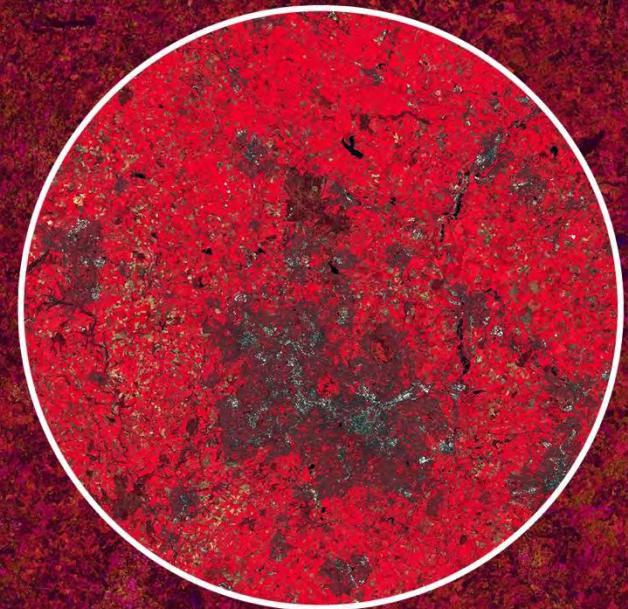


→ **8th ADVANCED TRAINING COURSE
ON LAND REMOTE SENSING**

10–14 September 2018
University of Leicester | United Kingdom



Forestry applications with Polarimetry
and Interferometry (Lecture)

Laurent FERRO-FAMIL, Eric POTTIER

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Eric POTTIER

eric.pottier@univ-rennes1.fr

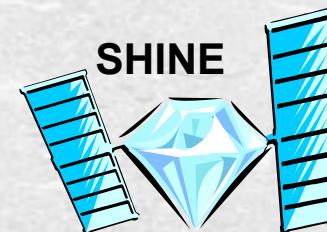


I.E.T.R. - UMR CNRS 6164
Université de Rennes I - Campus de Beaulieu
Pôle Micro Ondes Radar - Bat 11D
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CS 74205 - 35042 Rennes Cedex – France



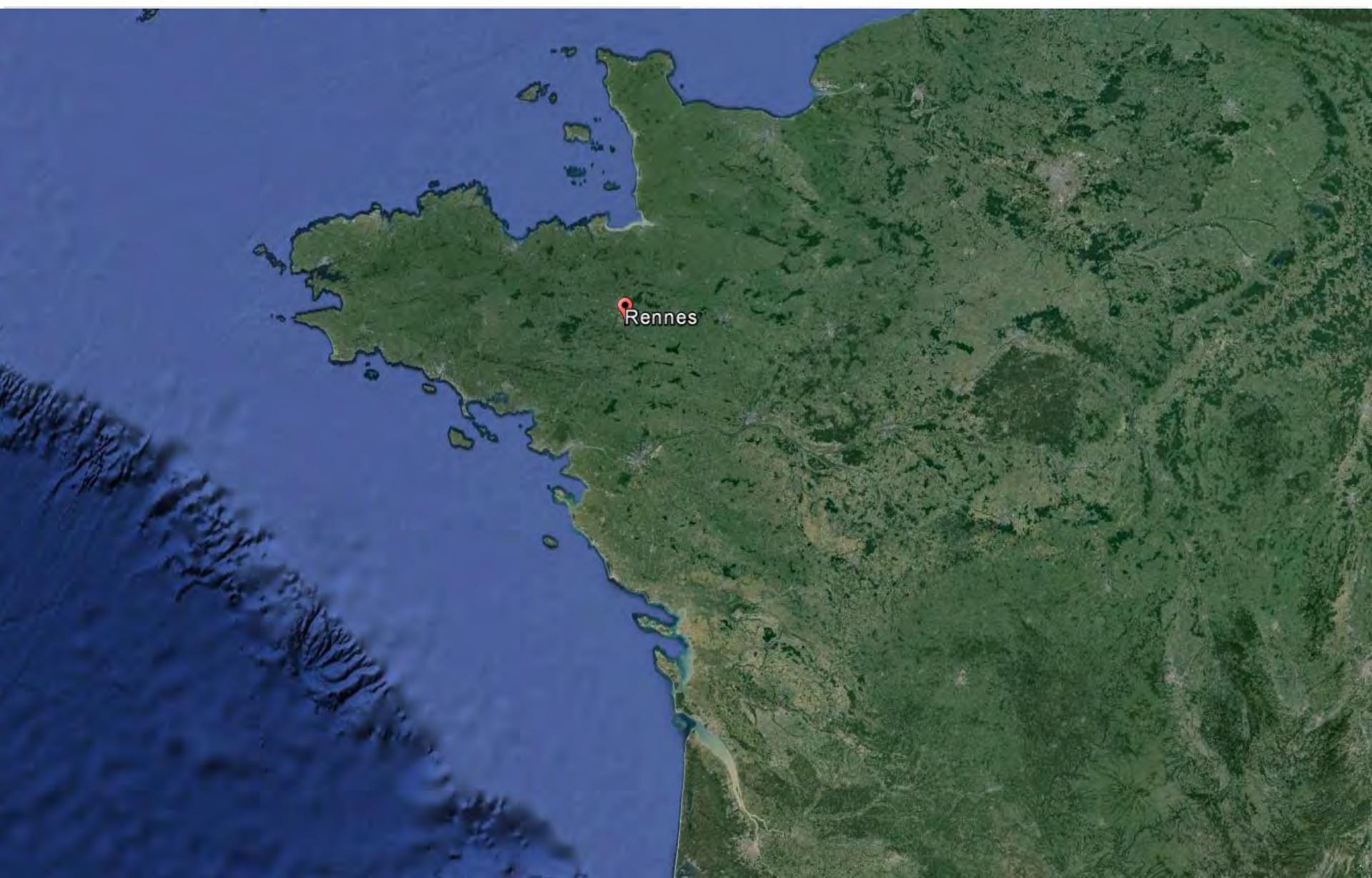
Laurent FERRO-FAMIL

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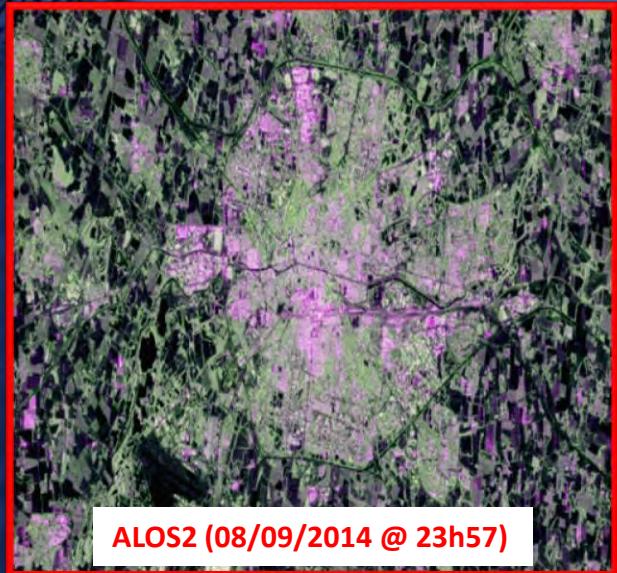
SAR & Hyperspectral multi-modal Imaging
and signal processing,
Electromagnetic modeling



