6th ADVANCED COURSE

ON RADAR POLARIMETRY 2021

SAR POLARIMETRY

Basic & Advanced Concepts and Applications

POTTIER Eric

IETR / University of Rennes 1, France

11-12 / 05 / 2021

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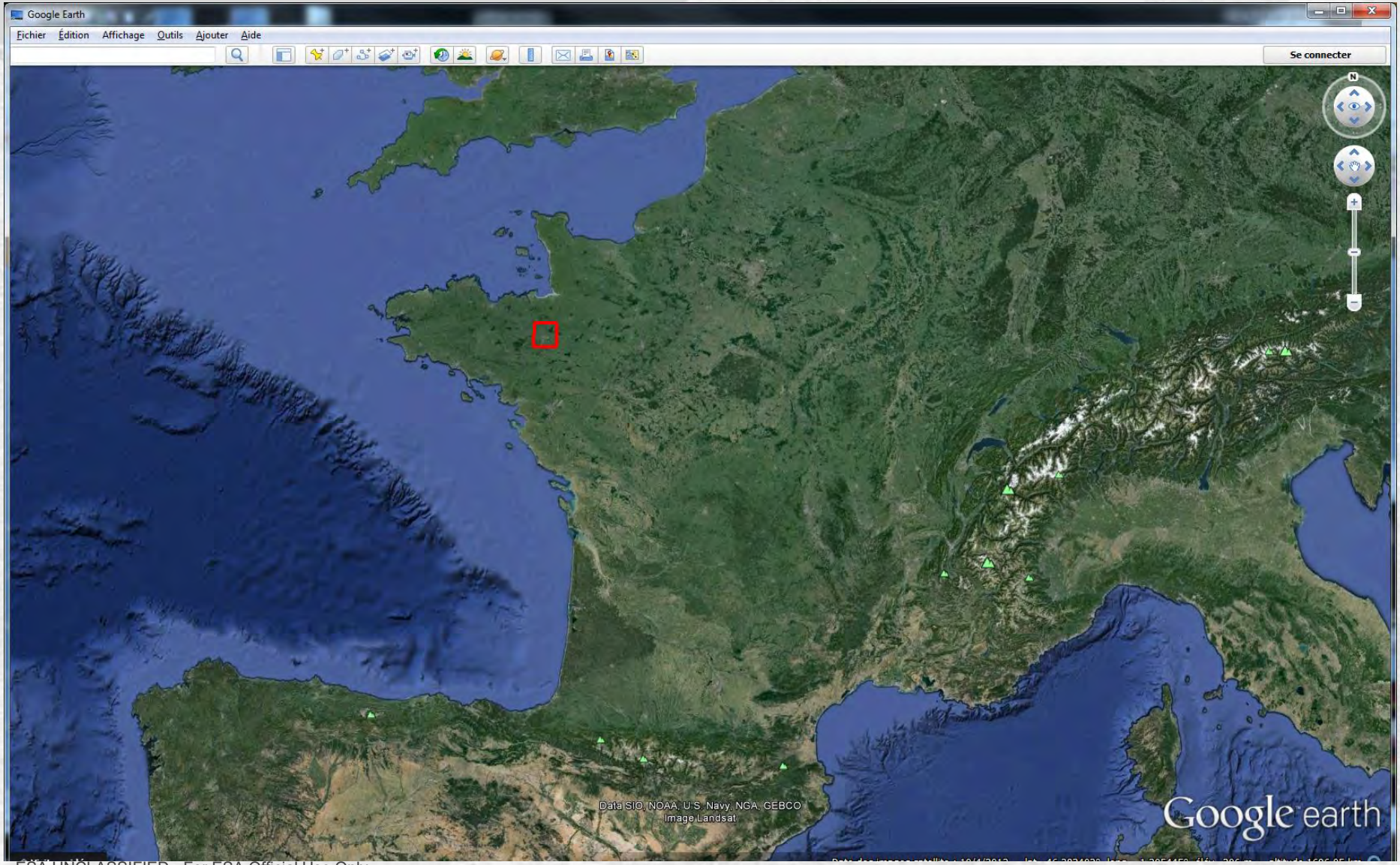
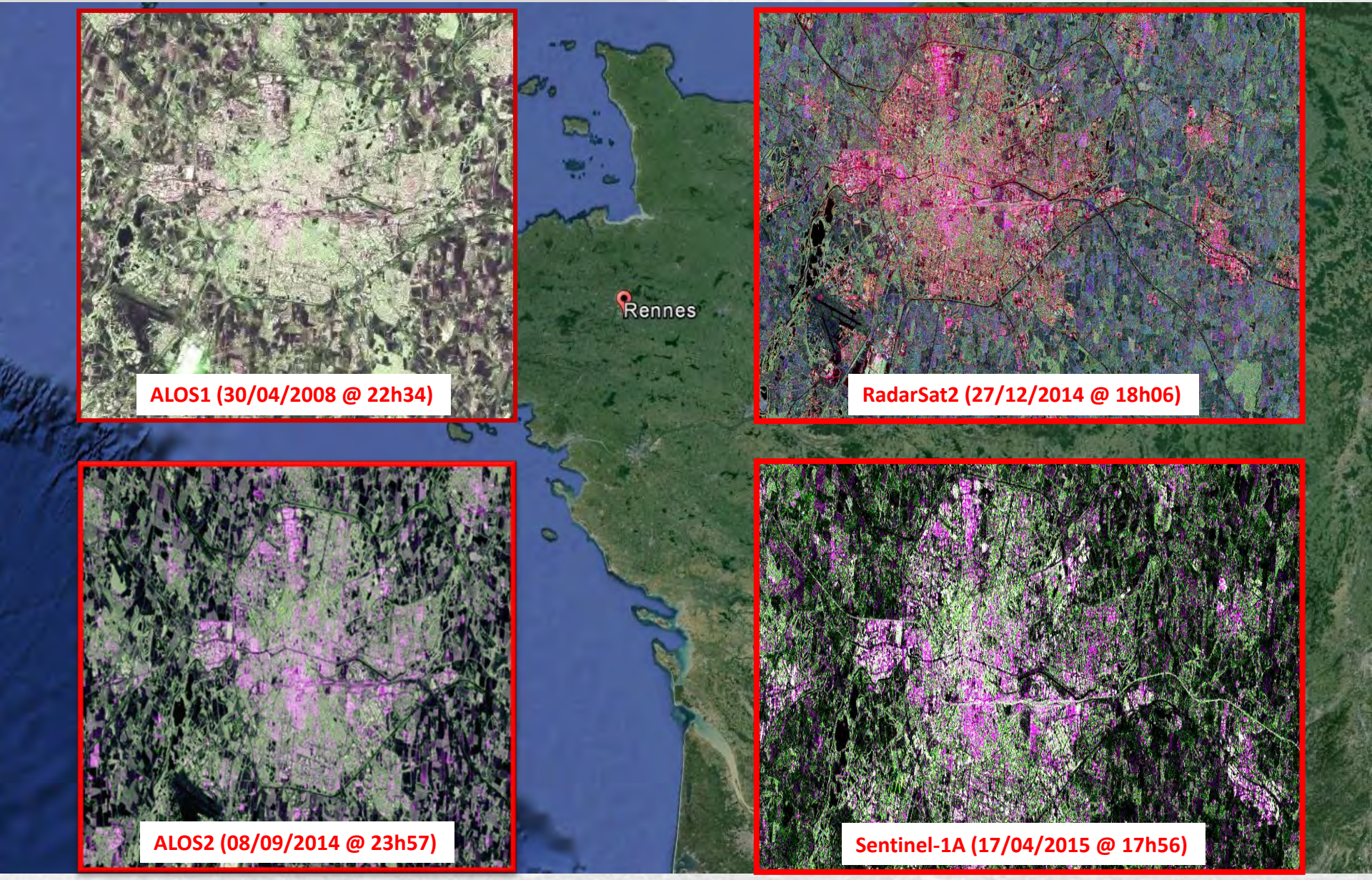
SHINE

SAR & Hyperspectral multi-modal Imaging and sigNal processing, Electromagnetic modeling

RENNES - BRITANNY - FRANCE



RENNES - BRITANNY - FRANCE



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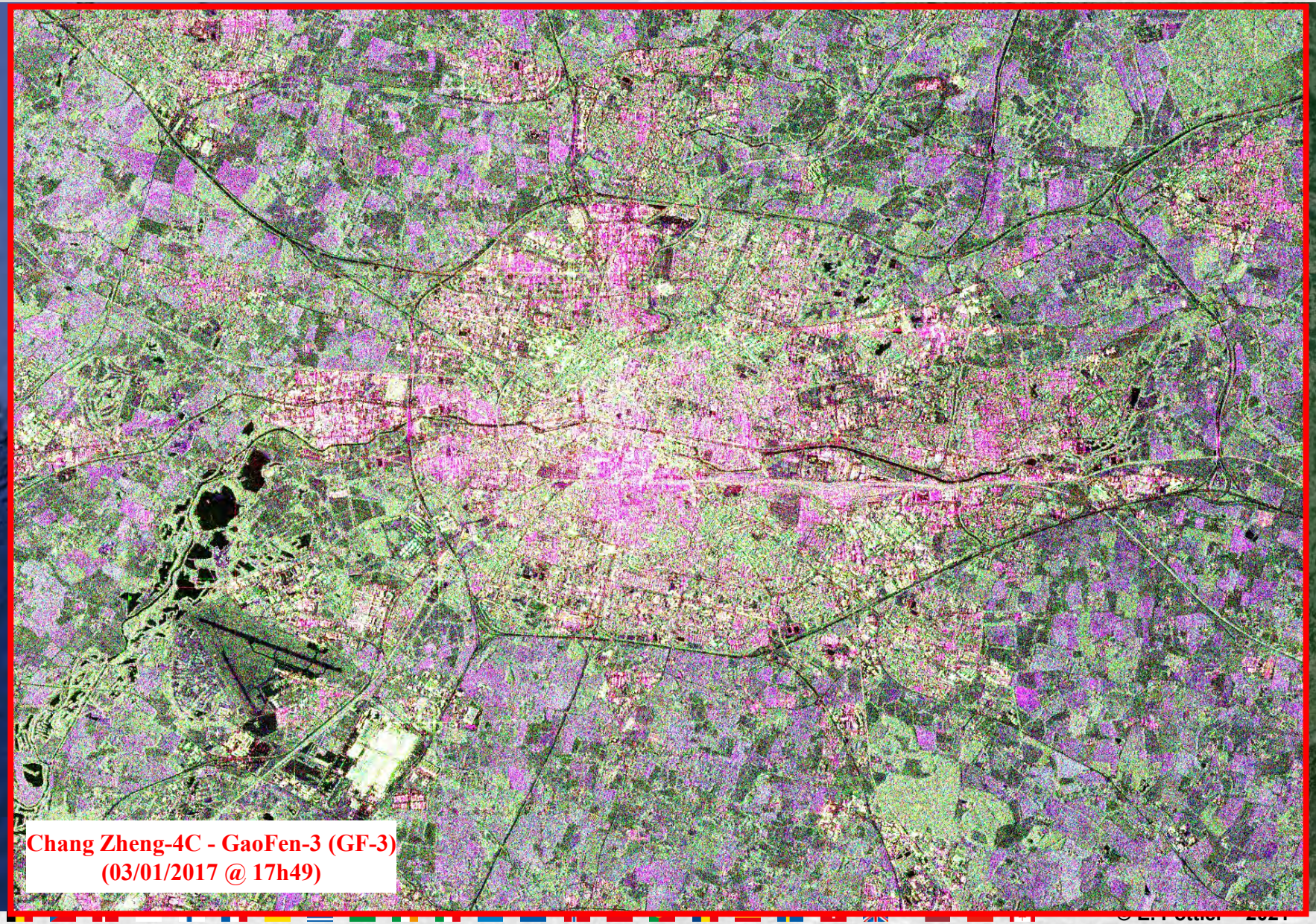

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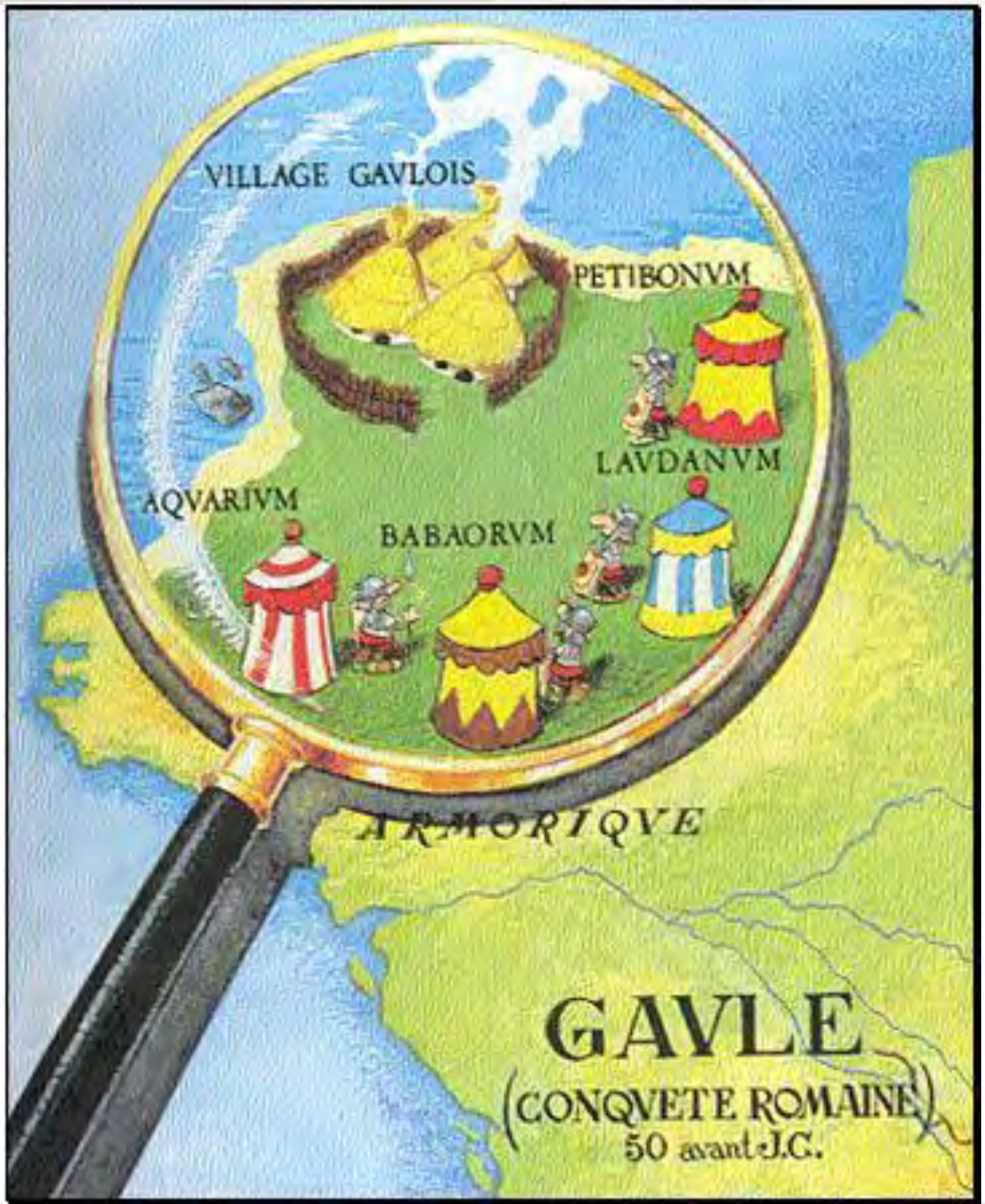
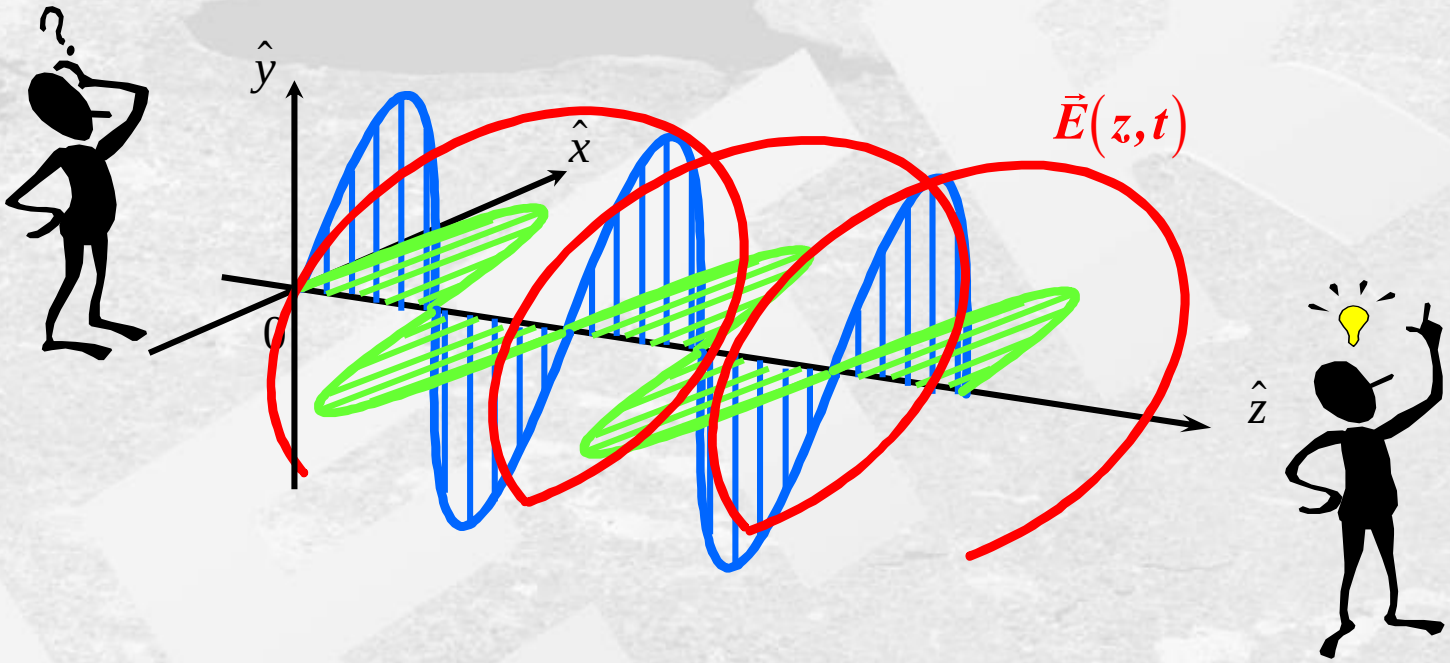




COVERED TOPICS

Objective

To provide

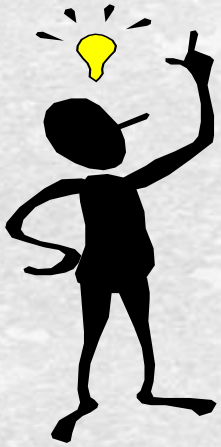
the minimum, but necessary,

amount of knowledge required

to understand

scientific works on

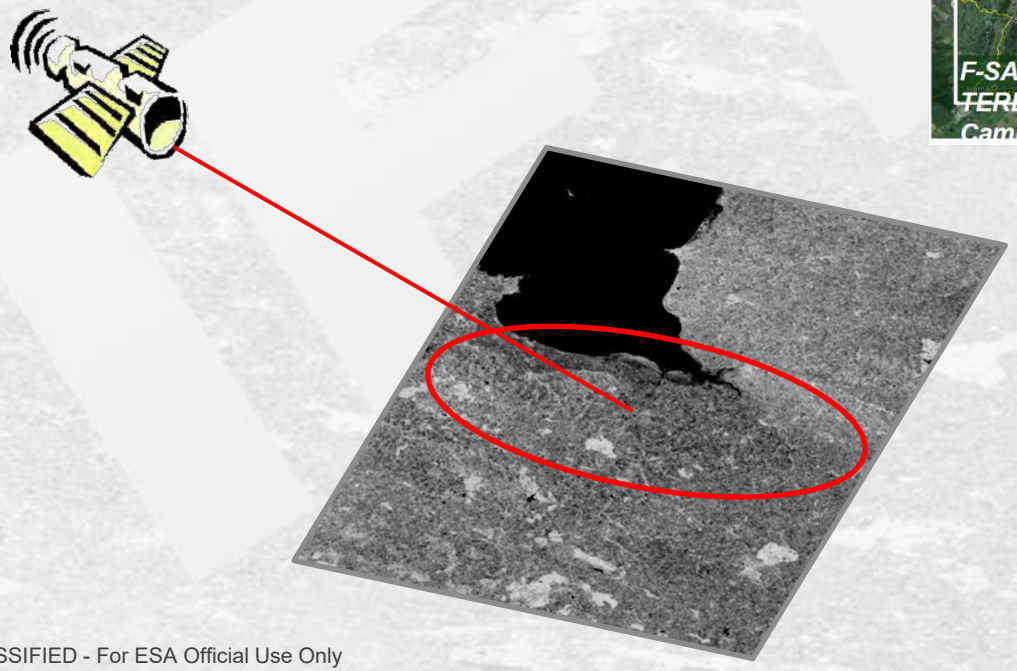
Radar Polarimetry

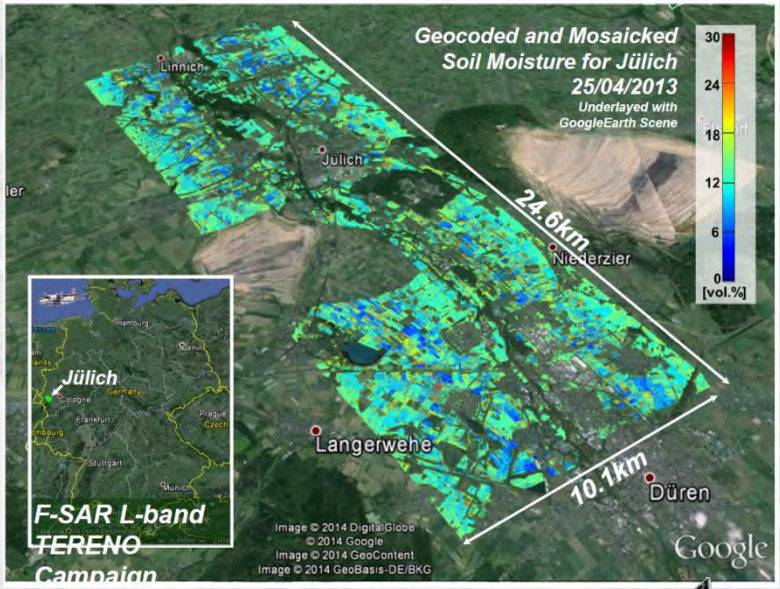


POLARIMETRY

PolSAR

Track$_1$

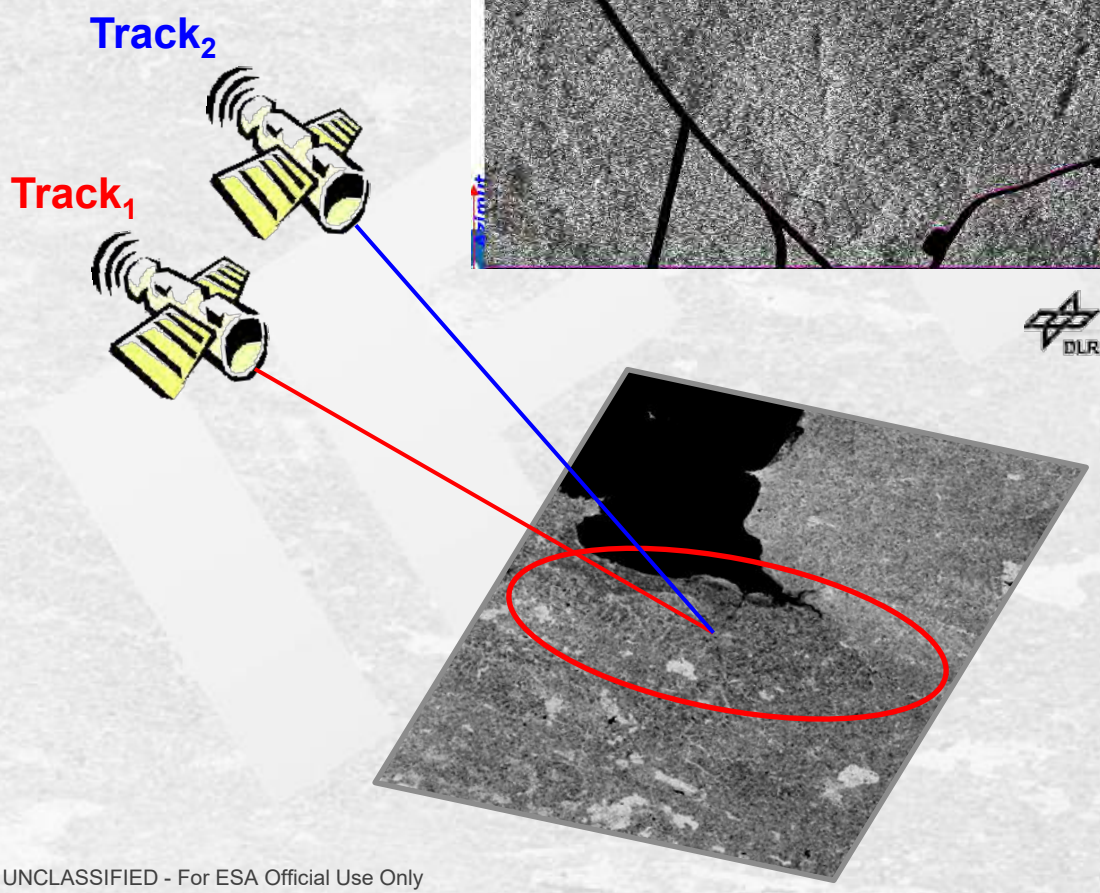


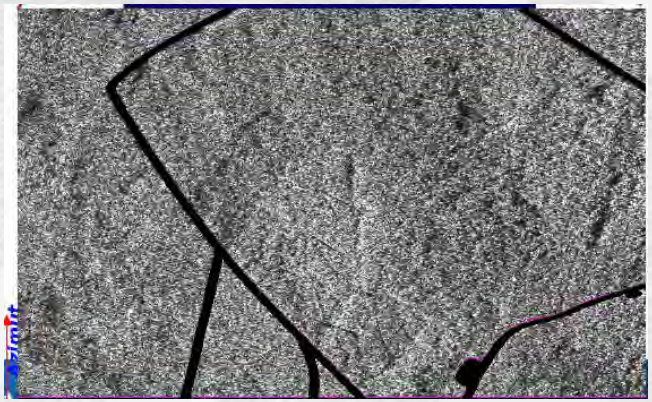


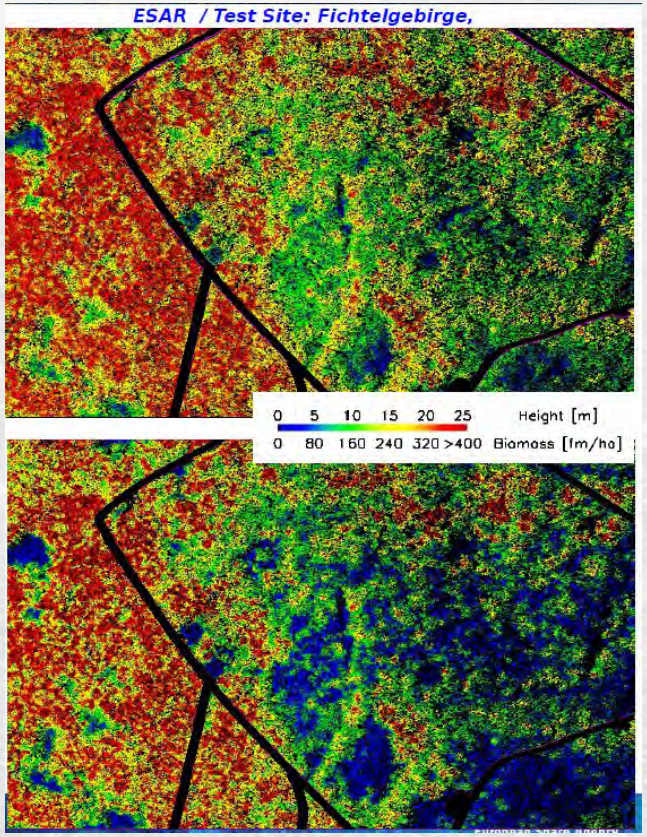
Soil moisture

Urban monitoring

Pol-InSAR

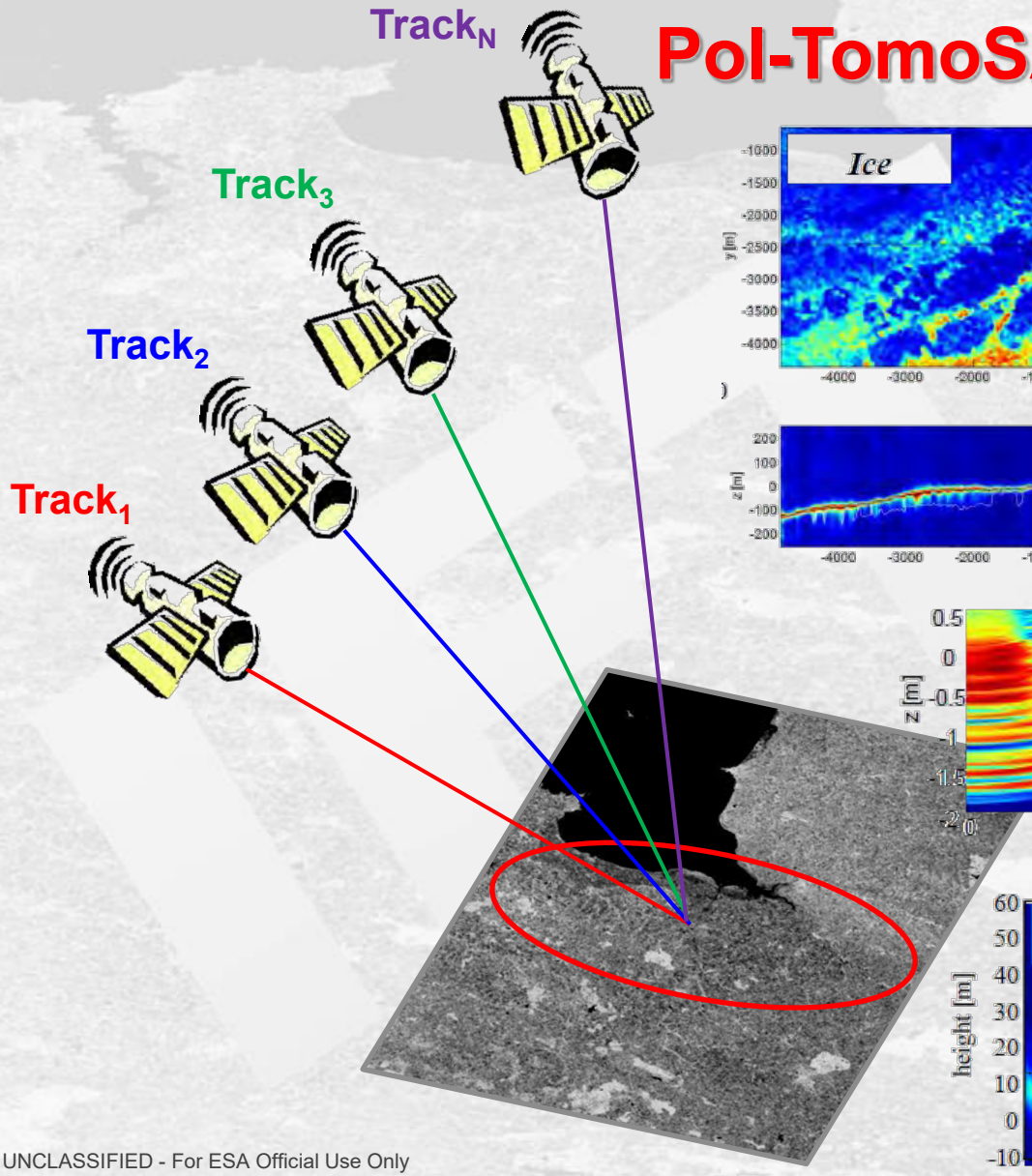


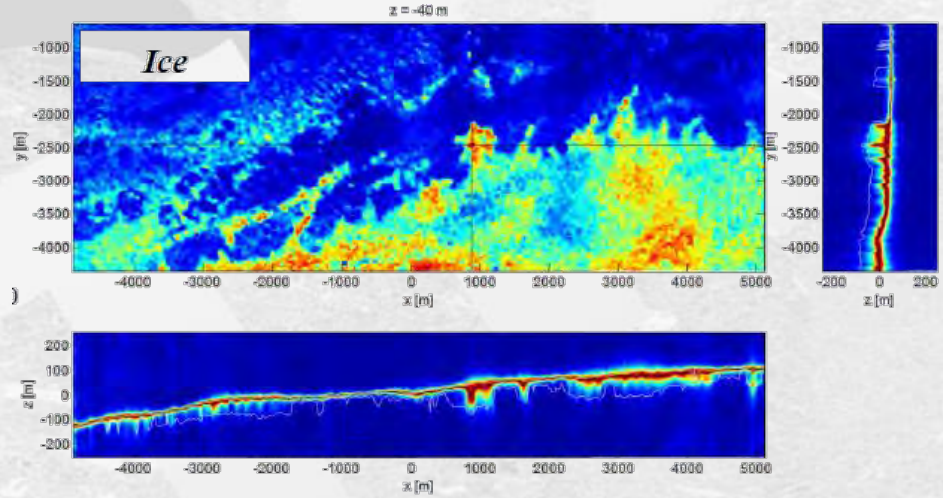


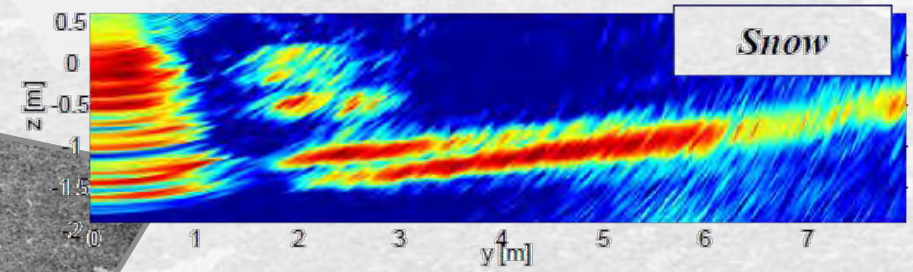


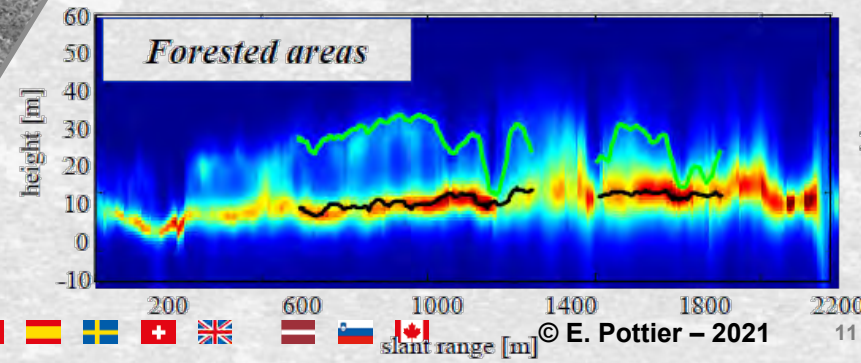
POLARIMETRY + TOMOGRAPHY

Pol-TomoSAR





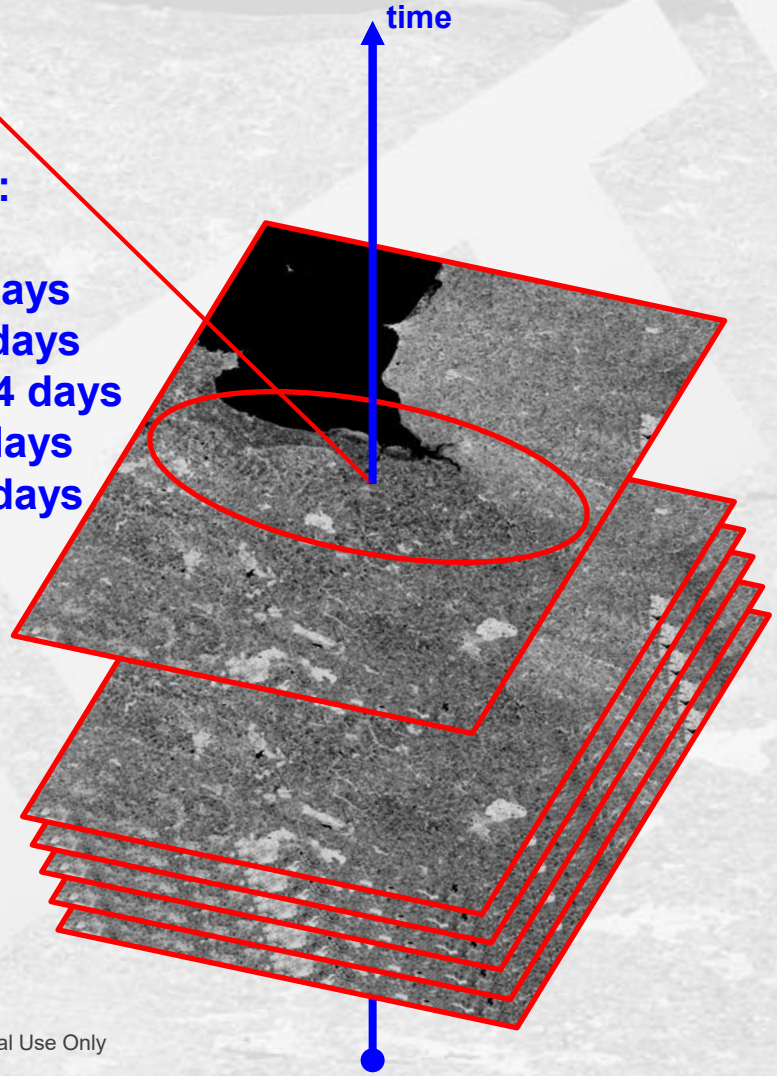




POLARIMETRY + TIME-SERIES

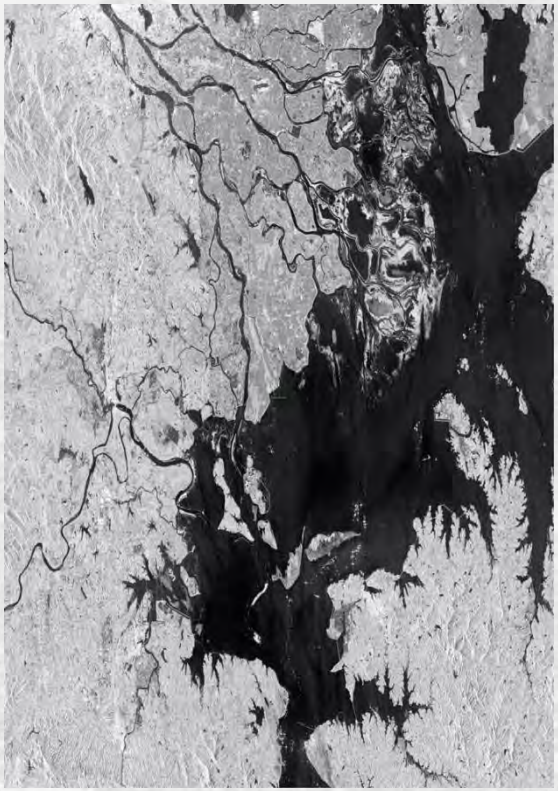
Track$_{1..N}$

Pol-TimeSAR



Revisit time :

- ALOS-2 = 14 days
- BIOMASS = 4 days
- RADARSAT2 = 24 days
- RISAT-1 = 25 days
- Sentinel-1 = 6 days

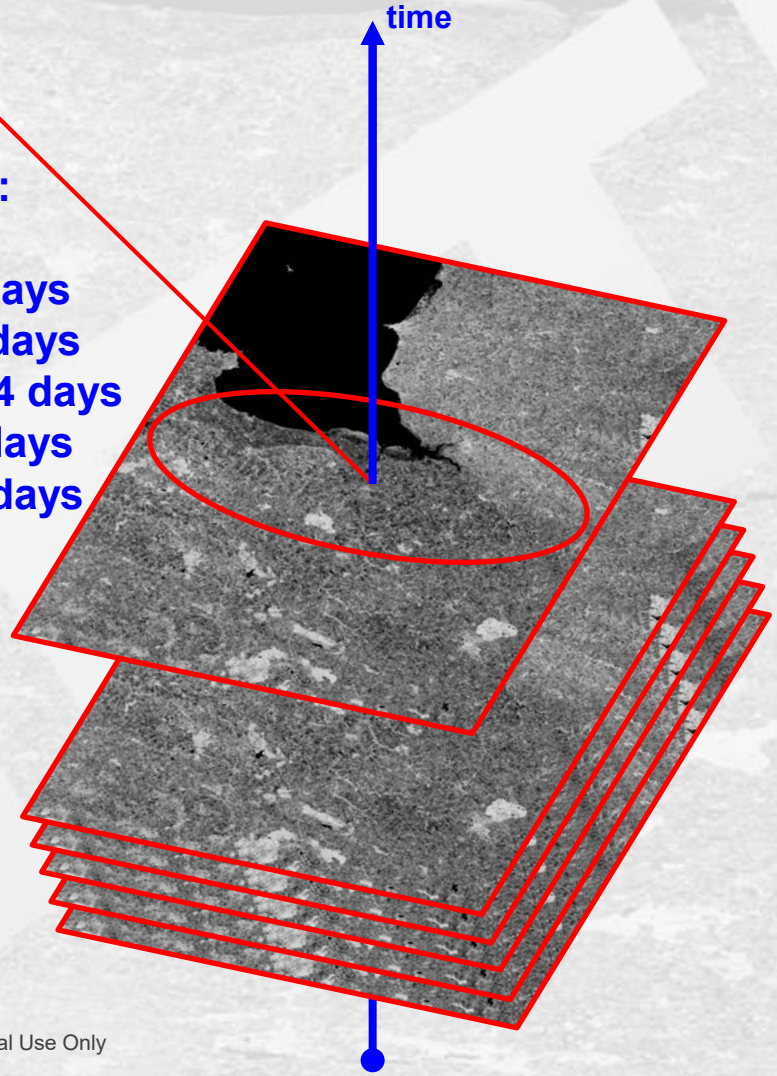


Polarimetric feature temporal evolution

POLARIMETRY + TIME-SERIES

Track$_{1..N}$

Pol-TimeSAR



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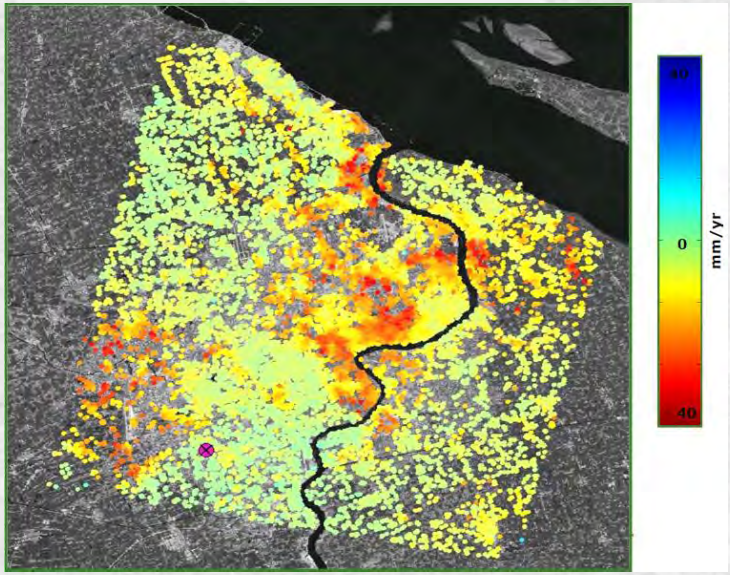


Subsidence Monitoring

Permanent scatterers
Coherent scatterers

Polarimetry = scatterer type

Basic / Advanced Concepts in PolSAR Analysis

Electromagnetic Wave Polarisation

- Wave Propagation
- Wave Polarisation
- Jones Vector
  - Polarisation Ratio
  - Complex Polarisation Plane
  - Orthogonal Jones Vector
  - Elliptical Basis Transformation

Scattering Polarimetry

- Scattering Problem
- Polarimetric Descriptors
  - Scattering / Sinclair Matrix
  - Target Vectors
  - Kennaugh Matrix / Huynen Parameters
  - Coherency Matrix
  - Covariance Matrix
- Elliptical Basis Transformations
- Polarimetric Target Dimension
  - MonostaticTarget Equations

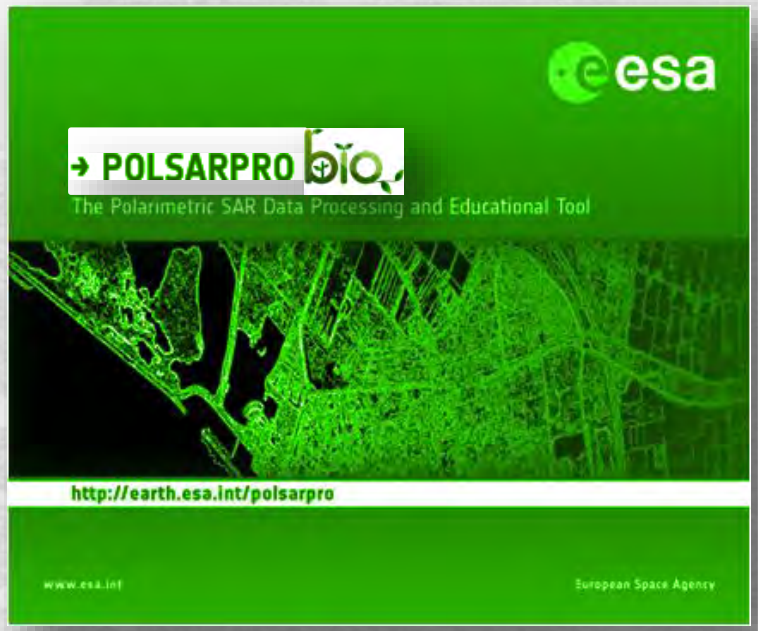
Basic / Advanced Concepts in PolSAR Analysis

- Polarimetric Speckle Filtering
- Target Decomposition Theorems
  - Krogager Decomposition
  - Huynen / Barnes Decompositions
  - Cloude / Holm Decompositions
  - Freeman / Yamaguchi Decompositions
  - Van Zyl / Arii Decomposition
  - H / A / α Decomposition
  - eigenvalues based parameters
  - TSVM Decomposition
- PolSAR Image Segmentation
  - H / α Unsupervised Classification
  - Wishart - H / A / α Classification
  - Wishart – Freeman Classification

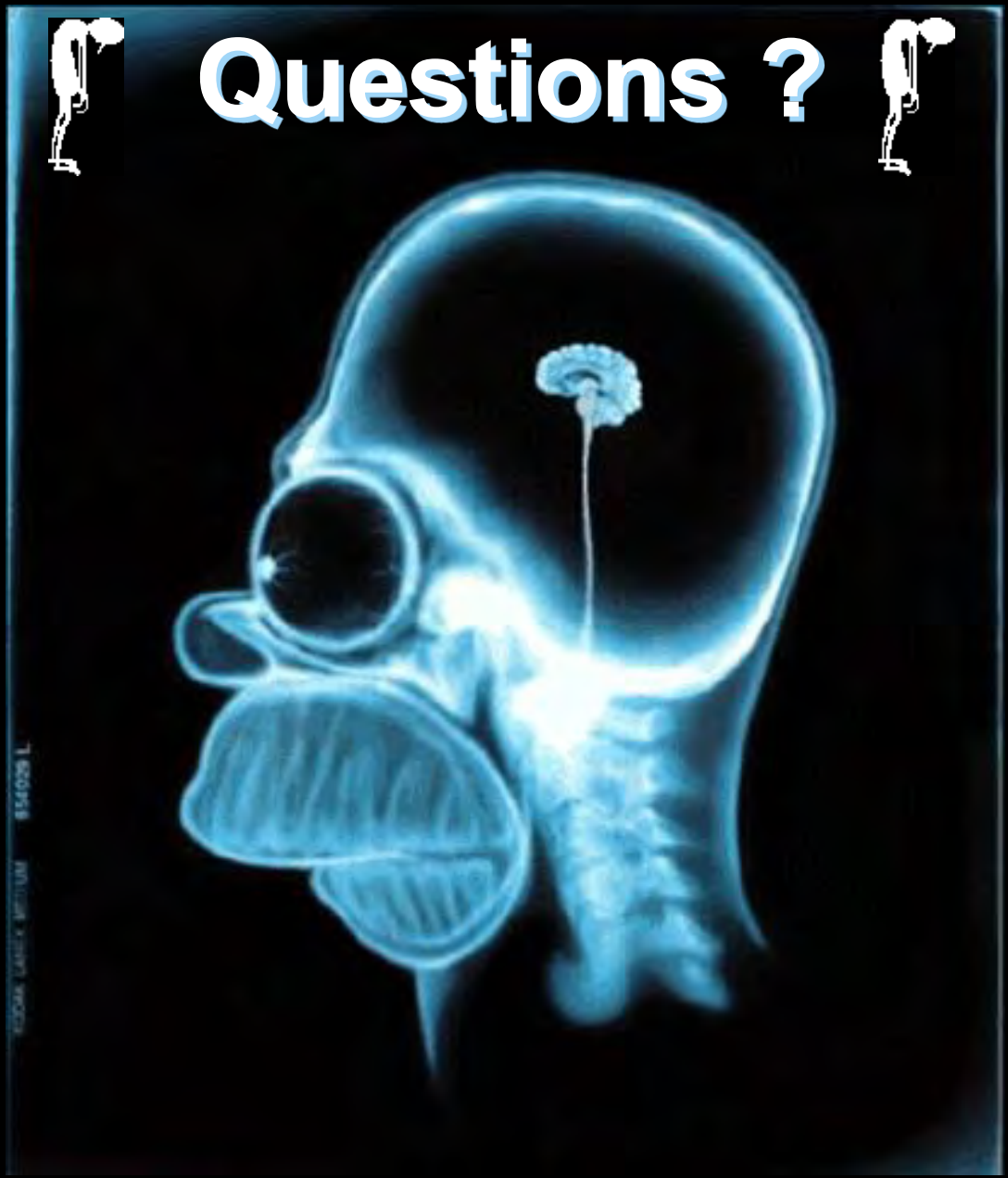
Wave Polarimetry

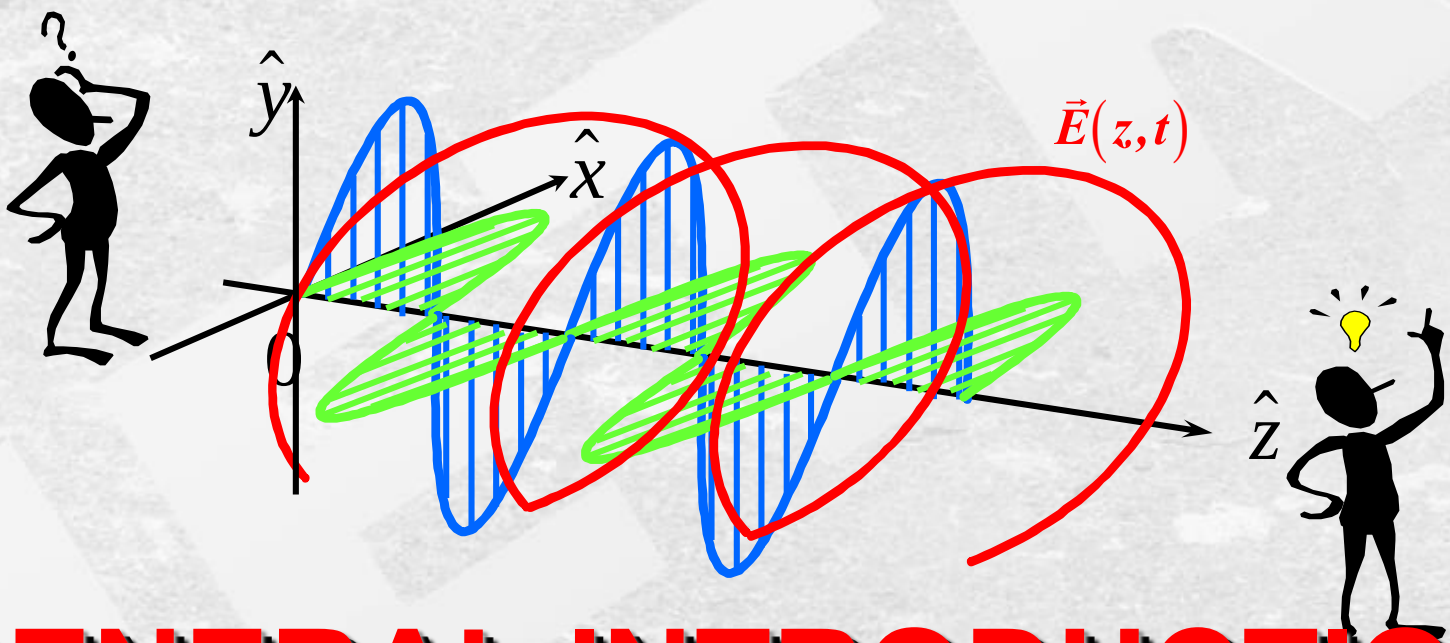
- Jones Vector
- Stokes Vector
  - Poincaré Sphere
  - Elliptical Basis Transformation
- Partially Polarised Waves
- Wave Covariance Matrix
- Wave Polarisation Dimension
- Wave Speckle Filtering
- Wave Decomposition

Practicals



Questions ?



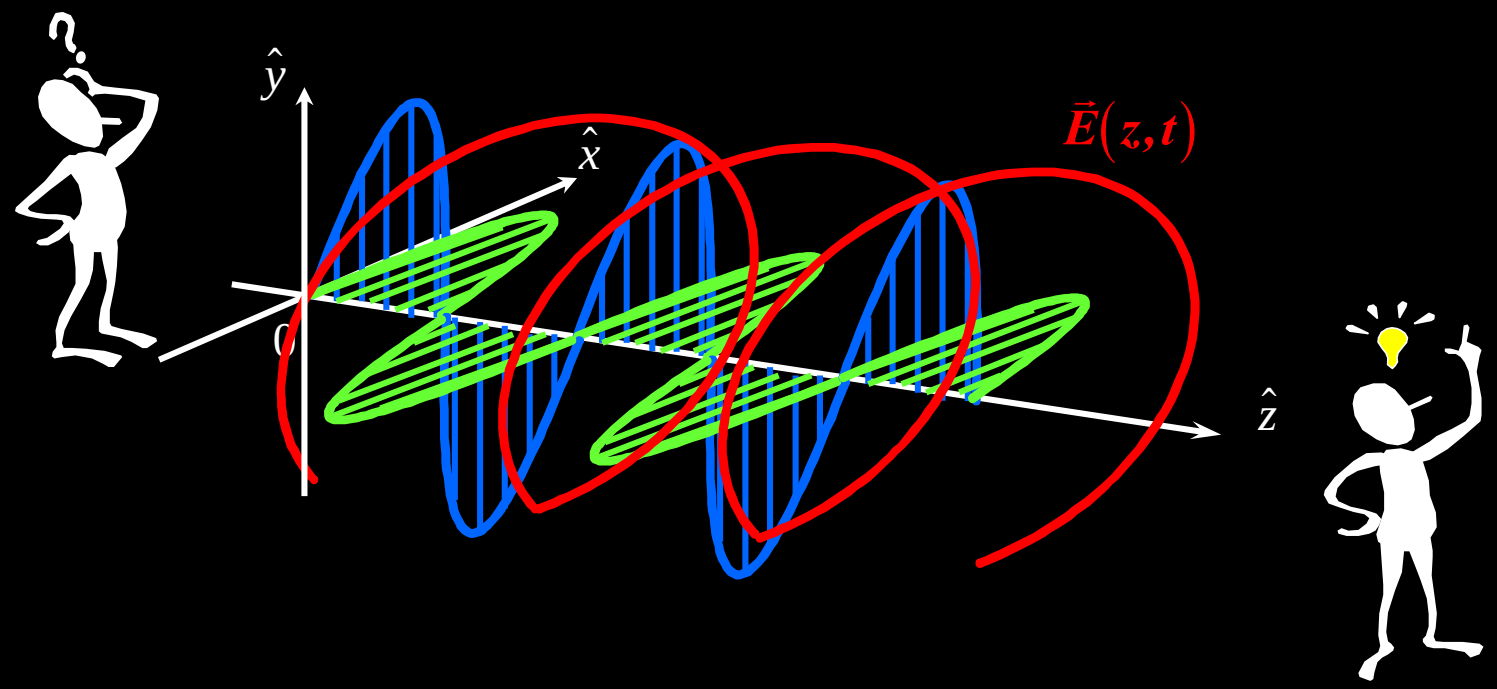


GENERAL INTRODUCTION

RADAR POLARIMETRY

- A bit of History
- Airborne and Space-borne Polarimetric SAR Sensors
- Software / Toolbox
- Learning / Training / Results

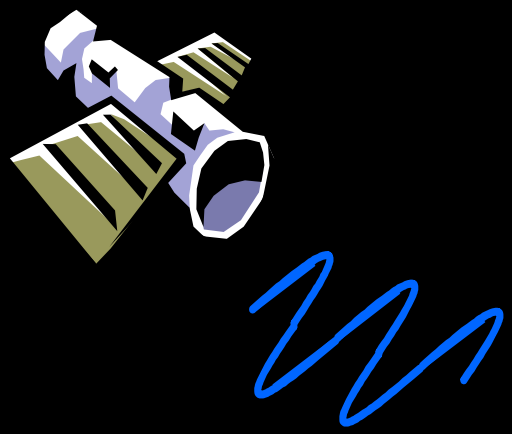
Radar Polarimetry



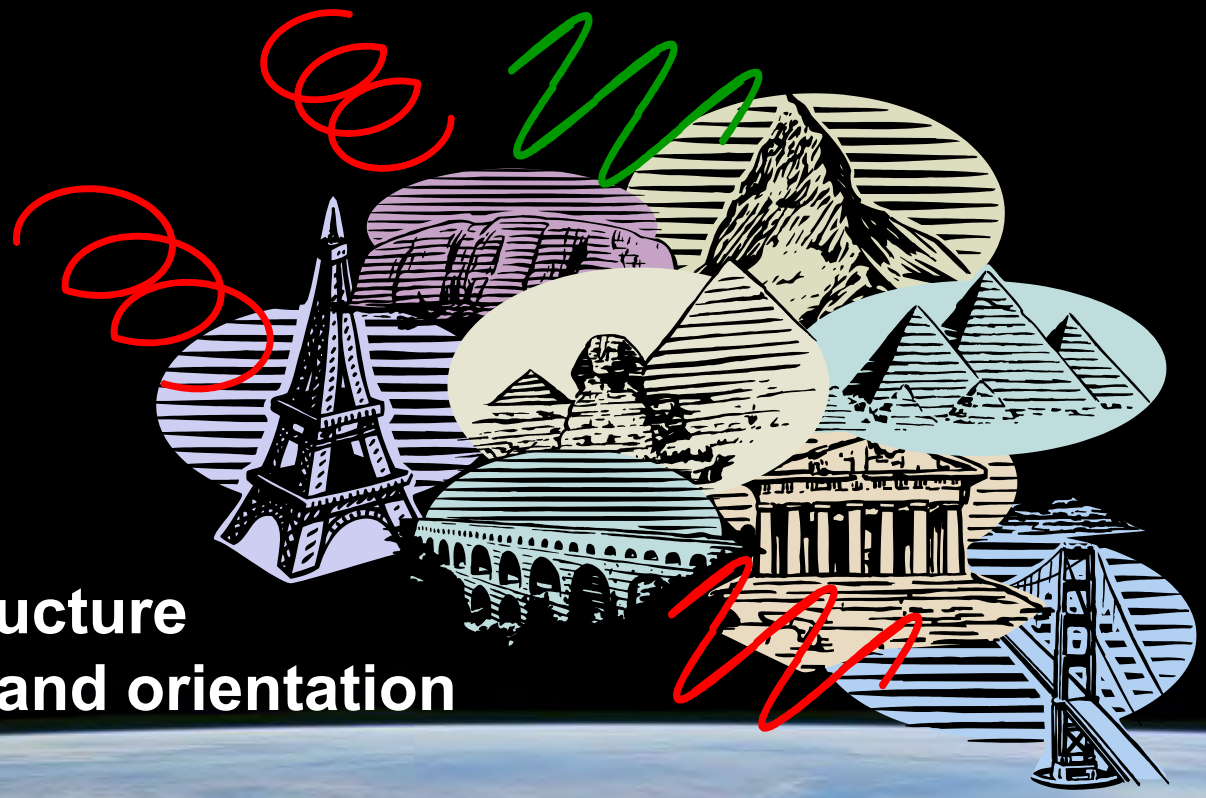
Radar Polarimetry (Polar : polarisation Metry: measure) is the science of acquiring, processing and analysing the polarization state of an electromagnetic field

Radar Polarimetry deals with the full vector nature of polarized electromagnetic waves

Radar Polarimetry



The POLARISATION information Contained in the waves backscattered from a given medium is highly related to:



its geometrical structure
reflectivity, shape and orientation

its geophysical properties such as humidity, roughness, …

SAR Polarimetry Applications

Forest Vegetation

- Forest Height
- Forest Biomass
- Forest Structure
- Canopy Extinction
- Underlying Topography

- Forest Ecology
- Forest Management
- Ecosystem Change
- Carbon Cycle

Agriculture

- Soil Moisture Content
- Soil roughness
- Height of Vegetation Layer
- Extinction of Vegetation Layer
- Moisture of Vegetation Layer

- Farming Management
- Water Cycle
- Desretification

Snow and Ice

- Topography
- Penetration Depth / Density
- Snow Ice Layer
- Snow Ice Extinction
- Water Equivalent

- Ecosystem Change
- Water Cycle
- Water Management

Urban Areas

- Geometric Properties
- Dielectric Properties

- Urban Monitoring

Courtesy of Dr. I. Hajnsek

A Bit Of History

Radar Polarimetry

Discovery of the Phenomena of Polarized Electromagnetic Energy

AD 1000
Use of the polarized skylight to locate a hidden sun

1669
First known Quantitative work on light observation

1677
Wave nature of light discovery
Explanation of the double refraction

Corpuscular model or « longitudinal » waves

1704
Corpuscular Model of light

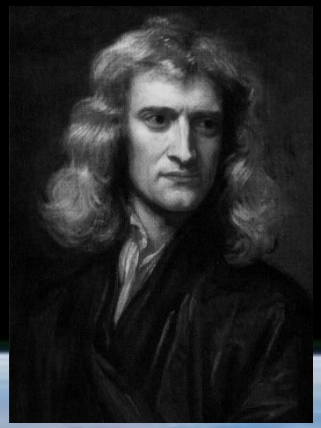
1808
Discovery of the polarization of light (intrinsic property of light and not of crystals)

Crystal of calcite
Iceland Spar
Sunstone

Bartholinus

Discovery of the double refraction in calcite

Huygens



Newton

Malus
X-1795

Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of **Polarization** leading to Radar Polarimetry

Brewster

1816

Fresnel

1820

Faraday

1832

« Transverse » nature of light waves

Stokes

1852

Maxwell

1873

Electromagnetic theory of light

Helmholtz

1881

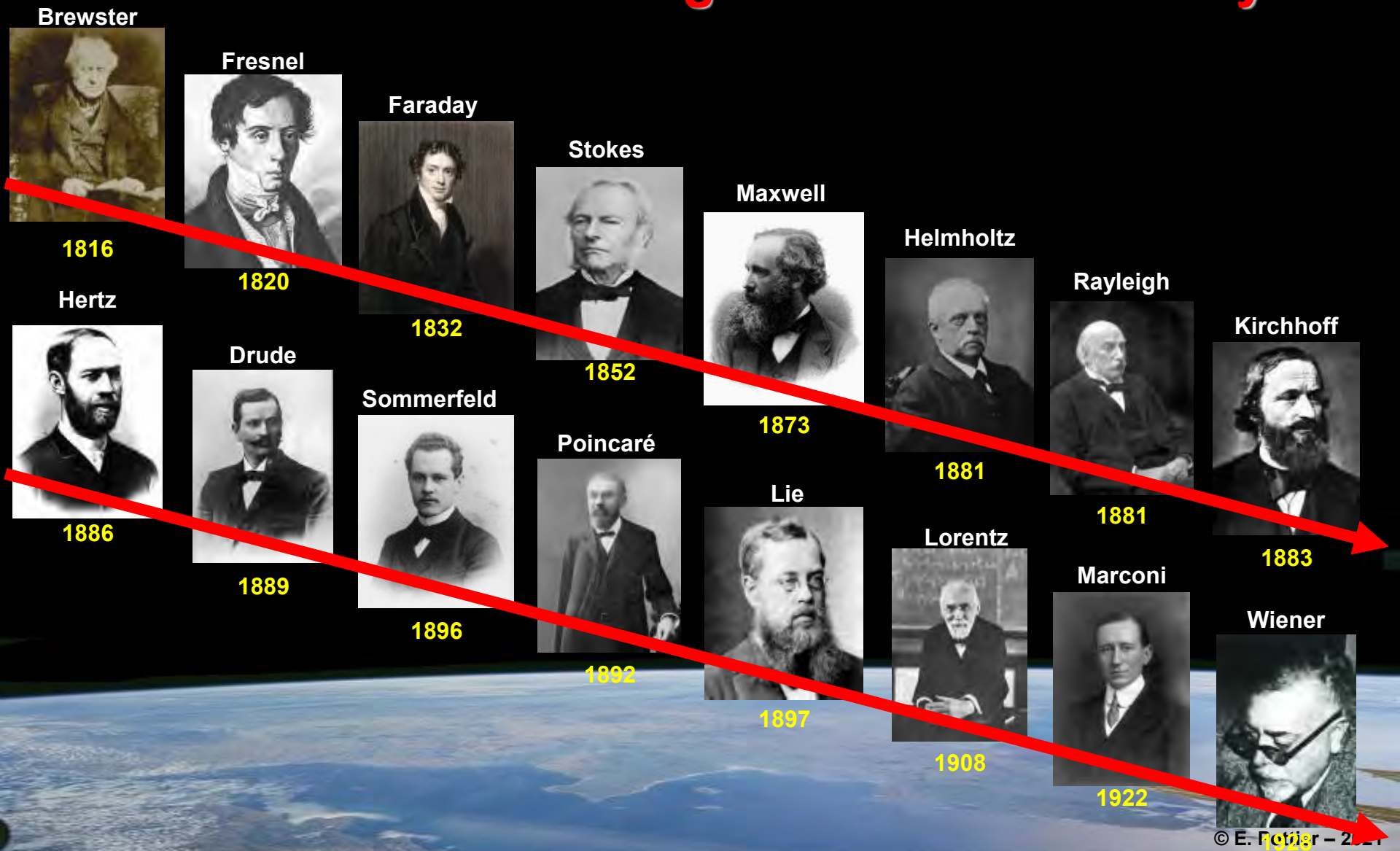
Rayleigh

1881

Kirchhoff

1883

Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of **Polarization** leading to Radar Polarimetry



Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of Polarization leading to Radar Polarimetry

Pauli

1950

Deschamp

1951

Born

1954

Wolf

1954

Kennaugh

1952

Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of Polarization leading to Radar Polarimetry

Kennaugh

1952

Huynen

1970

W. M. Boerner

1980 - 2018

The Radar Polarimetric Triptych

Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of Polarization leading to Radar Polarimetry


Kennaugh
1952


Huynen
1970


W. M. Boerner
1980 - 2018


K. Raney


J.J. Van Zyl


A. Freeman


R. Touzi


J.S. Lee


T. Ainsworth


S.R. Cloude


E. Pottier


P. Dubois


Y. Yamaguchi


C. Lopez

1990 - 2000
Radar Polarimetry
Scientific Progress


H. Mott


E. Lueneburg


E. Krogager


A. Moreira


Y.L. Desnos


Z. Czyz


K. Papathanassiou

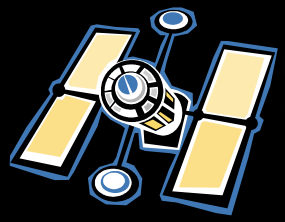

I. Hajnsek


T. Le Toan


L. Ferro-Famil

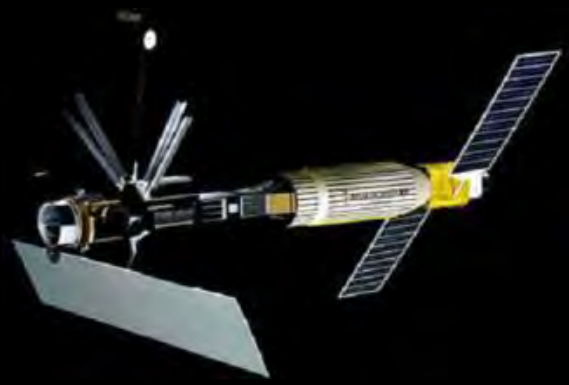

J.C. Souyris

Polarimetric Radar (SAR)

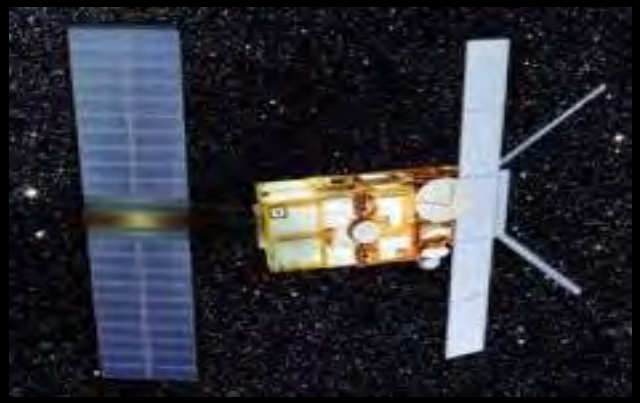


Spaceborne Sensors

Space-borne Sensors

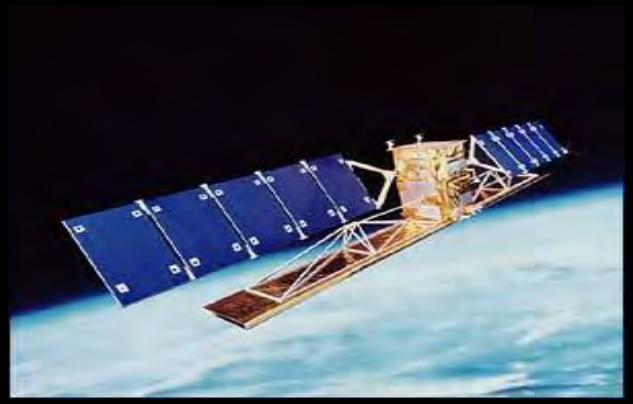


SEASAT
NASA/JPL (USA)
L-Band, 1978

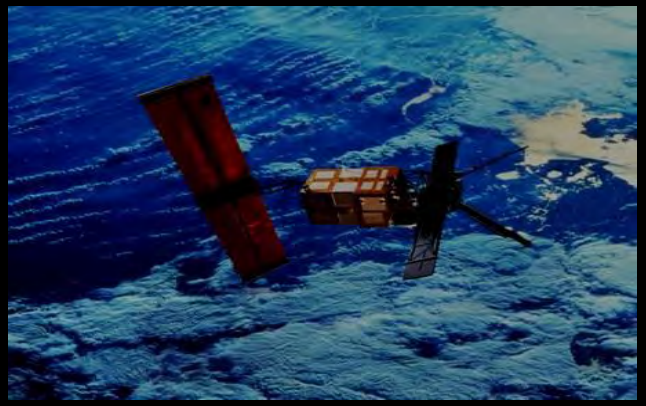


ERS-1
European Space Agency (ESA)
C-Band, 1991-2000

J-ERS-1
Japanese Space Agency (NASDA)
L-Band, 1992-1998



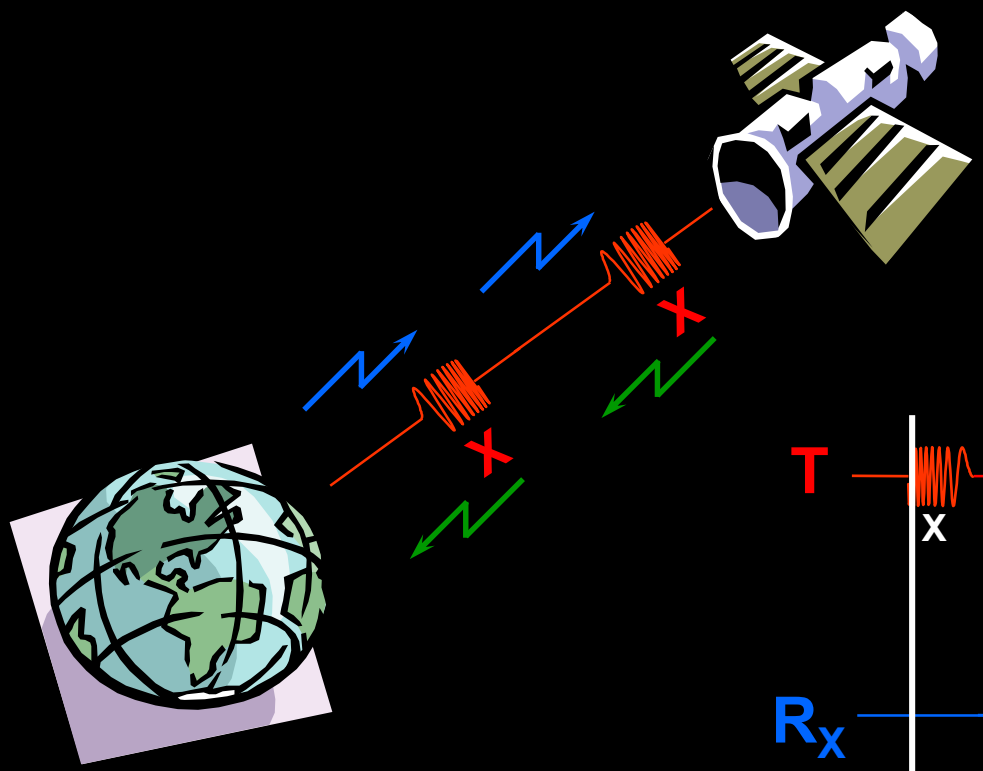
RadarSAT-1
Canadian Space Agency (CSA)
C-Band, 1995

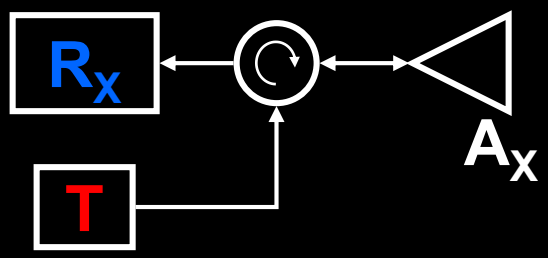


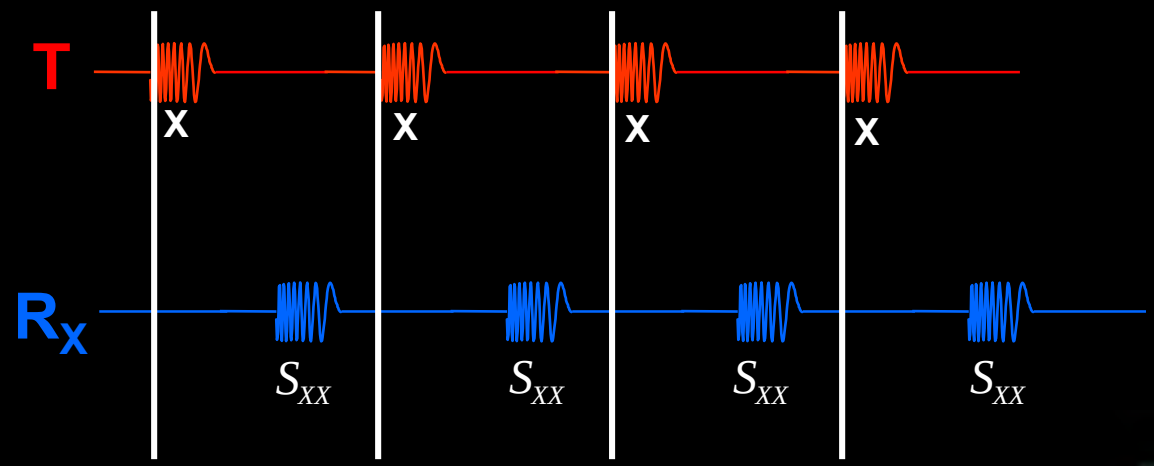
ERS-2
European Space Agency (ESA)
C-Band, 1995

Shuttle Radar Topography Mission
NASA/JPL (C-Band), DLR (X-Band)
February 2000

Scattering Coefficient





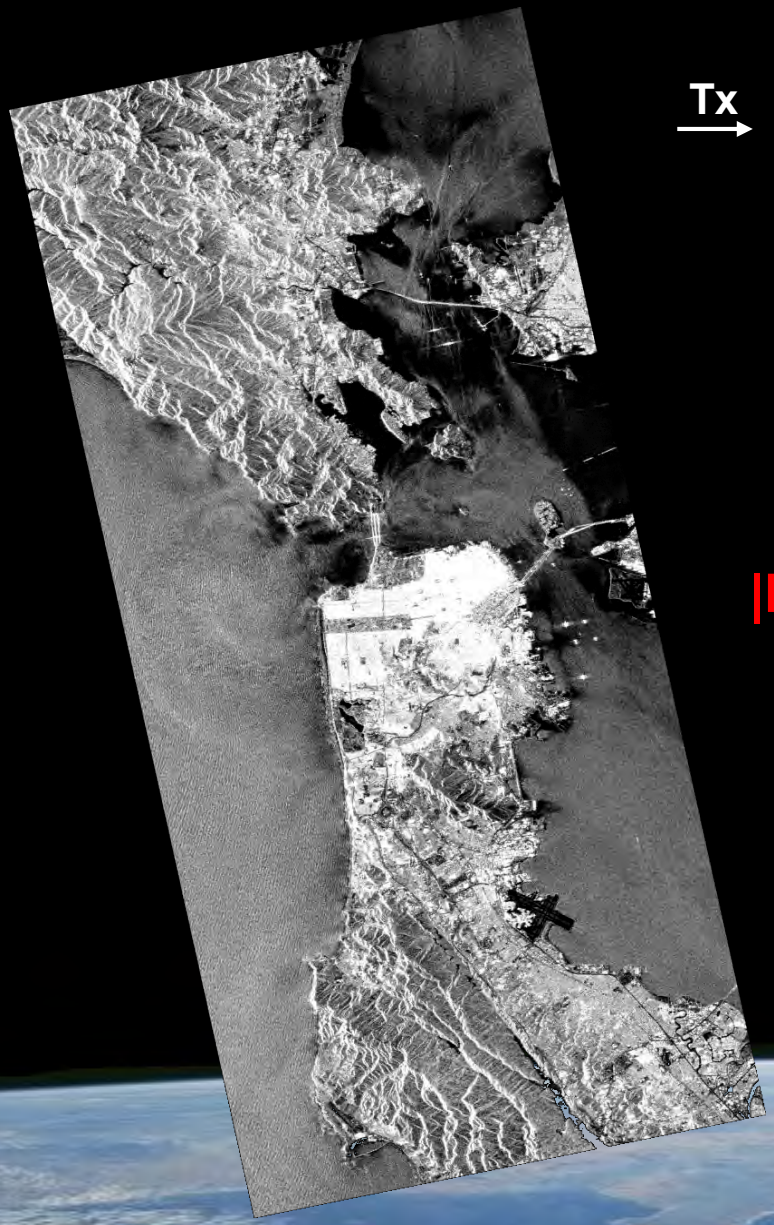


TRANSMITTER: X
RECEIVER: X

BACKSCATTERING COEFFICIENT $\{ S_{XX} \}$

NO POLARIMETRY

Space-borne Sensors

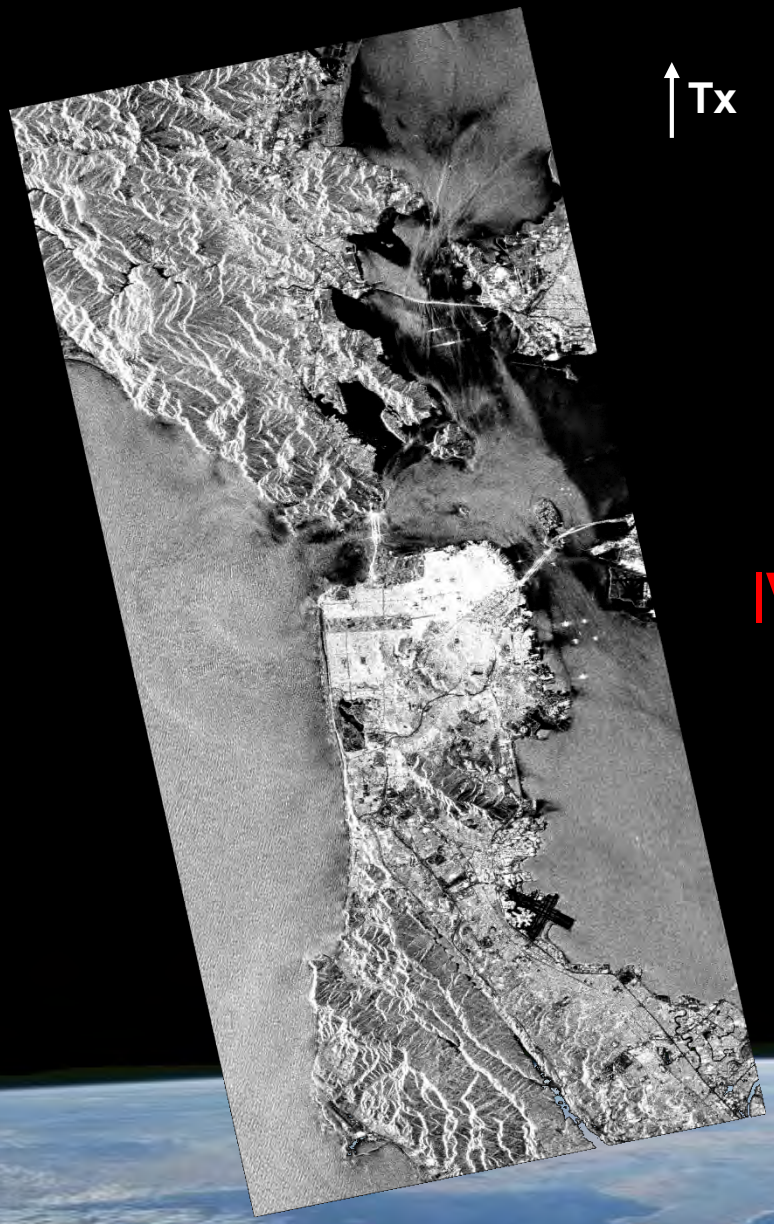


Tx → Rx →

$|HH|_{dB}$

San Francisco Bay – (L-Band)

Space-borne Sensors



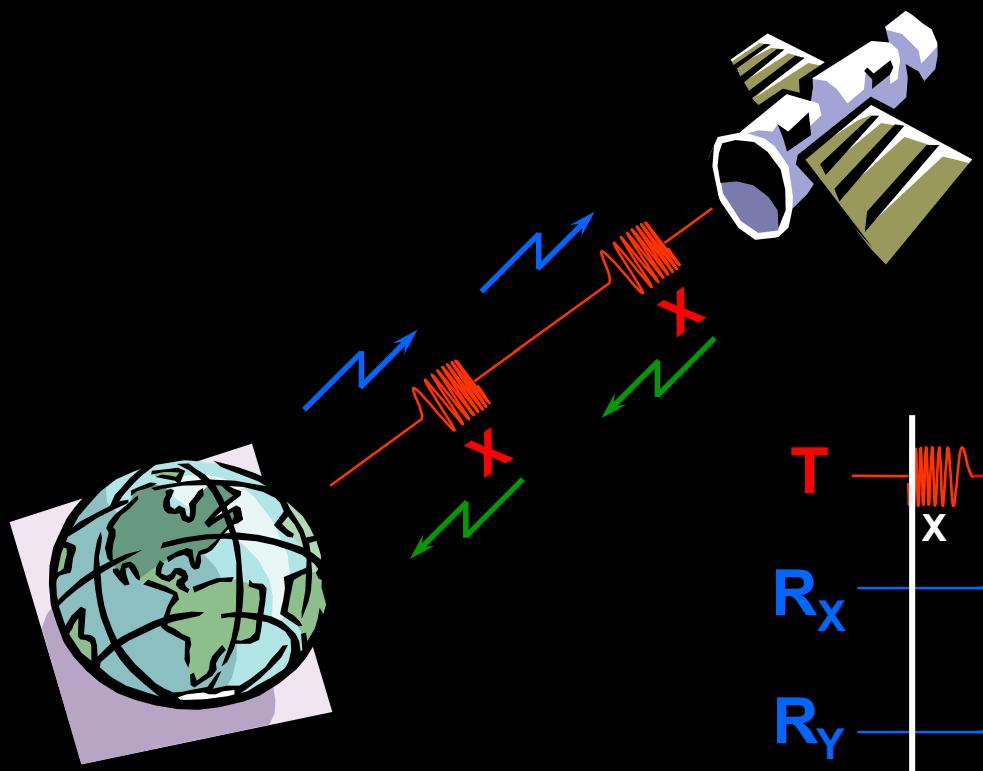
Tx

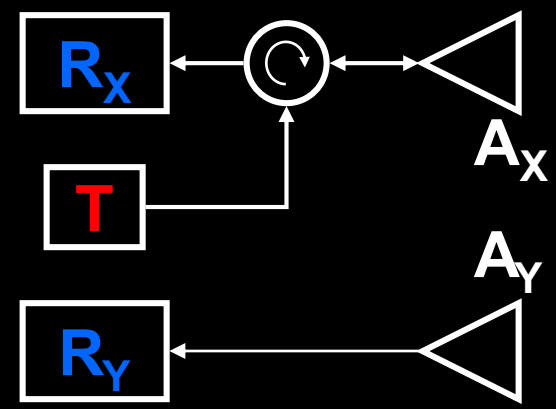
Rx

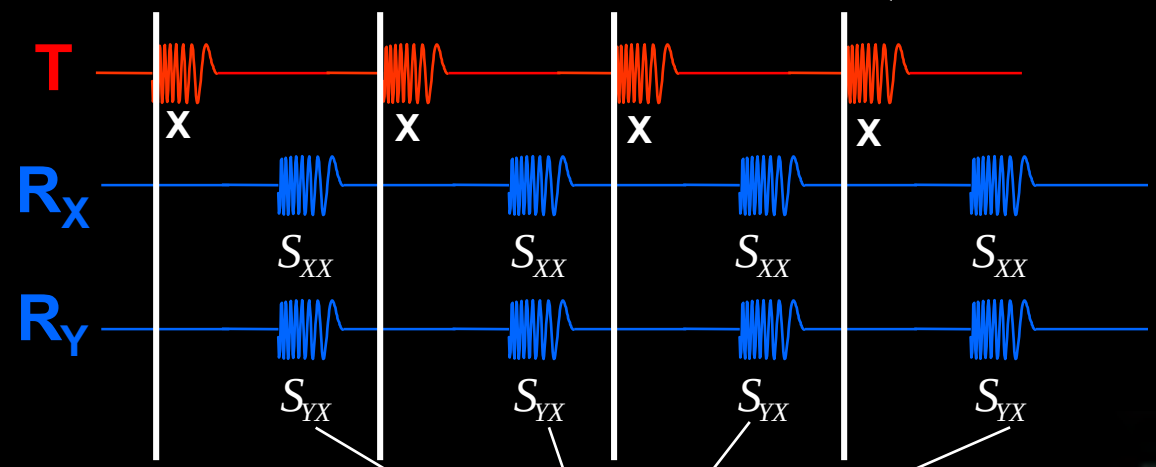
$|VV|_{dB}$

San Francisco Bay – (L-Band)

Wave Polarimetry





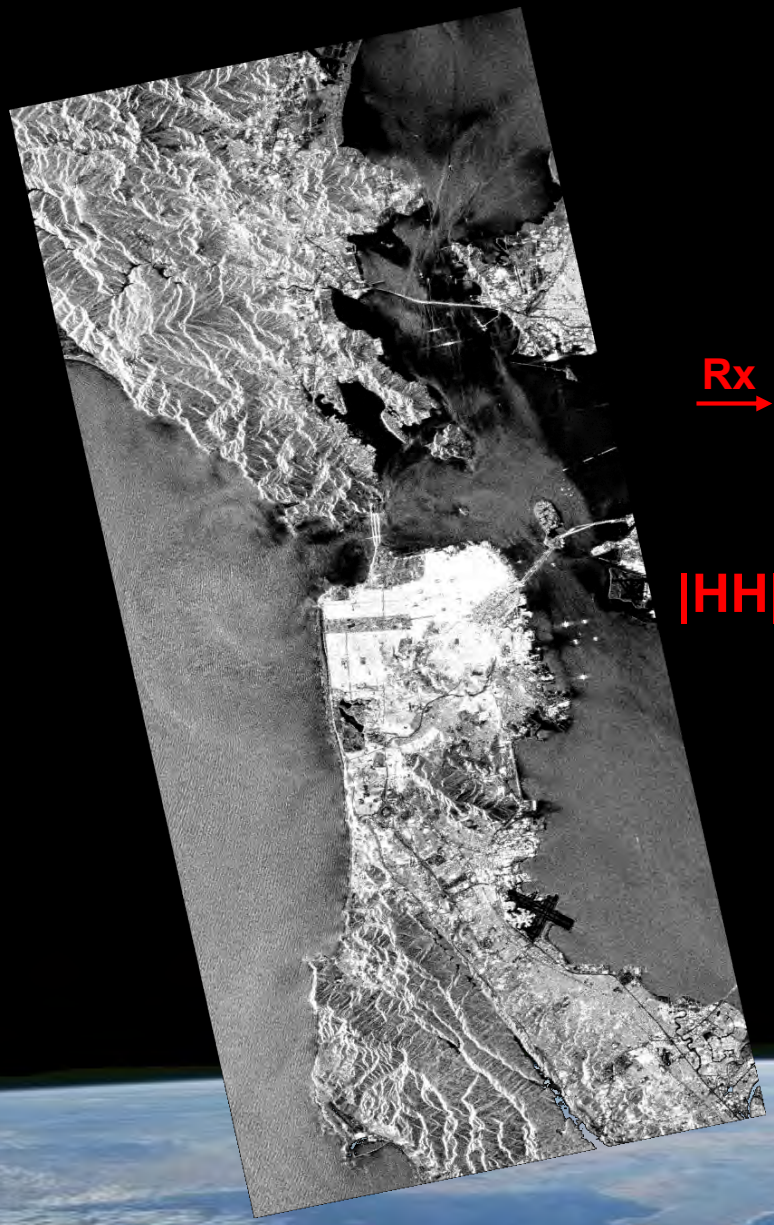


TRANSMITTER: X
RECEIVERS: X & Y

JONES VECTORS $\left\{ \underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix} \right\}$

WAVE POLARIMETRY

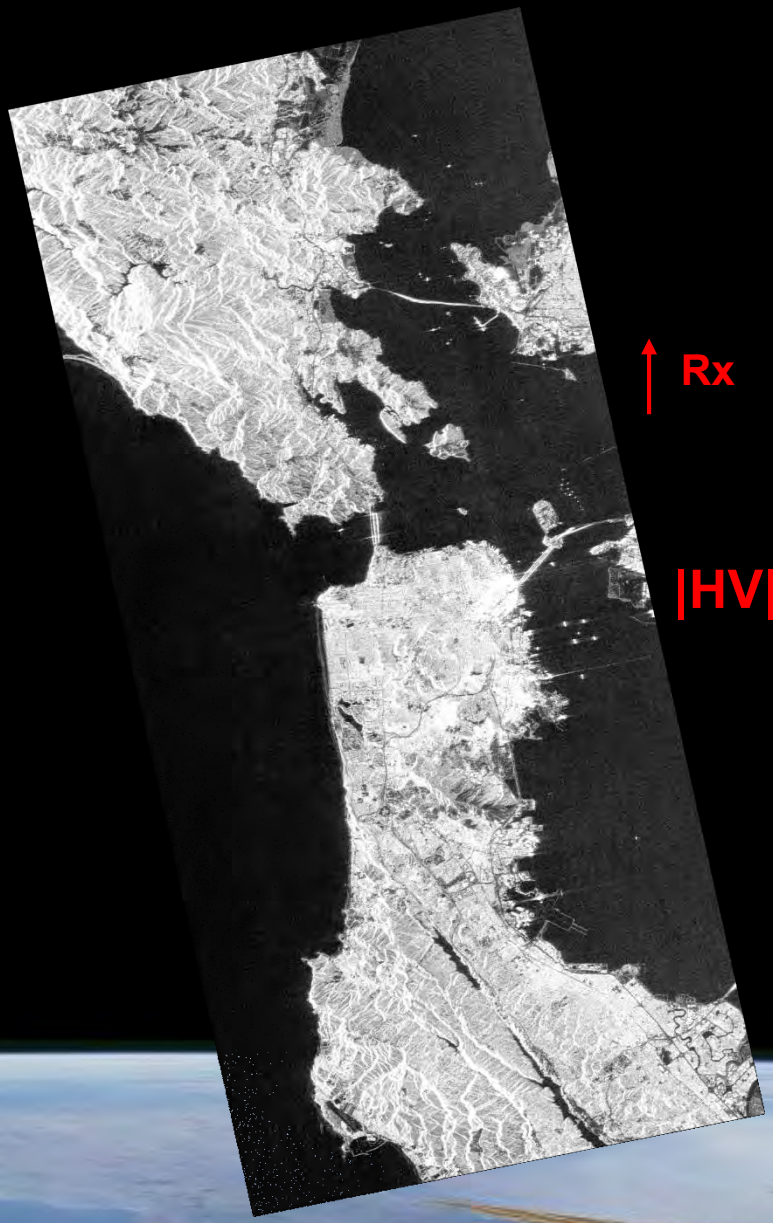
Space-borne PolSAR Sensors



Tx

Rx

$|HH|_{dB}$



Rx

$|HV|_{dB}$

San Francisco Bay – (L-Band)

Space-borne PolSAR Sensors

ENVISAT - ASAR

October 2001
C-Band (Sngl / Dual Inc)

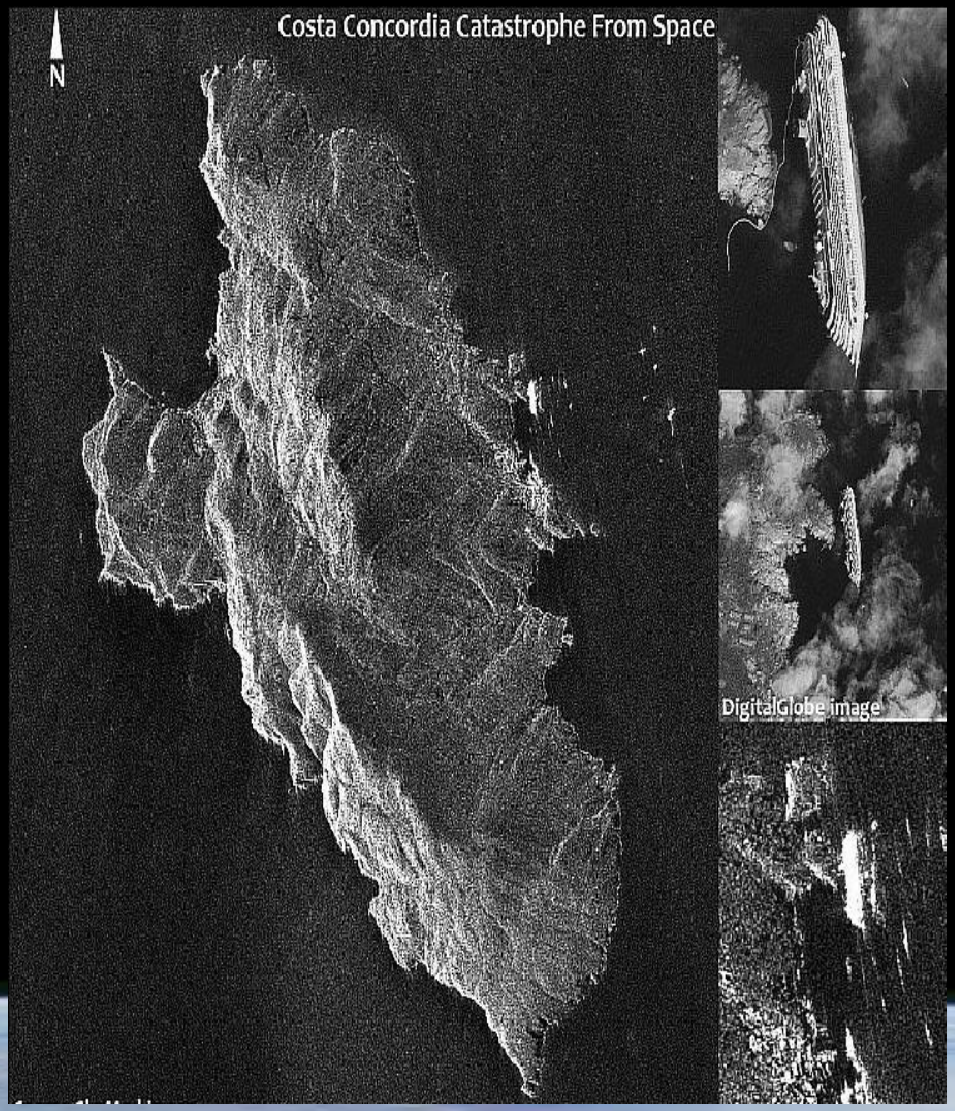
Space-borne PolSAR Sensors

COSMO - SkyMed

agenzia spaziale italiana

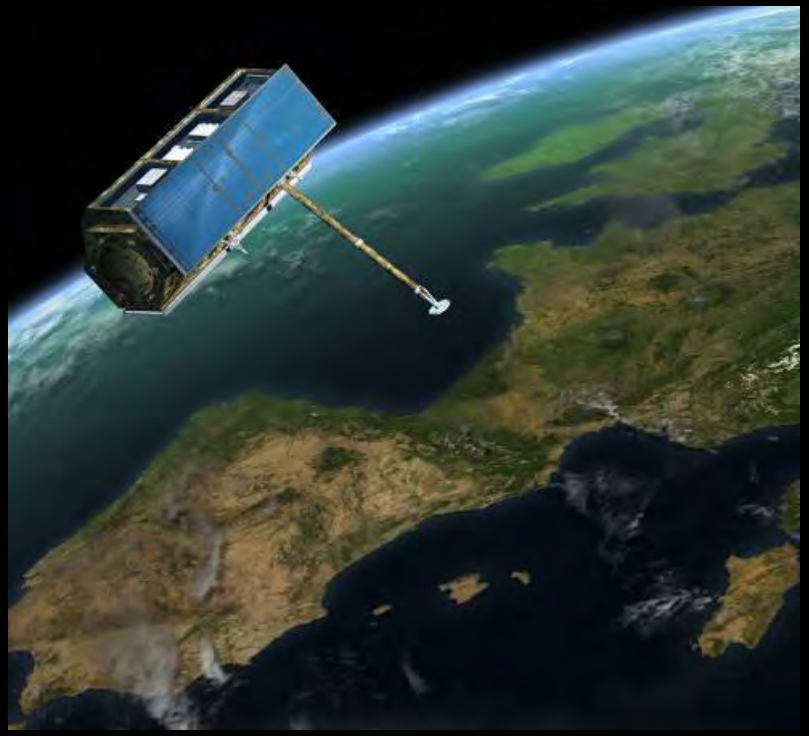
TELESPAZIO
A Finmeccanica / Thales Company

June 2007, Dec. 2007
Oct. 2008, Nov. 2010
X-Band (Sngl / Dual)
Revisit : 1 day



Space-borne PolSAR Sensors

TerraSAR - X

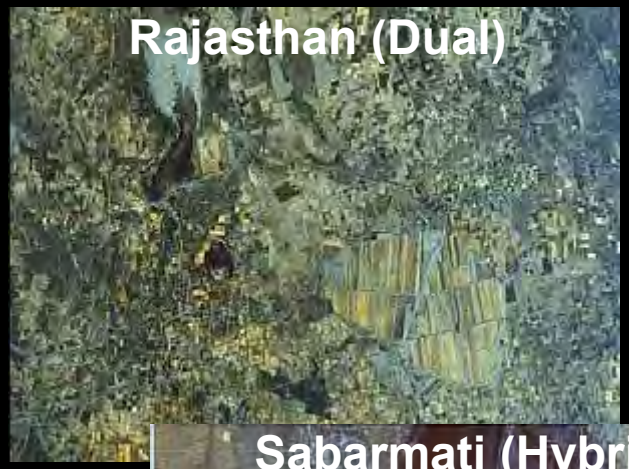


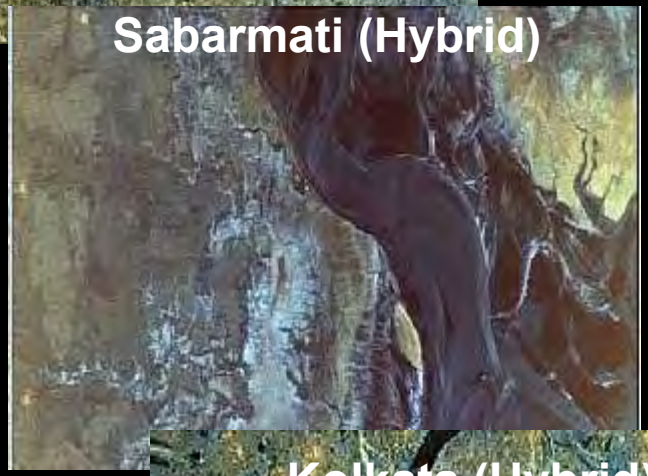
June 2007

X-Band (Sngl / Twin HH-VV / Quad Exp.)

