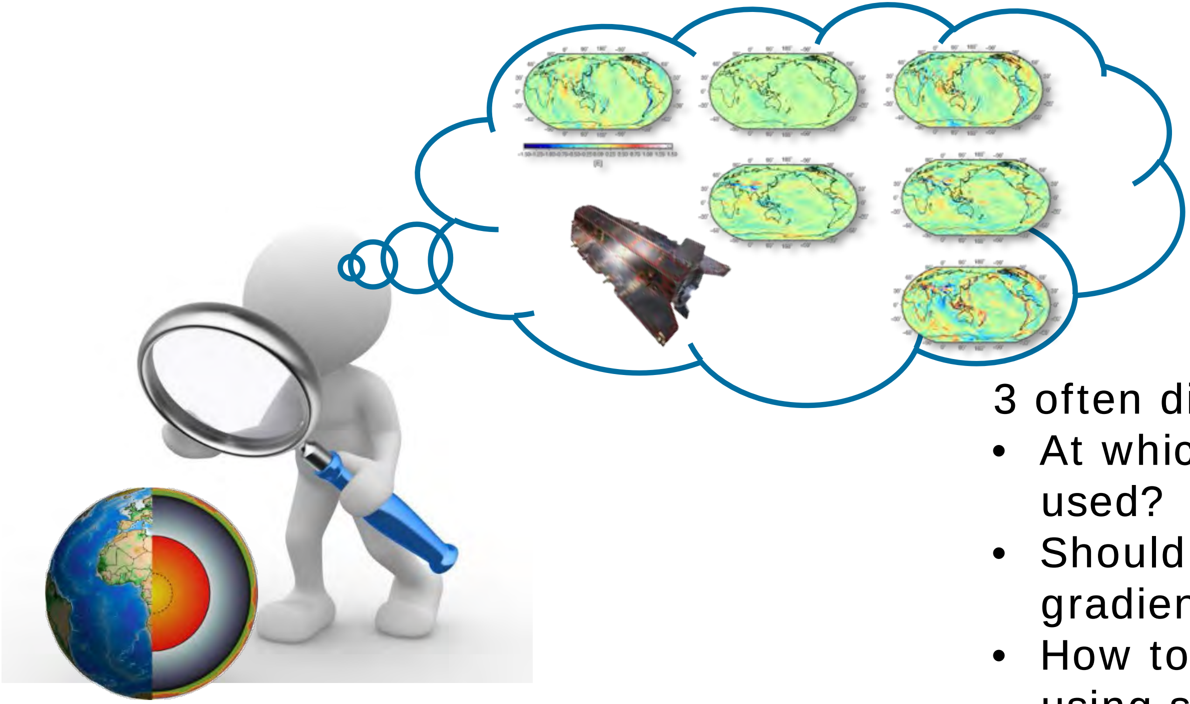

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# Model set-up to study sensitivity of gradient information

C A U
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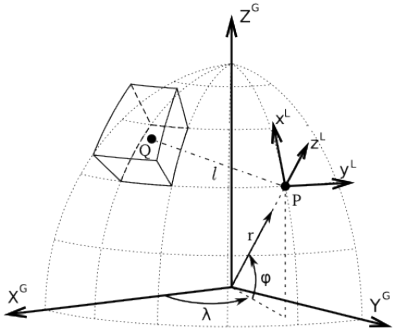
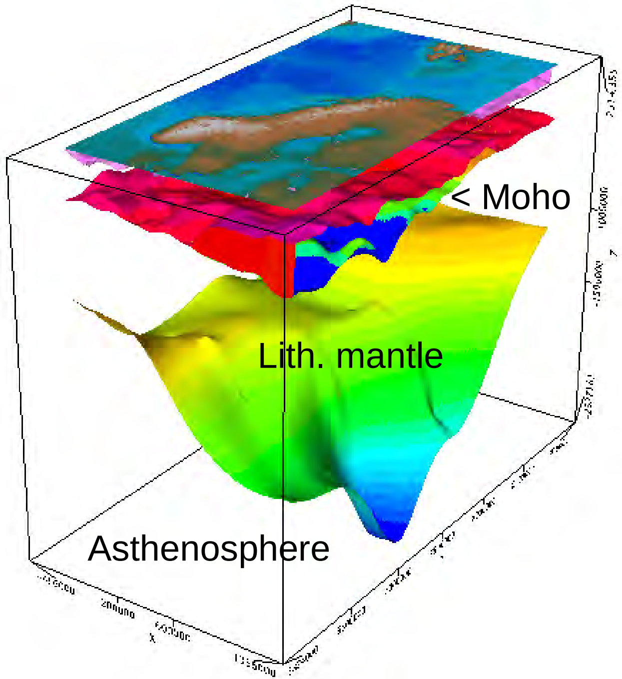
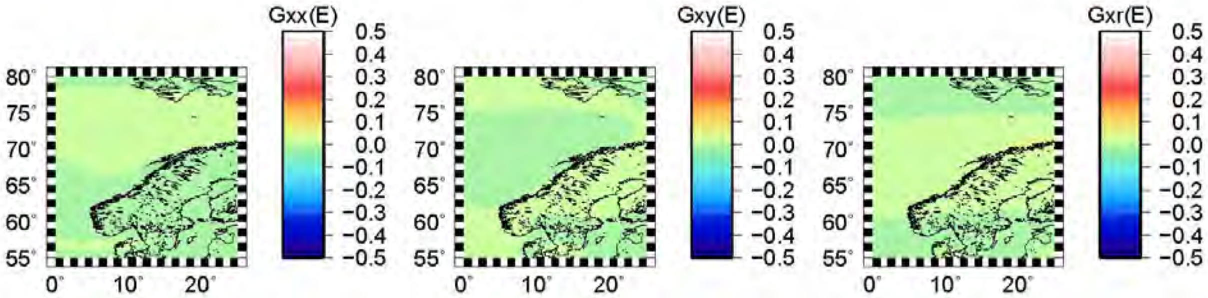
- Model geometry in tesseroids
- 25 km lateral resolution
- Vertical resolution variable as seen in depth slices following

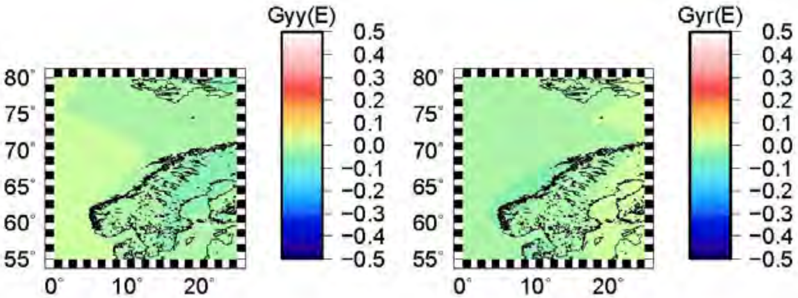


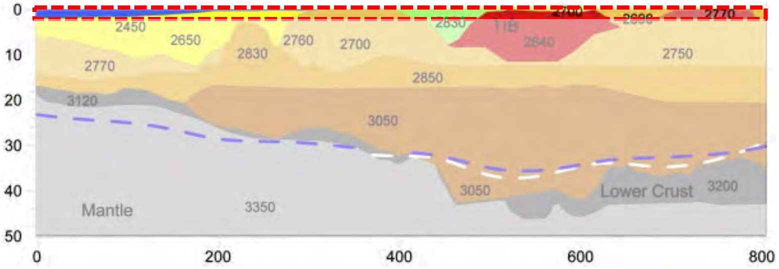



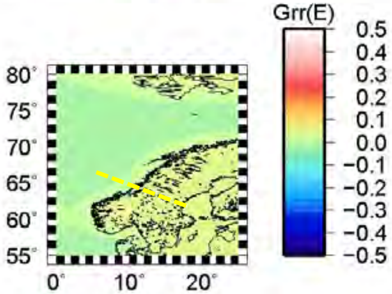

Model Name: 3D_NEAtlantic.g3d
Global Information
Vertical Density Function
Gravity
Magnetic
Layer 1: .\3DMod_Topography10.grd(GRD)
Density: 2.67
Susc: Constant
Layer 2: .\Null10.grd(GRD)
Density: 1.03
Susc: Constant
Layer 3: .\3DMod_Bathymetry10.grd(GRD)
Density: Vertical Density Function
Susc: Constant
Layer 4: .\BaseSed_NGU_NOAA_Laske.grd
Density: .\D_UC1.grd
Susc: Constant
Layer 5: .\UCMC.grd
Density: 2.8
Susc: Constant
Layer 6: .\UCLC.grd
Density: 2.95
Susc: Constant
Layer 7: .\3DMod_IsoTopLCBClipped.grd(GRD)
Density: 3.1
Susc: Constant
Layer 8: .\3DMod_Moho_Grad.grd
Density: .\MantleDensityGP250km.grd
Susc: Constant
Layer 9: .\LAB_Artemieva2.grd
Density: 3.3