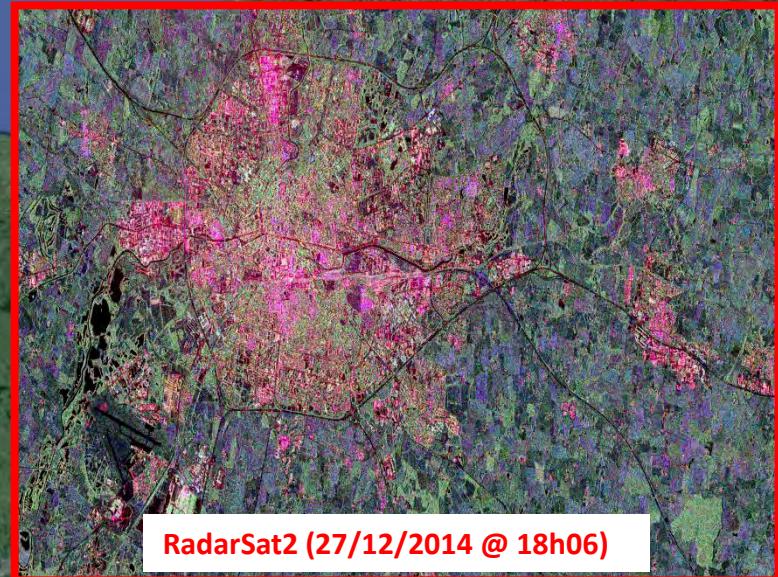




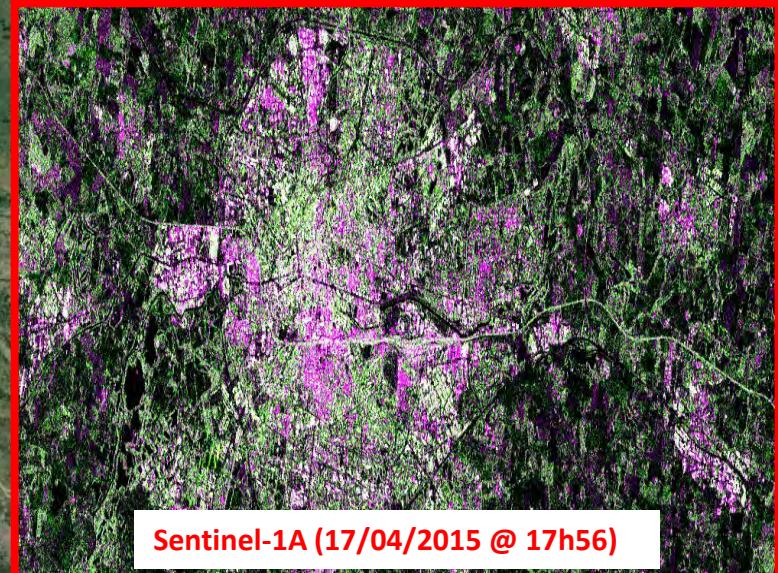
ALOS1 (30/04/2008 @ 22h34)



ALOS2 (08/09/2014 @ 23h57)



RadarSat2 (27/12/2014 @ 18h06)



Sentinel-1A (17/04/2015 @ 17h56)

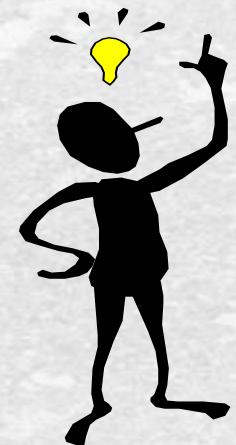


**To provide the minimum, but necessary,
amount of knowledge required
to understand and to practice :**

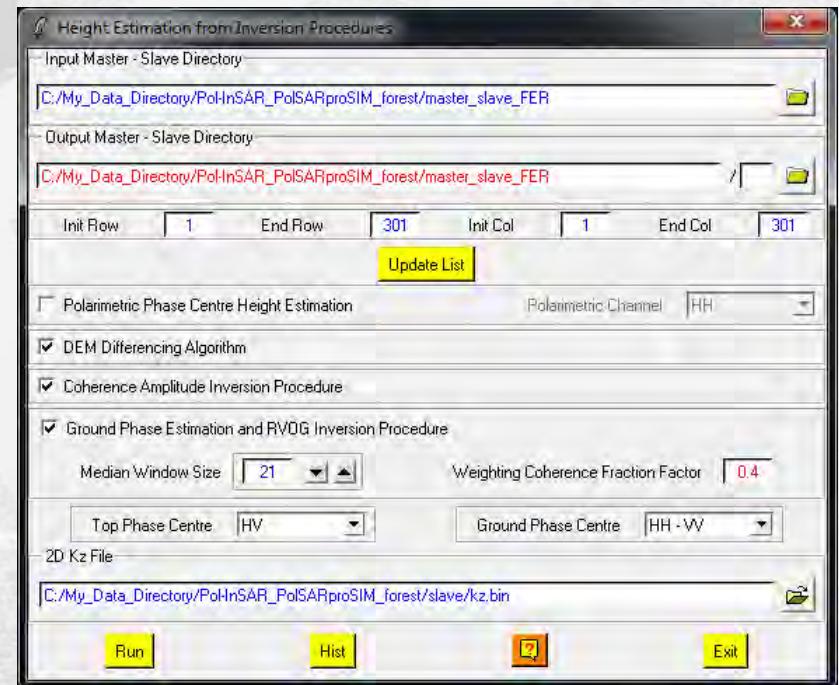


SAR Polarimetry + Interferometry (Pol-InSAR)

for forestry applications



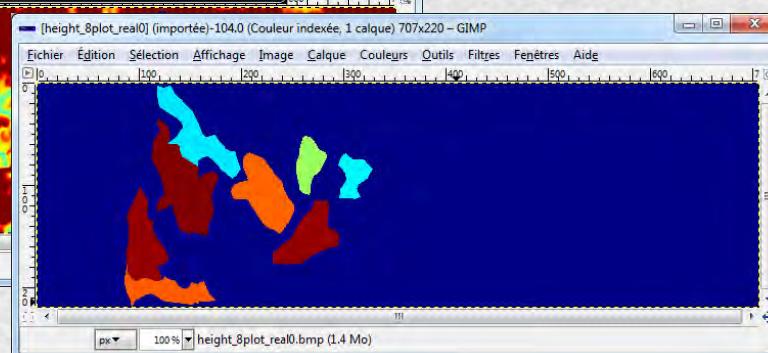
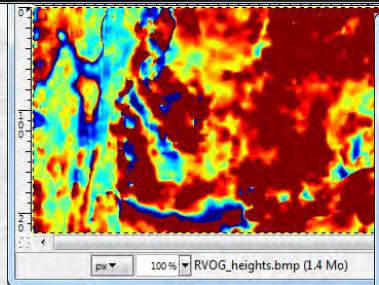
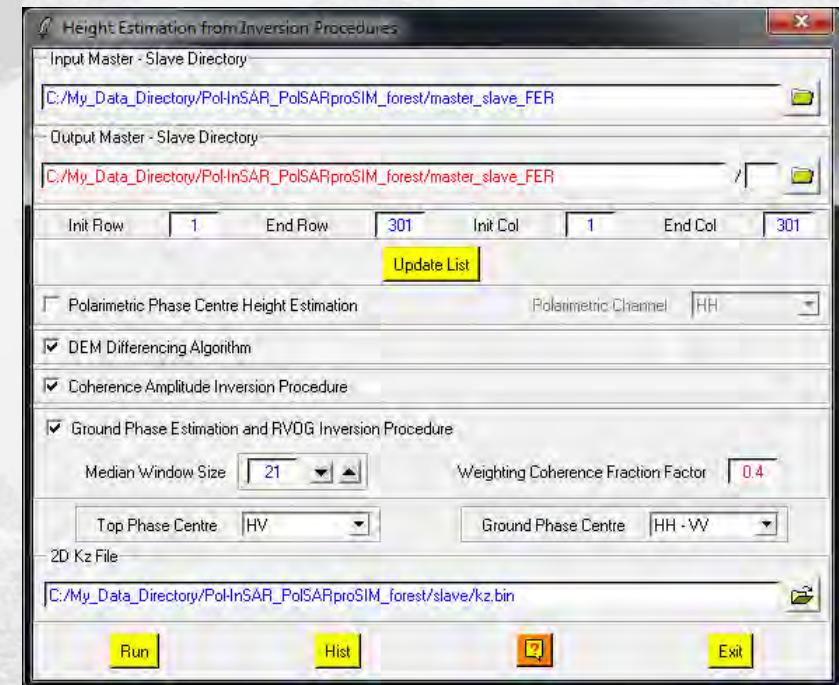
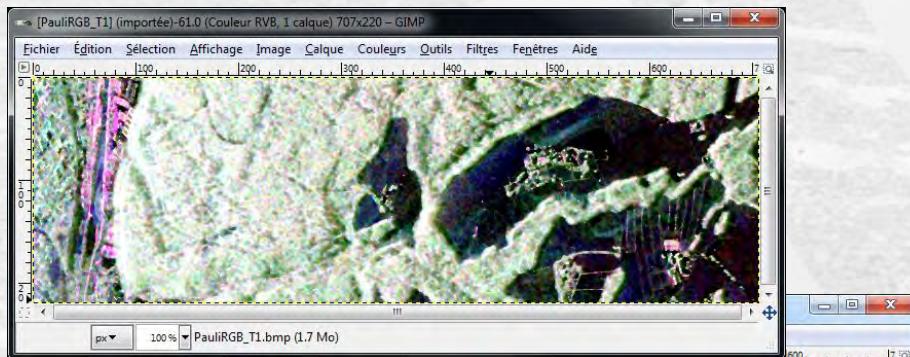
Objectives