Space-borne PolSAR Sensors

RISAT-1A





26 April 2012
C-Band (Sngl, Dual, Hybrid)
Operational since 2015

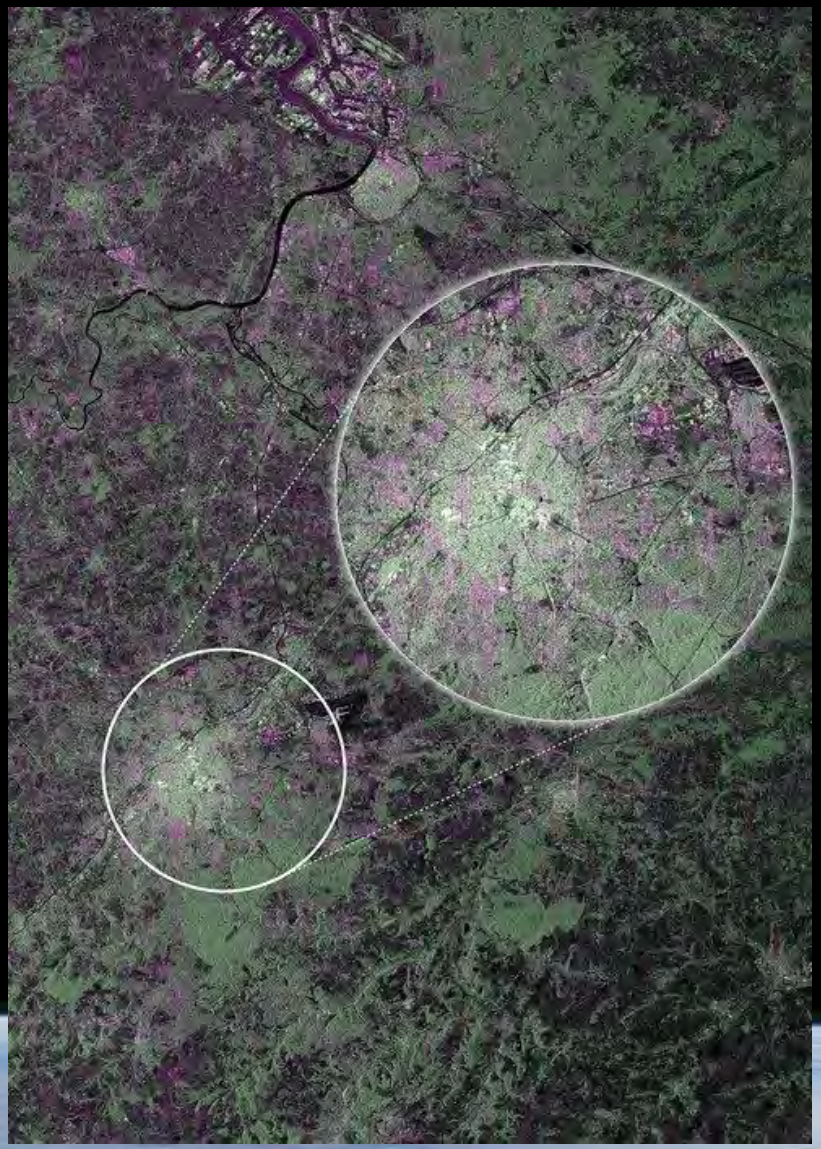
Space-borne PolSAR Sensors

SENTINEL – 1A

S1A : April 2014 S1B : April 2016

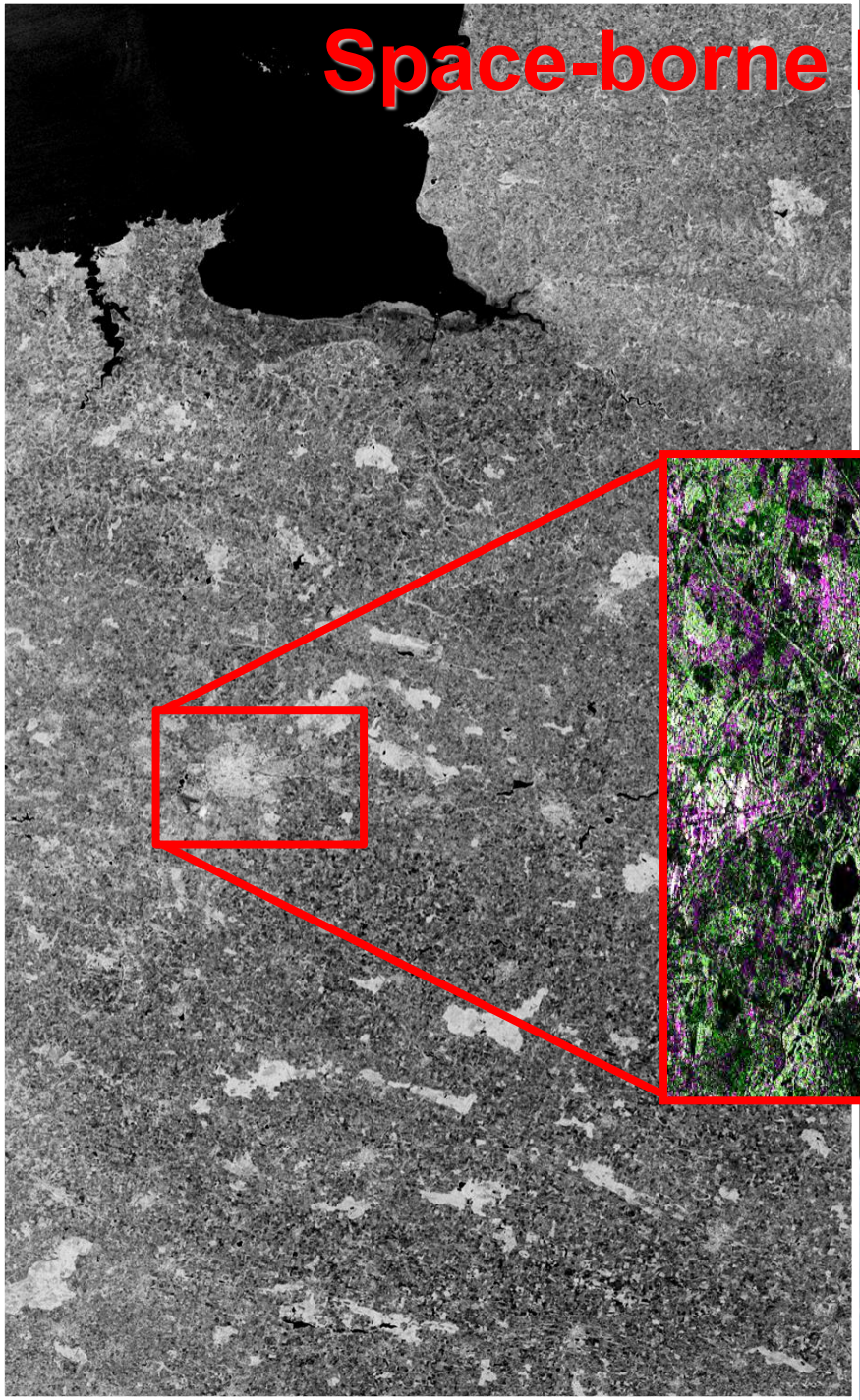
C-Band (Sngl, Dual)

Revisit : 6 days



Brussels – 12 April 2014

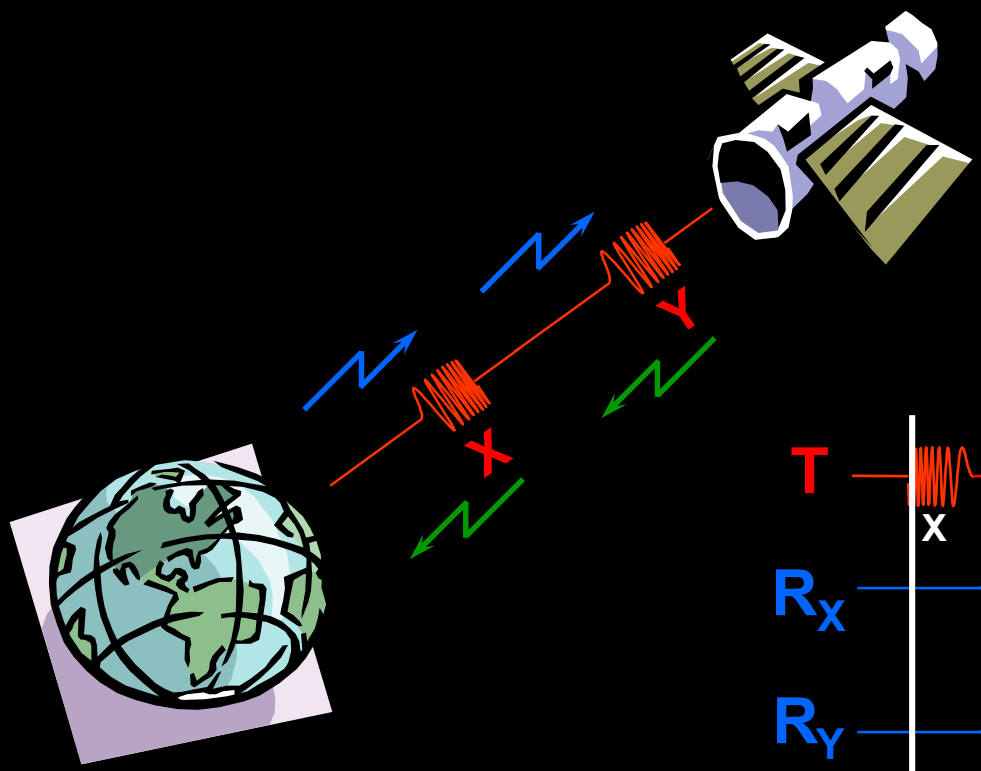
Space-borne PolSAR Sensors

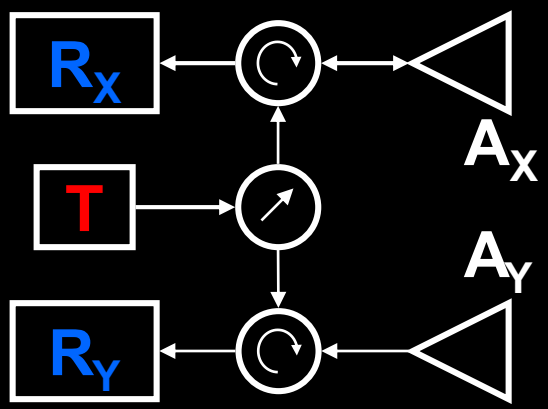


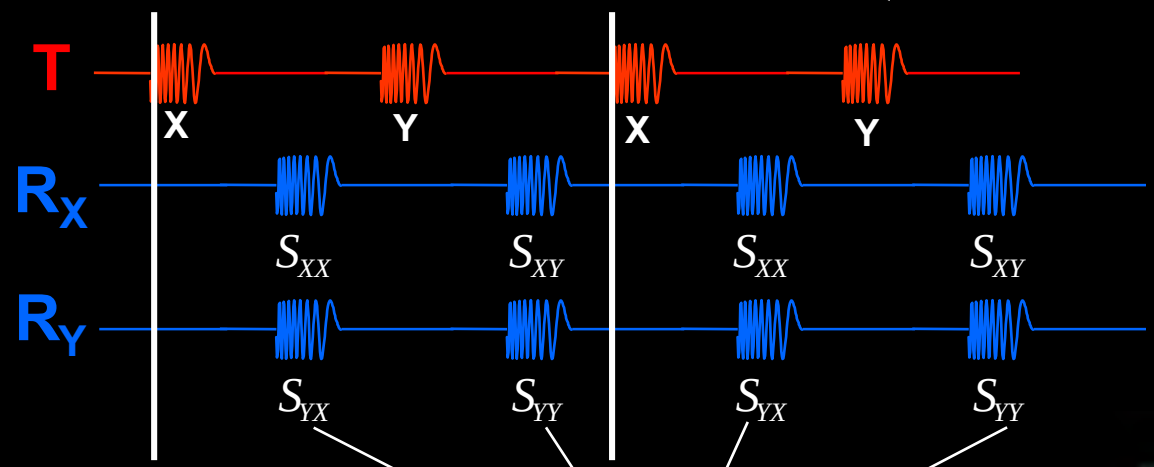
Rennes
Britanny
France

17/08/2016 @ 17h56

Scattering Polarimetry







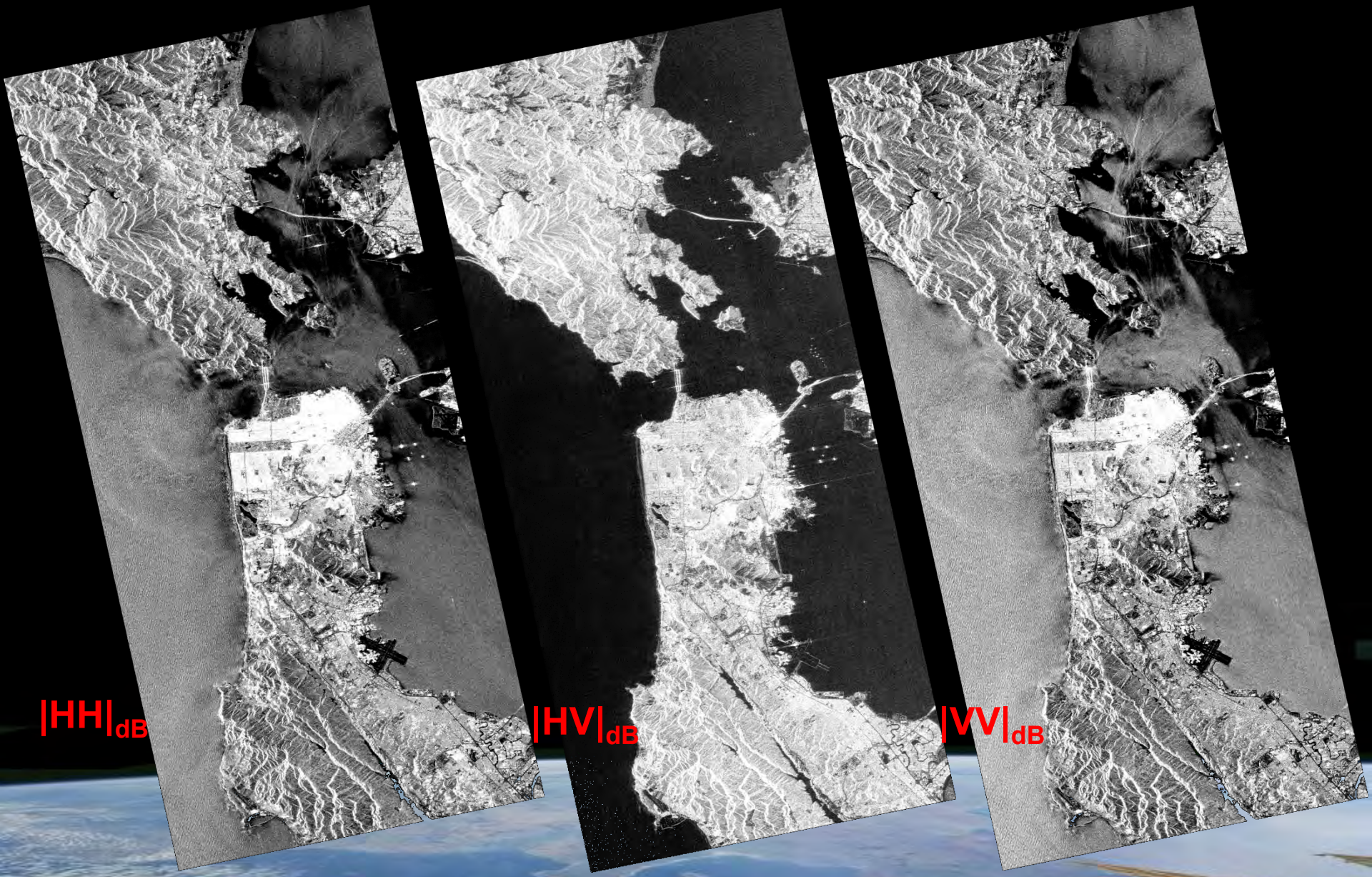
TRANSMITTER: X & Y
RECEIVERS: X & Y

SINCLAIR MATRICES

$$\left\{ [S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \right\}$$

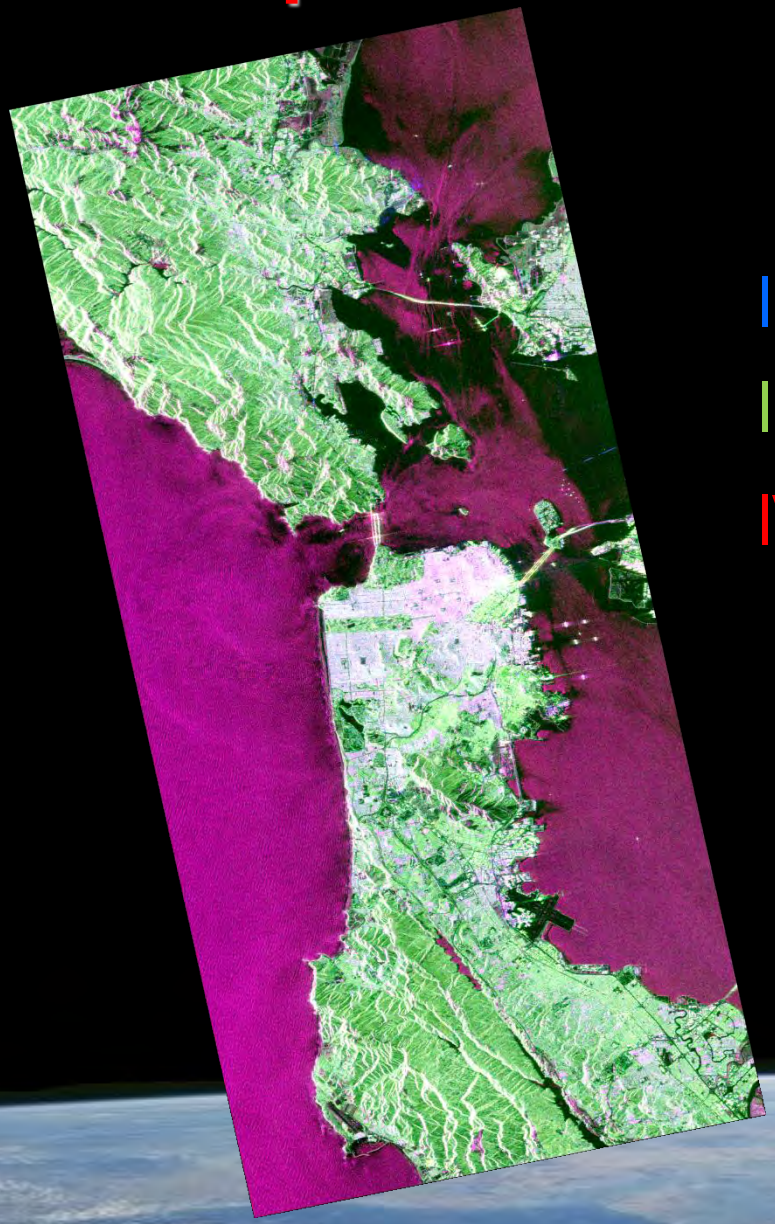
SCATTERING POLARIMETRY

Space-borne PolSAR Sensors



San Francisco Bay – (L-Band)

Space-borne PolSAR Sensors



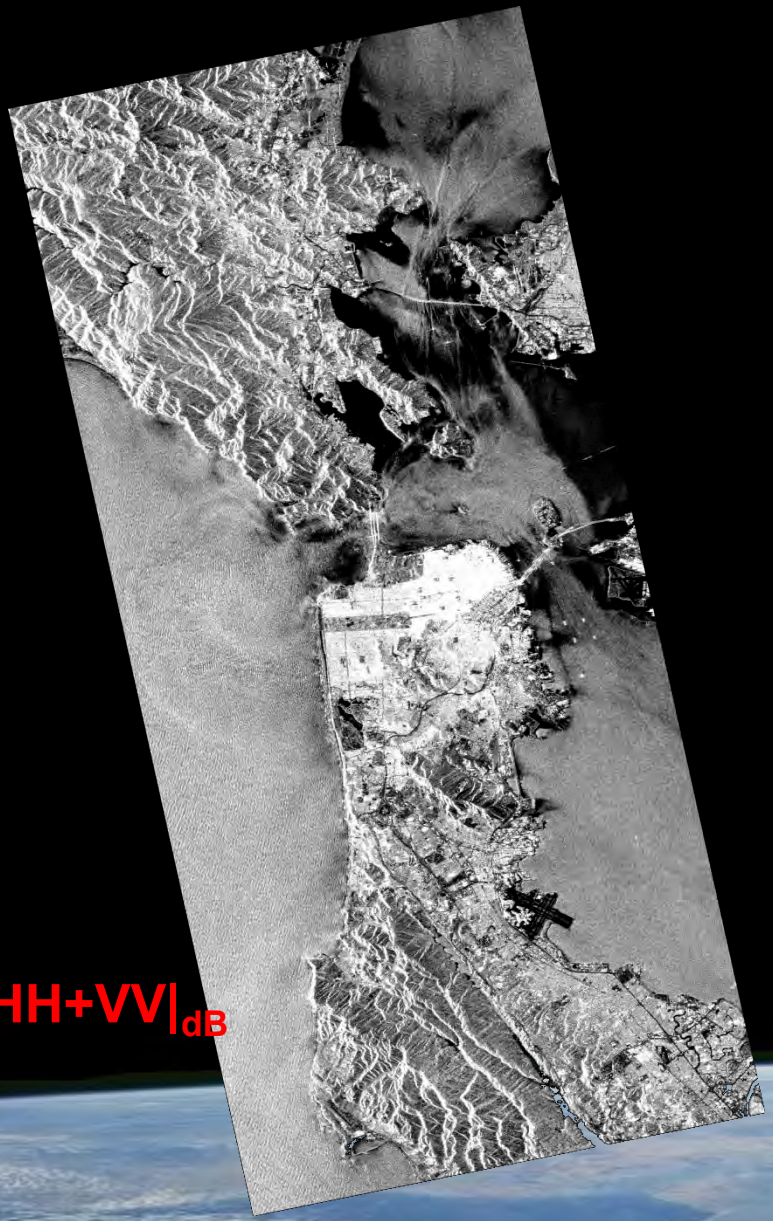
$|HH|_{dB}$

$|HV|_{dB}$

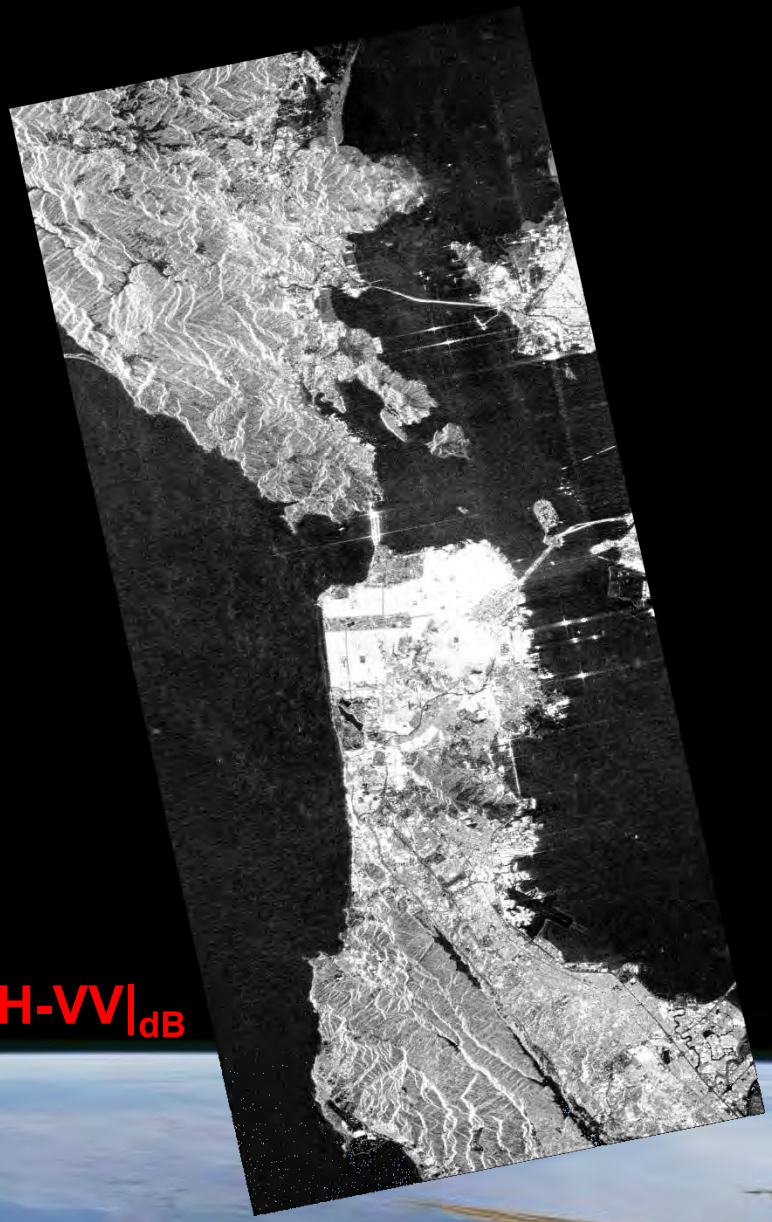
$|VV|_{dB}$

San Francisco Bay – (L-Band)

Space-borne PolSAR Sensors



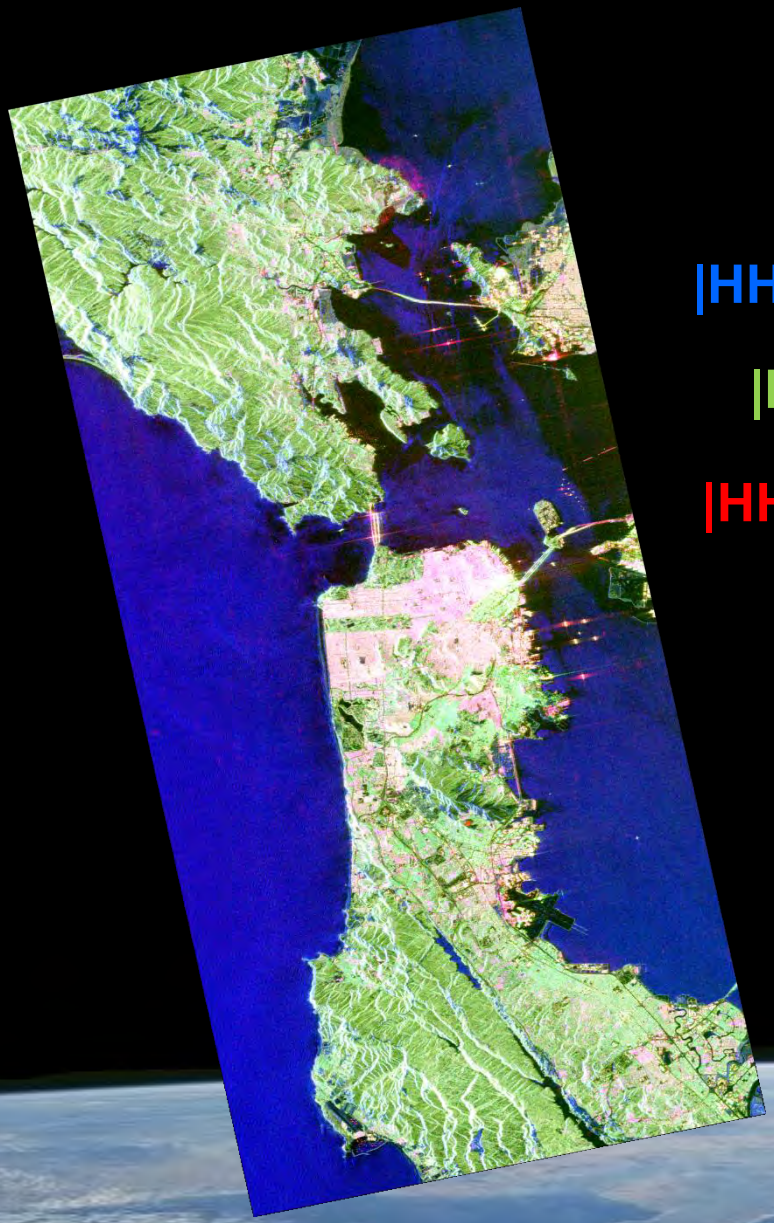
$|HH+VV|_{dB}$



$|HH-VV|_{dB}$

San Francisco Bay – (L-Band)

Space-borne PolSAR Sensors



$|HH+VV|_{dB}$

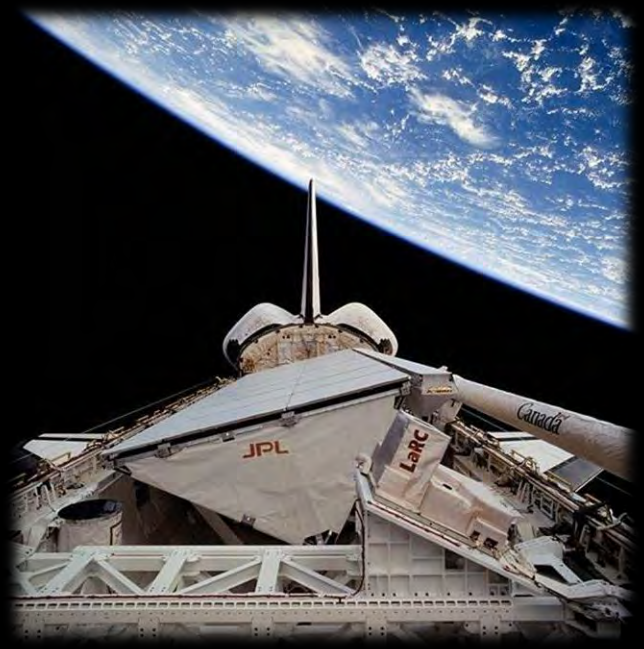
$|HV|_{dB}$

$|HH-VV|_{dB}$

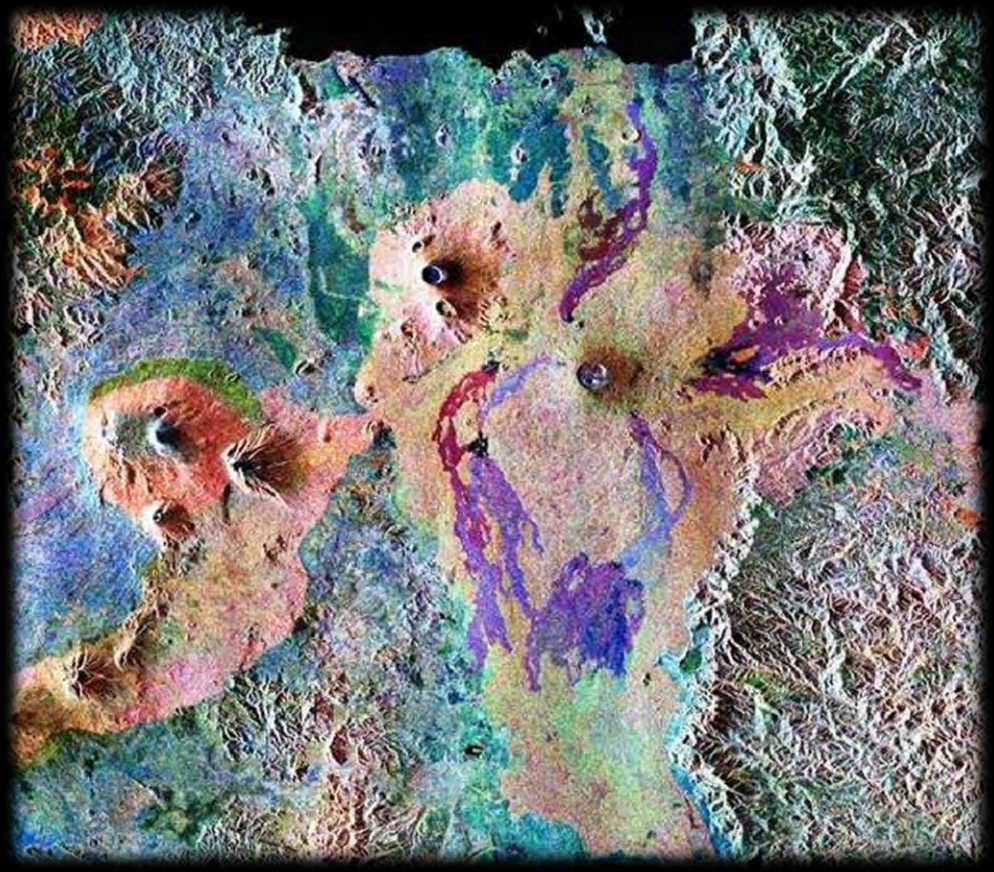
San Francisco Bay – (L-Band)

Space-borne PolSAR Sensors

SIR-C / X-SAR



April 1994
L- and C-Band (Quad)
X-Band (Sngl)



Rwanda, Zaire, Uganda

Space-borne PolSAR Sensors

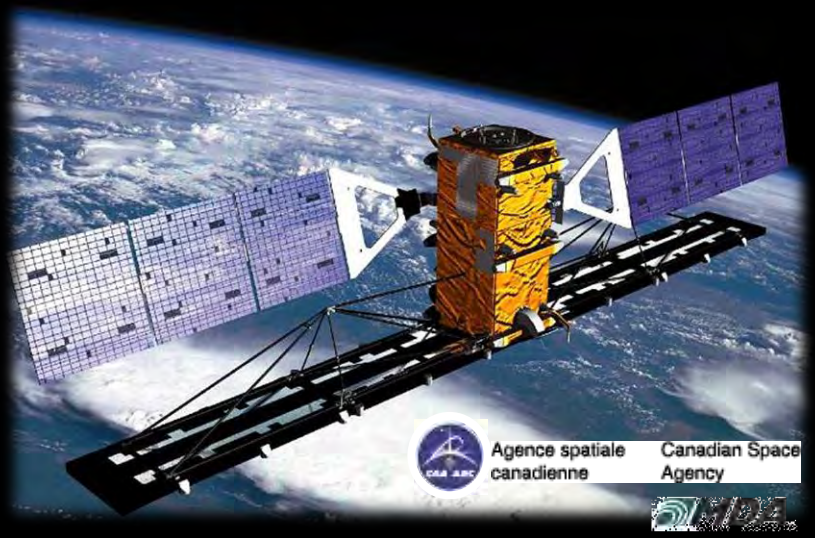
ALOS - PALSAR

January 2006
L-Band (Sngl / Twin / Quad)

ALOS : Advanced Land Observing Satellite
PALSAR : Phase Array L-Band SAR

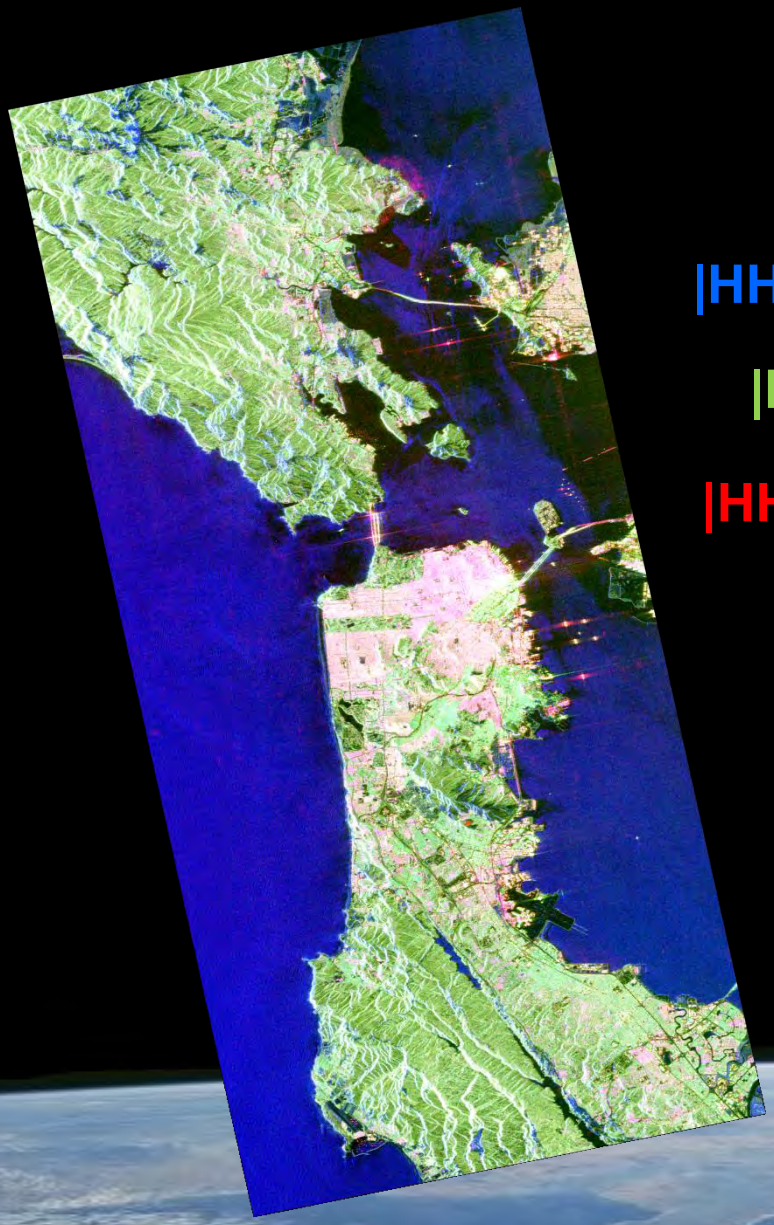
Space-borne PolSAR Sensors

RADARSAT - 2



December 2007
C-Band (Quad)

Space-borne PolSAR Sensors



$|HH+VV|_{dB}$

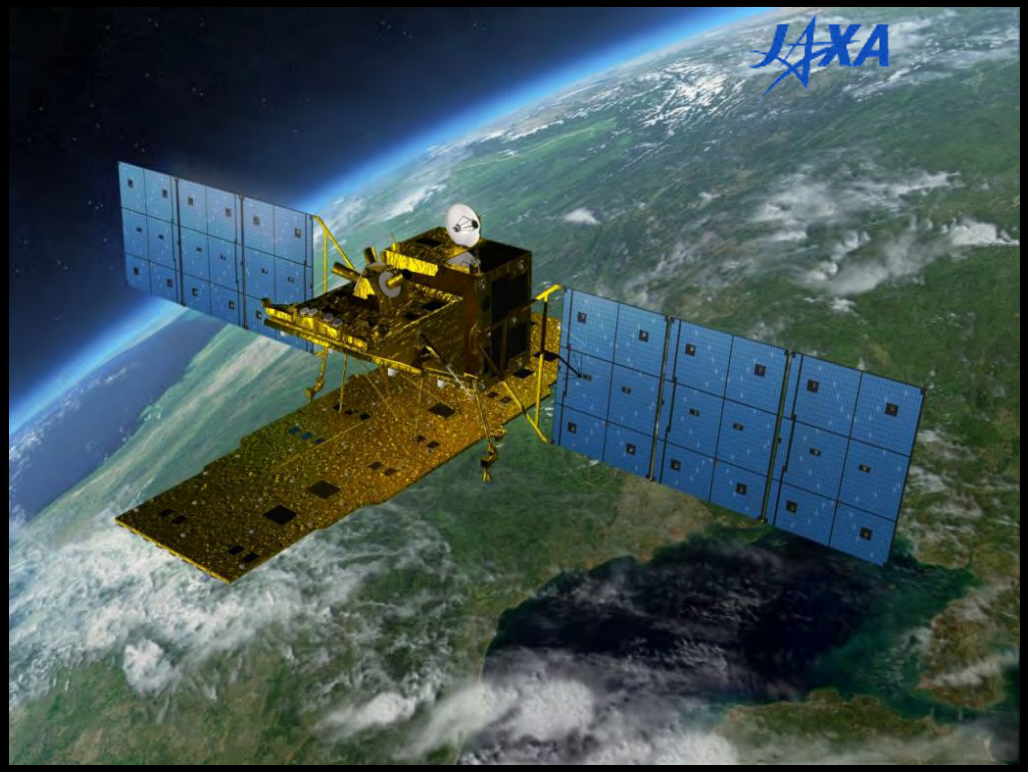
$|HV|_{dB}$

$|HH-VV|_{dB}$

San Francisco Bay – (L-Band and C-Band)

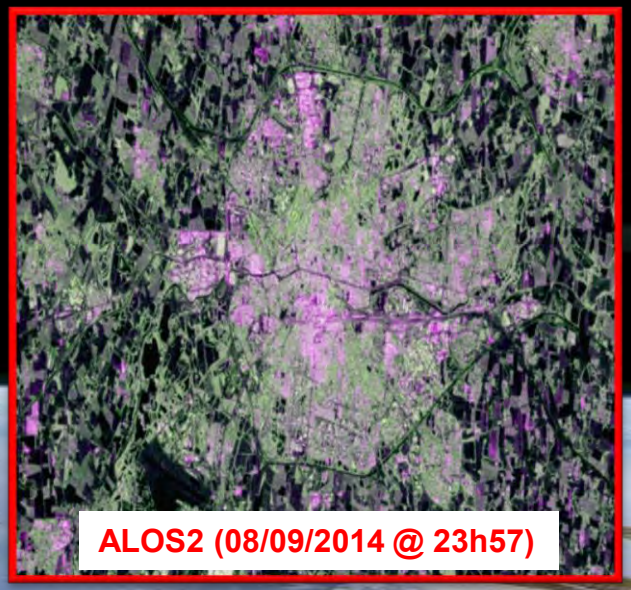
Space-borne PolSAR Sensors

ALOS - 2



May 2014

L-Band (Quad)



ALOS2 : Advanced Land Observing Satellite

PALSAR2 : Phase Array L-Band SAR

Space-borne PolSAR Sensors

$|HH+VV|_{dB}$

$|HV|_{dB}$

$|HH-VV|_{dB}$

San Francisco Bay – (L-Band : ALOS-1 and ALOS-2)

Space-borne PolSAR Sensors

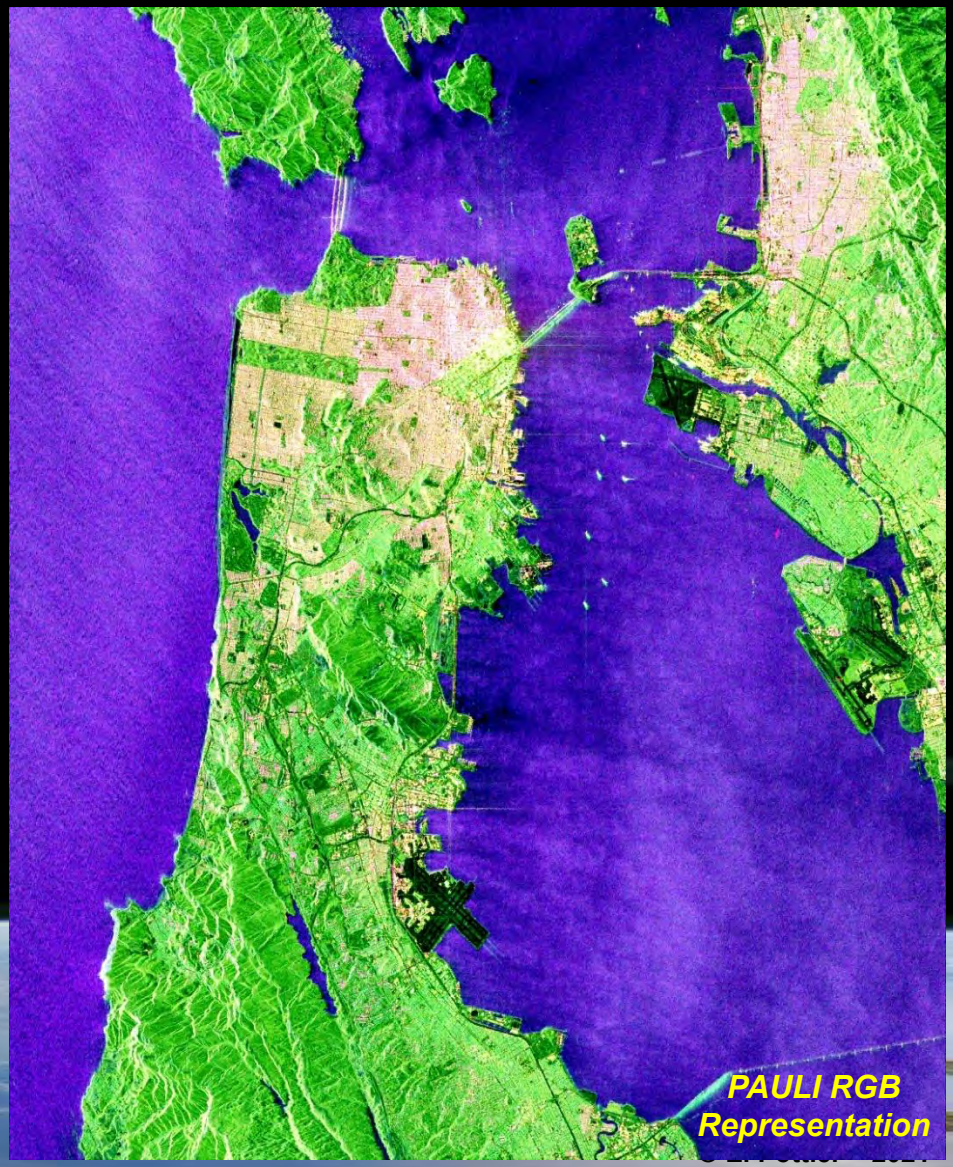
Chang Zheng-4C - GaoFen-3 (GF-3)

Long March-4C - High Resolution-3

August 2016

C-Band (Quad)

中国空间技术研究院
China Academy of Space Technology



IECAS

Courtesy of Dr. Qiu Xiaolan

IECAS / GIPAS

Space-borne PolSAR Sensors

SAOCOM – SAR-L

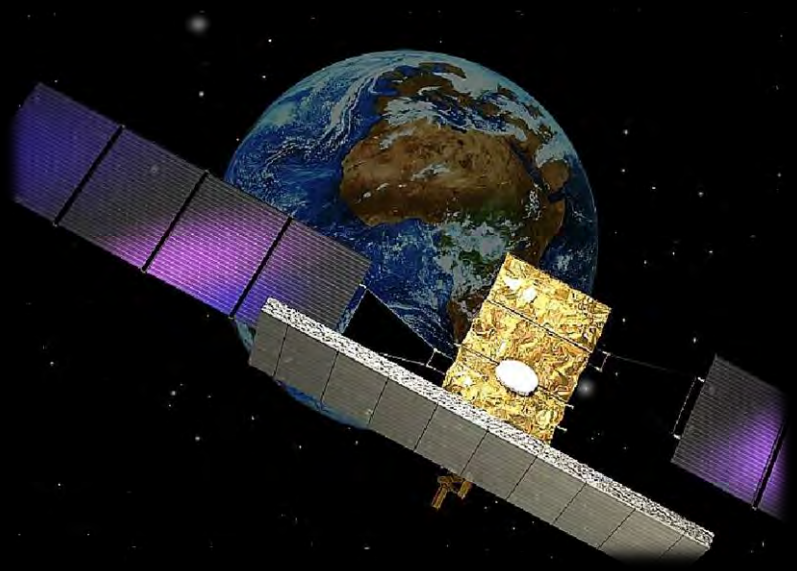
1A / 1B : Oct. 2018, August 2020
L-Band (Sngl, Dual, Quad)
Revisit : 4 days

RADARSAT Constellation Mission (RCM)

1A / 1B / 1C : June 2019
C-Band (Sngl, Dual, Quad, Hybrid)
Revisit : 4 days

Space-borne PolSAR Sensors

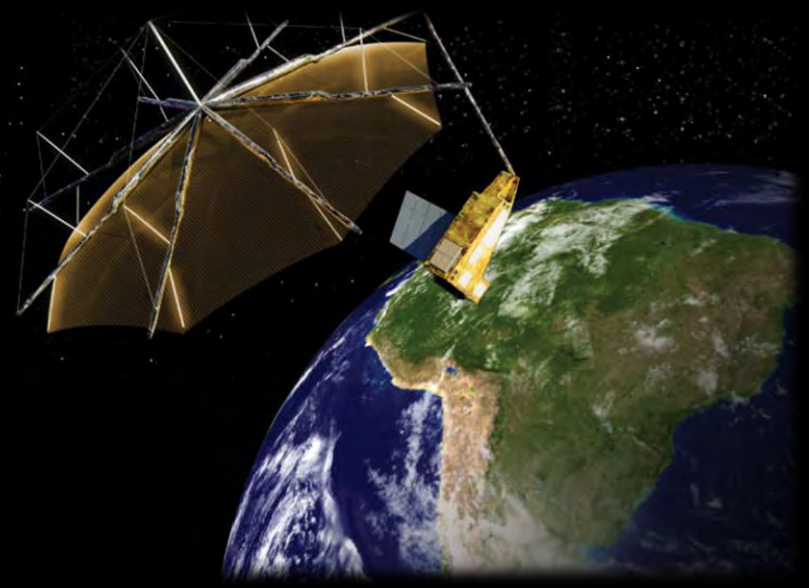
COSMO - SkyMed - CSG



2A / 2B : Dec. 2019, Dec. 2020

X-Band (Sngl / Dual / Quad)

7th Earth Explorer - BIOMASS

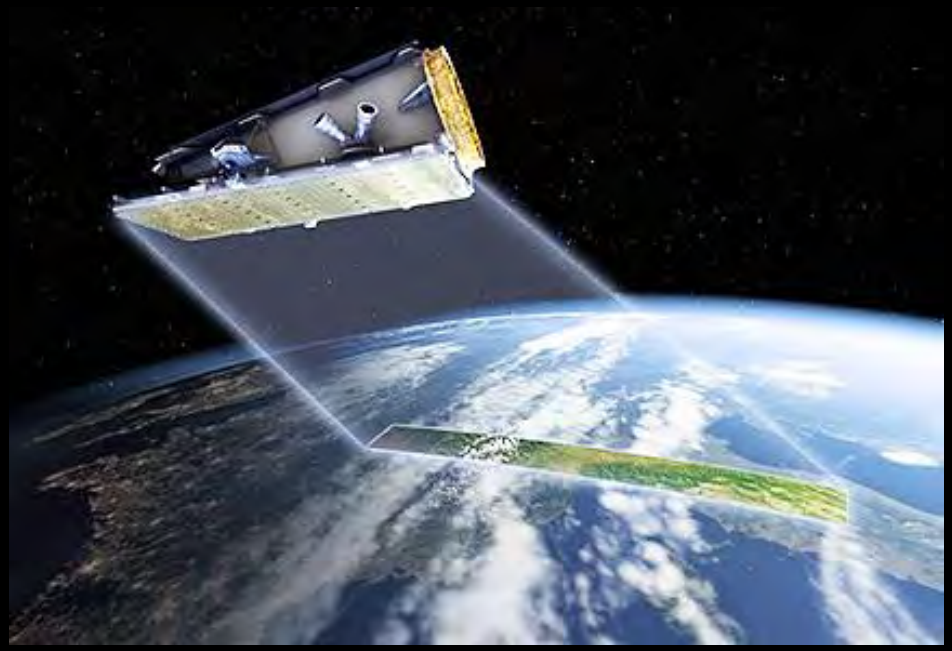


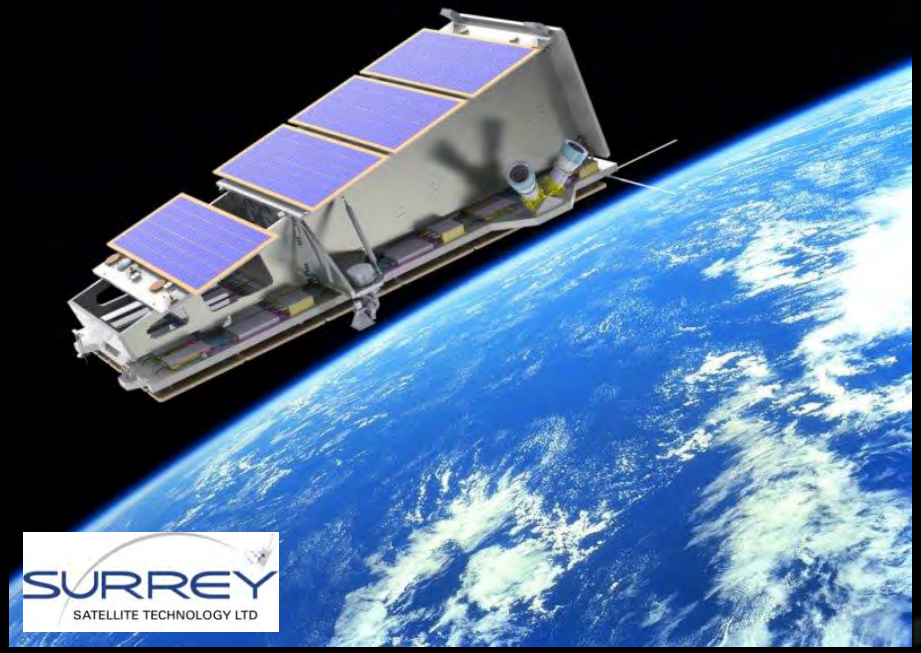
Planned on 2022

P-Band (Quad)

Space-borne PolSAR Sensors

NovaSAR - S





2018-2019

S-Band - resolution = 10-30m

Sngl (HH,VV) / Dual (HH,VH) or (VV, HV)

Tri (HH, VV, HV or VH) / Quad (HH, HV, VH, VV)

Space-borne PolSAR Sensors

NISAR
(NASA - ISRO SAR)

2020

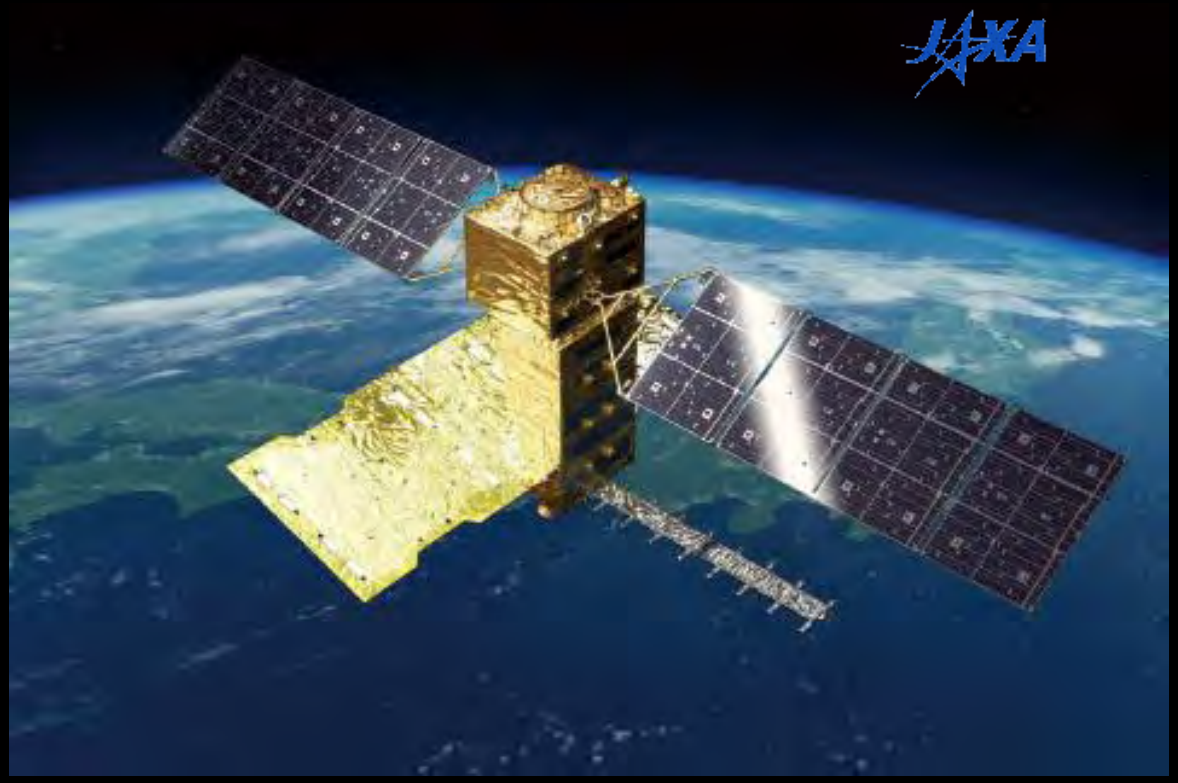
L- and S-Band

Sngl (HH,VV) / Dual (HH,VH) or (VV, HV)

Quad (HH, HV, VH, VV) / Compact (RH, RV)

Space-borne PolSAR Sensors

ALOS - 4



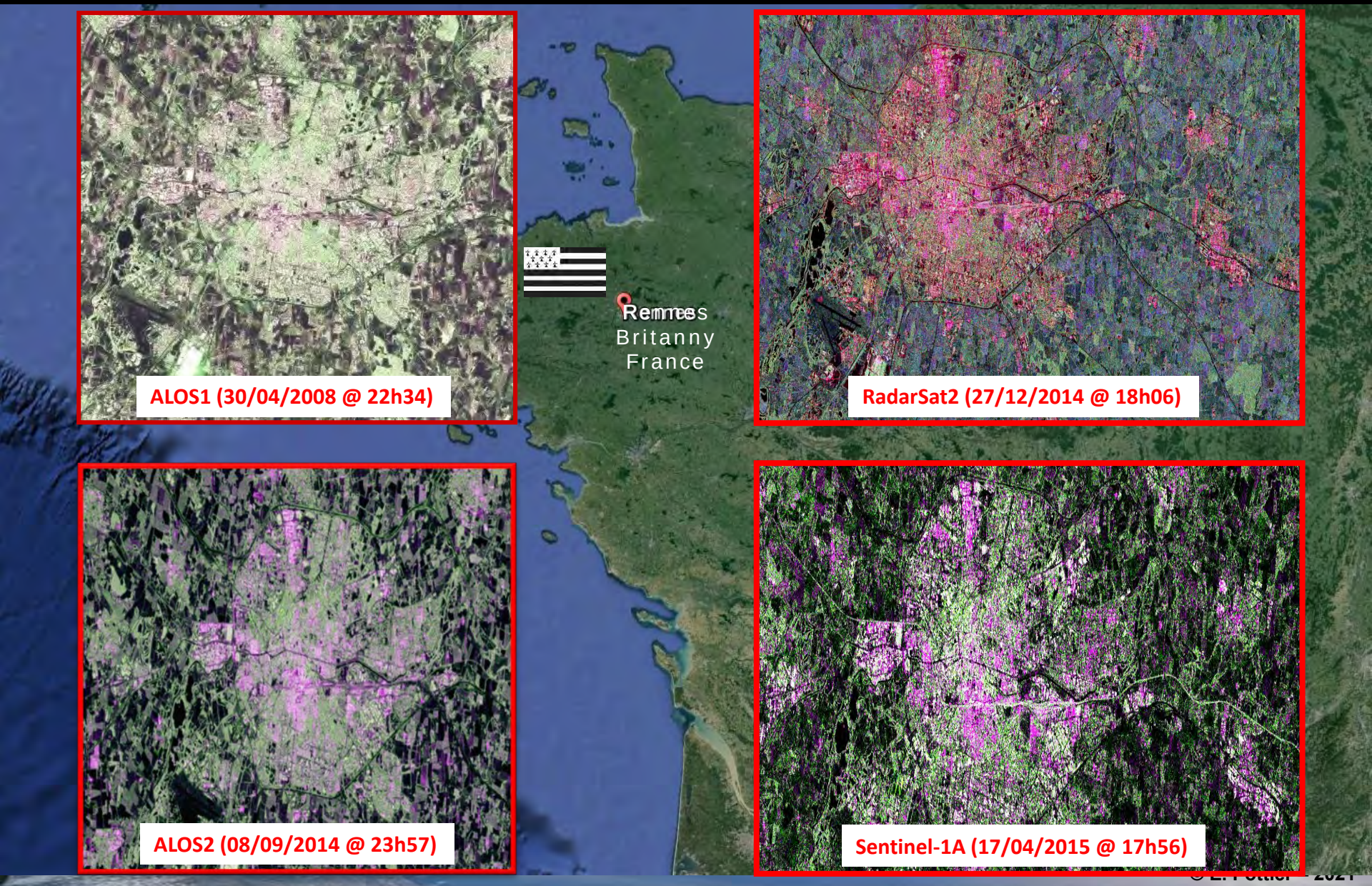
2020

L-Band (Quad) - Resolution 3 m - Footprint 100 km

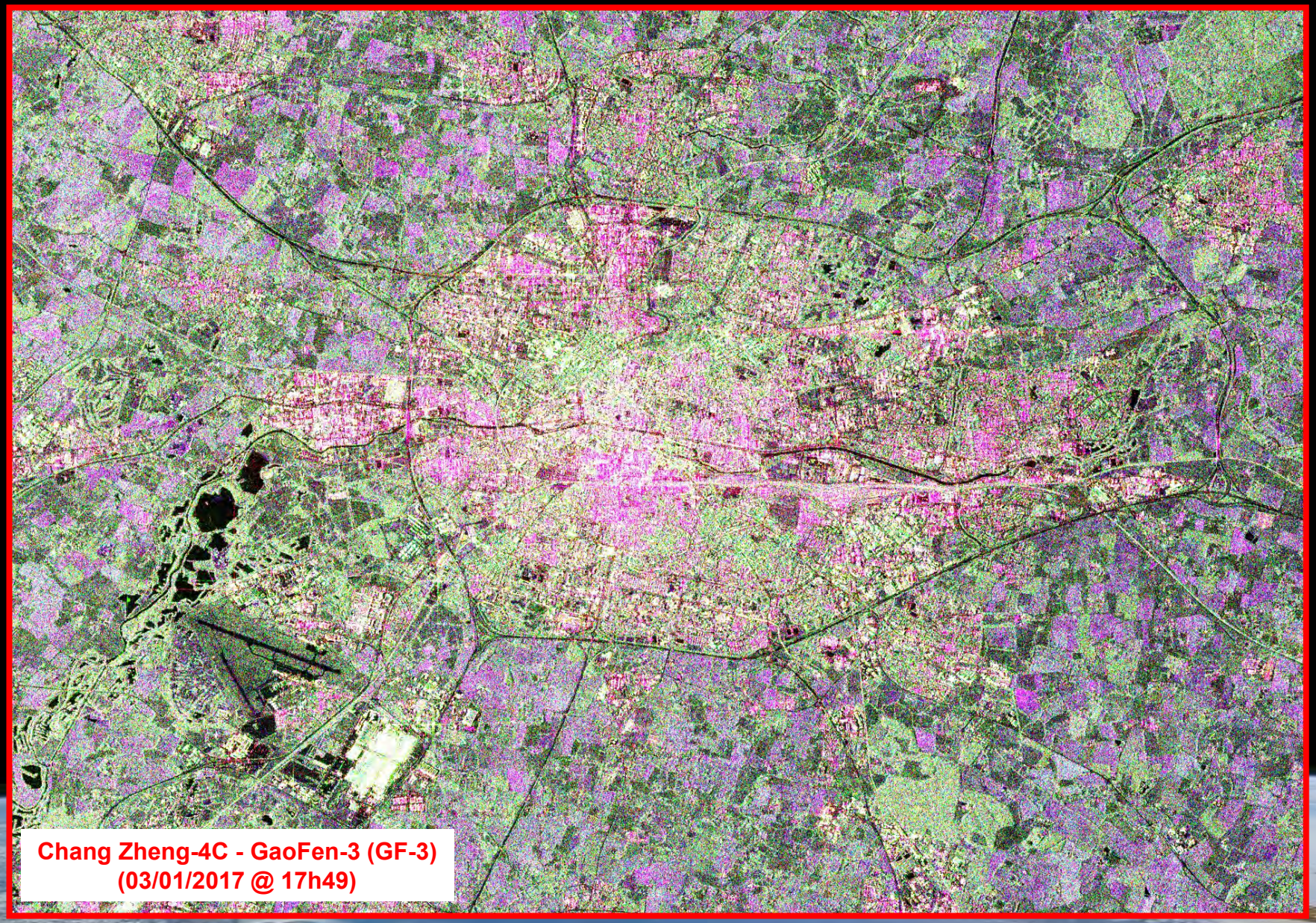
ALOS-4 : Advanced Land Observing Satellite
PALSAR-3 : Phase Array L-Band SAR

Space-borne PolSAR Sensors

Space-borne PolSAR Sensors



Space-borne PolSAR Sensors



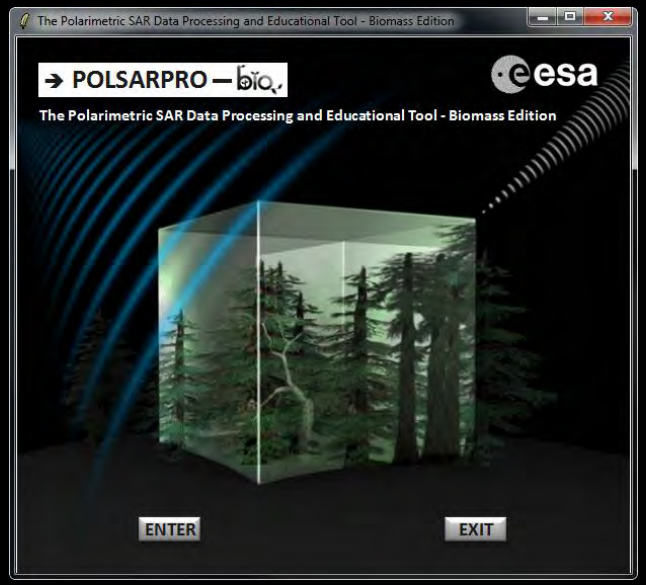
Chang Zheng-4C - GaoFen-3 (GF-3)
(03/01/2017 @ 17h49)

What About

Software / Toolbox ?

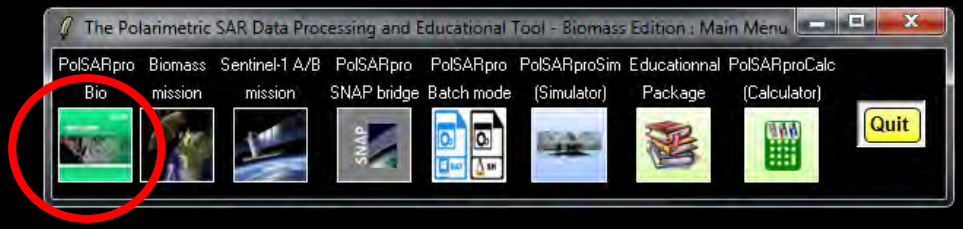
ESA PolSARpro - Biomass Edition Toolbox

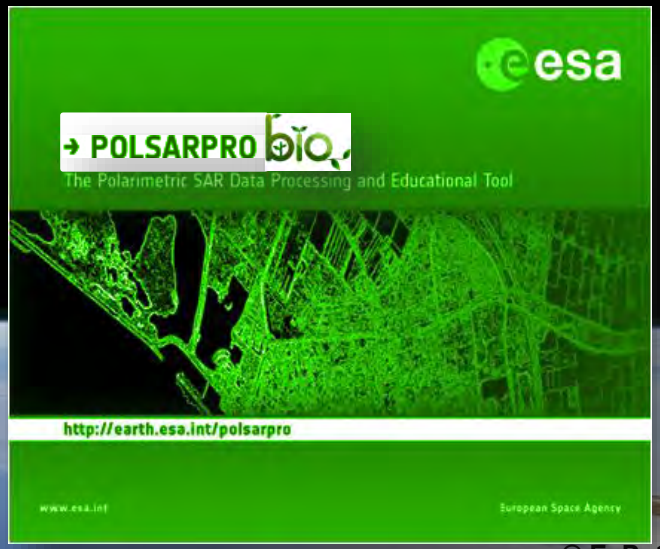
The Polarimetric SAR Data Processing and Educational Toolbox



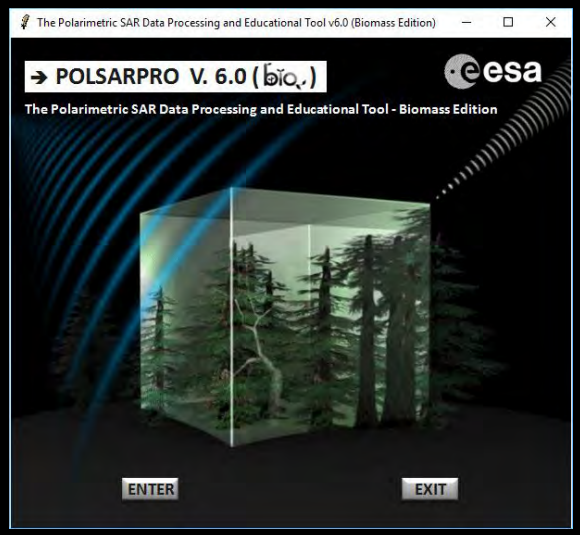
ENTRY SCREEN

MAIN WINDOW





ESA PolSARpro - Biomass Edition Toolbox



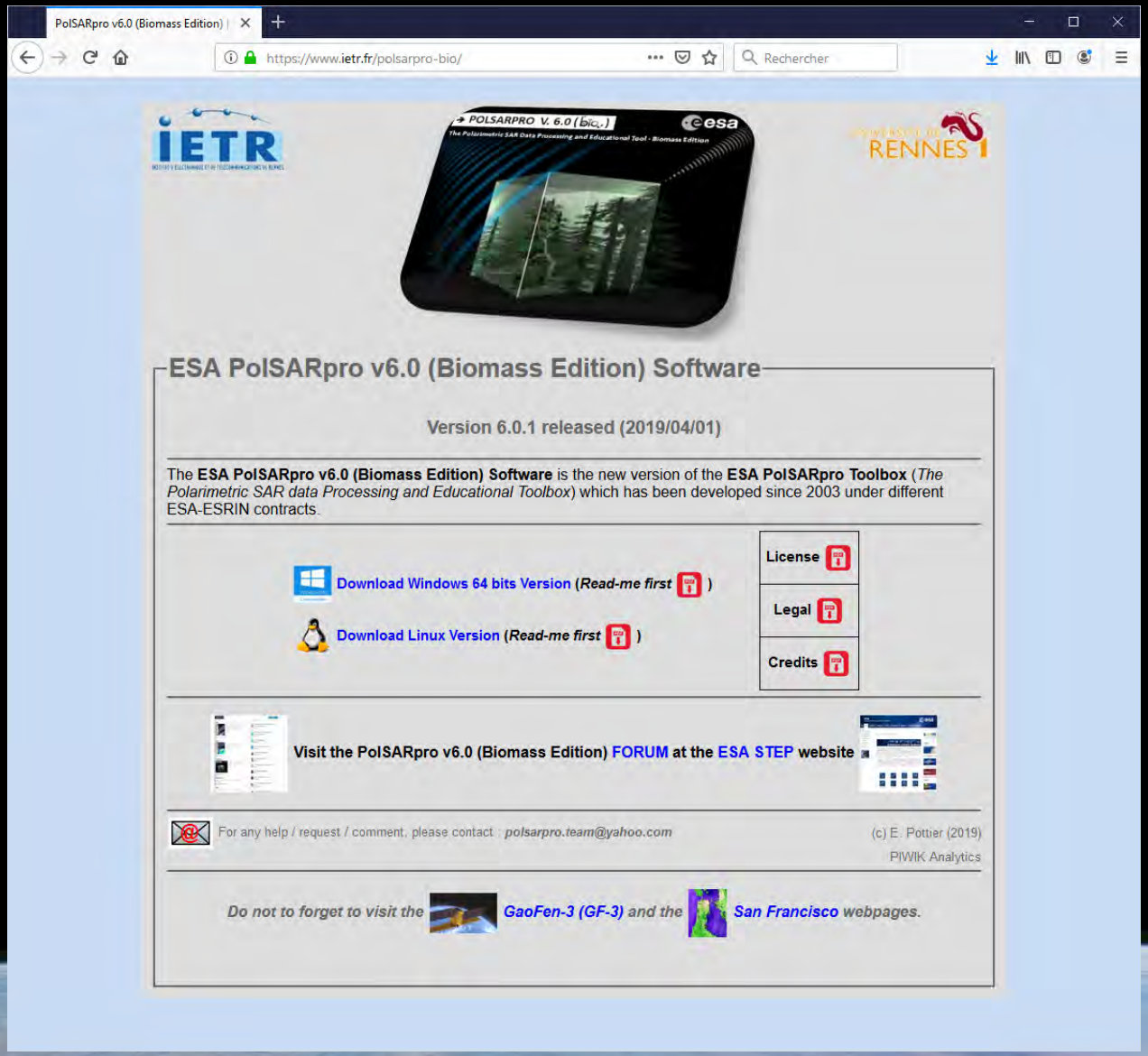
since **2003**

- **+3000** registered users
- **+70** foreign countries
- **+ 4000** downloads (2020)

International Collaborative Project

(4 Agencies, 19 Research Centres, 21 Universities)

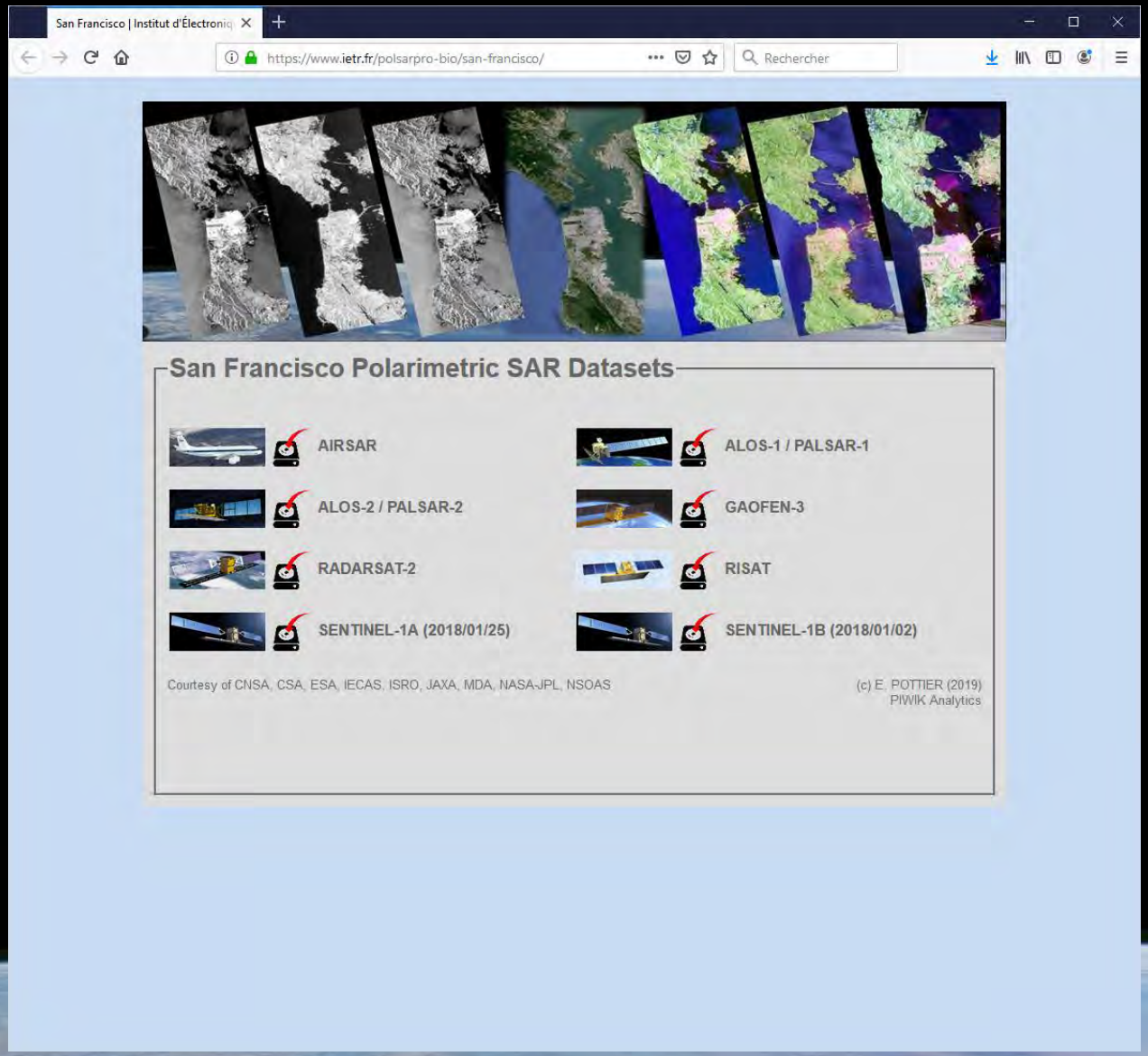
ESA PolSARpro - Biomass Edition Toolbox



https://www.ietr.fr/polsarpro-bio/

polsarpro.team@yahoo.com

ESA PolSARpro - Biomass Edition Toolbox

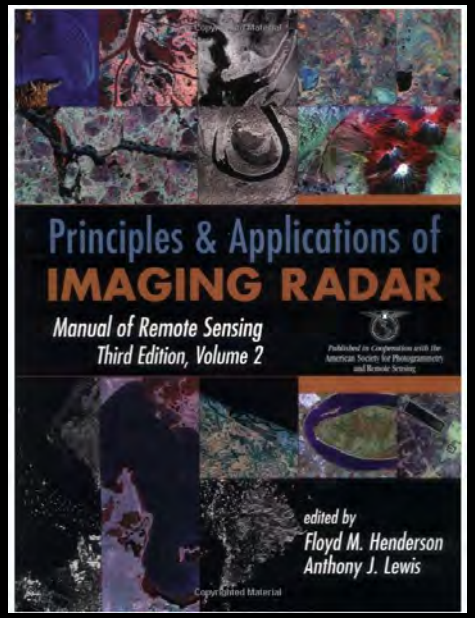


https://ietr-lab.univ-rennes1.fr/polsarpro-bio/san-francisco

Learning / Training

Next P.I Generations

Books On Polarimetric Radar, Polarimetric Interferometry SAR



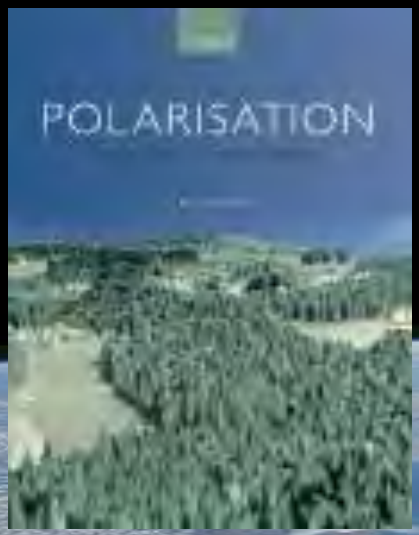
Polarimetric in Radar Remote Sensing : Basic and Applied Concepts

W-M. BOERNER, H. MOTT, E. LÜNEBURG,
C. LIVINGSTONE, B. BRISCO, R.J. BROWN, J. SCOTT PATERSON
Contributors : S.R. CLOUDE, E. KROGAGER, J.S. LEE,
D.L. SCHULER, J.J. VAN ZYL, D. RANDALL, P. BUDKEWITSCH
and E. POTTIER

Manual of Remote Sensing, Third Edition, Volume 2, 867p, 1998. Edited by Floyd M. Henderson and Anthony J. Lewis, John Wiley & Sons, Inc.,
ISBN 0-471-29406-3

Books On Polarimetric Radar, Polarimetric Interferometry SAR

Polarimetric Radar Imaging: From basics to applications
Jong-Sen LEE – Eric POTTIER
CRC Press; 1st ed., February 2009, pp 422
ISBN: 978-1420054972



Polarisation: Applications in Remote Sensing
Shane R. CLOUDE
Oxford University Press, October 2009, pp 352
ISBN: 978-0199569731

Books On Polarimetric Radar, Polarimetric Interferometry SAR

Polarimetric Radar Imaging: From basics to applications
Jong-Sen LEE – Eric POTTIER
CRC Press; 1st ed., February 2009, pp 422
ISBN: 978-1420054972

Prof. Wen HONG, Dr. Qiang YIN et al.