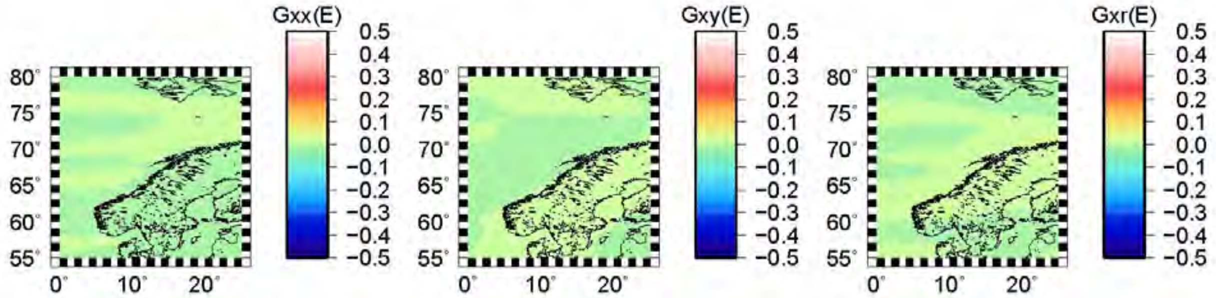


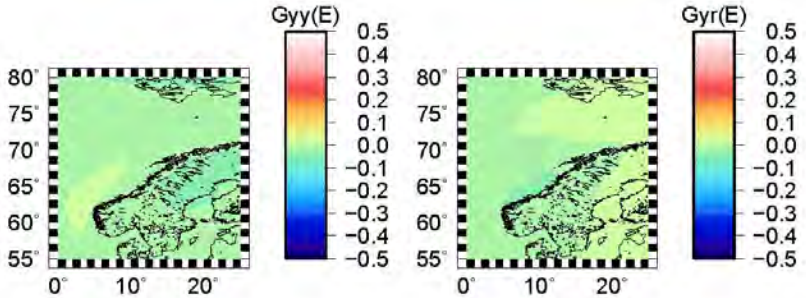

0-2.5 km depth
@225km height



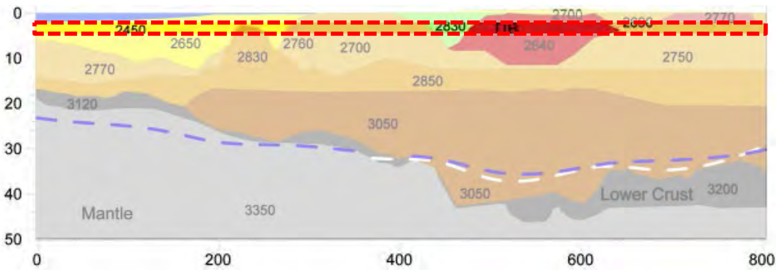



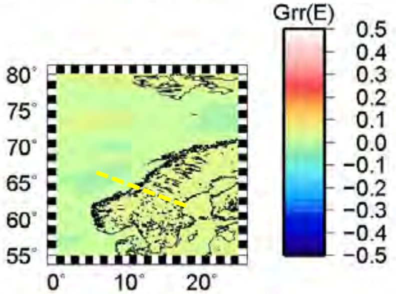



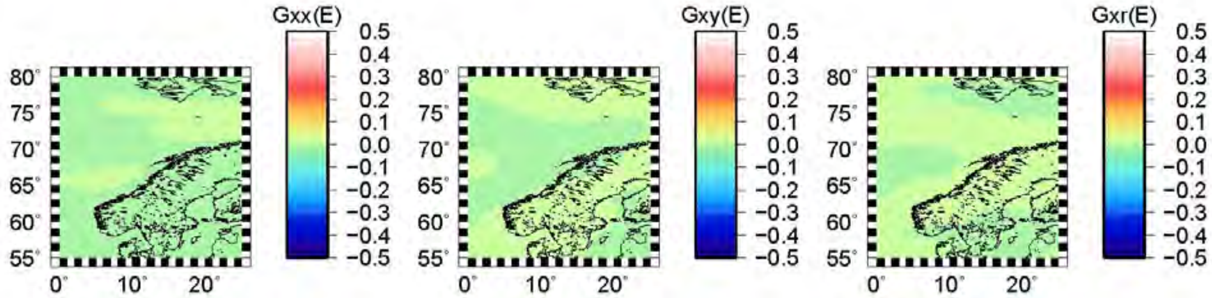


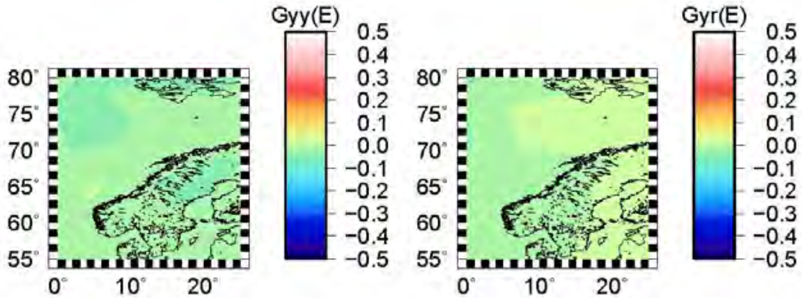




2.5-5 km depth
@225km height

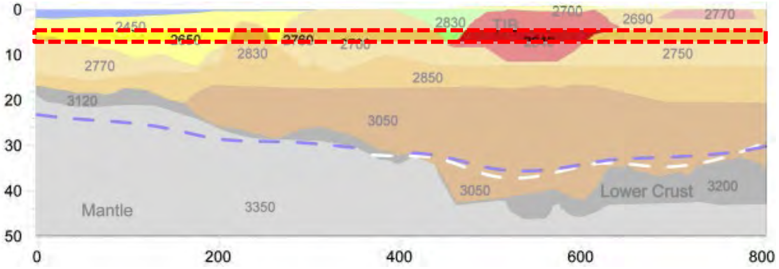


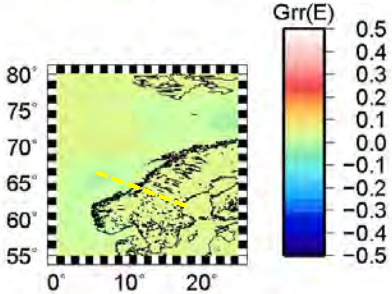



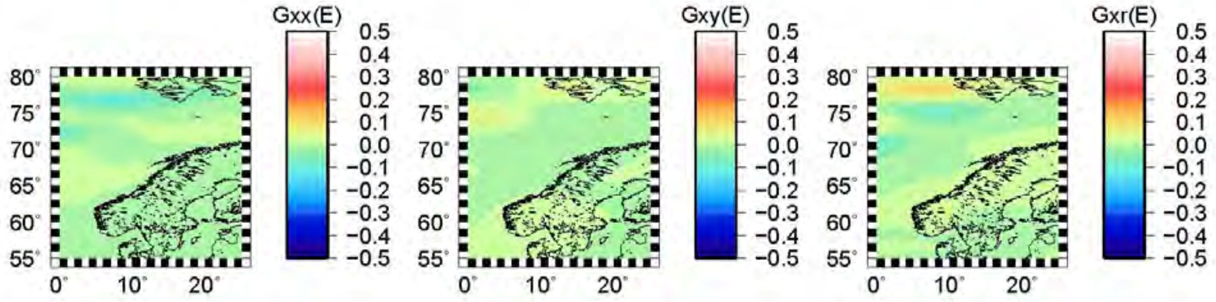


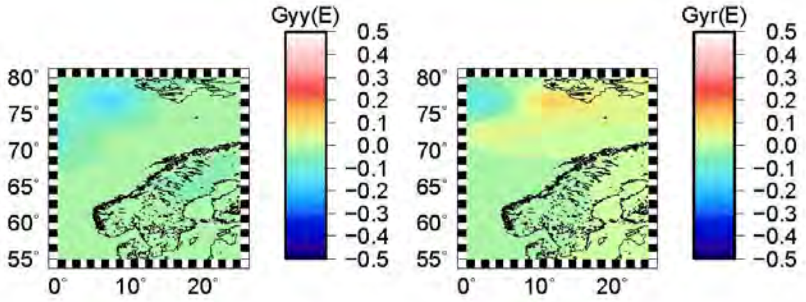




5-7.5 km depth
@225km height






7.5-10 km depth
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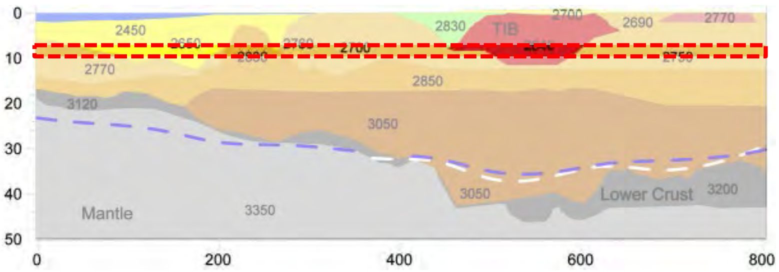
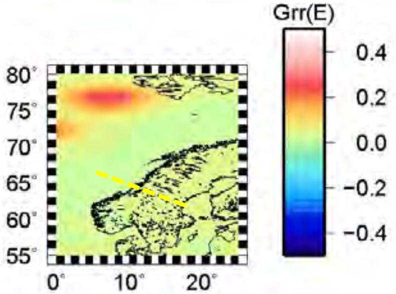
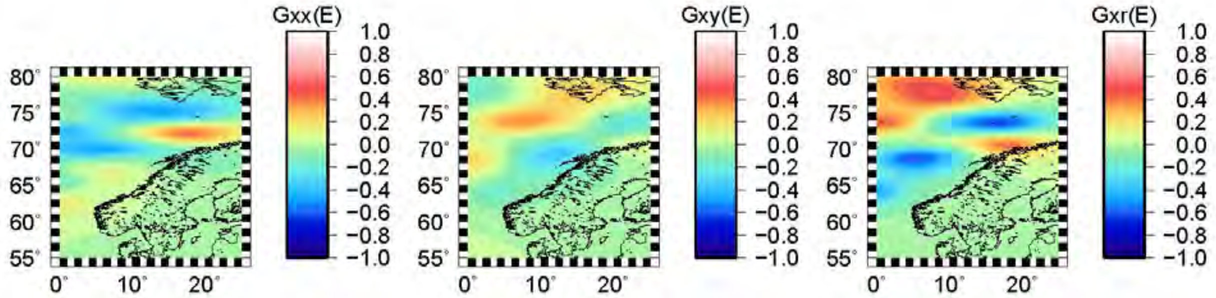
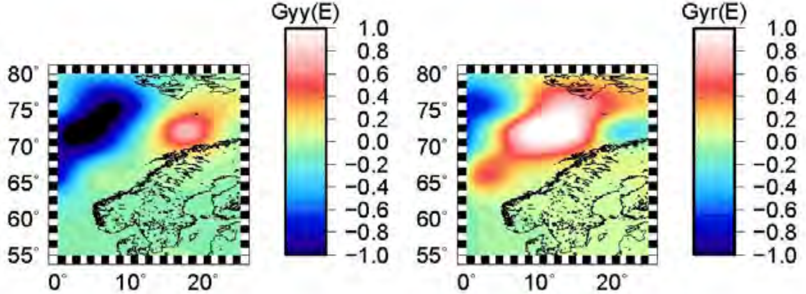
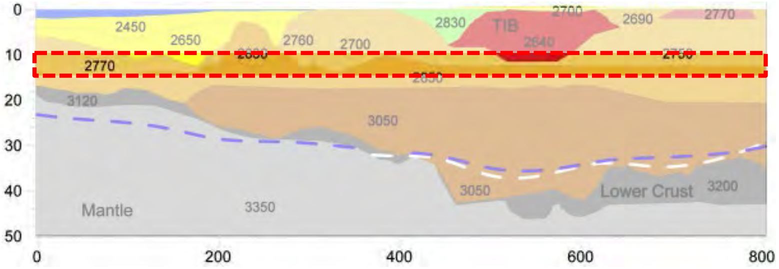