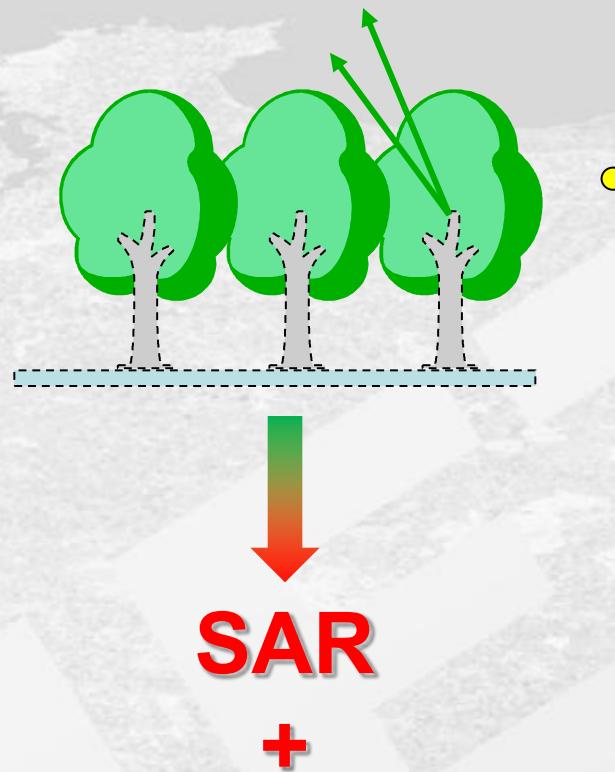


PolSARpro - practicals

INVERSION PROCEDURES

- DEM Differencing Algorithm
- Coherence Amplitude Inversion Procedure
- Ground Phase Estimation &
- RVOG Inversion Procedure