Polarisation: Applications in Remote Sensing
Shane R. CLOUDE
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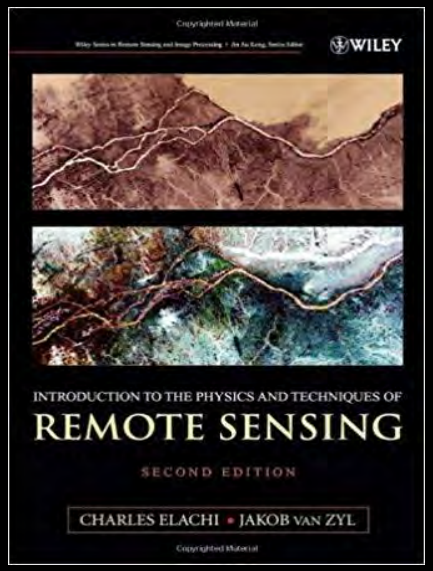
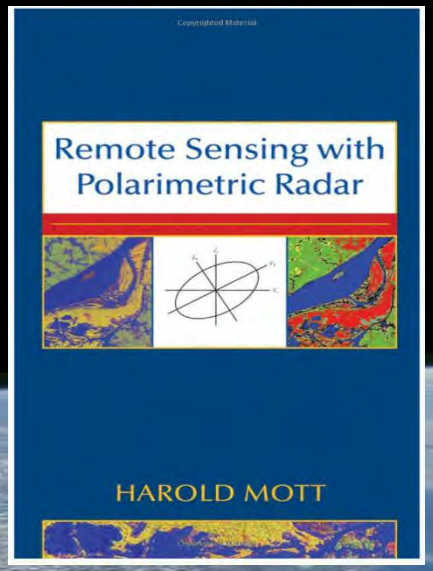
Books On Polarimetric Radar, Polarimetric Interferometry SAR

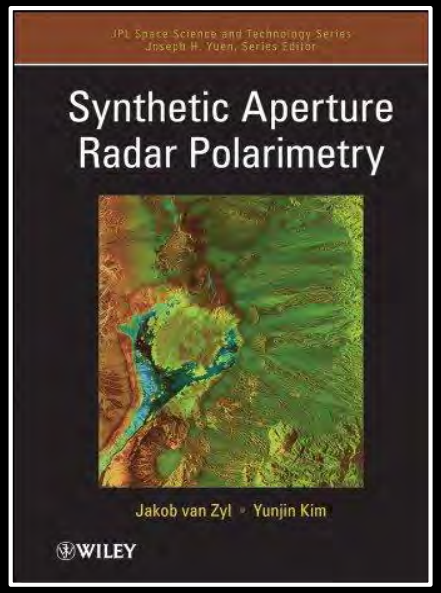


Introduction To The Physics and Techniques of Remote Sensing
Charles ELACHI – Jakob J. VAN ZYL
Wiley-Interscience; 2nd edition (July 31, 2007),
ISBN-10 0-471-47569-6
ISBN-13 978-0471475699

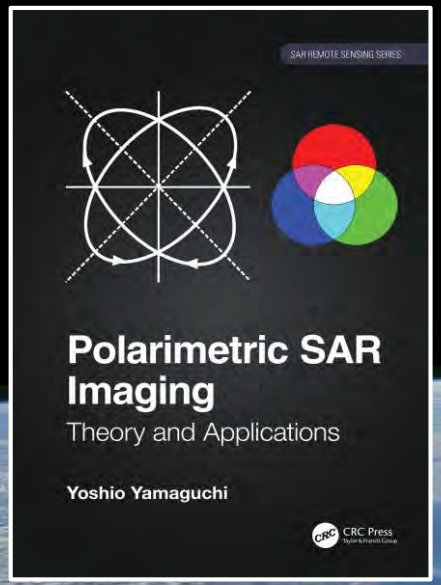


Remote Sensing with Polarimetric Radar
Harold MOTT
Wiley-IEEE Press; 1st edition (January 2, 2007),
ISBN-10 0-470-07476-0
ISBN-13 978-0470074763

Books On Polarimetric Radar, Polarimetric Interferometry SAR

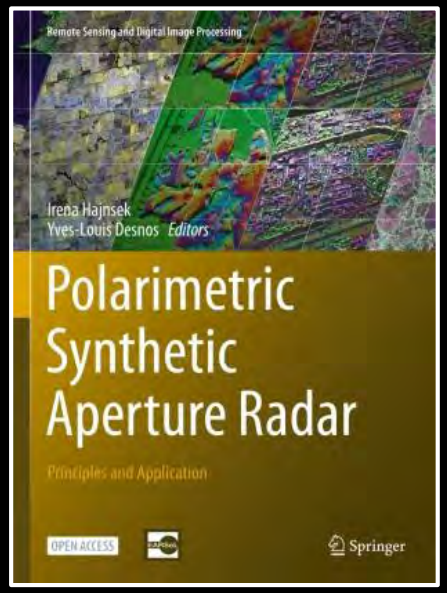


Synthetic Aperture Radar Polarimetry
Jakob J. VAN ZYL – Yunjin KIM
Wiley; 1st edition (October 14, 2011),
ISBN-10 1-118-11511-2
ISBN-13 978-1118115114



Polarimetric SAR Imaging : Theory and Applications
Yoshio Yamaguchi
CRC Press; 1st ed., August 2020, pp 350
ISBN: 978-1003049753

Books On Polarimetric Radar, Polarimetric Interferometry SAR



Polarimetric Synthetic Aperture Radar : Principles and applications

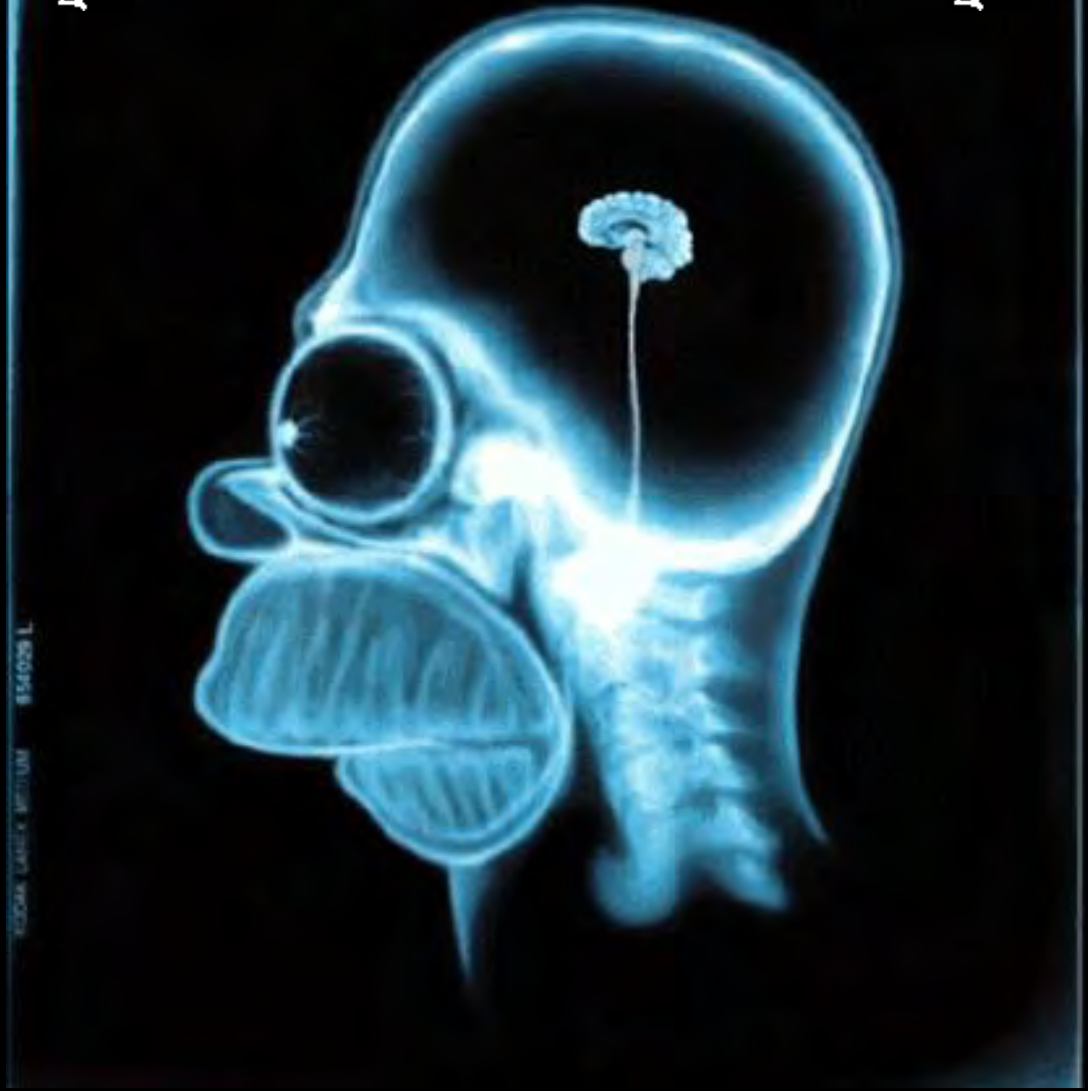
Irena HAJNSEK – Yves-Louis DESNOS editors

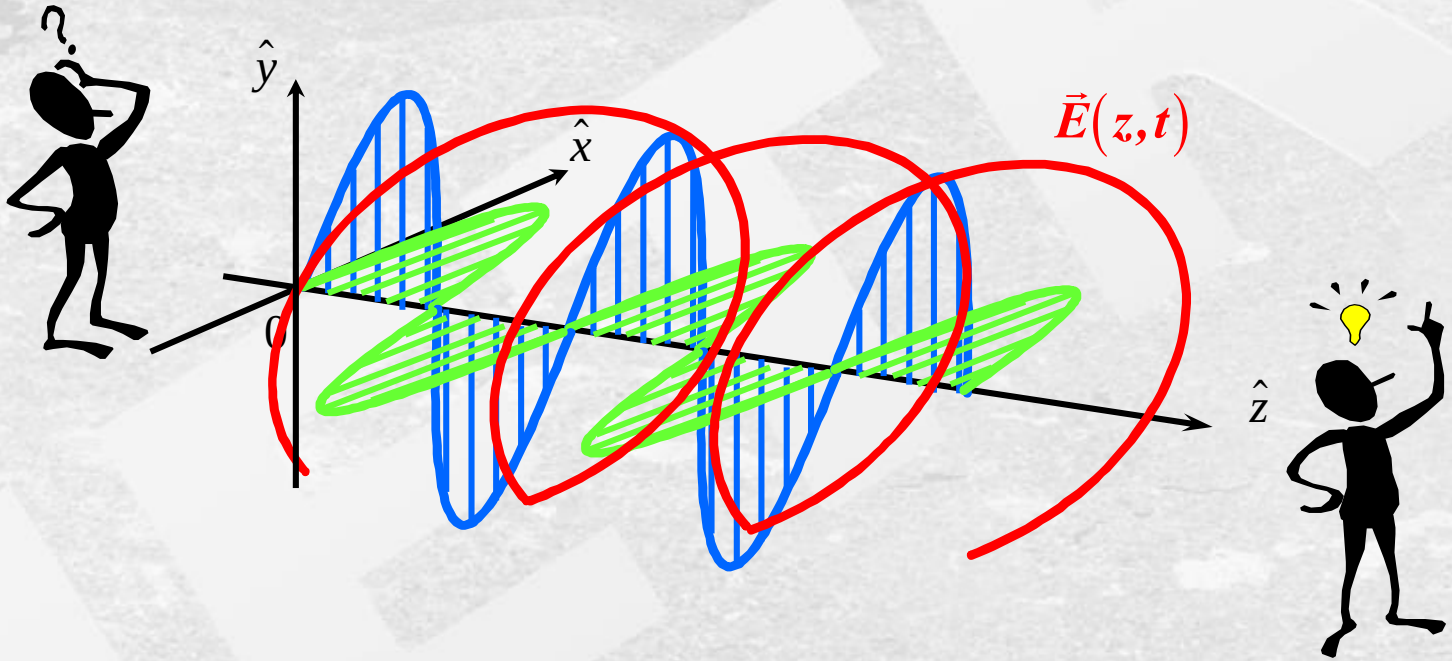
Springer; 1st edition (Marsh 30, 2021),

ISBN 978-3-030-56502-2

https://link.springer.com/content/pdf/10.1007%2F978-3-030-56504-6.pdf

Questions ?





DATASETS

SAN FRANCISCO BAY

AIRSAR JPL

DC8

P, L, C-Band (Quad)

© Google Earth

SAN FRANCISCO BAY

© Google Earth

|HH+VV|
$T_{11}=2A_0$

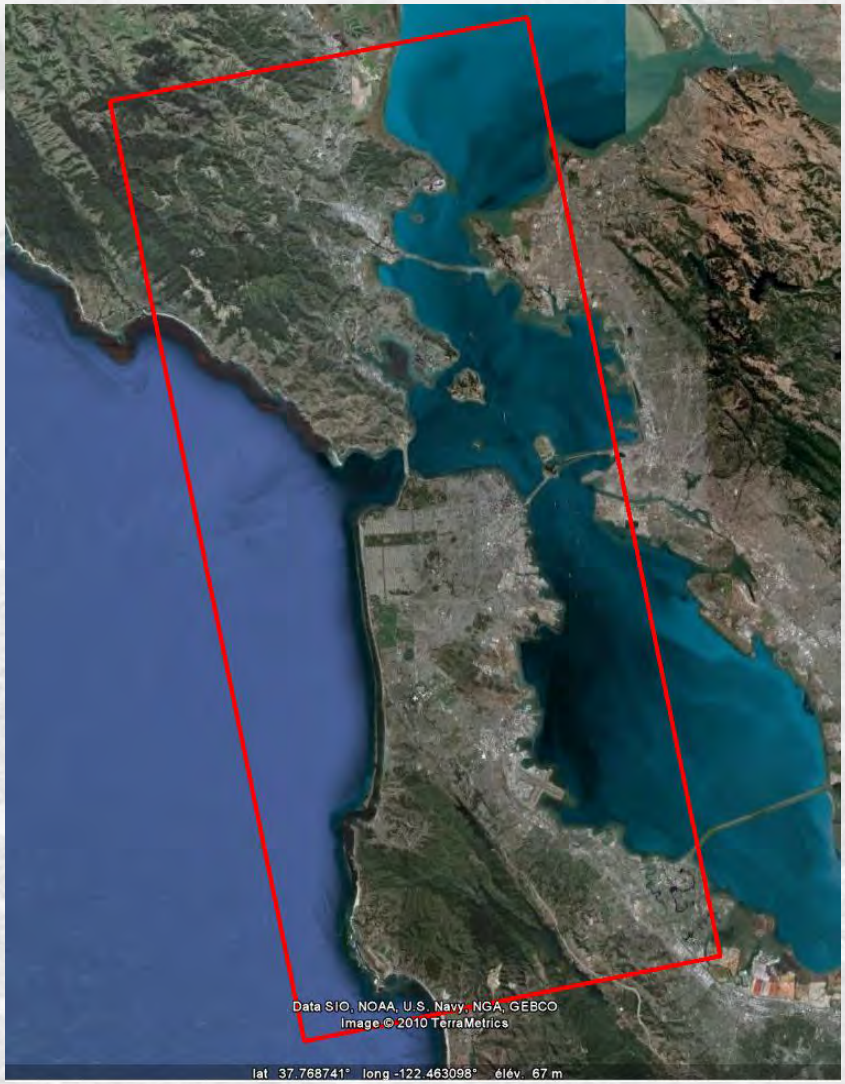
|HV|
$T_{33}=B_0-B$

|HH-VV|
$T_{22}=B_0+B$

SAN FRANCISCO BAY

ALOS - PALSAR

L-Band (Quad)



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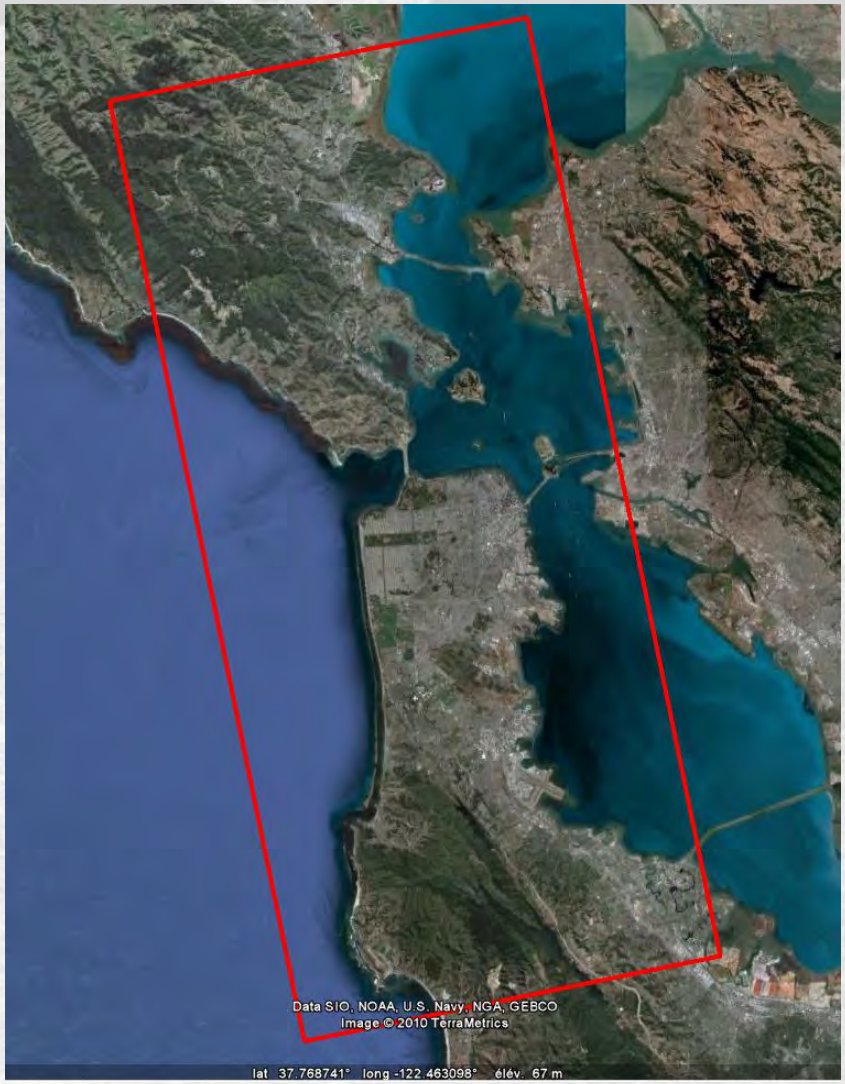
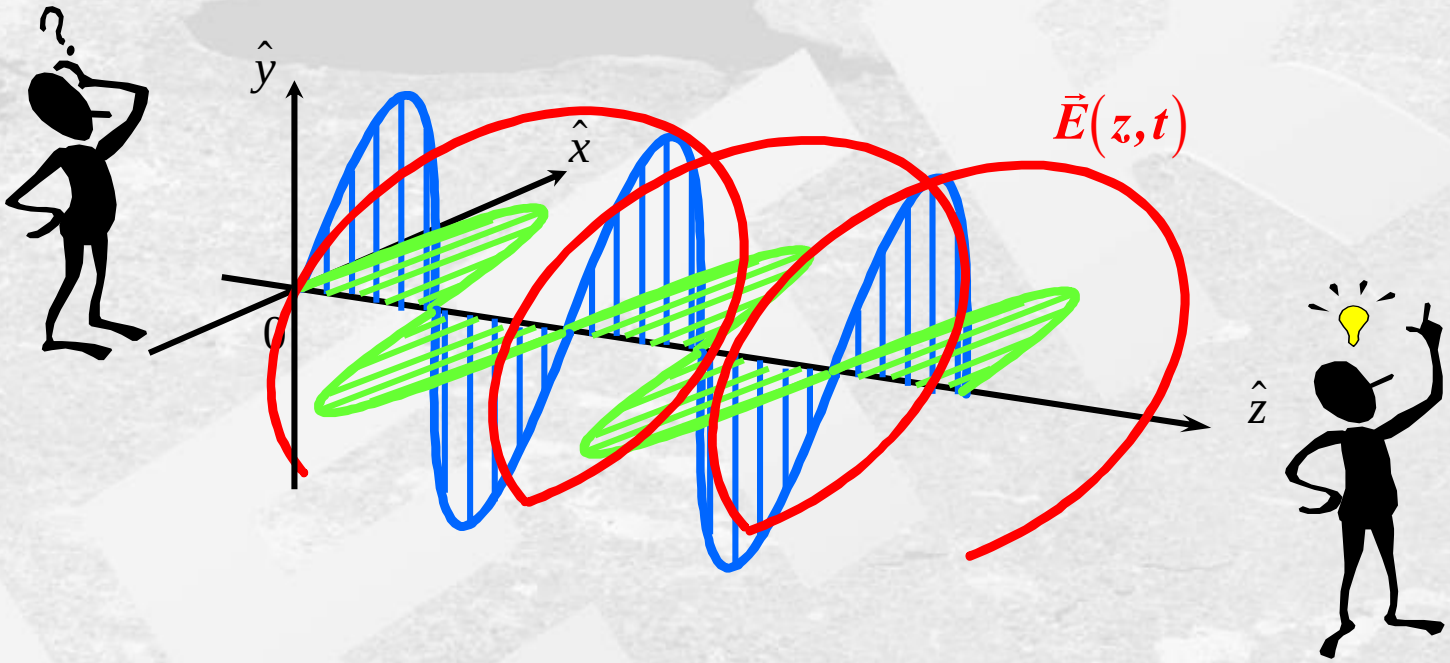
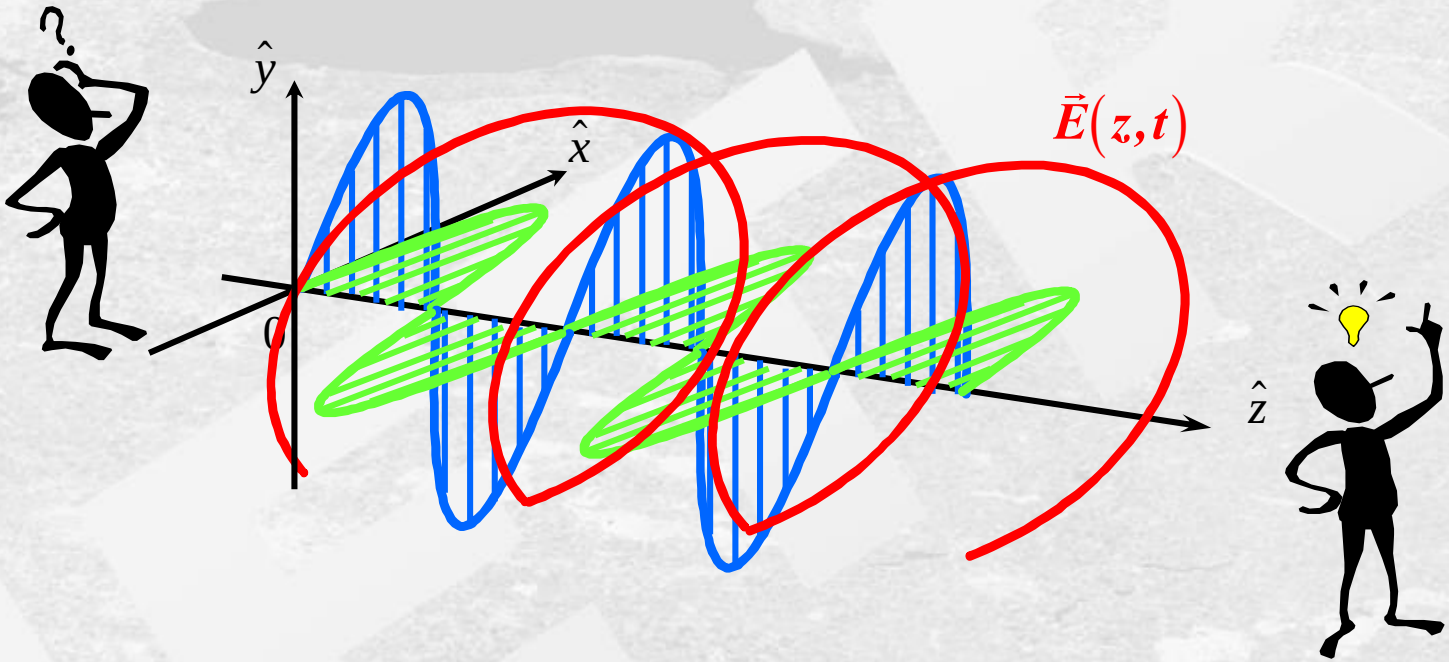
SAN FRANCISCO BAY





BASIC CONCEPTS



ELECTROMAGNETIC WAVE POLARISATION

PROPAGATION EQUATION

REAL ELECTRIC FIELD VECTOR $\vec{E}(z,t)$

MAXWELL EQUATIONS

MAXWELL – FARADAY EQUATION $\nabla \wedge \vec{E}(z,t) = -\dfrac{\partial \vec{B}(z,t)}{\partial t}$

MAXWELL – AMPERE EQUATION $\nabla \wedge \vec{H}(z,t) = \vec{J}_T(z,t)$

GAUSS THEOREM $\nabla \cdot \vec{D}(z,t) = \rho(z,t)$

$\nabla \cdot \vec{B}(z,t) = 0$

$\vec{B}(z,t) = \mu \vec{H}(z,t)$ $\quad \vec{J}_C(z,t) = \sigma \vec{E}(z,t)$

$\vec{D}(z,t) = \varepsilon \vec{E}(z,t)$ $\quad \vec{J}_T(z,t) = \vec{J}_C(z,t) + \dfrac{\partial \vec{D}(z,t)}{\partial t}$

σ (Conductivity)
μ (Permeability)
ε (Permittivity)

PROPAGATION EQUATION

$$\nabla \wedge \left(\nabla \wedge \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla \cdot \left(\nabla \vec{A}\right)$$

PROPAGATION EQUATION

$$\nabla^2 \vec{E}(z,t) - \mu\varepsilon \frac{\partial^2 \vec{E}(z,t)}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}(z,t)}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial \rho(z,t)}{\partial t}$$

HELMHOLTZ PROPAGATION EQUATION

$$\nabla^2 \vec{E}(z,t) - \mu\varepsilon \frac{\partial^2 \vec{E}(z,t)}{\partial t^2} = 0$$

Source Free, Linear, Homogeneous, Isotropic, Dielectric and lossless Medium

PROPAGATION EQUATION

COMPLEX ELECTRIC FIELD VECTOR $\underline{E}(z)$ **With:** $\vec{E}(z,t) = \Re\left(\underline{E}(z)e^{j\omega t}\right)$

HELMHOLTZ PROPAGATION EQUATION

$$\nabla^2 \underline{E}(z) + \underline{k}^2 \underline{E}(z) = 0$$

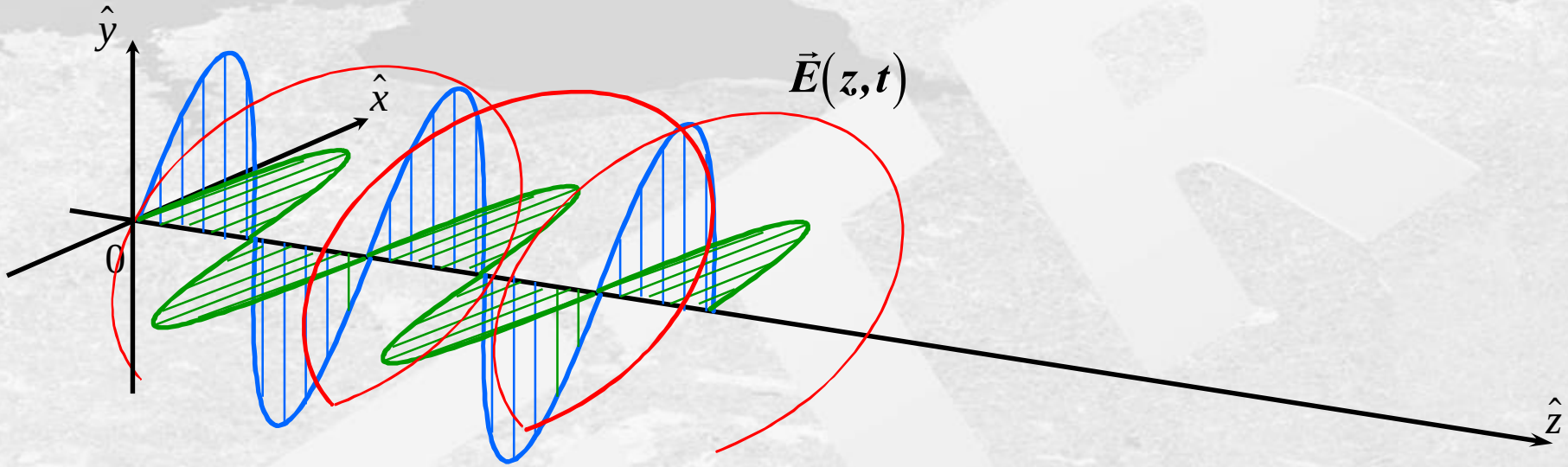
SOLUTION: $\underline{E}(z) = \underline{E}e^{-jkz}$

With: $\underline{E} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} E_{ox}e^{j\delta_x} \\ E_{oy}e^{j\delta_y} \\ E_{oz}e^{j\delta_z} \end{bmatrix}$

SINUSOIDAL PLANE WAVE

$$\nabla \cdot \vec{E}(z,t) = 0 \quad \Rightarrow \quad \frac{\partial E_z}{\partial z} = 0$$

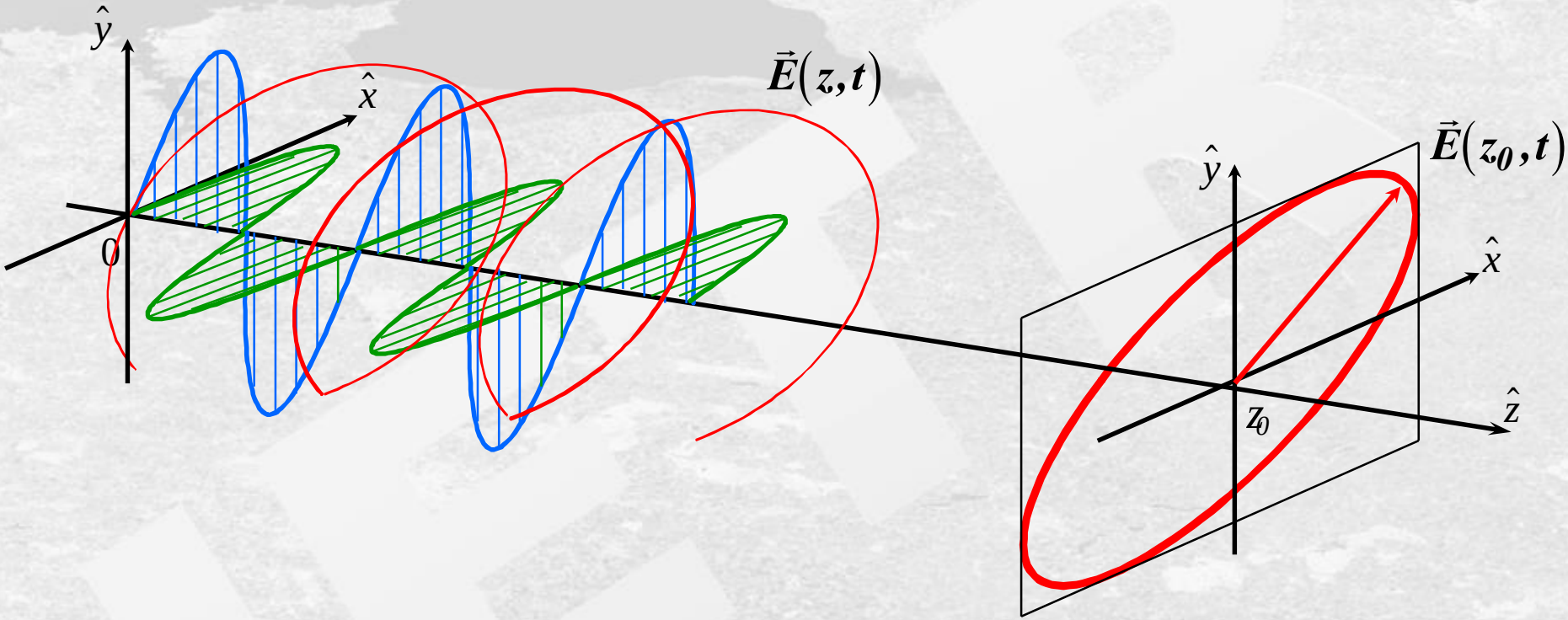
POLARISATION ELLIPSE



REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t) = \begin{cases} E_x = E_{0x} \cos(\omega t - kz - \delta_x) \\ E_y = E_{0y} \cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$

POLARISATION ELLIPSE

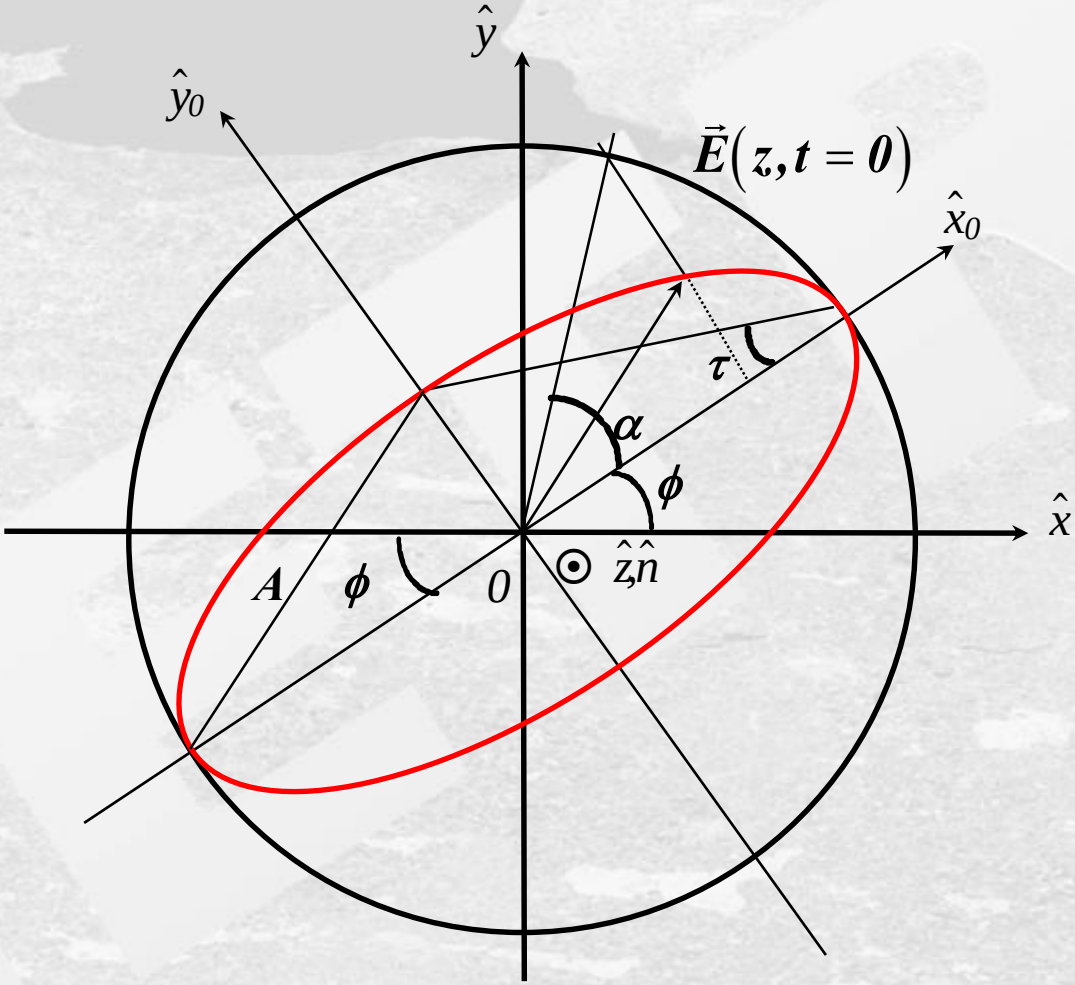


THE REAL ELECTRIC FIELD VECTOR MOVES IN TIME ALONG AN ELLIPSE

$$\left(\frac{E_x}{E_{0x}}\right)^2 - 2\frac{E_x E_y}{E_{0x}E_{0y}}\cos(\delta) + \left(\frac{E_y}{E_{0y}}\right)^2 = \sin^2(\delta)$$

With: $\delta = \delta_y - \delta_x$

POLARISATION ELLIPSE



A : WAVE AMPLITUDE

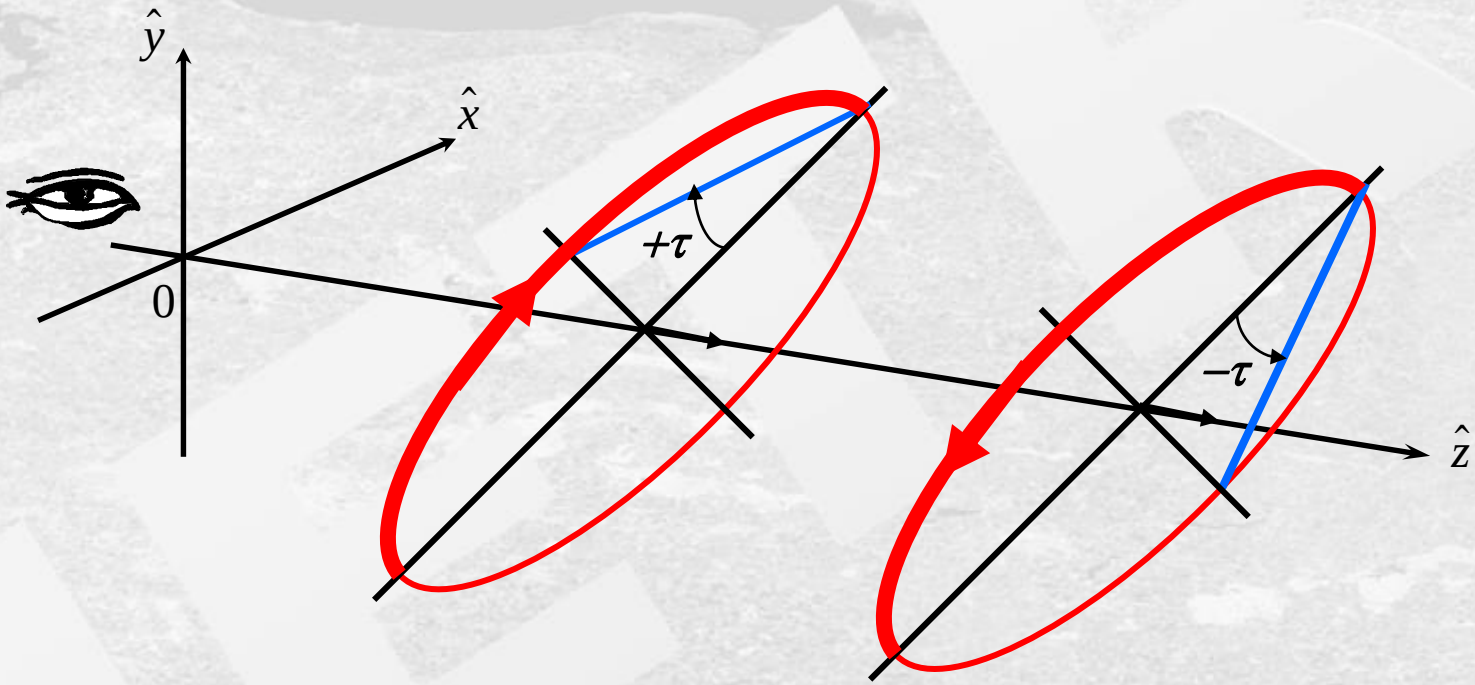
α : **ABSOLUTE PHASE**

ϕ : **ORIENTATION ANGLE** $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$

τ : **ELLIPTICITY ANGLE** $0 \leq \tau \leq \frac{\pi}{4}$

POLARISATION HANDENESS

ROTATION SENSE: LOOKING INTO THE DIRECTION OF THE WAVE PROPAGATION



ANTI-CLOCKWISE ROTATION

LEFT HANDED POLARISATION

ELLIPTICITY ANGLE : $\tau > 0$

CLOCKWISE ROTATION

RIGHT HANDED POLARISATION

ELLIPTICITY ANGLE : $\tau < 0$

$$-\frac{\pi}{4} \leq \tau \leq \frac{\pi}{4}$$

JONES VECTOR

REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t)=\begin{cases} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$

PHASOR = JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox}e^{j\delta_x} \\ E_y = E_{oy}e^{j\delta_y} \end{bmatrix}$$

With: $\vec{E}(z,t) = \Re\left(\underline{E}e^{j(\omega t - kz)}\right)$

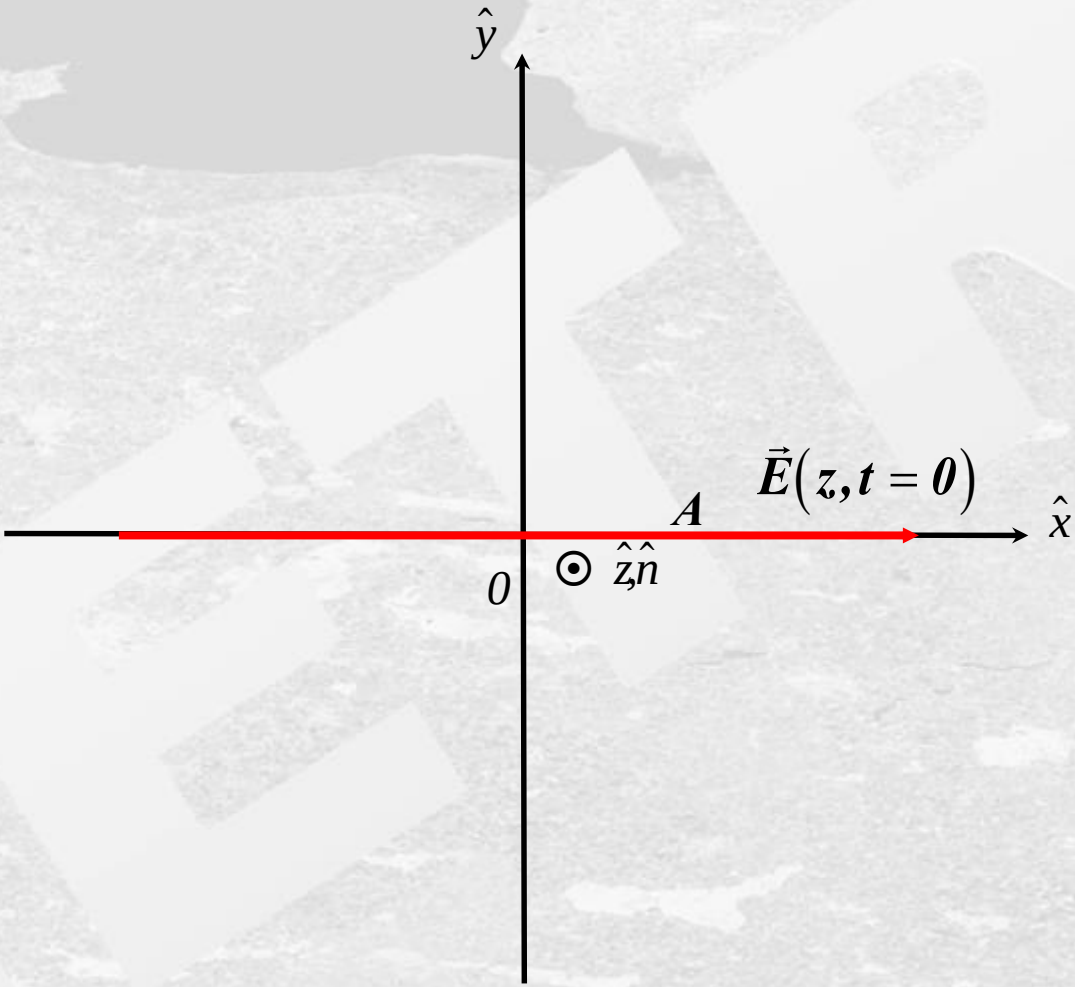
Special Unitary Pauli Matrices Group – SU(2)

$$\{\alpha \quad \phi \quad \tau\}$$

$$[U_2(\phi)] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}$$

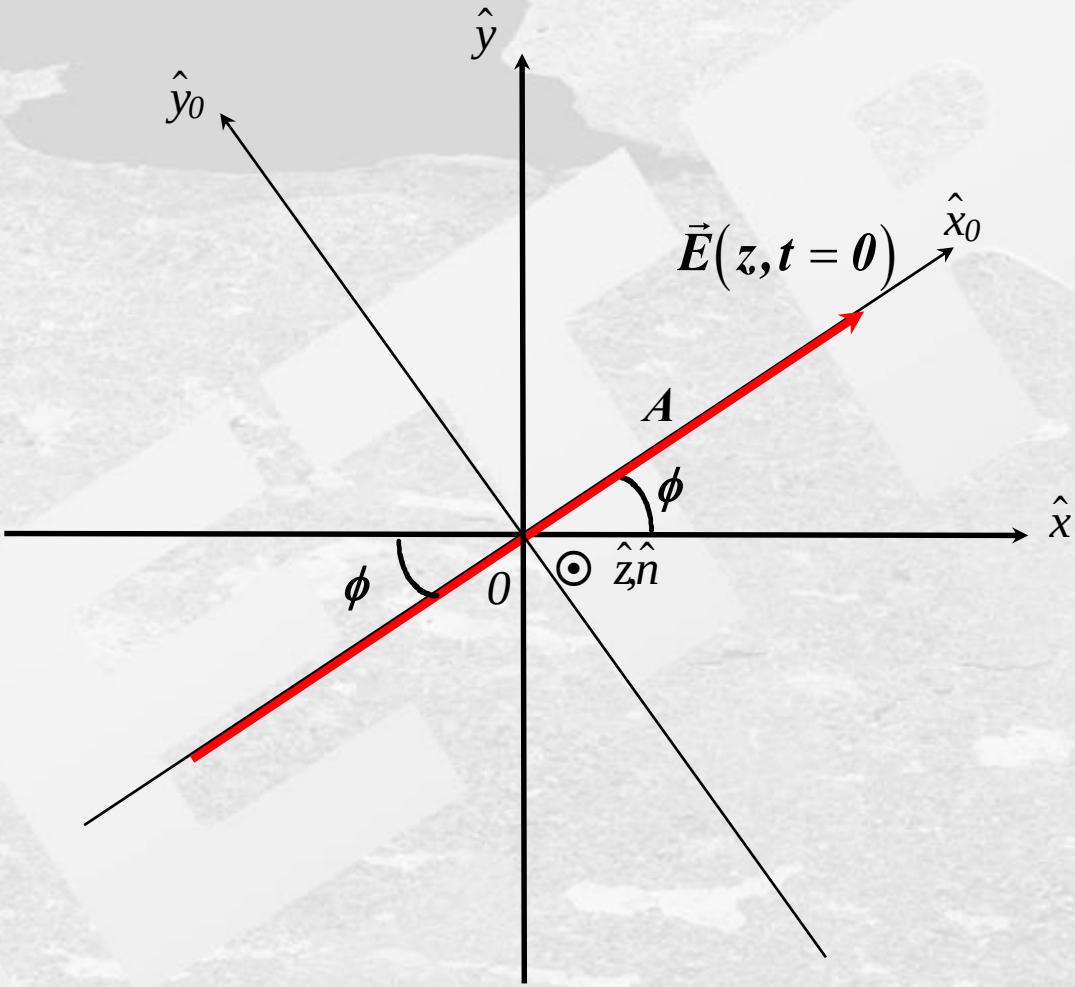
$$[U_2(\tau)] = \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \qquad [U_2(\alpha)] = \begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}$$

JONES VECTOR



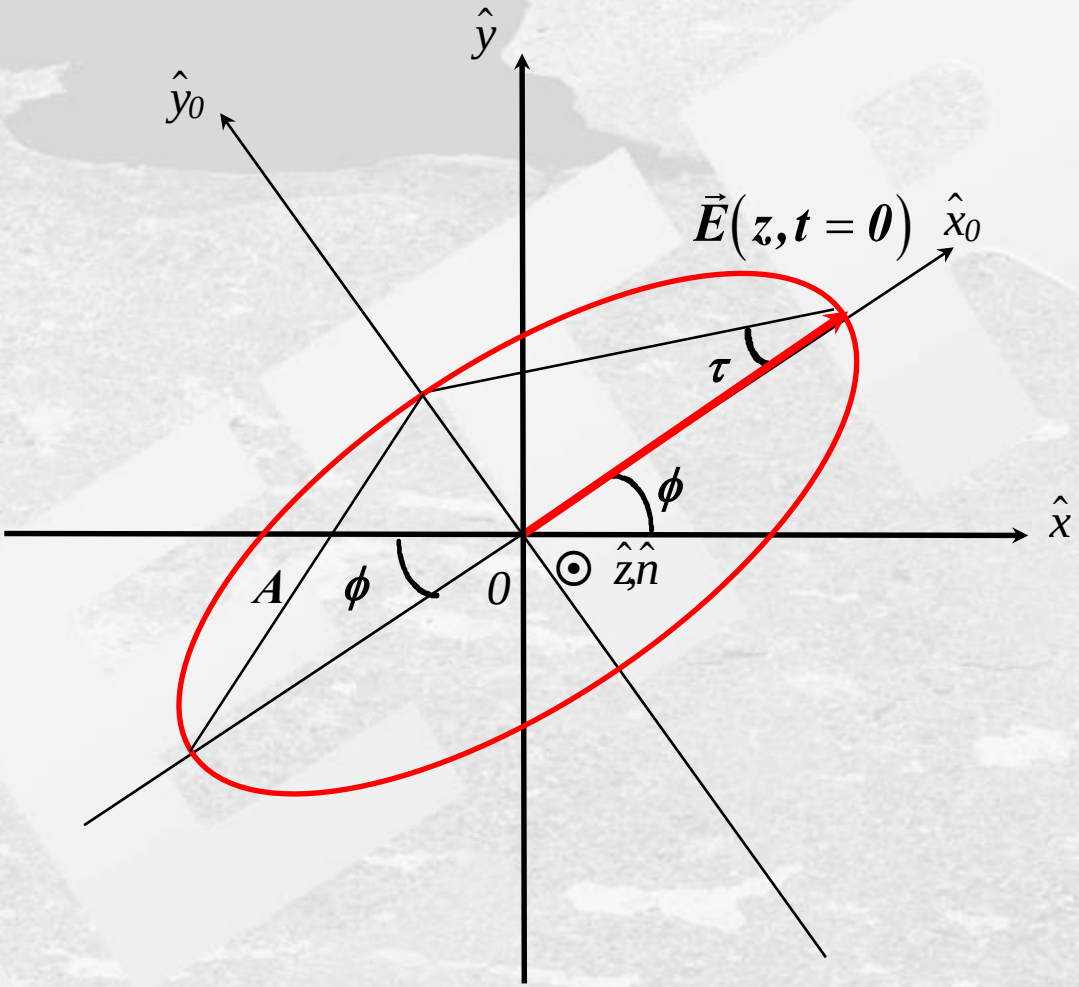
$$\underline{E} = A\ \hat{u}_x$$

JONES VECTOR



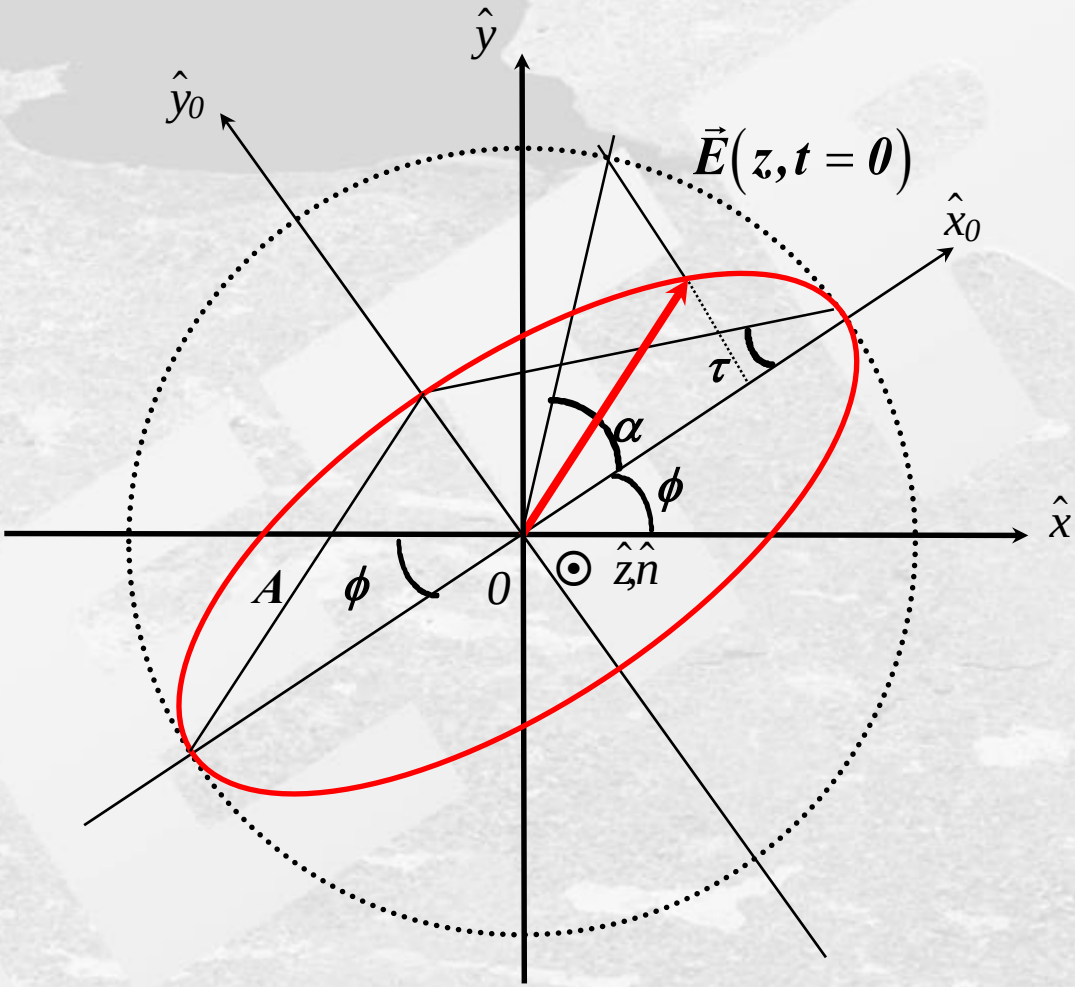
$$\underline{E} = A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \hat{u}_x$$

JONES VECTOR



$$\underline{E} = A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \hat{u}_x$$

JONES VECTOR



$$\underline{E} = A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \hat{u}_x$$

JONES VECTOR

$$\underline{E}_{(\hat{x},\hat{y})} = A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Downarrow$$

$$\underline{E}_{(\hat{x},\hat{y})} = Ae^{+j\alpha} \begin{bmatrix} \cos(\phi)\cos(\tau) - j\sin(\phi)\sin(\tau) \\ \sin(\phi)\cos(\tau) + j\cos(\phi)\sin(\tau) \end{bmatrix} = \begin{bmatrix} E_x = E_{ox}e^{j\delta_x} \\ E_y = E_{oy}e^{j\delta_y} \end{bmatrix}$$

GEOMETRICAL PARAMETERS

ABSOLUTE PHASE

$$\alpha = \delta_x$$

AMPLITUDE

$$A = \sqrt{E_{0x}^2 + E_{0y}^2}$$

ORIENTATION ANGLE

$$\tan 2\phi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta$$

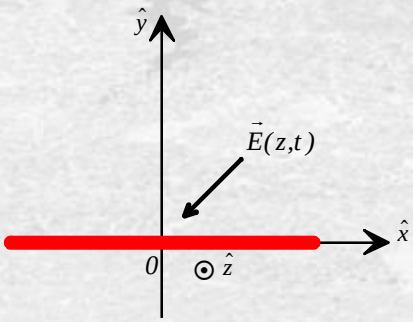
ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta$$

POLARISATION HANDENESS: $Sign(\tau)$

JONES VECTOR

HORIZONTAL POLARISATION STATE

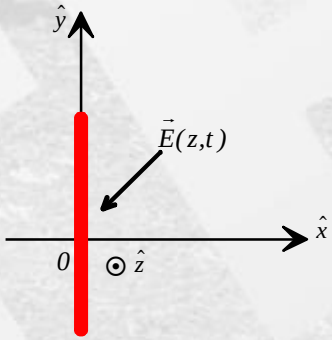


$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\phi = 0$$

$$\tau = 0$$

VERTICAL POLARISATION STATE

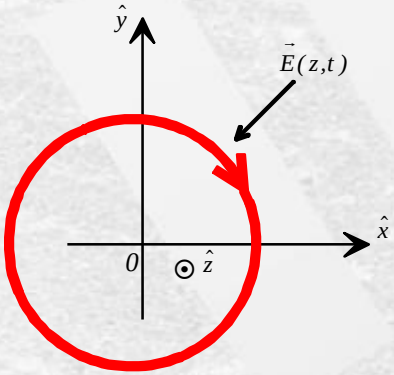


$$\underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\phi = \frac{\pi}{2}$$

$$\tau = 0$$

LEFT CIRCULAR POLARISATION STATE

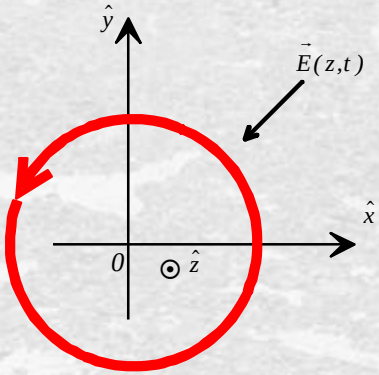


$$\underline{LC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}$$

$$-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2}$$

$$\tau = +\frac{\pi}{4}$$

RIGHT CIRCULAR POLARISATION STATE



$$\underline{RC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

$$-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2}$$

$$\tau = -\frac{\pi}{4}$$

ORTHOGONAL JONES VECTOR

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{ox} e^{j\delta_x} \\ E_{oy} e^{j\delta_y} \end{bmatrix}$$

$$= A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \hat{u}_x$$

POLARISATION ALGEBRA

NORM OF A JONES VECTOR $\|\underline{E}\| = \sqrt{E_{0x}^2 + E_{0y}^2}$

SCALAR PRODUCT $\langle \underline{A}, \underline{B} \rangle = \underline{A}^{T*} \underline{B}$

ORTHOGONALITY $\langle \underline{A}, \underline{A}_\perp \rangle = 0$

ORTHOGONAL JONES VECTOR

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR

$$\underline{E}_{\perp} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\hat{u}_y$$

ORTHOGONAL JONES VECTOR

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR

$$\underline{E}_{\perp} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\hat{u}_y$$

$$= A\begin{bmatrix} -\sin(\phi) & -\cos(\phi) \\ \cos(\phi) & -\sin(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\hat{u}_x$$

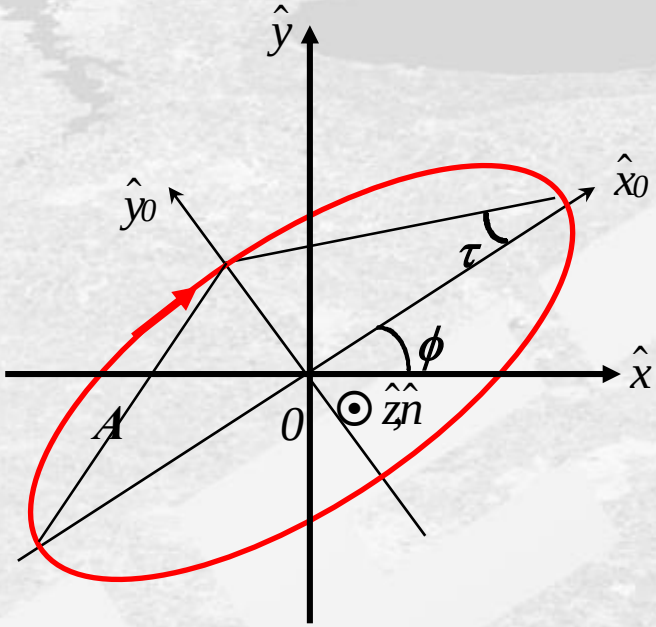
ORTHOGONAL JONES VECTOR

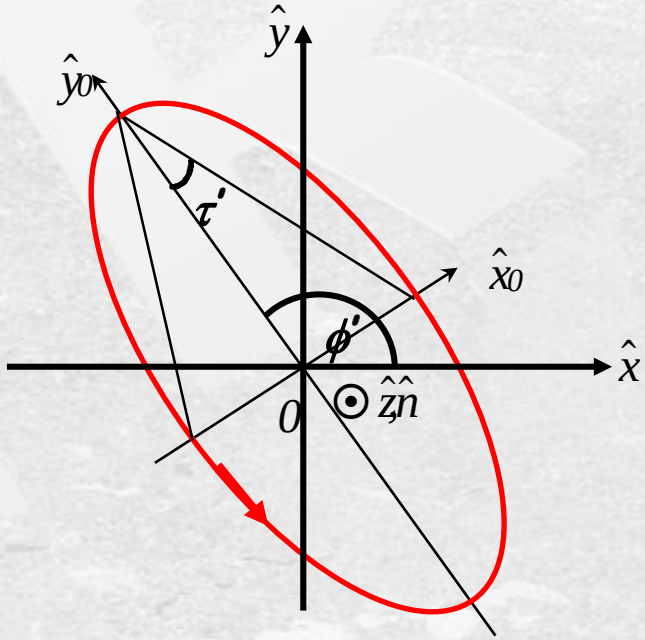
$$\underline{E}_\perp = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\hat{u}_y$$

$$= A\begin{bmatrix} -\sin(\phi) & -\cos(\phi) \\ \cos(\phi) & -\sin(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\hat{u}_x$$

$$\underline{E}_\perp = A\begin{bmatrix} \cos\left(\phi+\frac{\pi}{2}\right) & -\sin\left(\phi+\frac{\pi}{2}\right) \\ \sin\left(\phi+\frac{\pi}{2}\right) & \cos\left(\phi+\frac{\pi}{2}\right) \end{bmatrix}\begin{bmatrix} \cos(-\tau) & j\sin(-\tau) \\ j\sin(-\tau) & \cos(-\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR





ORTHOGONALITY CONDITIONS

$$(\phi, \tau) \mapsto \begin{cases} \phi' = \phi + \frac{\pi}{2} \\ \tau' = -\tau \end{cases}$$

➡ CHANGE OF POLARISATION HANDENESS

ELLIPTICAL BASIS TRANSFORMATION

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR

$$\underline{E}_{\perp} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\hat{u}_y$$

$$\Downarrow$$

$$\left[\underline{E}, \underline{E}_{\perp}\right] = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\left[\hat{u}_x, \hat{u}_y\right]$$

$$\Downarrow$$

ELLIPTICAL BASIS TRANSFORMATION

ELLIPTICAL BASIS TRANSFORMATION

JONES AND ORTHOGONAL JONES VECTORS

$$\underline{E},\underline{E}_{\perp}\Big] = A\begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}\Big[\hat{u}_x,\hat{u}_y$$

SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

$$\big[U(\varphi,\tau,\alpha)\big] = \underbrace{\begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}}_{[U_2(\phi)]}\underbrace{\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}}_{[U_2(\tau)]}\underbrace{\begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}}_{[U_2(\alpha)]}$$

$$[U_2][U_2]^{T*} = [I_{D2}]$$ CONSERVATION OF THE WAVE ENERGY

$$det([U_2]) = +1$$ ENSURES THE CORRECT PHASE DEFINITION

ELLIPTICAL BASIS TRANSFORMATION

SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

$$\left[U(\varphi,\tau,\alpha)\right]=\begin{bmatrix}\cos(\phi) & -\sin(\phi)\\ \sin(\phi) & \cos(\phi)\end{bmatrix}\begin{bmatrix}\cos(\tau) & j\sin(\tau)\\ j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}e^{+j\alpha} & 0\\ 0 & e^{-j\alpha}\end{bmatrix}$$

$$\Downarrow$$

ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\left[U_{(\underline{A},\underline{A}_\perp)\mapsto(\underline{B},\underline{B}_\perp)}\right]=\left[U(\phi,\tau,\alpha)\right]^{-1}$$

$$=\begin{bmatrix}e^{j\alpha} & 0\\ 0 & e^{-j\alpha}\end{bmatrix}\begin{bmatrix}\cos(\tau) & -j\sin(\tau)\\ -j\sin(\tau) & \cos(\tau)\end{bmatrix}\begin{bmatrix}\cos(\phi) & \sin(\phi)\\ -\sin(\phi) & \cos(\phi)\end{bmatrix}$$

ELLIPTICAL BASIS TRANSFORMATION

FROM LINEAR TO CIRCULAR BASIS

$$\underline{E} = E_H \underline{H} + E_V \underline{V} = E_{LC} \underline{LC} + E_{LC_\perp} \underline{LC}_\perp$$

$$\Downarrow$$

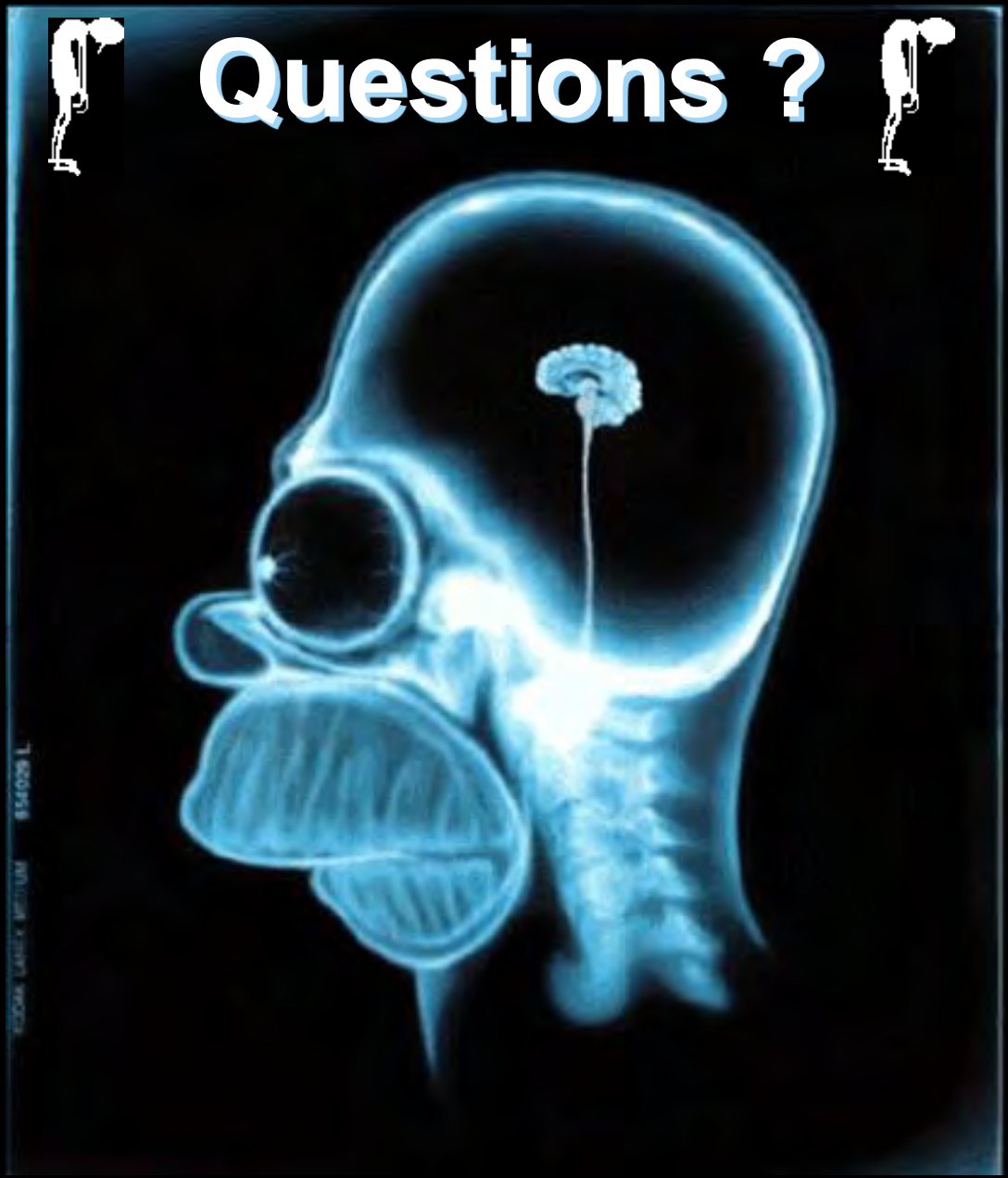
$$[\underline{LC}, \underline{LC}_\perp] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} = \left[U\left(0, \frac{\pi}{4}\right)\right] \Rightarrow \left[U_{(\underline{H},\underline{V}) \mapsto (\underline{LC},\underline{LC}_\perp)}\right] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}$$

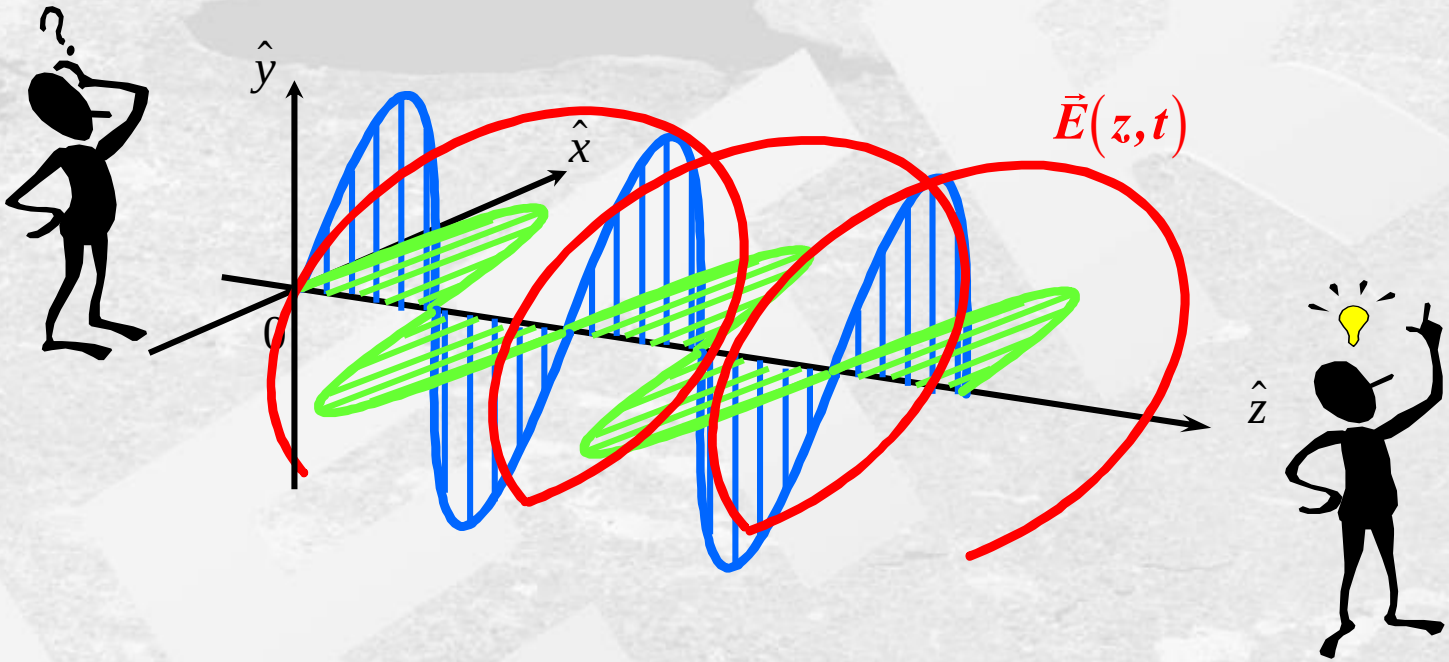
$$\Downarrow$$

$$\begin{bmatrix} E_{LC} \\ E_{LC_\perp} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix} \begin{bmatrix} E_H \\ E_V \end{bmatrix}$$

Ernst LÜNEBURG
(PIERS95 - Pasadena)

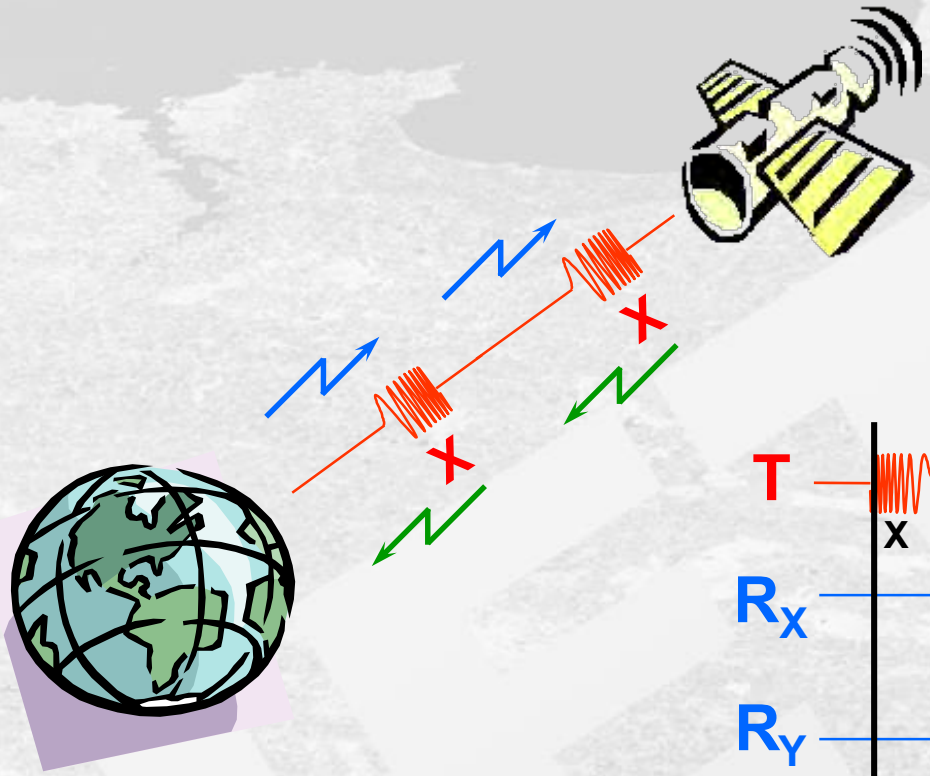
Questions ?

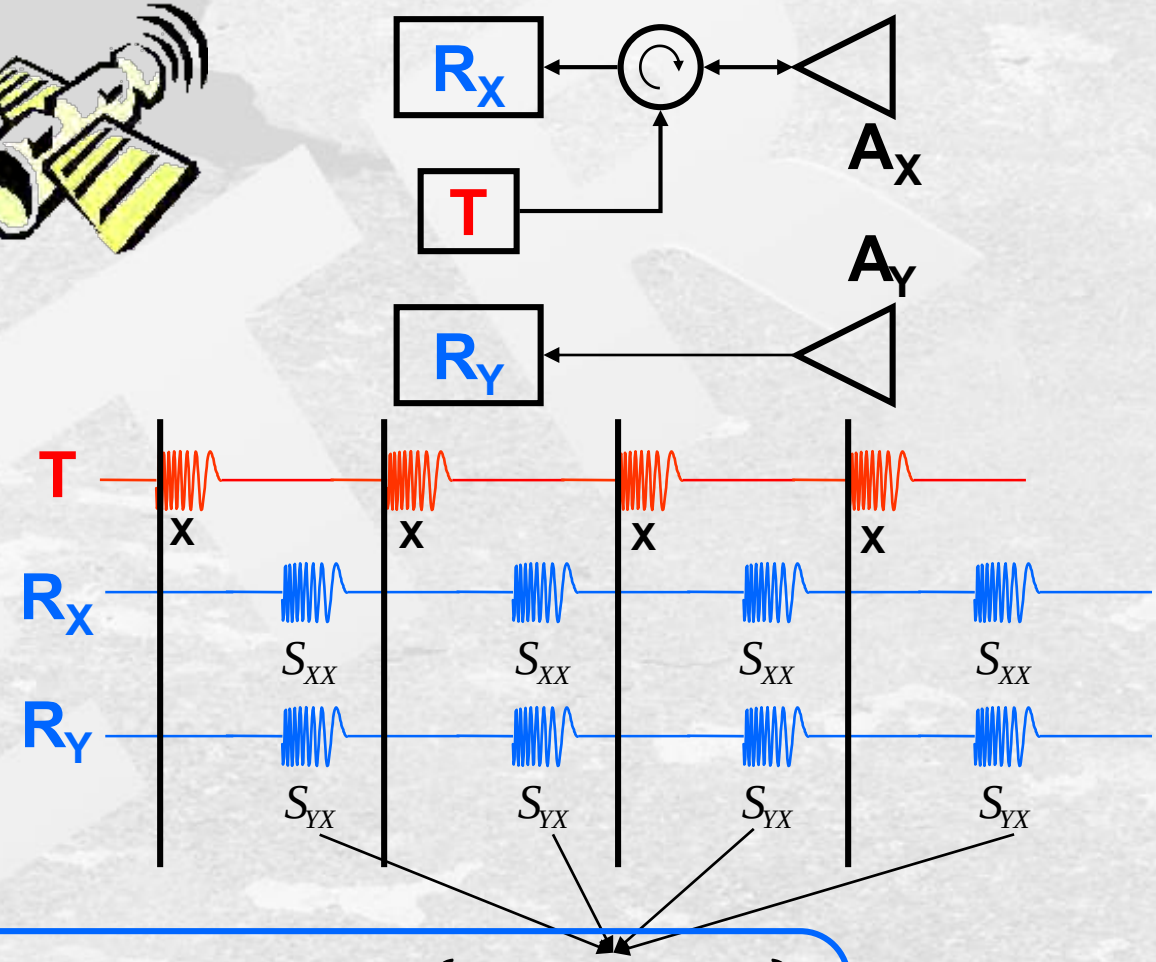




SCATTERING POLARIMETRY

WAVE POLARIMETRY



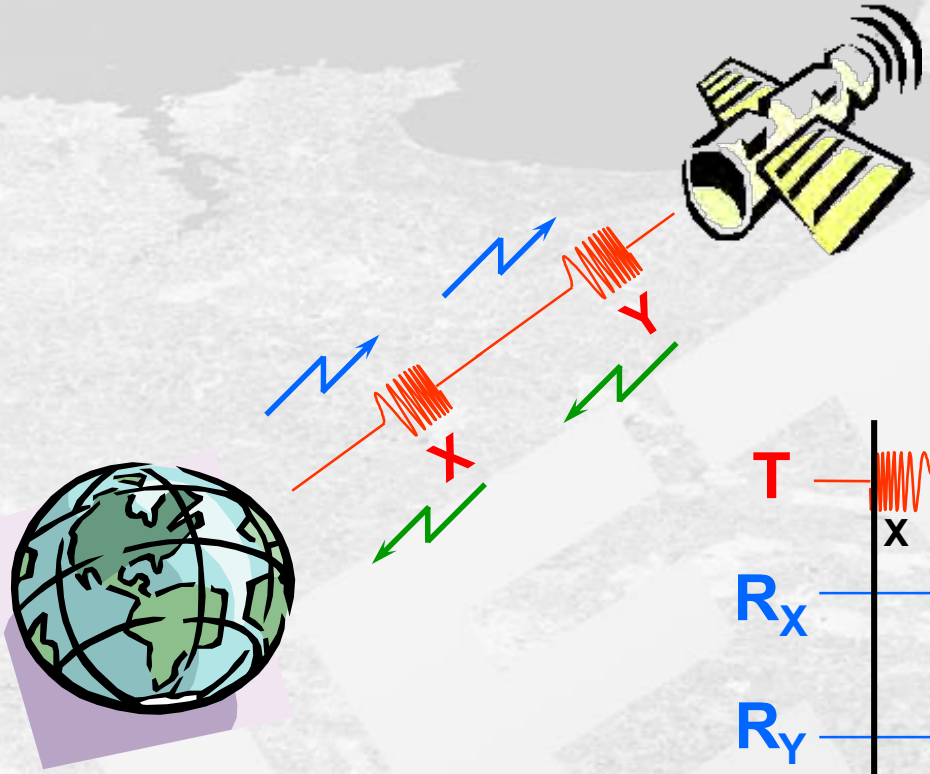


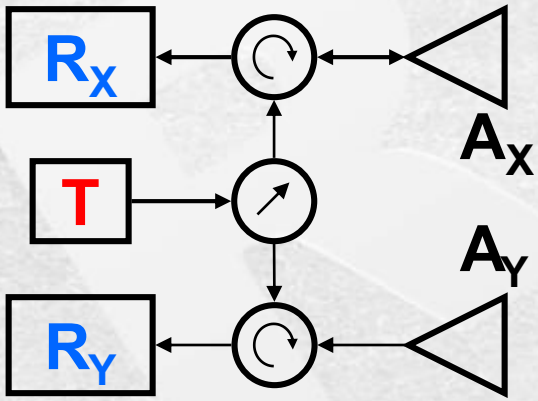
TRANSMITTER: X
RECEIVERS: X & Y

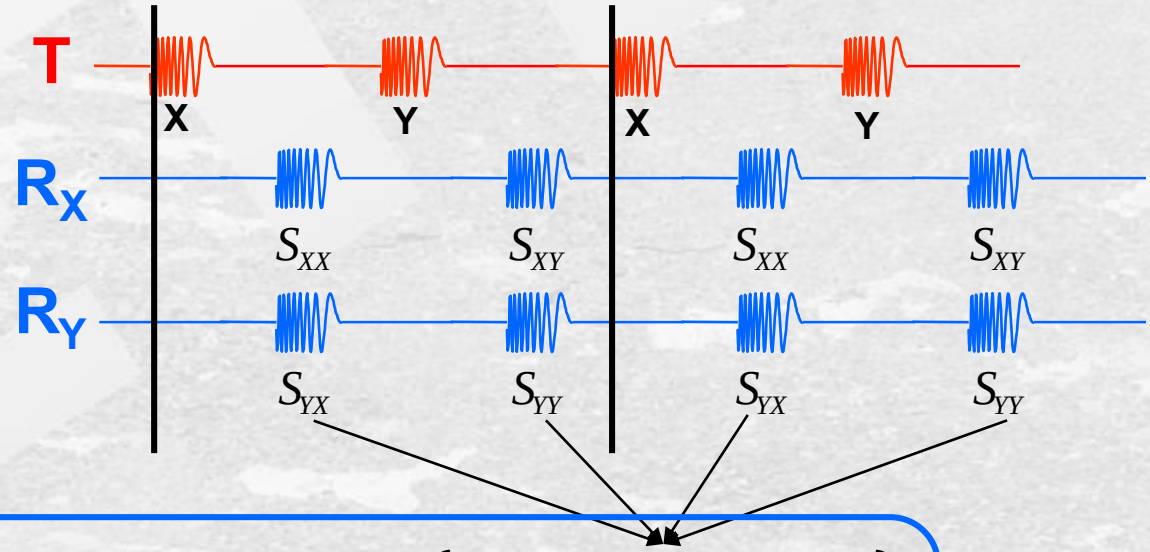
JONES VECTORS $\left\{ \underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix} \right\}$

WAVE POLARIMETRY

SCATTERING POLARIMETRY







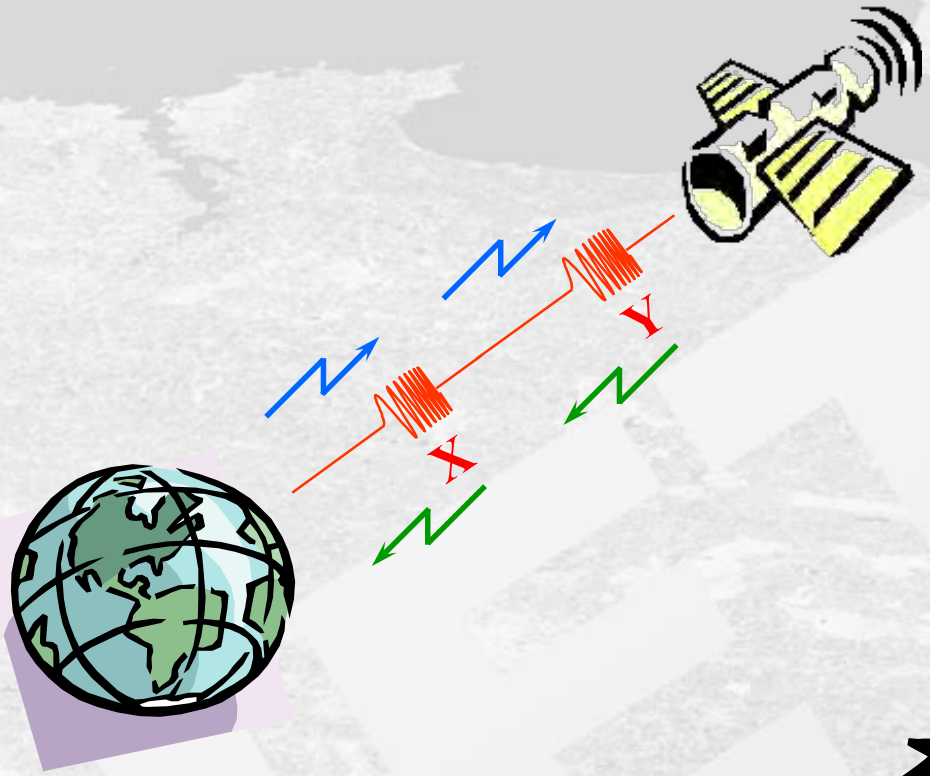
TRANSMITTER: X & Y
RECEIVERS: X & Y

SINCLAIR MATRICES

$$\left\{ [S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \right\}$$

SCATTERING POLARIMETRY

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

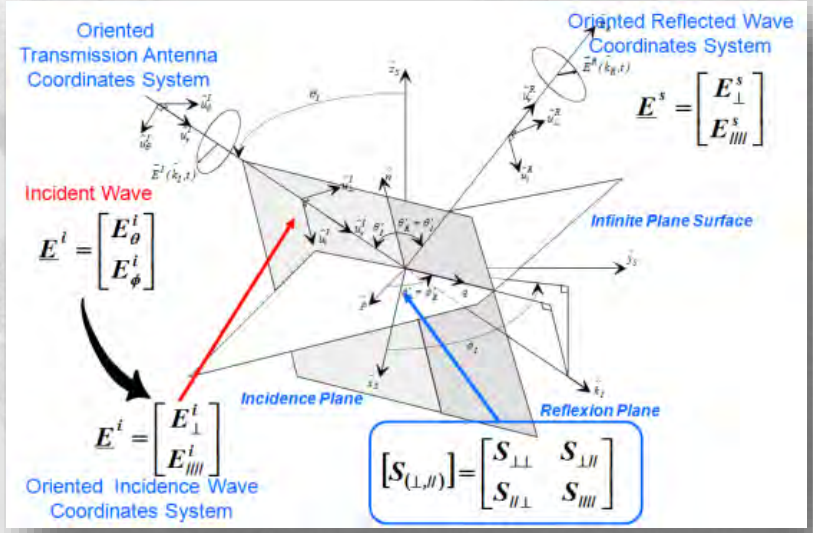
[S]	SINCLAIR Matrix
k, Ω	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

JONES / SINCLAIR MATRIX

BISTATIC CASE

SCATTERING MATRIX or JONES MATRIX

$$\begin{bmatrix} E_X^s \\ E_Y^s \end{bmatrix} = \frac{e^{jkr}}{r} \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \begin{bmatrix} E_X^i \\ E_Y^i \end{bmatrix}$$



RECIPROCITY THEOREM

$$S_{XY} = S_{YX}$$

MONOSTATIC CASE

BACKSCATTERING MATRIX or SINCLAIR MATRIX

$$\begin{bmatrix} E_X^s \\ E_Y^s \end{bmatrix} = \frac{e^{jkr}}{r} \begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix} \begin{bmatrix} E_X^i \\ E_Y^i \end{bmatrix}$$

BACKSCATTERING MATRIX

$$[S] = \frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix} = \frac{e^{jkr}}{r}\underbrace{\begin{bmatrix} |S_{XX}|e^{j\varphi_{XX}} & |S_{XY}|e^{j\varphi_{XY}} \\ |S_{XY}|e^{j\varphi_{XY}} & |S_{YY}|e^{j\varphi_{YY}} \end{bmatrix}}_{\text{ABSOLUTE BACKSCATTERING MATRIX}}$$

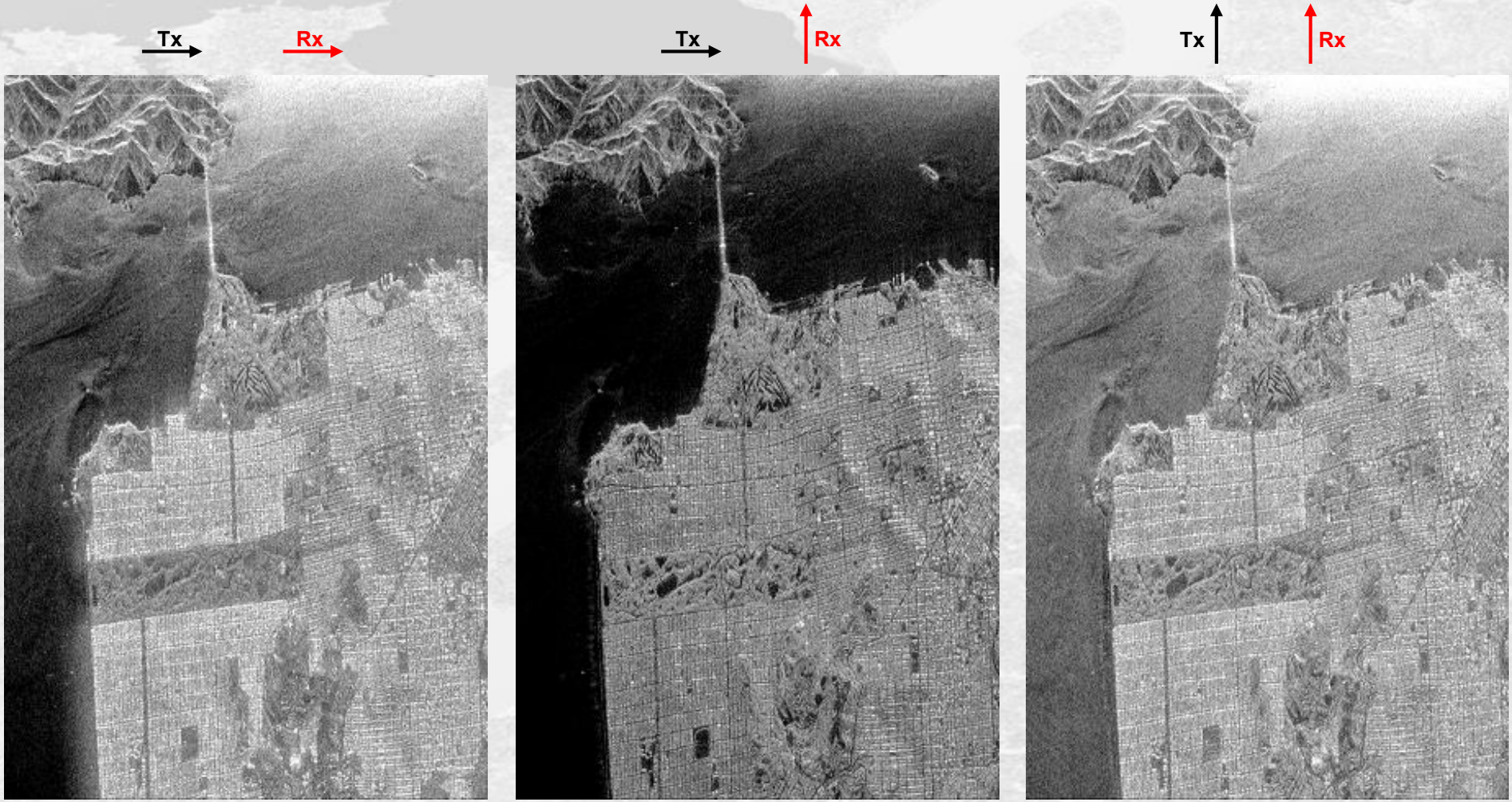
$$[S] = \underbrace{\frac{e^{jkr}e^{j\phi_{XX}}}{r}}_{\text{Absolute Phase Factor}}\underbrace{\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} \\ |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} & |S_{YY}|e^{j(\varphi_{YY}-\varphi_{XX})} \end{bmatrix}}_{\text{RELATIVE BACKSCATTERING MATRIX}}$$

RELATIVE BACKSCATTERING MATRIX

Five Parameters: 3 Amplitudes and 2 Phases

SCATTERER POLARIMETRIC DIMENSION = 5

SCATTERING POLARIMETRY



$|HH|_{dB}$ $|HV|_{dB}$ $|VV|_{dB}$

-30dB -15dB 0dB

SCATTERING POLARIMETRY

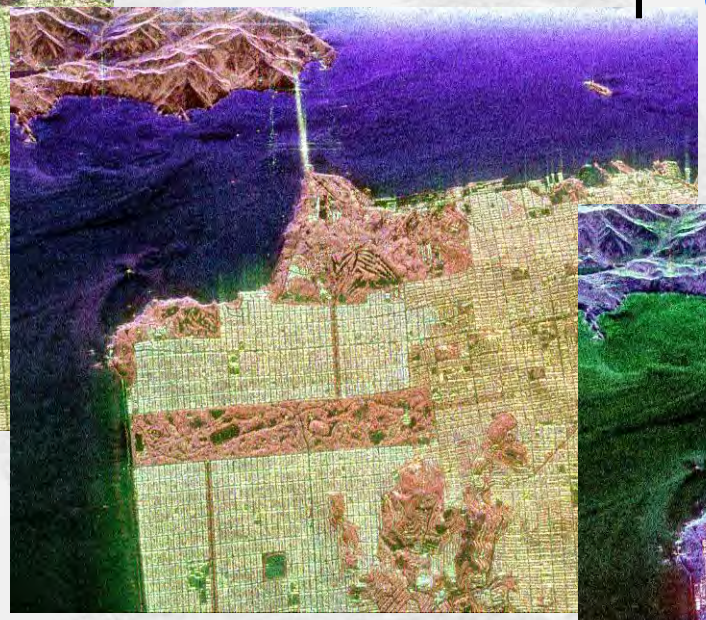
Sinclair Color Coding

© Google Earth

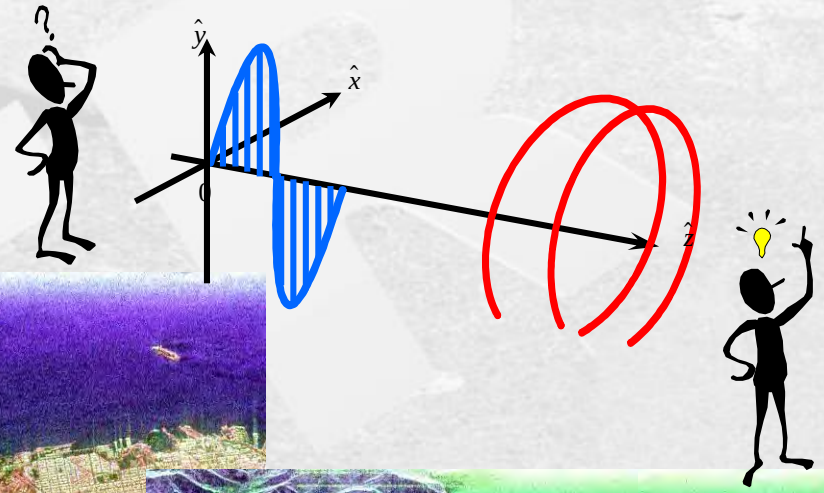
|HH| |HV| |VV|

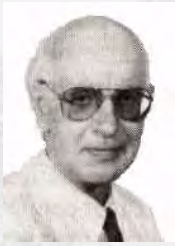
ELLIPTICAL BASIS TRANSFORMATION

Pauli Color Coding (H,V)

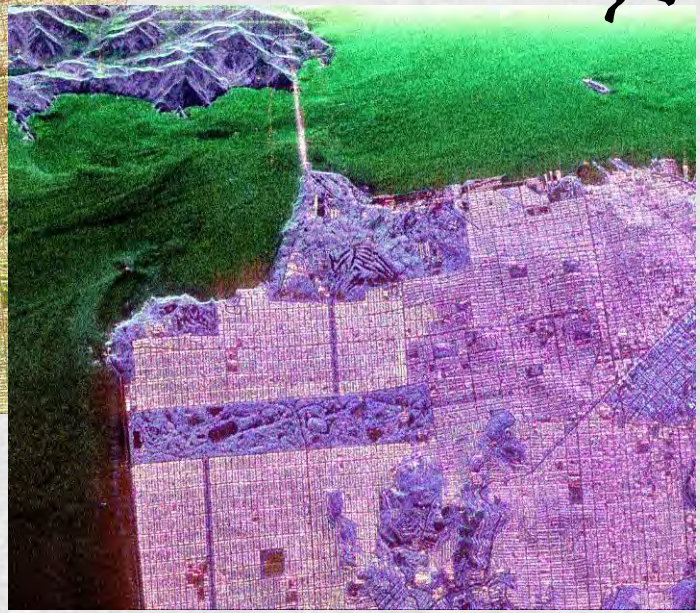


Pauli Color Coding (+45,-45)





Ernst LÜNEBURG
(PIERS95 - Pasadena)



Pauli Color Coding (L,R)

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T \left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

$$\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

SU(2) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\Downarrow$$

$$\left[U_{(\underline{A},\underline{A}_\perp)\mapsto(\underline{B},\underline{B}_\perp)}\right] = \left[U(\phi,\tau,\alpha)\right]^{-1}$$

$$= \begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$\left[U_2(-\alpha)\right] \qquad \left[U_2(-\tau)\right] \qquad \left[U_2(-\phi)\right]$$