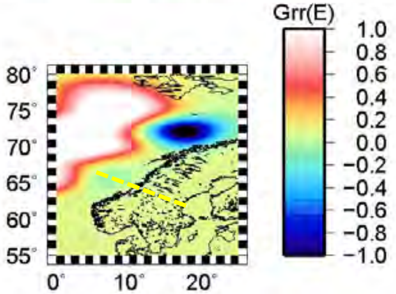



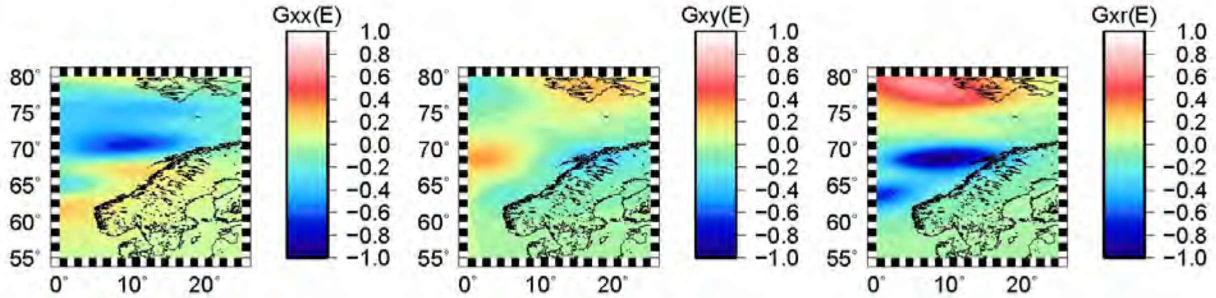


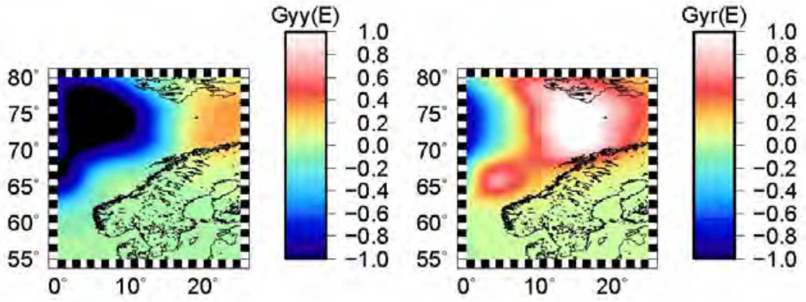




10-15 km depth
@225km height

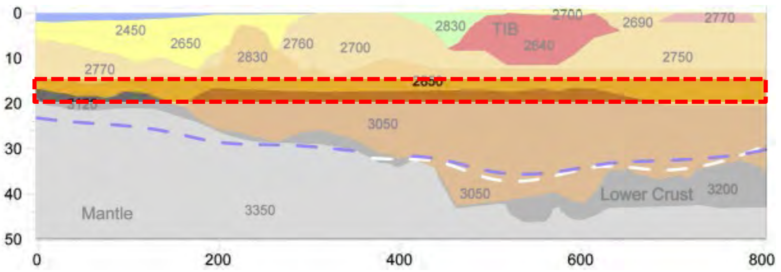


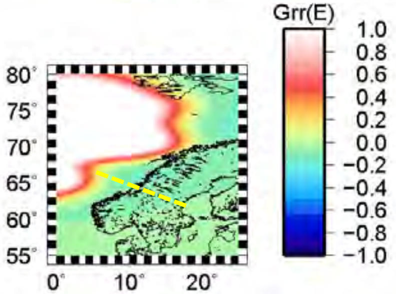



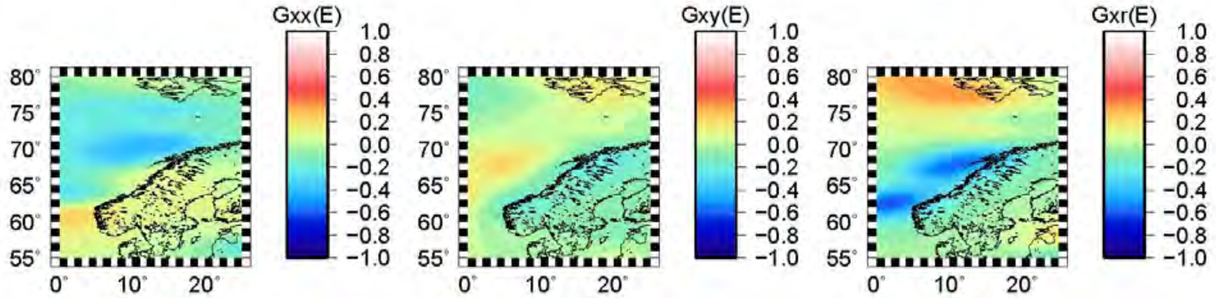


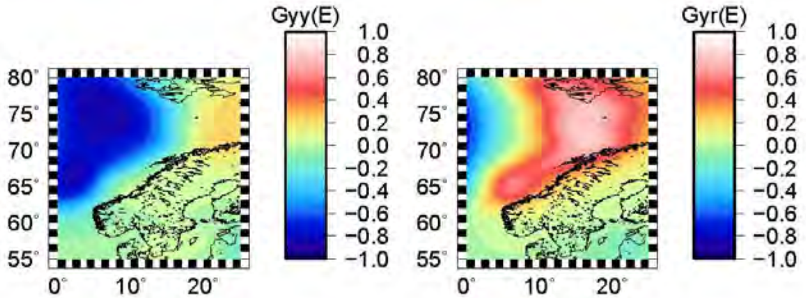




15-20 km depth
@225km height

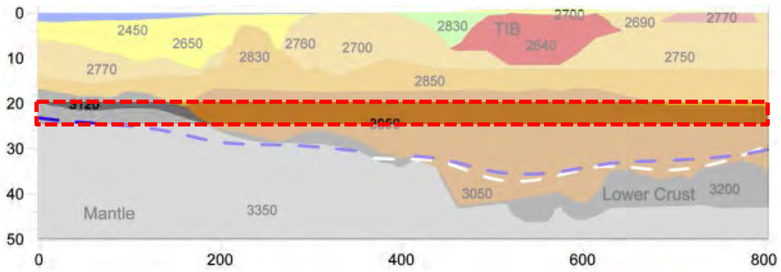


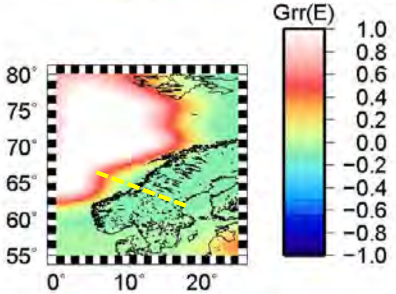



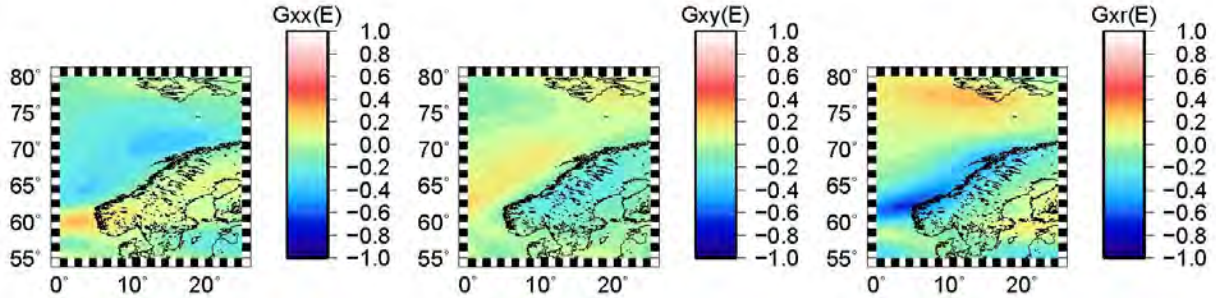


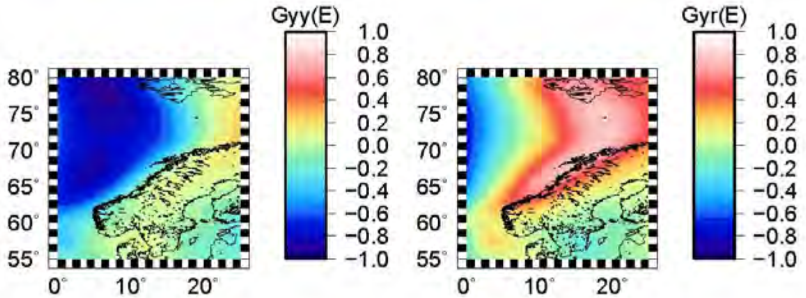




20-25 km depth
@225km height