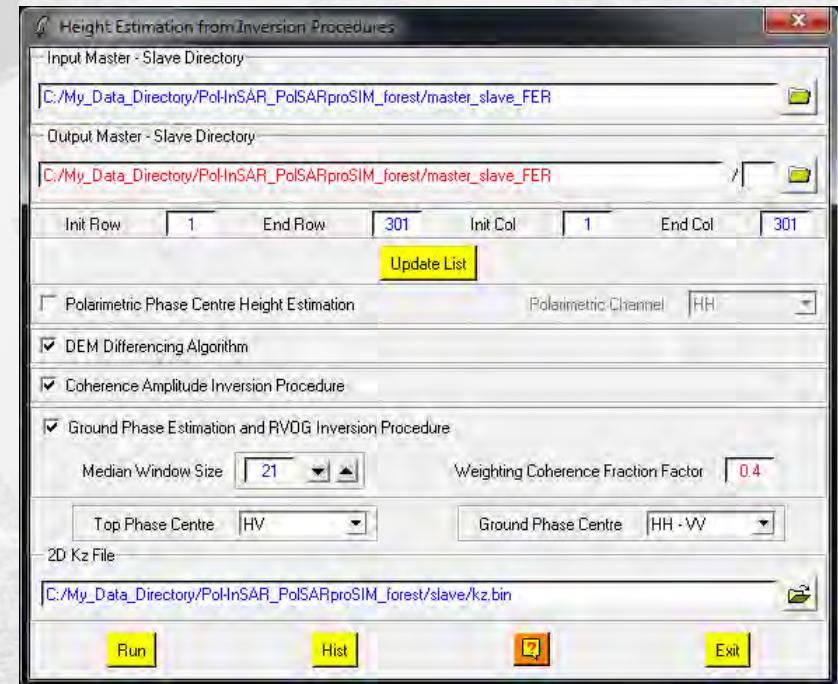




Interferometry
+
Polarimetry



Pol-InSAR





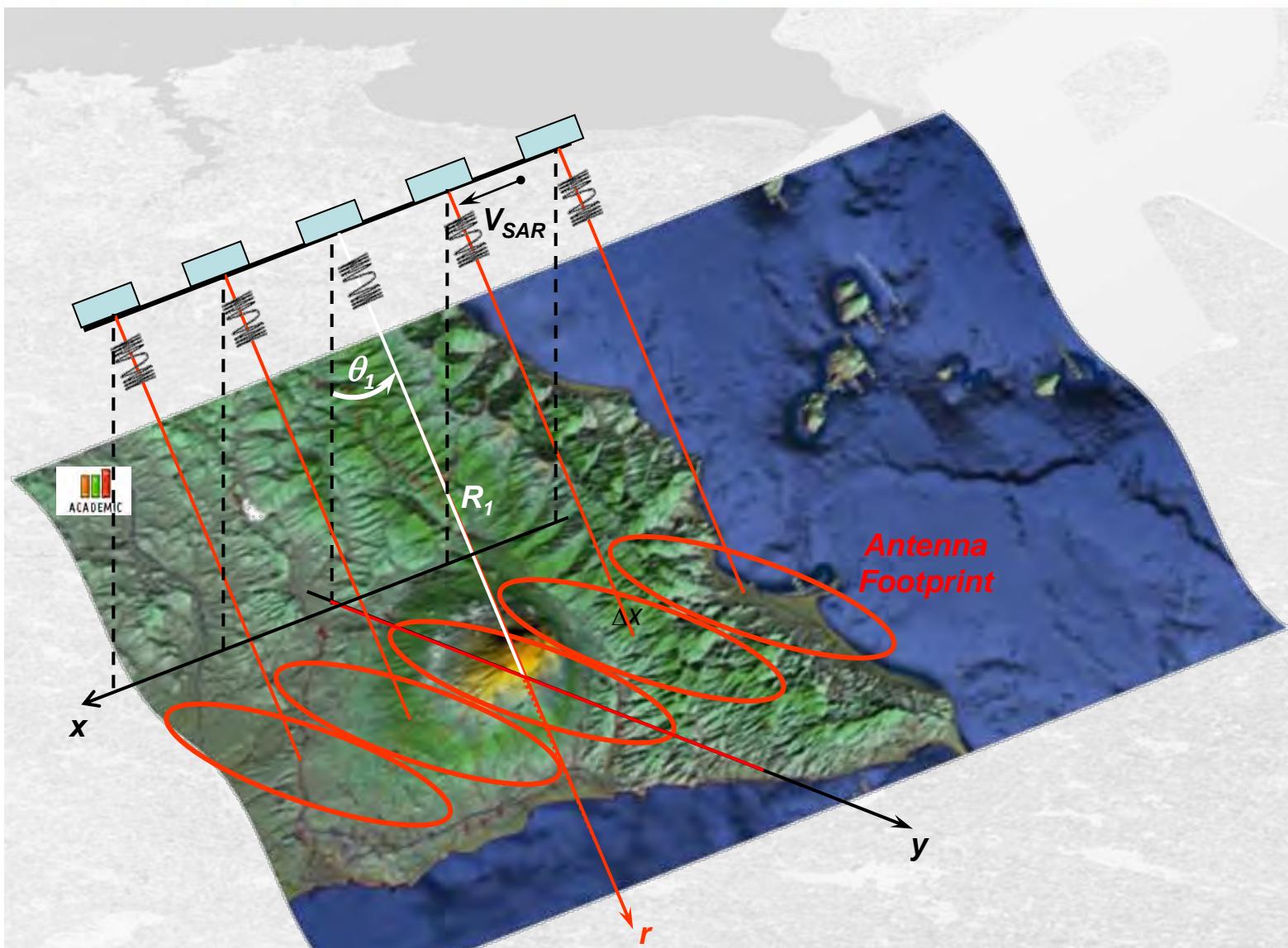
SAR

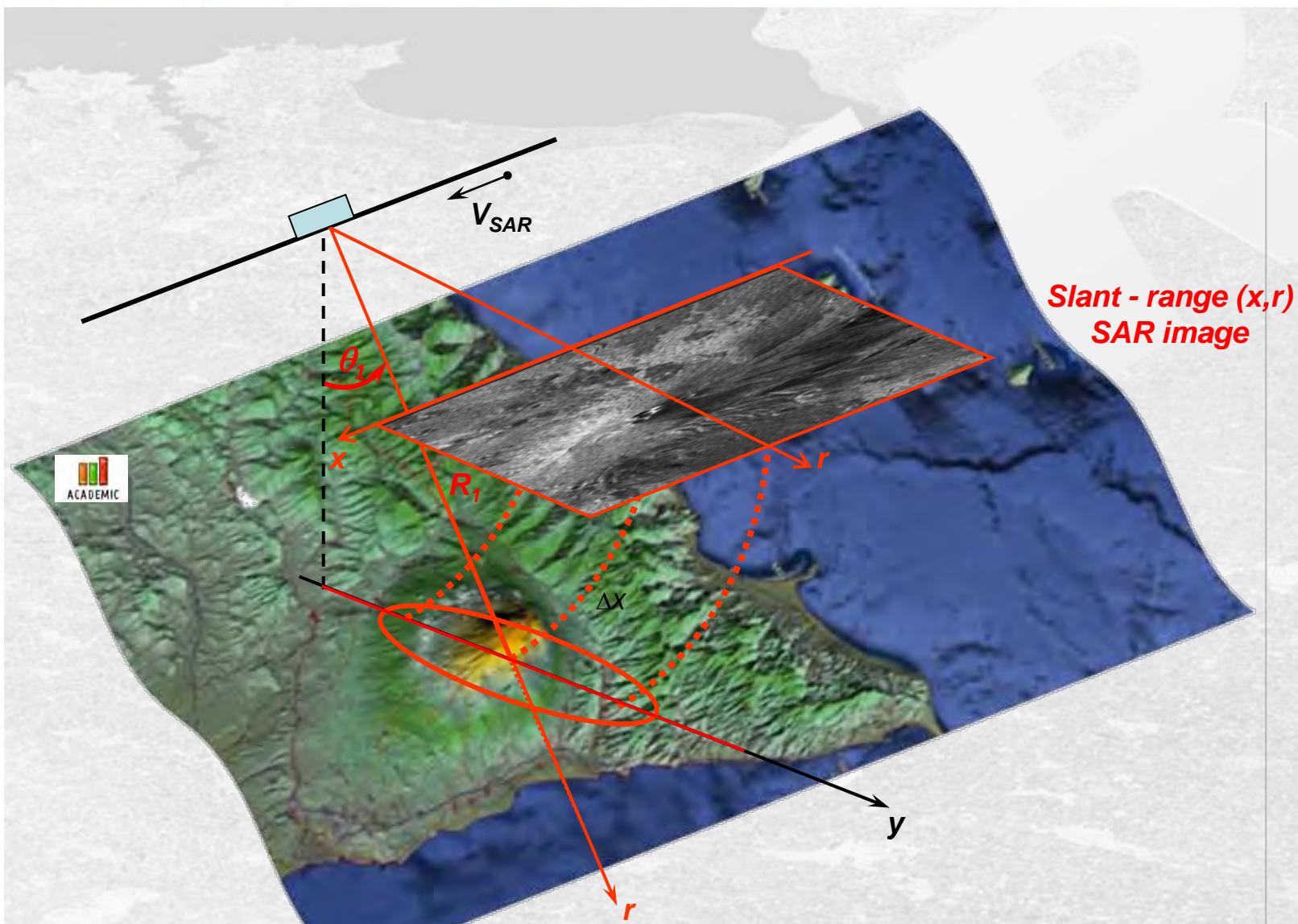
+

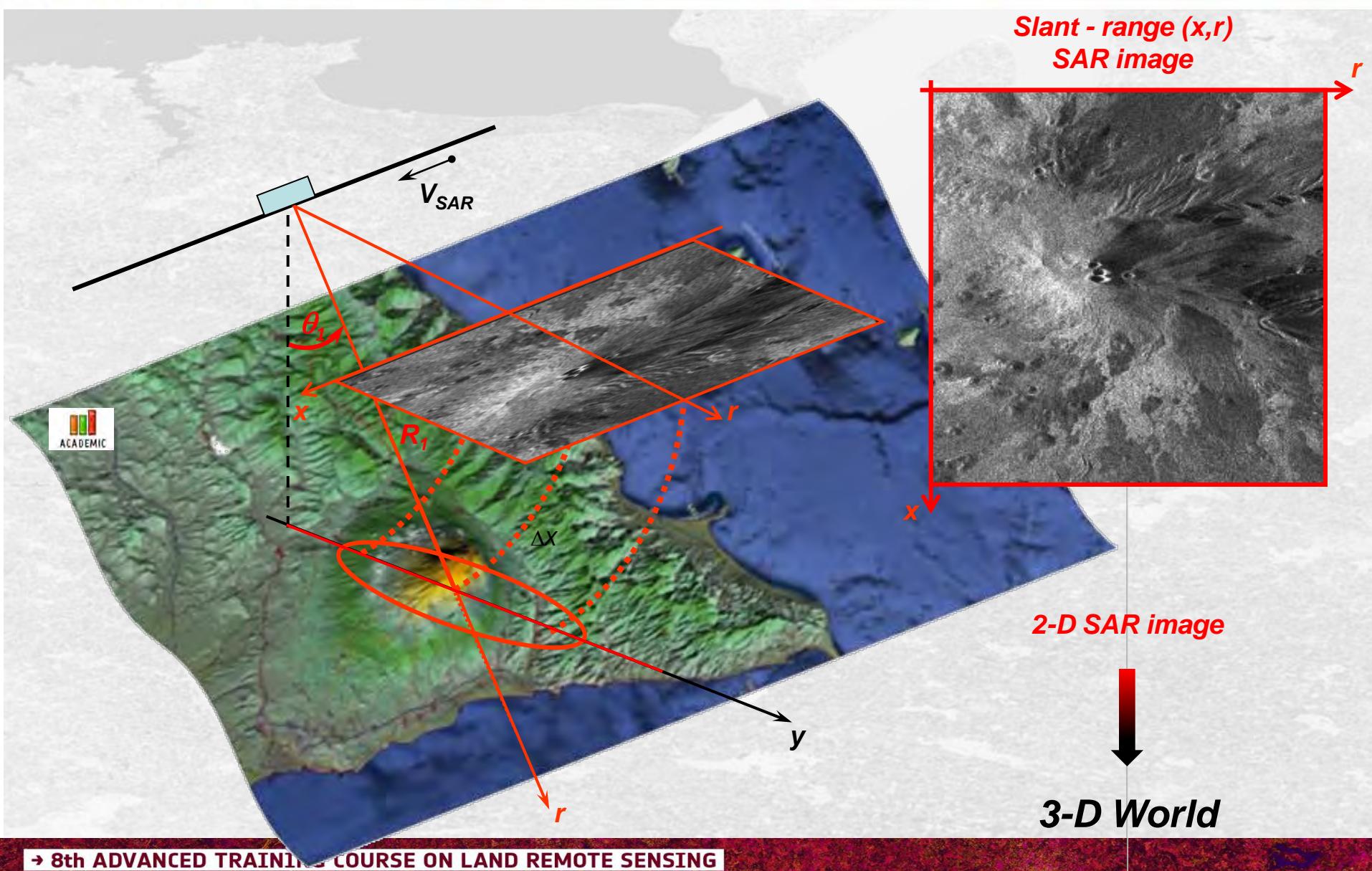
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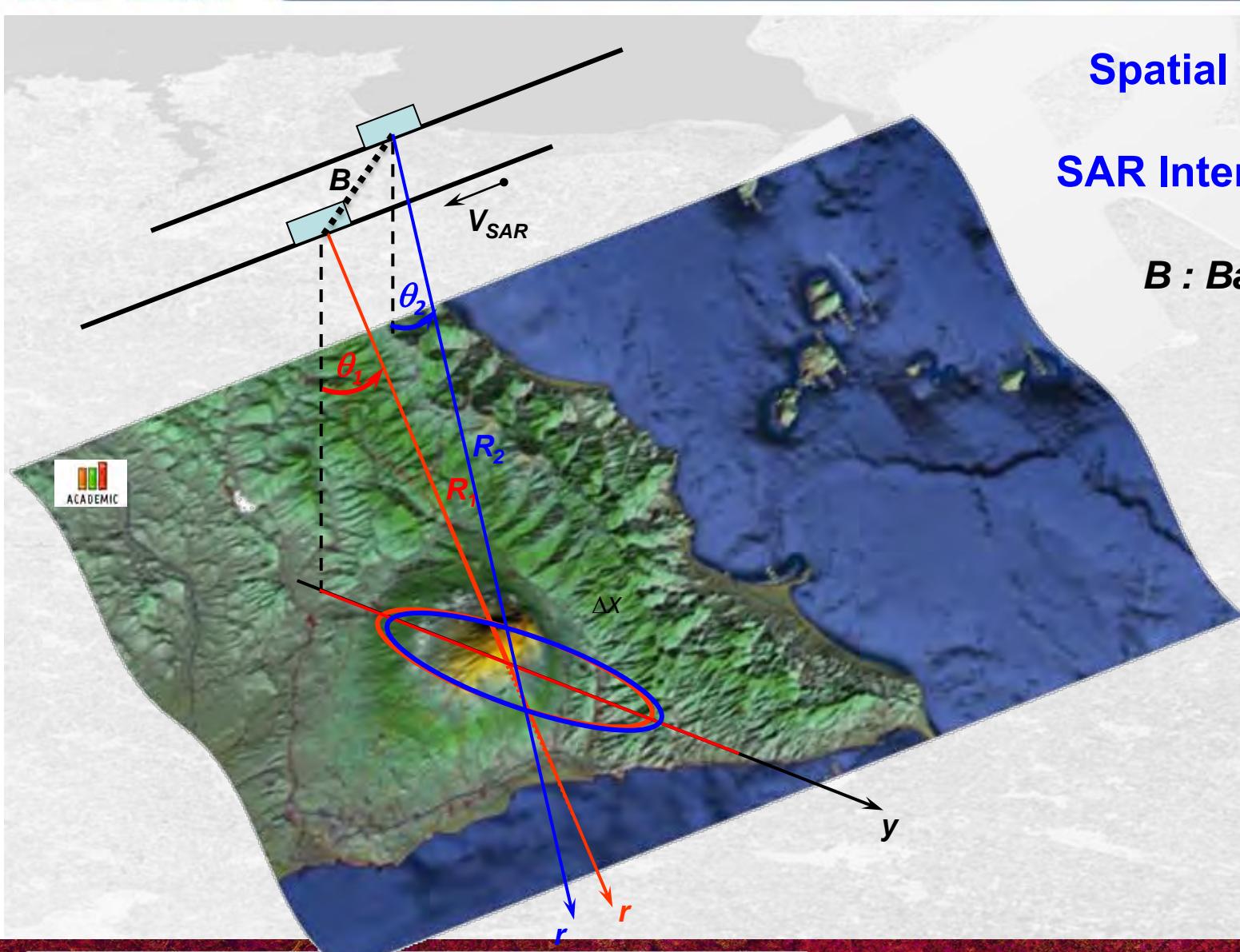
(In-SAR)