ELLIPTICAL BASIS TRANSFORMATION

(H,V) POLARISATION BASIS

© Google Earth

|HH+VV| |HV | |HH-VV|

ELLIPTICAL BASIS TRANSFORMATION

© Google Earth

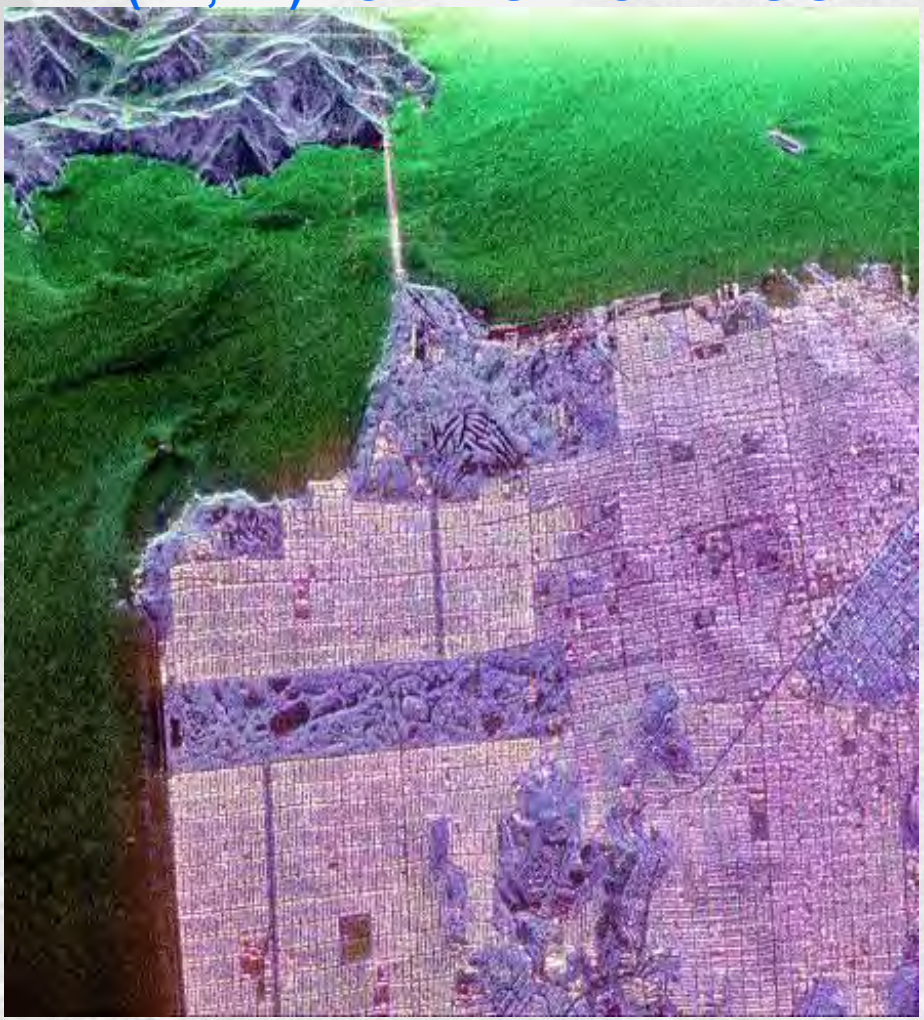
(+45°,-45°) POLARISATION BASIS

|AA+BB| |AB | |AA-BB|

With: A=Linear +45°, B=Linear –45°

ELLIPTICAL BASIS TRANSFORMATION

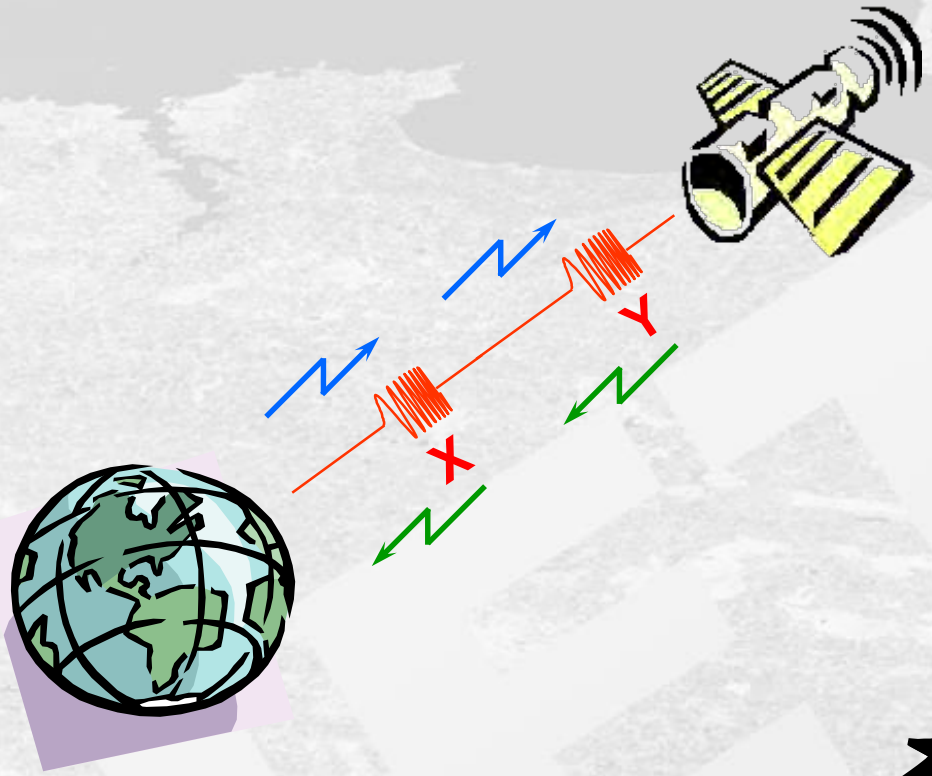
(LC,RC) POLARISATION BASIS



© Google Earth

|LL+RR| |LR | |LL-RR|

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
[K]	KENNAUGH Matrix
$\underline{k}$, $\underline{\Omega}$	Target Vectors
[T]	Coherency Matrix
[C]	Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

KENNAUGH MATRIX

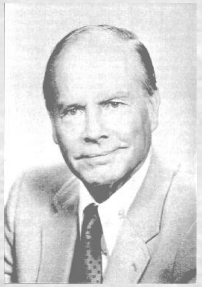
MONOSTATIC CASE

KENNAUGH MATRIX

$$[K]=\frac{1}{2}\left([V]^T\left[[S]\otimes[S]^*\right][V]\right) \qquad [V]=\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & 0 & 1 & -j\\ 0 & 0 & 1 & +j\\ 1 & -1 & 0 & 0\end{bmatrix}$$

HUYNEN PARAMETERS

$$[K]=\begin{bmatrix}A_0+B_0 & C & H & F\\ C & A_0+B & E & G\\ H & E & A_0-B & D\\ F & G & D & -A_0+B_0\end{bmatrix}$$



HUYNEN PARAMETERS

PHYSICAL INTERPRETATION

MAN-MADE TARGET DECOMPOSITION
IDENTIFICATION and ANALYSIS

« ***PHENOMENOLOGICAL THEORY OF RADAR TARGETS*** » (1970)

A0 : GENERATOR OF TARGET SYMMETRY

B0+B : GENERATOR OF TARGET NON-SYMMETRY

B0-B : GENERATOR OF TARGET IRREGULARITY

C : GENERATOR OF TARGET GLOBAL SHAPE (LINEAR)

D : GENERATOR OF TARGET LOCAL SHAPE (CURVATURE)

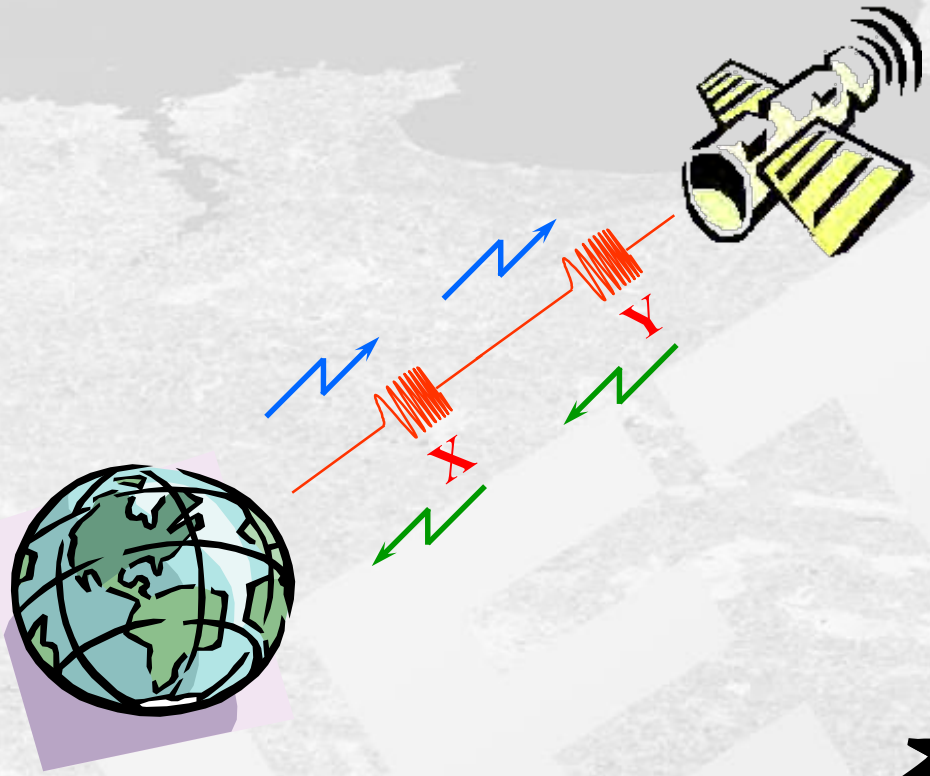
E : GENERATOR OF TARGET LOCAL TWIST (TORSION)

F : GENERATOR OF TARGET GLOBAL TWIST (HELICITY)

G : GENERATOR OF TARGET LOCAL COUPLING (GLUE)

H : GENERATOR OF TARGET GLOBAL COUPLING (ORIENTATION)

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
[K]	KENNAUGH Matrix
k, Ω	**Target Vectors**
[T]	Coherency Matrix
[C]	Covariance Matrix

TARGET VECTORS

VECTORIAL FORMULATION OF THE SCATTERING PROBLEM

SCATTERING MATRIX

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

SCATTERING VECTOR

$$\vec{S} := V([S]) = \frac{1}{2} Trace([S][\Psi]) = \begin{bmatrix} S1 \\ S2 \\ S3 \\ S4 \end{bmatrix} \in C_4$$

With: $V([S])$ MATRIX VECTORISATION OPERATOR

$[\Psi]$ SET OF ORTHOGONAL 2x2 MATRICES

FROBENIOUS NORM OF $\vec{S}$

$$\|\vec{S}\|^2 = \vec{S}^{T*} \cdot \vec{S} = |S_1|^2 + |S_2|^2 + |S_3|^2 + |S_4|^2$$

$$= Span([S]) = |S_{XX}|^2 + |S_{YX}|^2 + |S_{XY}|^2 + |S_{YY}|^2$$

TARGET VECTORS

PAULI SCATTERING VECTOR

$$\underline{k} = V([S]) = \frac{1}{2} Trace([S][\psi_P])$$

SET OF 2x2 COMPLEX MATRICES FROM THE PAULI MATRICES GROUP

$$[\psi_P] = \left\{ \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix} \right\}$$

$$\Downarrow$$

$$\boxed{\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad S_{XY} + S_{YX} \quad j(S_{XY} - S_{YX})\right]^T}$$

Advantage: Closer related to physical properties of the scatterer

Note: Also known as $\underline{k}_{4P}$

TARGET VECTORS

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega} = V([S]) = \frac{1}{2} Trace([S][\psi_L])$

SET OF 2x2 COMPLEX MATRICES FROM THE LEXICOGRAPHIC MATRICES GROUP

$$[\psi_L] = \left\{ 2\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\boxed{\underline{\Omega} = [S_{XX} \quad S_{XY} \quad S_{YX} \quad S_{YY}]^T}$$

Advantage: **Directly related to the system measurables**

Note: **Also known as $\underline{k}_{4L}$**

TARGET VECTORS

BISTATIC CASE → MONOSTATIC CASE

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ S_{XY} + S_{YX} \\ j(S_{XY} - S_{YX}) \end{bmatrix} \Rightarrow \underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ 2S_{XY} \end{bmatrix}$$

Note: Also known as $\underline{k}_{3P}$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ S_{XY} \\ S_{YX} \\ S_{YY} \end{bmatrix} \Rightarrow \underline{\Omega} = \begin{bmatrix} S_{XX} \\ \sqrt{2}S_{XY} \\ S_{YY} \end{bmatrix}$$

Note: Also known as $\underline{k}_{3L}$

SCATTERING VECTOR TRANSFORMATIONS

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ 2S_{XY} \end{bmatrix}$$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ \sqrt{2}S_{XY} \\ S_{YY} \end{bmatrix}$$

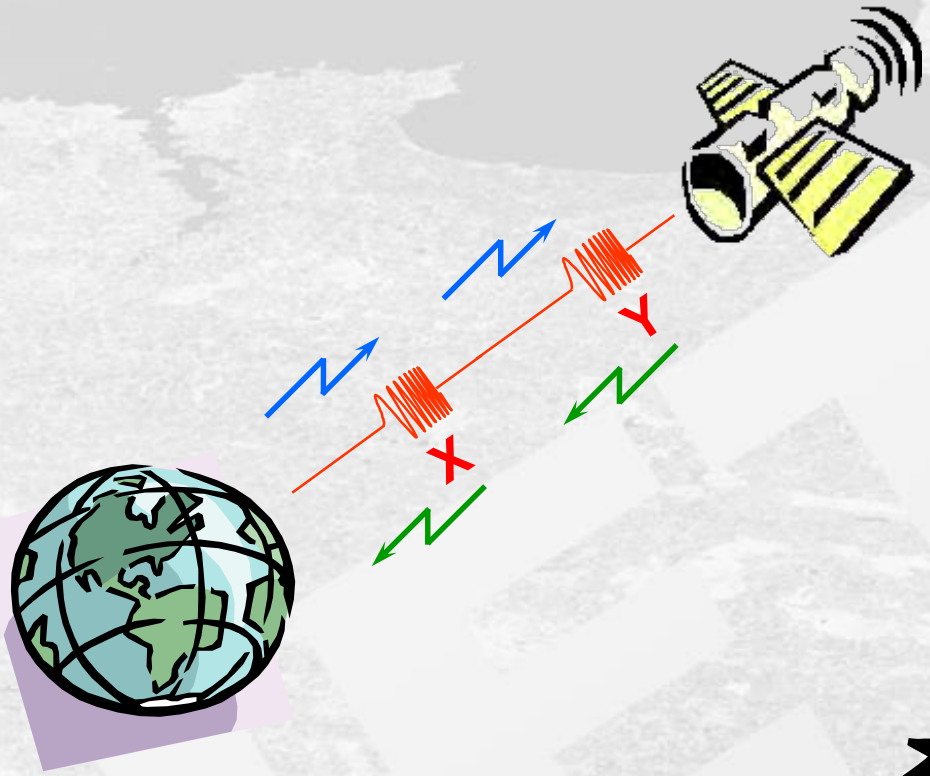
UNITARY TRANSFORMATION

$$\underline{k} = [D_3]\underline{\Omega} \quad and \quad \underline{\Omega} = [D_3]^{-1}\underline{k} = [D_3]^T\underline{k}$$

WHERE $[D_3]$ IS A SU(3) MATRIX IN ORDER TO PRESERVE THE NORM OF THE SCATTERING VECTOR

$$[D_3] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \end{bmatrix}$$

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
[K]	KENNAUGH Matrix
k, Ω	Target Vectors
[T]	Coherency Matrix
[C]	Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

COHERENCY MATRIX

MONOSTATIC CASE

PAULI SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} & S_{XX} - S_{YY} & 2S_{XY} \end{bmatrix}^T$$

COHERENCY MATRIX $[T]$

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

HERMITIAN POSITIVE SEMI-DEFINITE MATRIX - RANK 1

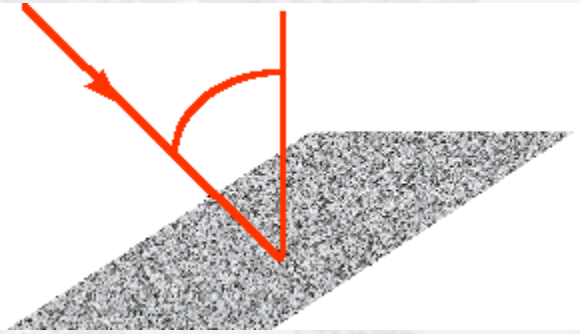
HUYNEN TARGET GENERATORS

$$T_{11} = 2A_0 = \left|S_{XX} + S_{YY}\right|^2 \qquad T_{22} = B_0 + B = \left|S_{XX} - S_{YY}\right|^2$$

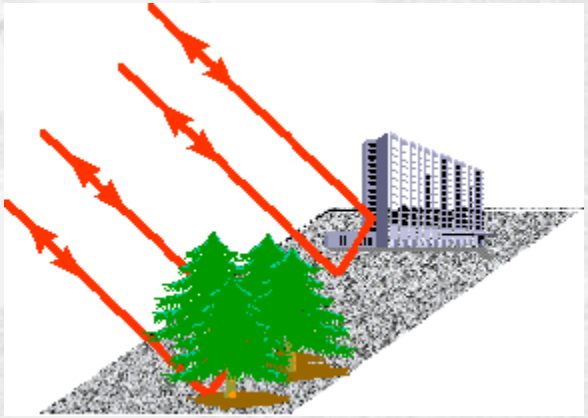
$$T_{33} = B_0 - B = 2\left|S_{XY}\right|^2$$

PHYSICAL INTERPRETATION

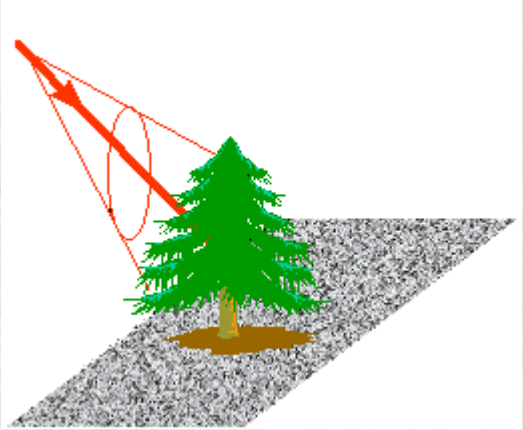
SINGLE BOUNCE SCATTERING (ROUGH SURFACE)



DOUBLE BOUNCE SCATTERING



VOLUME SCATTERING

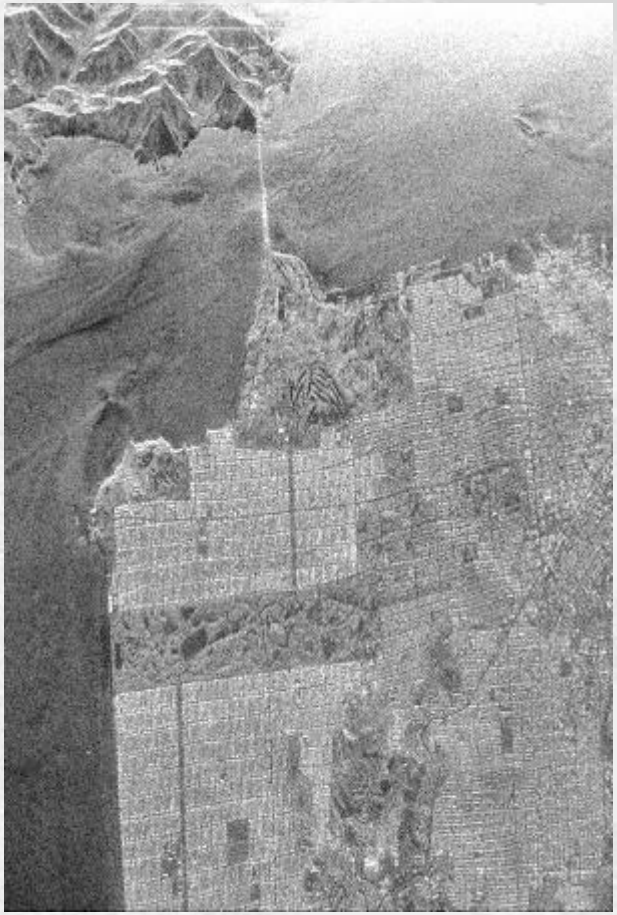


$$T_{11} = 2A_0 = |S_{XX} + S_{YY}|^2$$

$$T_{33} = B_0 - B = 2|S_{XY}|^2$$

$$T_{22} = B_0 + B = |S_{XX} - S_{YY}|^2$$

TARGET GENERATORS



$|HH+VV|_{dB}$

$|HV|_{dB}$

$|HH-VV|_{dB}$

-30dB -15dB 0dB

TARGET GENERATORS

(H,V) POLARISATION BASIS

© Google Earth

|HH+VV| |HV | |HH-VV|

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2]=\begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

SPECIAL UNITARY SU(3) GROUP

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(2\phi) & sin(2\phi) \\ 0 & -sin(2\phi) & cos(2\phi) \end{bmatrix}\begin{bmatrix} cos(2\tau) & 0 & j\,sin(2\tau) \\ 0 & 1 & 0 \\ j\,sin(2\tau) & 0 & cos(2\tau) \end{bmatrix}\begin{bmatrix} cos(2\alpha) & -j\,sin(2\alpha) & 0 \\ -j\,sin(2\alpha) & cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[U_3(2\phi)] \qquad [U_3(2\tau)] \qquad [U_3(2\alpha)]$$

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$\left[S_{(B,B_\perp)}\right]=\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T\left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

$$\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

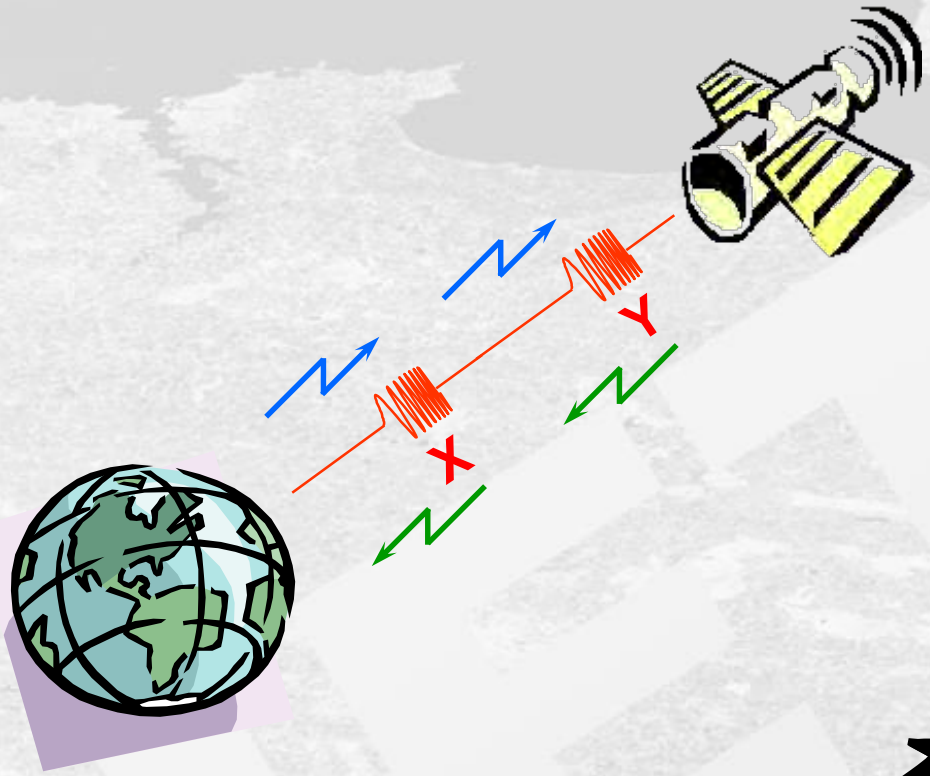
U(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

COHERENCY MATRIX

$$\left[T_{(B,B_\perp)}\right]=\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]\left[T_{(A,A_\perp)}\right]\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]^{-1}$$

SIMILARITY TRANSFORMATION

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
[K]	KENNAUGH Matrix
$\underline{k}$, $\underline{\Omega}$	Target Vectors
[T]	Coherency Matrix
[C]	**Covariance Matrix**

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

COVARIANCE MATRIX

BISTATIC CASE

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \begin{bmatrix} S_{XX} & S_{XY} & S_{YX} & S_{YY} \end{bmatrix}^T$$

$$\Downarrow$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX}S_{XX}^* & S_{XX}S_{XY}^* & S_{XX}S_{YX}^* & S_{XX}S_{YY}^* \\ S_{XY}S_{XX}^* & S_{XY}S_{XY}^* & S_{XY}S_{YX}^* & S_{XY}S_{YY}^* \\ S_{YX}S_{XX}^* & S_{YX}S_{XY}^* & S_{YX}S_{YX}^* & S_{YX}S_{YY}^* \\ S_{YY}S_{XX}^* & S_{YY}S_{XY}^* & S_{YY}S_{YX}^* & S_{YY}S_{YY}^* \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

COVARIANCE MATRIX

MONOSTATIC CASE

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \begin{bmatrix} S_{XX} & \sqrt{2}S_{XY} & S_{YY} \end{bmatrix}^T$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX}S_{XX}^* & \sqrt{2}S_{XX}S_{XY}^* & S_{XX}S_{YY}^* \\ \sqrt{2}S_{XY}S_{XX}^* & 2S_{XY}S_{XY}^* & \sqrt{2}S_{XY}S_{YY}^* \\ S_{YY}S_{XX}^* & \sqrt{2}S_{YY}S_{XY}^* & S_{YY}S_{YY}^* \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$\left[S_{(B,B_\perp)}\right]=\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T\left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

$$\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

U(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

COVARIANCE MATRIX

$$\left[C_{(B,B_\perp)}\right]=\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]\left[C_{(A,A_\perp)}\right]\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]^{-1}$$

SIMILARITY TRANSFORMATION

COVARIANCE-COHERENCY MATRICES

COHERENCY MATRIX

$$[T] = \underline{k} \cdot \underline{k}^{*T}$$

$$\underline{k} = [D_3]\underline{\Omega}$$

COVARIANCE MATRIX

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T}$$

UNITARY TRANSFORMATION

$$[T] = [D_3][C][D_3]^{T*}$$

$[T]$ **and** $[C]$ **HAVE THE SAME EIGENVALUES**

Both contain the same information about Polarimetric Scattering Amplitudes, Phase Angles and Correlations

$[T]$ is closer related to Physical and Geometrical Properties of the Scattering Process, and thus allows a better and direct physical interpretation

$[C]$ is directly related to the system measurables

$[T]$ is directly related to the Kennaugh matrix and the Huynen parameters

POLARIMETRIC DESCRIPTORS

SINCLAIR MATRIX

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

KENNAUGH MATRIX

$$[K] = \frac{1}{2}\left([V]^T\left[[S]\otimes[S]^*\right][V]\right)$$

EQUIVALENCE ?

SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX}+S_{YY} \quad S_{XX}-S_{YY} \quad 2S_{XY}\right]^T$$

COHERENCY MATRIX $[T]$

$$[T] = \underline{k}\cdot\underline{k}^{*T}$$

SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \left[S_{XX} \quad \sqrt{2}S_{XY} \quad S_{YY}\right]^T$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega}\,\underline{\Omega}^{T*}$$

POLARIMETRIC DESCRIPTORS

$$[\,S'\,]=[\,U_2\,]^T[\,S\,]\,[\,U_2\,]$$

$[\,S\,]$
SINCLAIR
SU(2)

$$[\,C'\,]=[\,U_3\,]\,[\,C\,]\,[\,U_3\,]^{-1}$$

$[\,C\,]$ COVARIANCE SU(3)

$[\,T\,]$ COHERENCY SU(3)

$[\,K\,]$
KENNAUGH
O(3)

$$[\,T'\,]=[\,U_3\,]\,[\,T\,]\,[\,U_3\,]^{-1}$$

$$[\,K'\,]=[\,O_3\,]\,[\,K]\,[\,O_3\,]^{-1}$$

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

SPECIAL UNITARY SU(3) GROUP (T Matrix)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\phi) & \sin(2\phi) \\ 0 & -\sin(2\phi) & \cos(2\phi) \end{bmatrix} \begin{bmatrix} \cos(2\tau) & 0 & j\sin(2\tau) \\ 0 & 1 & 0 \\ j\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} \cos(2\alpha) & -j\sin(2\alpha) & 0 \\ -j\sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[U_3(2\phi)] \qquad [U_3(2\tau)] \qquad [U_3(2\alpha)]$$

O(3) UNITARY GROUP

$$\begin{bmatrix} \cos 2\phi & -\sin 2\phi & 0 \\ \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 2\tau & 0 & -\sin 2\tau \\ 0 & 1 & 0 \\ \sin 2\tau & 0 & \cos 2\tau \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\alpha & -\sin 2\alpha \\ 0 & \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$[O_3(2\phi)] \qquad [O_3(2\tau)] \qquad [O_3(2\alpha)]$$

TARGET EQUATIONS

POLARIMETRIC GOLDEN NUMBER

POLARIMETRIC TARGET DIMENSION

TARGET EQUATIONS

$$[S]=\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\phi_{XY-XX}, \phi_{YY-XX}$$

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

II

5

KENNAUGH MATRIX $[K]$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(A0, B0, B, C, D, E, F, G, H)

COVARIANCE MATRIX $[C]$

9 REAL PARAMETERS

$|XX|$, $|XY|$, $|YY|$,
$\mathrm{Re}(XXXY^*)$, $\mathrm{Im}(XXXY^*)$
$\mathrm{Re}(XXYY^*)$, $\mathrm{Im}(XXYY^*)$
$\mathrm{Re}(XYYY^*)$, $\mathrm{Im}(XYYY^*)$

9 - 5 = 4 TARGET EQUATIONS

TARGET EQUATIONS

PURE TARGET – MONOSTATIC CASE

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

3x3 HERMITIAN MATRIX - RANK 1

↓

9 PRINCIPAL MINORS = 0

$$2A_0(B_0 + B) - C^2 - D^2 = 0 \qquad 2A_0(B_0 - B) - G^2 - H^2 = 0$$

$$-2A_0E + CH - DG = 0 \qquad B_0^2 - B^2 - E^2 - F^2 = 0$$

$$C(B_0 - B) - EH - GF = 0 \qquad -D(B_0 - B) + FH - GE = 0$$

$$2A_0F - CG - DH = 0 \qquad -G(B_0 + B) + FC - ED = 0$$

$$H(B_0 + B) - CE - DF = 0$$

TARGET EQUATIONS

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\phi_{XY-XX}, \phi_{YY-XX}$$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(*A0, B0, B, C, D, E, F, G, H*)

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

II

5

9 - 5 = 4 TARGET EQUATIONS

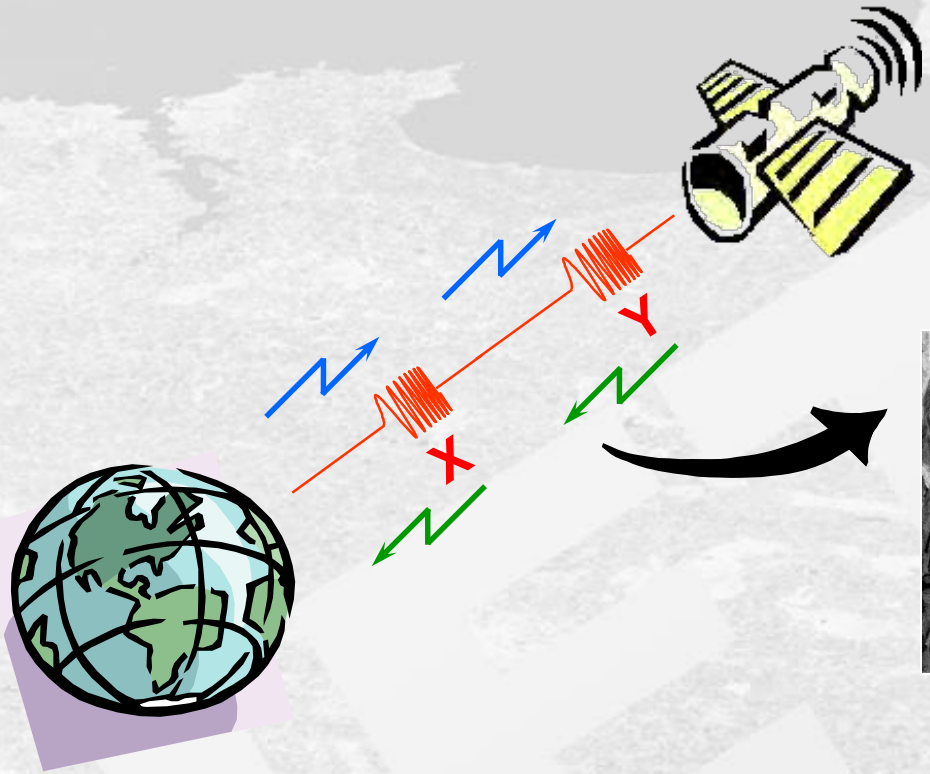
$$2A_0(B_0 + B) = C^2 + D^2$$

$$2A_0(B_0 - B) = G^2 + H^2$$

$$2A_0E = CH - DG$$

$$2A_0F = CG + DH$$

SCATTERING POLARIMETRY



TRANSMITTER: X & Y
RECEIVERS: X & Y

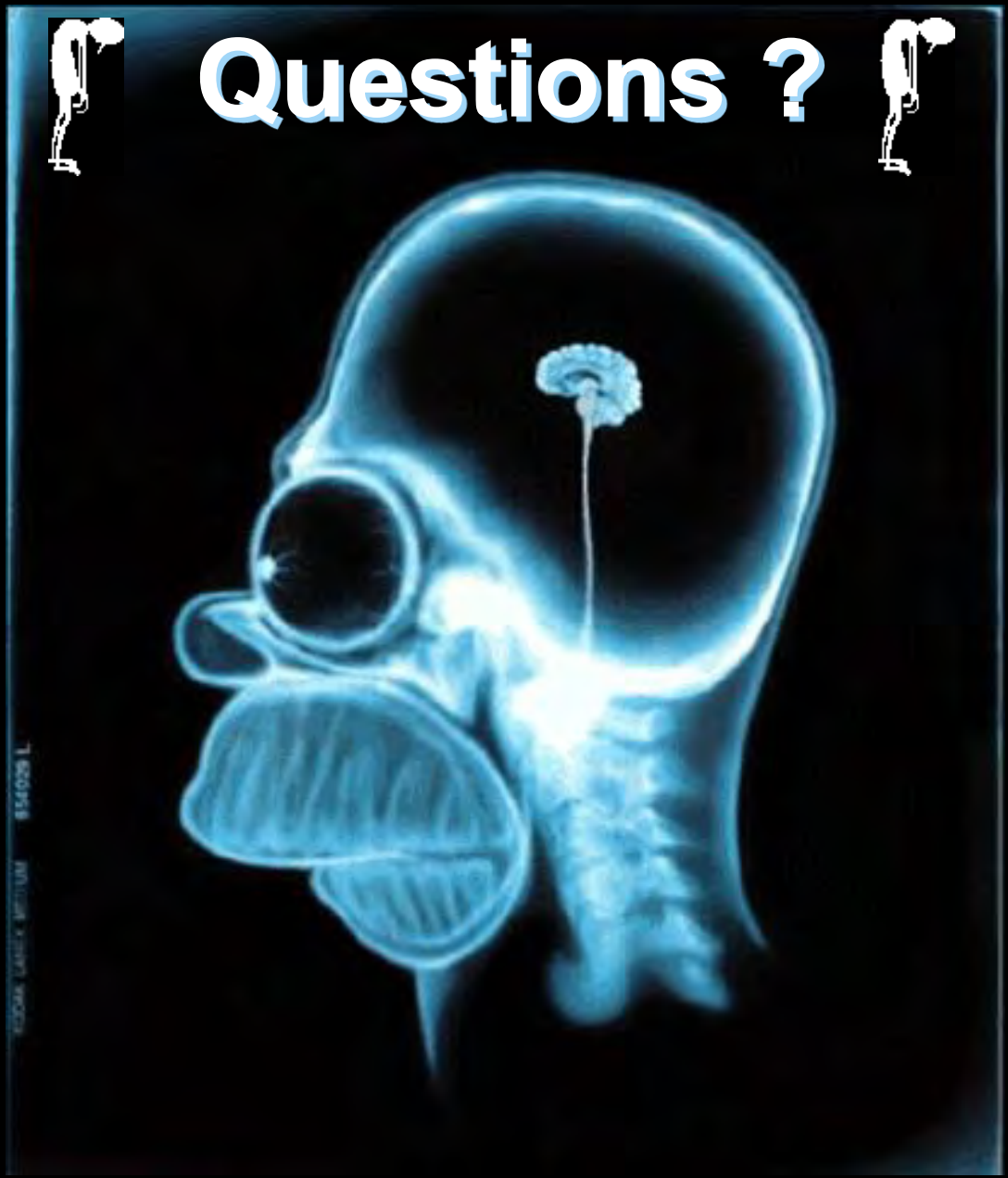
SINCLAIR MATRIX [S]

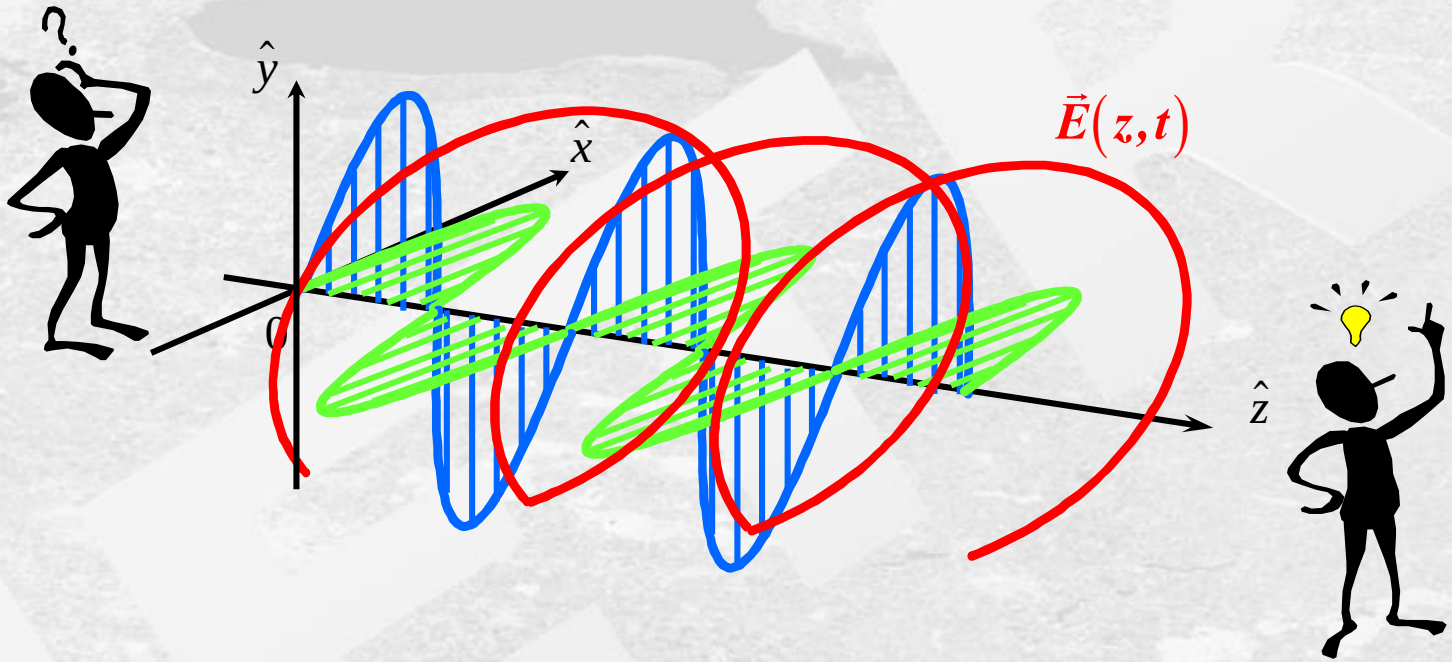
THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
k, Ω	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

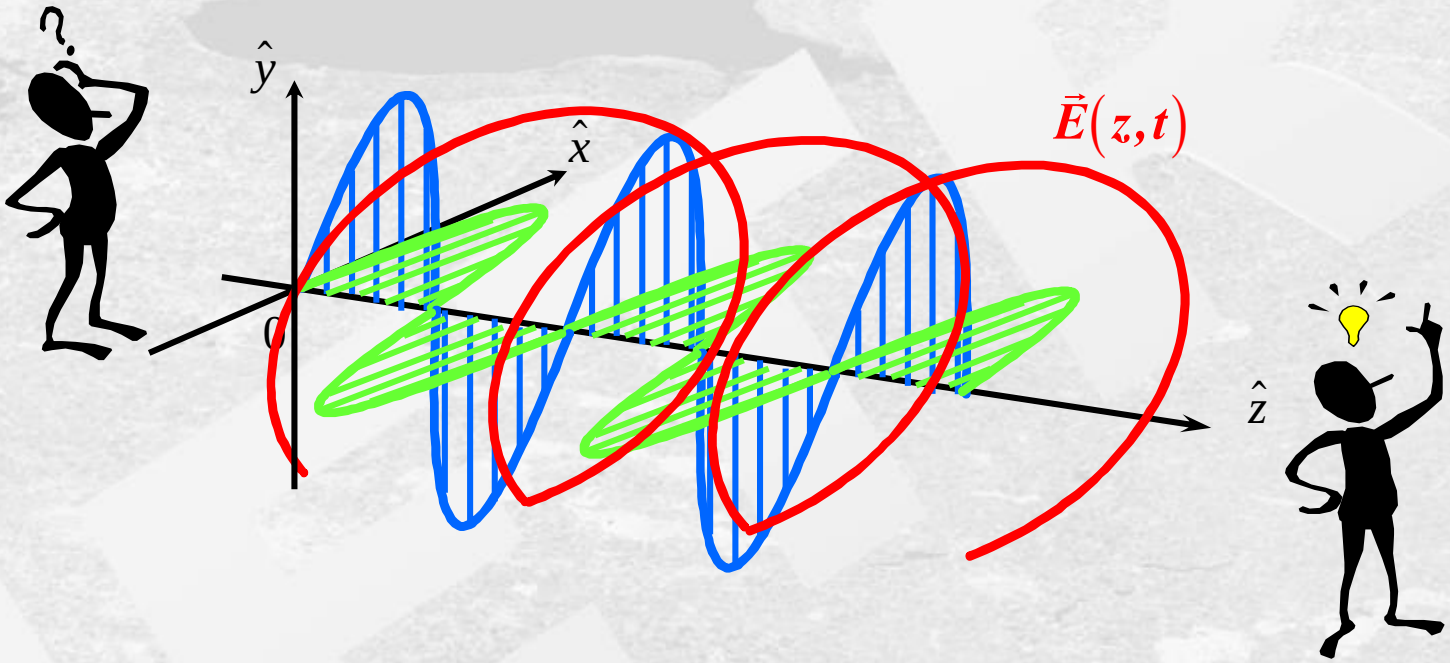
POLARIMETRIC REMOTE SENSING

Questions ?



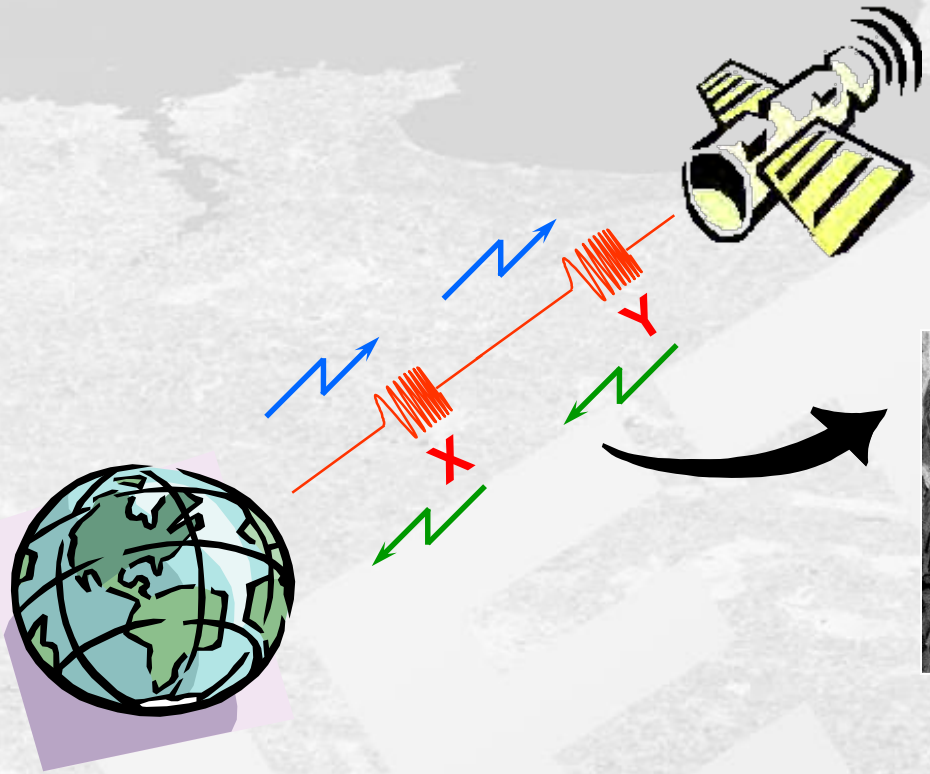


ADVANCED CONCEPTS



POLARIMETRIC REMOTE SENSING

SCATTERING POLARIMETRY



TRANSMITTER: X & Y
RECEIVERS: X & Y

SINCLAIR MATRIX [S]

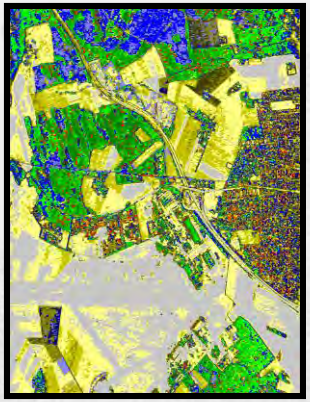
THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
k, Ω	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

POLARIMETRIC REMOTE SENSING

POLARIMETRIC REMOTE SENSING

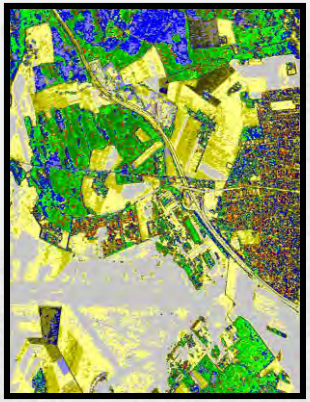
POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS



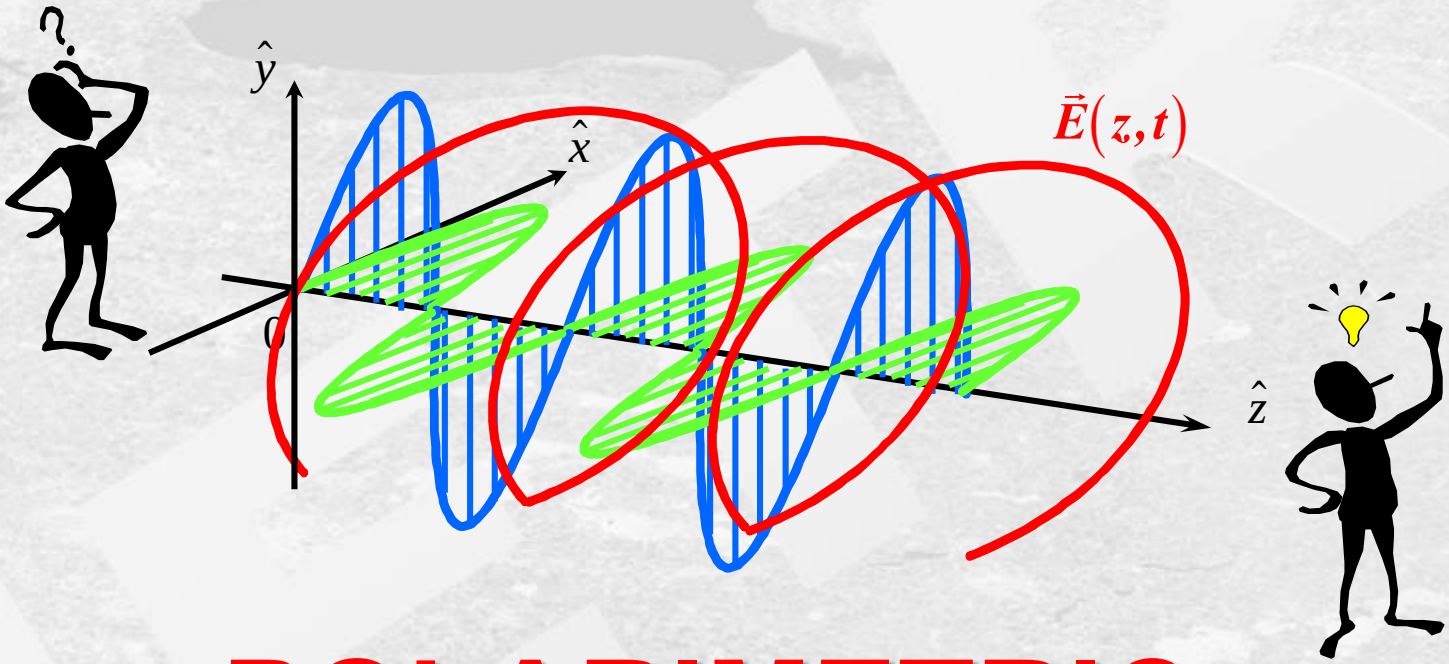
POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

POLARIMETRIC REMOTE SENSING

POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS



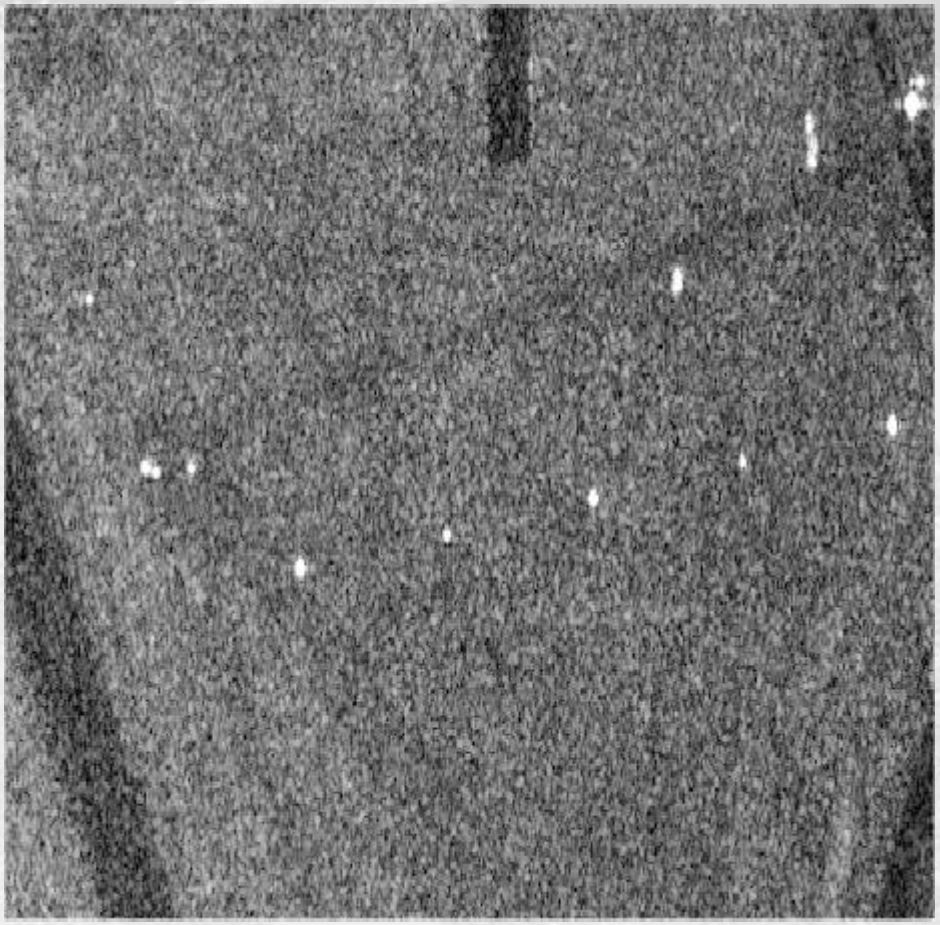
POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

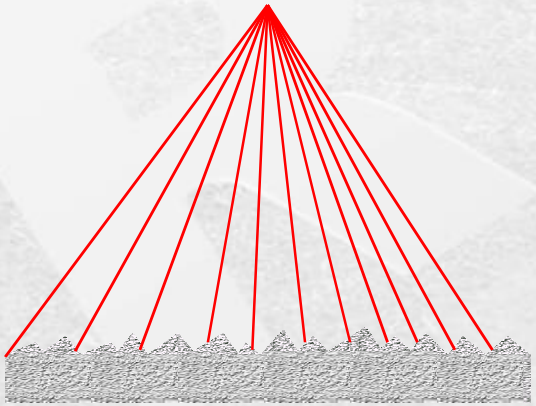


POLARIMETRIC SPECKLE FILTERING

An Introduction

SPECKLE PHENOMENON





SCATTERING FROM DISTRIBUTED SCATTERERS

↓

COHERENT INTERFERENCES OF WAVES SCATTERED FROM MANY RANDOMLY DISTRIBUTED ELEMENTARY SCATTERERS INSIDE THE RESOLUTION CELL

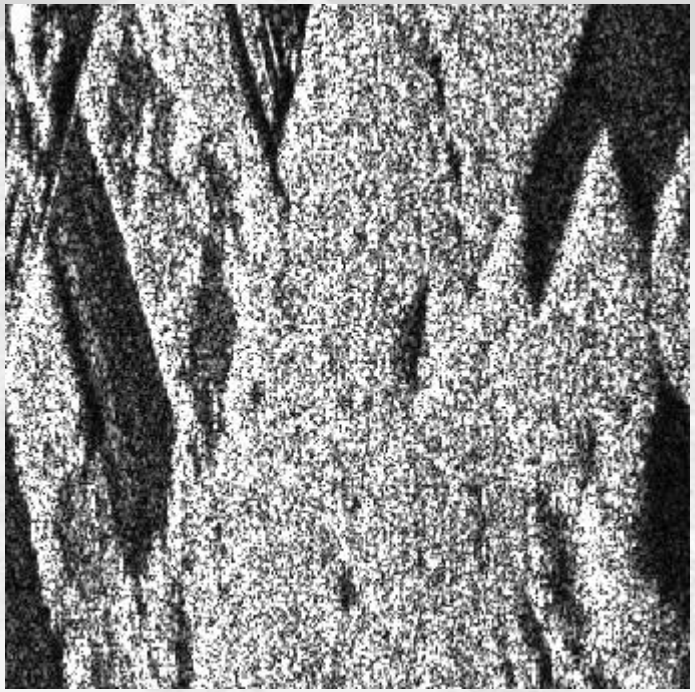
↓

GRANULAR NOISE

↓

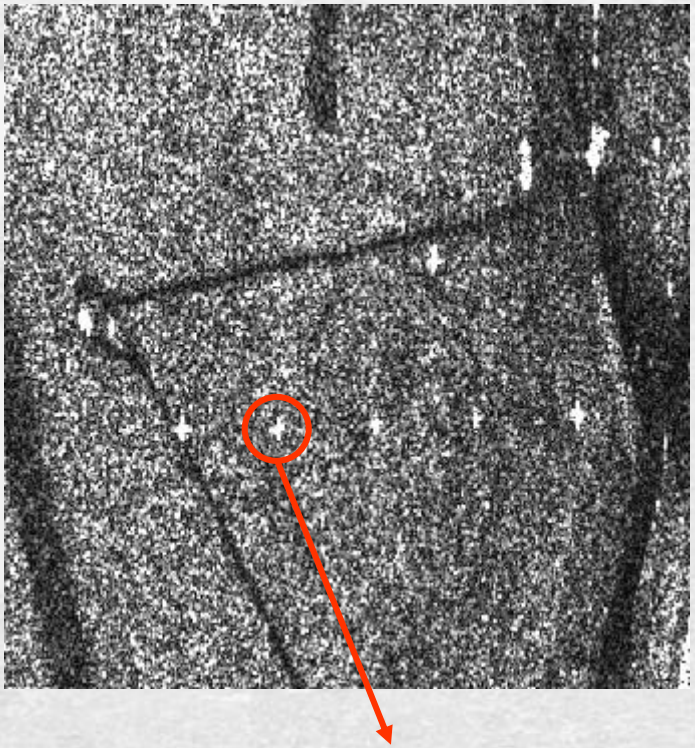
SPECKLE PHENOMENON

SPECKLE PHENOMENON



Fully Developed speckle

Bright points: Points where the interference is constructive
Dark points: Points where the interference is destructive



Corner reflector
Dominant scatter
No speckle

S_{hh} amplitude
E-SAR L-band system

Courtesy of Dr C. Lopez Martinez

SPECKLE FILTERING

SPECKLE PHENOMENON

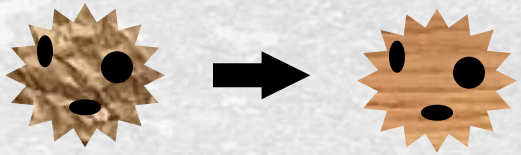
↓

DISTORTION OF THE INTERPRETATION

SPECKLE FILTERING

HOMOGENEOUS AREA

HETEROGENEOUS AREA



SPECKLE REDUCTION
(RADIOMETRIC RESOLUTION)

DETAILS PRESERVATION
(SPATIAL RESOLUTION)

LINEAR SPECKLE FILTERS
Intensity / Amplitude – Single / Multi Look – Single Pol Channel

Median Filter
MAP Filter (Kuan)
Gradient Filter
Nagao Filter (Nagao)
Sigma Filter (Lee)
Frost Filter (Frost)
Geometrical Filter (Crimmins)
Morphological Filter (Safa, Flouzat)

.

Local Statistics Filter (Lee 80)
Refined Lee Filter (Lee 81)

J.S. Lee, et al. "Speckle Filtering of SAR images: A Review," Remote Sensing Reviews, Vol. 8, pp. 313-340, 1994.

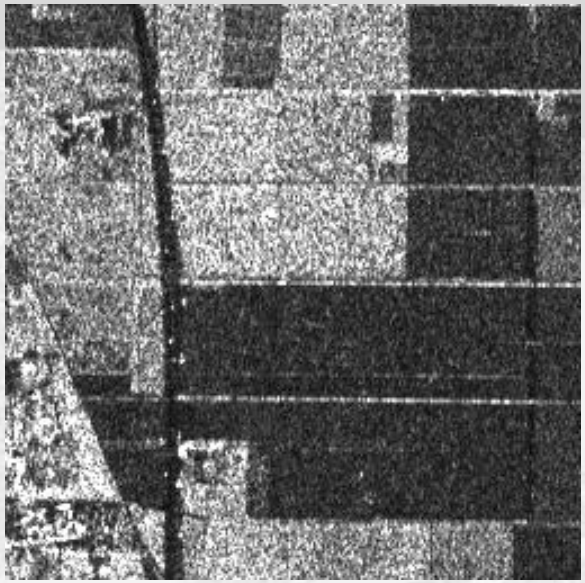
J.S. Lee,"Speckle analysis and smoothing of SAR images," Computer Graphics and Image Processing, Vol. 17, 1981.

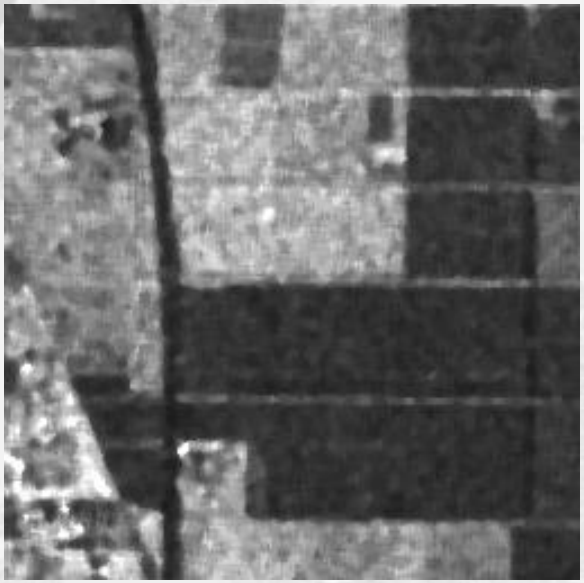
J.S. Lee,"Digital image enhancement and noise filtering by use of local statistics," IEEE PAMI, Vol. 2 No. 2, 1980.

J.S. Lee,"Refined filtering of image noise using local statistics," CVGIP, vol.15, 380-389, 1981.

SPECKLE FILTERING

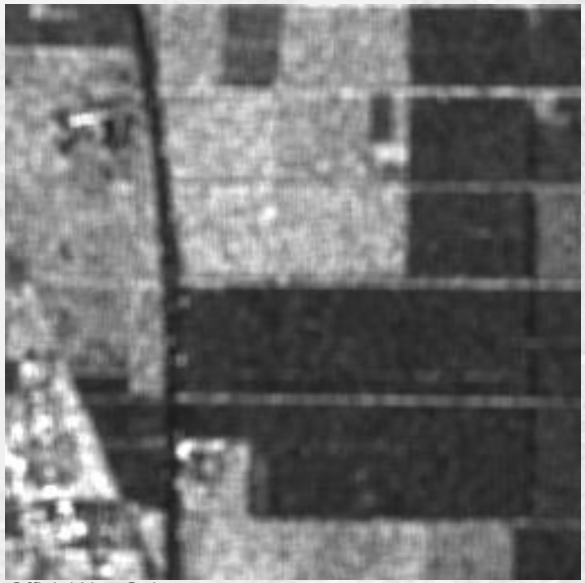
Original 4-look amplitude

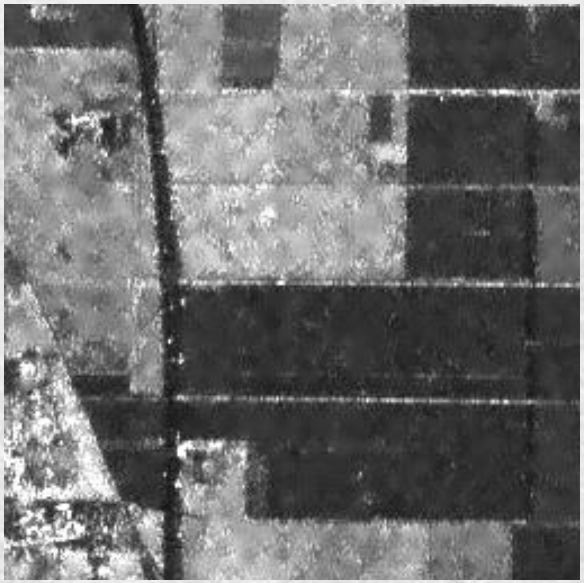




5x5 Median

5x5 Boxcar





Lee Refined (7x7)

POLSAR SPECKLE FILTERING

- Preserving polarimetric properties
  - Filter all elements equally like multi-look Processing
  - Select pixels with the same scattering property
- Introduce no cross-talk
  - Filter each element separately but equally
- Reduce speckle while preserving image quality

J.S. Lee, M.R. Grunes and G. De Grandi, "Polarimetric SAR Speckle Filtering and Its Impact on Terrain Classification" *IEEE TGRS*, September 1999

POLARIMETRIC VECTORIAL SPECKLE FILTER

$$\langle [T] \rangle = \frac{1}{N} \sum_{i=1}^{N} \underline{k}_i \underline{k}_i^{*T}$$

$$[\hat{T}] = E([T]) - k[E([T]) - [T]]$$

$$[\hat{T}]$$

SPAN IMAGE

$$S_S = \langle T_{11} \rangle + \langle T_{22} \rangle + \langle T_{33} \rangle$$

$$k = \frac{var(S)}{var(S_S)} = \frac{CV_{S_S}^2 - \sigma_v^2}{CV_{S_S}^2 \left[1 + \sigma_v^2\right]}$$

J.S. LEE

LINEAR SCALAR LEE FILTER

$$\hat{S} = E(S_S) - k[E(S_S) - S_S]$$

Homogeneous Areas

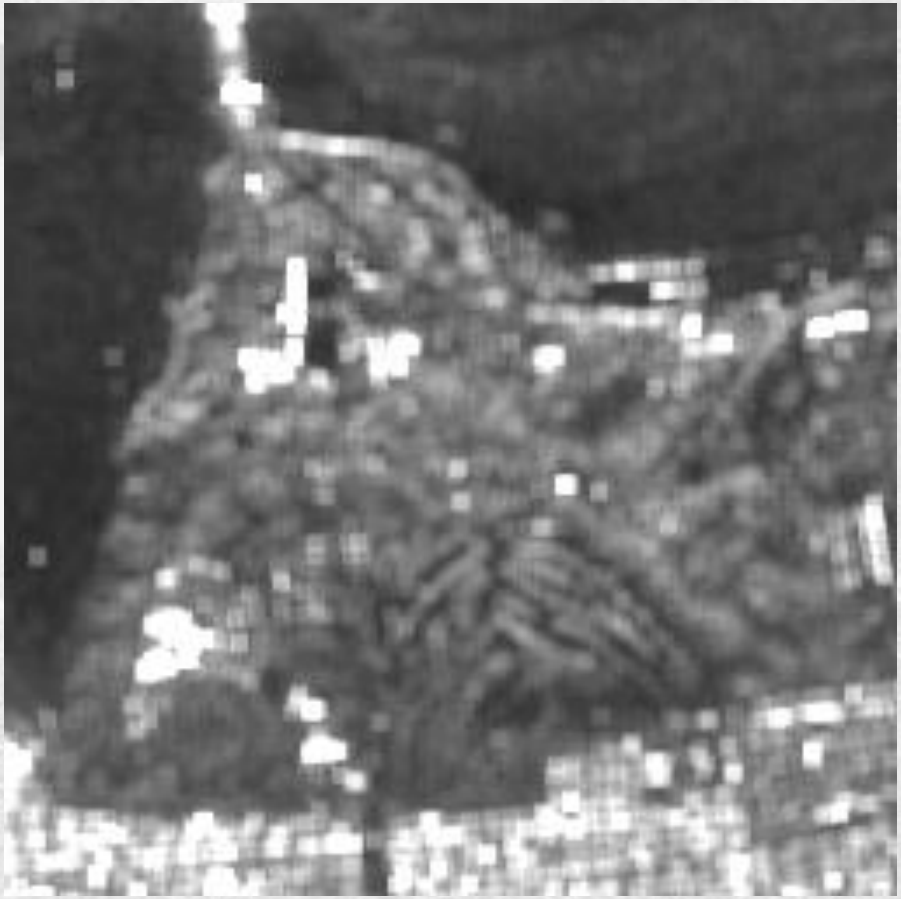
$$var(S) \approx 0 \quad \Rightarrow \quad k = 0 \quad \Rightarrow \hat{S} = E(S_S)$$

Highly Inhomogeneous Areas

$$var(S) \mapsto var(S_s) \quad \Rightarrow \quad k = 1 \quad \Rightarrow \hat{S} = S_S$$

REFINED FILTER

POLSAR SPECKLE FILTERING



SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

BoxCar Filter

POLSAR SPECKLE FILTERING

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

J.S. Lee, M.R. Grunes and G. De Grandi, "Polarimetric SAR Speckle Filtering and Its Impact on Terrain Classification" *IEEE TGRS*, September 1999

POLSAR SPECKLE FILTERING

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

J.S. Lee, D.L. Schuler, T.L. Ainsworth, M.R. Grunes, E Pottier, L. Ferro-Famil, "*Scattering Model Based Speckle Filetring of Polarimetric SAR Data*" IEEE – TGRS, vol 1, January 2006

POLSAR SPECKLE FILTERING

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

J.S. Lee, J.H Wen, T.L. Ainsworth, K.S. Chen, A.J. Chen, "*Improved Sigma Filter for Speckle Filtering of SAR Imagery*" IEEE – TGRS, vol 1, January 2009

POLSAR SPECKLE FILTERING

POLARIMETRIC SPECKLE FILTERING IS NOT AN EXACT SCIENCE
SUBJECTIVE, IMAGE DEPENDENT

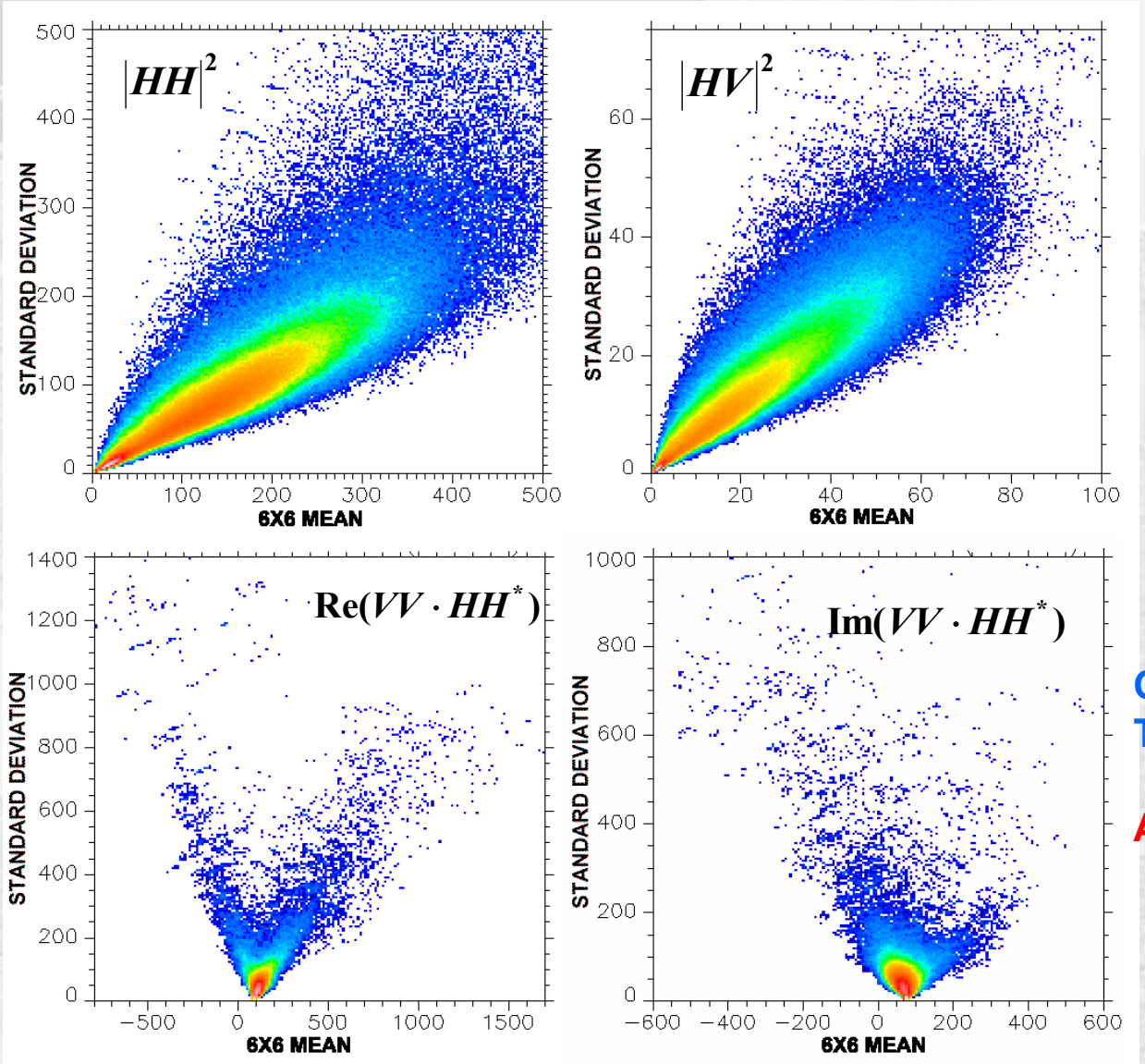
Quantitative Criteria (J.S. Lee - IGARSS 98)

- Speckle Reduction (E.N.L)
- Edge Sharpness Preservation
- Line and Point Target Contrast Preservation
- Retention of Mean Values in Homogeneous Regions
- Retention of Texture Information
- Retention of Polarimetric Information (co, cross-correlations)
- Computational Efficiency
- Implementation Complexity

$$[\hat{T}] = E([T]) - k[E([T]) - [T]]$$

THE POLARIMETRIC SPECKLE LEE FILTER IS TODAY A GOOD COMPROMISE

POLSAR SPECKLE NOISE MODEL



Diagonal Terms

Multiplicative

Off-Diagonal Terms

Additive/Multiplicative

J.S. Lee, M.R. Grunes and G. De Grandi, "Polarimetric SAR Speckle Filtering and Its Impact on Terrain Classification" *IEEE TGRS*, vol. 37, N°5, September 1999

MULTIPLICATIVE-ADDITIVE NOISE MODEL

C. LOPEZ MARTINEZ

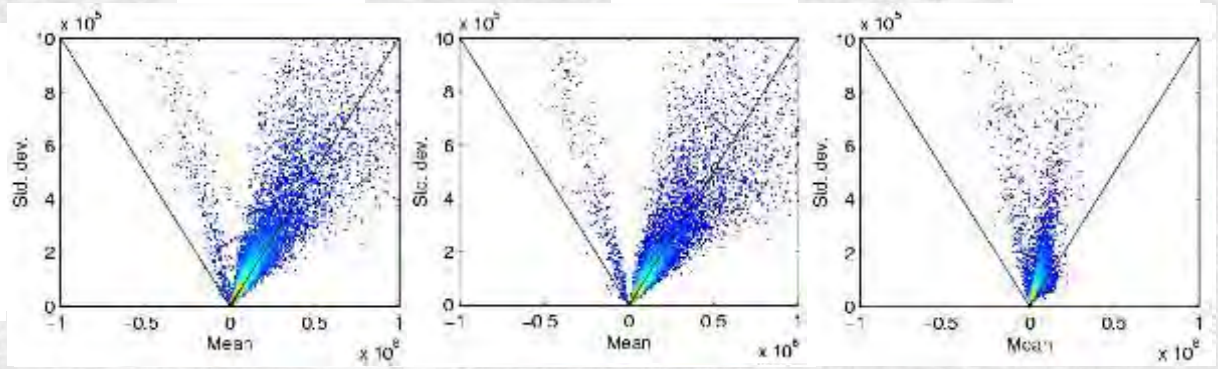
$$S_i S_j^* = \underbrace{\psi \bar{z}_n n_m N_c \exp(j\phi_x)}_{\text{Multiplicative term}} + \underbrace{\psi\left(|\rho| - N_c \bar{z}_n\right)\exp(j\phi_x) + \psi\left(n_{ar} + jn_{ai}\right)}_{\text{Additive term}}$$

Multiplicative speckle noise component: n_m ➡ Important for high coherence areas

Additive speckle noise component: $n_{ar}+jn_{ai}$ ➡ Important for low coherence areas

Combination controlled by complex coherence

$0.8 \leq |\rho| \leq 1$

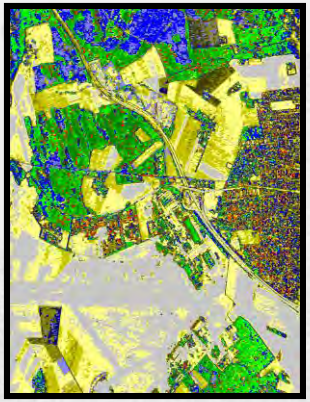


C. López-Martínez and X. Fàbregas, "Polarimetric SAR Speckle Noise Model," IEEE TGRS, vol. 41, no. 10, pp. 2232 – 2242, Oct. 2003

Courtesy of Dr C. Lopez Martinez

POLARIMETRIC REMOTE SENSING

POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS



POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

SPECKLE FILTERING

$$[T] = \underline{k}\,\underline{k}^{*T}$$

SPECKLE FILTER

AVERAGING DATA

SECOND ORDER STATISTICS

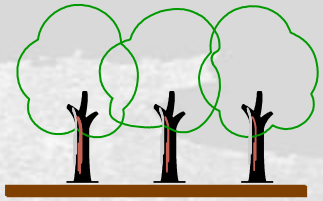
COHERENCY MATRICES

SMOOTHING AVERAGING

$$\langle [T] \rangle = \frac{1}{N} \sum_{i=1}^{N} \underline{k}_i \underline{k}_i^{*T}$$

CONCEPT OF THE DISTRIBUTED TARGET

TARGET DECOMPOSITIONS



PURE TARGET

COHERENCY MATRIX $[T]$

9 REAL DEPENDANT HUYNEN PARAMETERS (A0,B0,B,C,D,E,F,G,H)

POLARIMETRIC DISTRIBUTED TARGET « *DIMENSION* » = 5

9 - 5 = 4 TARGET EQUATIONS

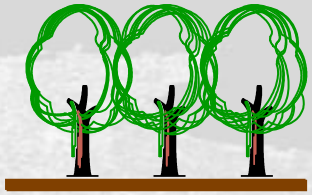
$$2A_0(B_0+B) = C^2+D^2$$

$$2A_0(B_0-B) = G^2+H^2$$

$$2A_0E = CH-DG$$

$$2A_0F = CG+DH$$

TARGET DECOMPOSITIONS



DISTRIBUTED TARGET

POLARIMETRIC DISTRIBUTED TARGET « *DIMENSION* » = 9

COHERENCY MATRIX <[*T*]>

9 REAL INDEPENDANT HUYNEN PARAMETERS

(<A0>,<B0>,<B>,<C>,<D>,<E>,<F>,<G>,<H>)

9 TARGET INEQUATIONS

$$2\langle A_0\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle^2+\langle D\rangle^2$$

$$2\langle A_0\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle G\rangle^2+\langle H\rangle^2$$

$$2\langle A_0\rangle\langle E\rangle\geq\langle C\rangle\langle H\rangle-\langle D\rangle\langle G\rangle$$

$$2\langle A_0\rangle\langle F\rangle\geq\langle C\rangle\langle G\rangle+\langle D\rangle\langle H\rangle$$

$$\langle B_0\rangle^2\geq\langle B\rangle^2+\langle E\rangle^2+\langle F\rangle^2$$

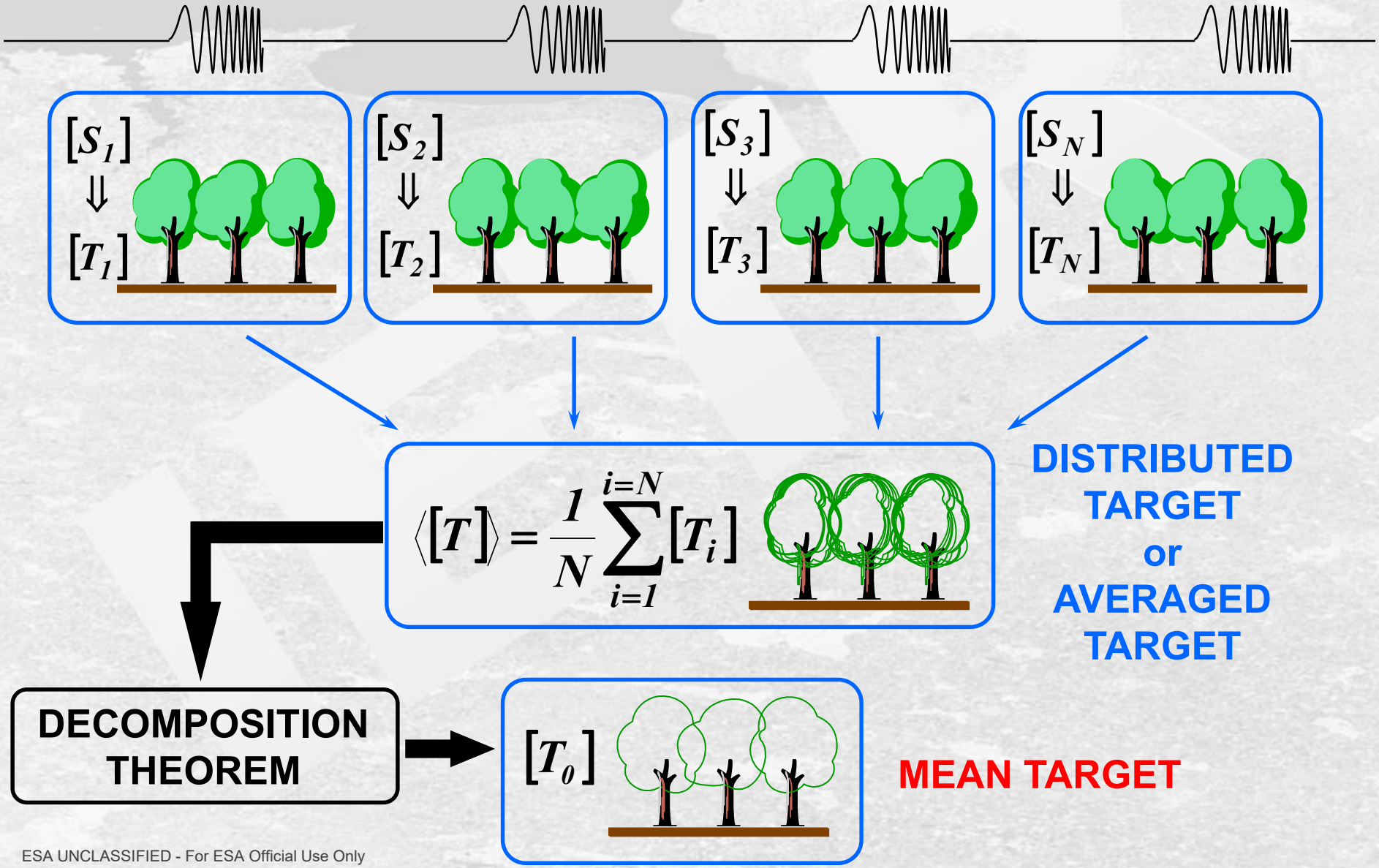
$$\langle H\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle\langle E\rangle+\langle D\rangle\langle F\rangle$$

$$\langle G\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle\langle F\rangle-\langle D\rangle\langle E\rangle$$

$$\langle C\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle H\rangle\langle E\rangle+\langle F\rangle\langle G\rangle$$

$$\langle D\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle F\rangle\langle H\rangle-\langle G\rangle\langle E\rangle$$

TARGET DECOMPOSITIONS



TARGET DECOMPOSITIONS

[S]

COHERENT DECOMPOSITION

- E. KROGAGER (1990)
- W.L. CAMERON (1990)

[K]

TARGET DICHOTOMY

- J.R. HUYNEN (1970)
- R.M. BARNES (1988)
- D. LI – Y, ZHANG (2016)

[T]

AZIMUTHAL SYMMETRY

MODEL BASED DECOMPOSITION

- A.J. FREEMAN – S.L. DURDEN (1992)
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- G. SINGH (2013), H. AGHABABAEE (2016)
- A. BHATTACHARYA – A. FRERY (2015)
- F. XU – Y.Q.JIN (2015) …

EIGENVECTORS BASED DECOMPOSITION

- S.R. CLOUDE (1985)
- W.A. HOLM (1988)

EIGENVECTORS / EIGENVALUES ANALYSIS & MODEL BASED DECOMPOSITION

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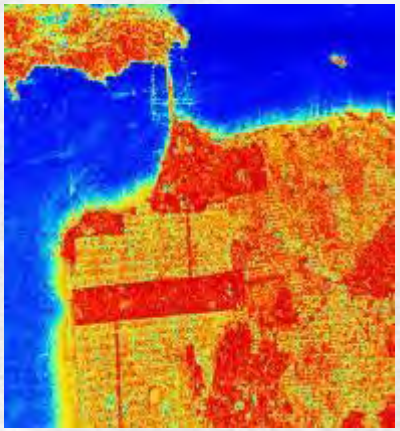
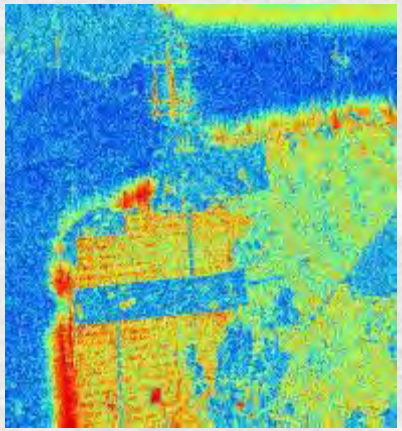
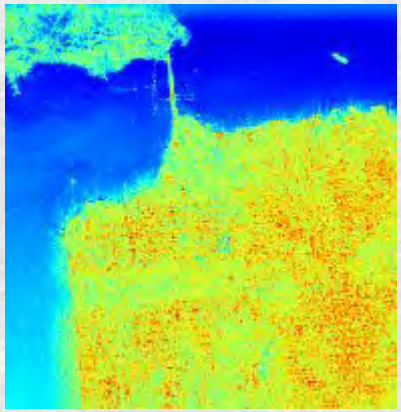
THE H / A / α POLARIMETRIC TARGET DECOMPOSITION THEOREM







S.R. CLOUDE - E. POTTIER (1995 - 1996)

H / A / α DECOMPOSITION

TARGET VECTOR

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

LOCAL ESTIMATE OF THE COHERENCY MATRIX

$$\langle [T] \rangle = \frac{1}{N}\sum_{i=1}^{N} \underline{k}_i \cdot \underline{k}_i^{*T} = \frac{1}{N}\sum_{i=1}^{N} [T_i]$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES
$\lambda_1 > \lambda_2 > \lambda_3$

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3} \lambda_k}$$

H / A / α DECOMPOSITION

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES
$\lambda_1 > \lambda_2 > \lambda_3$

PARAMETERISATION OF THE SU(3) UNITARY MATRIX

$$[U_3] = \begin{bmatrix} \cos\alpha_1 e^{j\phi_1} & \cos\alpha_2 e^{j\phi_2} & \cos\alpha_3 e^{j\phi_3} \\ \sin\alpha_1 \cos\beta_1 e^{j\phi_1} e^{j\delta_1} & \sin\alpha_2 \cos\beta_2 e^{j\phi_2} e^{j\delta_2} & \sin\alpha_3 \cos\beta_3 e^{j\phi_3} e^{j\delta_3} \\ \sin\alpha_1 \sin\beta_1 e^{j\phi_1} e^{j\gamma_1} & \sin\alpha_2 \sin\beta_2 e^{j\phi_2} e^{j\gamma_2} & \sin\alpha_3 \sin\beta_3 e^{j\phi_3} e^{j\gamma_3} \end{bmatrix}$$

TARGET 1 TARGET 2 TARGET 3

H / A / $\underline{\alpha}$ DECOMPOSITION

PROBABILITIES

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3} \lambda_k}$$

AVERAGED PARAMETERS

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3 \qquad \underline{\beta} = P_1\beta_1 + P_2\beta_2 + P_3\beta_3$$

$$\underline{\gamma} = P_1\gamma_1 + P_2\gamma_2 + P_3\gamma_3 \qquad \underline{\delta} = P_1\delta_1 + P_2\delta_2 + P_3\delta_3$$

UNITARY TARGET VECTOR ($\underline{u}_0$) OF THE MEAN DOMINANT MECHANISM

$$\underline{u}_0 = \left[\cos(\underline{\alpha}) \quad \sin(\underline{\alpha})\cos(\underline{\beta})\, e^{j\underline{\delta}} \quad \sin(\underline{\alpha})\sin(\underline{\beta})\, e^{j\underline{\gamma}} \right]^T$$

MEAN SCATTERING MECHANISM

UNITARY VECTOR $\underline{u}_0$

$$\underline{u}_0 = \begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha})\cos(\underline{\beta})e^{j\underline{\delta}} \\ \sin(\underline{\alpha})\sin(\underline{\beta})e^{j\underline{\gamma}} \end{bmatrix}$$

TARGET MAGNITUDE

$$\underline{\lambda} = P_1\lambda_1 + P_2\lambda_2 + P_3\lambda_3 = \frac{\sum_{i=1}^{3}\lambda_i^2}{\sum_{k=1}^{3}\lambda_k}$$

TARGET VECTOR $\underline{k}_0$

$$\underline{k}_0 = \sqrt{\underline{\lambda}} \begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha})\cos(\underline{\beta})e^{j\underline{\delta}} \\ \sin(\underline{\alpha})\sin(\underline{\beta})e^{j\underline{\gamma}} \end{bmatrix}$$

H / A / α DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$ $\sqrt{\lambda}\cos(\alpha)$ $\sqrt{\lambda}\sin(\alpha)\cos(\beta)$

$\sqrt{\lambda}\sin(\alpha)\sin(\beta)$

H / A / α DECOMPOSITION

ROLL INVARIANCE PROPERTY

SAME PHYSICAL PHENOMENOUS **WHATEVER THE ANTENNA** ***ORIENTATION ANGLE*** **AROUND THE** ***RADAR LINE OF SIGHT***

ORIENTED (θ) COHERENCY MATRIX

$$\langle [T(\theta)] \rangle = [U_R(\theta)] \langle [T] \rangle [U_R(\theta)]^{-1}$$

SU(3) UNITARY ROTATION MATRIX (θ)

$$[U_R(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle [T(\theta)] \rangle = [U_3(\theta)][\Sigma][U_3(\theta)]^{-1}$$

EIGENVALUES λ_1 λ_2 λ_3 : ROLL INVARIANT

PROBABILITIES P_1 P_2 P_3 : ROLL INVARIANT

H / A / α DECOMPOSITION

EIGENVECTORS UNITARY MATRIX

$$[U_3(\theta)] = [U_R(\theta)][U_3]$$

PARAMETERIZATION OF THE UNITARY MATRIX

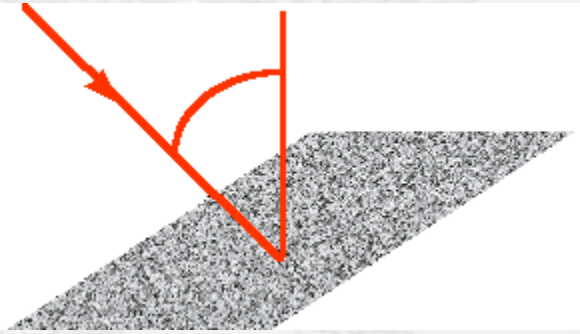
$$[U_3] = \begin{bmatrix} \cos\alpha_1 e^{j\phi'_1} & \cos\alpha_2 e^{j\phi'_2} & \cos\alpha_3 e^{j\phi'_3} \\ \sin\alpha_1 \cos\beta'_1 e^{j\phi'_1} e^{j\delta'_1} & \sin\alpha_2 \cos\beta'_2 e^{j\phi'_2} e^{j\delta'_2} & \sin\alpha_3 \cos\beta'_3 e^{j\phi'_3} e^{j\delta'_3} \\ \sin\alpha_1 \sin\beta'_1 e^{j\phi'_1} e^{j\gamma'_1} & \sin\alpha_2 \sin\beta'_2 e^{j\phi'_2} e^{j\gamma'_2} & \sin\alpha_3 \sin\beta'_3 e^{j\phi'_3} e^{j\gamma'_3} \end{bmatrix}$$

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3$$: ROLL INVARIANT

PHYSICAL INTERPRETATION

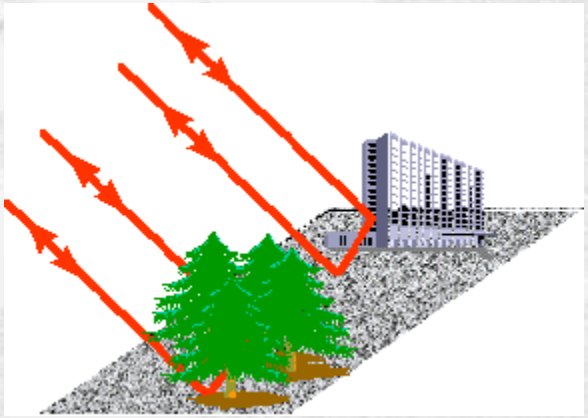
α PHYSICAL INTERPRETATION

SINGLE BOUNCE SCATTERING (ROUGH SURFACE)



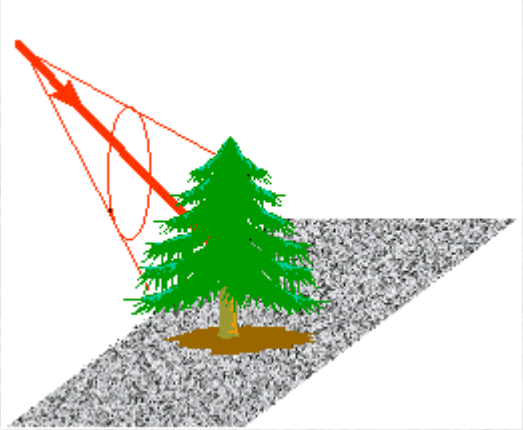
$$a \mapsto b \Rightarrow \nu \mapsto 0$$
$$\Downarrow$$
$$\alpha \mapsto 0$$