25-30 km depth
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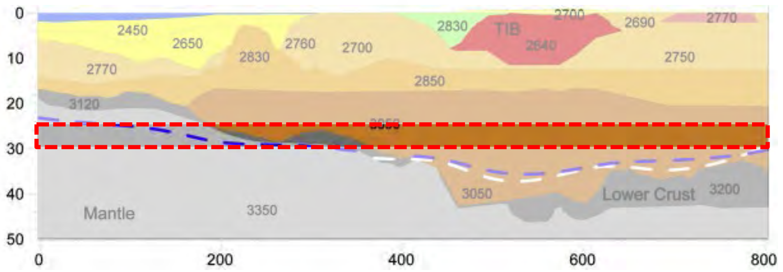


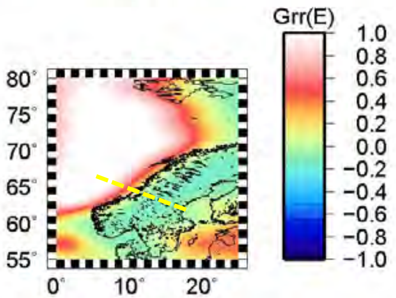

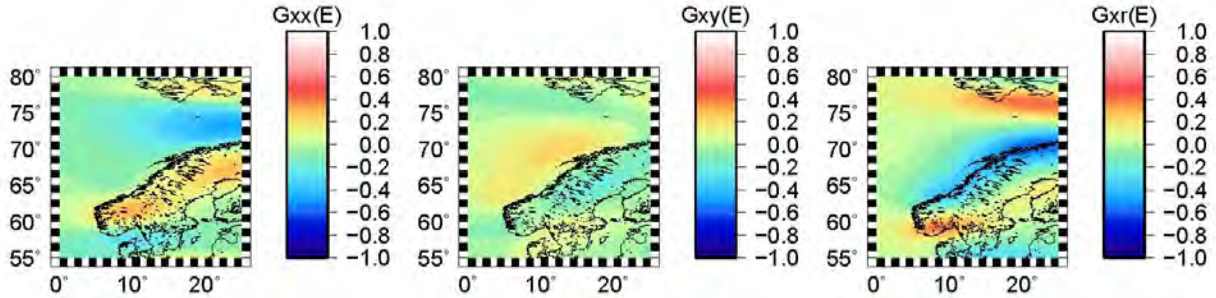


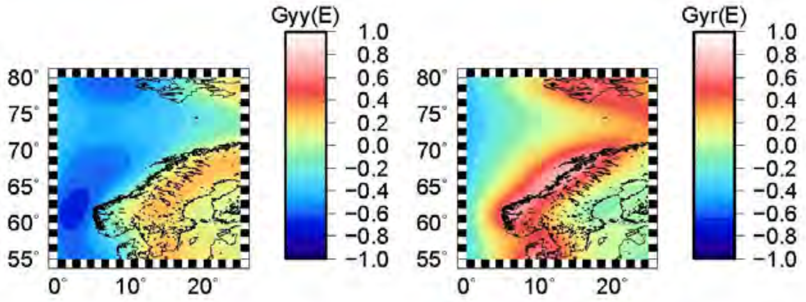


30-35 km depth
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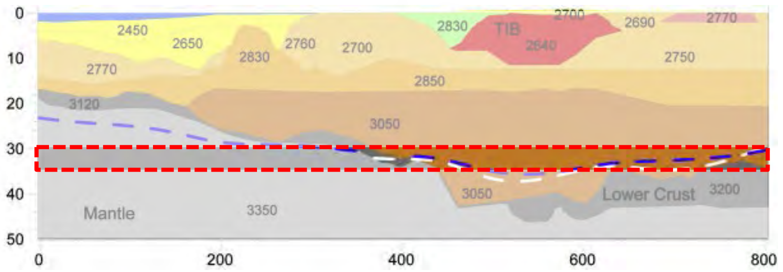


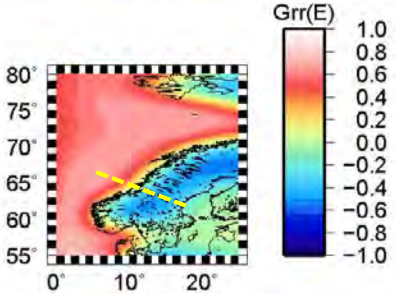

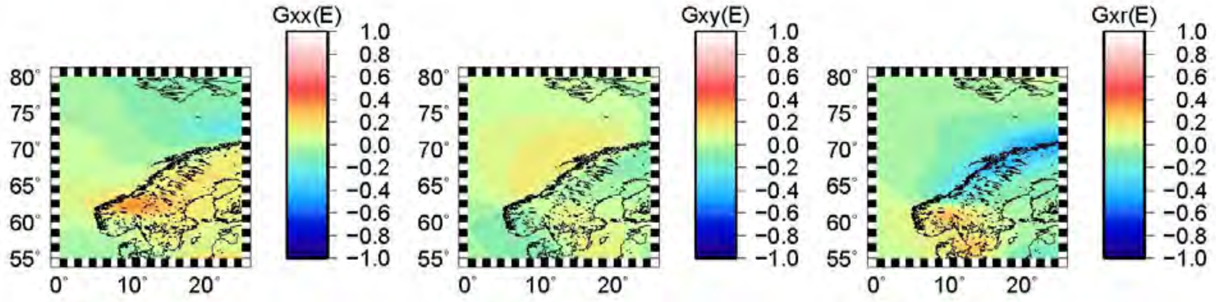


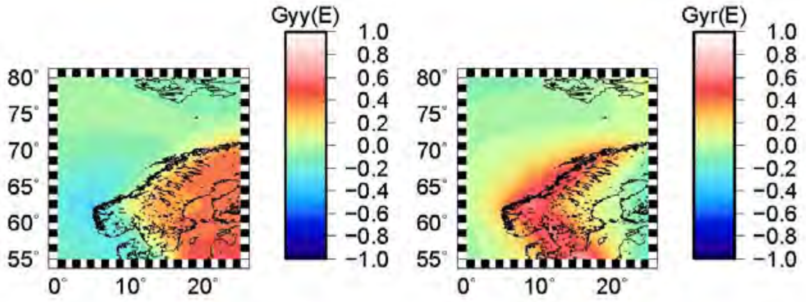


35-40 km depth
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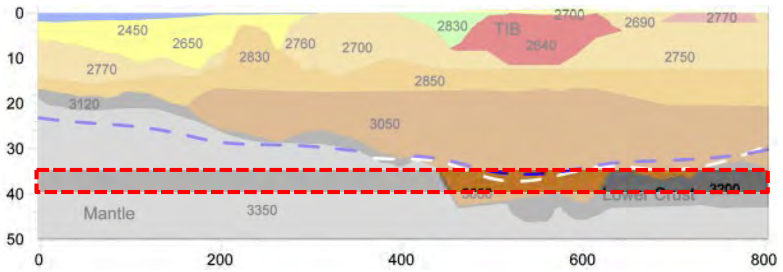