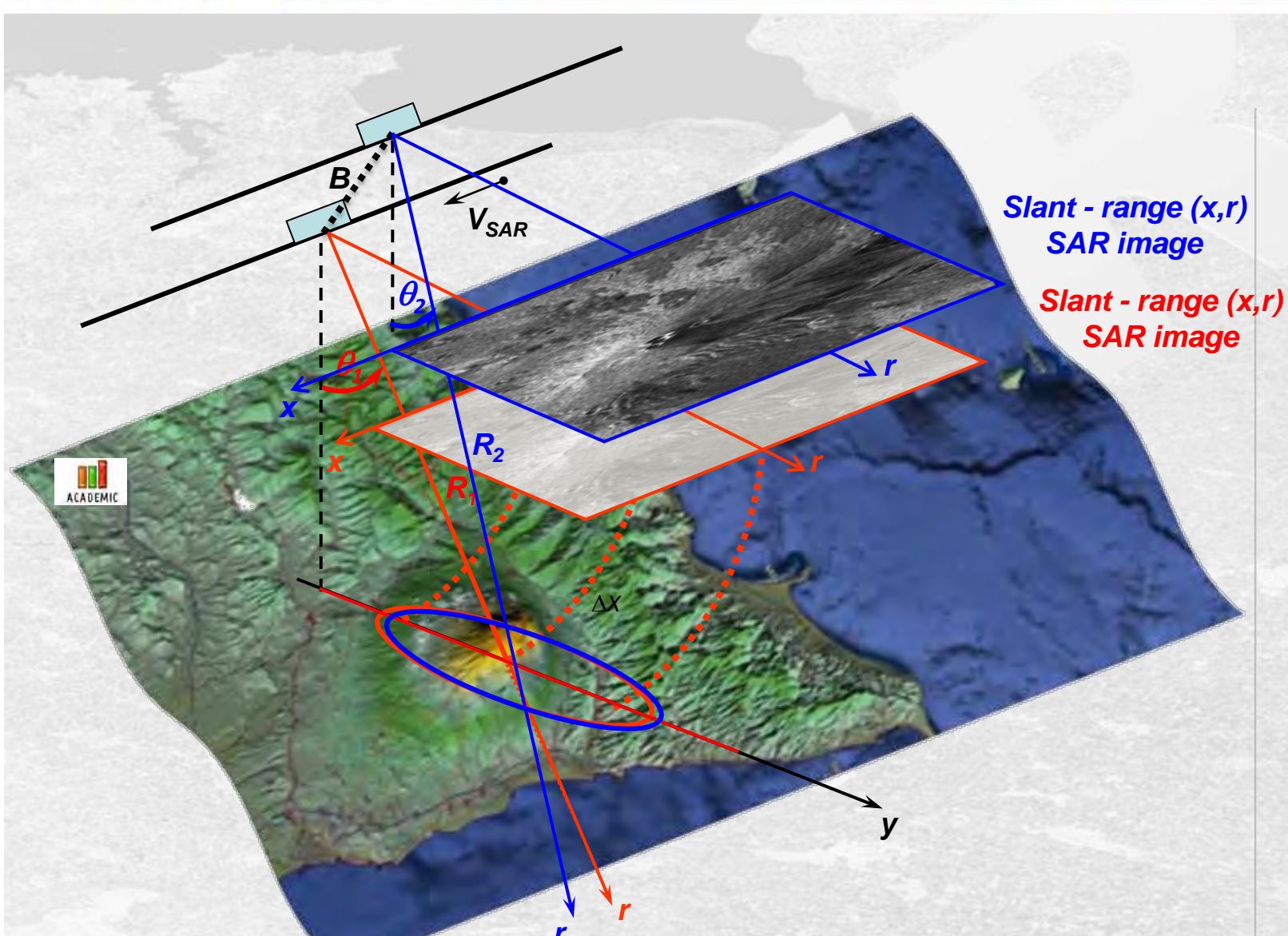


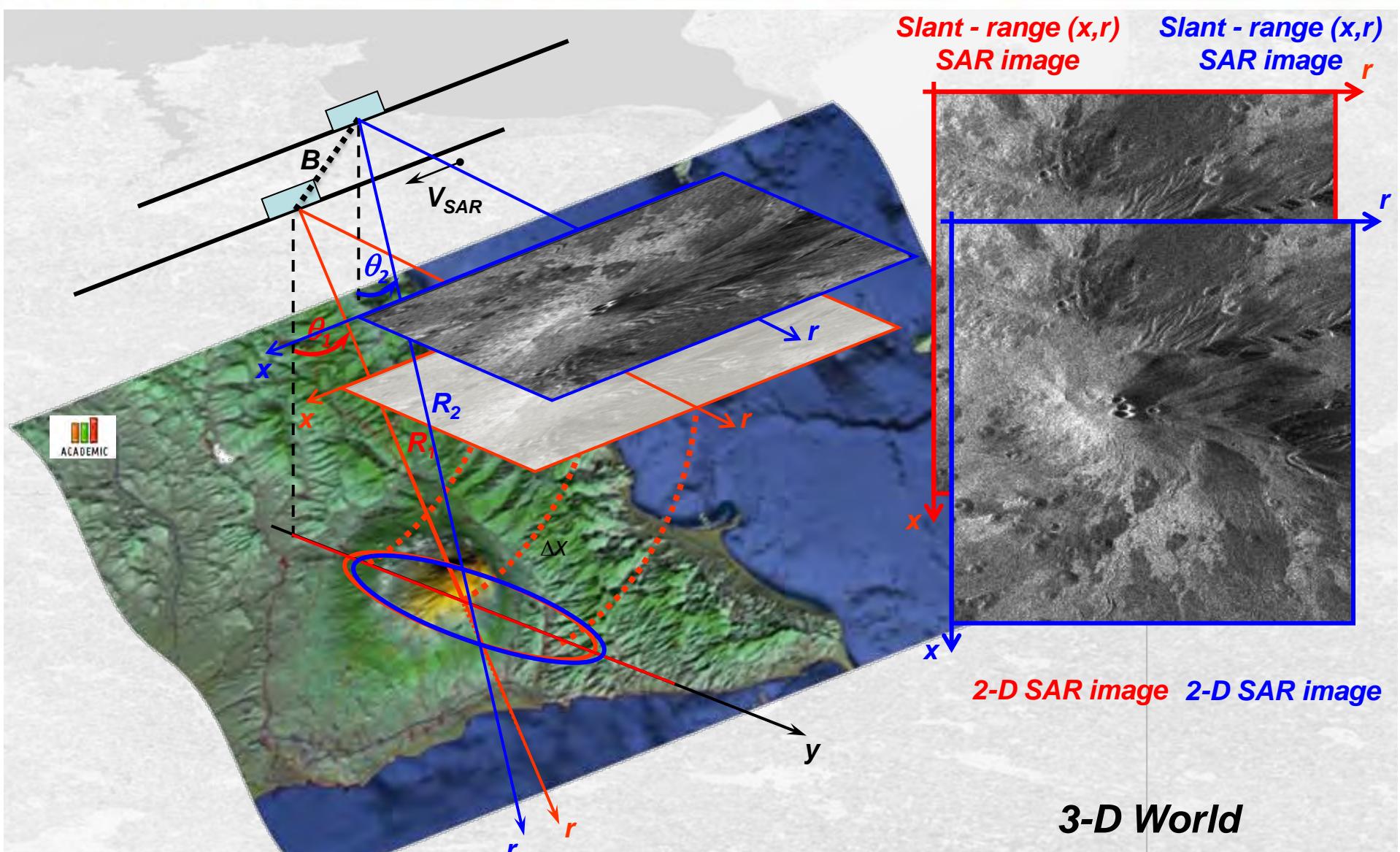


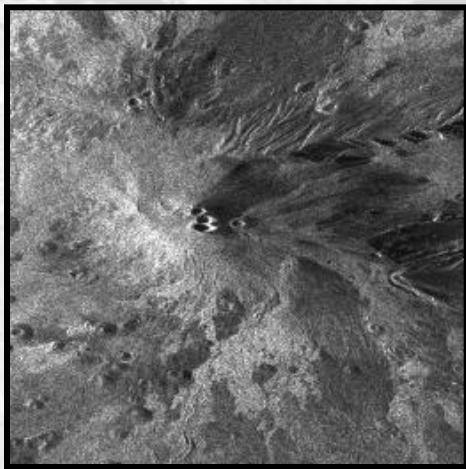
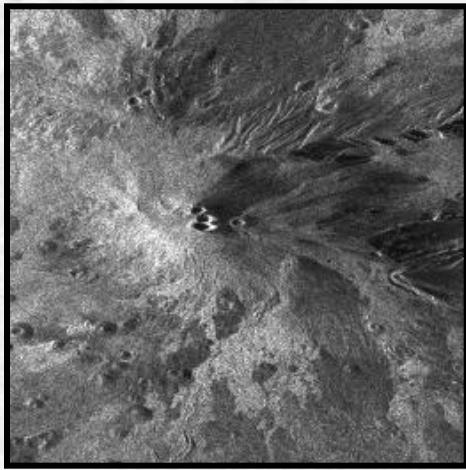
Spatial diversity