DOUBLE BOUNCE SCATTERING



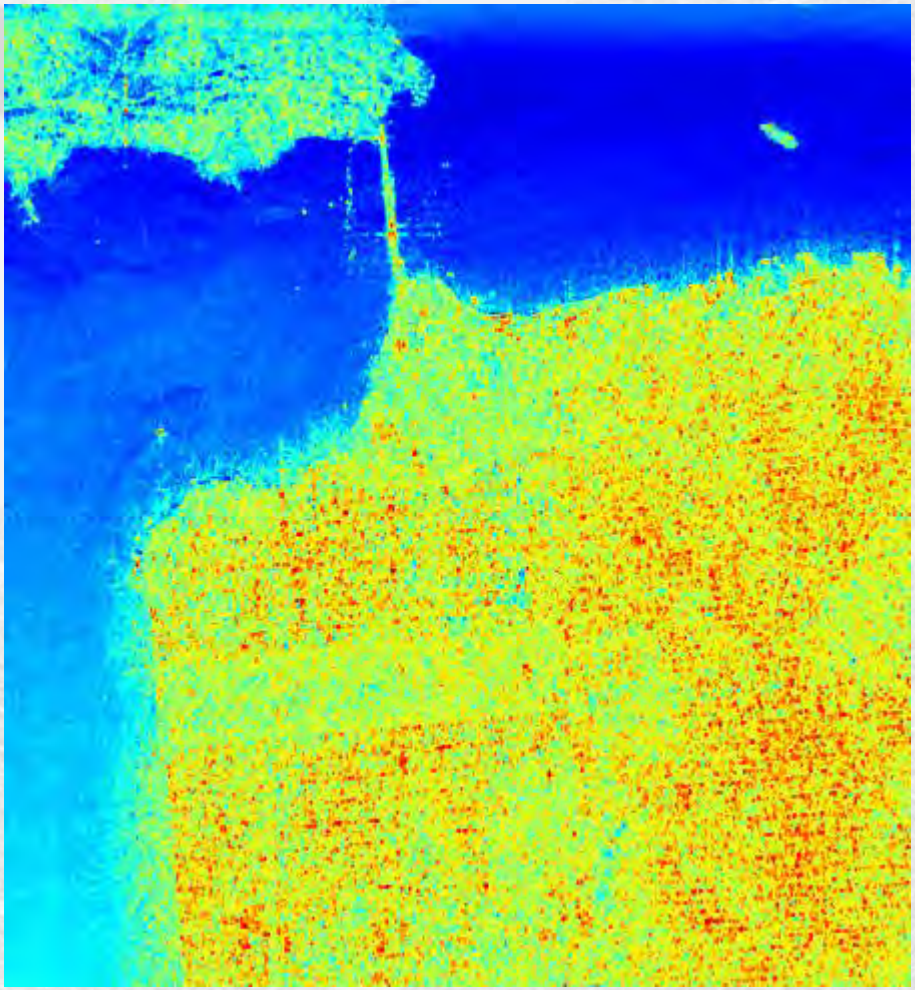
$$a \mapsto -b \Rightarrow \varepsilon \mapsto 0$$
$$\Downarrow$$
$$\alpha \mapsto \frac{\pi}{2}$$

VOLUME SCATTERING

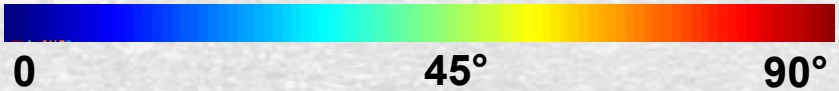


$$a >> b \Rightarrow \varepsilon \approx \nu$$
$$\Downarrow$$
$$\alpha \mapsto \frac{\pi}{4}$$

H / A / α DECOMPOSITION



$2A_0$ $B_0 + B$ $B_0 - B$



α PARAMETER

H / A / α DECOMPOSITION

EIGENVALUES λ_1 λ_2 λ_3 : ROLL INVARIANT

PROBABILITIES P_1 P_2 P_3 : ROLL INVARIANT

ENTROPY

**(DEGREE OF RANDOMNESS
STATISTICAL DISORDER)**

$$H = -\sum_{i=1}^{3} P_i \log_3(P_i)$$

PURE TARGET

$\lambda_1 = SPAN \quad \lambda_2 = 0 \quad \lambda_3 = 0$

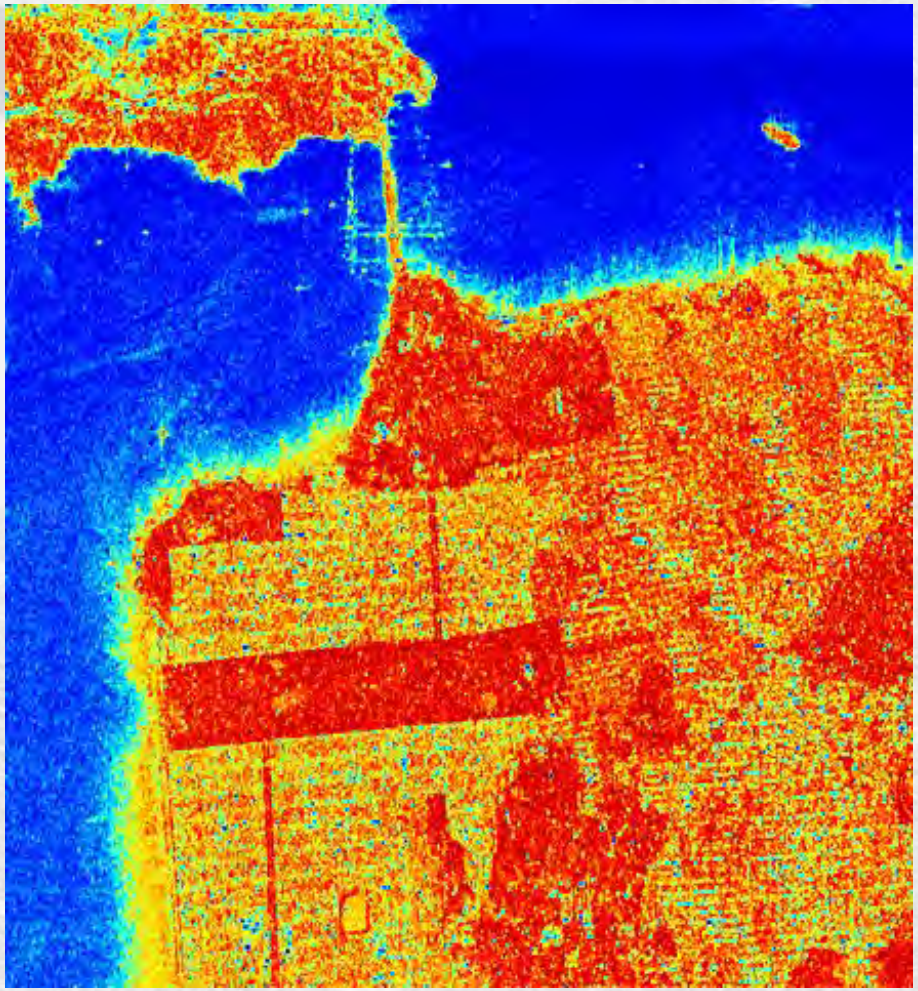
$H = 0$

DISTRIBUTED TARGET

$\lambda_1 = \lambda_2 = \lambda_3 = SPAN / 3$

$H = 1$

H / A / α DECOMPOSITION



$2A_0$ $B_0 + B$ $B_0 - B$

ENTROPY (H)

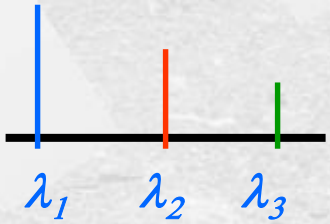
H / A / α DECOMPOSITION

DIFFICULT MECHANISM DISCRIMINATION WHEN : H > 0.7

ANISOTROPY

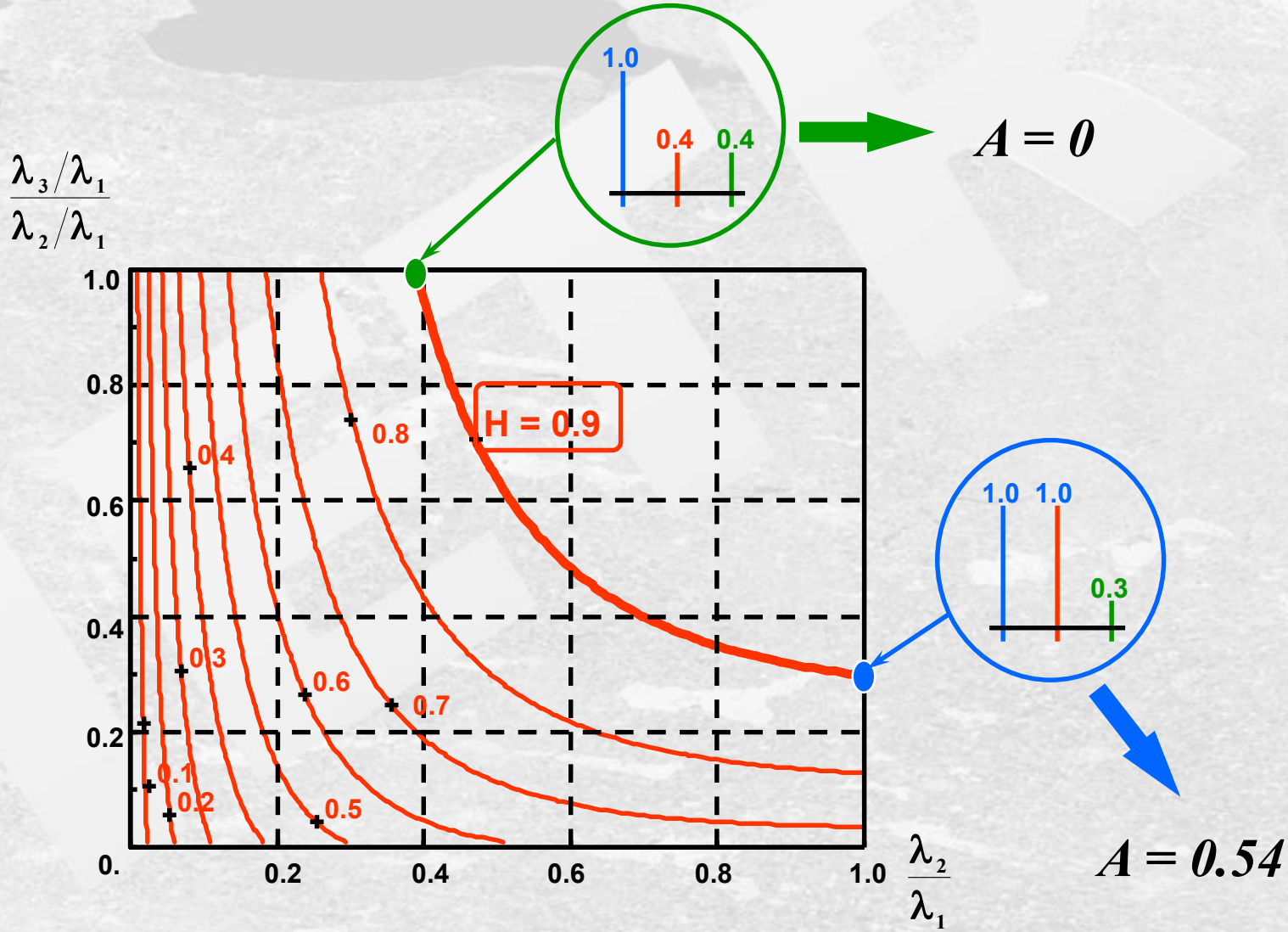
(EIGENVALUES SPECTRUM)

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$

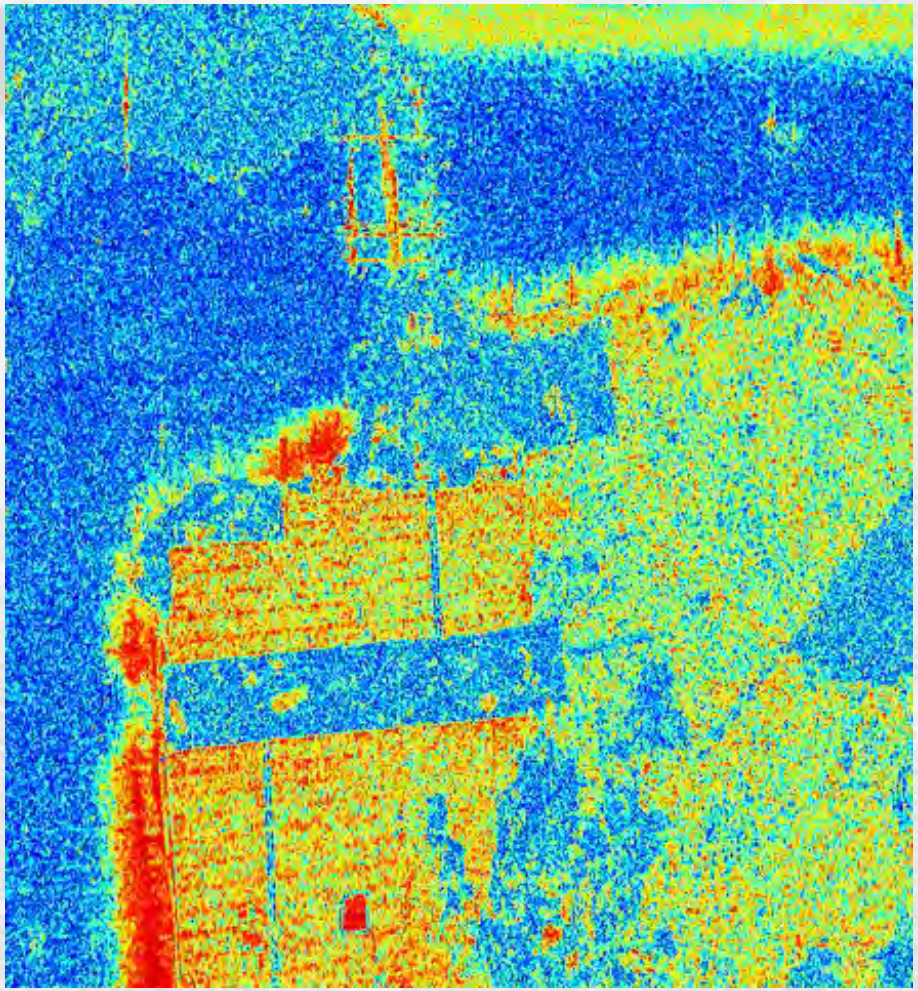


- **COMPLEMENTARY TO ENTROPY**
- **DISCRIMINATION WHEN H > 0.7**
- **ROLL INVARIANT**

H / A / α DECOMPOSITION



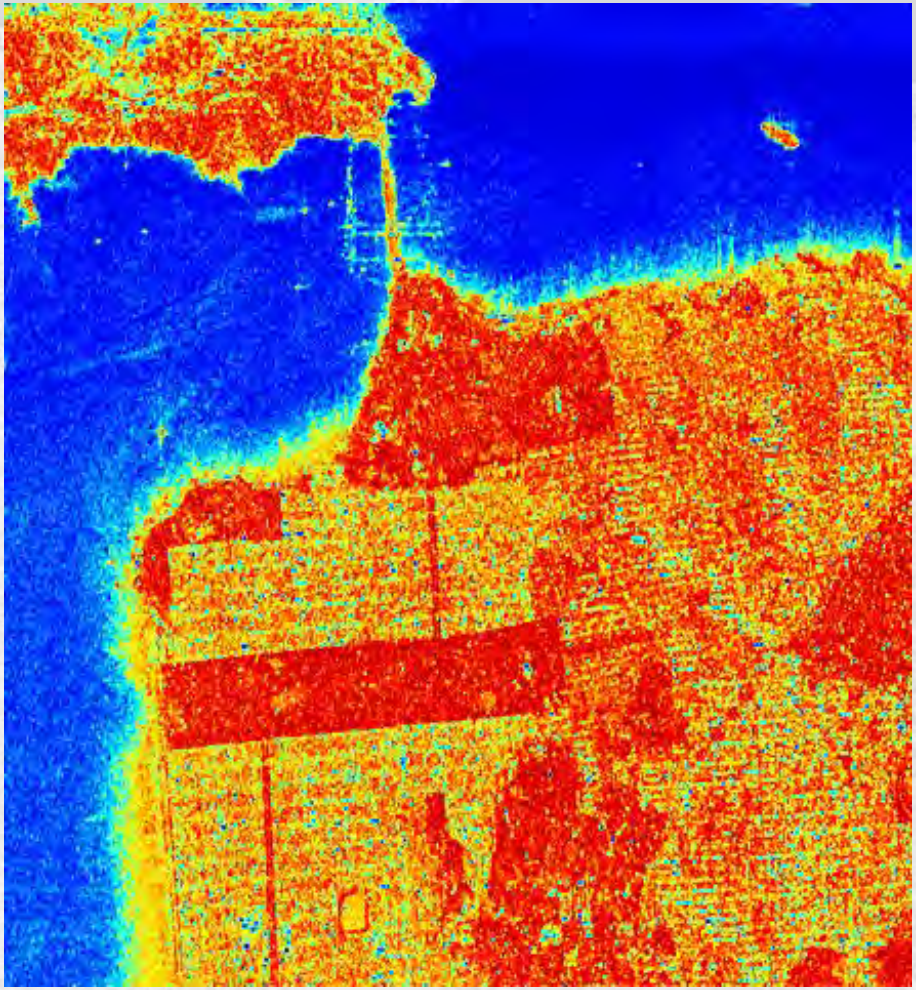
H / A / α DECOMPOSITION



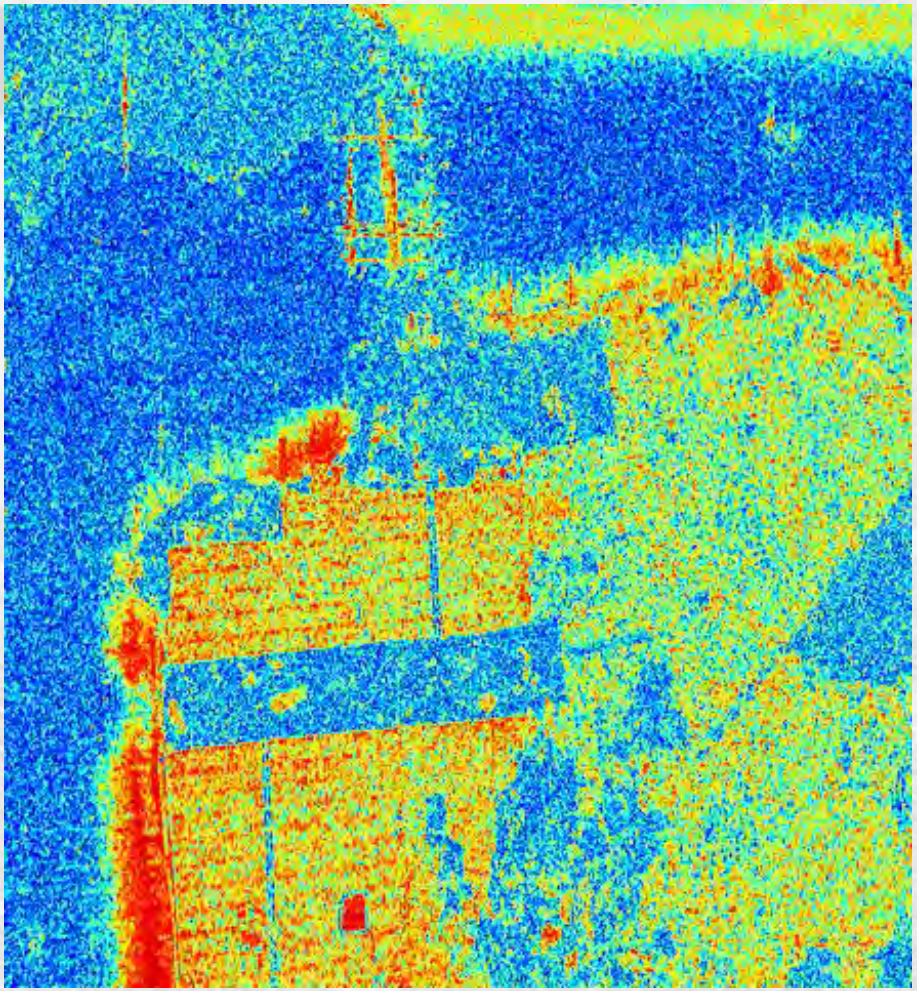
$2A_0$ $B_0 + B$ $B_0 - B$

ANISOTROPY (A)

H / A / α DECOMPOSITION



ENTROPY (H)



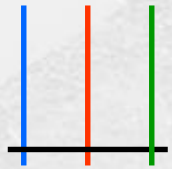
ANISOTROPY (A)

H / A / α DECOMPOSITION

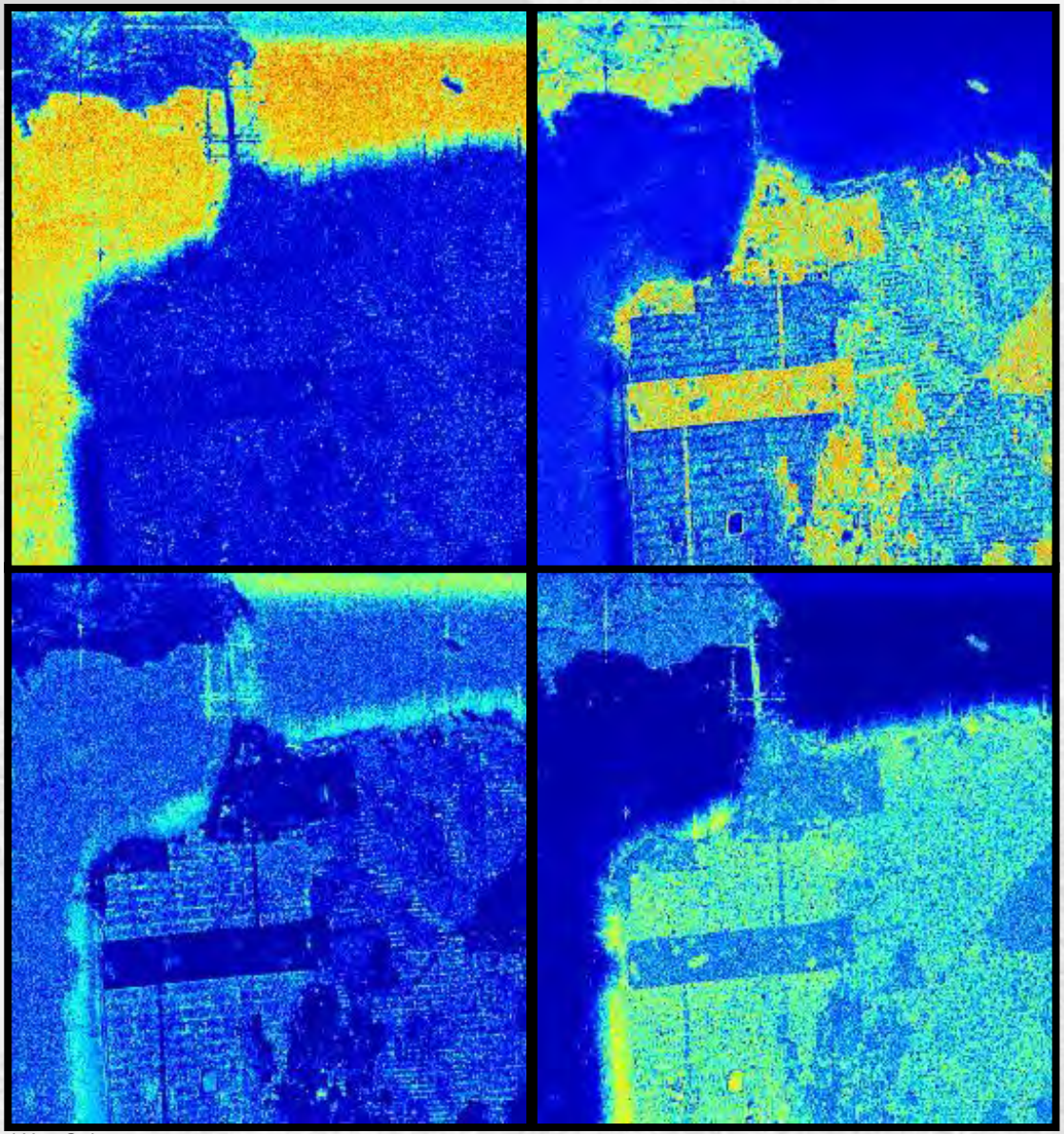
(1-H)(1-A)

1 MECHANISM

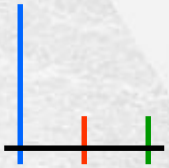
H(1-A)



3 MECHANISMS

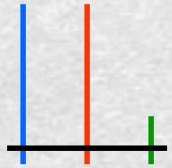


A(1-H)

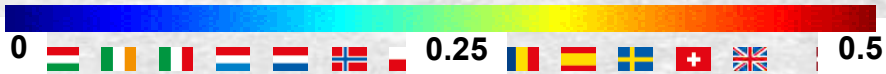


2 MECHANISMS

HA



2 MECHANISMS



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COHERENT

TARGET DECOMPOSITION

(1990)

ERNST KROGAGER

(1990)

DECOMPOSITION

$[S]$ → THREE COHERENT COMPONENTS

↓

$$[S]=\begin{bmatrix} a+b & c \\ c & a-b \end{bmatrix}=e^{j\phi}\left\{k_S[S_S]+e^{j\phi_R}\left(k_D[S_D]+k_H[S_H]\right)\right\}$$

SINGLE BOUNCE SCATTERING | DOUBLE BOUNCE SCATTERING | HELICAL SCATTERING

COHERENT DECOMPOSITION

$$[S] = e^{j\phi}\left\{k_S\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + e^{j\phi_R}\left(k_D\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} + \frac{k_H}{2}\begin{bmatrix}1 & \pm j\\ \pm j & -1\end{bmatrix}\right)\right\}$$

ROTATION AROUND THE RADAR LINE OF SIGHT

$$[U] = \begin{bmatrix}\cos(\theta) & \sin(\theta)\\ -\sin(\theta) & \cos(\theta)\end{bmatrix}$$

$$\begin{aligned}[S(\theta)] &= [U]^T[S][U]\\ &= e^{j\phi}\left\{k_S\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + e^{j\phi_R}\left(k_D\begin{bmatrix}\cos(2\theta) & \sin(2\theta)\\ \sin(2\theta) & -\cos(2\theta)\end{bmatrix}\cdots\right.\right.\\ &\qquad\left.\left.\cdots + \frac{k_H e^{\mp j2\theta}}{2}\begin{bmatrix}1 & \pm j\\ \pm j & -1\end{bmatrix}\right)\right\}\end{aligned}$$

COHERENT DECOMPOSITION

$$[S] = e^{j\phi}\begin{bmatrix} k_S + e^{j\phi_R}\left\{k_D\cos(2\theta) + \frac{\hat{k}_H}{2}\right\} & e^{j\phi_R}\left\{k_D\sin(2\theta) \pm j\frac{\hat{k}_H}{2}\right\} \\ e^{j\phi_R}\left\{k_D\sin(2\theta) \pm j\frac{\hat{k}_H}{2}\right\} & k_S - e^{j\phi_R}\left\{k_D\cos(2\theta) + \frac{\hat{k}_H}{2}\right\} \end{bmatrix}$$

AVEC : $\hat{k}_H = k_H e^{\mp j2\theta}$

$$\underline{k} = \sqrt{2}e^{j\phi}\begin{bmatrix} k_S & e^{j\phi_R}\left\{k_D\cos(2\theta) + \frac{\hat{k}_H}{2}\right\} & e^{j\phi_R}\left\{k_D\sin(2\theta) \pm j\frac{\hat{k}_H}{2}\right\} \end{bmatrix}^T$$

$$\underline{k} = \sqrt{2}k_S e^{j\phi}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \hat{k}_H e^{j(\phi+\phi_R)}\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ \pm j \end{bmatrix} + \sqrt{2}k_D e^{j(\phi+\phi_R)}\begin{bmatrix} 0 \\ \cos(2\theta) \\ \sin(2\theta) \end{bmatrix}$$

COHERENT DECOMPOSITION

SINGLE SCATTERING CONTRIBUTION

$$k_S = \sqrt{A_0} \qquad [S_S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

DOUBLE SCATTERING CONTRIBUTION

$$k_D = \sqrt{B_0 - |F|} \qquad [S_D] = \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$$

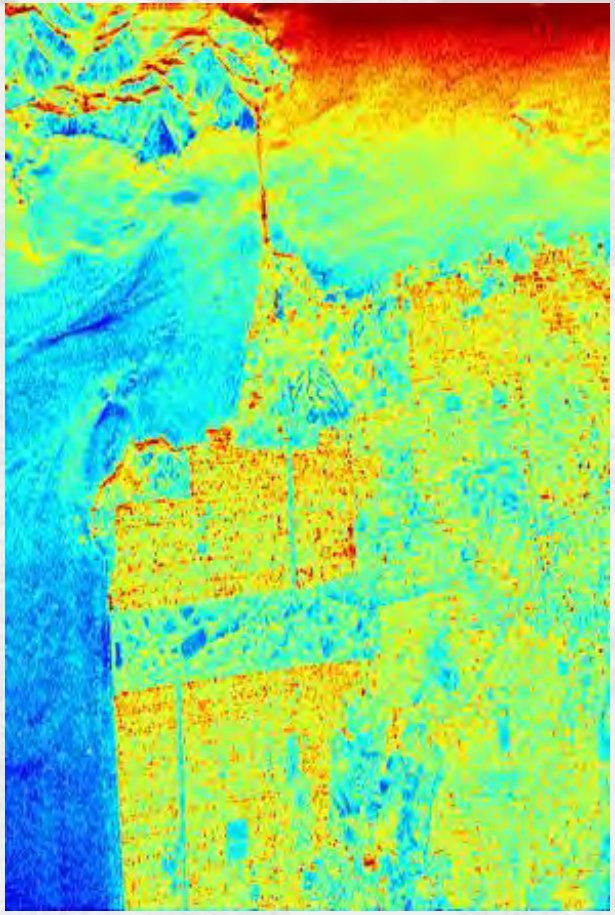
$$\tan(4\theta) = \frac{E}{B}$$

DIPLANE ORIENTATION ANGLE
INSIDE THE TARGET

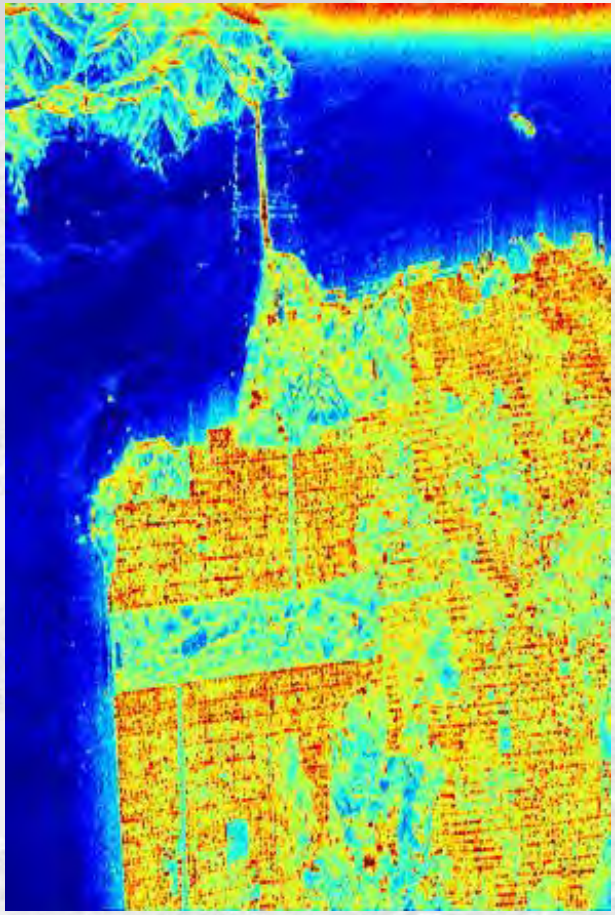
HELICAL SCATTERING CONTRIBUTION

$$k_H = \sqrt{B_0 + |F|} - \sqrt{B_0 - |F|} \qquad [S_H] = \frac{1}{2}\begin{bmatrix} 1 & \pm j \\ \pm j & -1 \end{bmatrix}$$

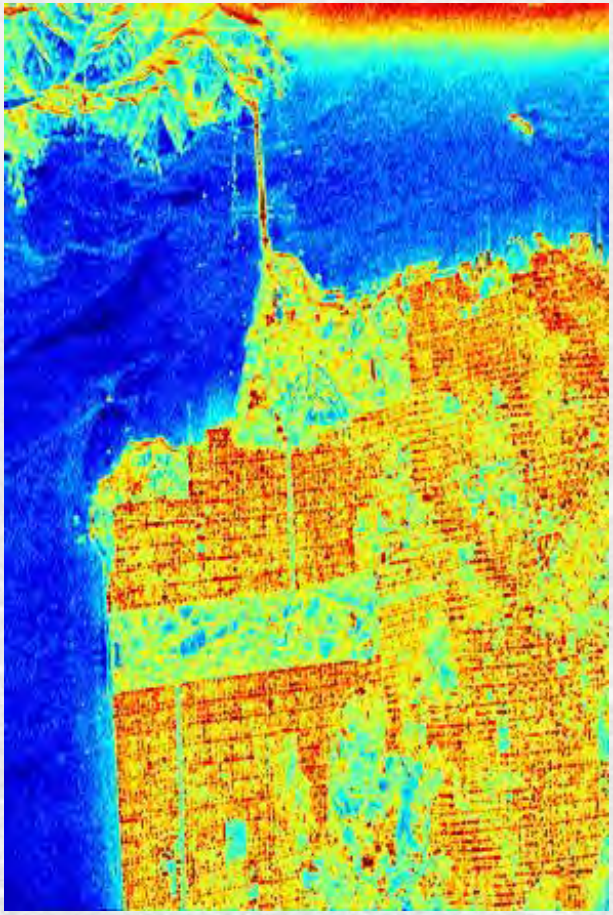
COHERENT DECOMPOSITION



$(k_S)_{dB}$



$(k_D)_{dB}$



$(k_H)_{dB}$

COHERENT DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$ k_S k_D k_H

COHERENT DECOMPOSITION

$$\underline{k} = \sqrt{2}k_S e^{j\phi}\begin{bmatrix}1\\0\\0\end{bmatrix} + \hat{k}_H e^{j(\phi+\phi_R)}\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\\varepsilon j\end{bmatrix} + \sqrt{2}k_D e^{j(\phi+\phi_R)}\begin{bmatrix}0\\\cos(2\theta)\\\sin(2\theta)\end{bmatrix}$$

EIGENVECTORS OF $[U_{3R}(\phi)]$ (ROLL INVARIANCE)

→ NO ORTHOGONALITY OF THE TARGETS COMPONENTS

→ COHERENT DECOMPOSITION and SPECKLE FILTERING ?

TARGET DECOMPOSITIONS

[S]

COHERENT DECOMPOSITION

- E. KROGAGER (1990)
- W.L. CAMERON (1990)

[K]

TARGET DICHOTOMY

- J.R. HUYNEN (1970)
- R.M. BARNES (1988)
- D. LI – Y, ZHANG (2016)

[T]

EIGENVECTORS BASED DECOMPOSITION

- S.R. CLOUDE (1985)
- W.A. HOLM (1988)

EIGENVECTORS / EIGENVALUES ANALYSIS

- S.R. CLOUDE - E. POTTIER (1996-1997)
- R. TOUZI (TSVM 2007)

[C]

AZIMUTHAL SYMMETRY

MODEL BASED DECOMPOSITION

- A.J. FREEMAN – S.L. DURDEN (1992)
- Y. YAMAGUSHI (2005 - 2012), AN (2010)
- G. SINGH (2013), H. AGHABABAEE (2016)
- A. BHATTACHARYA – A. FRERY (2015)
- F. XU – Y.Q.JIN (2015) …

EIGENVECTORS / EIGENVALUES ANALYSIS & MODEL BASED DECOMPOSITION

- J.J. VAN ZYL (1992-2008), M. ARII (2010)

TARGET DICHOTOMY

DECOMPOSITIONS

- **J. R. HUYNEN**
  (1970)

- **R. M. BARNES**
  (1988)

- **D. LI – Y. ZHANG**
  (2016)

J.R. HUYNEN DECOMPOSITION

$$\langle [T] \rangle = \begin{bmatrix} \langle 2A_0 \rangle & \langle C \rangle - j\langle D \rangle & \langle H \rangle + j\langle G \rangle \\ \langle C \rangle + j\langle D \rangle & \langle B_0 \rangle + \langle B \rangle & \langle E \rangle + j\langle F \rangle \\ \langle H \rangle - j\langle G \rangle & \langle E \rangle - j\langle F \rangle & \langle B_0 \rangle - \langle B \rangle \end{bmatrix} = [T_0] + [T_N]$$

$$\Downarrow$$

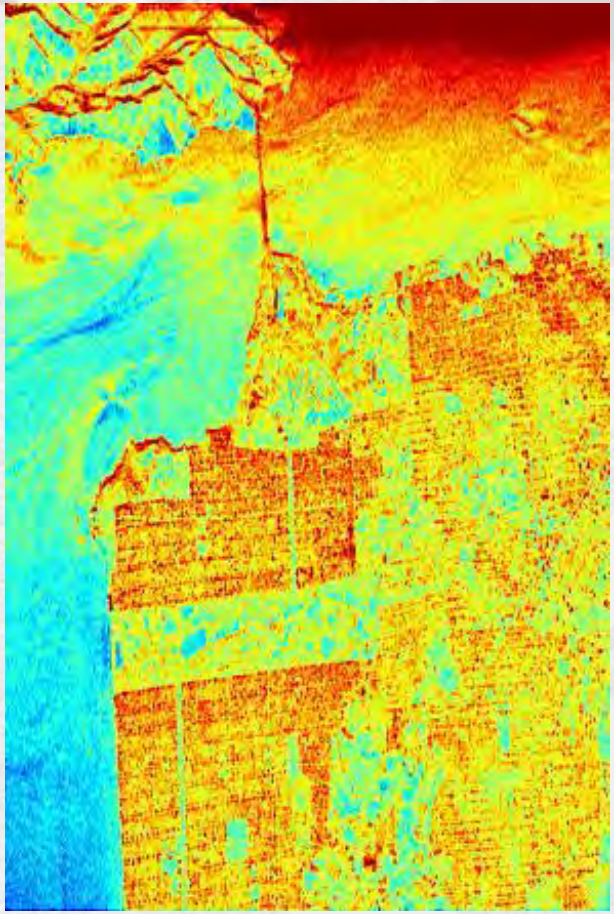
→ **PURE TARGET**

$$[T_0] = \begin{bmatrix} \langle 2A_0 \rangle & \langle C \rangle - j\langle D \rangle & \langle H \rangle + j\langle G \rangle \\ \langle C \rangle - j\langle D \rangle & B_{0T} + B_T & E_T + jF_T \\ \langle H \rangle - j\langle G \rangle & E_T - jF_T & B_{0T} - B_T \end{bmatrix}$$

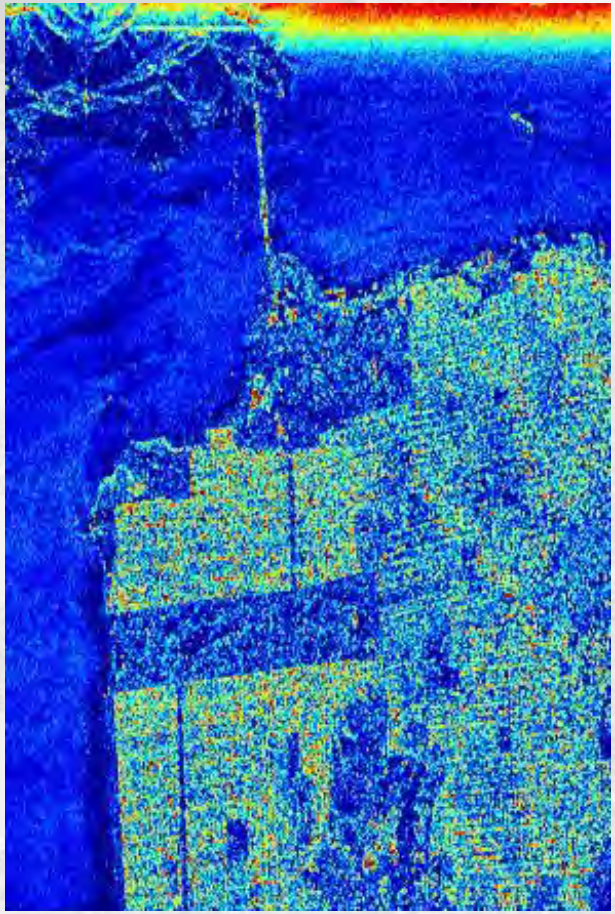
→ **N-TARGET**

$$[T_N] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_{0N} + B_N & E_N + jF_N \\ 0 & E_N - jF_N & B_{0N} - B_N \end{bmatrix}$$

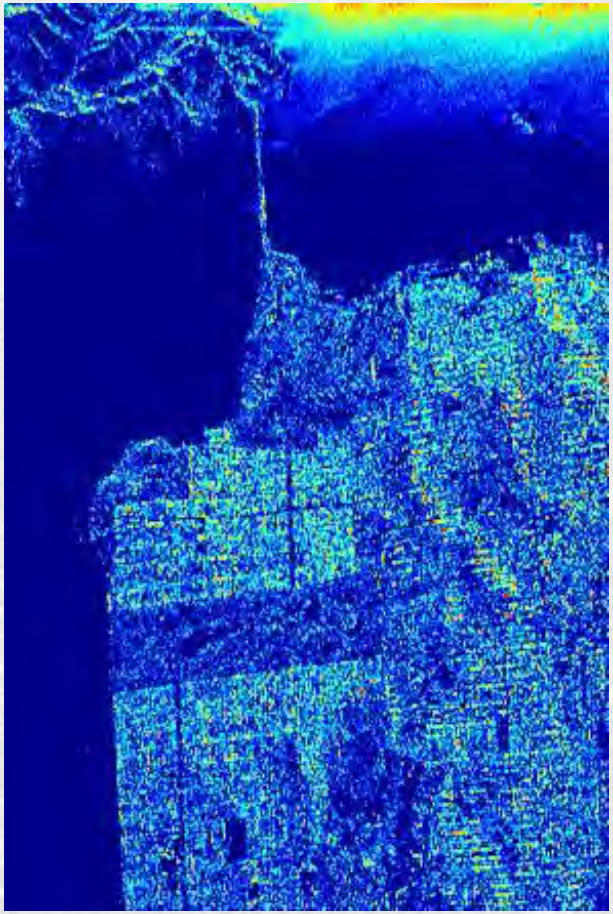
J.R. HUYNEN DECOMPOSITION



$$\langle 2A_0 \rangle$$



$$\left(\langle C \rangle^2 + \langle D \rangle^2\right) / 2A_0$$



$$\left(\langle H \rangle^2 + \langle G \rangle^2\right) / 2A_0$$

J.R. HUYNEN DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

$\langle 2A_0 \rangle$ $\left(\langle C \rangle^2 + \langle D \rangle^2\right) / 2A_0$

$\left(\langle H \rangle^2 + \langle G \rangle^2\right) / 2A_0$

BARNES DECOMPOSITION

3 SINGLE TARGET VECTORS

$$\underline{k}_{01} = \frac{\langle [T] \rangle \underline{q}_1}{\sqrt{\underline{q}_1^{T*} \langle [T] \rangle \underline{q}_1}} = \frac{1}{\sqrt{\langle 2A_0 \rangle}} \begin{bmatrix} \langle 2A_0 \rangle \\ \langle C \rangle + j\langle D \rangle \\ \langle H \rangle - j\langle G \rangle \end{bmatrix}$$

→ **HUYNEN DECOMPOSITION**

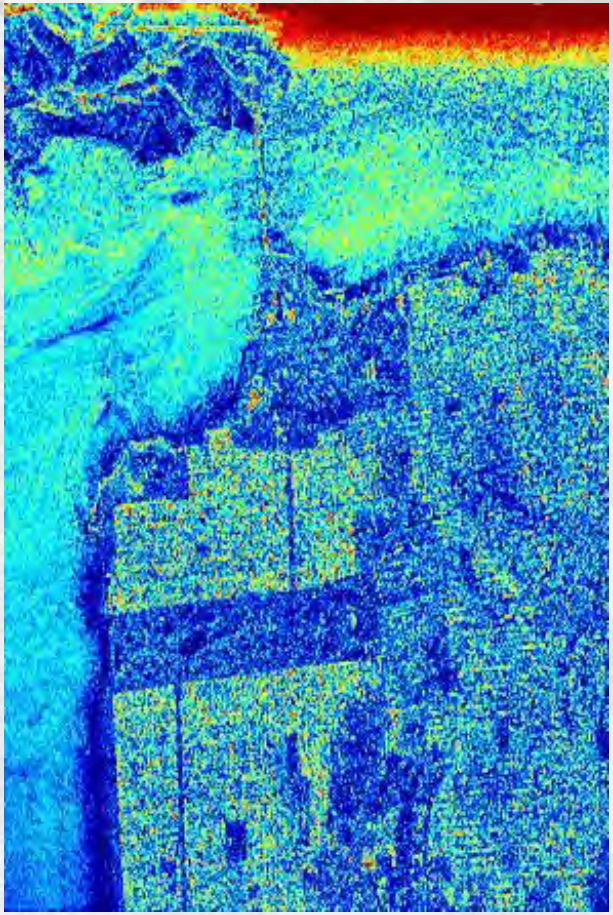
(SYMMETRIC WORLD)

$$\underline{k}_{02} = \frac{\langle [T] \rangle \underline{q}_2}{\sqrt{\underline{q}_2^{T*} \langle [T] \rangle \underline{q}_2}} = \frac{1}{\sqrt{2\left(\langle B_0 \rangle - \langle F \rangle\right)}} \begin{bmatrix} \langle C \rangle - \langle G \rangle + j\langle H \rangle - j\langle D \rangle \\ \langle B_0 \rangle + \langle B \rangle - \langle F \rangle + j\langle E \rangle \\ \langle E \rangle + j\langle B_0 \rangle - j\langle B \rangle - j\langle F \rangle \end{bmatrix}$$

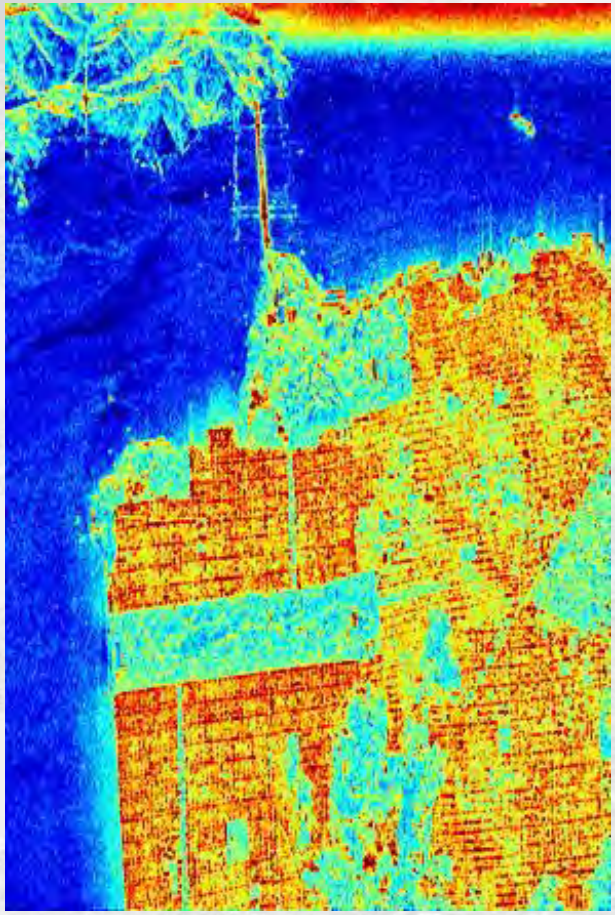
$$\underline{k}_{03} = \frac{\langle [T] \rangle \underline{q}_3}{\sqrt{\underline{q}_3^{T*} \langle [T] \rangle \underline{q}_3}} = \frac{1}{\sqrt{2\left(\langle B_0 \rangle + \langle F \rangle\right)}} \begin{bmatrix} \langle H \rangle + \langle D \rangle + j\langle C \rangle + j\langle G \rangle \\ \langle E \rangle + j\langle B_0 \rangle + j\langle B \rangle + j\langle F \rangle \\ \langle B_0 \rangle - \langle B \rangle + \langle F \rangle + j\langle E \rangle \end{bmatrix}$$

(HELICITY - NON SYMMETRIC COMPONENTS = EXOTIC WORLD)

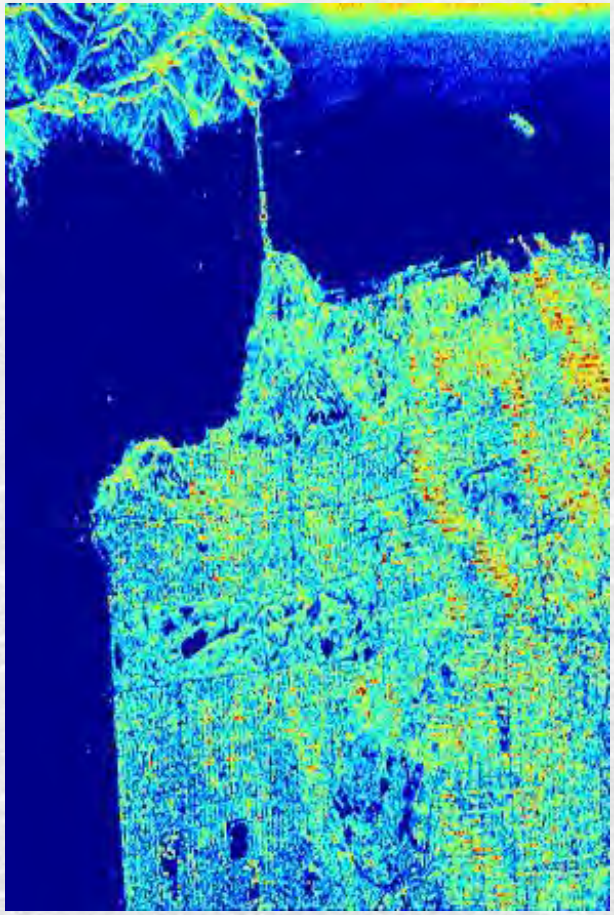
BARNES DECOMPOSITION



$$\frac{\sqrt{\left(\langle C\rangle-\langle G\rangle\right)^2+\left(\langle H\rangle-\langle D\rangle\right)^2}}{\sqrt{2\left(\langle B_0\rangle-\langle F\rangle\right)}}$$



$$\frac{\sqrt{\left(\langle B_0\rangle+\langle B\rangle-\langle F\rangle\right)^2+\langle E\rangle^2}}{\sqrt{2\left(\langle B_0\rangle-\langle F\rangle\right)}}$$



$$\frac{\sqrt{\left(\langle B_0\rangle-\langle B\rangle-\langle F\rangle\right)^2+\langle E\rangle^2}}{\sqrt{2\left(\langle B_0\rangle-\langle F\rangle\right)}}$$

BARNES DECOMPOSITION

$2A_0$ $\qquad$ $B_0 + B$ $\qquad$ $B_0 - B$

$$\frac{\sqrt{(\langle C\rangle-\langle G\rangle)^2+(\langle H\rangle-\langle D\rangle)^2}}{\sqrt{2(\langle B_0\rangle-\langle F\rangle)}}$$

$$\frac{\sqrt{(\langle B_0\rangle+\langle B\rangle-\langle F\rangle)^2+\langle E\rangle^2}}{\sqrt{2(\langle B_0\rangle-\langle F\rangle)}}$$

$$\frac{\sqrt{(\langle B_0\rangle-\langle B\rangle-\langle F\rangle)^2+\langle E\rangle^2}}{\sqrt{2(\langle B_0\rangle-\langle F\rangle)}}$$

J.R. HUYNEN DECOMPOSITION

HUYNEN DECOMPOSITION

TARGET DICHOTOMY : PURE TARGET + N TARGET

ROLL INVARIANCE OF THE FORM OF THE N-TARGET

NO UNICITY : 3 DIFFERENT DECOMPOSITIONS

↓

MAN-MADE TARGET DECOMPOSITION
IDENTIFICATION - ANALYSIS

TARGET DECOMPOSITIONS

[S]

COHERENT DECOMPOSITION

- E. KROGAGER (1990)
- W.L. CAMERON (1990)

[K]

TARGET DICHOTOMY

- J.R. HUYNEN (1970)
- R.M. BARNES (1988)
- D. LI – Y, ZHANG (2016)

[T]

EIGENVECTORS BASED DECOMPOSITION

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- W.A. HOLM (1988)

EIGENVECTORS / EIGENVALUES ANALYSIS

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- R. TOUZI (TSVM 2007)

[C]

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- A. BHATTACHARYA – A. FRERY (2015)
- F. XU – Y.Q.JIN (2015) …

EIGENVECTORS / EIGENVALUES ANALYSIS & MODEL BASED DECOMPOSITION

- J.J. VAN ZYL (1992-2008), M. ARII (2010)

EIGENVECTOR BASED DECOMPOSITION

- **SHANE R. CLOUDE** (1985-1992)
- **WILLIAM A. HOLM** (1988)

EIGENVECTOR-BASED DECOMPOSITION

PROPRIETY

EIGENVALUE PROBLEM IS AUTOMATICALLY

BASIS INVARIANT

- **GENERATE A DIAGONAL FORM OF THE COHERENCY MATRIX**
- **PHYSICALLY INTERPRETATION AS STATISTICAL INDEPENDENCE BETWEEN A SET OF VECTORS**
- **GENERAL DECOMPOSITION INTO INDEPENDENT SCATTERING PROCESSES**

EIGENVECTOR-BASED DECOMPOSITION

COHERENCY MATRIX

$$\langle [T] \rangle = \begin{bmatrix} \langle 2A_0 \rangle & \langle C \rangle - j\langle D \rangle & \langle H \rangle + j\langle G \rangle \\ \langle C \rangle + j\langle D \rangle & \langle B_0 \rangle + \langle B \rangle & \langle E \rangle + j\langle F \rangle \\ \langle H \rangle - j\langle G \rangle & \langle E \rangle - j\langle F \rangle & \langle B_0 \rangle - \langle B \rangle \end{bmatrix}$$

$$\Downarrow$$

$$\boxed{\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1}}$$

$$[\Sigma] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}_{\lambda_1 \geq \lambda_2 \geq \lambda_3}$$

3x3 DIAGONAL MATRIX OF EIGENVALUES

$$[U_3] = [\underline{u}_1, \underline{u}_2, \underline{u}_3]$$

SU(3) UNITARY MATRIX (EIGENVECTORS)

SHANE R. CLOUDE

(1985-1992)

DECOMPOSITION

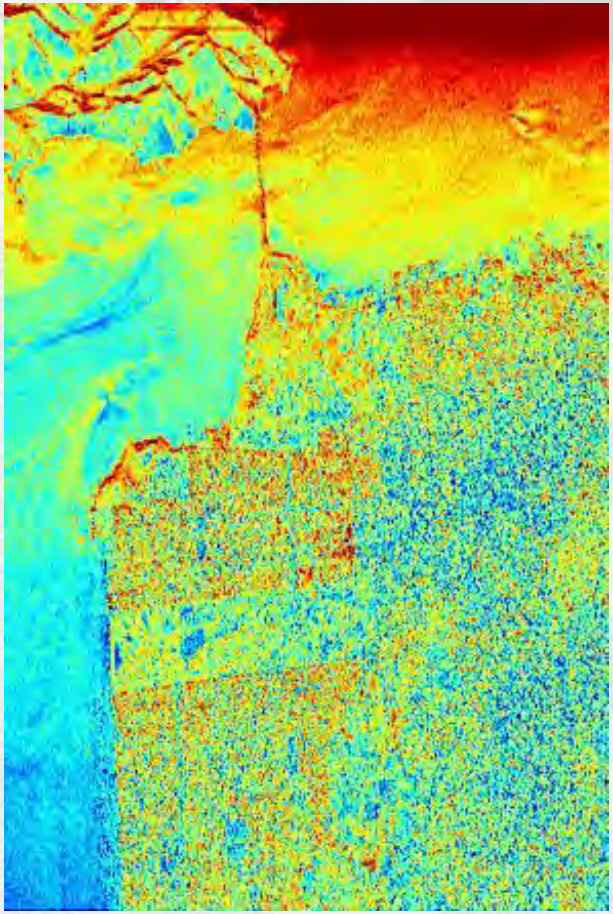
IDENTIFICATION OF THE DOMINANT SCATTERING MECHANISM

VIA THE

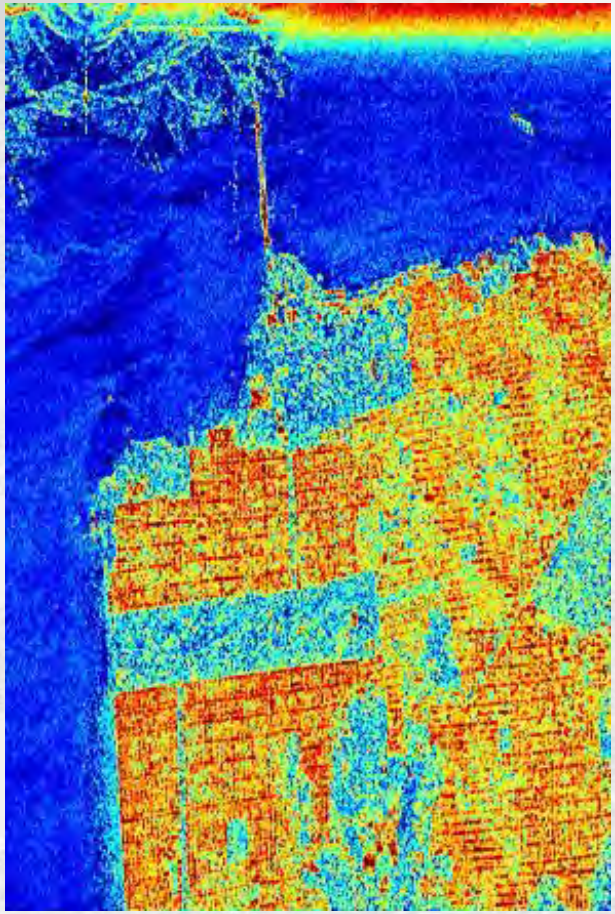
EXTRACTION OF THE LARGEST EIGENVALUE

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} \quad \Rightarrow \quad [T_1] = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} = \underline{k}_1 \underline{k}_1^{T*}$$

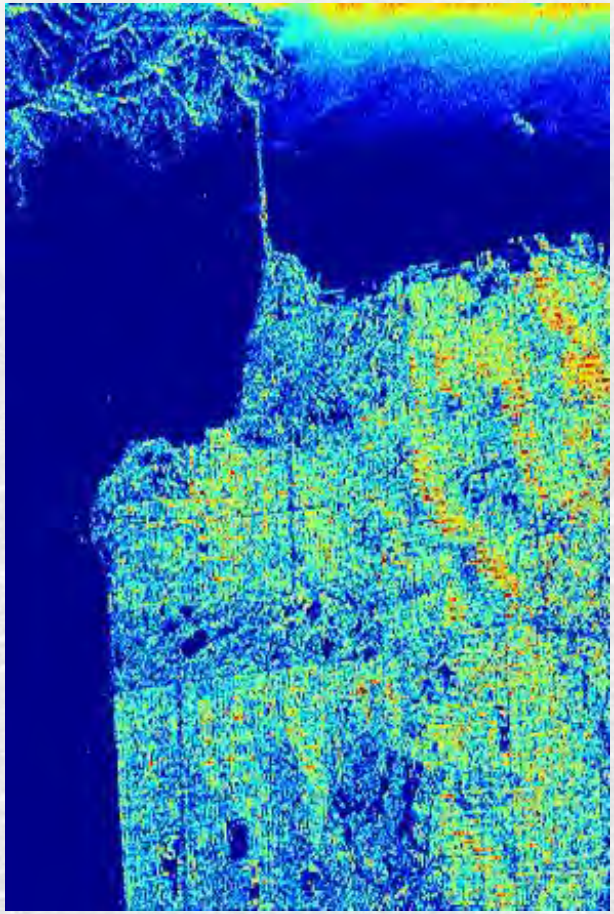
S.R. CLOUDE DECOMPOSITION



$\sqrt{\lambda_1}\,|u_{11}|$



$\sqrt{\lambda_1}\,|u_{12}|$



$\sqrt{\lambda_1}\,|u_{13}|$

S.R. CLOUDE DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$ $\sqrt{\lambda_1}\,|u_{11}|$ $\sqrt{\lambda_1}\,|u_{12}|$ $\sqrt{\lambda_1}\,|u_{13}|$

WILLIAM A. HOLM

(1988)

DECOMPOSITION

ALTERNATIVE PHYSICAL INTERPRETATION

OF THE EIGENVALUES SPECTRUM

$$[\Sigma] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$\Downarrow$$

$$[\Sigma] = \begin{bmatrix} \lambda_1 - \lambda_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \lambda_2 - \lambda_3 & 0 & 0 \\ 0 & \lambda_2 - \lambda_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \lambda_3 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

W.A HOLM DECOMPOSITION

$$\langle [T] \rangle = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} + \lambda_2 \underline{u}_2 \underline{u}_2^{T*} + \lambda_3 \underline{u}_3 \underline{u}_3^{T*}$$

$$\langle [T] \rangle = (\lambda_1 - \lambda_2) \underline{u}_1 \underline{u}_1^{T*} + (\lambda_2 - \lambda_3)\left(\underline{u}_1 \underline{u}_1^{T*} + \underline{u}_2 \underline{u}_2^{T*}\right) + \lambda_3 [I_{3D}]$$

PURE TARGET
(AVERAGE)

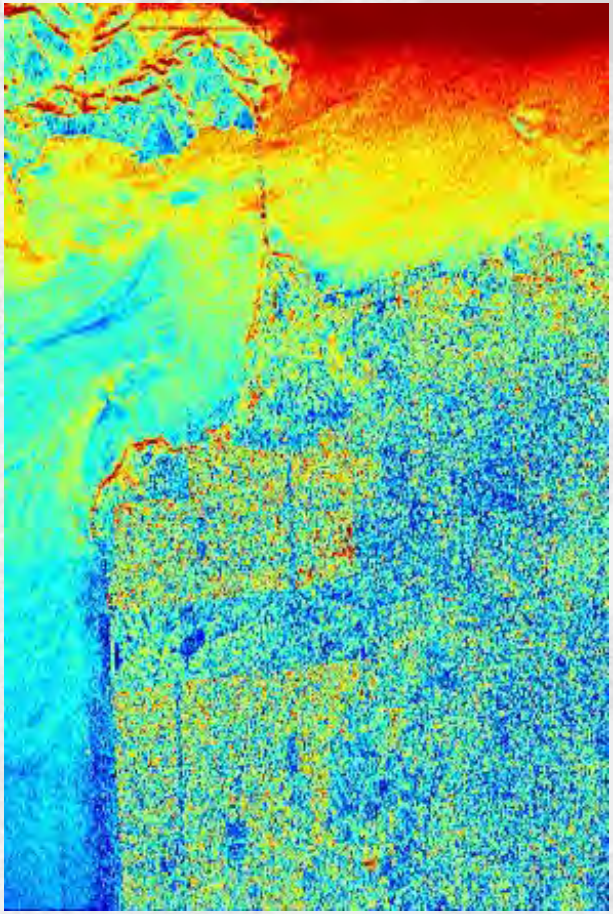
MIXED TARGET
(VARIANCE)

NOISE
(UNPOLARIZED)

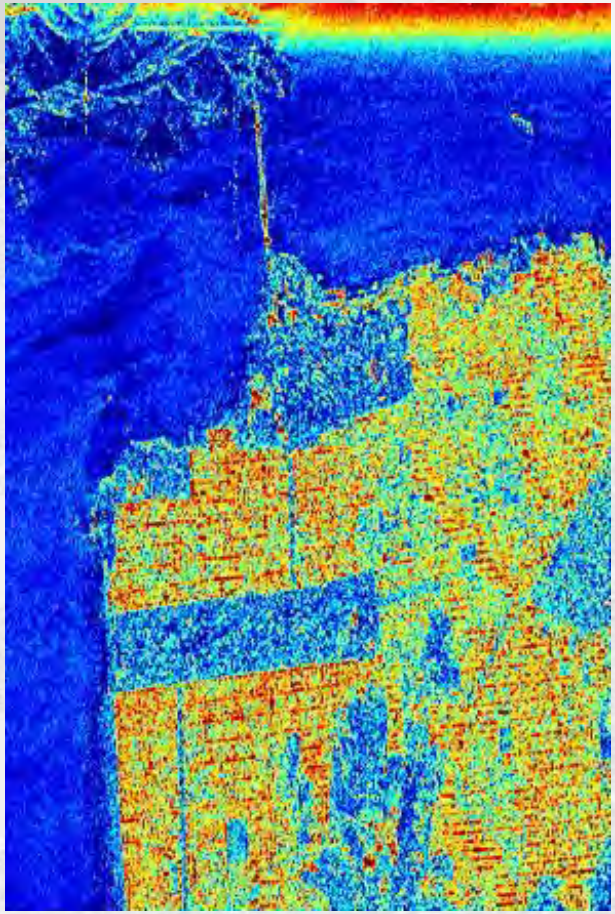
CONCEPT OF : **TARGET** **PLUS** **NOISE**

HYBRID APPROACH OF THE HUYNEN MODEL

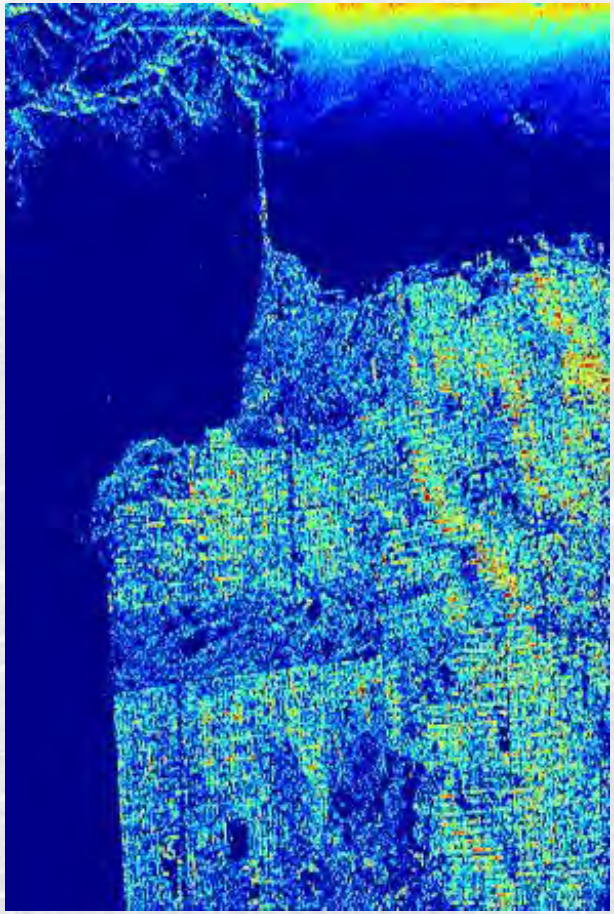
W.A HOLM DECOMPOSITION



$$\sqrt{\lambda_1 - \lambda_2}\,|u_{11}|$$



$$\sqrt{\lambda_1 - \lambda_2}\,|u_{12}|$$



$$\sqrt{\lambda_1 - \lambda_2}\,|u_{13}|$$

W.A HOLM DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$ $\sqrt{\lambda_1 - \lambda_2}\,|u_{11}|$ $\sqrt{\lambda_1 - \lambda_2}\,|u_{12}|$

$\sqrt{\lambda_1 - \lambda_2}\,|u_{13}|$

TARGET DECOMPOSITIONS

[S]

COHERENT DECOMPOSITION

- E. KROGAGER (1990)
- W.L. CAMERON (1990)

[K]

TARGET DICHOTOMY

- J.R. HUYNEN (1970)
- R.M. BARNES (1988)
- D. LI – Y, ZHANG (2016)

[T]

AZIMUTHAL SYMMETRY

MODEL BASED DECOMPOSITION

- A.J. FREEMAN – S.L. DURDEN (1992)
- Y. YAMAGUSHI (2005 - 2012), AN (2010)
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EIGENVECTORS BASED DECOMPOSITION

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EIGENVECTORS / EIGENVALUES ANALYSIS & MODEL BASED DECOMPOSITION

- J.J. VAN ZYL (1992-2008), M. ARII (2010)

EIGENVECTORS / EIGENVALUES ANALYSIS

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- R. TOUZI (TSVM 2007)

TARGET DECOMPOSITION FOR TARGETS WITH REFLECTION SYMMETRY

MEDIUM WITH REFLECTION SYMMETRY

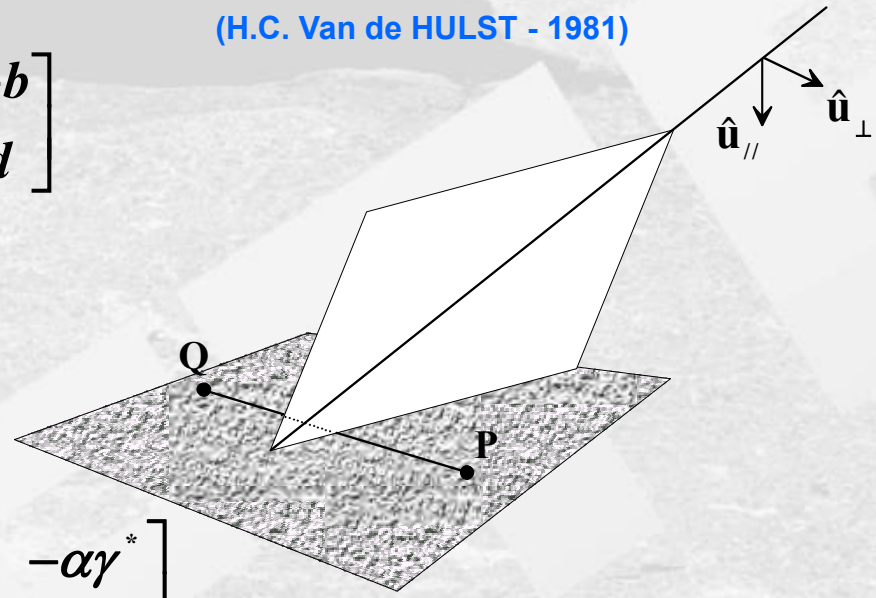
(H.C. Van de HULST - 1981)

$$[S_Q]=\begin{bmatrix} a & -b \\ -b & d \end{bmatrix}$$

$$[S_P]=\begin{bmatrix} a & b \\ b & d \end{bmatrix}$$

$$\underline{k}_Q=\begin{bmatrix} \alpha \\ \beta \\ -\gamma \end{bmatrix}$$

$$\underline{k}_P=\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$



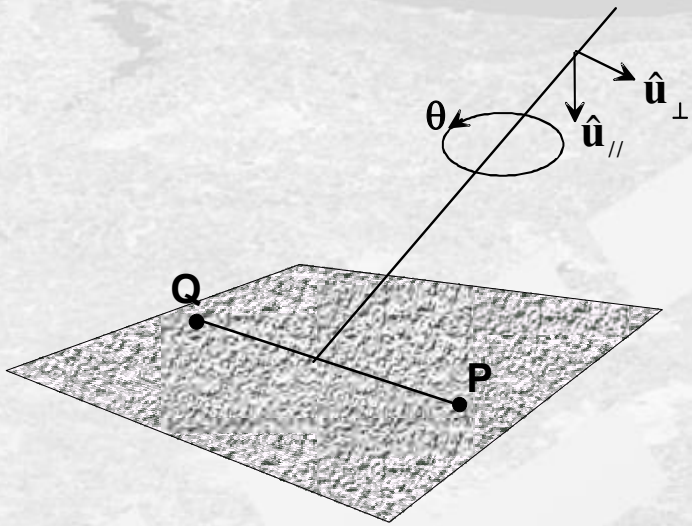
$$[T_Q]=\begin{bmatrix} |\alpha|^2 & \alpha\beta^* & -\alpha\gamma^* \\ \beta\alpha^* & |\beta|^2 & -\beta\gamma^* \\ -\gamma\alpha^* & -\gamma\beta^* & |\gamma|^2 \end{bmatrix}$$

$$[T_P]=\begin{bmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \beta\alpha^* & |\beta|^2 & \beta\gamma^* \\ \gamma\alpha^* & \gamma\beta^* & |\gamma|^2 \end{bmatrix}$$

$$\langle[T]\rangle=[T_P]+[T_Q]=\begin{bmatrix} |\alpha|^2 & \alpha\beta^* & 0 \\ \beta\alpha^* & |\beta|^2 & 0 \\ 0 & 0 & |\gamma|^2 \end{bmatrix}$$

MEDIUM WITH ROTATION SYMMETRY

(H.C. Van de HULST - 1981)



$$\langle [T(\theta)] \rangle = [R_3(\theta)] \langle [T] \rangle [R_3(\theta)]^{-1}$$

With:

$$[R_3(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\theta) & \sin(2\theta) \\ 0 & -\sin(2\theta) & \cos(2\theta) \end{bmatrix}$$

EIGENVECTORS OF $[U^R_{3P}]$

$$[U^R_{3P}]\underline{q} = \lambda \underline{q}$$

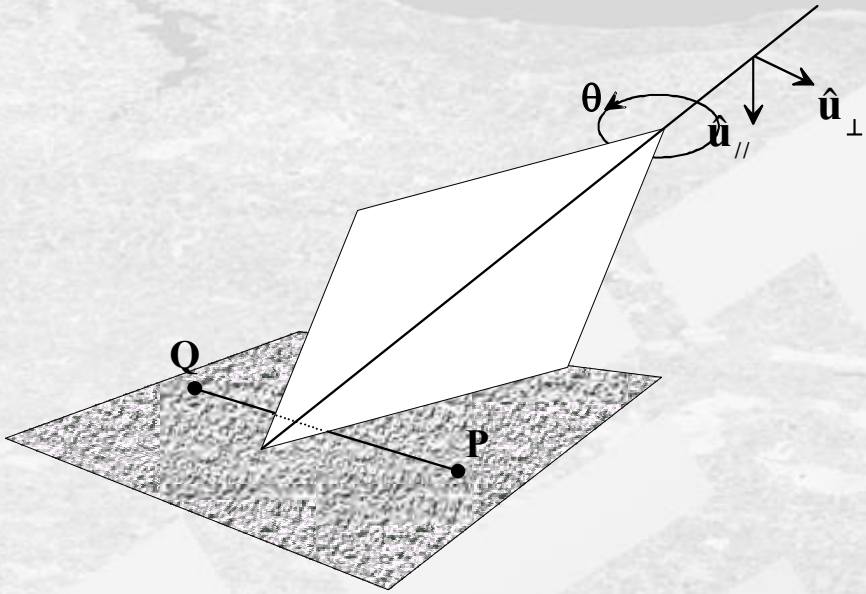
$$\underline{q}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \underline{q}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ j \end{bmatrix} \quad \underline{q}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ j \\ 1 \end{bmatrix}$$

$$\langle [T_R] \rangle = \alpha \underline{q}_1 \underline{q}_1^{T*} + \beta \underline{q}_2 \underline{q}_2^{T*} + \gamma \underline{q}_3 \underline{q}_3^{T*}$$

$$= \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & -j(\beta-\gamma) \\ 0 & j(\beta-\gamma) & \beta+\gamma \end{bmatrix}$$

MEDIUM WITH AZIMUTHAL SYMMETRY

(S.H. NGHIEM et al. - 1992)



$$[T_{PR}] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & -j(\beta-\gamma) \\ 0 & j(\beta-\gamma) & \beta+\gamma \end{bmatrix}$$

$$[T_{QR}] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & j(\beta-\gamma) \\ 0 & -j(\beta-\gamma) & \beta+\gamma \end{bmatrix}$$

AZIMUTHAL SYMMETRY =
REFLECTION + ROTATION SYMMETRIES

$$\langle [T] \rangle = [T_{PR}] + [T_{QR}] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & 0 \\ 0 & 0 & \beta+\gamma \end{bmatrix}$$

COHERENCY MATRIX

General Case

$$\langle [T] \rangle = \begin{bmatrix} T_1 & T_2 & T_3 \\ T_2^* & T_4 & T_5 \\ T_3^* & T_5^* & T_6 \end{bmatrix}$$

Reflection Symmetry

$$\langle [T] \rangle = \begin{bmatrix} T_1 & T_2 & 0 \\ T_2^* & T_4 & 0 \\ 0 & 0 & T_6 \end{bmatrix}$$

Rotation Symmetry

$$\langle [T] \rangle = \begin{bmatrix} T_1 & 0 & 0 \\ 0 & T_4 & T_5 \\ 0 & T_5^* & T_4 \end{bmatrix}$$

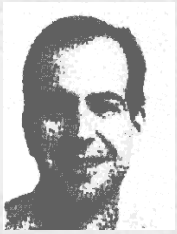
Azimuthal Symmetry

$$\langle [T] \rangle = \begin{bmatrix} T_1 & 0 & 0 \\ 0 & T_4 & 0 \\ 0 & 0 & T_4 \end{bmatrix}$$

MODEL BASED DECOMPOSITION

- **A. FREEMAN – S. DURDEN**

  (1992)

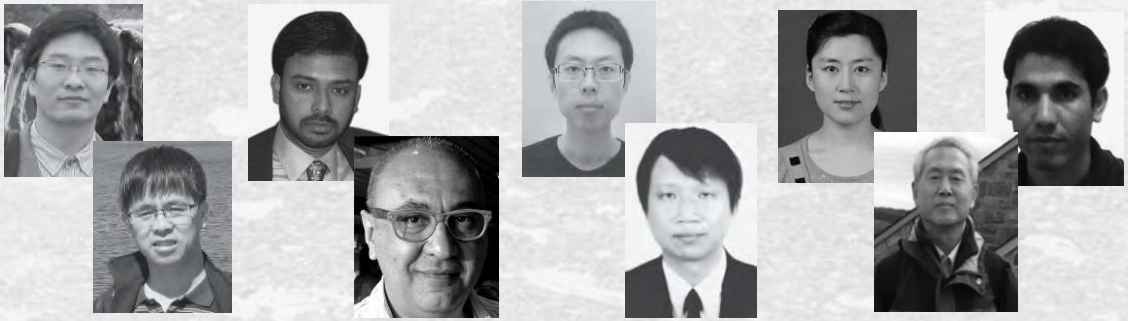



- **Y. YAMAGUCHI – S. SINGH**

  (2005 - 2018)


- **And others …**

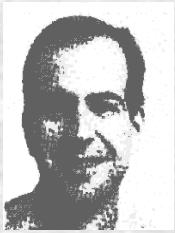
  (2015 - 2017)



TARGET DECOMPOSITION FOR TARGETS WITH REFLECTION SYMMETRY

MODEL BASED DECOMPOSITION

A. FREEMAN – S. DURDEN (1992)



A. Freeman and S.L. Durden, "A Three-Component Scattering Model for Polarimetric SAR Data"
IEEE TGRS, vol. 36, no. 3, May 1998

MODEL BASED DECOMPOSITION

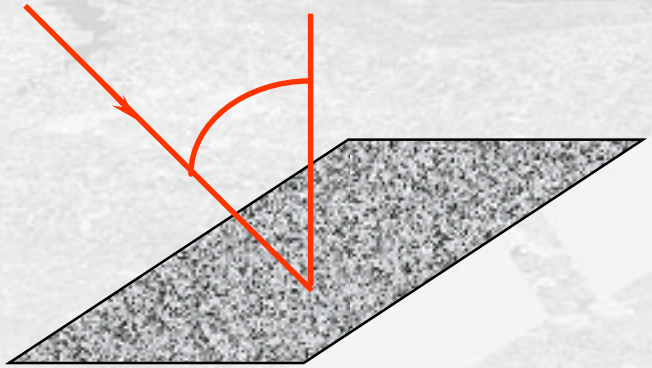
3 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_S] + [T_D] + [T_V]$$

$[T_S]$	$[T_D]$	$[T_V]$
SINGLE SCATTERING	DOUBLE SCATTERING	VOLUME SCATTERING

MODEL BASED DECOMPOSITION

SINGLE SCATTERING
(ROUGH SURFACE)



MECHANISM

$$[S_s] = \begin{bmatrix} R_H & 0 \\ 0 & R_V \end{bmatrix} \Rightarrow \underline{k}_s = \begin{bmatrix} R_H + R_V \\ R_H - R_V \\ 0 \end{bmatrix}$$

COHERENCY MATRIX

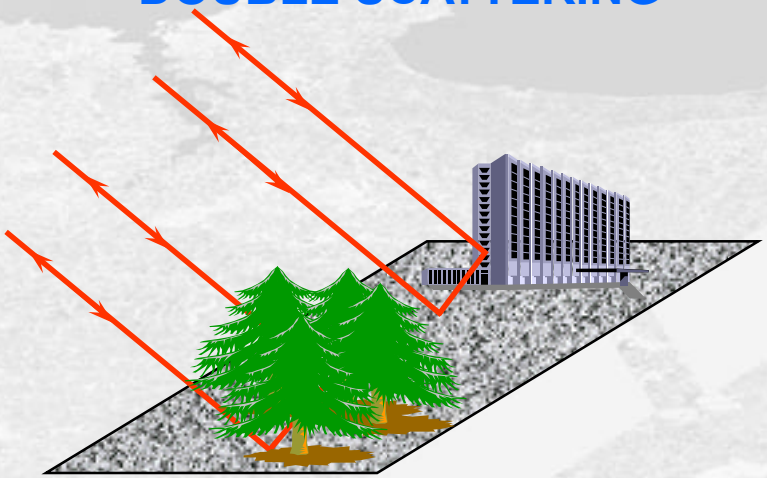
$$[T_s] = f_s \begin{bmatrix} |\beta+1|^2 & (\beta+1)(\beta-1)^* & 0 \\ (\beta+1)^*(\beta-1) & |\beta-1|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$f_s = |R_V|^2$$

$$\beta = \frac{R_H}{R_V}$$

MODEL BASED DECOMPOSITION

DOUBLE SCATTERING



MECHANISM

$$[S_D] = \begin{bmatrix} R_{GH} R_{TH} & 0 \\ 0 & -R_{GV} R_{TV} \end{bmatrix}$$

$$\Rightarrow \underline{k}_D = \begin{bmatrix} R_{GH} R_{TH} - R_{GV} R_{TV} \\ R_{GH} R_{TH} + R_{GV} R_{TV} \\ 0 \end{bmatrix}$$

COHERENCY MATRIX

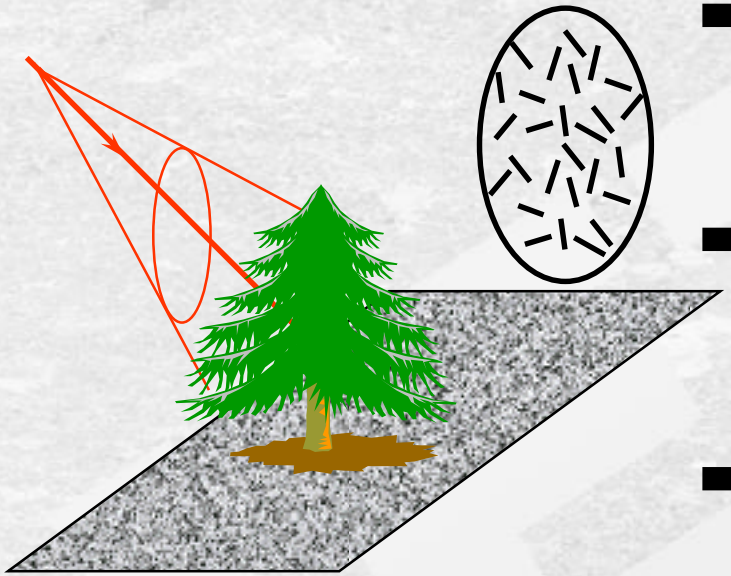
$$[T_D] = f_D \begin{bmatrix} |\alpha - 1|^2 & (\alpha-1)(\alpha+1)^* & 0 \\ (\alpha-1)^*(\alpha+1) & |\alpha+1|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$f_D = |R_{GV} R_{TV}|^2$$

$$\alpha = \frac{R_{GH} R_{TH}}{R_{GV} R_{TV}}$$

MODEL BASED DECOMPOSITION

VOLUME SCATTERING
(RANDOMLY ORIENTED VERY THIN CYLINDER-LIKE SCATTERERS)



MECHANISM
(CYLINDER)

$$[S]=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

ORIENTED MECHANISM

$$[S_\theta]=[U_2(\theta)]^T[S][U_2(\theta)]$$

UNIFORM ORIENTATION

$$P(\theta)=\frac{1}{2\pi}$$

SECOND-ORDER STATISTICS

$$[T_V]=\langle[T_\theta]\rangle=\int_0^{2\pi}[T_V]P(\theta)d\theta$$

COVARIANCE MATRIX

(THIN CYLINDERS)

$$[T_V]=\frac{f_V}{3}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$a \mapsto 1 \quad b \mapsto 0$$

MODEL BASED DECOMPOSITION

3 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_S] + [T_D] + [T_V]$$

SINGLE SCATTERING — DOUBLE SCATTERING — VOLUME SCATTERING

$$T_{11} = f_S |\beta + 1|^2 + f_D |\alpha - 1|^2 + \frac{4 f_V}{3}$$

$$T_{12} = f_S (\beta + 1)(\beta - 1)^* + f_D (\alpha - 1)(\alpha + 1)^*$$

$$T_{22} = f_S |\beta - 1|^2 + f_D |\alpha + 1|^2 + \frac{2 f_V}{3}$$

$$T_{33} = \frac{2 f_V}{3}$$

→ 5 UNKNOWN REAL COEFFICIENTS

→ 4 OBSERVED EQUATIONS

MODEL BASED DECOMPOSITION

$$if\ \Re\left(\left\langle S_{XX}S_{YY}^{*}\right\rangle - \frac{f_V}{3}\right) \geq 0 \quad \Rightarrow \quad \alpha = +1$$

$$if\ \Re\left(\left\langle S_{XX}S_{YY}^{*}\right\rangle - \frac{f_V}{3}\right) \leq 0 \quad \Rightarrow \quad \beta = +1$$

$$\boxed{\left\{f_S, |\beta|, f_D, |\alpha|, f_V\right\}}$$

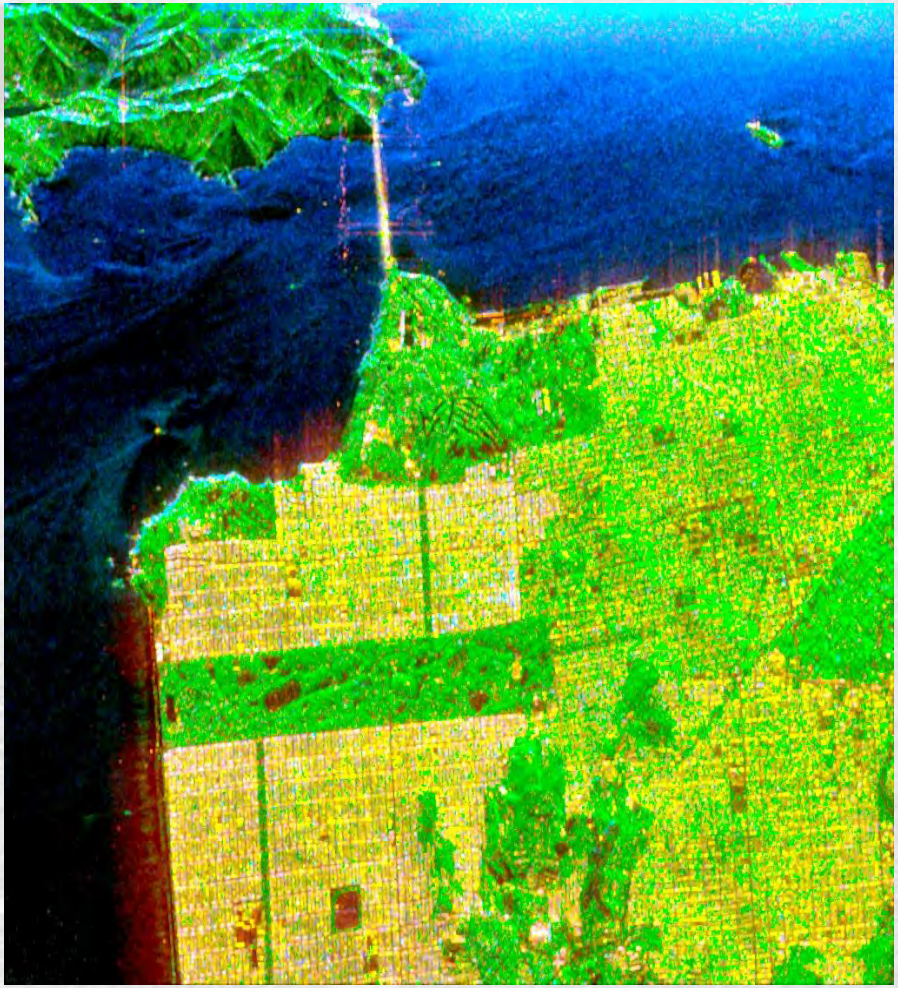
$$span = \langle T_{11}\rangle + \langle T_{22}\rangle + \langle T_{33}\rangle = f_S\left(1+\beta^2\right) + f_D\left(1+|\alpha|^2\right) + \frac{2}{3}f_V$$

SINGLE BOUNCE SCATTERING (ODD)

DOUBLE DOUBLE SCATTERING (DBL)

VOLUME SCATTERING (VOL)

MODEL BASED DECOMPOSITION



$2A_0$ $B_0 + B$ $B_0 - B$

$$ODD = f_S\left(1+\beta^2\right)$$

$$DBL = f_D\left(1+\alpha^2\right)$$

$$VOL = \frac{2 f_V}{3}$$

TARGET DECOMPOSITION FOR TARGETS WITHOUT REFLECTION SYMMETRY

MODEL BASED - 4 COMPONENTS DECOMPOSITION

Y. YAMAGUCHI et al. (2005 - 2013)

MODEL BASED DECOMPOSITION

MEDIUM WITHOUT ANY REFLECTION SYMMETRY

4 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_S] + [T_D] + [T_V] + [T_H]$$

$[T_S]$	$[T_D]$	$[T_V]$	$[T_H]$
SINGLE SCATTERING	DOUBLE SCATTERING	VOLUME SCATTERING	HELIX SCATTERING

$$[S]_{\pm Helix} = \frac{1}{2}\begin{bmatrix} 1 & \pm j \\ \pm j & -1 \end{bmatrix} \qquad \langle [T] \rangle_{Helix} = \frac{1}{2}\left\langle \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & \pm j \\ 0 & \mp j & 1 \end{bmatrix} \right\rangle$$

Non reflection Symmetric cases

Yamaguchi Y., Moriyama T., Ishido M. and Yamada H., "*Four-Component Scattering Model for Polarimetric SAR Image Decomposition*", IEEE Trans. Geos. Remote Sens., vol. 43, no. 8, August 2005.

Yamaguchi Y., Yajima Y. and Yamada H., "A Four-Component Decomposition of POLSAR Images Based on the Coherency Matrix", IEEE Geos. Rem. Sens. Letters, vol. 3, no. 3, July 2006.

MODEL BASED DECOMPOSITION

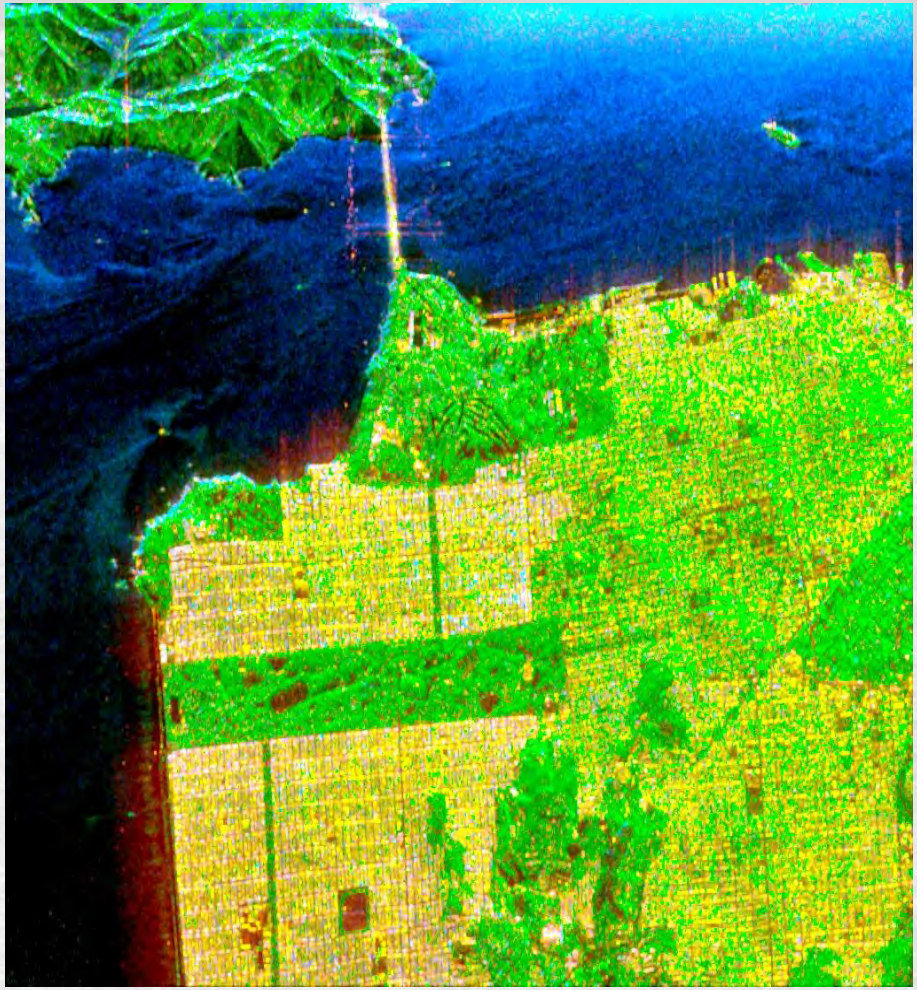
$2A_0$ $B_0 + B$ $B_0 - B$

$$ODD = f_S\left(1+\beta^2\right)$$

$$DBL = f_D\left(1+\alpha^2\right)$$

$$VOL = \frac{2 f_V}{3}$$

MODEL BASED DECOMPOSITION



ODD DBL VOL

Freeman decomposition **Yamaguchi decomposition**

MODEL BASED DECOMPOSITION

Y. Yamaguchi, A. Sato, W.M. Boerner, R. Sato, H. Yamada, "*4-component scattering power decomposition with rotation of coherency matrix*", IEEE TGRS vol. 49, no. 6, June 2011.