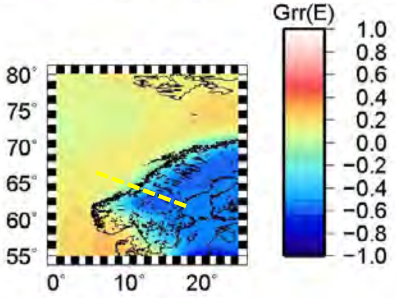

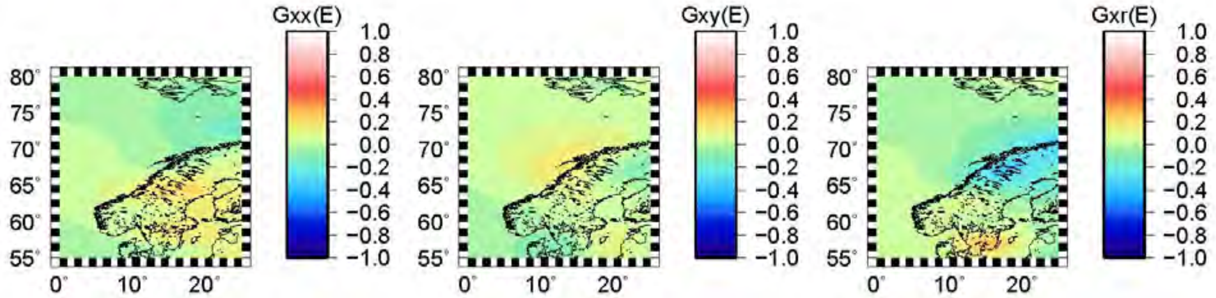


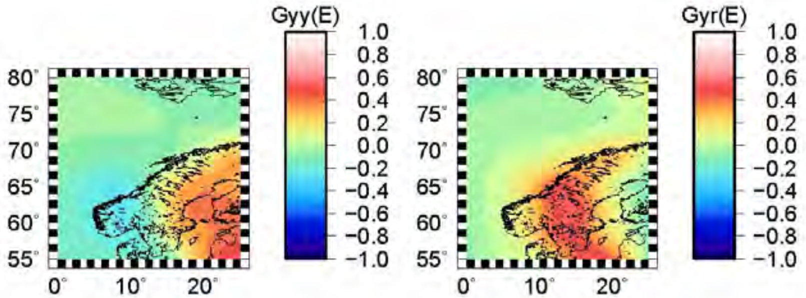


40-45 km depth
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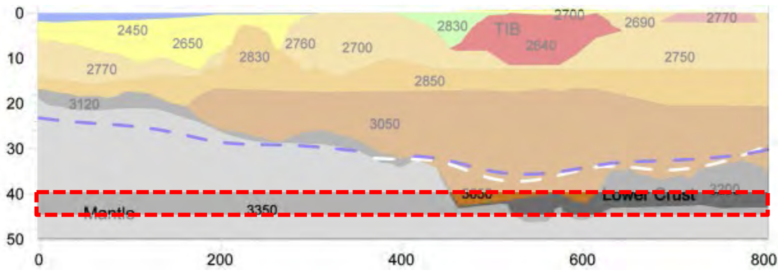


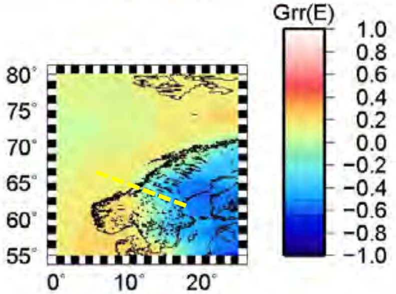

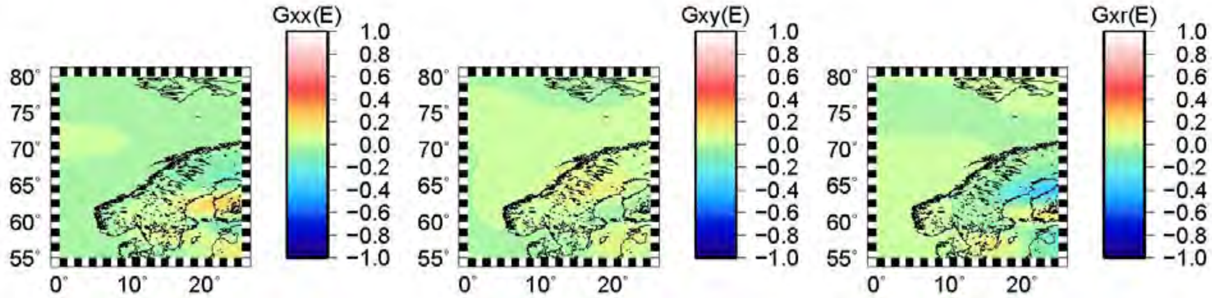


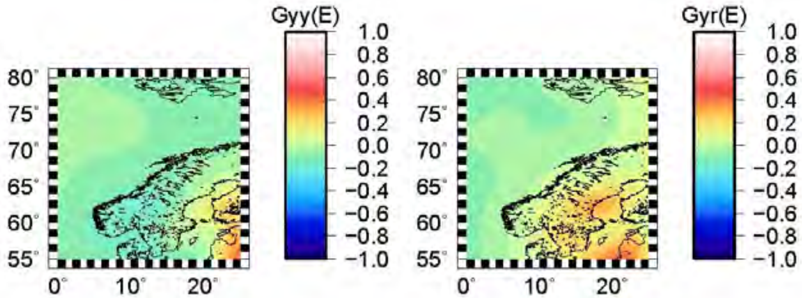


45-50 km depth
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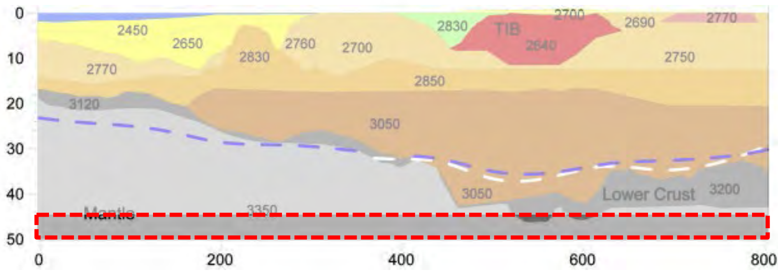


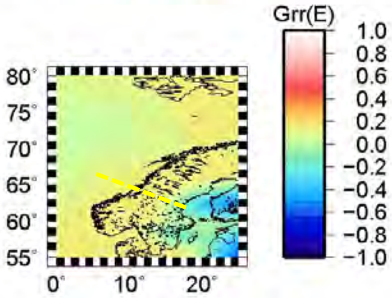

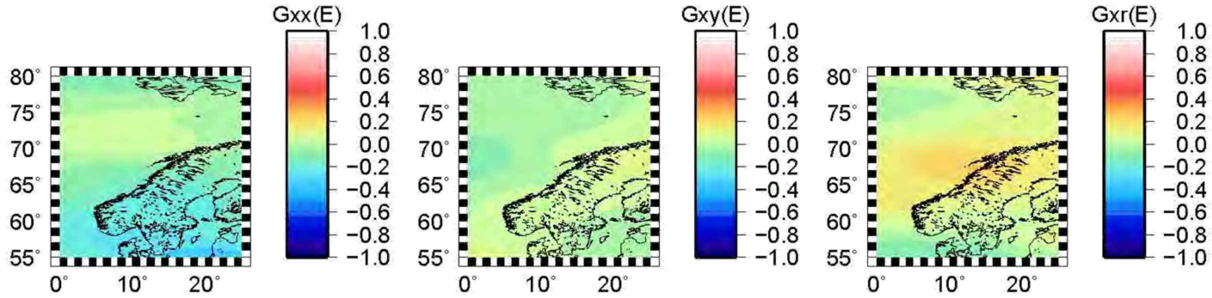


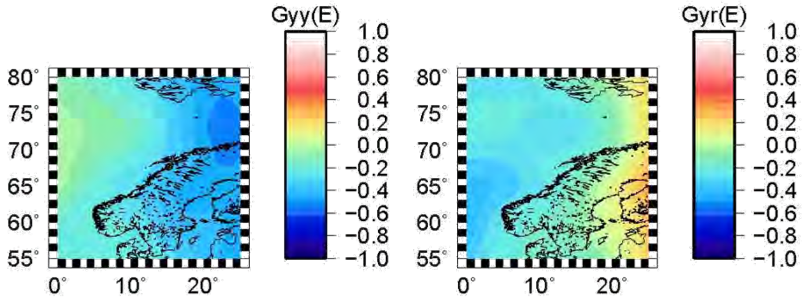


50-75 km depth
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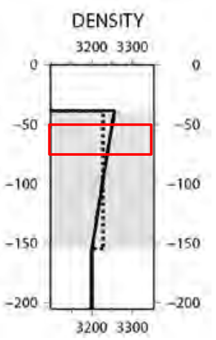