SAR Interferometry

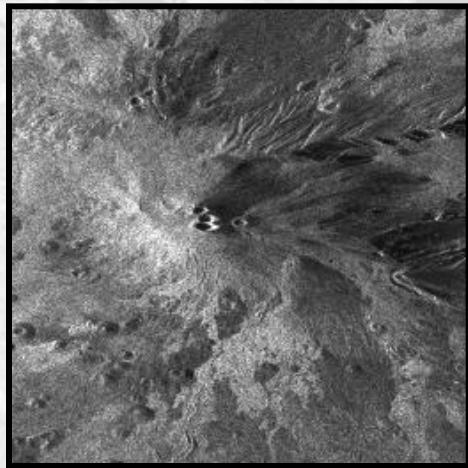
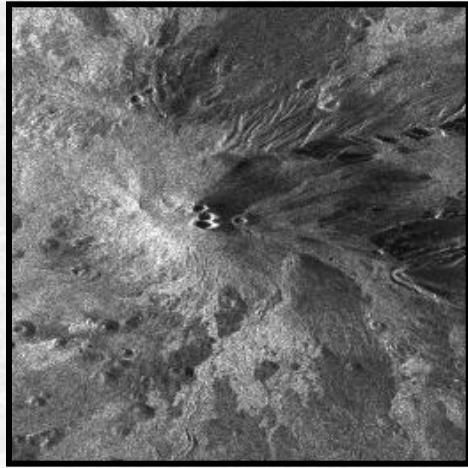
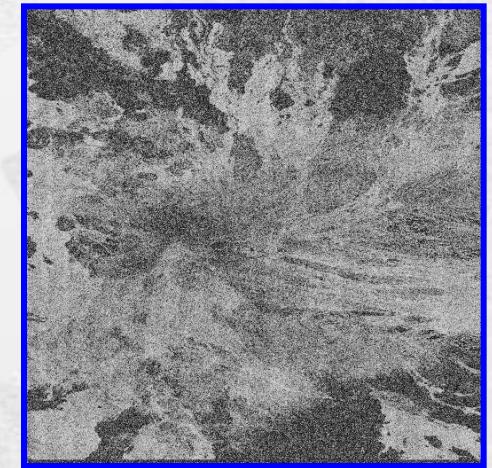
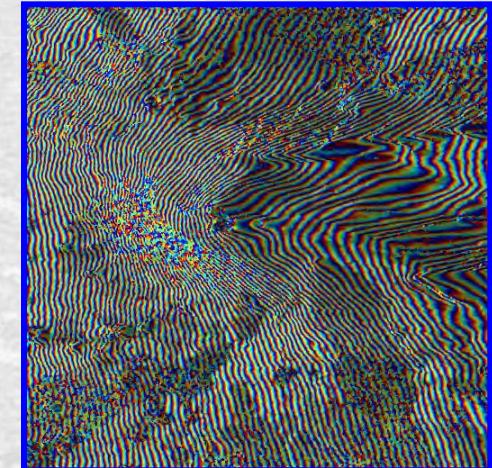
B : Baseline



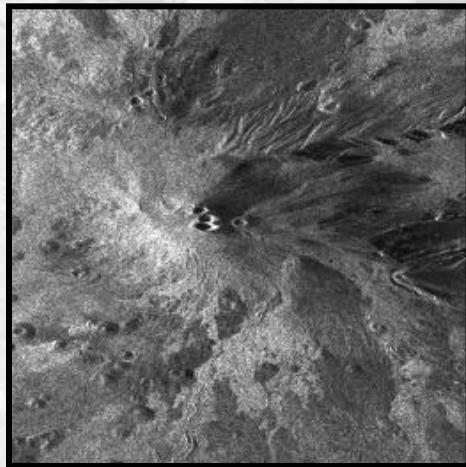
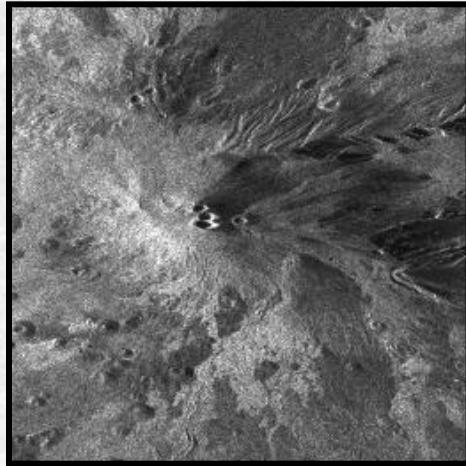


s_1 Interferometric coherence γ s_2 

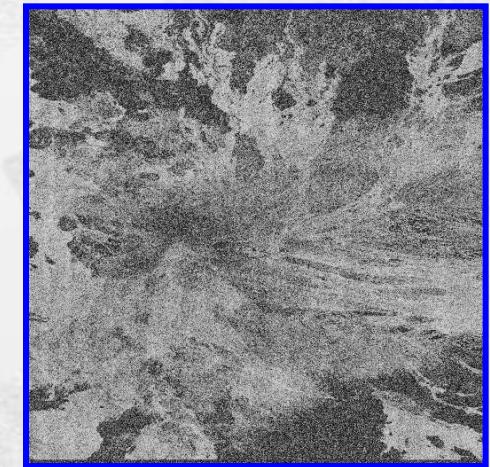
$$\gamma = \frac{E(s_1 s_2^*)}{\sqrt{E(s_1 s_1^*) E(s_2 s_2^*)}} = \frac{E(s_1 s_2^*)}{\sqrt{I_1 I_2}} = |\gamma| e^{j\phi}$$

s_1 Interferometric coherence γ s_2  $|\gamma|$  ϕ 

$$\gamma = \frac{E(s_1 s_2^*)}{\sqrt{E(s_1 s_1^*) E(s_2 s_2^*)}} = \frac{E(s_1 s_2^*)}{\sqrt{I_1 I_2}} = |\gamma| e^{j\phi}$$

s_1 Interferometric coherence γ s_2 

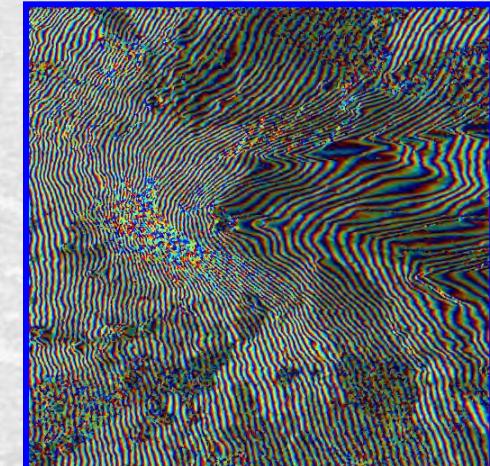
$$\gamma = \frac{E(s_1 s_2^*)}{\sqrt{E(s_1 s_1^*) E(s_2 s_2^*)}} = \frac{E(s_1 s_2^*)}{\sqrt{I_1 I_2}} = |\gamma| e^{j\phi}$$