A. Sato, Y. Yamaguchi, G. Singh, and S.-E. Park, "*4-component scattering power decomposition with extended volume scattering model*", IEEE GRS Letters, vol. 9, no. 2, pp. 166–170, March 2012.

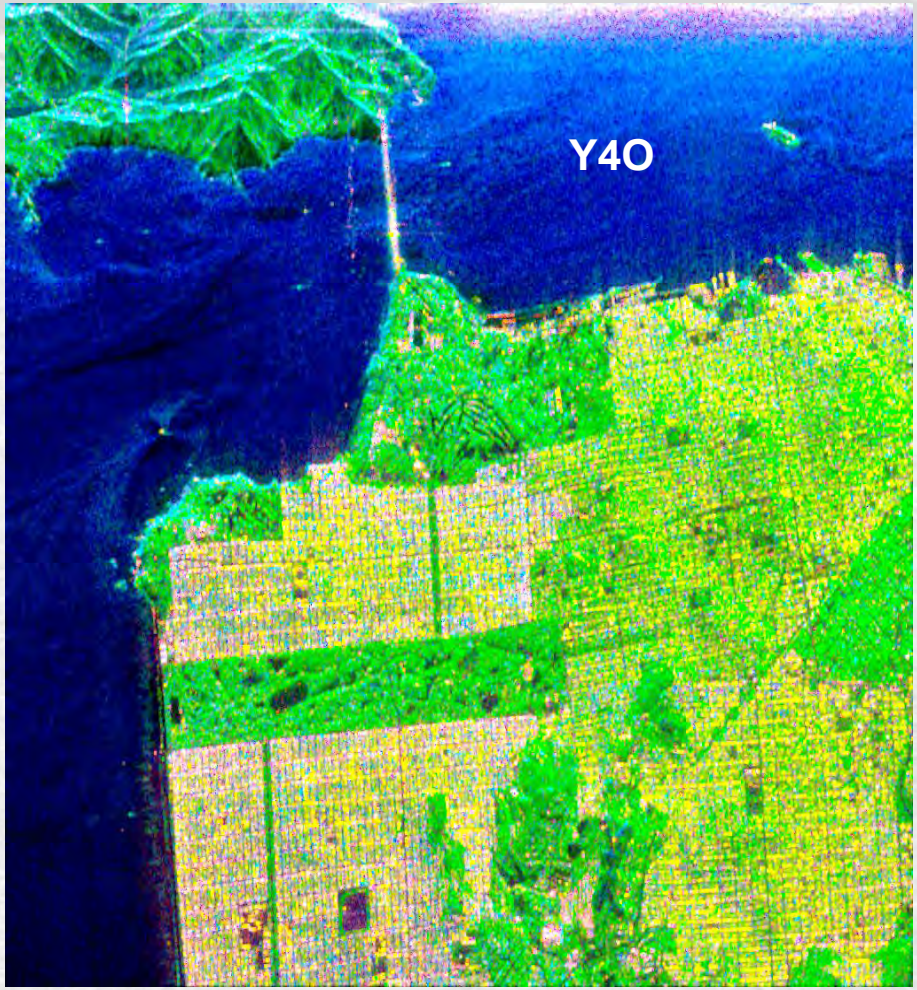
G. Singh, Y. Yamaguchi, S.E. Park, *Y. Cui, H. Kobayashi, « Hybrid Freeman/Eigenvalue Decomposition Method With Extended Volume Scattering Model »* IEEE GRS Letters, vol. 10, no. 1, January 2013.

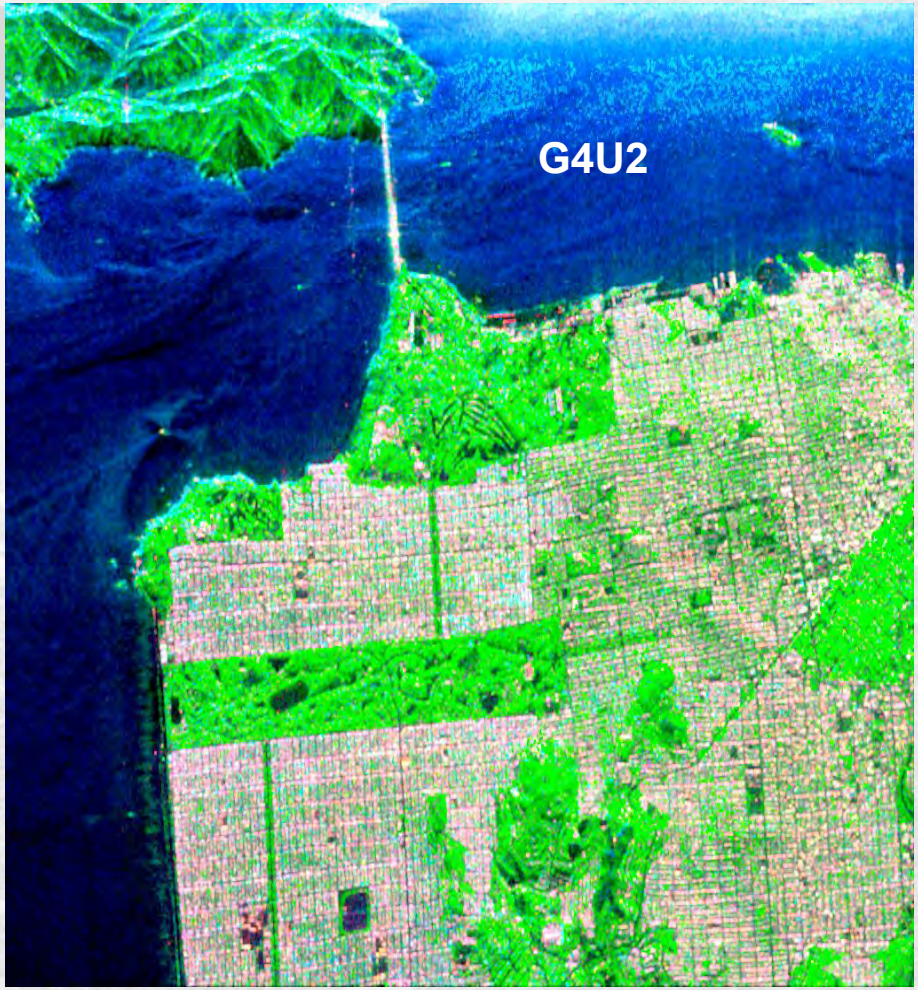
G. Singh, Y. Yamaguchi, S.E. Park, *« General Four-Component Scattering Power Decomposition With Unitary Transformation of Coherency Matrix »* IEEE TGRS vol. 51, no. 5, May 2013.

MODEL BASED DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$ ODD DBL VOL

MODEL BASED DECOMPOSITION



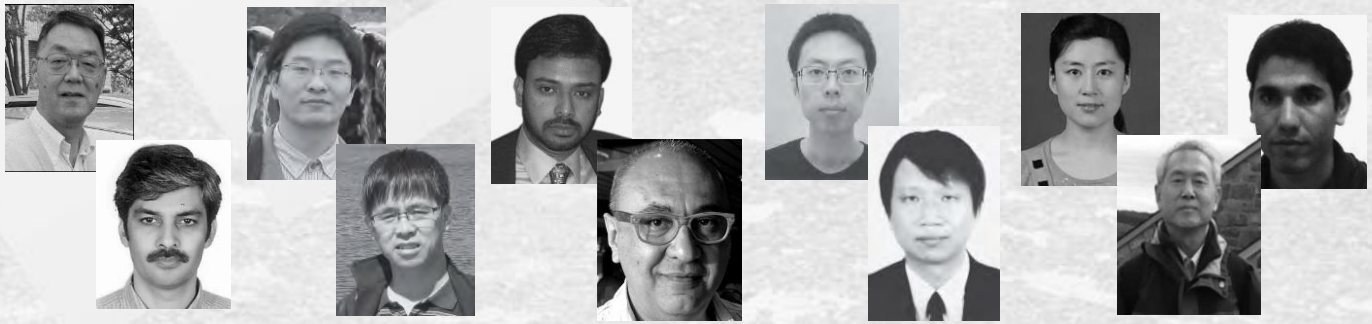


ODD DBL VOL

TARGET DECOMPOSITION FOR TARGETS WITHOUT REFLECTION SYMMETRY

MODEL BASED - 4 / 5 / 6 COMPONENTS DECOMPOSITION

(2015 - 2017)



MODEL BASED DECOMPOSITION

A. BHATTACHARYA, A. FRERY, "*Modifying the Yamaguchi 4-component decomposition scattering powers using a stochastic distance*", IEEE JSTARS, vol. 8, pp 3497-3506, July 2015.

F. XU, Y.Q. JIN, "*Deorientation theory of Polarimetric scattering targets and application to terrain surface classification*", IEEE TGRS Vol 43, n° 10, October 2015.

B. ZOU, D. LU, L. ZHANG, W.M. Moon, *« Eigen-decomposition-based Four Component Decomposition for PoSAR Data"*. IEEE JSTARS, vol. 9, pp 1286-1296, March 2016.

H. AGHABABAEE, M. Reza SAHEBI, *"Incoherent Target Scattering Decomposition of Polarimetric SAR Data Based on Vector Model Roll-Invariant Parameters"*. IEEE TGRS, vol. 54, no 8, August 2016.

G. SINGH, Y. YAMAGUCHI, "*Model-based Six-Component Scattering Matrix Power Decomposition*", IEEE TGRS Vol 56, n° 10, October 2018.

MODEL BASED DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

ODD DBL VOL

Singh decomposition – 6 components

TARGET DECOMPOSITIONS

[S]

COHERENT DECOMPOSITION

- E. KROGAGER (1990)
- W.L. CAMERON (1990)

[K]

TARGET DICHOTOMY

- J.R. HUYNEN (1970)
- R.M. BARNES (1988)
- D. LI – Y, ZHANG (2016)

[T]

AZIMUTHAL SYMMETRY

MODEL BASED DECOMPOSITION

- A.J. FREEMAN – S.L. DURDEN (1992)
- Y. YAMAGUSHI (2005 - 2012), AN (2010)
- G. SINGH (2013), H. AGHABABAEE (2016)
- A. BHATTACHARYA – A. FRERY (2015)
- F. XU – Y.Q.JIN (2015) …

EIGENVECTORS BASED DECOMPOSITION

- S.R. CLOUDE (1985)
- W.A. HOLM (1988)

EIGENVECTORS / EIGENVALUES ANALYSIS & MODEL BASED DECOMPOSITION

- J.J. VAN ZYL (1992-2008), M. ARII (2010)

EIGENVECTORS / EIGENVALUES ANALYSIS

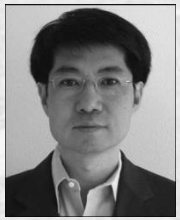
- S.R. CLOUDE - E. POTTIER (1996-1997)
- R. TOUZI (TSVM 2007)

TARGET DECOMPOSITION FOR TARGETS WITH / WITHOUT REFLECTION SYMMETRY

REQUIEREMENTS FOR MODEL BASED POLARIMETRIC DECOMPOSITIONS

J.J. VAN ZYL – M. ARII – Y. KIM (2010)





J. J. Van Zyl, M. Arii, Y. Kim, "*Model-Based Decomposition of Polarimetric SAR Covariance Matrices Constrained for Nonnegative Eigenvalues*" IEEE TGRS, vol. 49, n°9, Sept. 2011.

ADAPTATIVE MODEL-BASED DECOMPOSITION

$$[C'_{\text{remainder}}] = \langle [C] \rangle - f_v \langle [C_{\text{vol}}(\theta_0, \sigma)] \rangle$$

$$\langle [C_{\text{vol}}(\theta_0, \sigma)] \rangle = [C_\alpha] + p(\sigma)[C_\beta] + q(\sigma)[C_\gamma].$$

$$[C_\alpha] = \frac{1}{8}\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

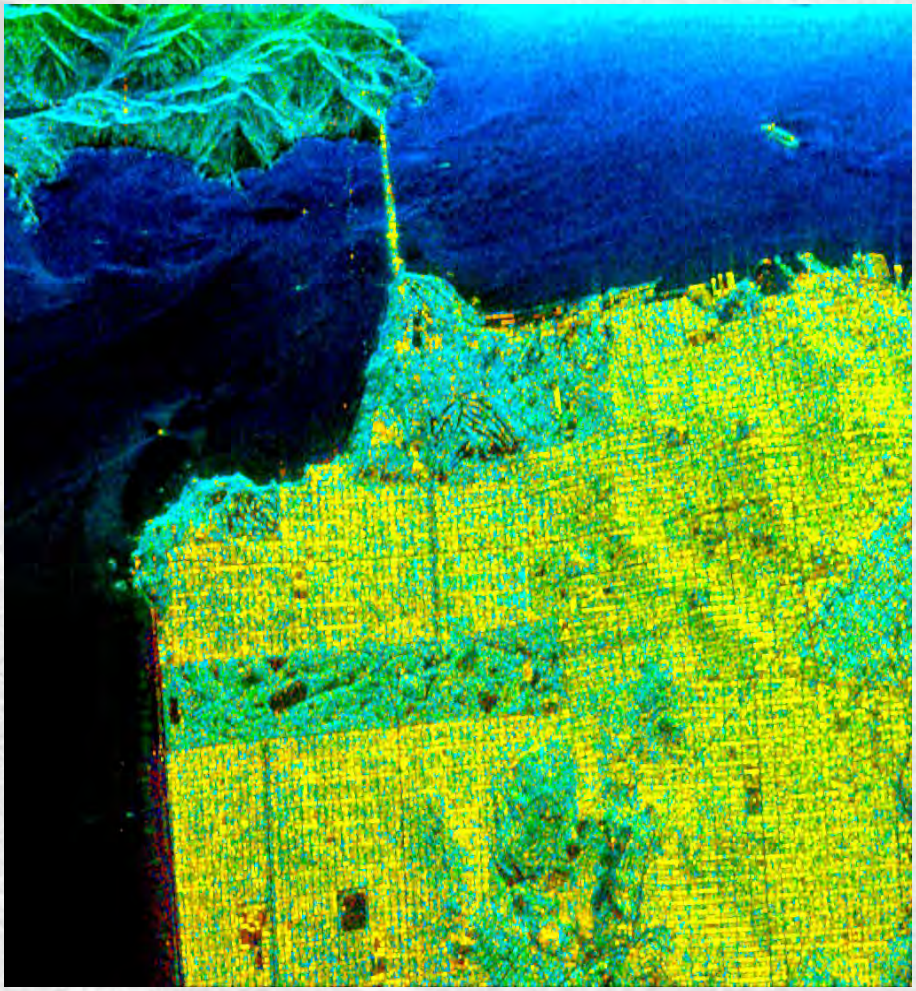
$$[C_\beta(2\theta_0)] = \frac{1}{8}\begin{bmatrix} -2\cos 2\theta_0 & \sqrt{2}\sin 2\theta_0 & 0 \\ \sqrt{2}\sin 2\theta_0 & 0 & \sqrt{2}\sin 2\theta_0 \\ 0 & \sqrt{2}\sin 2\theta_0 & 2\cos 2\theta_0 \end{bmatrix}$$

$$[C_\gamma(4\theta_0)] = \frac{1}{8}\begin{bmatrix} \cos 4\theta_0 & -\sqrt{2}\sin 4\theta_0 & -\cos 4\theta_0 \\ -\sqrt{2}\sin 4\theta_0 & -2\cos 4\theta_0 & \sqrt{2}\sin 4\theta_0 \\ -\cos 4\theta_0 & \sqrt{2}\sin 4\theta_0 & \cos 4\theta_0 \end{bmatrix}.$$

$$\begin{aligned} p(\sigma) &= 2.0806\sigma^6 - 6.3350\sigma^5 + 6.3864\sigma^4 \\ &\quad - 0.4431\sigma^3 - 3.9638\sigma^2 - 0.0008\sigma + 2.000 \\ q(\sigma) &= 9.0166\sigma^6 - 18.7790\sigma^5 + 4.9590\sigma^4 \\ &\quad + 14.5629\sigma^3 - 10.8034\sigma^2 + 0.1902\sigma + 1.000. \end{aligned}$$

M. Arii, J. J. Van Zyl, Y. Kim, "Adaptative *Model-Based Decomposition of Polarimetric SAR Covariance Matrices*" IEEE TGRS, vol. 49, n°9, Sept. 2011.

MODEL BASED DECOMPOSITION



$2A_0$ $B_0 + B$ $B_0 - B$ ODD DBL VOL

TARGET DECOMPOSITIONS

[S]

COHERENT DECOMPOSITION

- E. KROGAGER (1990)
- W.L. CAMERON (1990)

[K]

TARGET DICHOTOMY

- J.R. HUYNEN (1970)
- R.M. BARNES (1988)
- D. LI – Y, ZHANG (2016)

[T]

EIGENVECTORS BASED DECOMPOSITION

- S.R. CLOUDE (1985)
- W.A. HOLM (1988)

EIGENVECTORS / EIGENVALUES ANALYSIS

- S.R. CLOUDE - E. POTTIER (1996-1997)
- R. TOUZI (TSVM 2007)

[C]

AZIMUTHAL SYMMETRY

MODEL BASED DECOMPOSITION

- A.J. FREEMAN – S.L. DURDEN (1992)
- Y. YAMAGUSHI (2005 - 2012), AN (2010)
- G. SINGH (2013), H. AGHABABAEE (2016)
- A. BHATTACHARYA – A. FRERY (2015)
- F. XU – Y.Q.JIN (2015) …

EIGENVECTORS / EIGENVALUES ANALYSIS & MODEL BASED DECOMPOSITION

- J.J. VAN ZYL (1992-2008), M. ARII (2010)

H / A / α DECOMPOSITION

TARGET VECTOR

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

LOCAL ESTIMATE OF THE COHERENCY MATRIX

$$\langle [T] \rangle = \frac{1}{N}\sum_{i=1}^{N} \underline{k}_i \cdot \underline{k}_i^{*T} = \frac{1}{N}\sum_{i=1}^{N} [T_i]$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES
$\lambda_1 > \lambda_2 > \lambda_3$

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3} \lambda_k}$$

EIGENVALUE-BASED PARAMETERS

S. Allain

S.E.R.D and D.E.R.D PARAMETERS
(*Single- and Double-bounce Eigenvalue Relative Difference*)

$$SERD = \frac{\lambda_S - \lambda_{3_{NOS}}}{\lambda_S + \lambda_{3_{NOS}}} \qquad DERD = \frac{\lambda_D - \lambda_{3_{NOS}}}{\lambda_D + \lambda_{3_{NOS}}}$$

T. Ainsworth

POLARIZATION FRACTION

$$PF = 1 - \frac{3\lambda_3}{Span} = 1 - \frac{3\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \qquad 0 \le PF \le 1$$

POLARIZATION ASYMMETRY

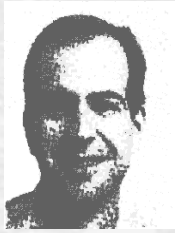
$$PA = \frac{(\lambda_1 - \lambda_3) - (\lambda_2 - \lambda_3)}{(\lambda_1 - \lambda_3) + (\lambda_2 - \lambda_3)} = \frac{\lambda_1 - \lambda_2}{Span - 3\lambda_3} \qquad 0 \le PA \le 1$$

EIGENVALUE-BASED PARAMETERS

J. Van Zyl

RADAR VEGETATION INDEX

$$RVI = \frac{4\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \qquad 0 \le RVI \le \frac{4}{3}$$



S.L. Durden

PEDESTAL HEIGHT

$$PH = \frac{min(\lambda_1, \lambda_2, \lambda_3)}{max(\lambda_1, \lambda_2, \lambda_3)} = \frac{\lambda_3}{\lambda_1} \qquad 0 \le PH \le 1$$

E. Luneuburg

TARGET RANDOMNESS

$$p_R = \sqrt{\frac{3}{2}}\sqrt{\frac{\lambda_2^2 + \lambda_3^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}} \qquad 0 \le p_R \le 1$$

J. Praks

E. Colin

ALTERNATIVE ENTROPY PARAMETERS DERIVATION

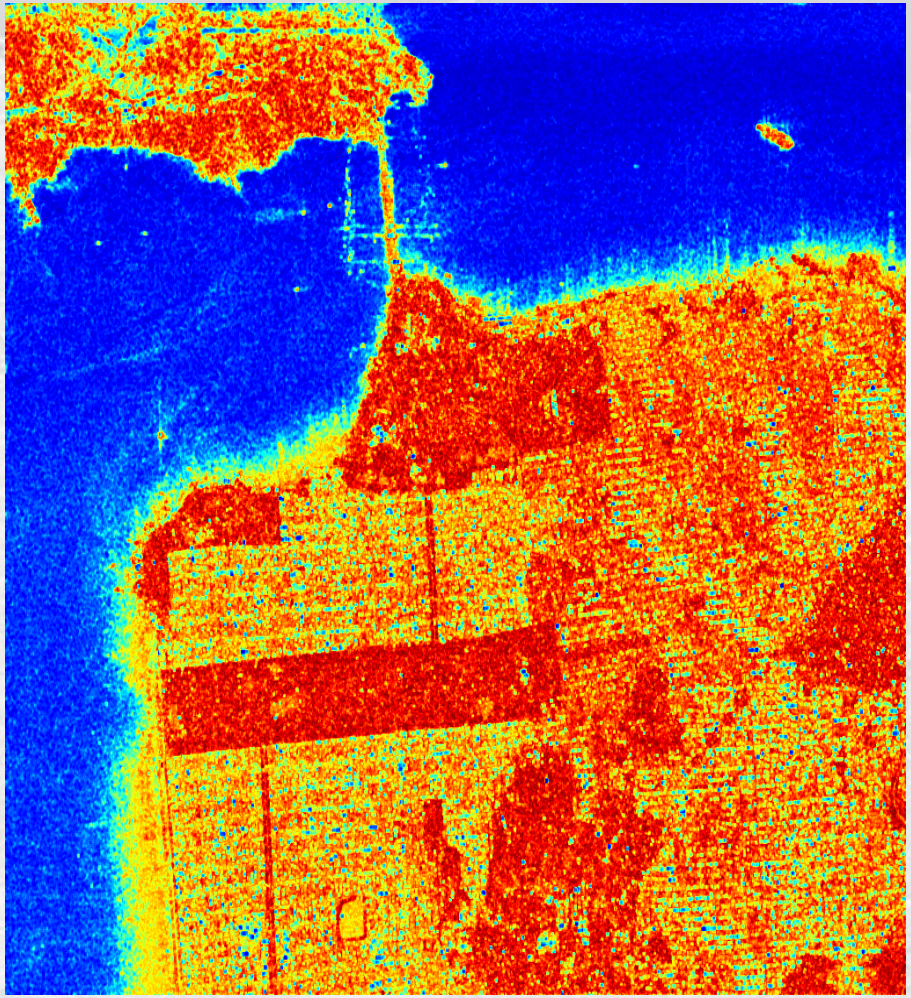
Normalized Coherency Matrix

$$\mathbf{N}_3 = \left\langle \underline{\mathbf{k}}^{T*} \cdot \underline{\mathbf{k}} \right\rangle^{-1} \left\langle \underline{\mathbf{k}} \cdot \underline{\mathbf{k}}^{T*} \right\rangle = \frac{\mathbf{T}_3}{\mathrm{Tr}(\mathbf{T}_3)}$$

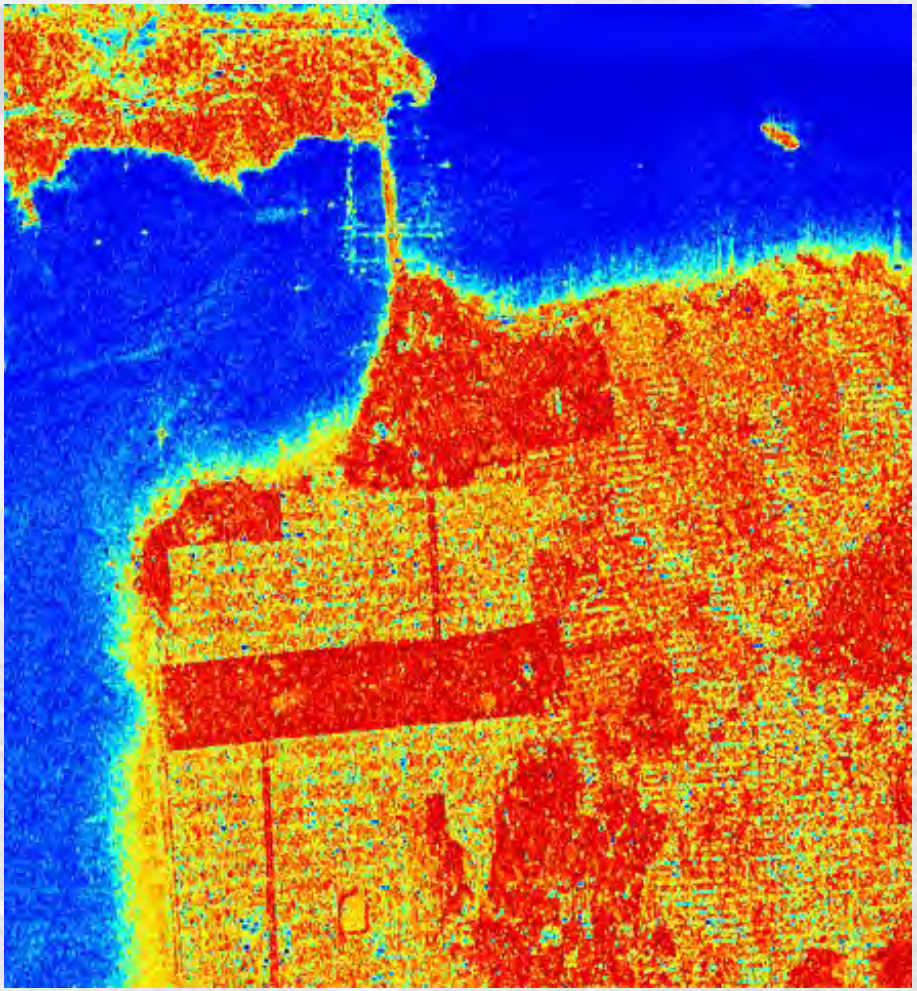
$$\boxed{H \approx 2.52 + 0.78\, log_3\left(\left|\mathbf{N}_3 + 0.16\mathbf{I}_{D3}\right|\right)}$$

ENTROPY

EIGENVALUE-BASED PARAMETERS



ENTROPY – Praks Colin



ENTROPY (H)

J. Praks

E. Colin

J. Morio

P. Réfrégier

ALTERNATIVE ENTROPY PARAMETERS DERIVATION

Normalized Coherency Matrix

$$\mathbf{N}_3 = \left\langle \underline{\mathbf{k}}^{T*} \cdot \underline{\mathbf{k}} \right\rangle^{-1} \left\langle \underline{\mathbf{k}} \cdot \underline{\mathbf{k}}^{T*} \right\rangle = \frac{\mathbf{T}_3}{\mathrm{Tr}(\mathbf{T}_3)}$$

$$H \approx 2.52 + 0.78\, log_3\left(\left|\mathbf{N}_3 + 0.16\mathbf{I}_{D3}\right|\right)$$

ENTROPY

SHANNON POLARIMETRIC ENTROPY (2006)

$$SE = log\left(\pi^3 e^3 \left|\mathbf{T}_3\right|\right) = SE_I + SE_P$$

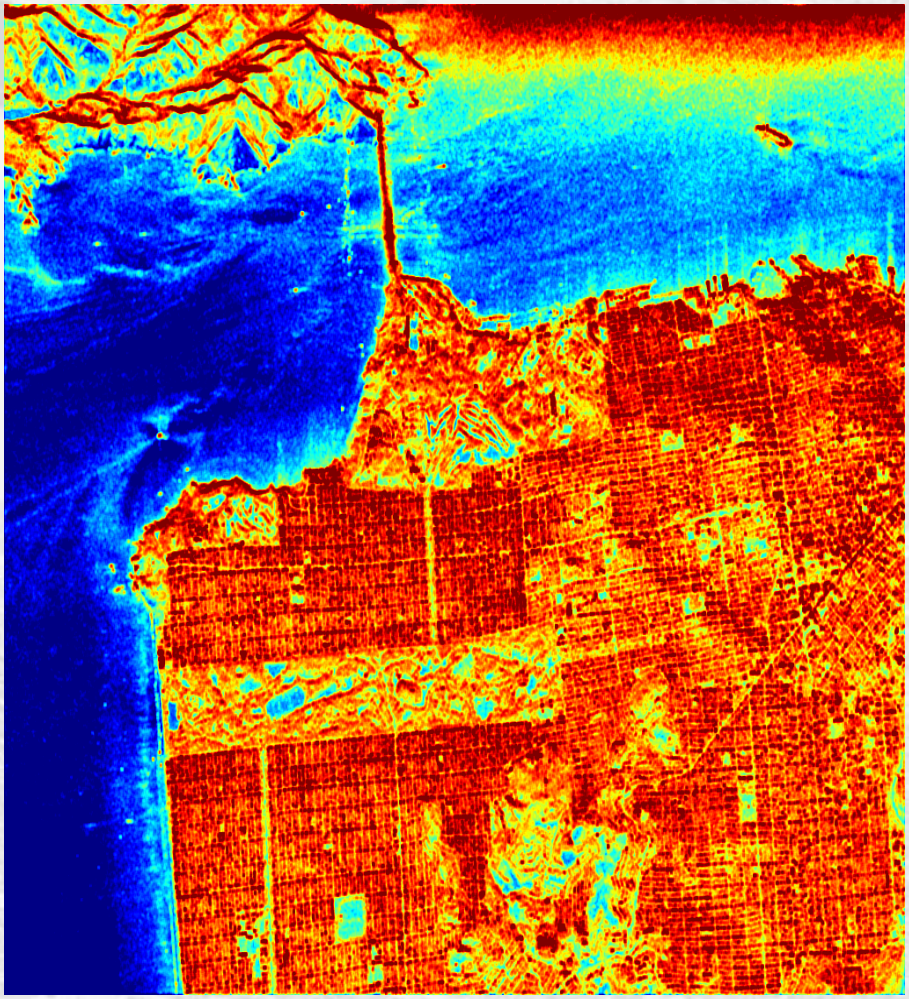
$$SE_I = 3\, log\left(\frac{\pi\, e\, I_T}{3}\right) = 3\, log\left(\frac{\pi\, e\, \mathrm{Tr}(\mathbf{T}_3)}{3}\right)$$

INTENSITY

$$SE_P = log\left(1 - p_T^2\right) = log\left(27 \frac{\left|\mathbf{T}_3\right|}{\mathrm{Tr}(\mathbf{T}_3)^3}\right)$$

DEGREE OF POLARIZATION

EIGENVALUE-BASED PARAMETERS

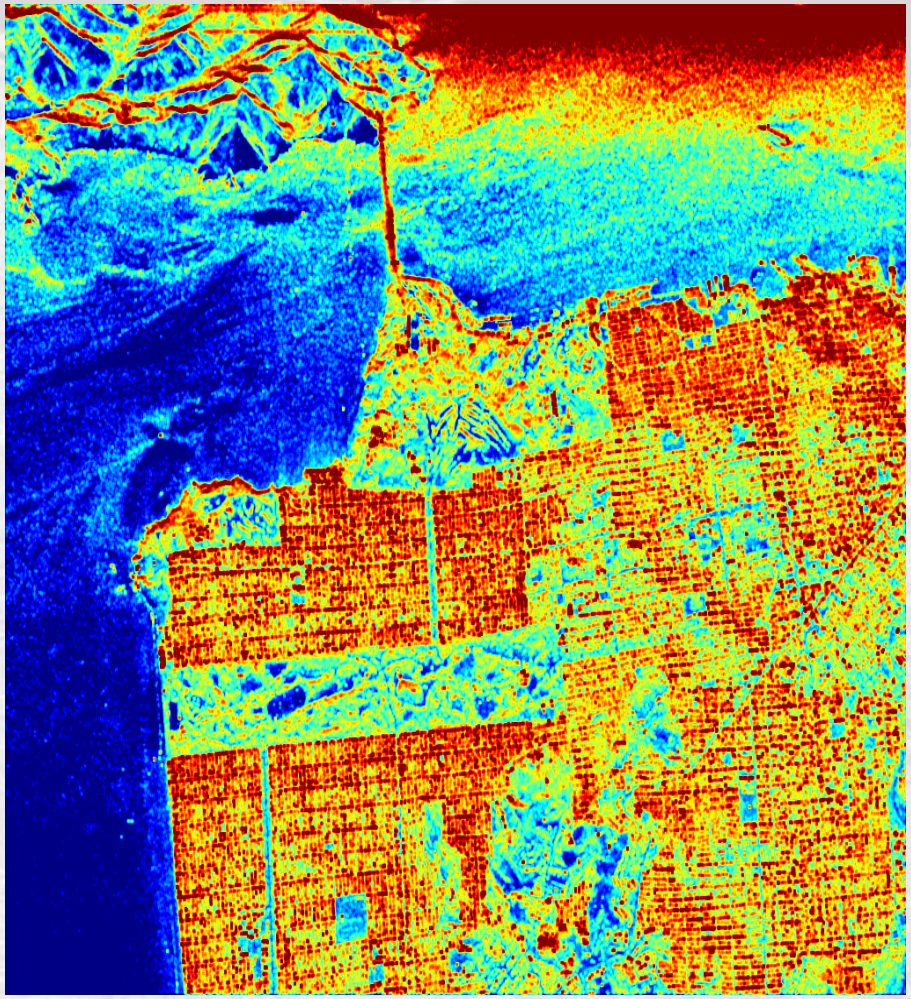


$2A_0$ $B_0 + B$ $B_0 - B$

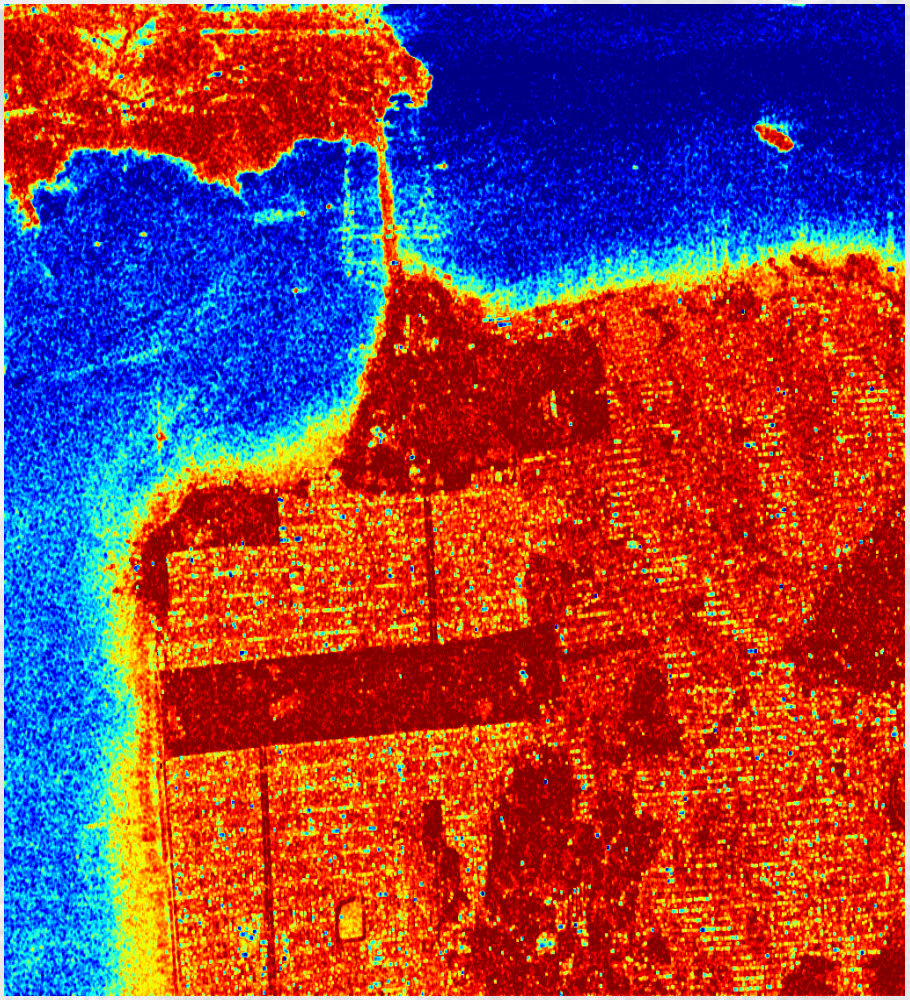
-13.0 1.0

SHANNON ENTROPY (SE-norm)

EIGENVALUE-BASED PARAMETERS



SHANNON ENTROPY (SE-I)



SHANNON ENTROPY (SE-P)

TARGET SCATTERING VECTOR

MODEL DECOMPOSITION

R. TOUZI (2010)

T.S.V.M DECOMPOSITION

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES
$\lambda_1 > \lambda_2 > \lambda_3$

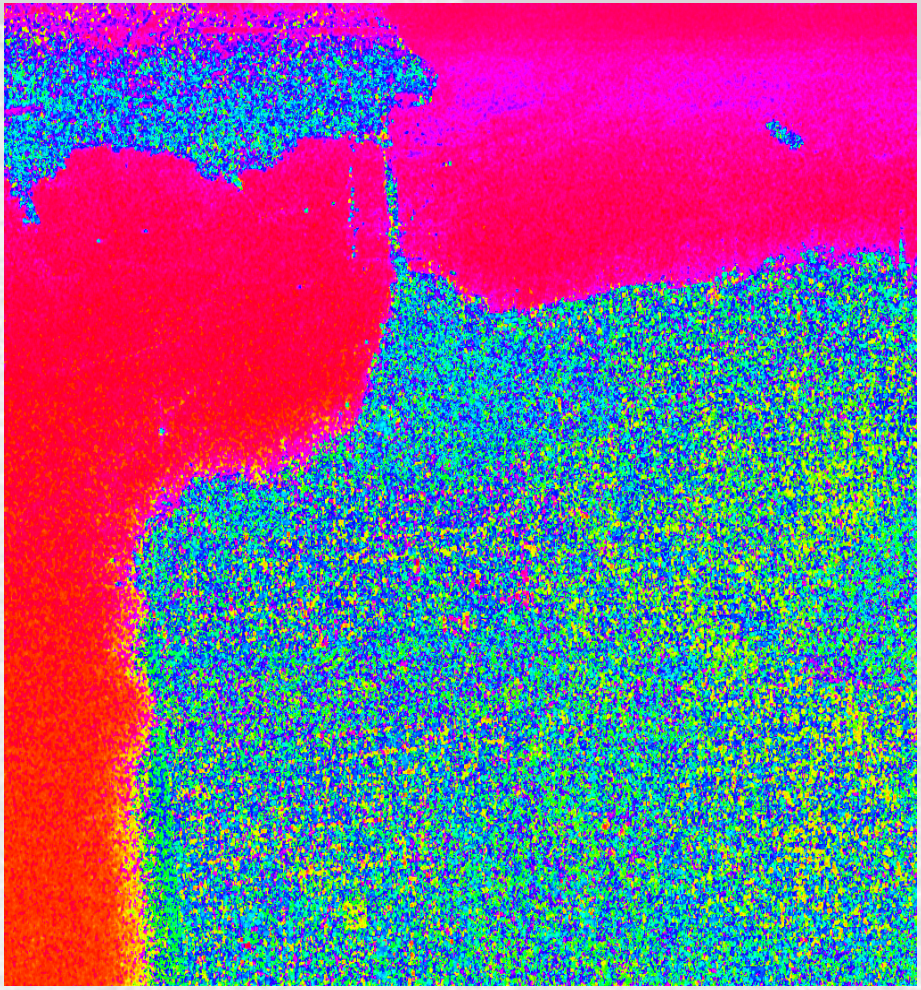
PARAMETERISATION OF THE EIGENVECTOR

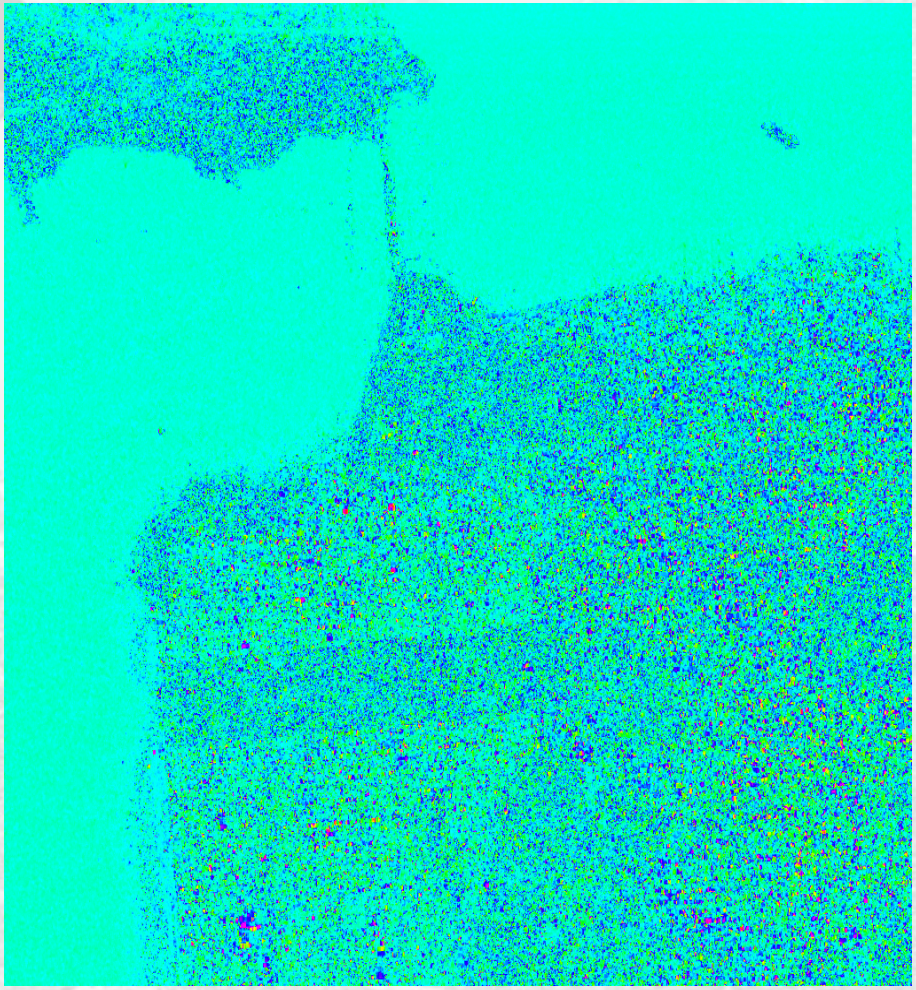
$$\begin{bmatrix} \cos\alpha\, e^{j\phi} \\ \sin\alpha \cos\beta\, e^{j\phi} e^{j\delta} \\ \sin\alpha \sin\beta\, e^{j\phi} e^{j\gamma} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\psi & -\sin 2\psi \\ 0 & \sin 2\psi & \cos 2\psi \end{bmatrix} \begin{bmatrix} \cos\alpha_s \cos 2\tau_m \\ \sin\alpha_s\, e^{j\phi_{\alpha_s}} \\ -j\cos\alpha_s \sin 2\tau_m \end{bmatrix}$$

ψ : Target Orientation τ_m : Target Helicity

α_s , ϕ_{α_s} : Symmetric scattering type vector parameters

T.S.V.M DECOMPOSITION





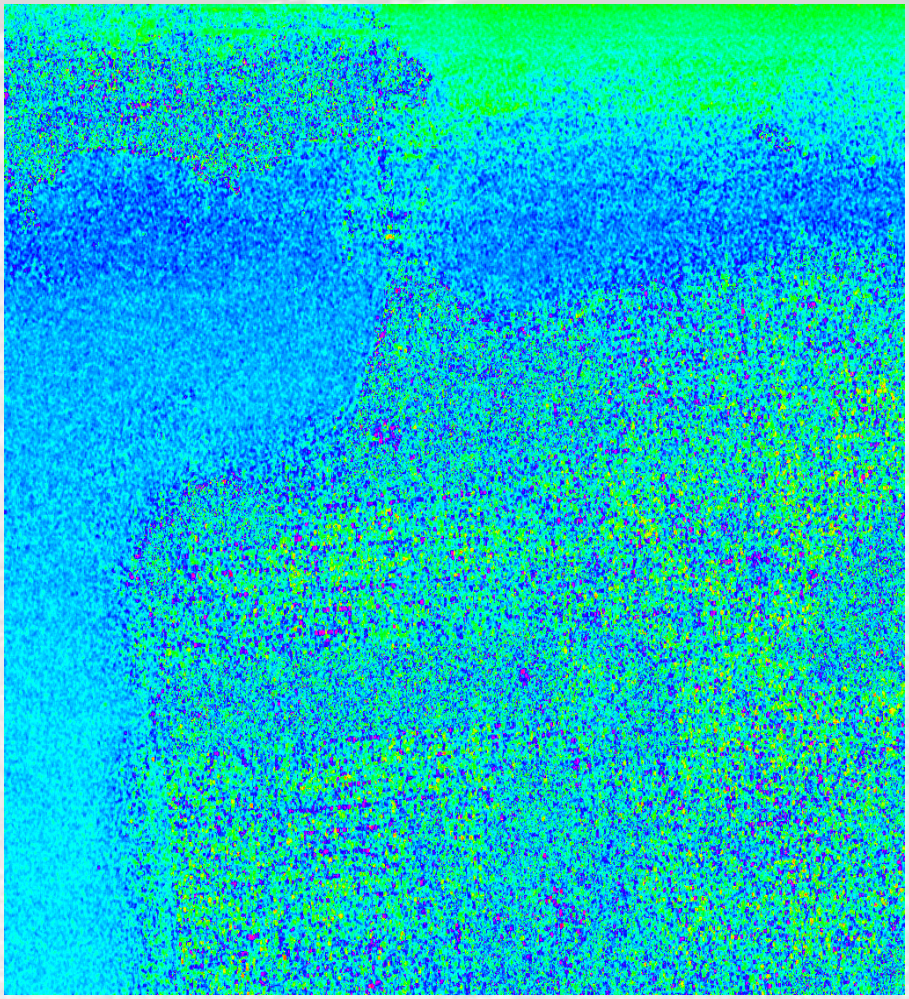
-90 0 +90

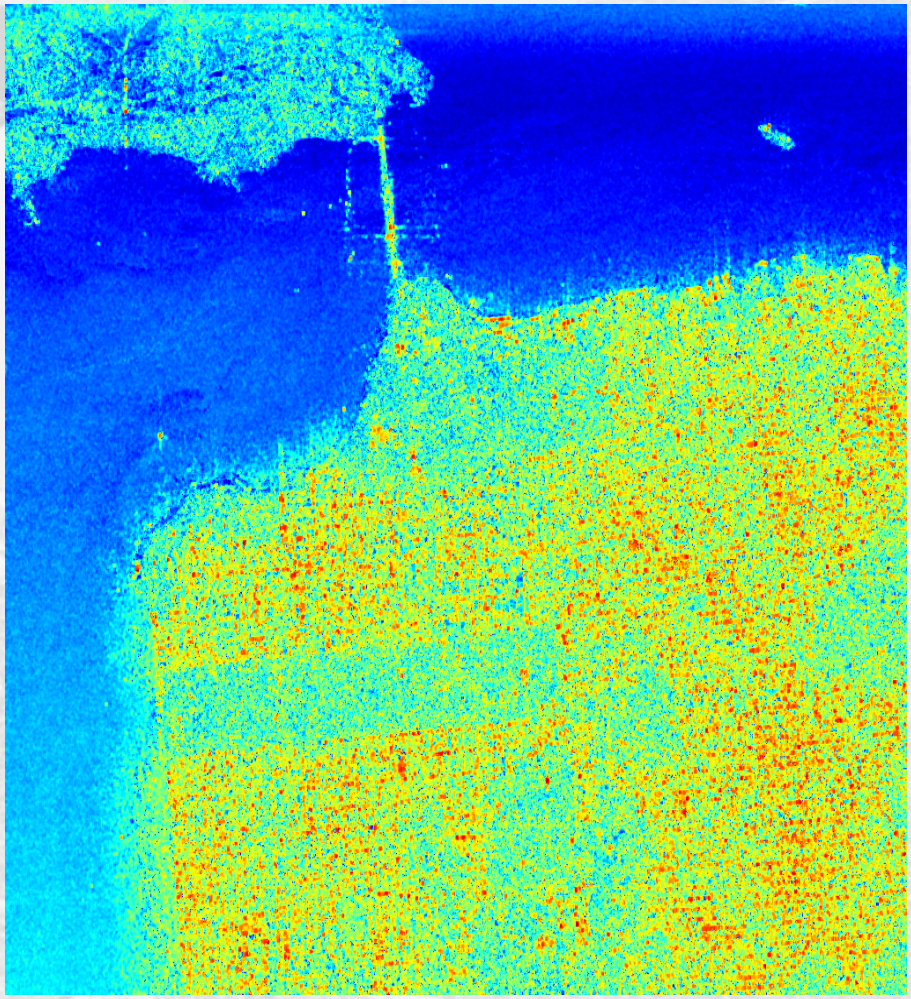
Target Orientation (ψ)

-45 0 +45

Target Helicity (τ_m)

T.S.V.M DECOMPOSITION





-90 0 +90

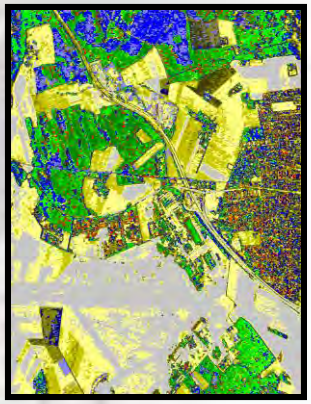
(ϕ_{α_s})

0 45 90

(α_s)

POLARIMETRIC REMOTE SENSING

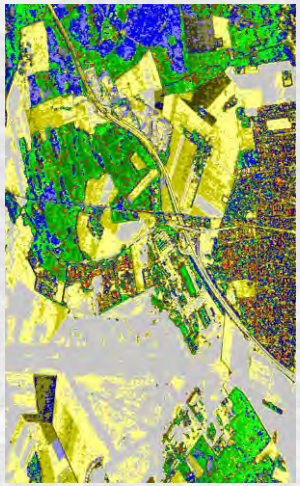
POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS

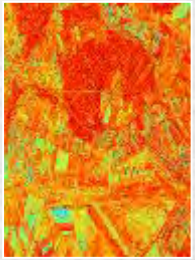


POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

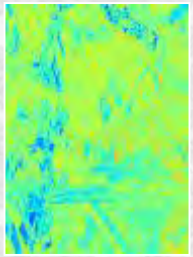
UNSUPERVISED POLSAR CLASSIFICATION

S.R. CLOUDE, E.POTTIER (1996)





H



α

H / α DECOMPOSITION THEOREM

H / α CLASSIFICATION

ENTROPY

$$H = -\sum_{i=1}^{3} P_i \log_3(P_i)$$

α PARAMETER

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3$$

ANISOTROPY

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$

3 ROLL INVARIANT PARAMETERS

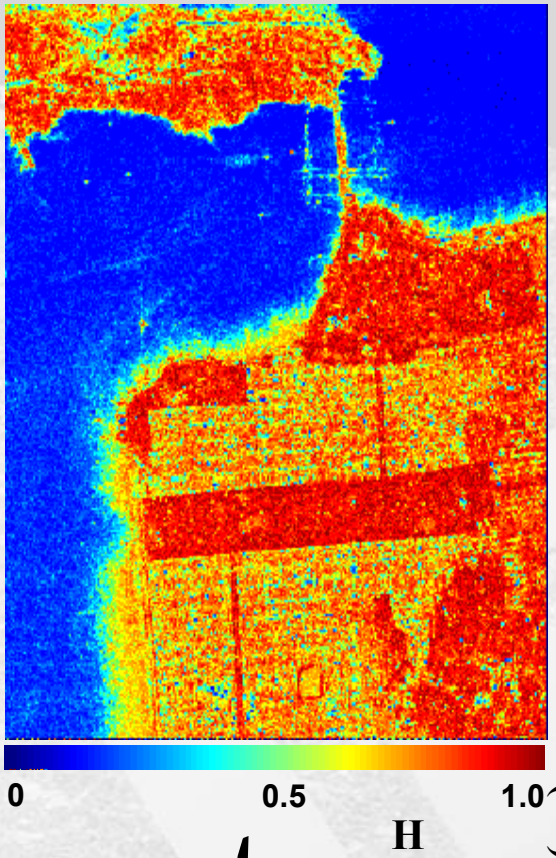
$$\underline{I} = \begin{bmatrix} \underline{\alpha} \\ HA \\ H(1-A) \\ (1-H)A \\ (1-H)(1-A) \end{bmatrix}$$

→ **PHYSICAL SCATTERING MECHANISM**

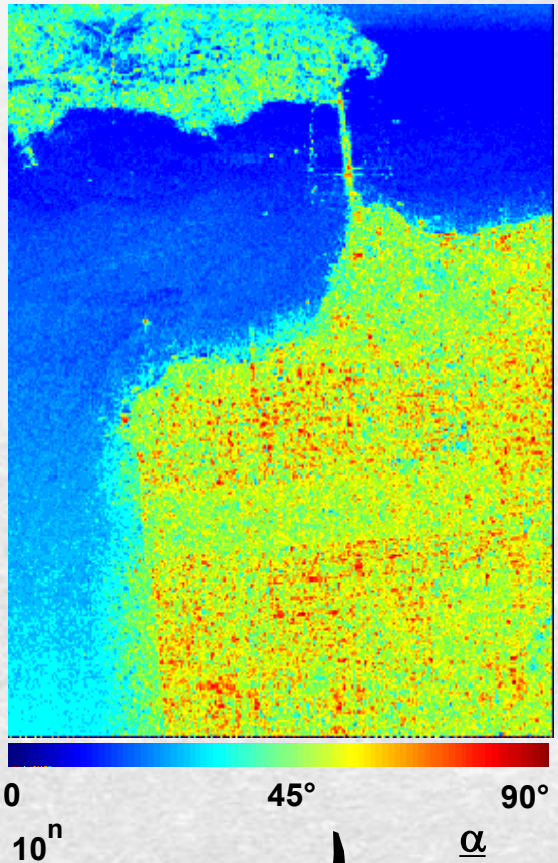
→ **TYPE OF SCATTERING PROCESS**

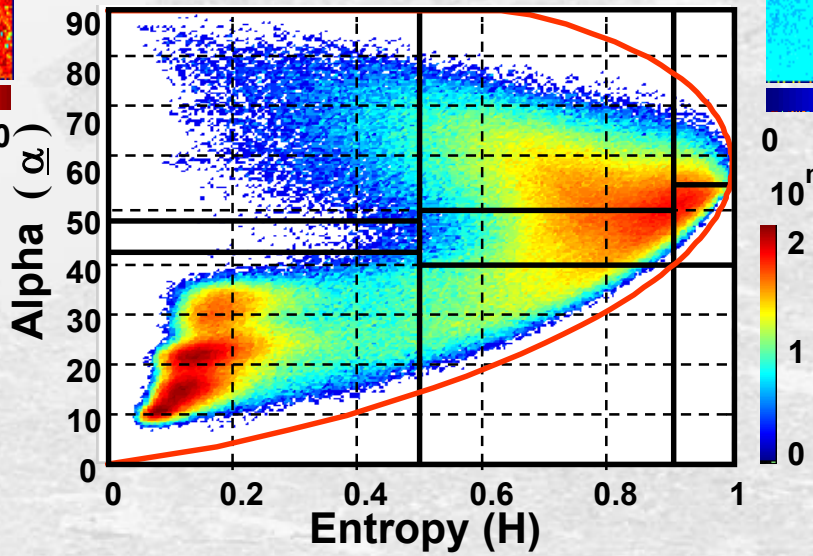
SEGMENTATION / CLASSIFICATION

H / α CLASSIFICATION



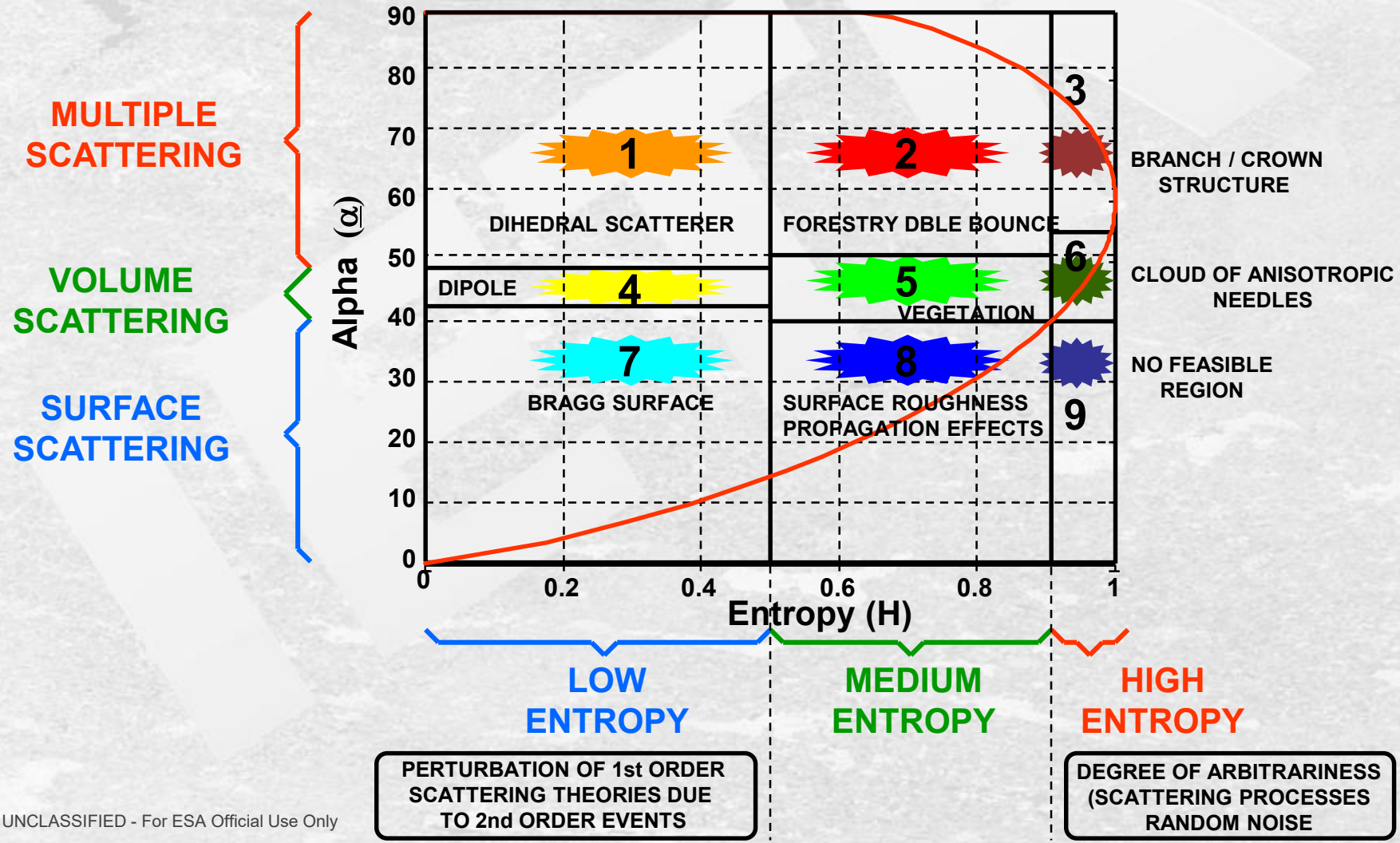
POLSAR DATA DISTRIBUTION IN THE H / α PLANE





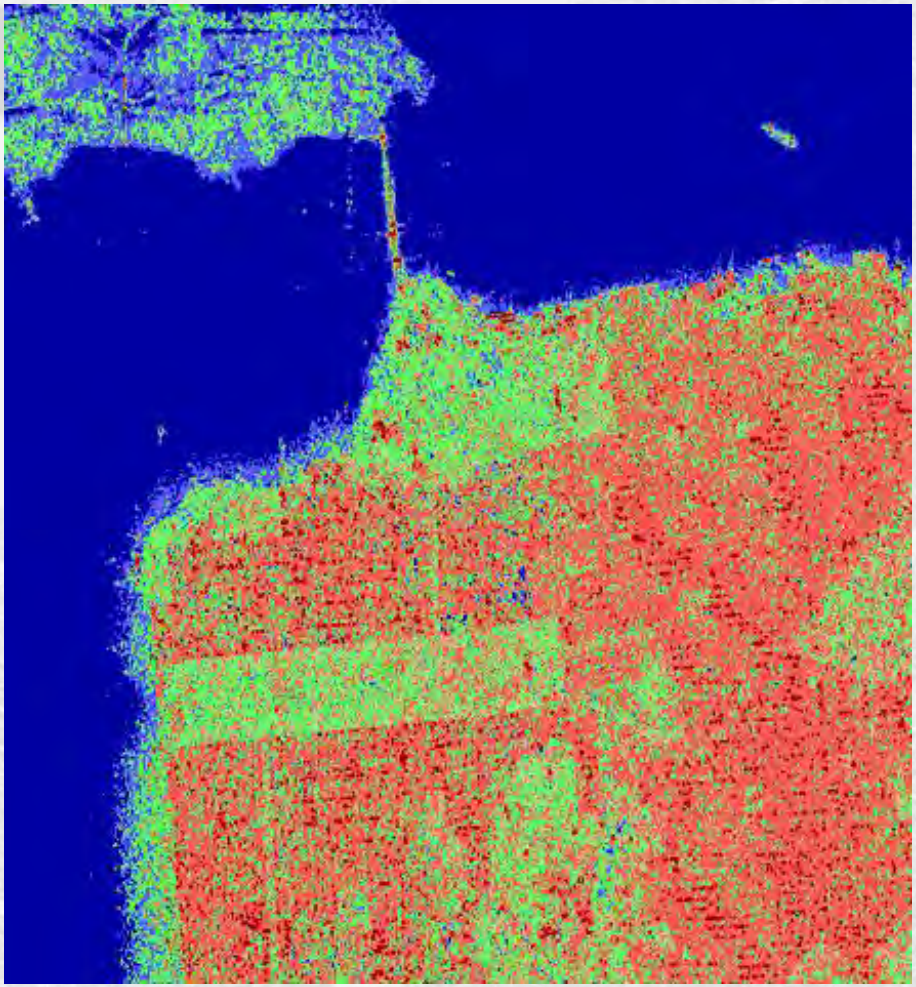
H / α CLASSIFICATION

SEGMENTATION OF THE H / α SPACE

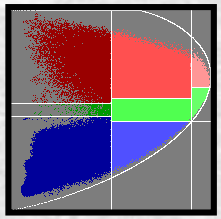


H / α CLASSIFICATION

H - α classification

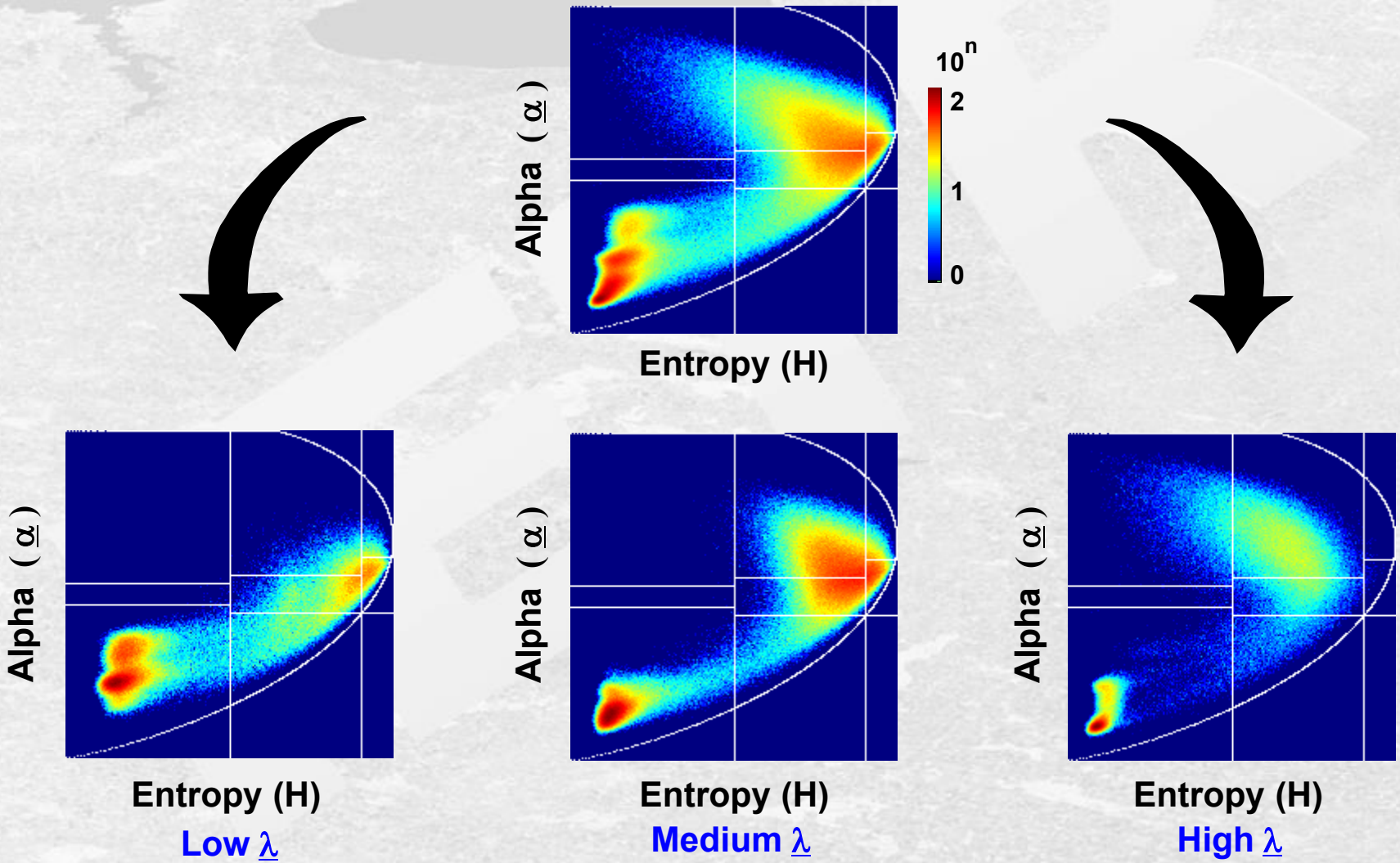


$2A_0$ $B_0 + B$ $B_0 - B$



H / α / span CLASSIFICATION

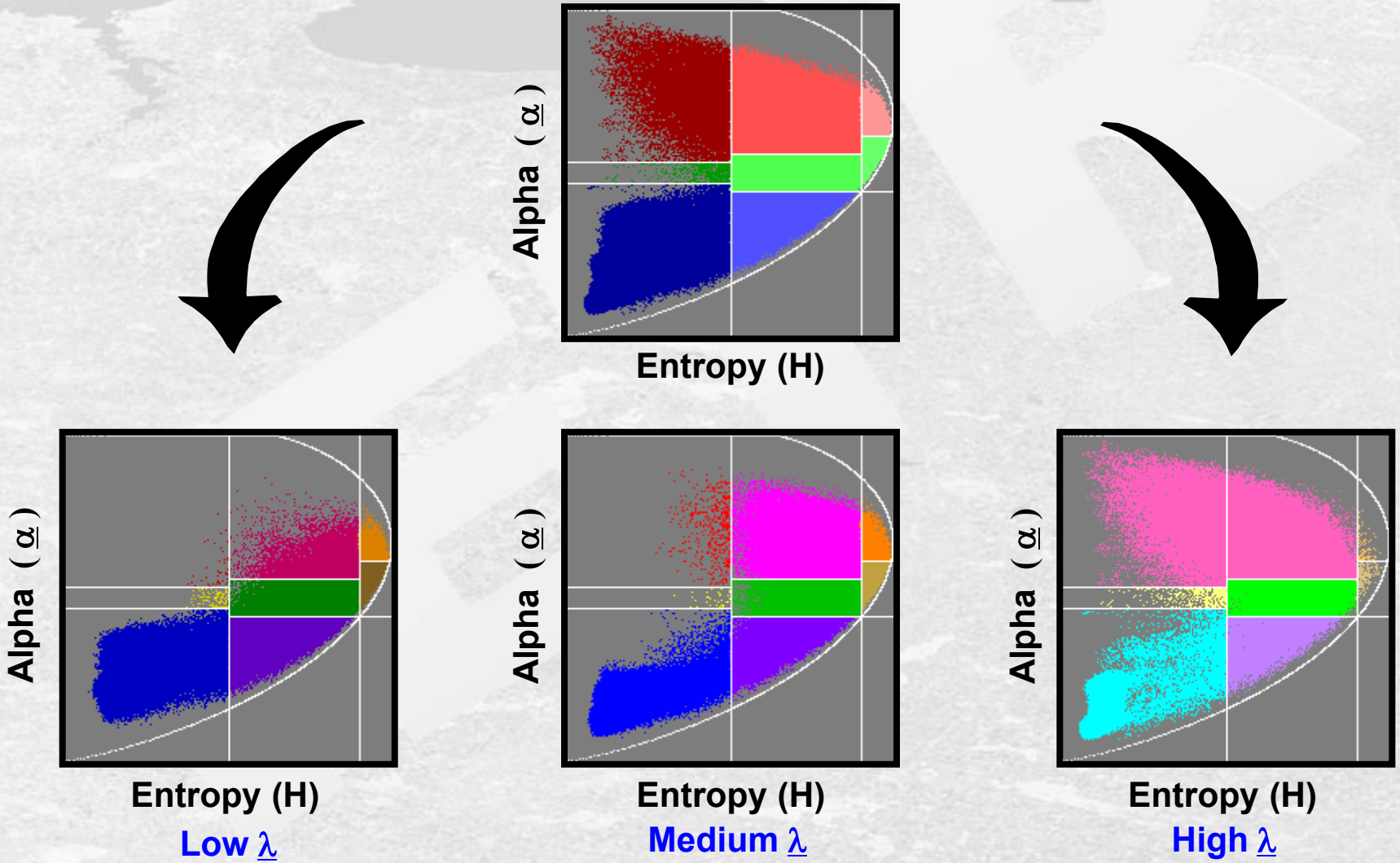
POLSAR DATA DISTRIBUTION IN THE H / α PLANE



Cao Fang, Hong Wen *A New Classification Method Based on Cloude-Pottier Eigenvalue / Eigenvector Decomposition,* ICARSS 05, Seoul, Korea

H / α / span CLASSIFICATION

POLSAR DATA DISTRIBUTION IN THE H / α PLANE



Cao Fang, Hong Wen *A New Classification Method Based on Cloude-Pottier Eigenvalue / Eigenvector Decomposition,* IGARSS 05, Seoul, Korea

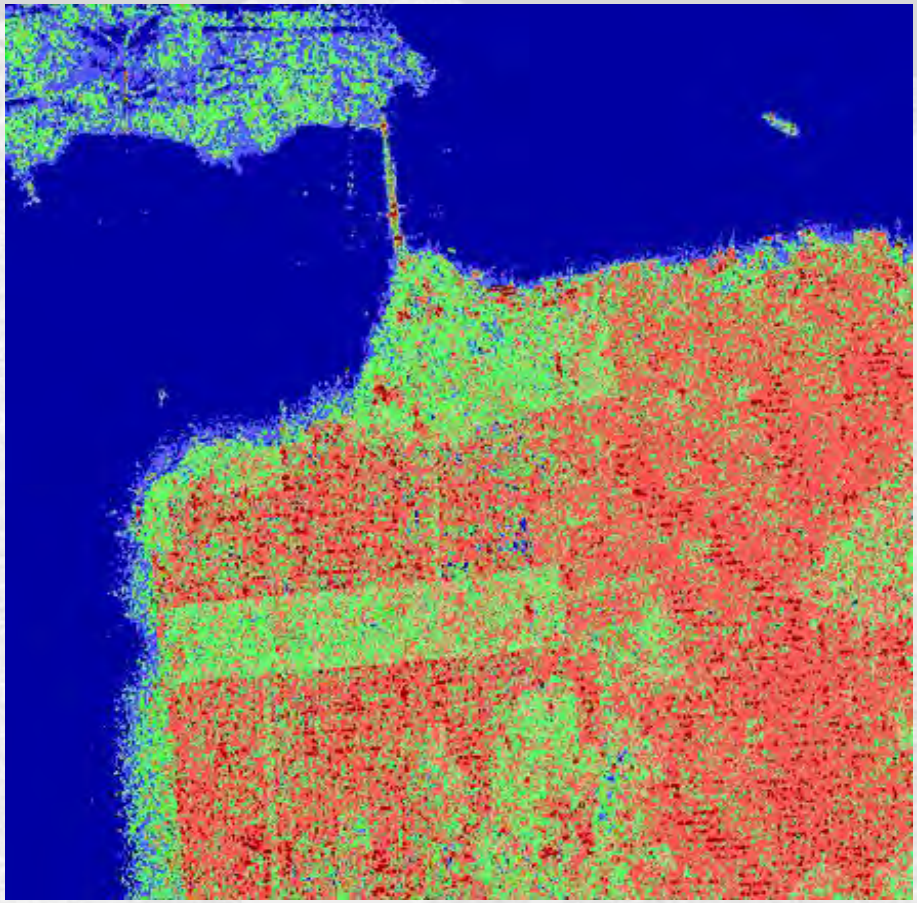
H / α / span CLASSIFICATION

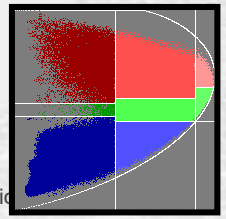
H – α (λ) classification

$2A_0$ $B_0 + B$ $B_0 - B$

H / α CLASSIFICATION

H-α classification





H / α Classification Space
Sub-divised into 9 basic zones

↓

Location of the boundaries is arbitrary and generically

Degree of arbitrariness on the setting of these boundaries

↓

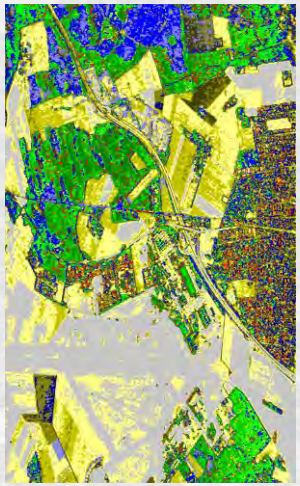
Segmentation is offered merely to illustrate the unsupervised classification strategy and to emphasize the geometrical segmentation of physical scattering processes

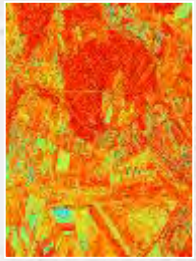
WISHART PDF

$$P(\langle[T]\rangle/[T_m]) = \frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p} e^{-LTr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)[T_m]^L}$$

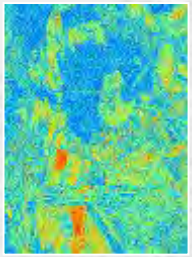
UNSUPERVISED POLSAR CLASSIFICATION

E.POTTIER, J.S LEE (2000)

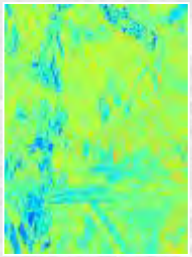




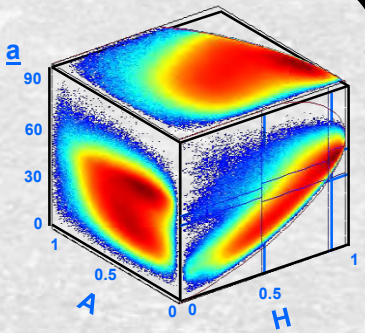
H



A



α



H / A / α DECOMPOSITION THEOREM

PolSAR TERRAIN and LAND-USE CLASSIFICATION

J.S. Lee, M.R. Grunes, E. Pottier, L. Ferro-Famil, "Unsupervised terrain classification preserving scattering characteristics," IEEE Transactions on Geoscience and Remote Sensing,vol. 42, no.4, pp. 722-731, April, 2004.

J.S. Lee, M. R. Grunes and E. Pottier, "Quantitative Comparison of Classification Capability: Fully polarimetric versus Dual- and Single polarization SAR," IEEE TGRS, November 2002

E. Pottier and J.S. Lee, "Application of the « H / A / α » polarimetric decomposition theorem for unsupervised classification of fully polarimetric SAR data based on the Wishart distribution" Proceedings of EUSAR2000

J.S. Lee, M.R. Grunes, T.L. Ainsworth, L. Du, D.L. Schuler, and S.R. Cloude, " Unsupervised Classification of Polarimetric SAR Imagery Based on Target Decomposition and Wishart Distribution," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 37, no. 5, 2249-2258, September 1999.

J.S. Lee, M. R. Grunes and R. Kwok," Classification of Polarimetric SAR Images Based on the Complex Wishart Distribution," ***Int. J. Remote Sensing, vol.32, No. 5, Sept. 1994.***

J.S. Lee, E. Pottier, *Polarimetric Radar Imaging: From Basics to Applications,* Taylor & Francis/CRC, 2009

WISHART CLASSIFIER

Target Vector

$$\underline{X} = \begin{bmatrix} S_{HH} & \sqrt{2}S_{HV} & S_{VV} \end{bmatrix}^T \qquad P(\underline{X}) = \frac{1}{\pi^3 |[C]|} e^{-\underline{X}^{*T}[C]^{-1}\underline{X}}$$

$$\underline{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH} + S_{VV} & S_{HH} - S_{VV} & 2S_{HV} \end{bmatrix}^T \qquad P(\underline{k}) = \frac{1}{\pi^3 |[T]|} e^{-\underline{k}^{*T}[T]^{-1}\underline{k}}$$

Coherency Matrix

$$\langle [T] \rangle = \frac{1}{N} \sum_{i=1}^{N} \underline{k}_i \cdot \underline{k}_i^{*T} = \frac{1}{N} \sum_{i=1}^{N} [T_i]$$

$$P(\langle [T] \rangle / [T_m]) = \frac{L^{Lp} \left| \langle [T] \rangle \right|^{L-p} e^{-LTr\left([T_m]^{-1} \langle [T] \rangle\right)}}{\pi^{\frac{p(p-1)}{2}} \Gamma(L) \ldots \Gamma(L-p+1) |[T_m]|^L}$$

COMPLEX WISHART DISTRIBUTION

L: Number of Look p: Polarimetric Dimension

WISHART CLASSIFIER

$$P(\langle[T]\rangle/[T_m]) = \frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p} e^{-LTr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)[T_m]^L}$$

BAYES MAXIMUM LIKELIHOOD CLASSIFICATION PROCEDURE

$$\langle[T]\rangle \in [T_m] \quad if \quad P([T_m]/\langle[T]\rangle) \geq P([T_j]/\langle[T]\rangle) \quad \forall\ j \neq m$$

Applying Bayes rule $P([T_m]/\langle[T]\rangle) = \dfrac{P(\langle[T]\rangle/[T_m])}{P(\langle[T]\rangle)} P([T_m])$

It follows

$$\langle[T]\rangle \in [T_m] \quad if \quad P(\langle[T]\rangle/[T_m])P([T_m]) \geq P(\langle[T]\rangle/[T_j])P([T_j]) \quad \forall\ j \neq m$$

$[T_m]$: Cluster Center of the class m

WISHART CLASSIFIER

$$P(\langle[T]\rangle/[T_m]) = \frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p} e^{-LTr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)[T_m]^L}$$

BAYES MAXIMUM LIKELIHOOD CLASSIFICATION PROCEDURE

$$\langle[T]\rangle \in [T_m] \quad if \quad d_m(\langle[T]\rangle) < d_j(\langle[T]\rangle) \quad \forall\ j \neq m$$

with

$$d_m(\langle[T]\rangle) = LTr\left([T_m]^{-1}\langle[T]\rangle\right) + L\ln([T_m]) - \ln(P([T_m])) + K$$

$[T_m]$: Cluster Center of the class m

ROBUSTENESS OF WISHART CLASSIFIER

$$d_m(\langle[T]\rangle) = LTr\left([T_m]^{-1}\langle[T]\rangle\right) + L\ln([T_m]) - \ln(P([T_m])) + K$$

INDEPENDENT OF # OF LOOKS

INDEPENDENT OF POLARIZATION BASIS

[T] or [C] IDENTICAL CLASSIFICATION RESULTS

For Dual-Pol ($p=2$), PolSAR ($p=3$), Pol-InSAR ($p=6$)

J.S. Lee, E. Pottier, *Polarimetric Radar Imaging: From Basics to Applications,* Taylor & Francis/CRC, 2009

H / α - WISHART CLASSIFIER

k - mean CLASSIFICATION PROCEDURE

PROVIDE INITIAL $[T_m]^{(0)}$ FOR EACH CLASS

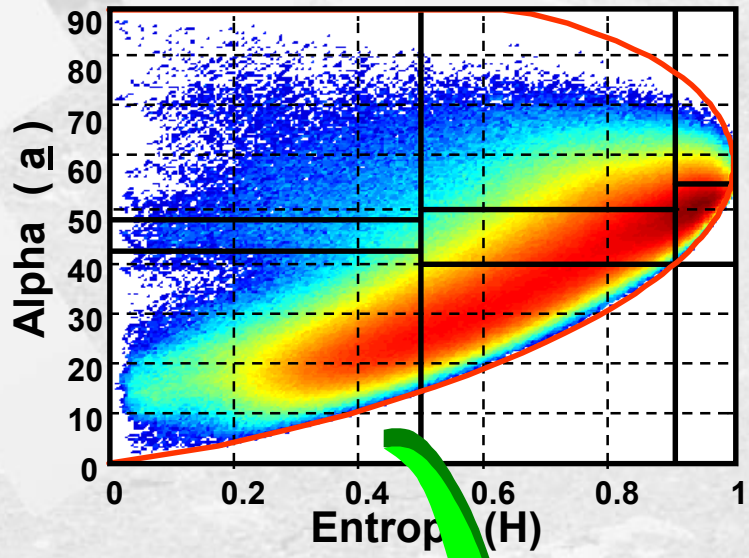
CLASSIFY THE WHOLE IMAGE WITH THE DISTANCE PROCEDURE

$$\langle [T] \rangle \in [T_m] \quad if \quad d_m(\langle [T] \rangle) < d_j(\langle [T] \rangle) \quad \forall\ j \neq m$$

COMPUTE $[T_m]^{(k+1)}$ FOR EACH CLASS USING THE CLASSIFIED PIXELS OF STEP 2

$$[T_m]^{(k+1)} = \frac{1}{N_m} \sum_{j=1}^{j=N_m} \langle [T] \rangle_j$$

TERMINAISON CRITERION ?