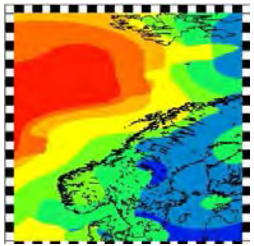

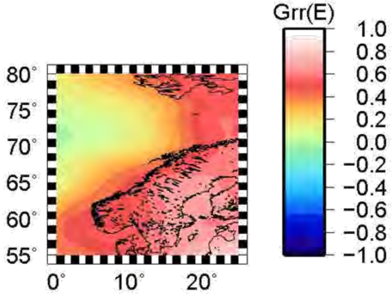

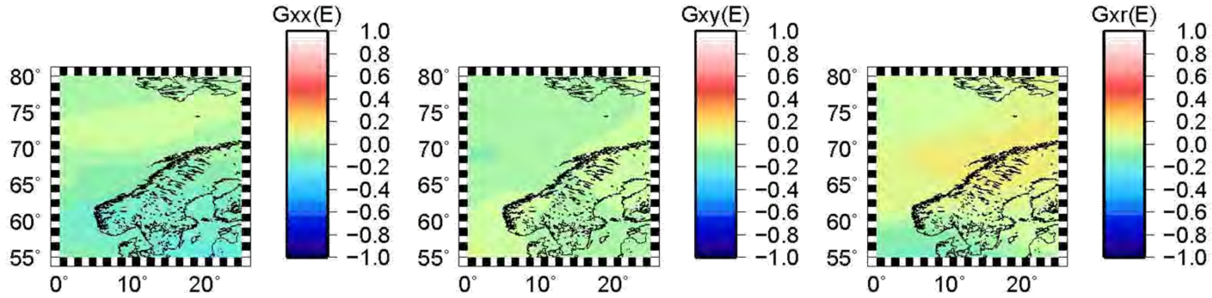


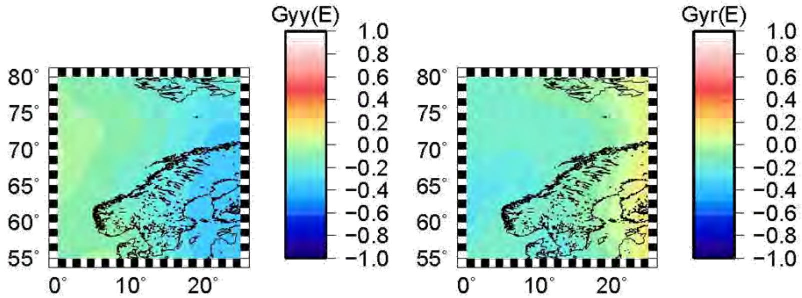


75-100 km depth
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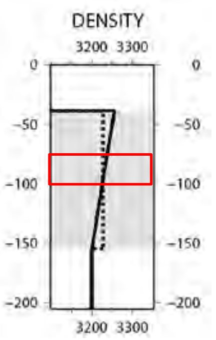
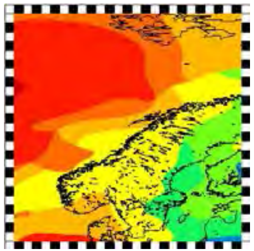
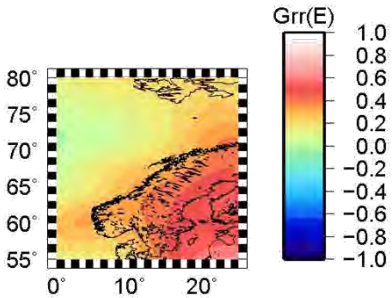
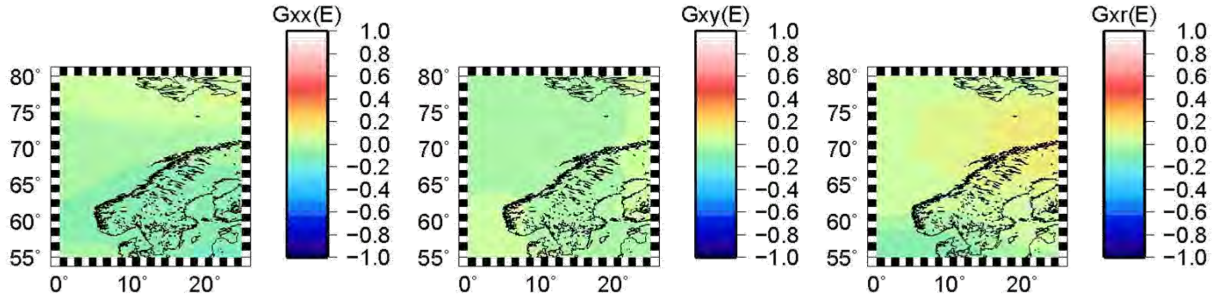
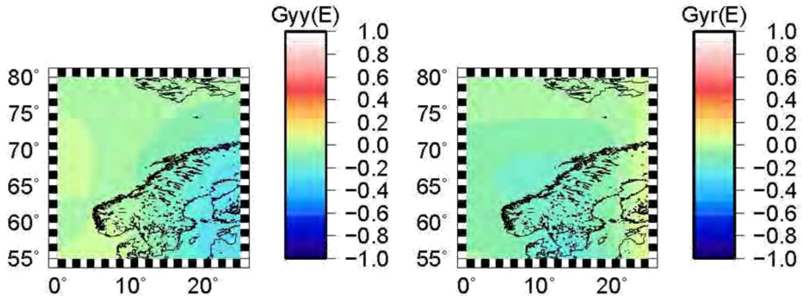