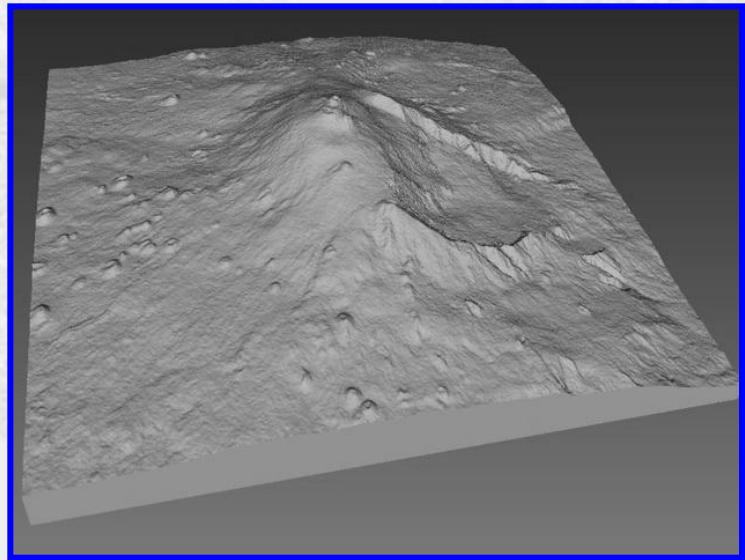
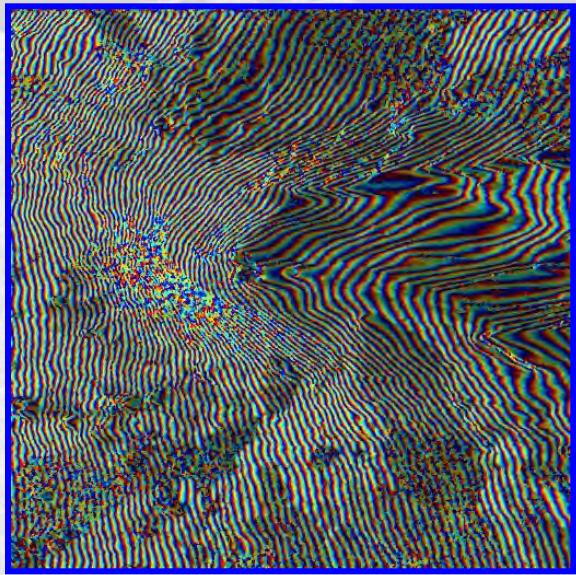
 $|\gamma|$  ϕ

Phase fringes

Contour lines

3-D World



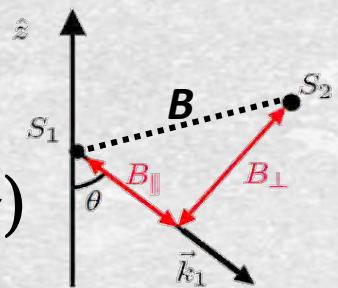


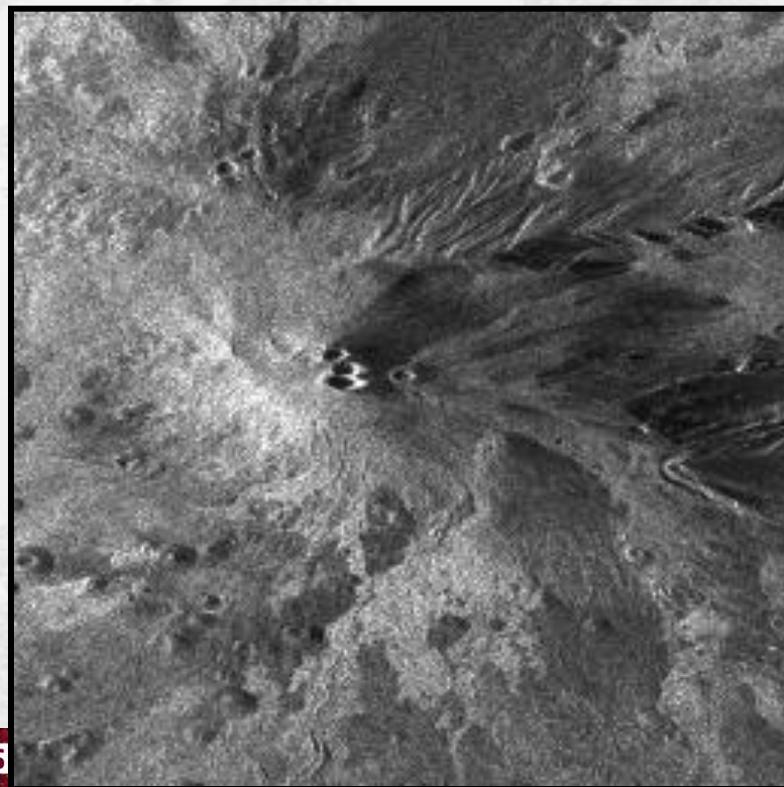
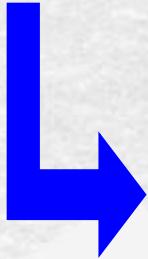
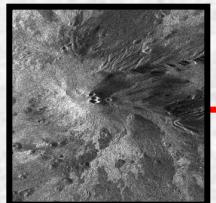
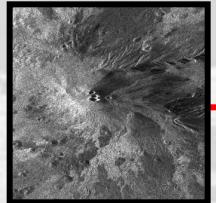
$$\phi = \Delta\phi_{1.2} \approx \Delta\phi_{topo} + \Delta\phi_{fe}$$

$$\Delta\phi_{topo} \propto \frac{k_c B_\perp}{R_I \sin(\theta_I)} \Delta h$$

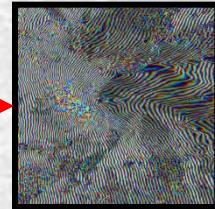
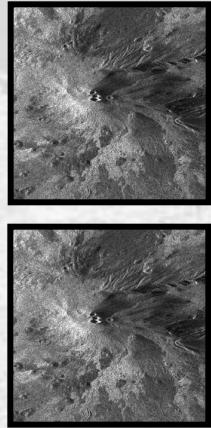
$$k_c = \frac{4\pi f_c}{c} = \frac{4\pi}{\lambda_c}$$

$$B_\perp = B \cos(\theta_I - \alpha)$$

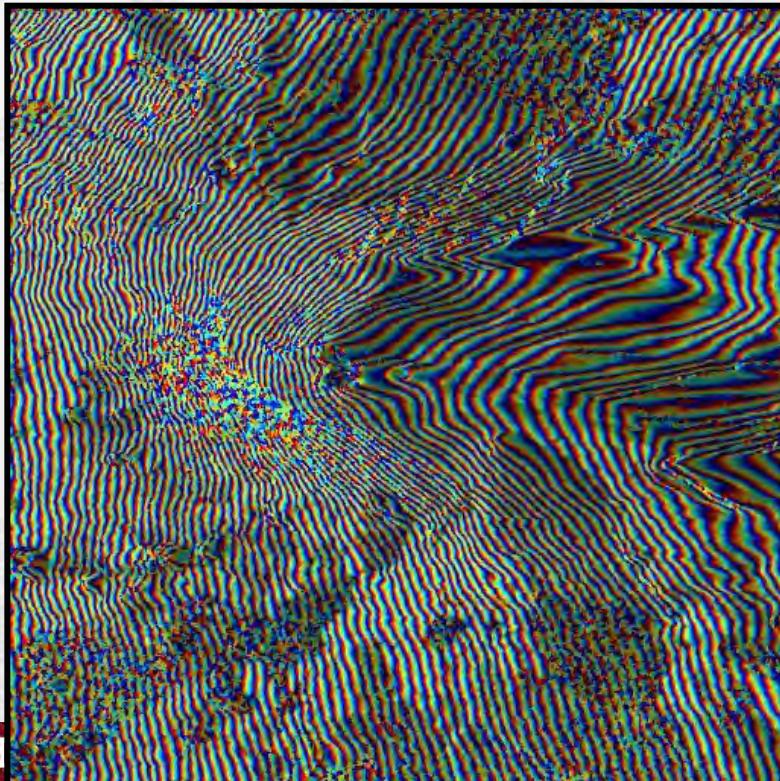


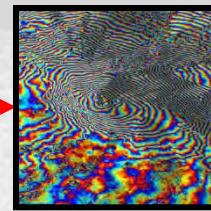
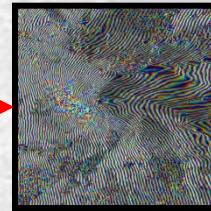
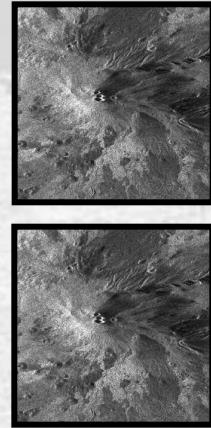