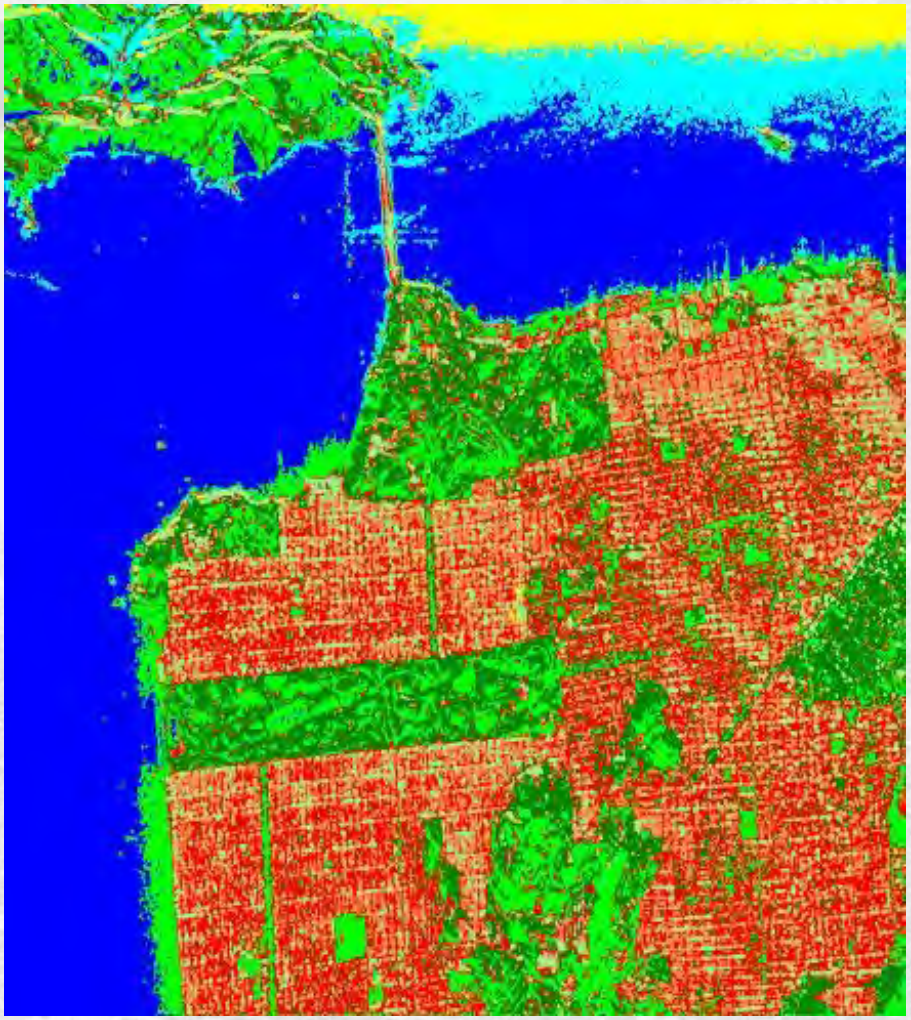
$$[T_m]^{(0)} = \frac{1}{N_m} \sum_{k=1}^{k=N_m} \langle [T] \rangle_k$$

Cluster Center of the class *m* (Lee 1998)

H / α - WISHART CLASSIFIER

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

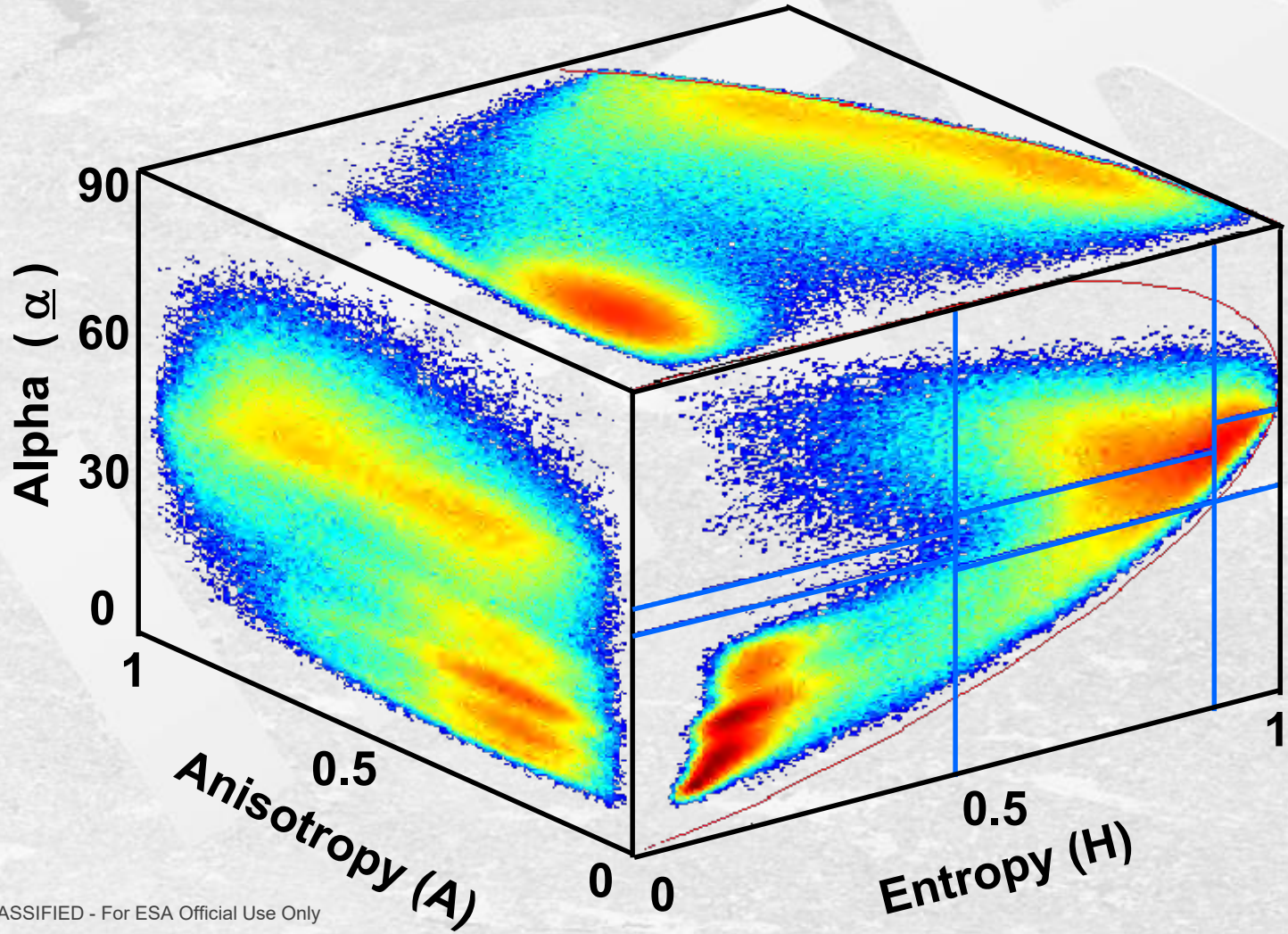
4th ITERATION



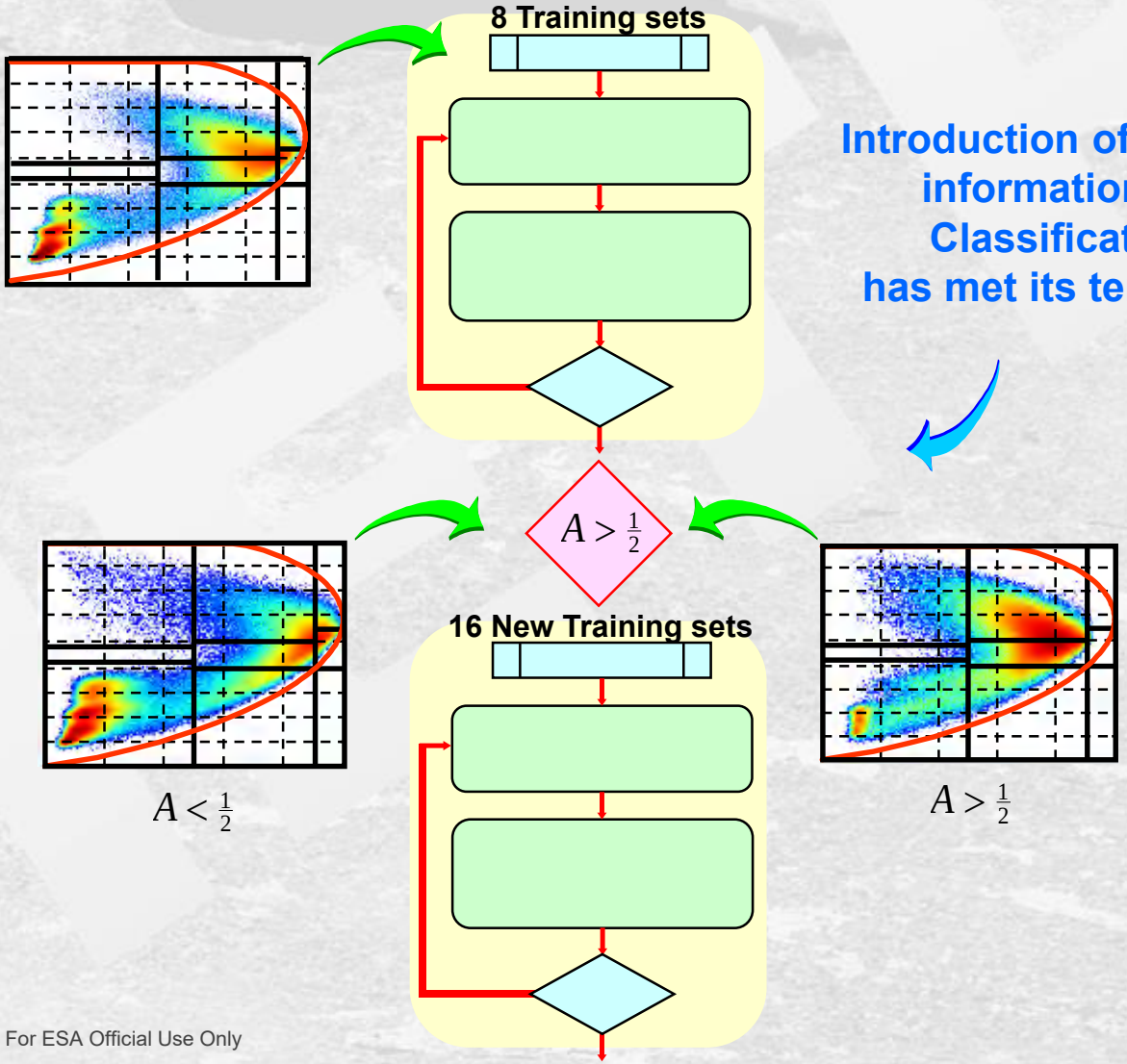
$2A_0$ $B_0 + B$ $B_0 - B$

C1 C2 C3 C4 C5 C6 C7 C8

POLSAR DATA DISTRIBUTION IN THE H / A / α SPACE



2 Successive k - mean Classification procedures

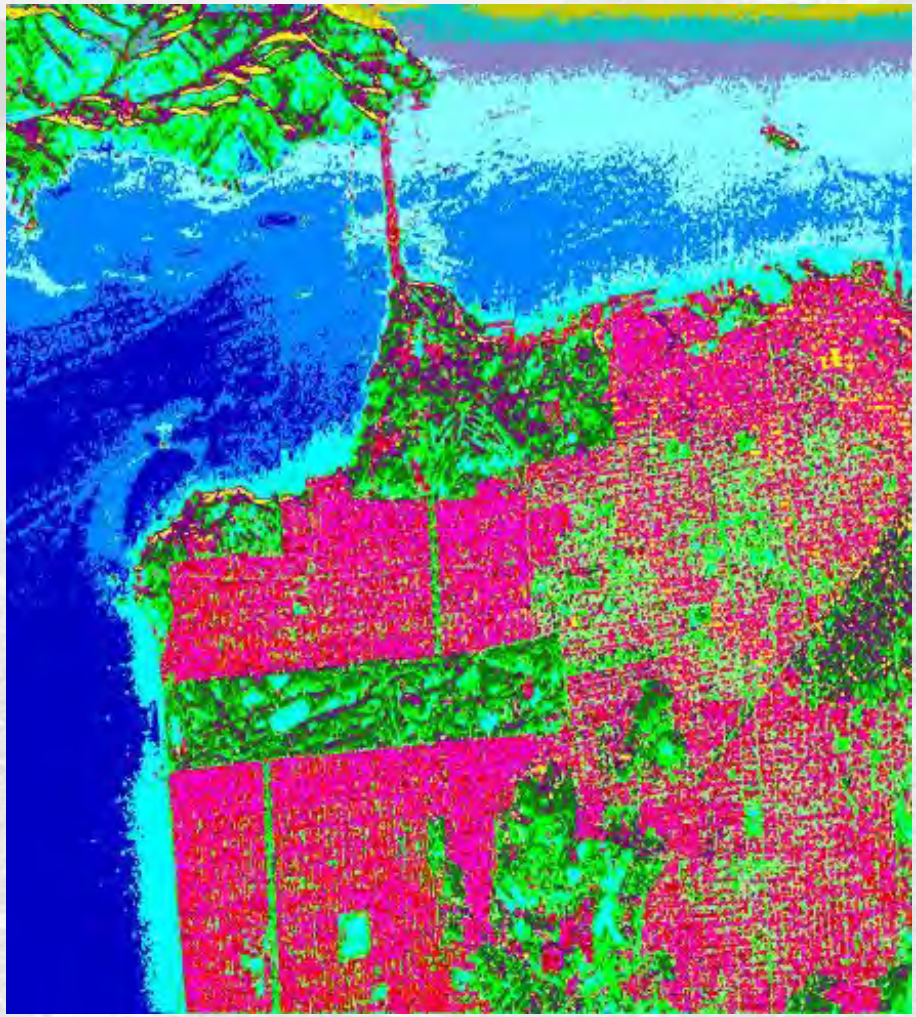


Introduction of the Anisotropy (A) information once the first Classification procedure has met its termination criterion

H / A / α - WISHART CLASSIFIER

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

4th ITERATION

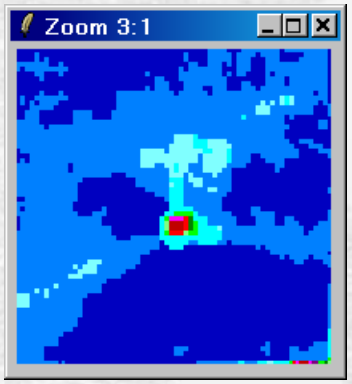


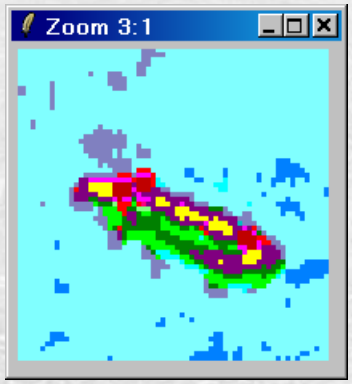
$2A_0$ $B_0 + B$ $B_0 - B$

C1	C2	C3	C4	C5	C6	C7	C8
C9	C10	C11	C12	C13	C14	C15	C16

H / A / α - WISHART CLASSIFIER

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

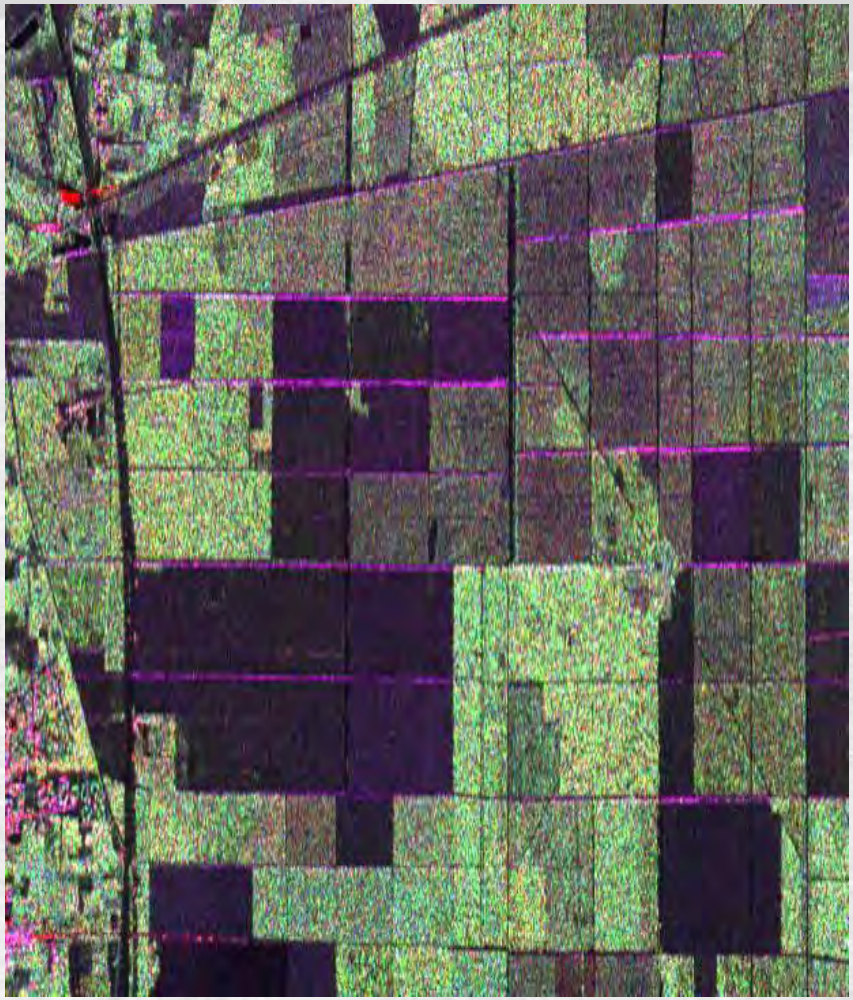


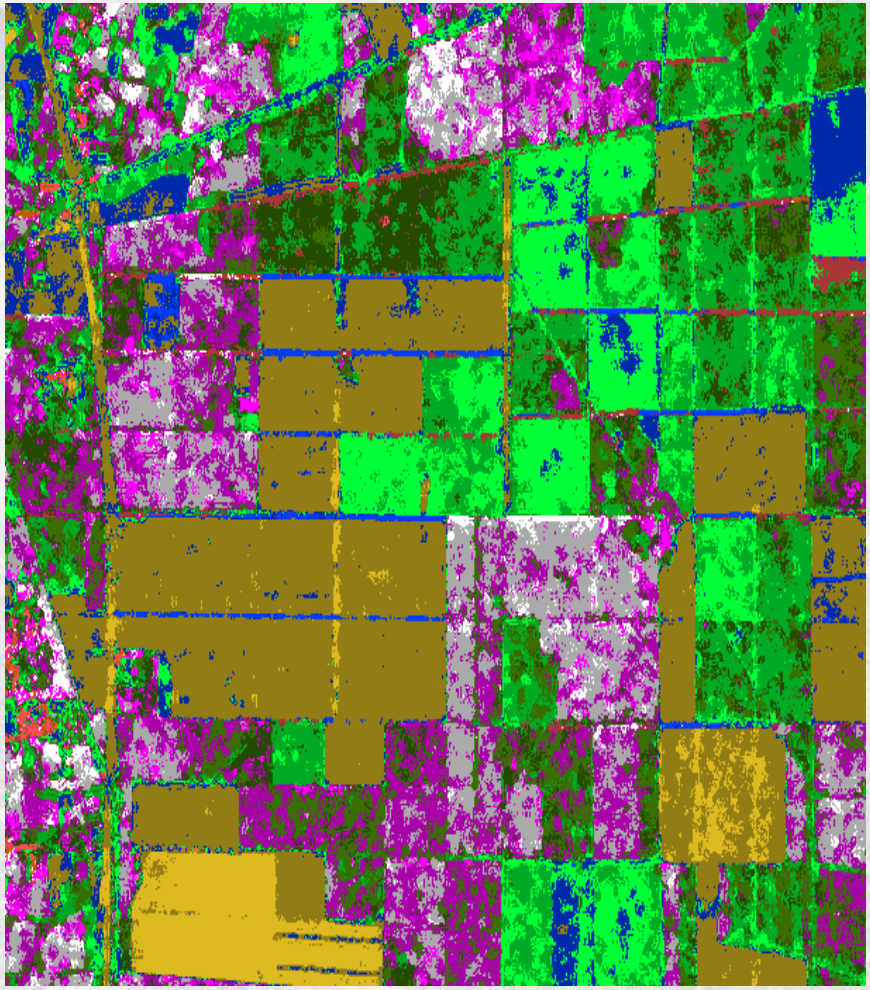


$2A_0$ $B_0 + B$ $B_0 - B$

H / A / α - WISHART CLASSIFIER

NEZER FOREST JPL - AIRSAR L-band



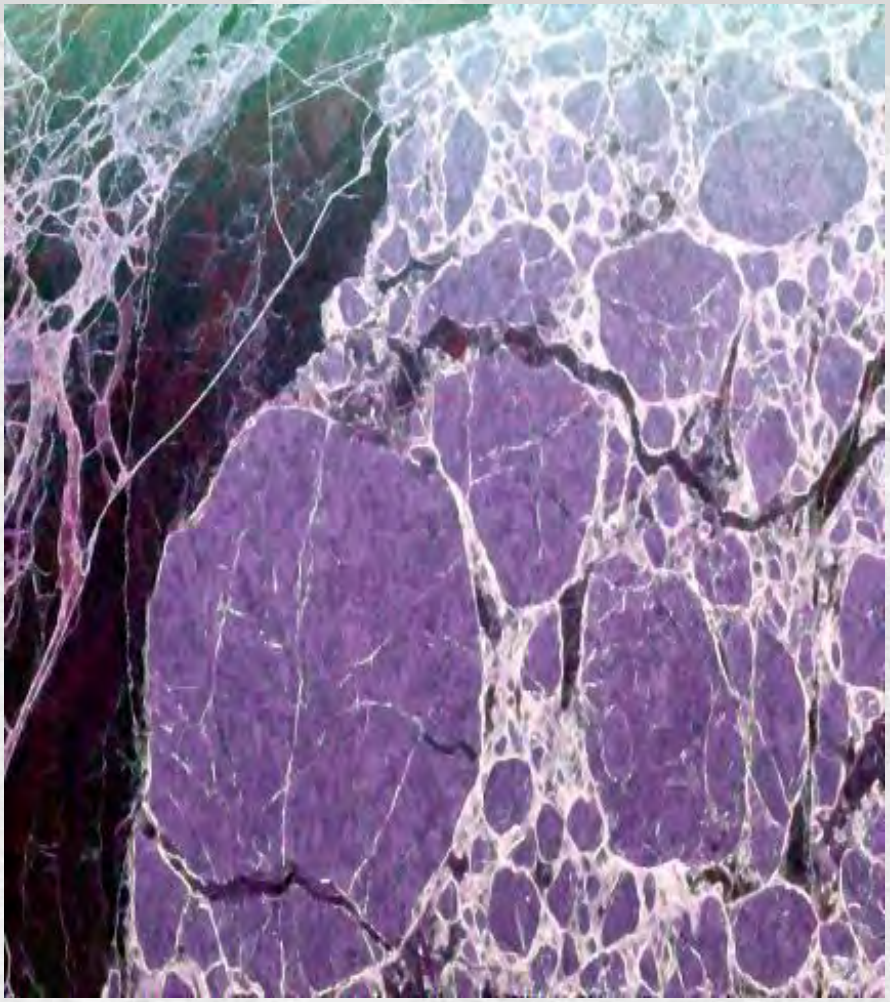


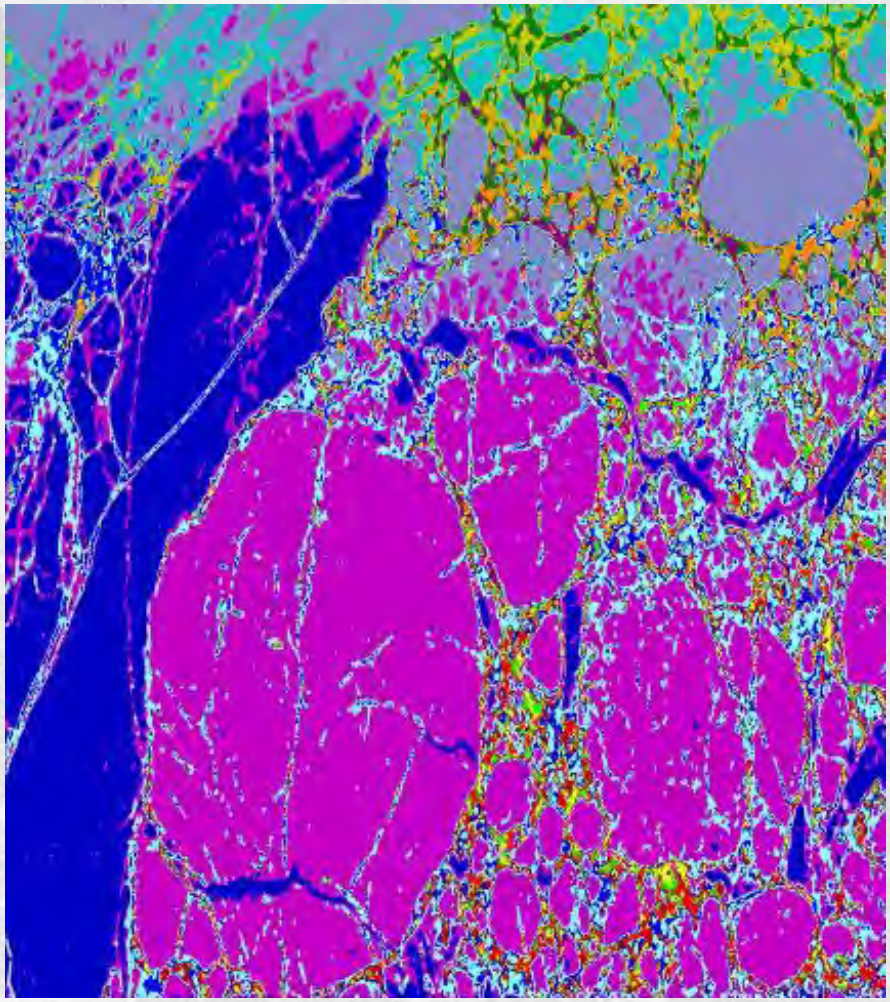
$2A_0$ $B_0 + B$ $B_0 - B$

C1	C2	C3	C4	C5	C6	C7	C8
C9	C10	C11	C12	C13	C14	C15	C16

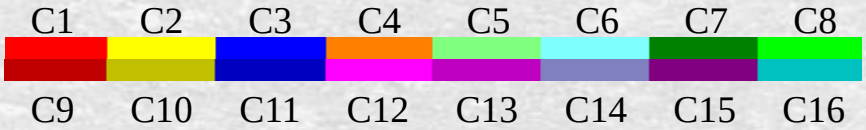
H / A / α - WISHART CLASSIFIER

ICE AREA JPL - AIRSAR L-band



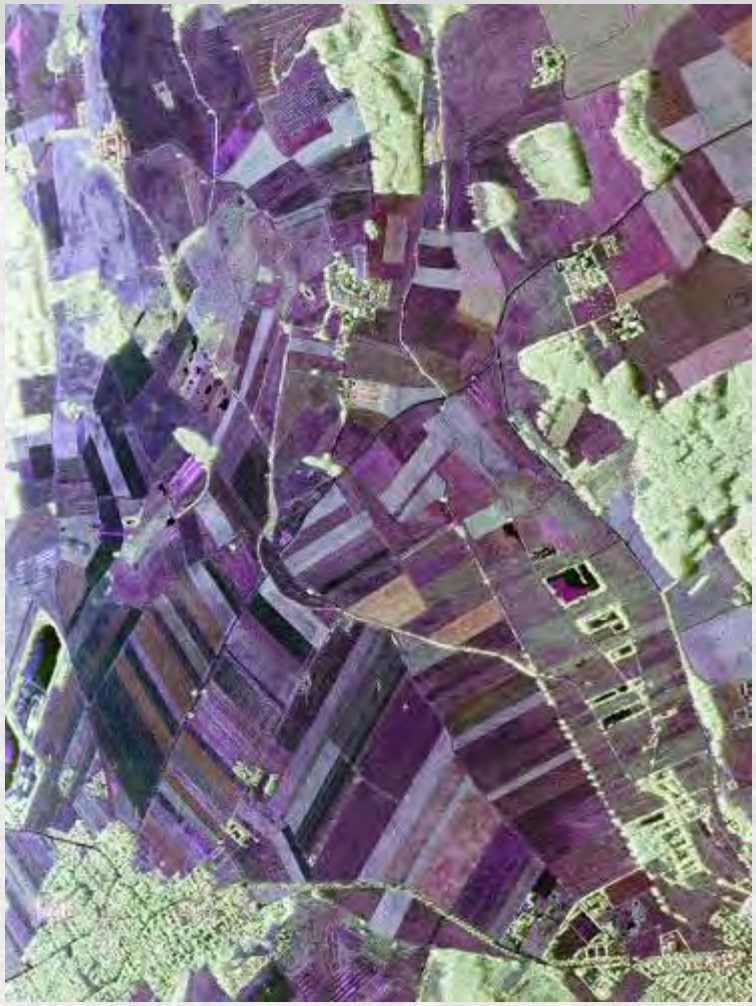


$2A_0$ $B_0 + B$ $B_0 - B$



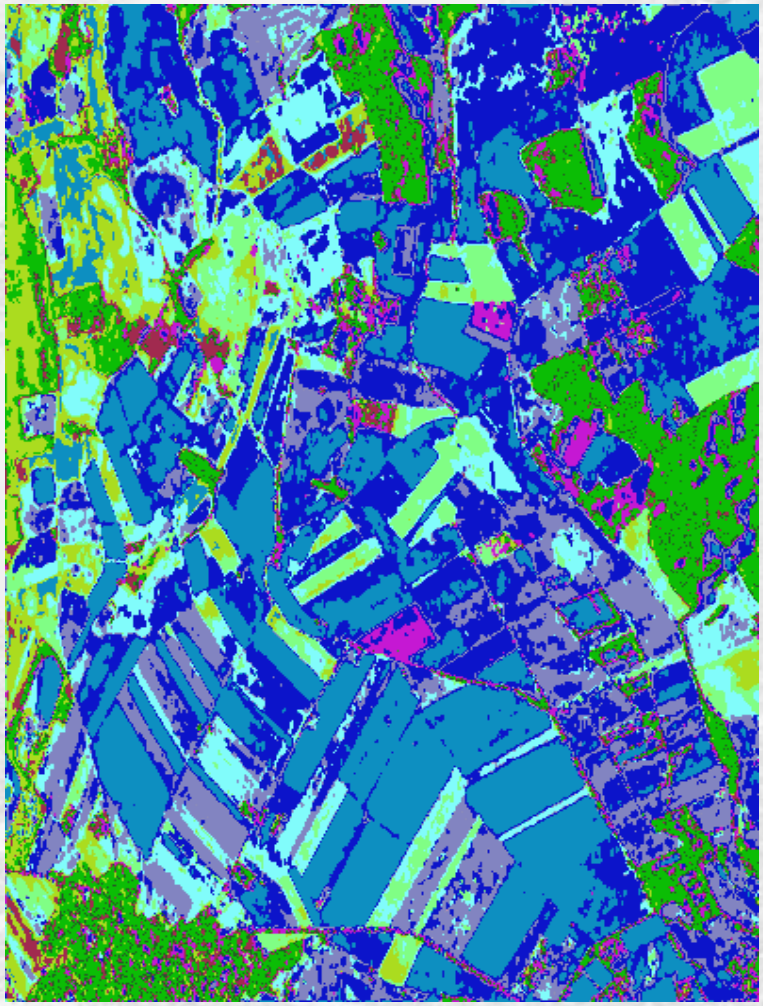
H / A / α - WISHART CLASSIFIER

ALLING - ESAR L-band



$2A_0$ $B_0 + B$ $B_0 - B$

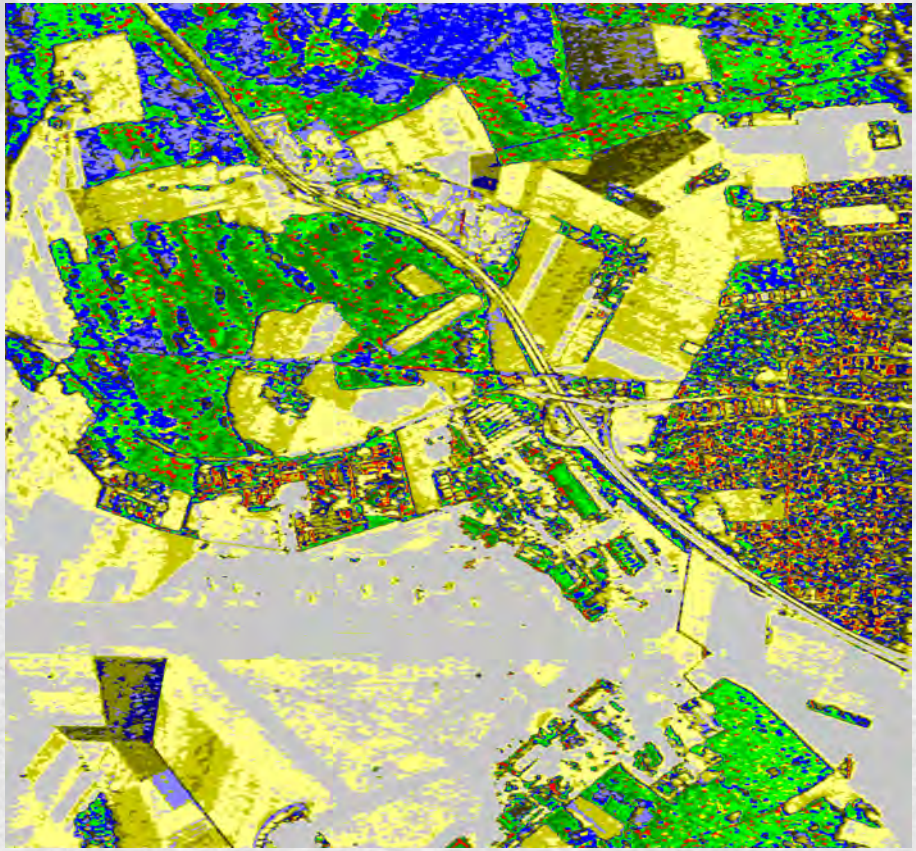
H / A / α and WISHART CLASSIFIER



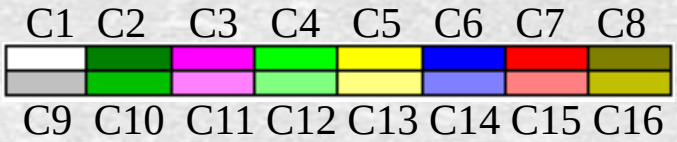
H / A / α - WISHART CLASSIFIER

OBERPFAFFENHOFEN - ESAR L-band

H / A / α and WISHART CLASSIFIER

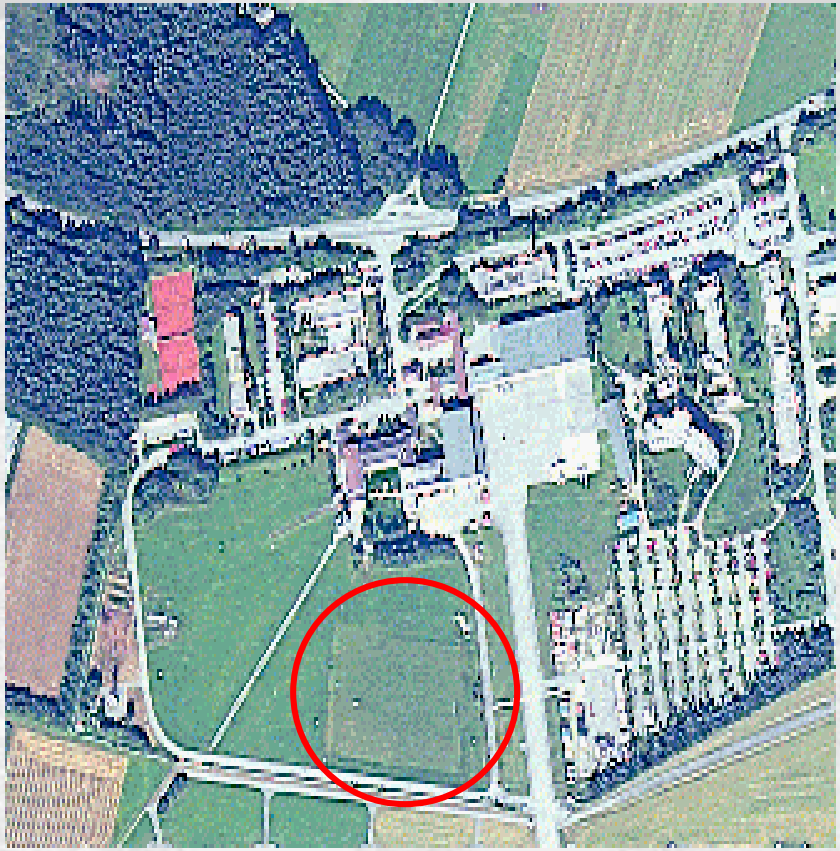


$2A_0$ $B_0 + B$ $B_0 - B$

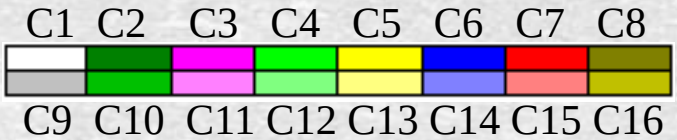


H / A / α - WISHART CLASSIFIER

OBERPFAFFENHOFEN - ESAR L-band



H / A / α and WISHART CLASSIFIER

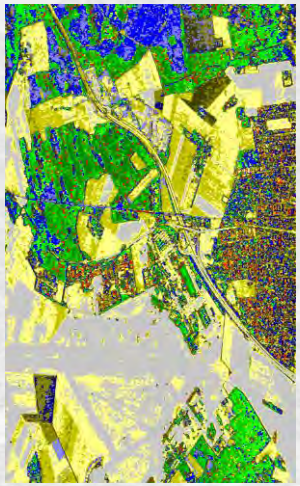


WISHART PDF

$$P(\langle[T]\rangle/[T_m]) = \frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p} e^{-LTr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)[T_m]^L}$$

UNSUPERVISED POLSAR CLASSIFICATION

J.S LEE et al. (2002)

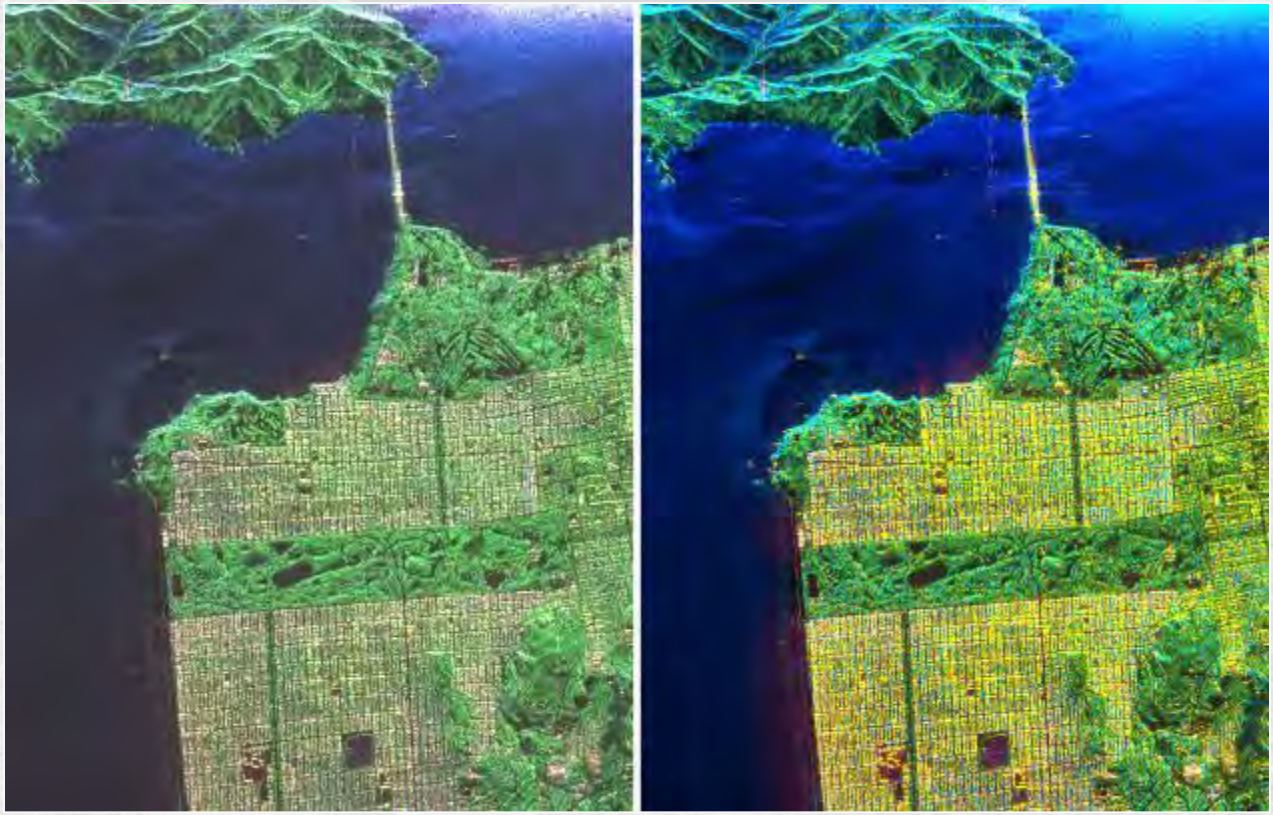


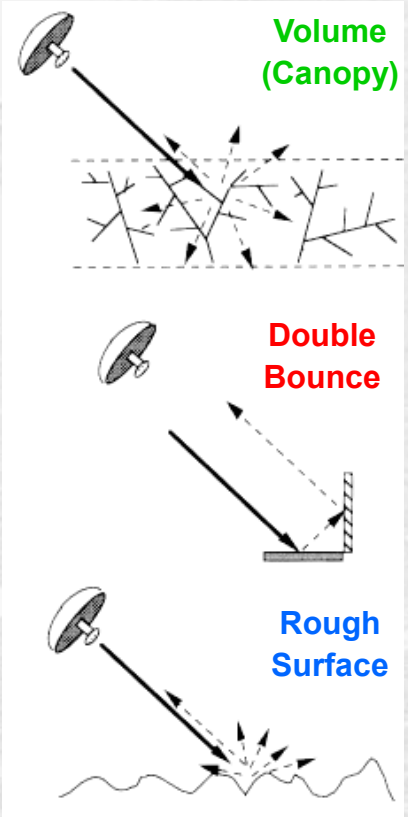
Unsupervised Classification
Preserving Scattering Mechanisms

J.S. Lee, M.R. Grunes, E. Pottier and L. Ferro-Famil,"Segmentation of polarimetric SAR images that preserves scattering mechanisms" Proceedings of EUSAR2002

FREEMAN DECOMPOSITION

Courtesy of Dr J.S Lee



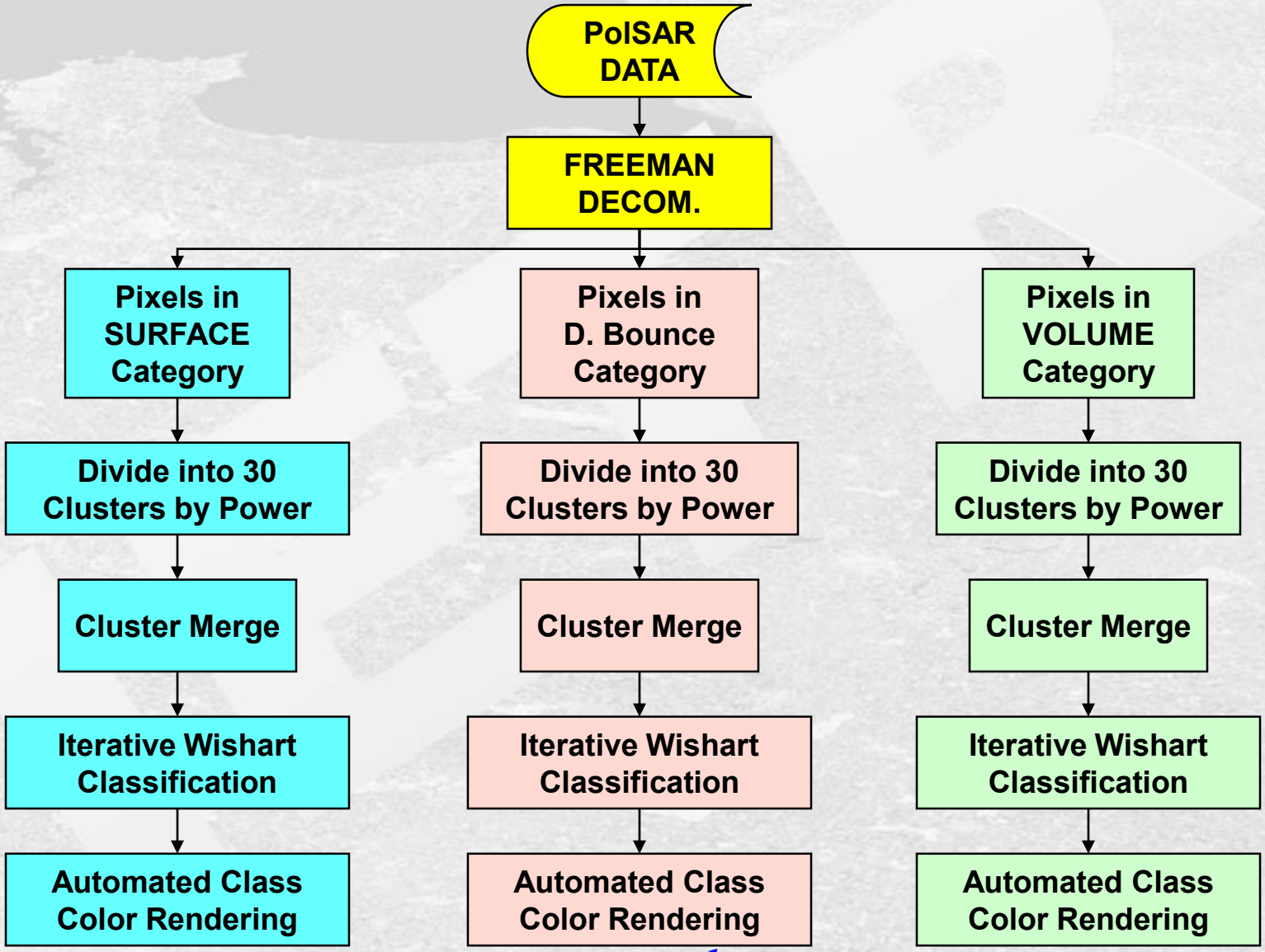


|HH-VV|, |HV|, |HH+VV|

Freeman and Durden

A. Freeman and S.L. Durden, "A Three-Component Scattering Model for Polarimetric SAR Data" IEEE TGRS, vol. 36, no. 3, May 1998

PROCEDURE – FLOW CHART



Cluster Merging $D_{ij} = \frac{1}{2}\{ln(|V_i|) + ln(|V_j|) + Tr(V_i^{-1}V_j + V_j^{-1}V_i)\}$

Wishart Iteration – After Class Merge

Classification Maps

First Iteration

Second Iteration

Third Iteration

Note: Stability insures good convergence

FREEMAN - WISHART CLASSIFIER

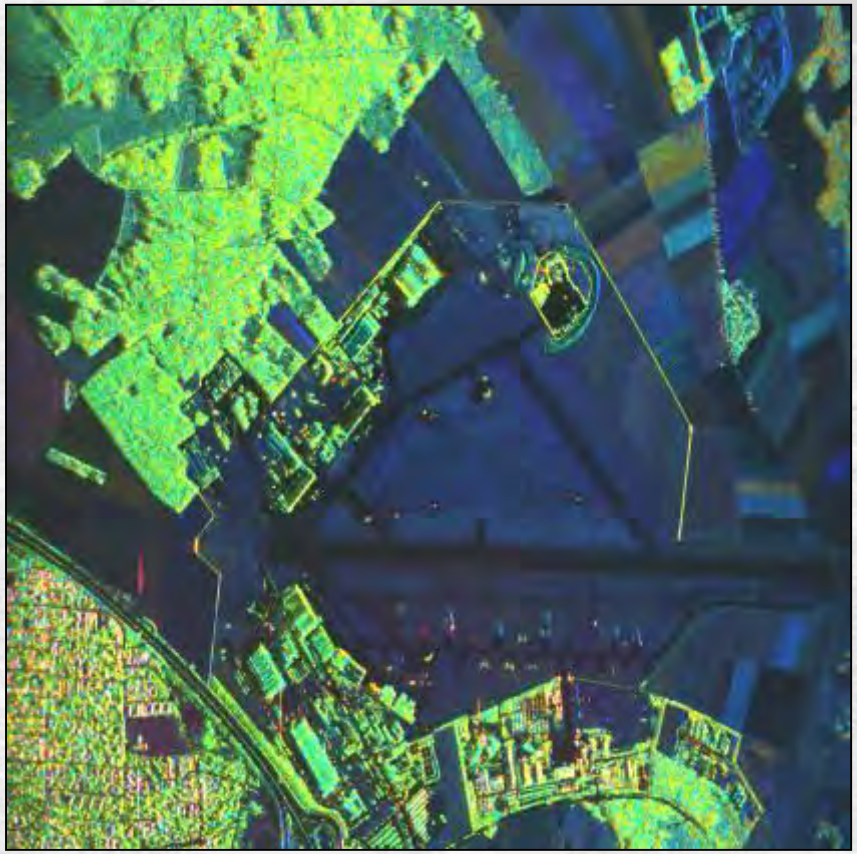
Courtesy of Dr J.S Lee

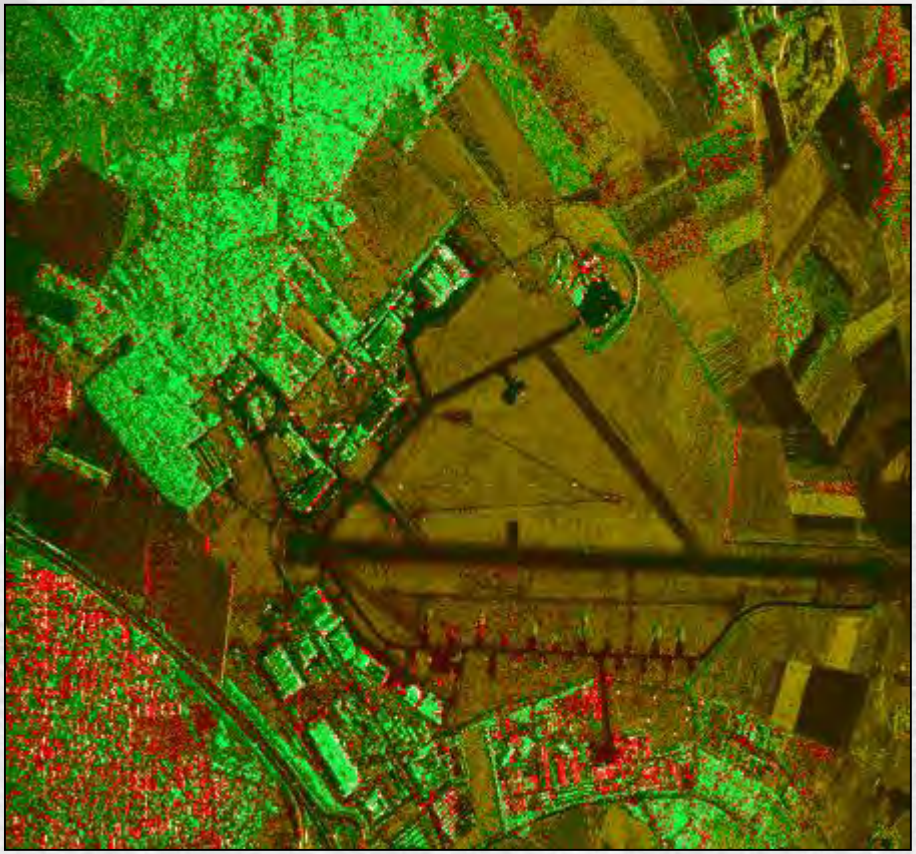
|HH-VV|, |HV|, |HH+VV|

4th Iteration (15 classes)

FREEMAN - WISHART CLASSIFIER

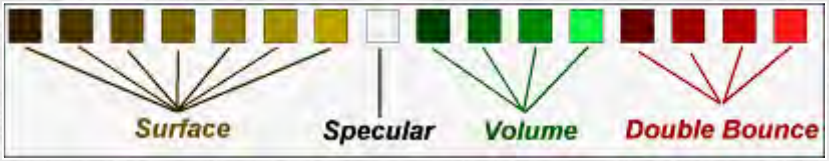
Courtesy of Dr J.S Lee





$2A_0$ $B_0 + B$ $B_0 - B$

4th Iteration (15 classes)

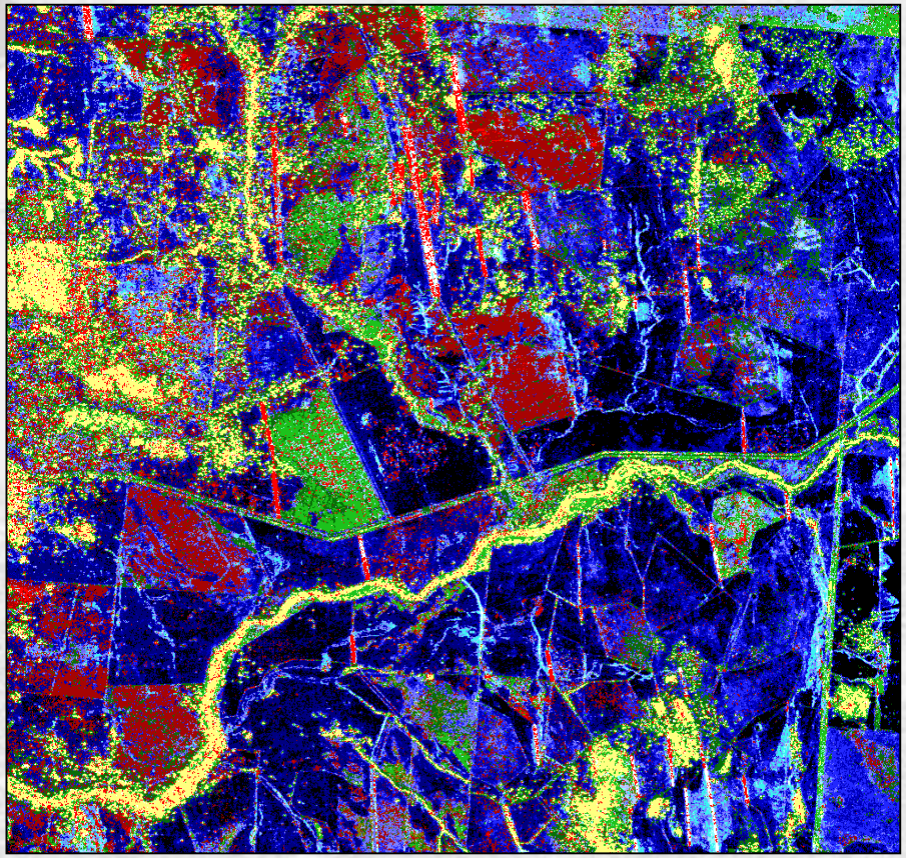


FREEMAN - WISHART CLASSIFIER

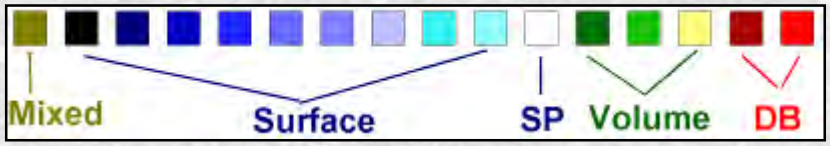
Courtesy of Dr J.S Lee

$2A_0$ $B_0 + B$ $B_0 - B$

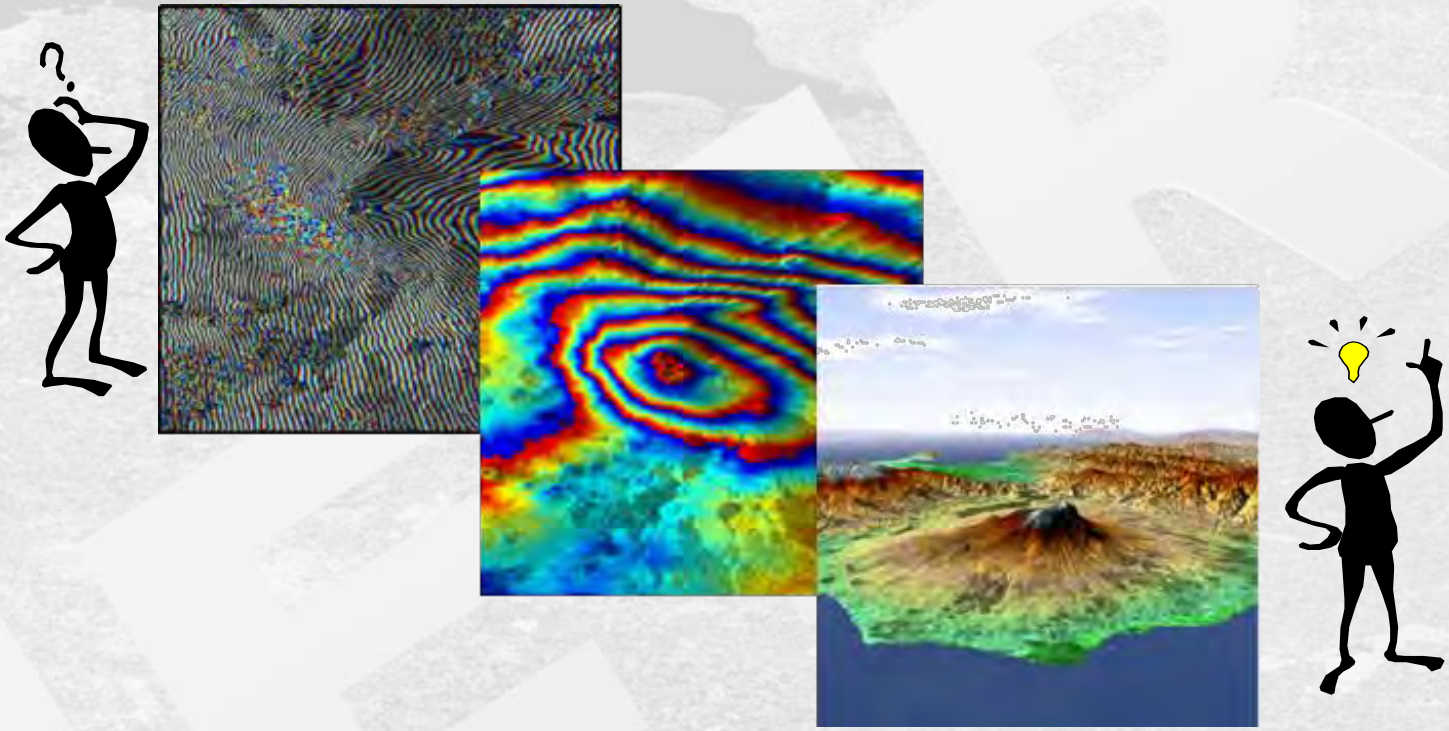
Australian Pasture



4th Iteration (15 classes)

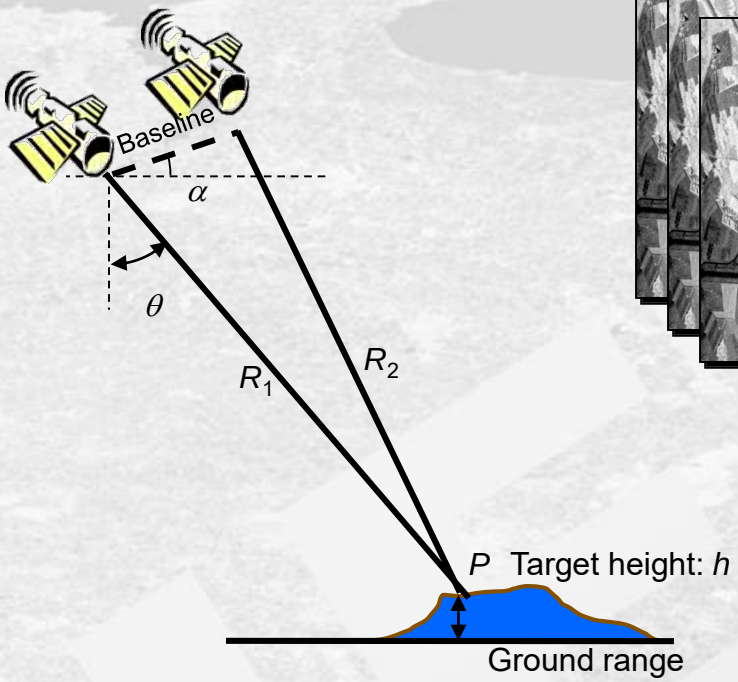


L-Band Volume Dominated



POLARIMETRIC INTERFEROMETRIC SAR (Pol-InSAR)

POL-InSAR



$$\underline{k}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX_1} + S_{YY_1} \\ S_{XX_1} - S_{YY_1} \\ 2S_{XY_1} \end{bmatrix}$$

$$\underline{\underline{k}} = \begin{bmatrix} \underline{k}_1 \\ \underline{k}_2 \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC TARGET VECTOR

$$\underline{k}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX_2} + S_{YY_2} \\ S_{XX_2} - S_{YY_2} \\ 2S_{XY_2} \end{bmatrix}$$

POL-InSAR

$$\underline{\underline{k}} = \begin{bmatrix} \underline{k}_1 \\ \underline{k}_2 \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC TARGET VECTOR

$$\langle [T_6] \rangle = \langle \underline{\underline{k}} \cdot \underline{\underline{k}}^{T*} \rangle = \begin{bmatrix} \langle \underline{k}_1 \cdot \underline{k}_1^{T*} \rangle & \langle \underline{k}_1 \cdot \underline{k}_2^{T*} \rangle \\ \langle \underline{k}_2 \cdot \underline{k}_1^{T*} \rangle & \langle \underline{k}_2 \cdot \underline{k}_2^{T*} \rangle \end{bmatrix} = \begin{bmatrix} \langle [T_1] \rangle & \langle [\Omega_{12}] \rangle \\ \langle [\Omega_{12}]^{T*} \rangle & \langle [T_2] \rangle \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC COHERENCY MATRIX (6x6)

$\langle [T_1] \rangle$ **HERMITIAN POLARIMETRIC COHERENCY MATRIX (3x3)**

$\langle [T_2] \rangle$ **HERMITIAN POLARIMETRIC COHERENCY MATRIX (3x3)**

$\langle [\Omega_{12}] \rangle$ **NON HERMITIAN POLARIMETRIC INTER-COHERENCY MATRIX (3x3)**

DUAL CHANNELS POLINSAR UNSUPERVISED SEGMENTATION

$$\langle [T_6] \rangle = \langle \underline{\underline{k}} \cdot \underline{\underline{k}}^{T*} \rangle = \begin{bmatrix} \langle \underline{k}_1 \cdot \underline{k}_1^{T*} \rangle & \langle \underline{k}_1 \cdot \underline{k}_2^{T*} \rangle \\ \langle \underline{k}_2 \cdot \underline{k}_1^{T*} \rangle & \langle \underline{k}_2 \cdot \underline{k}_2^{T*} \rangle \end{bmatrix} = \begin{bmatrix} \langle [T_1] \rangle & \langle [\Omega_{12}] \rangle \\ \langle [\Omega_{12}]^{T*} \rangle & \langle [T_2] \rangle \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC COHERENCY MATRIX (6x6)

$\langle [T_6] \rangle$ **FOLLOWS A WISHART DISTRIBUTION**

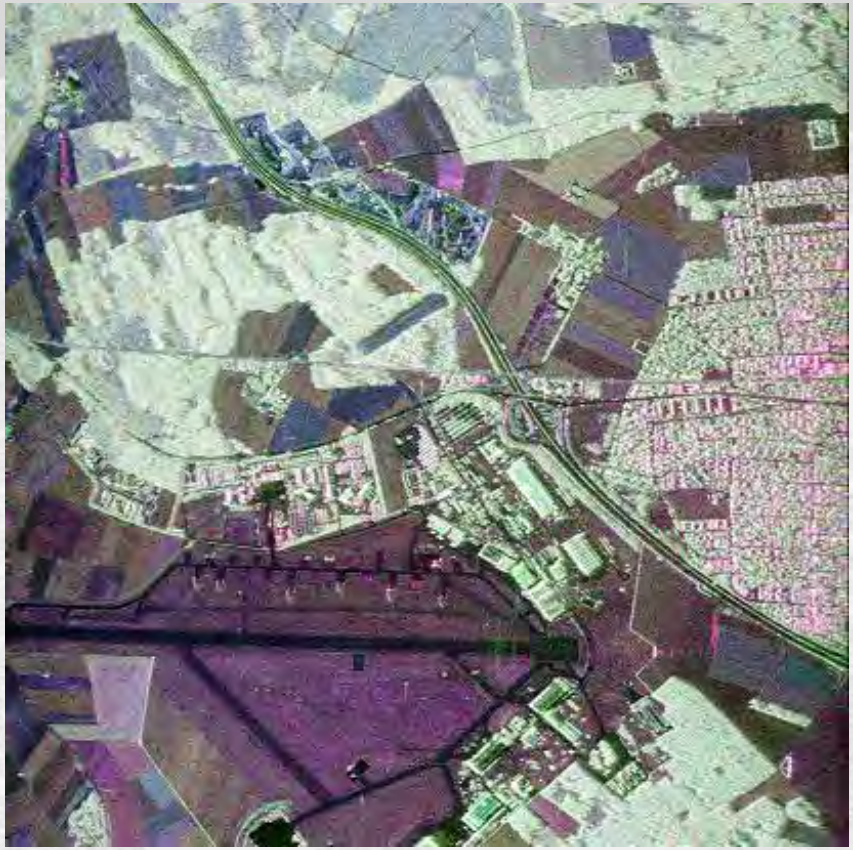
$$P(\langle [T_6] \rangle / [\Sigma_m]) = \frac{|\langle [T_6] \rangle|^{L-p} \, exp\left(-tr\left([\Sigma_m]^{-1} \langle [T_6] \rangle\right)\right)}{K(L,p) |[\Sigma_m]|^L} = W_C(L, [\Sigma_m])$$

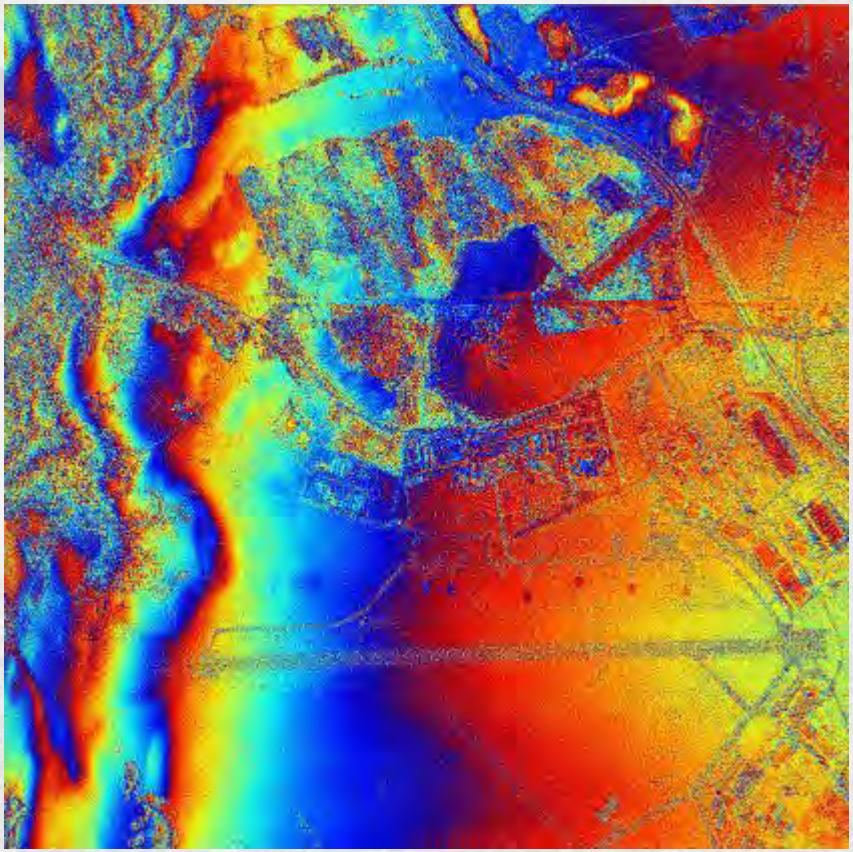
L: Number of Look
p: Polarimetric Dimension

With: $$K(L,p) = \frac{\pi^{\frac{p(p-1)}{2}}}{L^{Lp}} \Gamma(L) \ldots \Gamma(L-p+1)$$

$[\Sigma m]$: Cluster Center of the class *m*

POL-InSAR



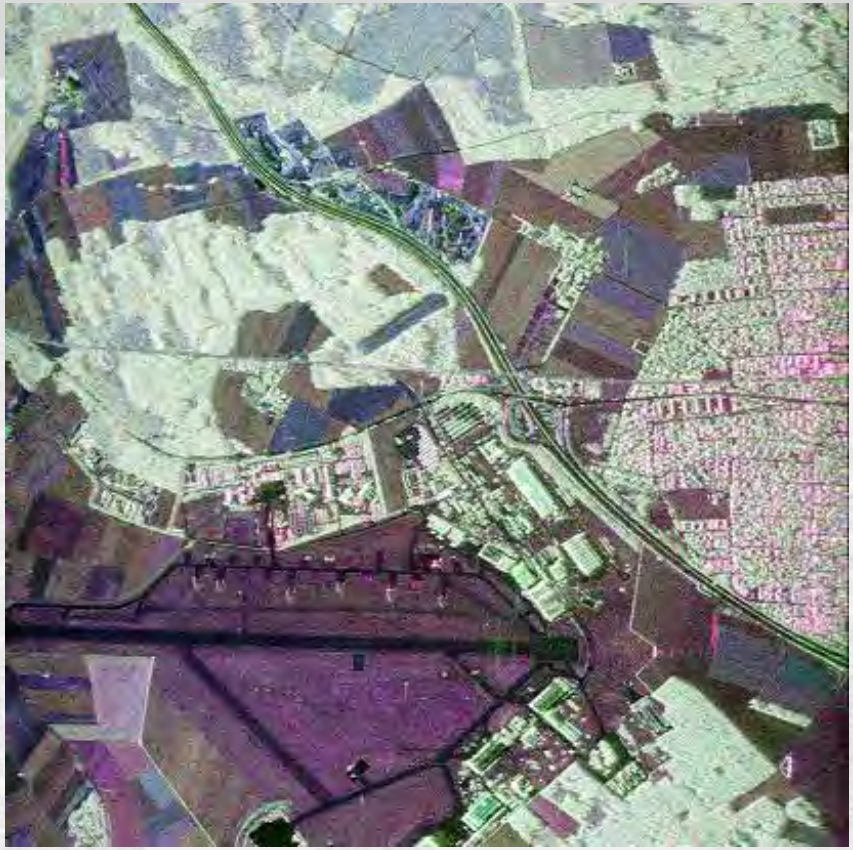


DLR E-SAR L Band
Pol-In SAR (1.5m x 3m) – Baseline 15m

POL-SAR INFORMATION

IN-SAR INFORMATION **Arg(γ)**

POL-InSAR



DLR E-SAR L Band
Pol-In SAR (1.5m x 3m) – Baseline 5m

POL-SAR INFORMATION

IN-SAR INFORMATION $|\gamma|$

COMPLEMENTARY INFORMATION

POL-InSAR

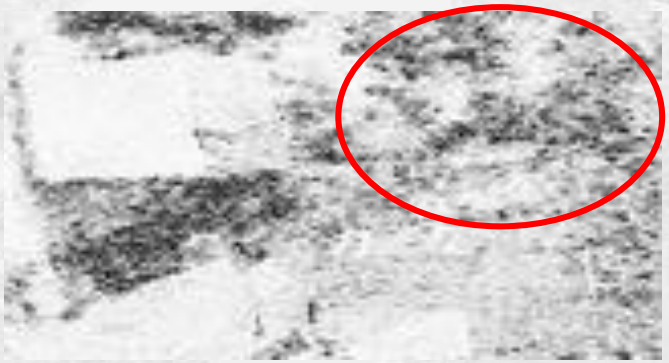
HETEROGENEOUS AREA

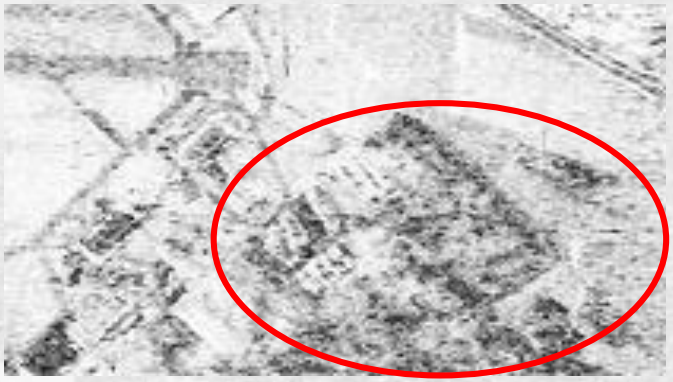
HOMOGENEOUS AREA

DIFFERENT POLARIMETRIC SCATTERING MECHANISMS

CONSTANT INTERFEROMETRIC COHERENCE

POL-InSAR





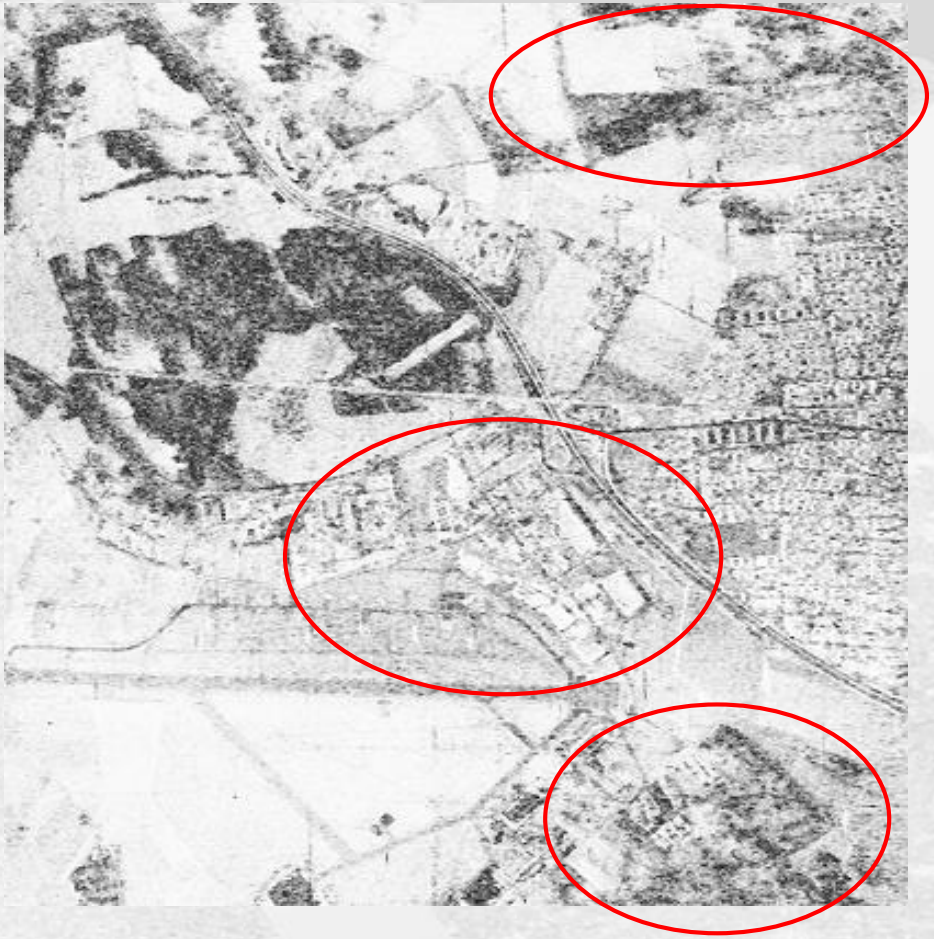
HOMOGENEOUS AREA

HETEROGENEOUS AREA

SAME POLARIMETRIC SCATTERING MECHANISMS

DIFFERENT INTERFEROMETRIC COHERENCE

POL-InSAR



INTERFEROMETRIC COHERENCE γ

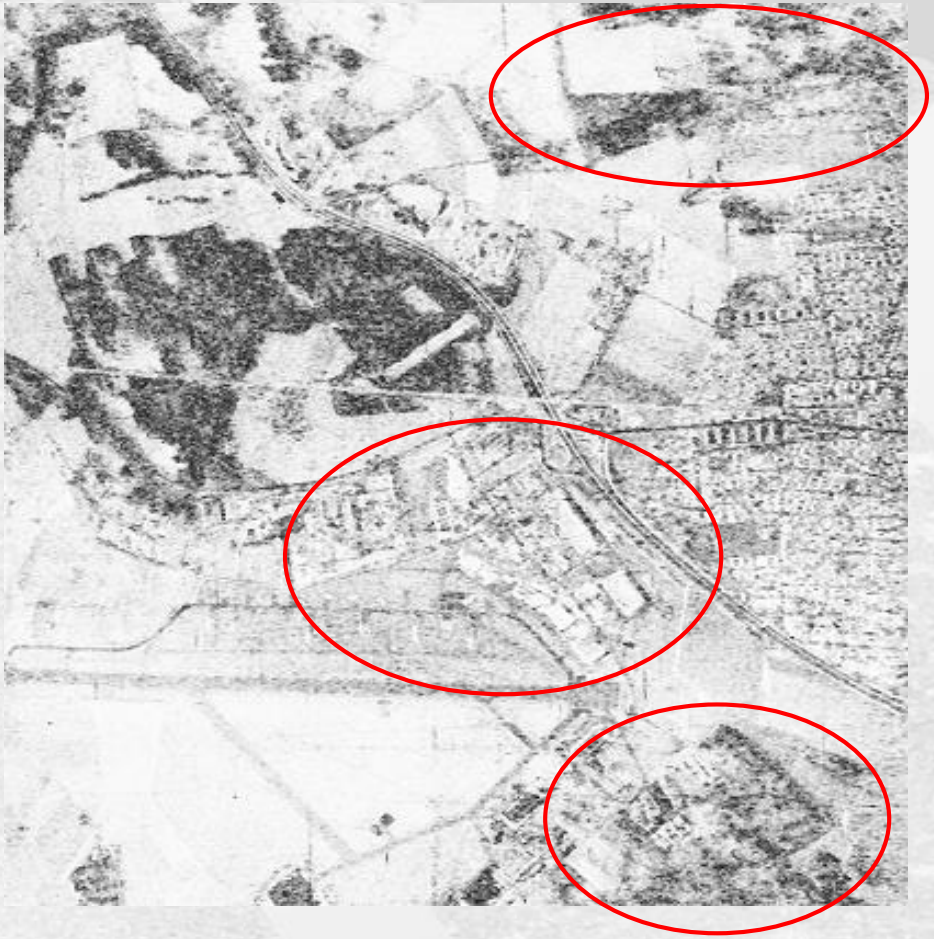
$2A_0$

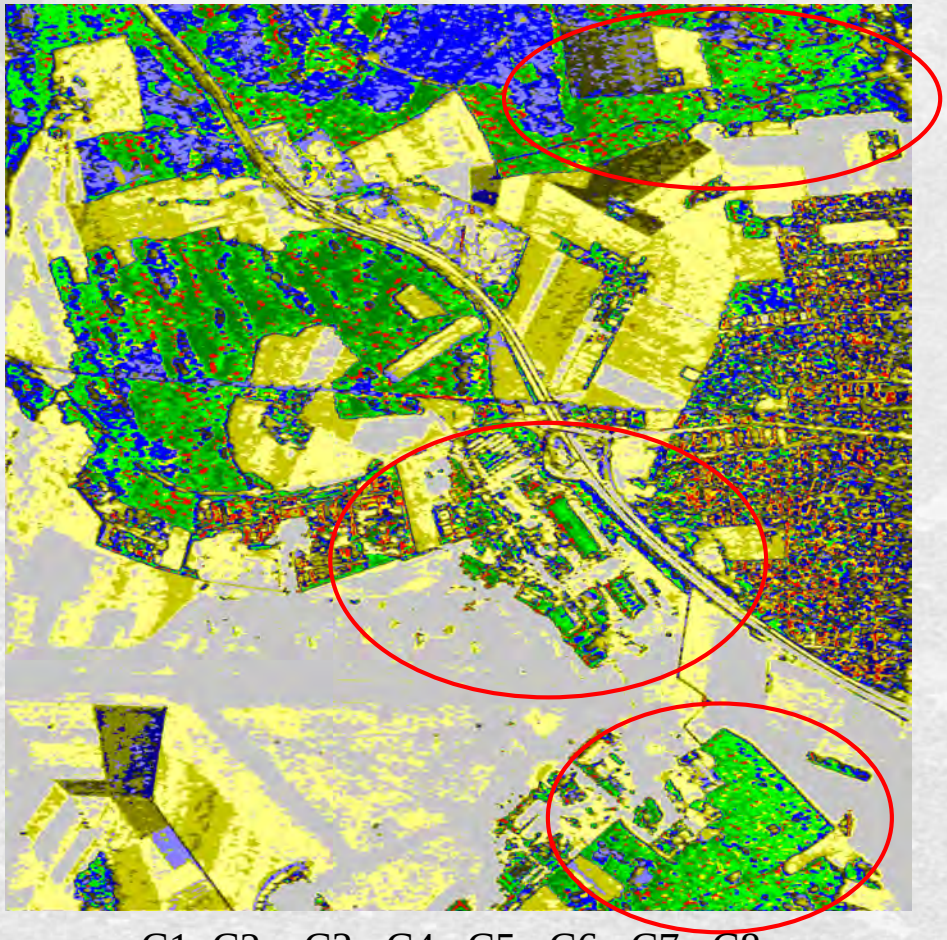
$B_0 + B$

$B_0 - B$

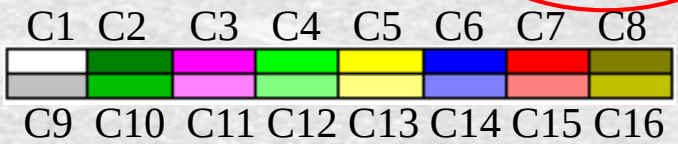
POL-InSAR

Wishart H-A-α segmentation





INTERFEROMETRIC COHERENCE γ

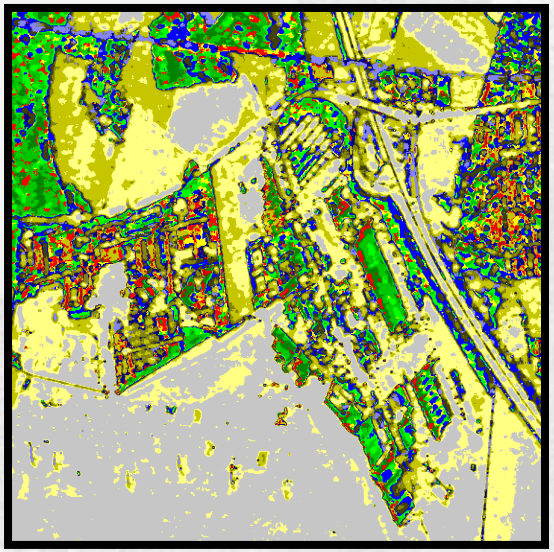


POL-InSAR

Optical Image

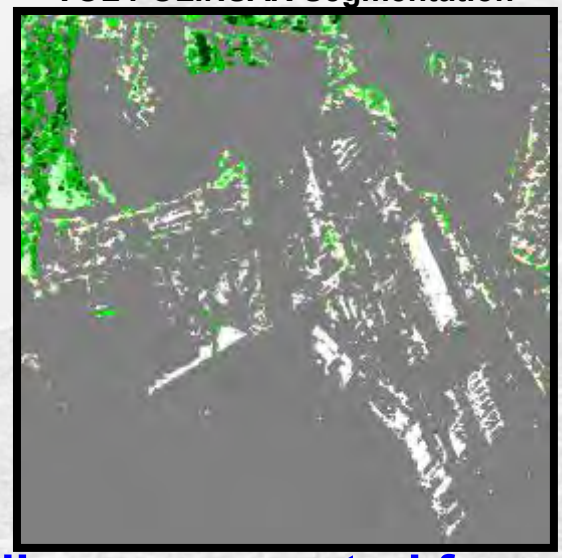
POLSAR Image

POLSAR Segmentation

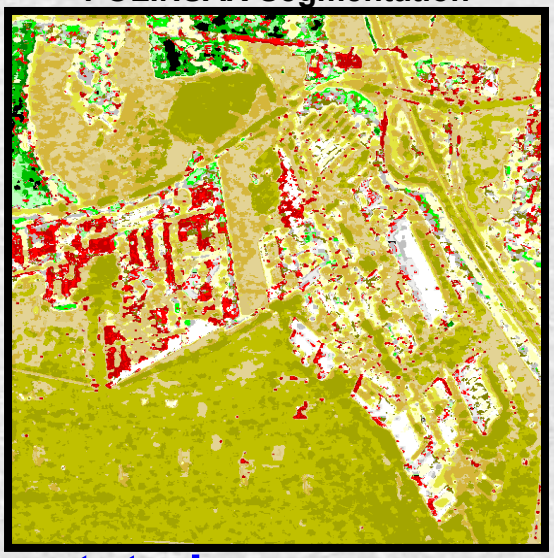


INSAR Image

VOL POLINSAR Segmentation



POLINSAR Segmentation



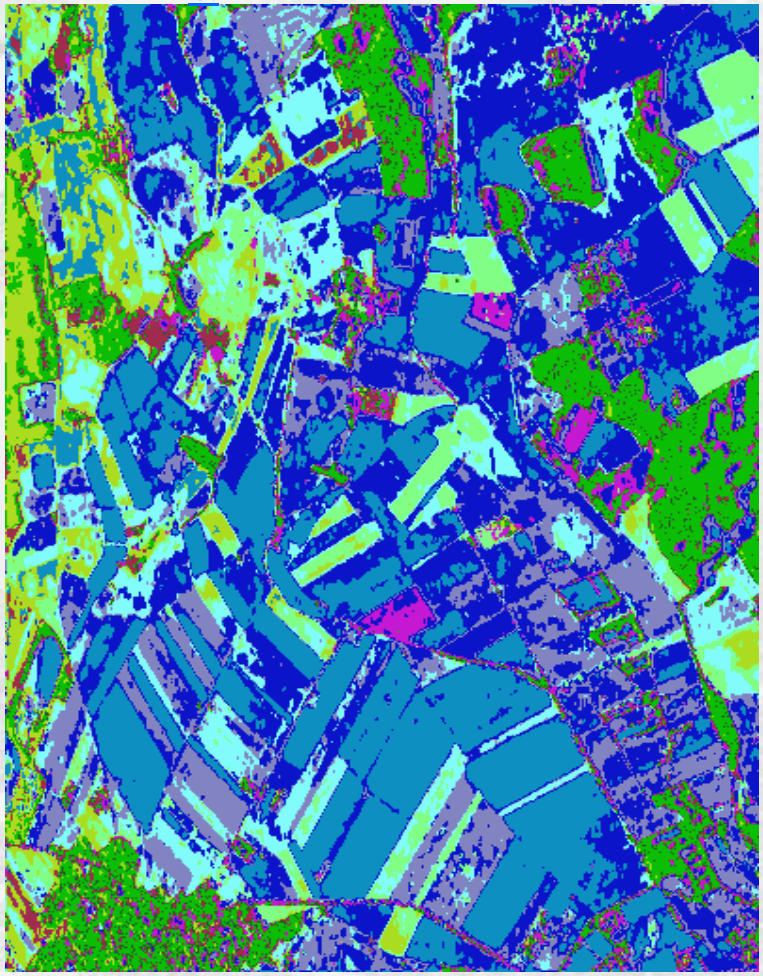
Oriented buildings segmented from vegetated areas

POL-InSAR

ALLING - ESAR L-band

$2A_0$ $B_0 + B$ $B_0 - B$

H / A / α and WISHART CLASSIFIER



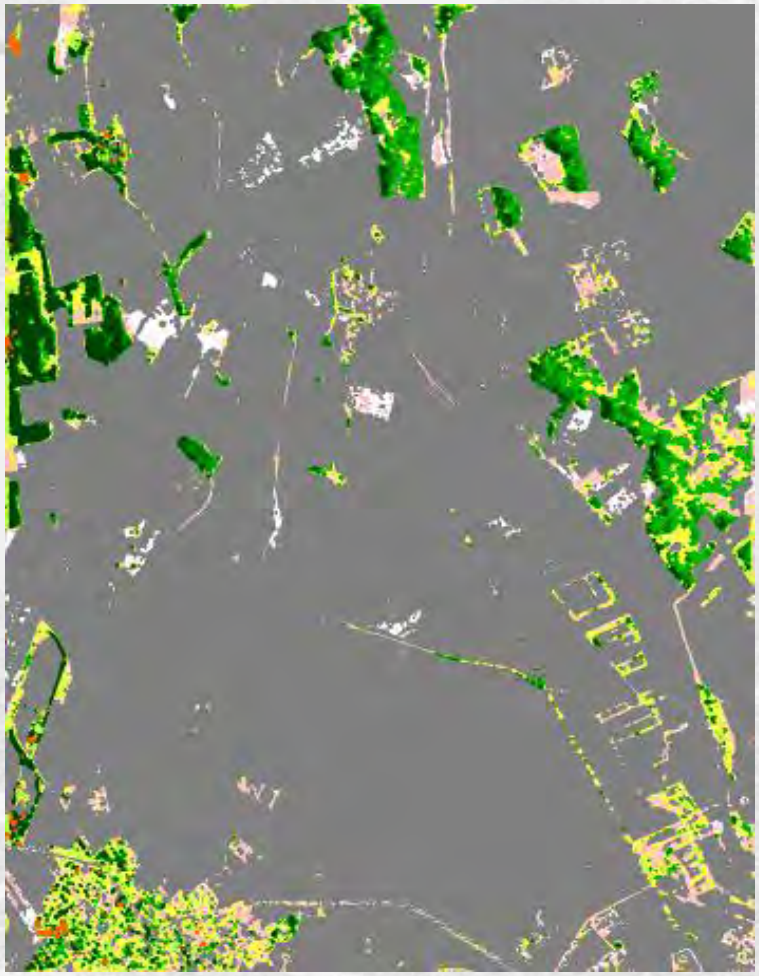
POL-InSAR

DLR E-SAR L Band – Pol-InSAR (1.5m x 3m) – Baseline 5m

POL-SAR INFORMATION

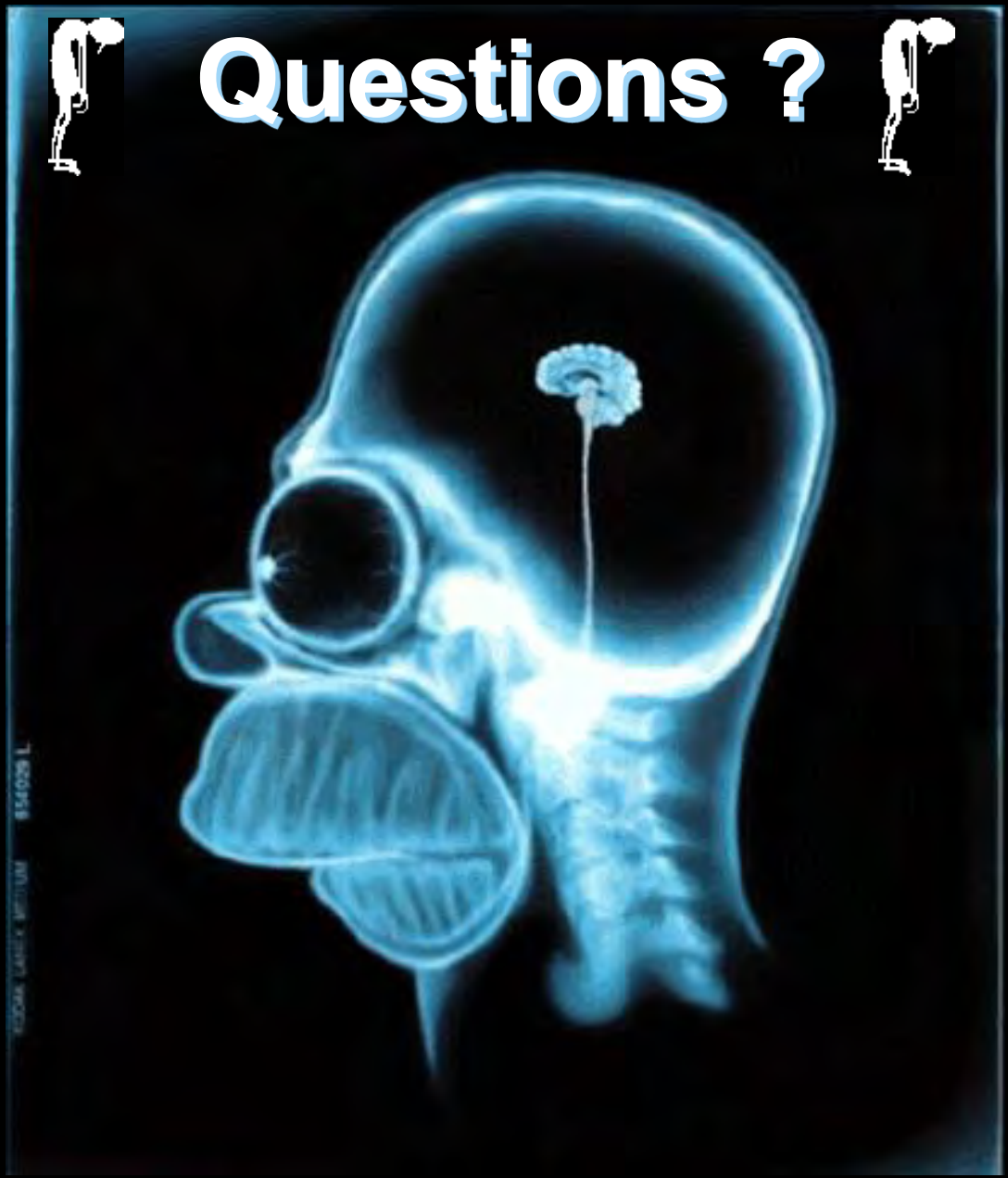
IN-SAR INFORMATION $|\gamma|$

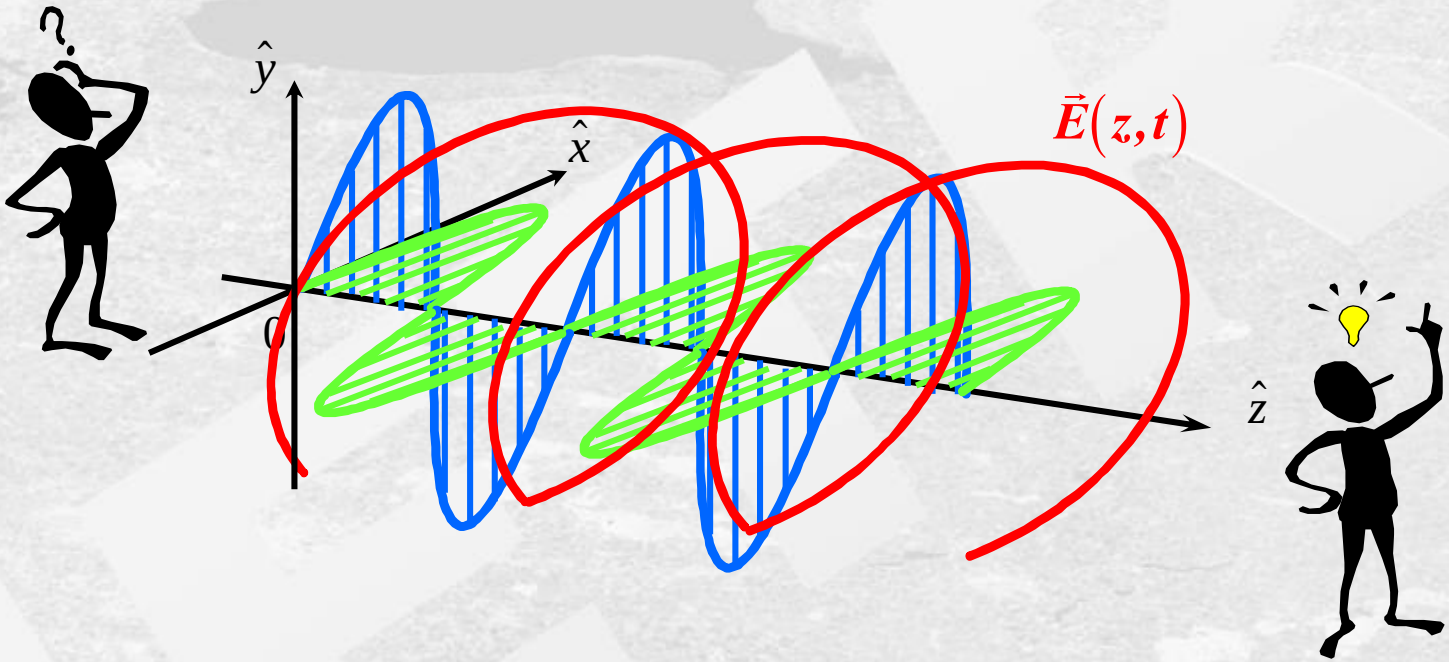
POL-InSAR



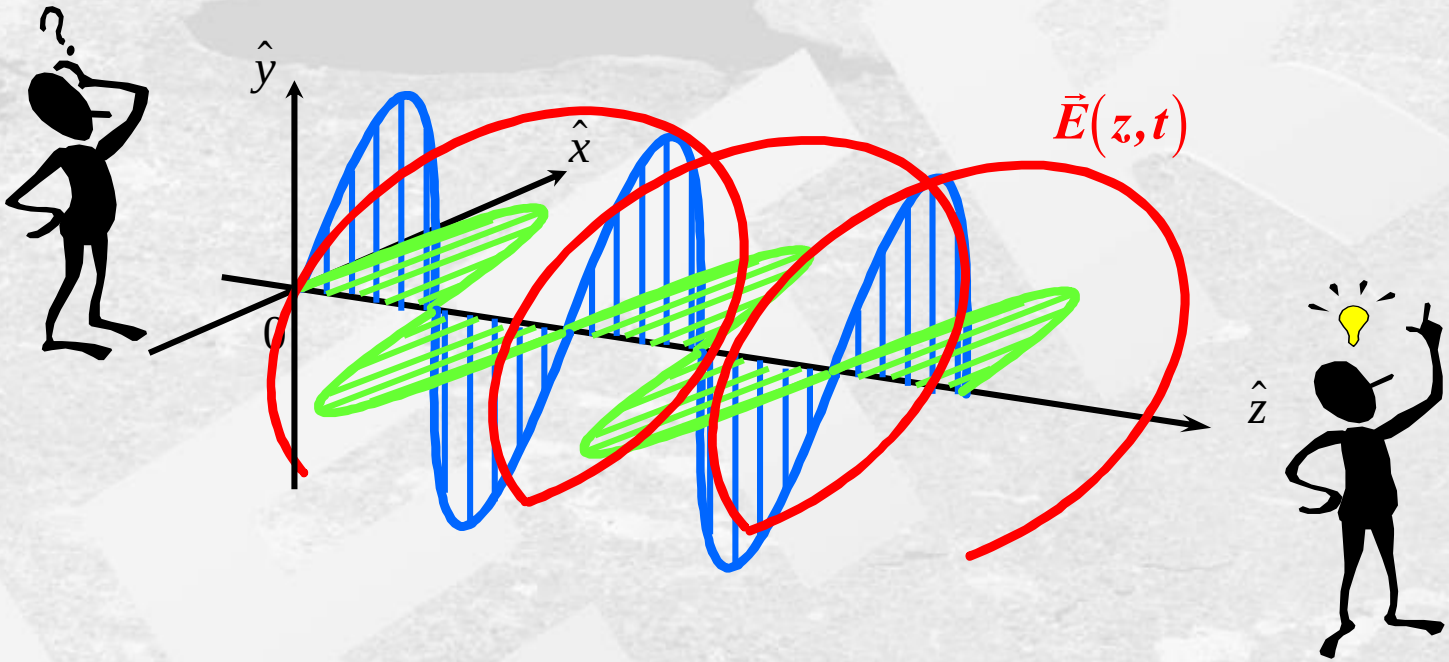
Oriented Targets segmented from Vegetated Areas

Questions ?