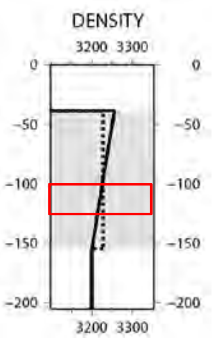



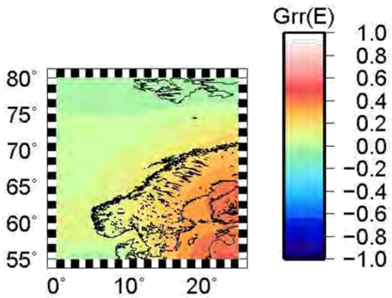


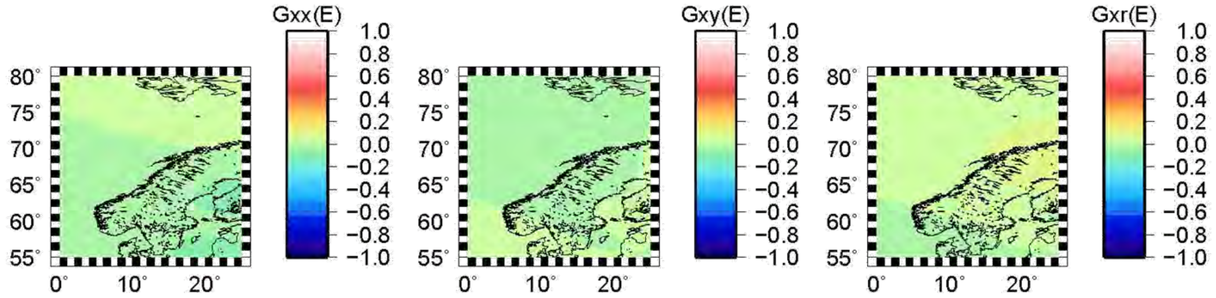


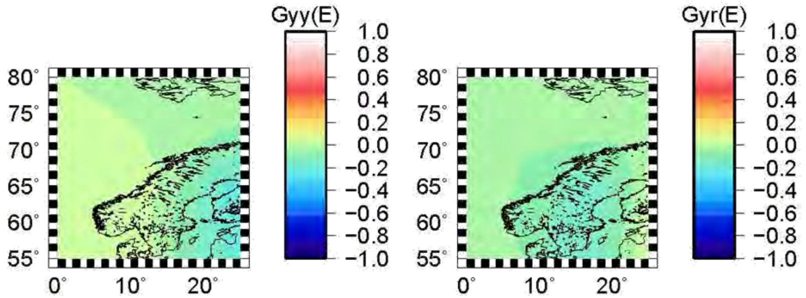




100-125 km depth
@225km height

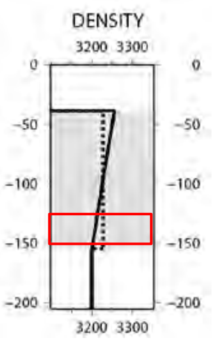


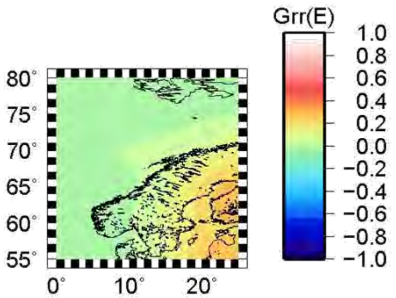



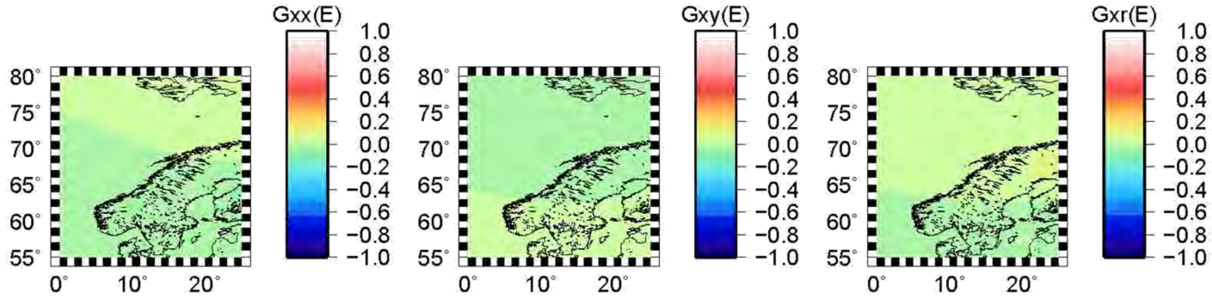


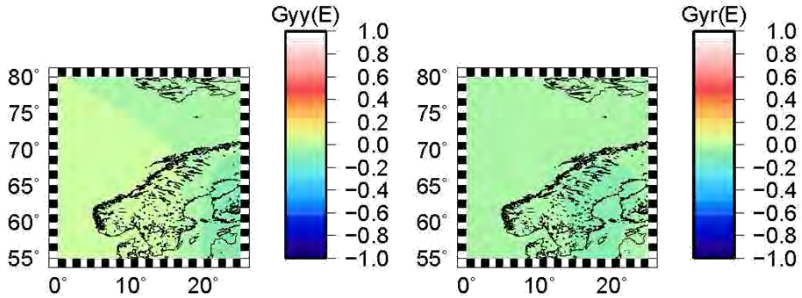




125-150 km depth
@225km height

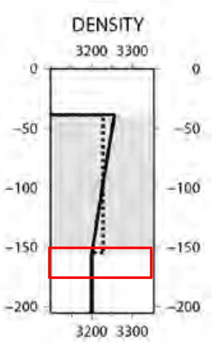


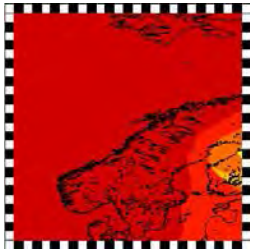



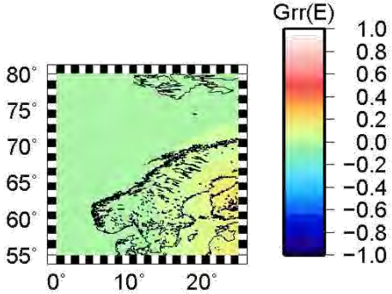


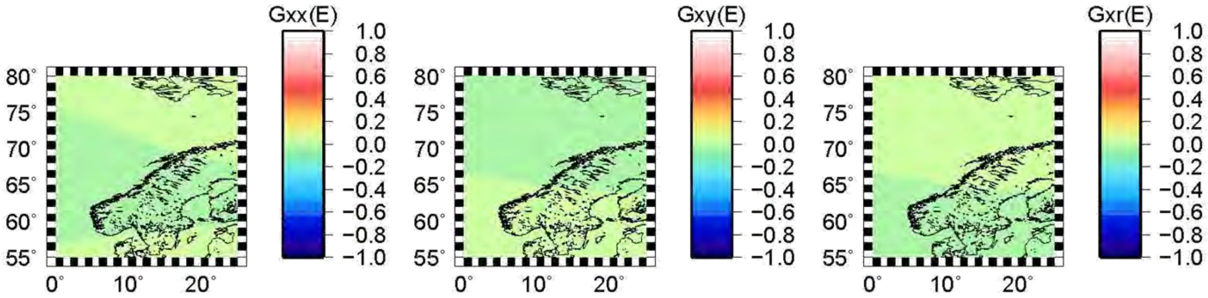



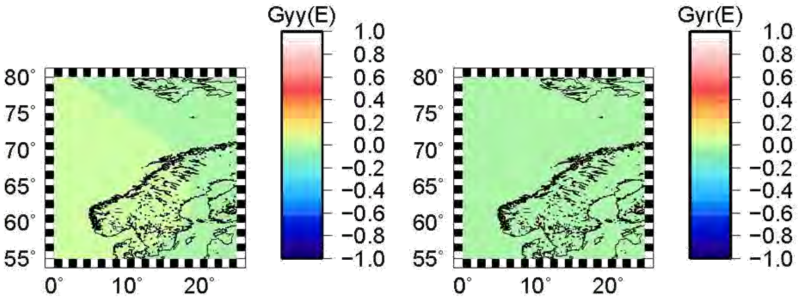

150-175 km depth
@225km height



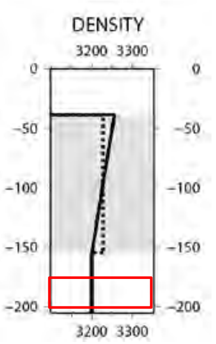



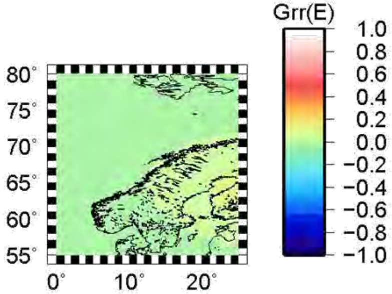




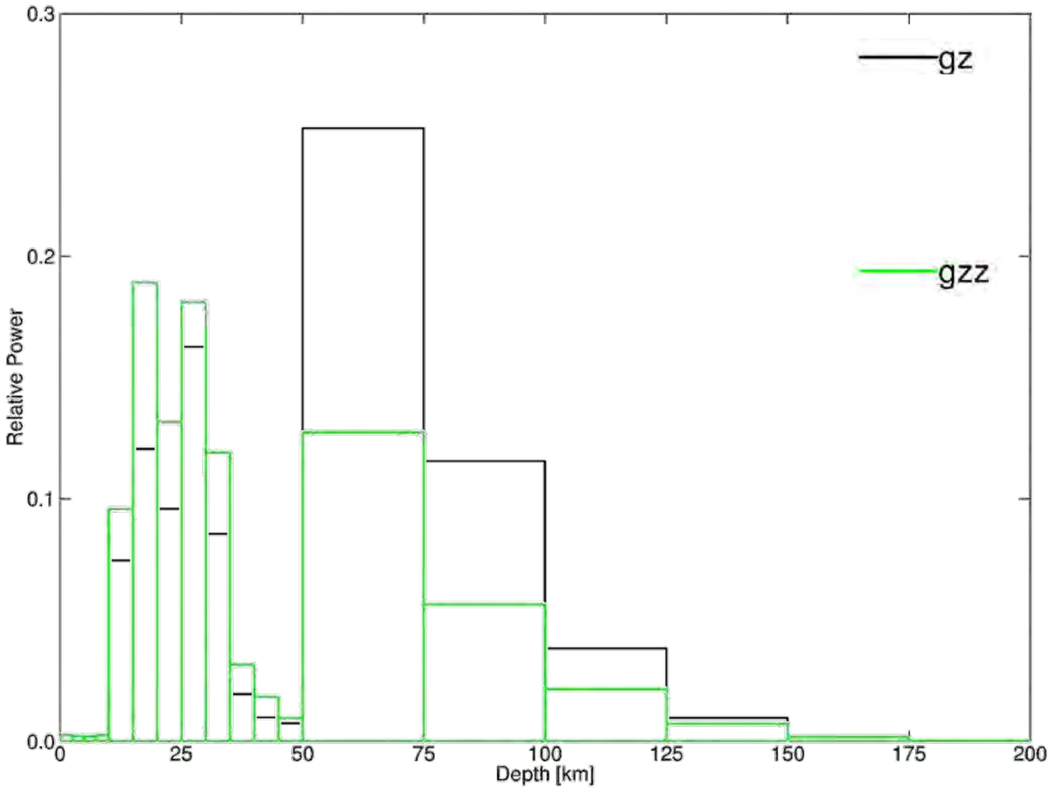

175-200 km depth
@225km height



# Relative signal power @225 km height

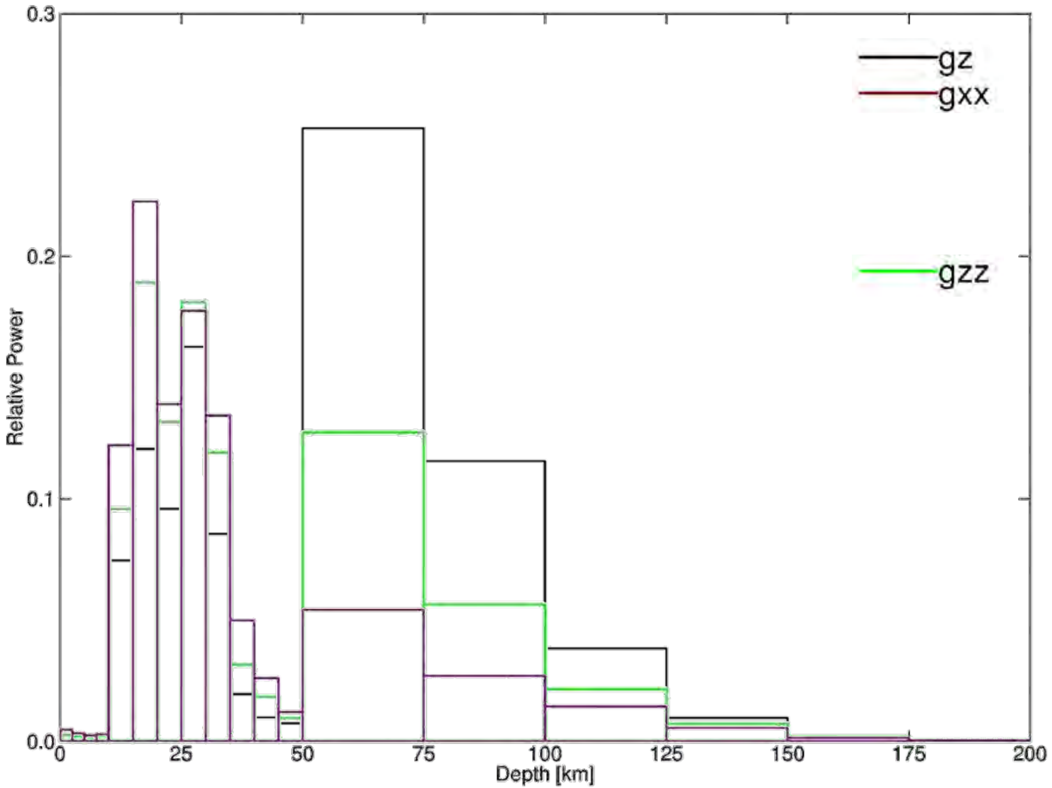
Christian-Albrechts-Universität zu Kiel




- Vertical gradient is compared to the gravity field shifted to intra-crustal sources
- And less affected by regional trends

# Relative signal power @225 km height

Christian-Albrechts-Universität zu Kiel

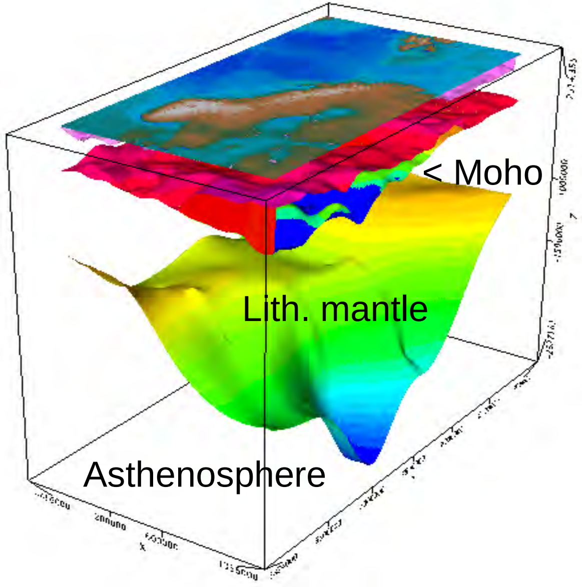


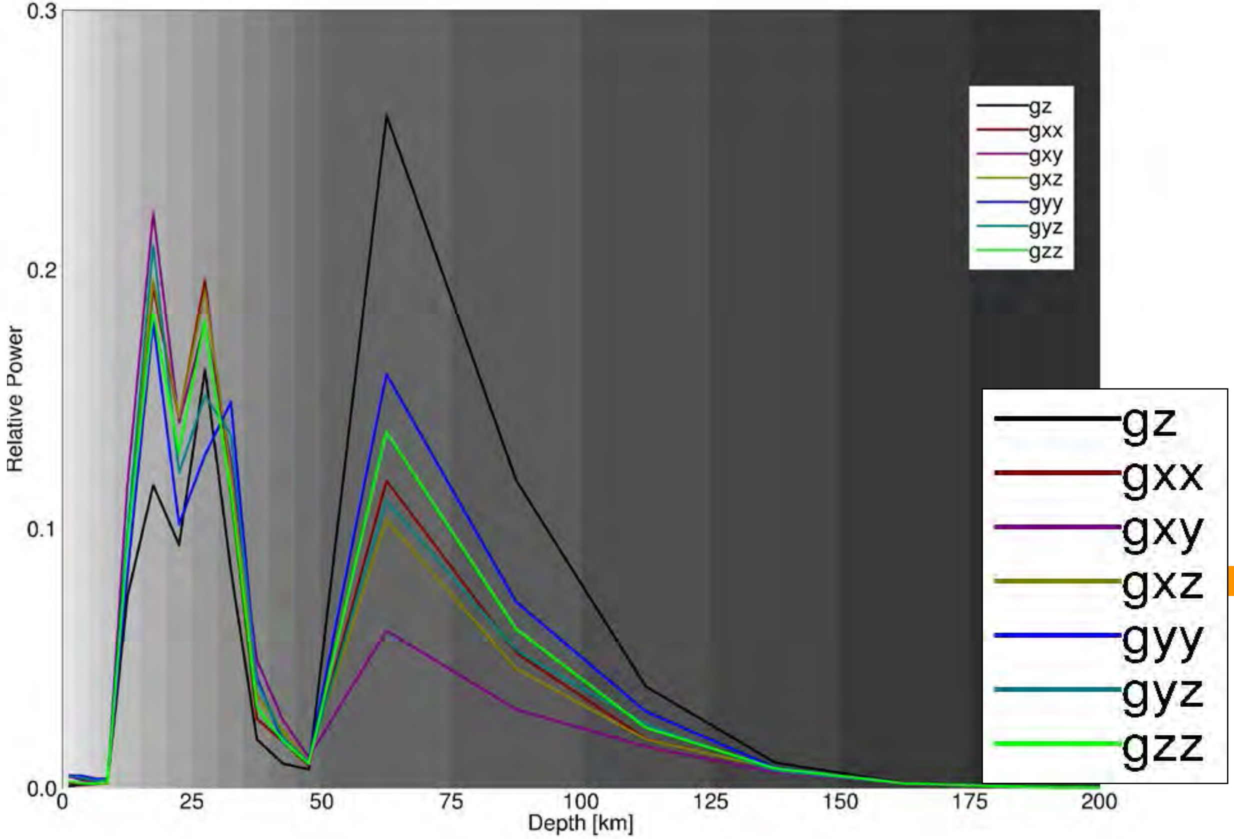

- Vertical gradient is compared to the gravity field shifted to intra-crustal sources
- And less affected by regional trends
- Non-vertical component is even more sensitive to shallow sources

=> Different depth sensitivity of gradients can be exploited

# Relative signal power @225 km height





# Geophysics from GOCE

# Summary

- GOCE satellite mission provided gravity gradients at satellite height of 225 and 255 km
    - For model inversion it is probably best to use data close to their original point of acquisition
    - Gravity gradients might contain additional information even though they are all derivatives of the potential
- Geophysical modelling of satellite gravity gradients requires a spherical Earth representation
- Each gravity gradient component has a characteristic depth sensitivity that can be exploited in geophysical inversion