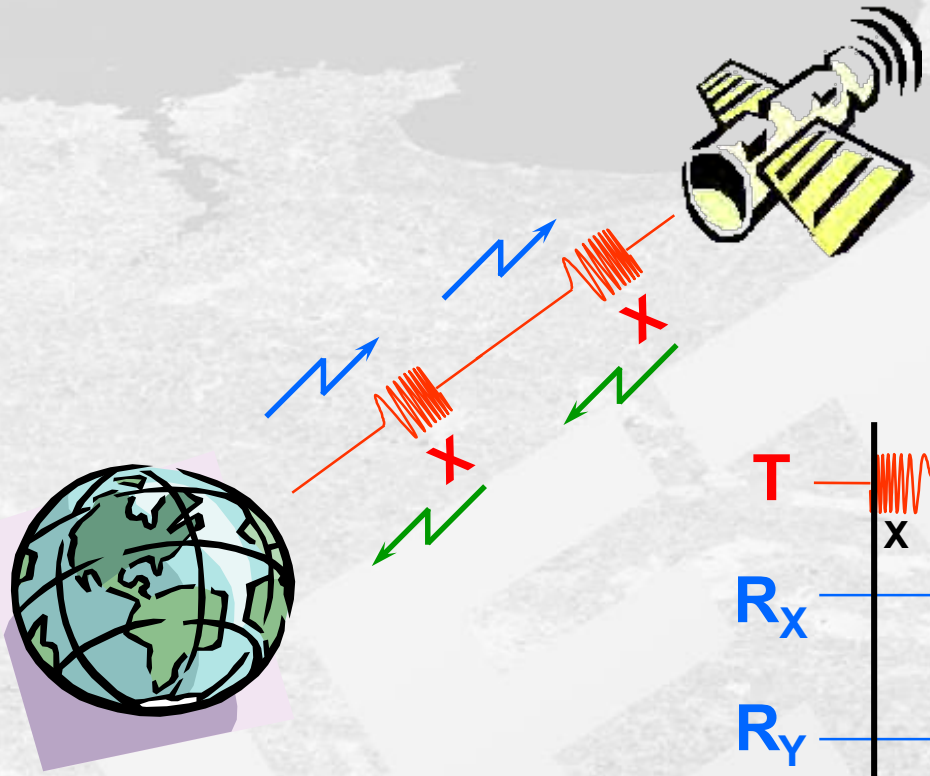


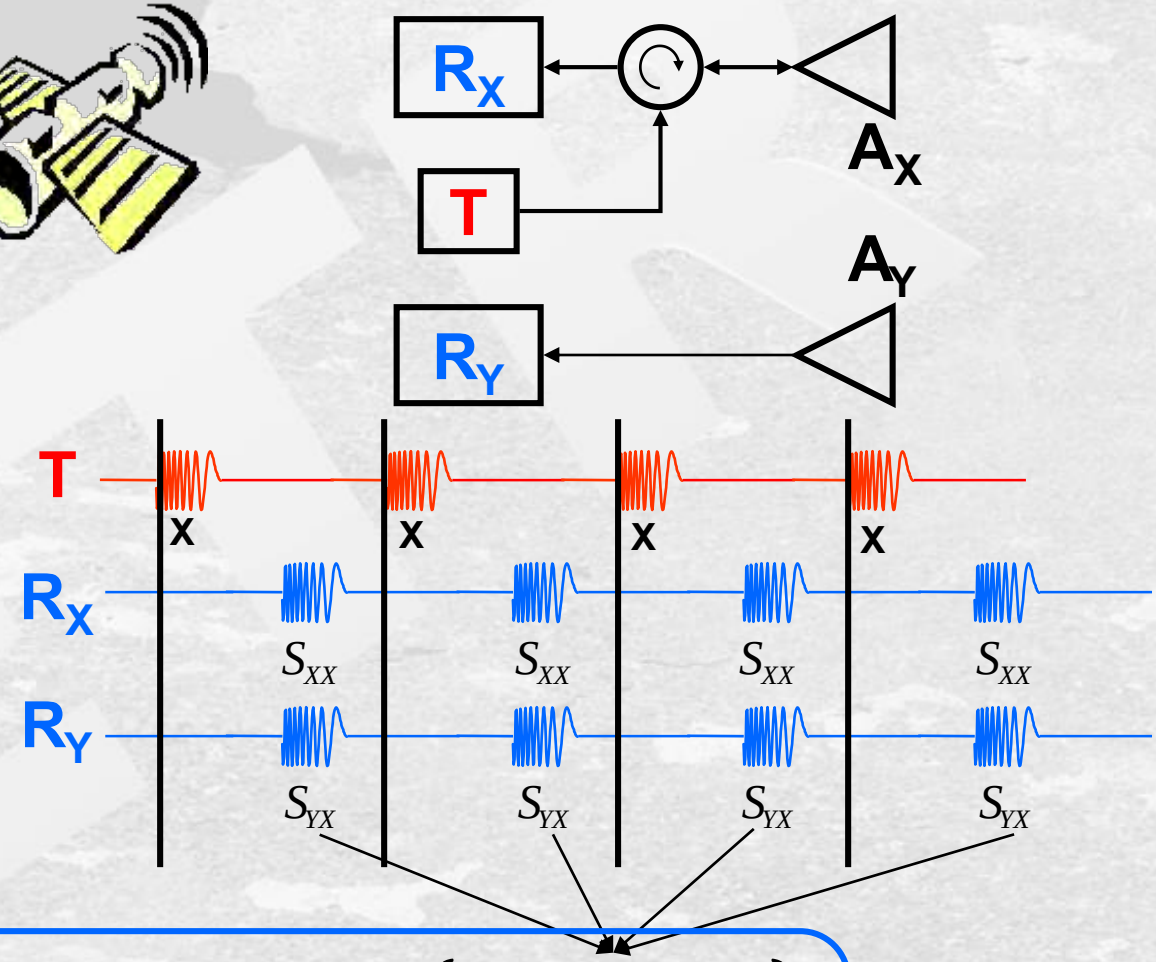
BASIC CONCEPTS



WAVE POLARIMETRY

WAVE POLARIMETRY



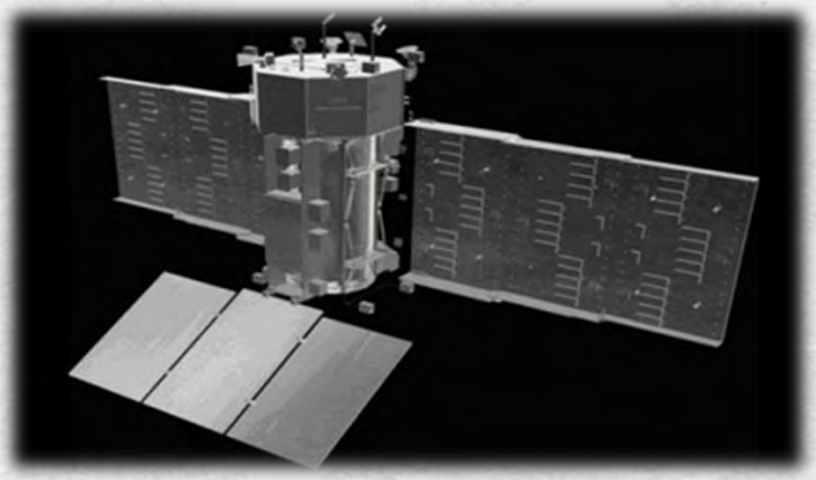


TRANSMITTER: X
RECEIVERS: X & Y

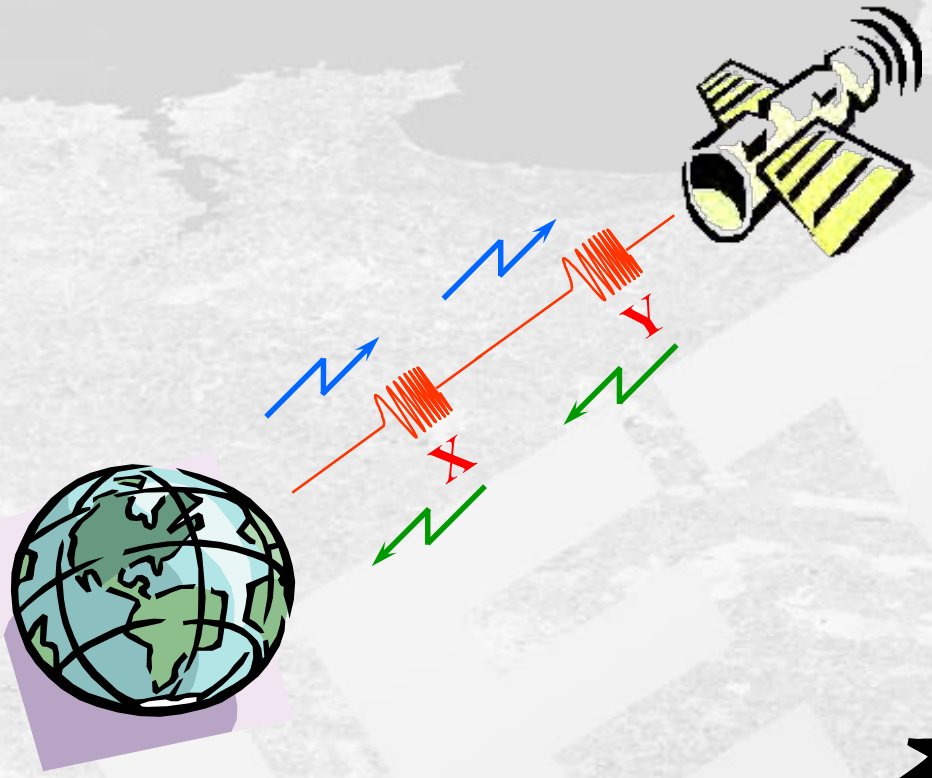
JONES VECTORS $\left\{ \underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix} \right\}$

WAVE POLARIMETRY

CURRENT / FUTURE SENSORS



POLARIMETRIC DESCRIPTORS



TRANSMITTER: X
RECEIVERS: X & Y

THE DIFFERENT WAVE POLARIMETRIC DESCRIPTORS

E	Jones Vector
g	Stokes Vector
[C]	Covariance Matrix

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{ox} e^{j\delta_x} \\ E_{oy} e^{j\delta_y} \end{bmatrix}$$

$$= A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \hat{u}_x$$

Special Unitary Pauli Matrices Group – SU(2)

$$[U_2(\phi)] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$[U_2(\tau)] = \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \qquad [U_2(\alpha)] = \begin{bmatrix} e^{+j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}$$

STOKES VECTOR

REAL REPRESENTATION OF THE POLARISATION STATE OF A MONOCHROMATIC WAVE

$$\underline{E} \cdot \underline{E}^{T*} = \begin{bmatrix} E_x E_x^* & E_x E_y^* \\ E_y E_x^* & E_y E_y^* \end{bmatrix}$$

PAULI MATRICES GROUP

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}$$

$$\Downarrow$$

$$\underline{E} \cdot \underline{E}^{T*} = \frac{1}{2}\{g_0\sigma_0 + g_1\sigma_1 + g_2\sigma_2 + g_3\sigma_3\} = \frac{1}{2}\begin{bmatrix} g_0 + g_1 & g_2 - jg_3 \\ g_2 + jg_3 & g_0 - g_1 \end{bmatrix}$$

$\{g_0, g_1, g_2, g_3\}$ **STOKES PARAMETERS**

STOKES VECTOR

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix}$$

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 = |E_x|^2 + |E_y|^2 \\ g_1 = |E_x|^2 - |E_y|^2 \\ g_2 = 2\Re\left(E_x E_y^*\right) \\ g_3 = -2\Im\left(E_x E_y^*\right) \end{bmatrix}$$

WAVE POLARISATION STATE ESTIMATION FROM INTENSITIES MEASUREMENTS

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 = E_{0x}^2 + E_{0y}^2 \\ g_1 = E_{0x}^2 - E_{0y}^2 \\ g_2 = 2E_{0x}E_{0y}\cos(\delta) \\ g_3 = 2E_{0x}E_{0y}\sin(\delta) \end{bmatrix} = \begin{bmatrix} g_0 = A^2 \\ g_1 = A^2\cos 2\phi \cos 2\tau \\ g_2 = A^2 \sin 2\phi \cos 2\tau \\ g_3 = A^2 \sin 2\tau \end{bmatrix}$$

GEOMETRICAL PARAMETERS

ORIENTATION ANGLE

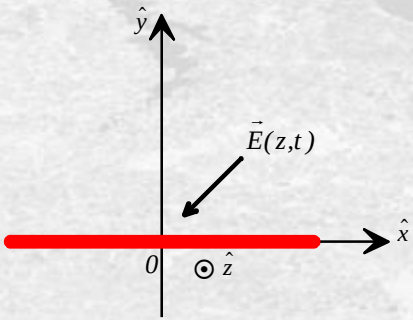
$$\tan 2\phi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta = \frac{g_2}{g_1}$$

ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta = \frac{g_3}{g_0}$$

STOKES VECTOR

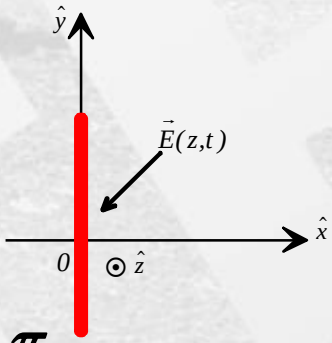
HORIZONTAL POLARISATION STATE



$$\underline{g}_{\underline{H}} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\phi = 0 \quad \tau = 0$$

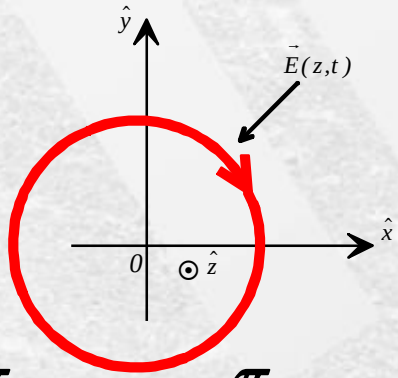
VERTICAL POLARISATION STATE



$$\underline{g}_{\underline{V}} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\phi = \frac{\pi}{2} \quad \tau = 0$$

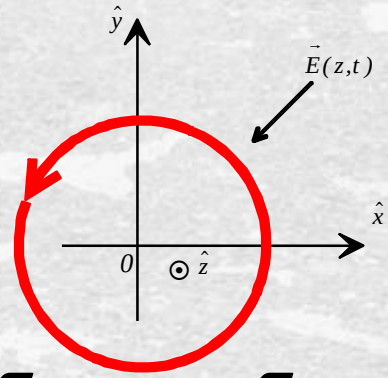
LEFT CIRCULAR POLARISATION STATE



$$\underline{g}_{\underline{LC}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$-\frac{\pi}{2} \le \phi \le +\frac{\pi}{2} \quad \tau = +\frac{\pi}{4}$$

RIGHT CIRCULAR POLARISATION STATE



$$\underline{g}_{\underline{RC}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$-\frac{\pi}{2} \le \phi \le +\frac{\pi}{2} \quad \tau = -\frac{\pi}{4}$$

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2} Tr\left([U_2(\theta)]^{T*} \sigma_p [U_2(\theta)] \sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

O(3) UNITARY GROUP

$$\begin{bmatrix} \cos 2\phi & -\sin 2\phi & 0 \\ \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 2\tau & 0 & -\sin 2\tau \\ 0 & 1 & 0 \\ \sin 2\tau & 0 & \cos 2\tau \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\alpha & -\sin 2\alpha \\ 0 & \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$[O_3(2\phi)] \qquad [O_3(2\tau)] \qquad [O_3(2\alpha)]$$

O(4) UNITARY ROTATION ROUP

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

$[U_2(\phi)]$ $[U_2(\tau)]$ $[U_2(\alpha)]$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2}Tr\left([U_2(\theta)]^{T*}\sigma_p[U_2(\theta)]\sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

STOKES VECTOR

$$\underline{g}_{\underline{E}} = A^2\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}\underline{g}_{\hat{u}_x}$$

$[O_3(2\phi)]$ $[O_3(2\tau)]$ $[O_3(2\alpha)]$

POINCARE SPHERE

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} |E_x|^2 + |E_y|^2 \\ |E_x|^2 - |E_y|^2 \\ 2\Re\left(E_x E_y^*\right) \\ -2\Im\left(E_x E_y^*\right) \end{bmatrix} = \begin{bmatrix} E_{0x}^2 + E_{0y}^2 \\ E_{0x}^2 - E_{0y}^2 \\ 2E_{0x}E_{0y}\cos(\delta) \\ 2E_{0x}E_{0y}\sin(\delta) \end{bmatrix} = \begin{bmatrix} A^2 \\ A^2\cos 2\phi \cos 2\tau \\ A^2 \sin 2\phi \cos 2\tau \\ A^2 \sin 2\tau \end{bmatrix}$$

$\{g_0\}$ TOTAL WAVE INTENSITY

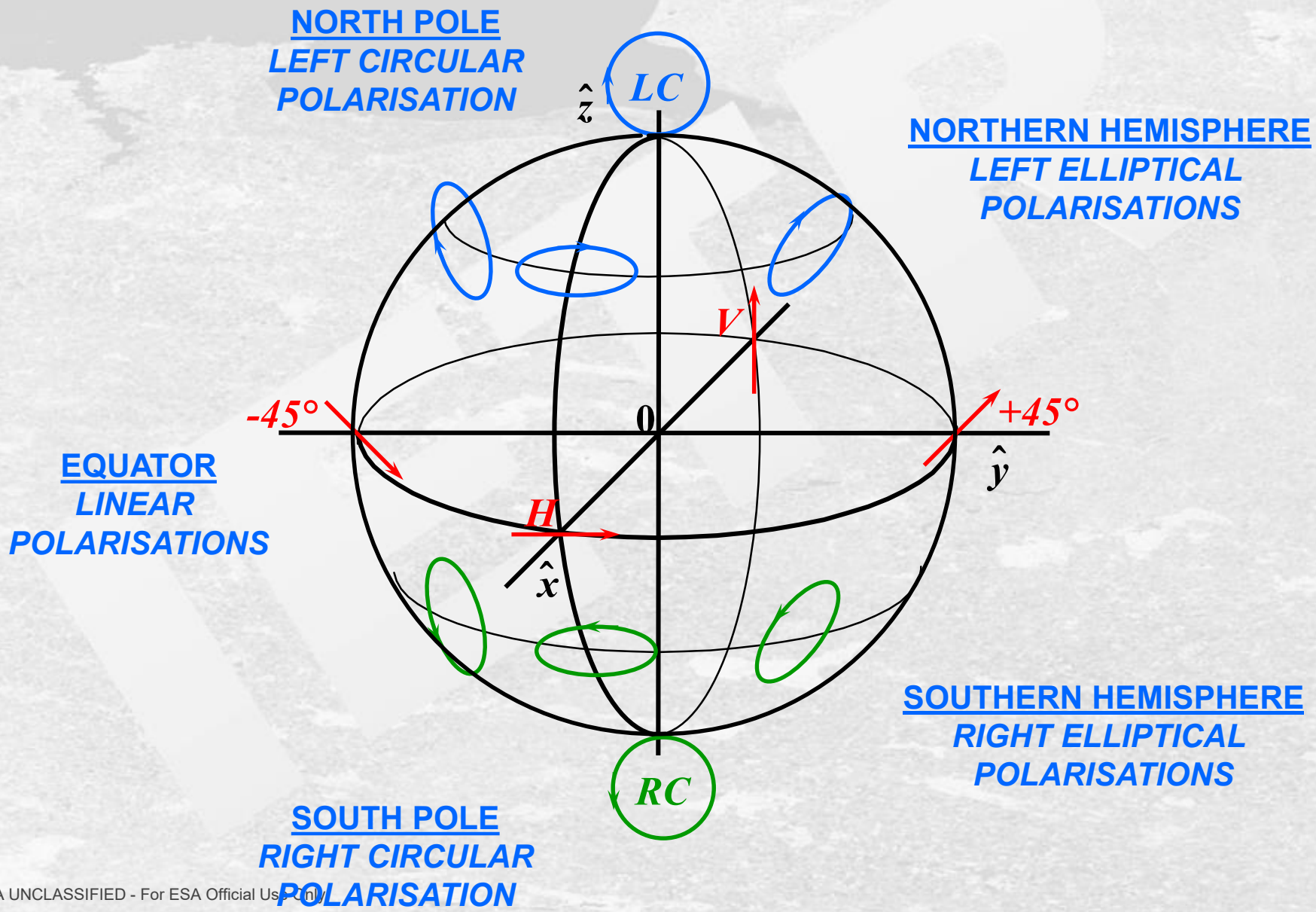
$\{g_1, g_2, g_3\}$ POLARISED WAVE INTENSITIES

$$\boxed{g_0^2 = g_1^2 + g_2^2 + g_3^2}$$

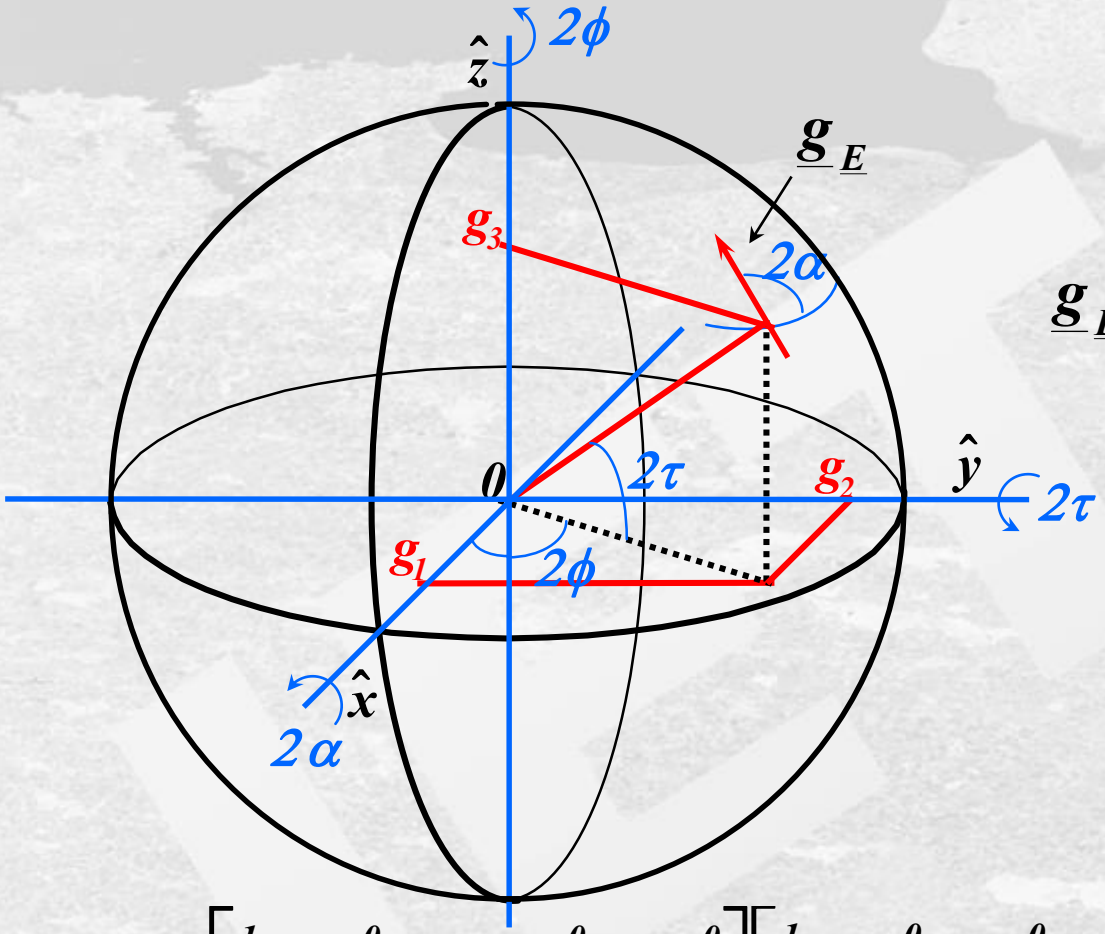
WAVE FULLY POLARISED

$\{g_1, g_2, g_3\}$ Spherical Coordinates of a point P on a sphere with radius g_0

POINCARE SPHERE



POINCARE SPHERE



$$\underline{g}_E = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A^2 \\ A^2 \cos 2\phi \cos 2\tau \\ A^2 \sin 2\phi \cos 2\tau \\ A^2 \sin 2\tau \end{bmatrix}$$

$$\underline{g}_E = A^2 \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{[O_3(2\phi)]} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix}}_{[O_3(2\tau)]} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}}_{[O_3(2\alpha)]} \underline{g}_{\hat{u}_y}$$

POINCARE SPHERE

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

ORTHOGONAL JONES VECTOR

$$\underline{E}_{\perp} = \begin{bmatrix} E'_x \\ E'_y \end{bmatrix}$$

ORTHOGONALITY CONDITIONS

$$(\phi, \tau) \mapsto \begin{cases} \phi' = \phi + \frac{\pi}{2} \\ \tau' = -\tau \end{cases}$$

STOKES VECTOR

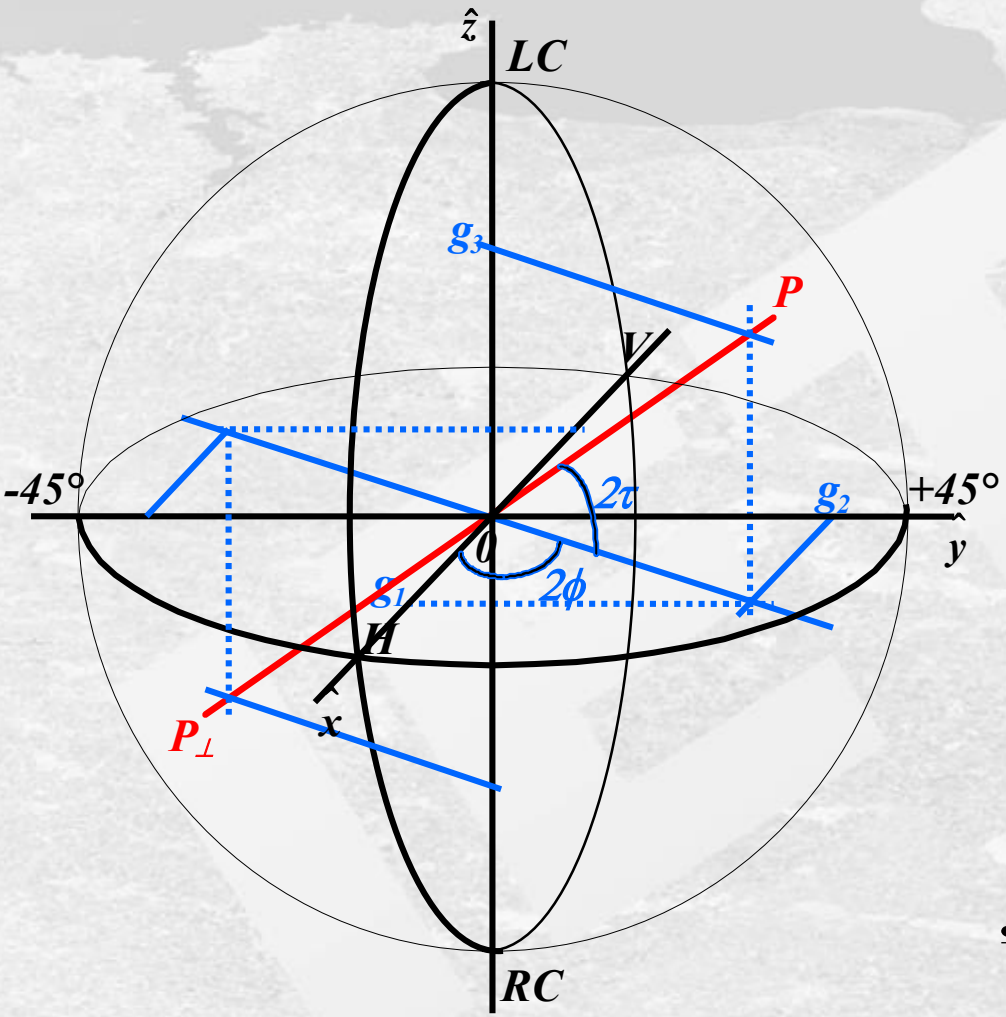
$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ A\cos 2\phi \cos 2\tau \\ A\sin 2\phi \cos 2\tau \\ A\sin 2\tau \end{bmatrix}$$

ORTHOGONAL STOKES VECTOR

$$\underline{g}_{\underline{E}_{\perp}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ -A\cos 2\phi \cos 2\tau \\ -A\sin 2\phi \cos 2\tau \\ -A\sin 2\tau \end{bmatrix}$$

ORTHOGONALITY = ANTIPODALITY

POINCARE SPHERE



STOKES VECTOR

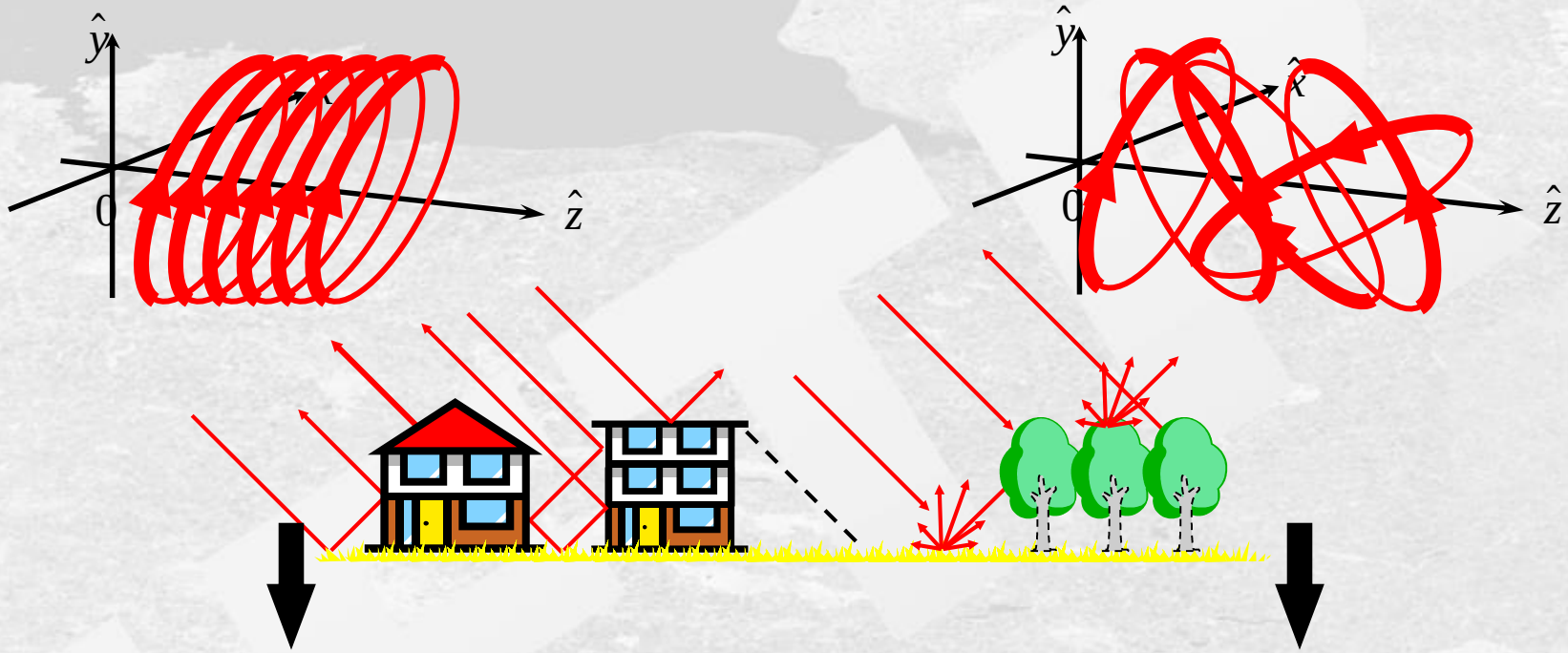
$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ A\cos 2\phi \cos 2\tau \\ A\sin 2\phi \cos 2\tau \\ A\sin 2\tau \end{bmatrix}$$

ORTHOGONAL STOKES VECTOR

$$\underline{g}_{\underline{E}_\perp} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ -A\cos 2\phi \cos 2\tau \\ -A\sin 2\phi \cos 2\tau \\ -A\sin 2\tau \end{bmatrix}$$

ORTHOGONALITY = ANTIPODALITY

PARTIALLY POLARISED WAVES



DETERMINISTIC SCATTERING

COMPLETELY POLARISED WAVE

RANDOM SCATTERING

PARTIALLY POLARISED WAVE

Polarisation Ellipse varies in time
Amplitude, Phase: Random processes

STATISTICAL DESCRIPTION

PARTIALLY POLARISED WAVES

JONES VECTORS $\{\underline{E}\}$

$$\Downarrow$$

WAVE COVARIANCE MATRIX

$$\langle [J] \rangle = \langle \underline{E}\,\underline{E}^{T*} \rangle = \begin{bmatrix} \langle |E_x|^2 \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle |E_y|^2 \rangle \end{bmatrix}$$

$$\Updownarrow$$

$$\langle [J] \rangle = \frac{1}{2} \begin{bmatrix} \langle g_0 \rangle + \langle g_1 \rangle & \langle g_2 \rangle - j\langle g_3 \rangle \\ \langle g_2 \rangle + j\langle g_3 \rangle & \langle g_0 \rangle - \langle g_1 \rangle \end{bmatrix}$$

$$\langle g_0 \rangle^2 \geq \langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2$$

PARTIALLY POLARISED WAVES

EIGENVALUES DECOMPOSITION

$$\langle [J] \rangle = [U_2] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} [U_2]^{-1} = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} + \lambda_2 \underline{u}_2 \underline{u}_2^{T*}$$

➡ **2 ORTHOGONAL EIGENVECTORS** $[U_2] = [\underline{u}_1, \underline{u}_2]$

➡ **2 REAL EIGENVALUES**

$$\lambda_1 = \frac{1}{2}\left\{ \langle g_0 \rangle + \sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2} \right\}$$

$$\lambda_2 = \frac{1}{2}\left\{ \langle g_0 \rangle - \sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2} \right\}$$

PARTIALLY POLARISED WAVES DESCRIPTORS

Degree of Polarisation

$$DoP = \frac{\sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2}}{\langle g_0 \rangle} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \left(1 - \frac{4\,det([J])}{Trace^2([J])}\right)$$

Polarised Wave Power / Total Wave Power

Anisotropy

$$0 \le DoP \le 1$$

Wave Entropy

$$0 \le H \le 1$$

$$H = -\sum_{i=1}^{i=2} p_i \log_2(p_i)$$

With: $p_i = \frac{\lambda_i}{\lambda_1 + \lambda_2}$

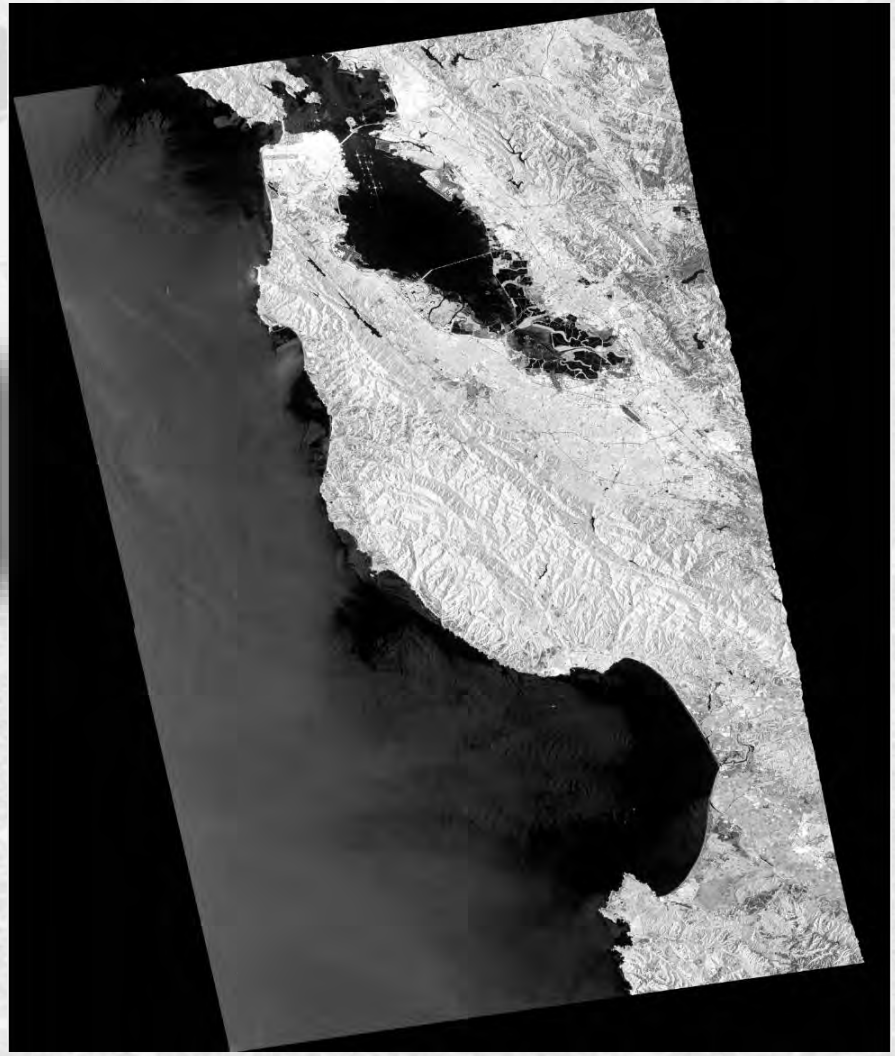
Degree of randomness, statistical disorder

PARTIALLY POLARISED WAVES

13590 x 25896

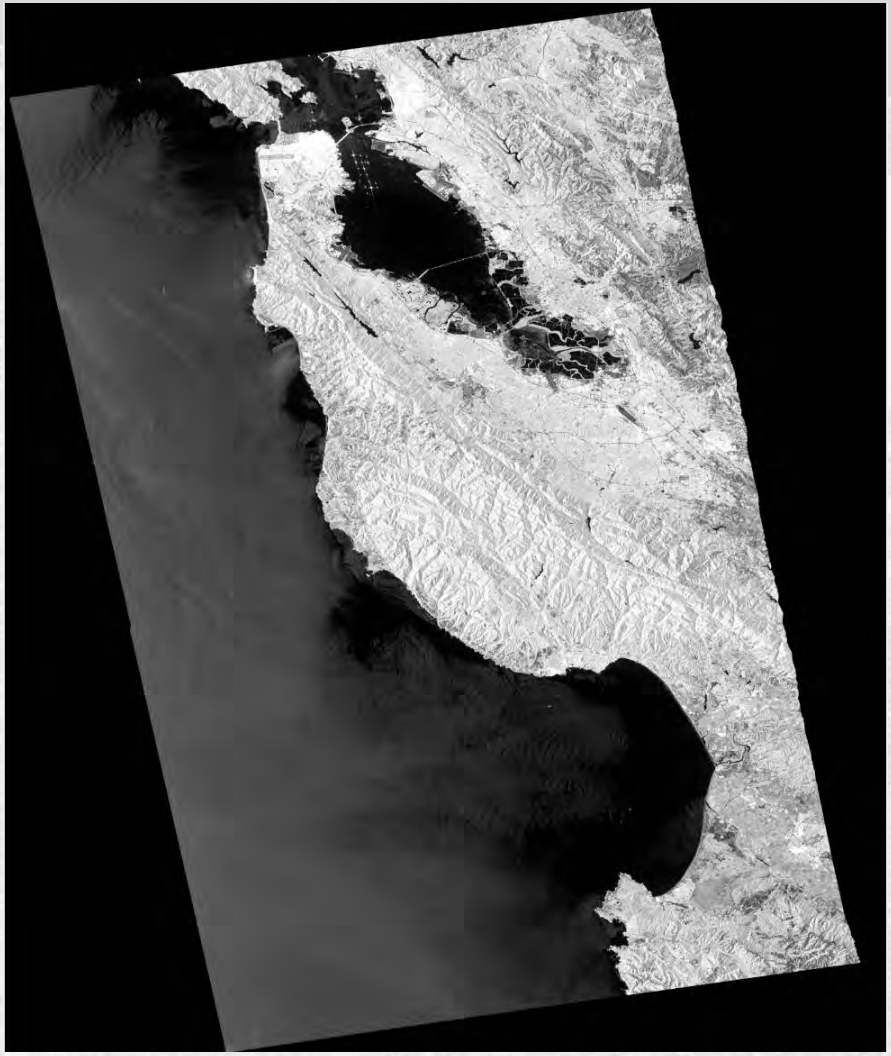
PARTIALLY POLARISED WAVES

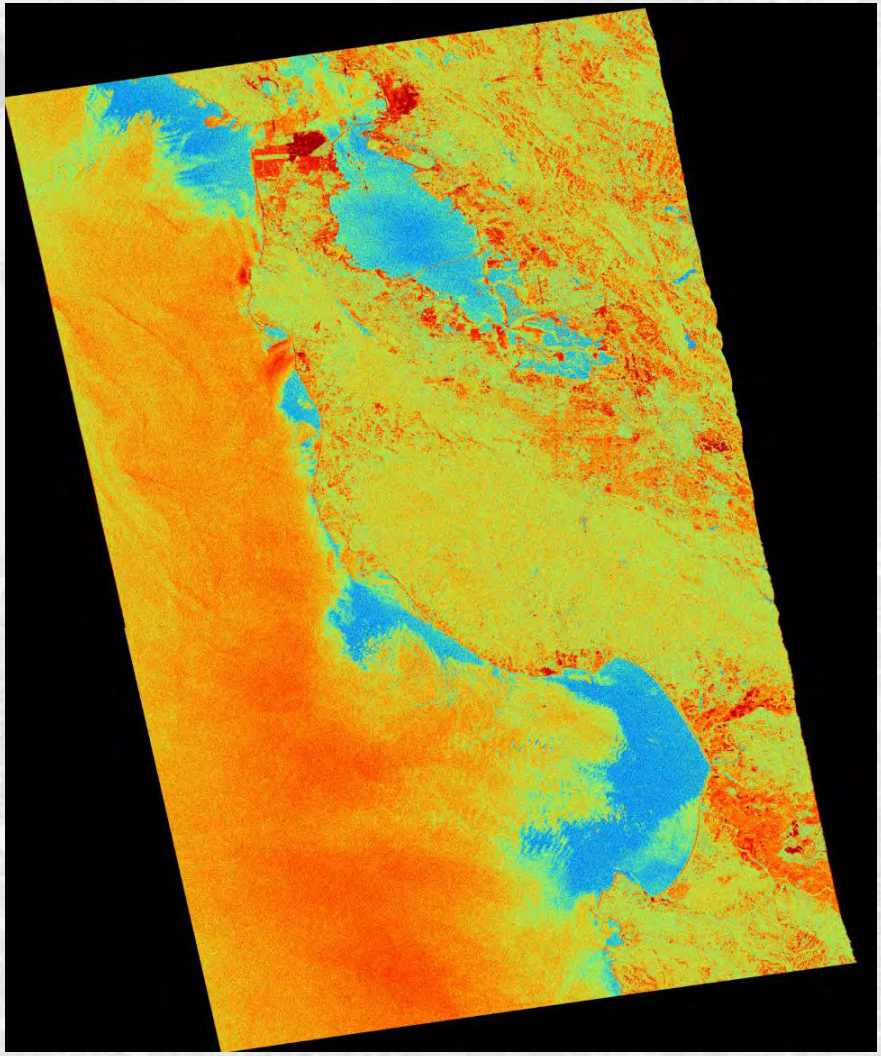
13590 x 25896



6638 x 5567

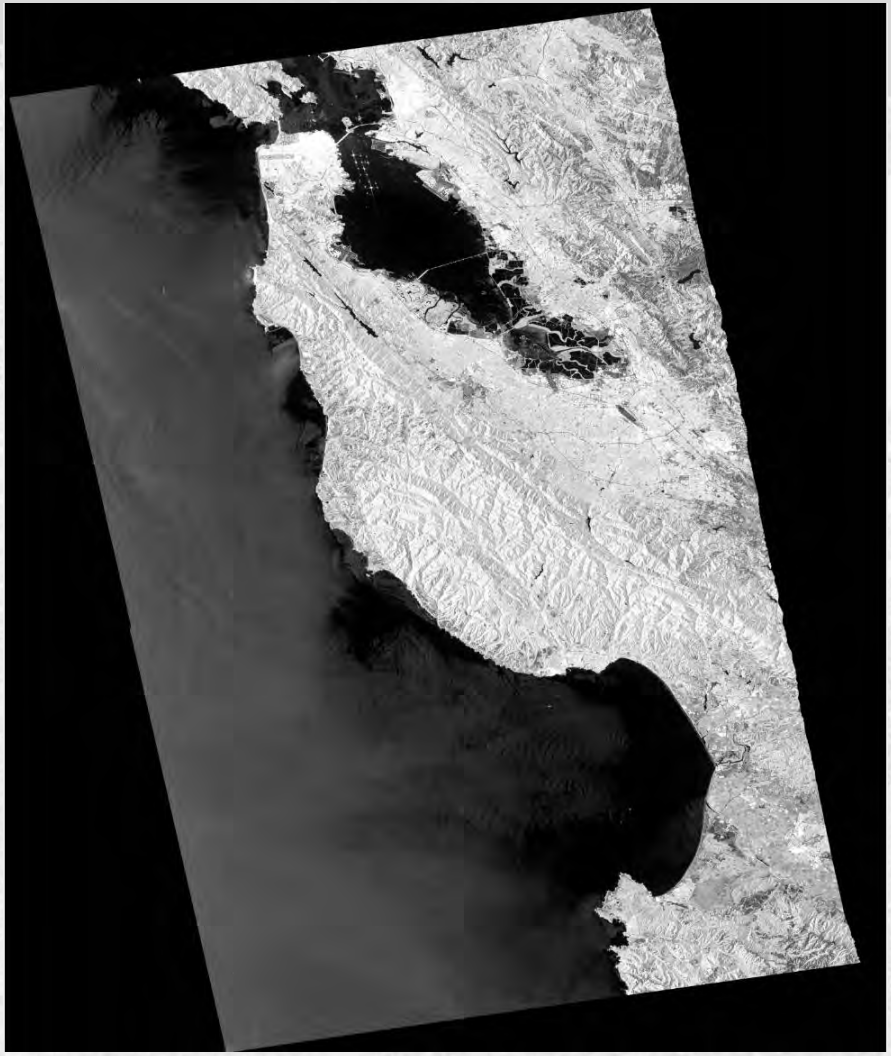
PARTIALLY POLARISED WAVES

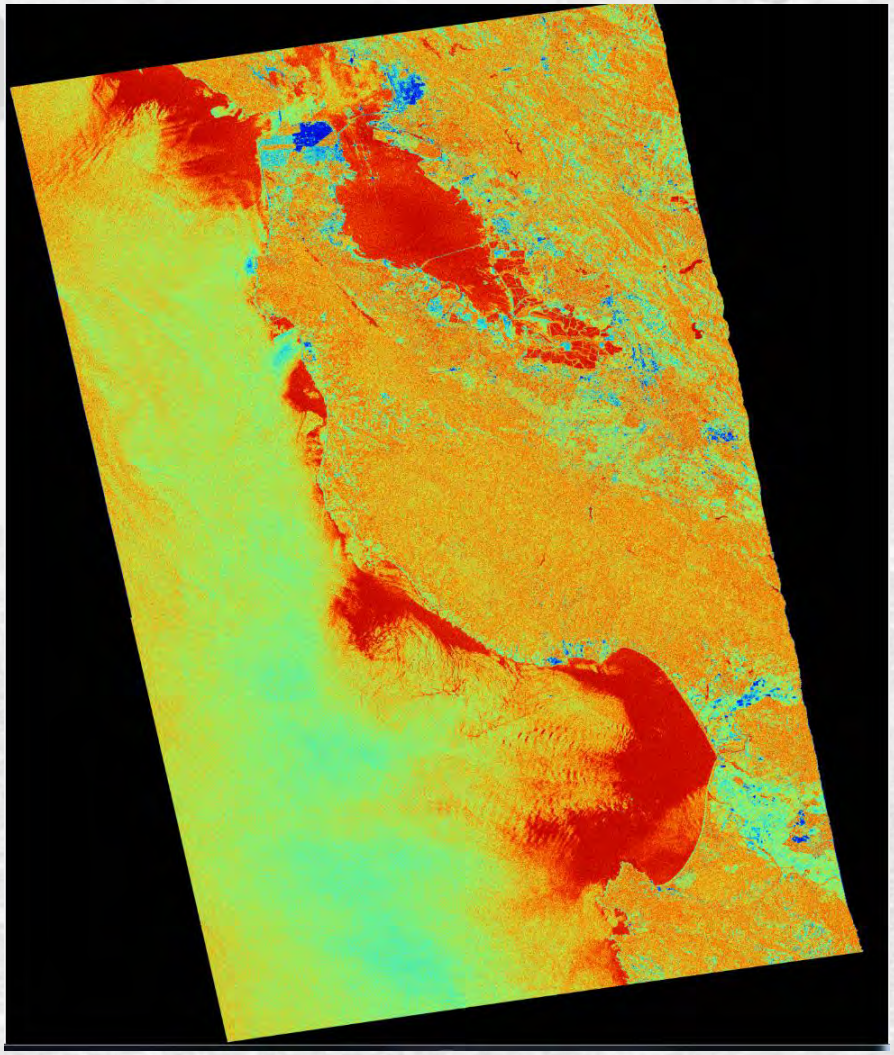




Anisotropy

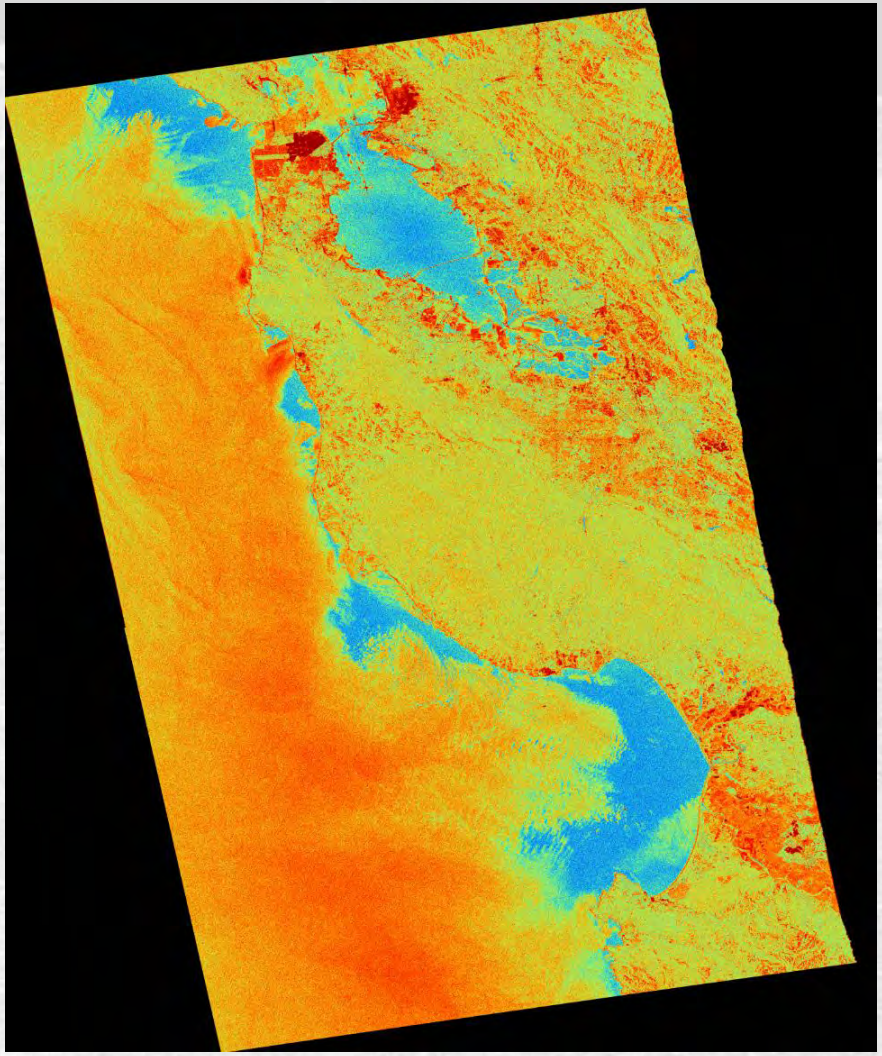
PARTIALLY POLARISED WAVES



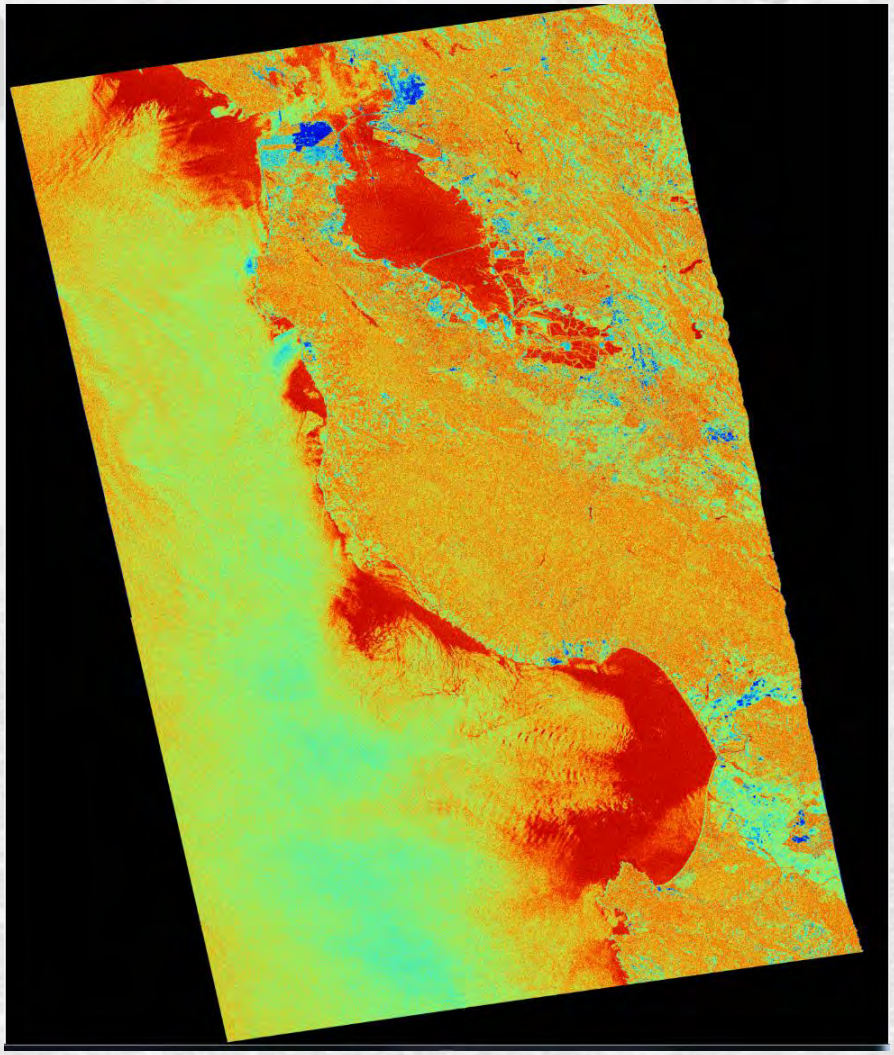


Entropy

PARTIALLY POLARISED WAVES



Anisotropy



Entropy

WAVE DESCRIPTORS

MONOCHROMATIC PLANE WAVES

COMPLEX DOMAIN

JONES VECTOR $\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$

REAL DOMAIN

STOKES VECTOR $\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix}$

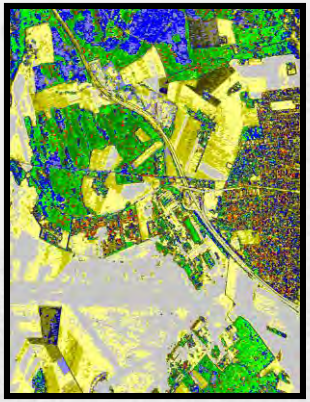
PLANE WAVES FULLY DESCRIBED BY 3 INDEPENDANT PARAMETERS

- $E_{0x}, E_{0x}, \delta = \delta_y - \delta_x$
- (A, ϕ, τ) or (A, γ, δ)
- $\{g_1, g_2, g_3\}$

WAVE POLARIMETRIC DIMENSION = 3

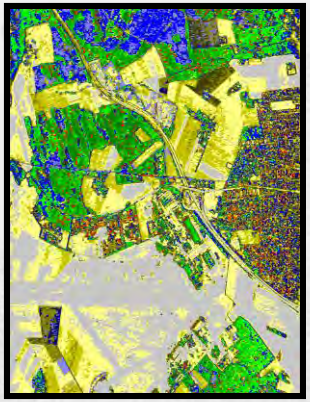
POLARIMETRIC REMOTE SENSING

POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS



POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS



$$\left[\hat{C}_2\right] = E\left(\left[C_2\right]\right) - k\left[E\left(\left[C_2\right]\right) - \left[C_2\right]\right]$$

POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

WAVE DESCRIPTORS

PARTIALLY POLARISED PLANE WAVES

COMPLEX DOMAIN

COVARIANCE MATRIX $\langle [J] \rangle = \langle \underline{E}\,\underline{E}^{T*} \rangle$

REAL DOMAIN

STOKES VECTOR $\langle \underline{g}_{\underline{E}} \rangle = \begin{bmatrix} \langle g_0 \rangle \\ \langle g_1 \rangle \\ \langle g_2 \rangle \\ \langle g_3 \rangle \end{bmatrix}$

PLANE WAVES FULLY DESCRIBED BY 4 INDEPENDANT PARAMETERS

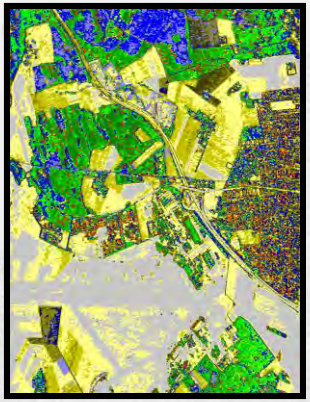
$$\cdot \left\langle |E_x|^2 \right\rangle, \left\langle E_x E_y^* \right\rangle, \left\langle E_y E_x^* \right\rangle, \left\langle |E_y|^2 \right\rangle$$

$$\cdot \left\{ \langle g_0 \rangle, \langle g_1 \rangle, \langle g_2 \rangle, \langle g_3 \rangle \right\}$$

WAVE POLARIMETRIC DIMENSION = 4

POLARIMETRIC REMOTE SENSING

POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS



POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

MODEL BASED DECOMPOSITION

3 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_S] + [T_D] + [T_V]$$

UNITARY TRANSFORMATION

$$[T] = [D_3][C][D_3]^{T*}$$

$$\langle [C] \rangle = [C_S] + [C_D] + [C_V]$$

$[C_S]$	$[C_D]$	$[C_V]$
SINGLE SCATTERING	DOUBLE SCATTERING	VOLUME SCATTERING

COVARIANCE MATRIX

$$[C_3] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX}S_{XX}^* & \sqrt{2}S_{XX}S_{XY}^* & S_{XX}S_{YY}^* \\ \sqrt{2}S_{XY}S_{XX}^* & 2S_{XY}S_{XY}^* & \sqrt{2}S_{XY}S_{YY}^* \\ S_{YY}S_{XX}^* & \sqrt{2}S_{YY}S_{XY}^* & S_{YY}S_{YY}^* \end{bmatrix}$$

$$\Downarrow$$

$$[J] = \underline{E} \cdot \underline{E}^{T*} = \begin{bmatrix} E_x E_x^* & E_x E_y^* \\ E_y E_x^* & E_y E_y^* \end{bmatrix} = [C_2] = \begin{bmatrix} S_{XX}S_{XX}^* & S_{XX}S_{XY}^* \\ S_{XY}S_{XX}^* & S_{XY}S_{XY}^* \end{bmatrix}$$

COVARIANCE MATRIX

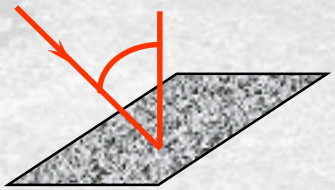
$$[C_3] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX}S_{XX}^* & \sqrt{2}S_{XX}S_{XY}^* & S_{XX}S_{YY}^* \\ \sqrt{2}S_{XY}S_{XX}^* & 2S_{XY}S_{XY}^* & \sqrt{2}S_{XY}S_{YY}^* \\ S_{YY}S_{XX}^* & \sqrt{2}S_{YY}S_{XY}^* & S_{YY}S_{YY}^* \end{bmatrix}$$

$$[J] = \underline{E} \cdot \underline{E}^{T*} = \begin{bmatrix} E_x E_x^* & E_x E_y^* \\ E_y E_x^* & E_y E_y^* \end{bmatrix} = [C_2] = \begin{bmatrix} S_{XX}S_{XX}^* & S_{XX}S_{XY}^* \\ S_{XY}S_{XX}^* & S_{XY}S_{XY}^* \end{bmatrix}$$

MODEL BASED DECOMPOSITION

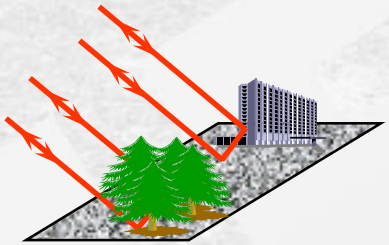
DUAL - POL (XX, XY)

SINGLE SCATTERING
(ROUGH SURFACE)



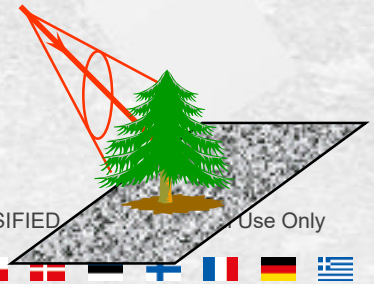
$$[C_{3S}] = f_S \begin{bmatrix} \beta^2 & 0 & \beta \\ 0 & 0 & 0 \\ \beta & 0 & 1 \end{bmatrix} \Rightarrow [C_{2S}] = f_S \begin{bmatrix} \beta^2 & 0 \\ 0 & 0 \end{bmatrix}$$

DOUBLE SCATTERING



$$[C_{3D}] = f_D \begin{bmatrix} \alpha^2 & 0 & -\alpha \\ 0 & 0 & 0 \\ -\alpha & 0 & 1 \end{bmatrix} \Rightarrow [C_{2D}] = f_D \begin{bmatrix} \alpha^2 & 0 \\ 0 & 0 \end{bmatrix}$$

VOLUME SCATTERING



$$[C_{3V}] = f_V \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & 1 \end{bmatrix} \Rightarrow [C_{2V}] = f_V \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

MODEL BASED DECOMPOSITION

$$\langle [C_3] \rangle = \begin{bmatrix} f_S\beta^2 + f_D\alpha^2 + f_V & 0 & f_S\beta - f_D\alpha + \frac{f_V}{3} \\ 0 & \frac{2f_V}{3} & 0 \\ f_S\beta - f_D\alpha + \frac{f_V}{3} & 0 & f_S + f_D + f_V \end{bmatrix}$$

DUAL - POL (XX, XY)

$$\langle [C_2] \rangle = \begin{bmatrix} f_S\beta^2 + f_D\alpha^2 + f_V & 0 \\ 0 & \frac{f_V}{3} \end{bmatrix}$$

TWIN - POL (XX, YY)

$$\langle [C_2] \rangle = \begin{bmatrix} f_S\beta^2 + f_D\alpha^2 + f_V & f_S\beta - f_D\alpha + \frac{f_V}{3} \\ f_S\beta - f_D\alpha + \frac{f_V}{3} & f_S + f_D + f_V \end{bmatrix}$$

H / A / α DECOMPOSITION

WAVE COVARIANCE MATRIX

$$\langle [C_2] \rangle = \langle \underline{E}\,\underline{E}^{T*} \rangle = \begin{bmatrix} \langle |E_x|^2 \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle |E_y|^2 \rangle \end{bmatrix}$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle [C_2] \rangle = [U_2] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} [U_2]^{-1} = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} + \lambda_2 \underline{u}_2 \underline{u}_2^{T*}$$

2 ORTHOGONAL EIGENVECTORS $[U_2] = [\underline{u}_1, \underline{u}_2]$ $\quad \underline{u}_i = \begin{bmatrix} \cos(\alpha_i) \\ \sin(\alpha_i)\, e^{j\delta_i} \end{bmatrix}$

2 REAL EIGENVALUES $\lambda_1 > \lambda_2$

H / A / $\underline{\alpha}$ DECOMPOSITION

PROBABILITIES

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{2} \lambda_k}$$

AVERAGED PARAMETERS

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2$$

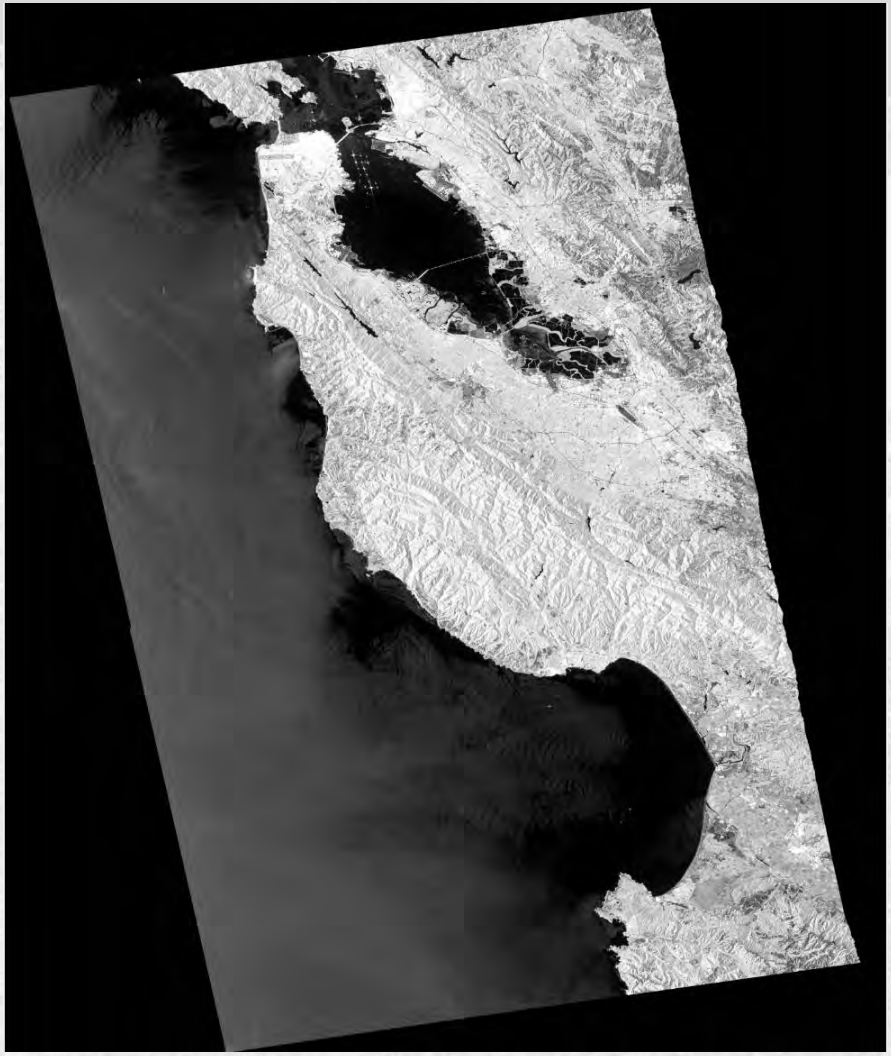
$$\underline{\delta} = P_1\delta_1 + P_2\delta_2$$

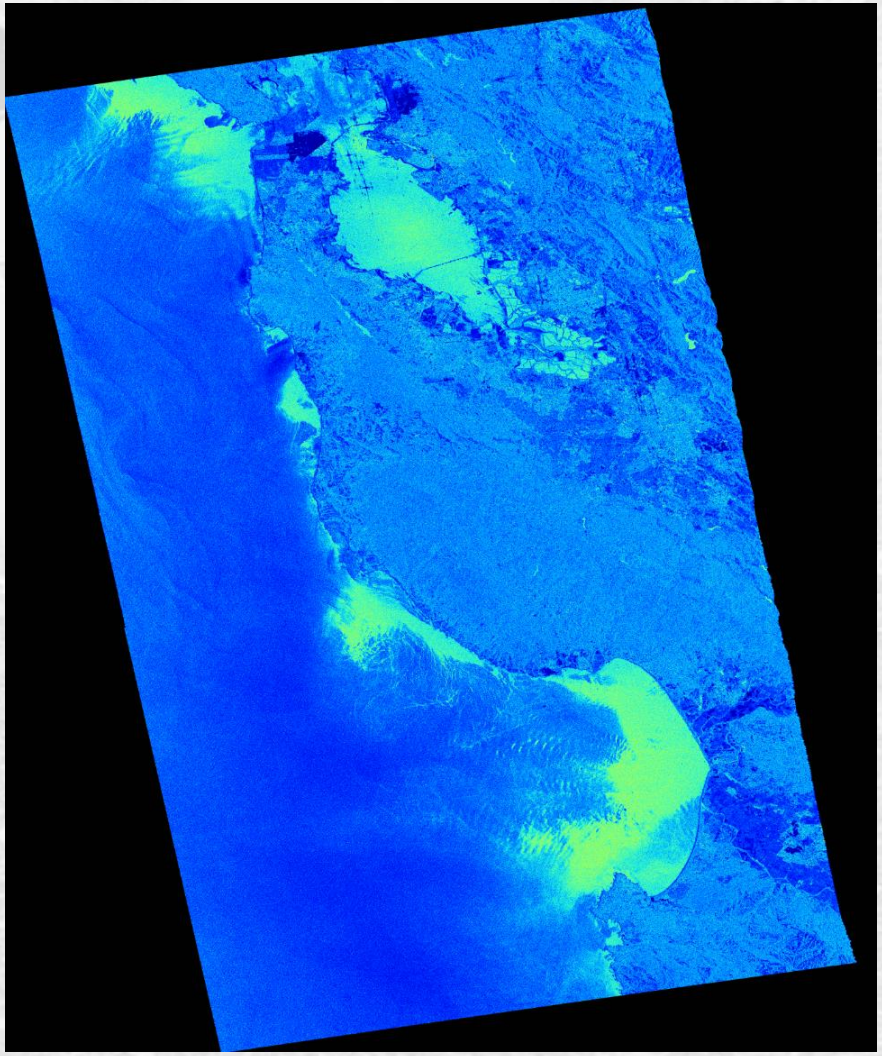
$$\underline{\lambda} = P_1\lambda_1 + P_2\lambda_2$$

JONES VECTOR $\underline{E}_0$

$$\underline{E}_0 = \sqrt{\underline{\alpha}} \begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha}) e^{j\underline{\delta}} \end{bmatrix}$$

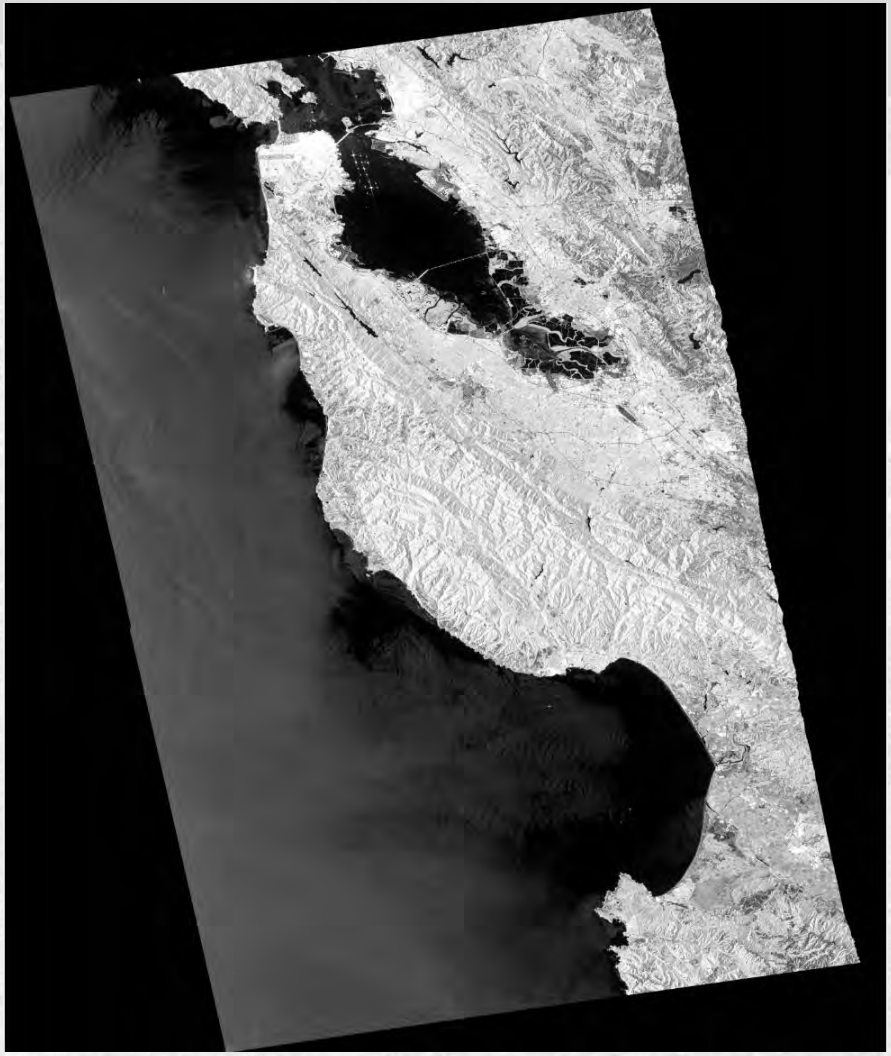
H / A / α DECOMPOSITION

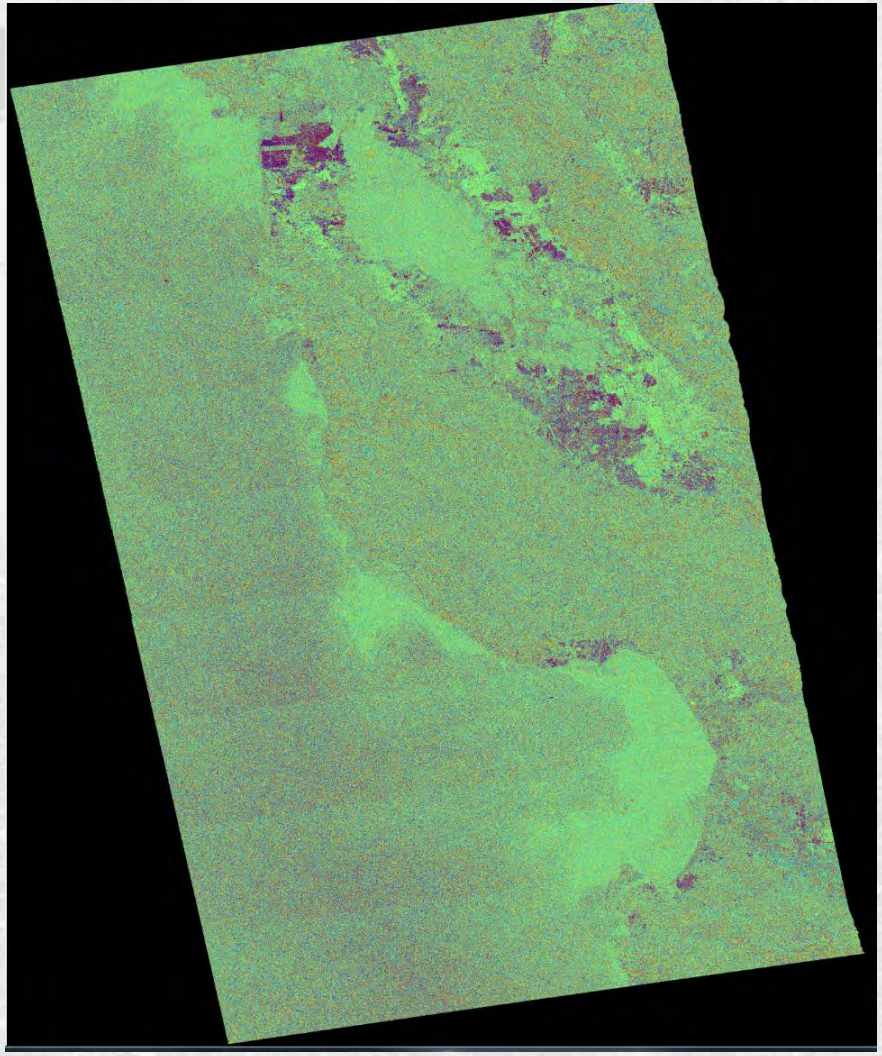




Alpha

H / A / α DECOMPOSITION





Delta

H / A / α DECOMPOSITION

Degree of Polarisation = Anisotropy

$$DoP = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$$

Wave Entropy

$$H = -\sum_{i=1}^{i=2} p_i \log_2(p_i)$$

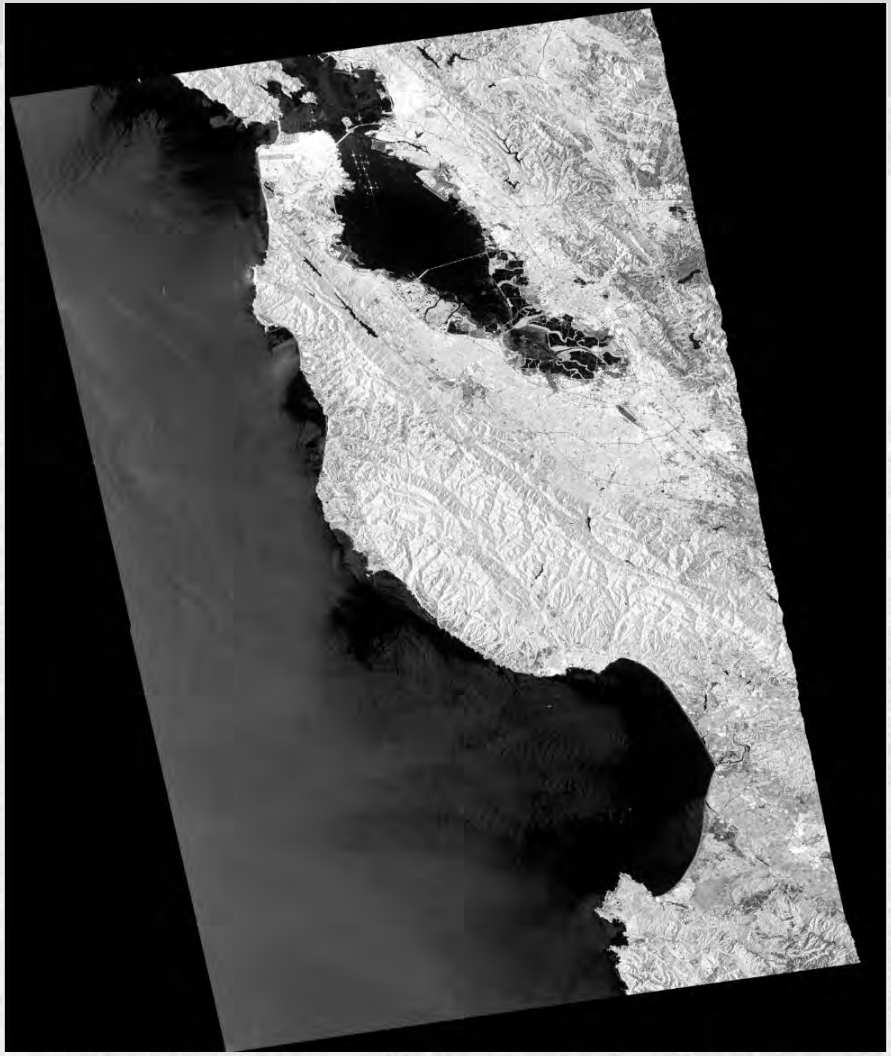
Shannon Entropy

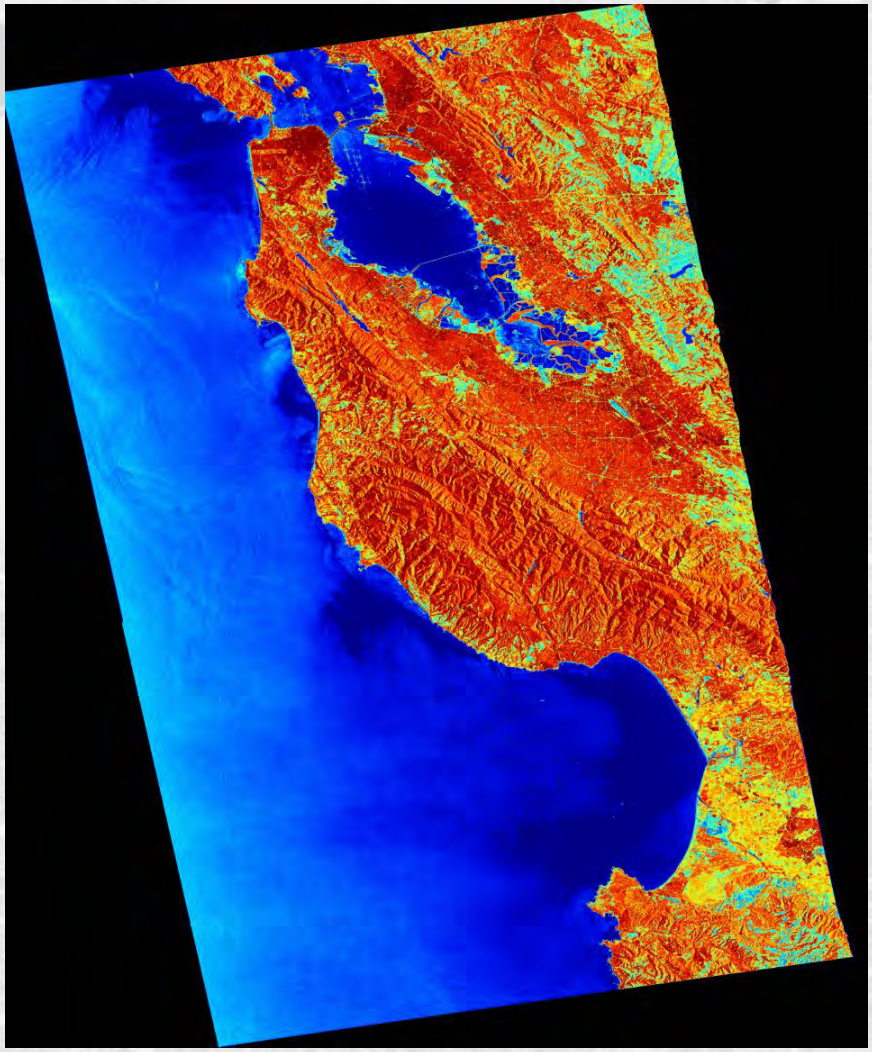
$$SE = \log\left(\pi^2 e^2 |C_2|\right) = SE_I + SE_P$$

$$SE_I = 2\log\left(\frac{\pi e I_T}{2}\right) = 2\log\left(\frac{\pi e Tr(C_2)}{2}\right)$$

$$SE_P = \log\left(1 - p_T^2\right) = \log\left(4\frac{|C_2|}{Tr(C_2)^2}\right)$$

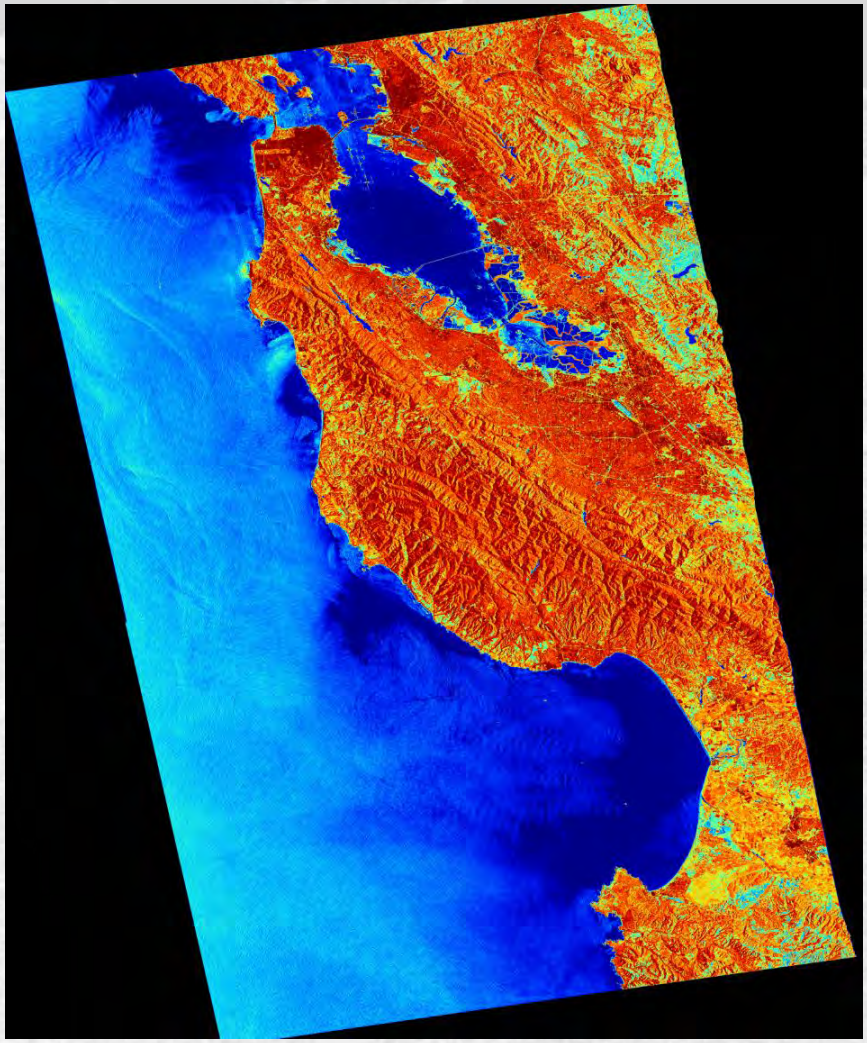
H / A / α DECOMPOSITION



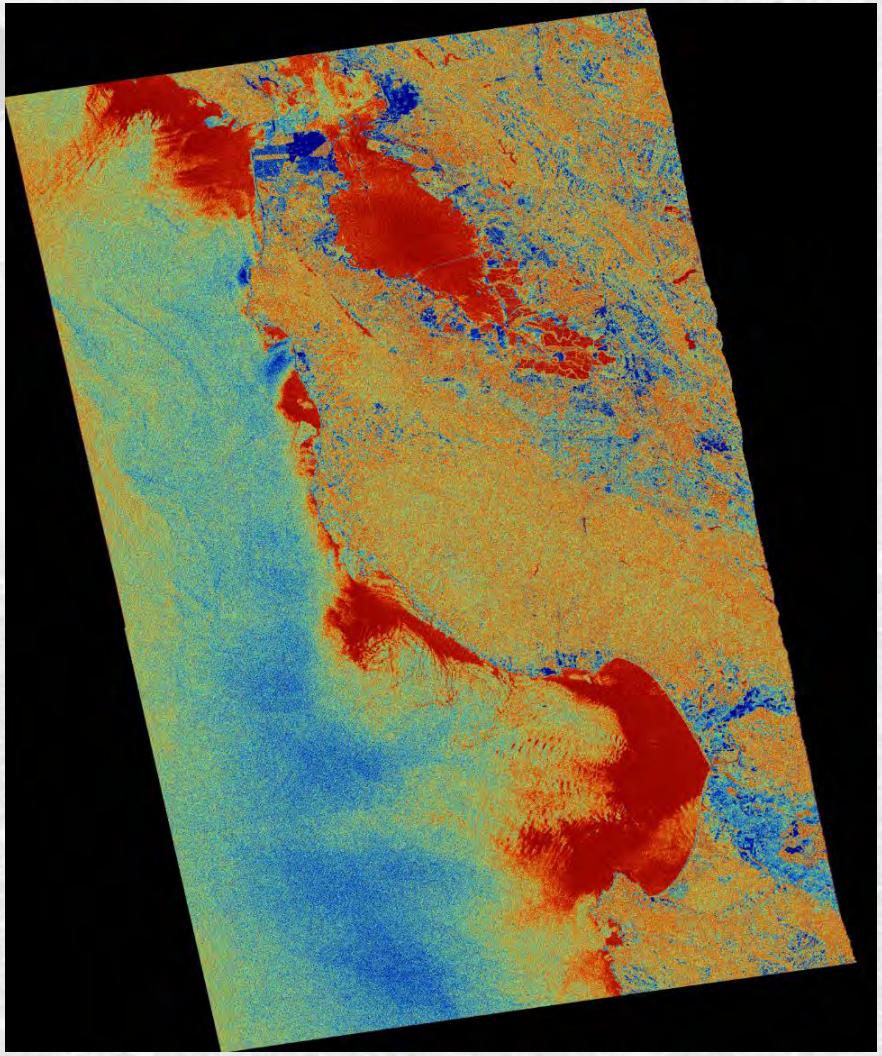


Shannon - Entropy

H / A / α DECOMPOSITION

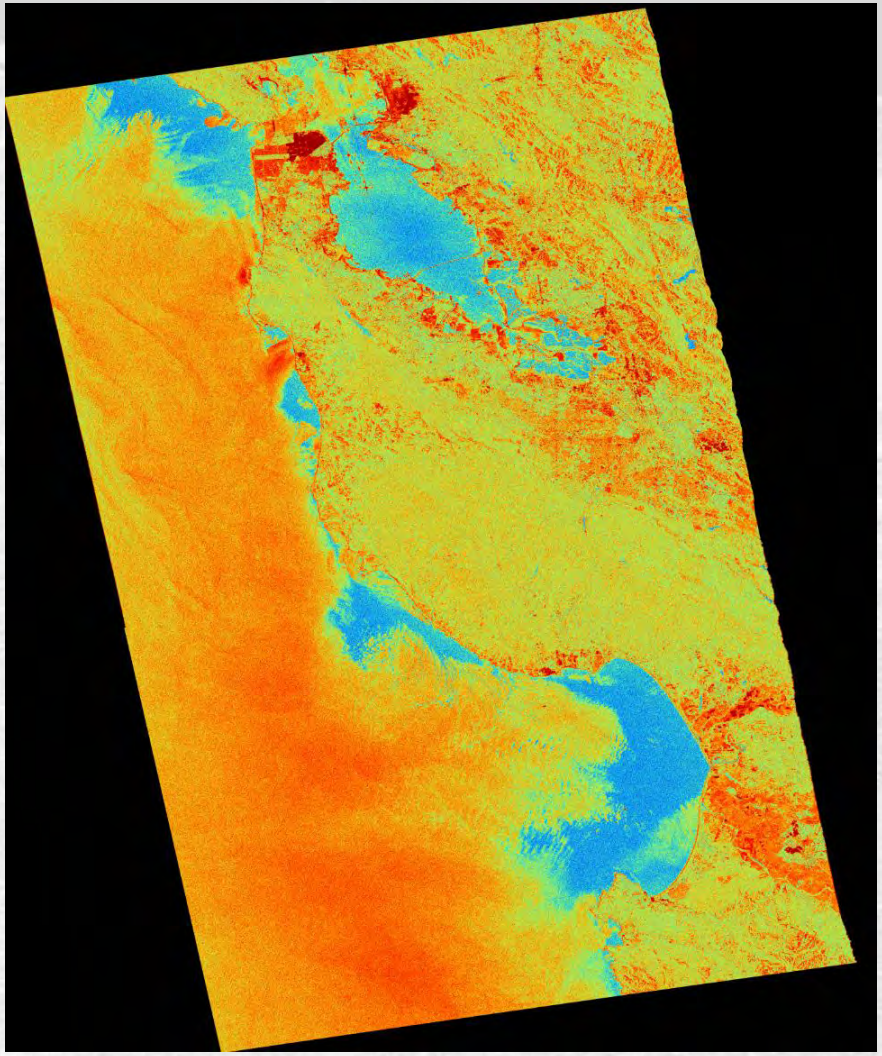


Shannon - Entropy - I

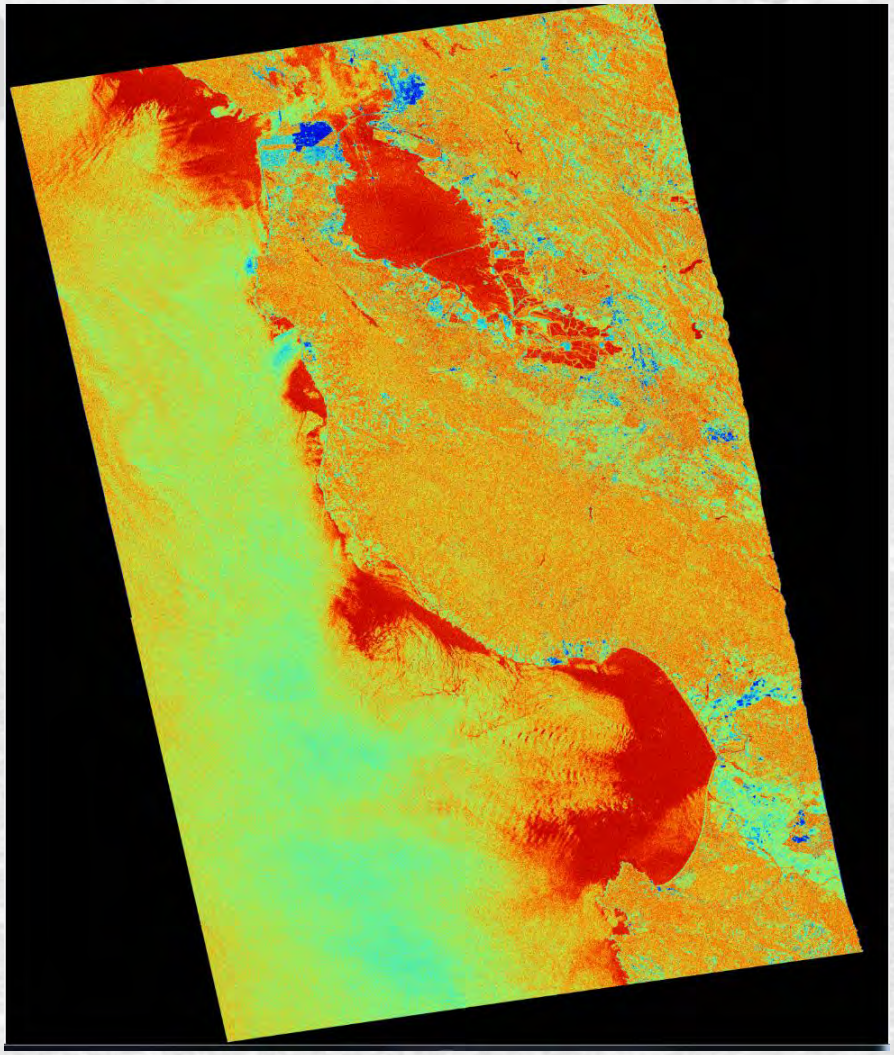


Shannon - Entropy - P

H / A / α DECOMPOSITION



Anisotropy



Entropy

DUAL-POL EIGENVALUES / EIGENVECTORS ANALYSIS

$$\langle [C_2] \rangle \equiv \{\lambda_i, \underline{v}_i\} \equiv \{\alpha_i, \delta_i, \lambda_i\}$$

$$\Downarrow$$

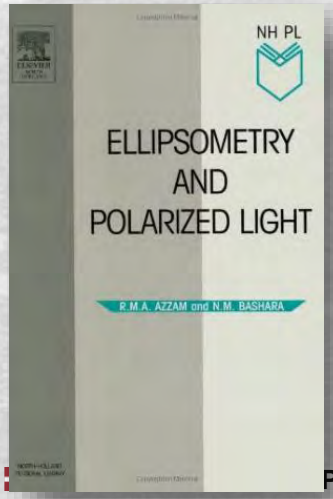
$$[C_2]_0 \equiv \sum_i p_i \{\alpha_i, \delta_i, \lambda_i\} = \{\alpha_0, \delta_0, \lambda_0\}$$

DUAL-POL MODEL BASED

$$\langle [C_2] \rangle \equiv [C_2]_? \oplus \cdots$$

ELLIPSOMETRIC DECOMPOSITION ?

PHYSICAL MEANINGS ?



Questions ?