SAR
IMAGES

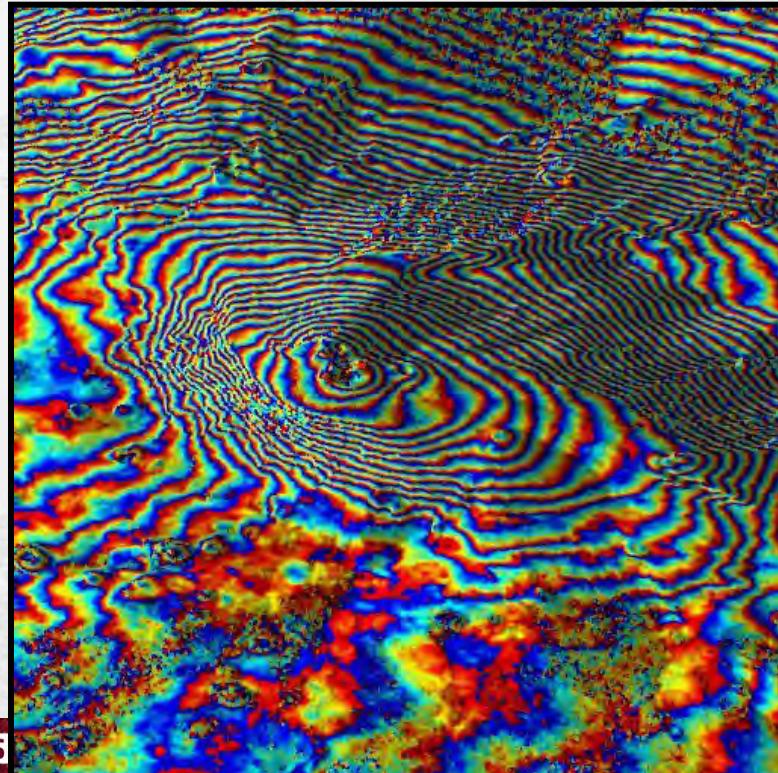


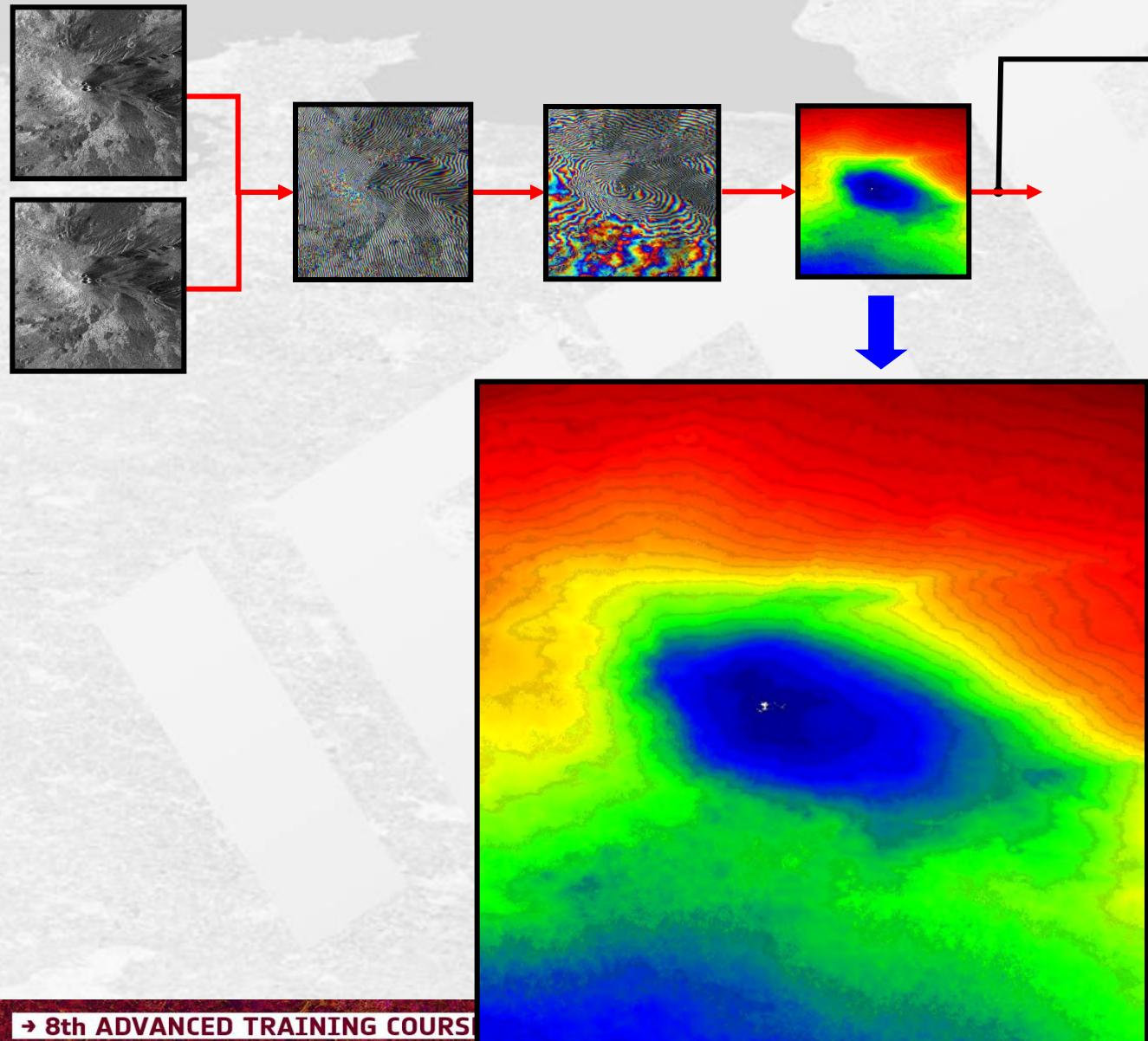
INTERFEROMETRIC
PHASE



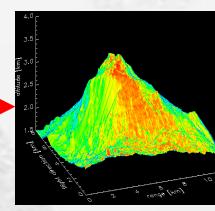
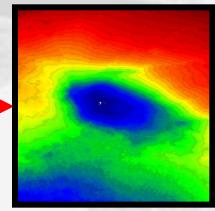
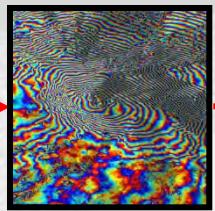
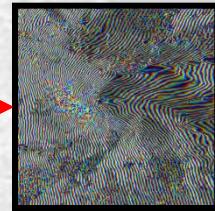
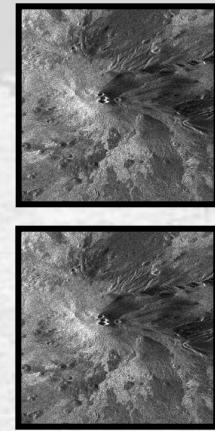


FLAT EARTH
REMOVAL

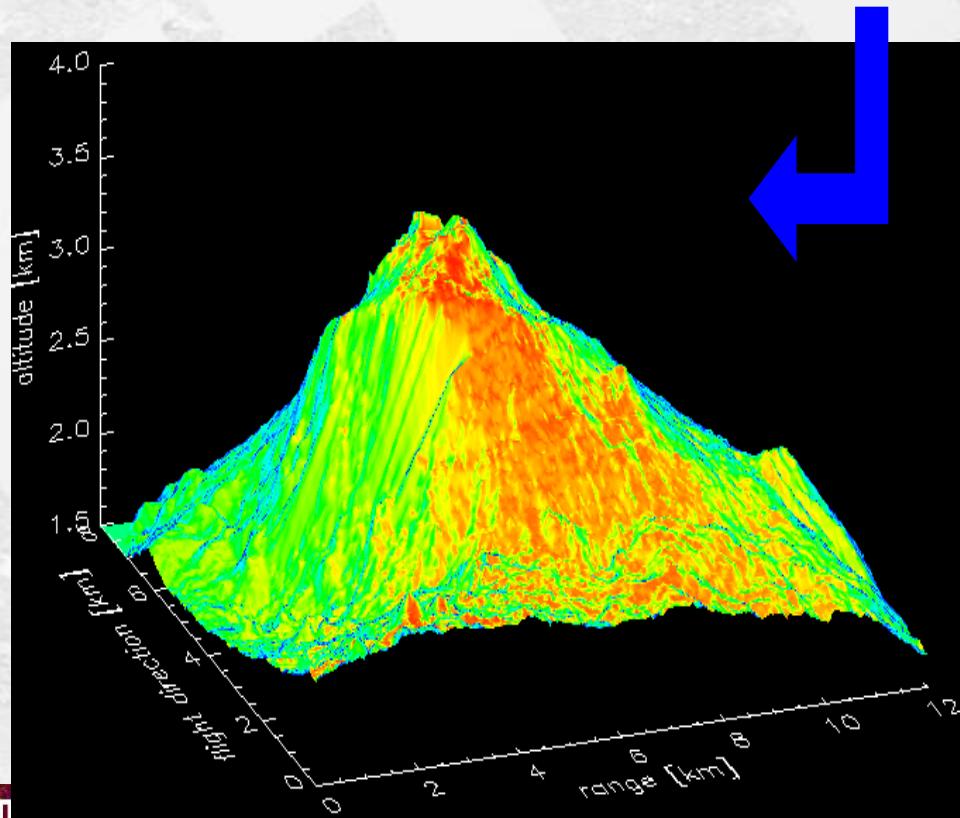


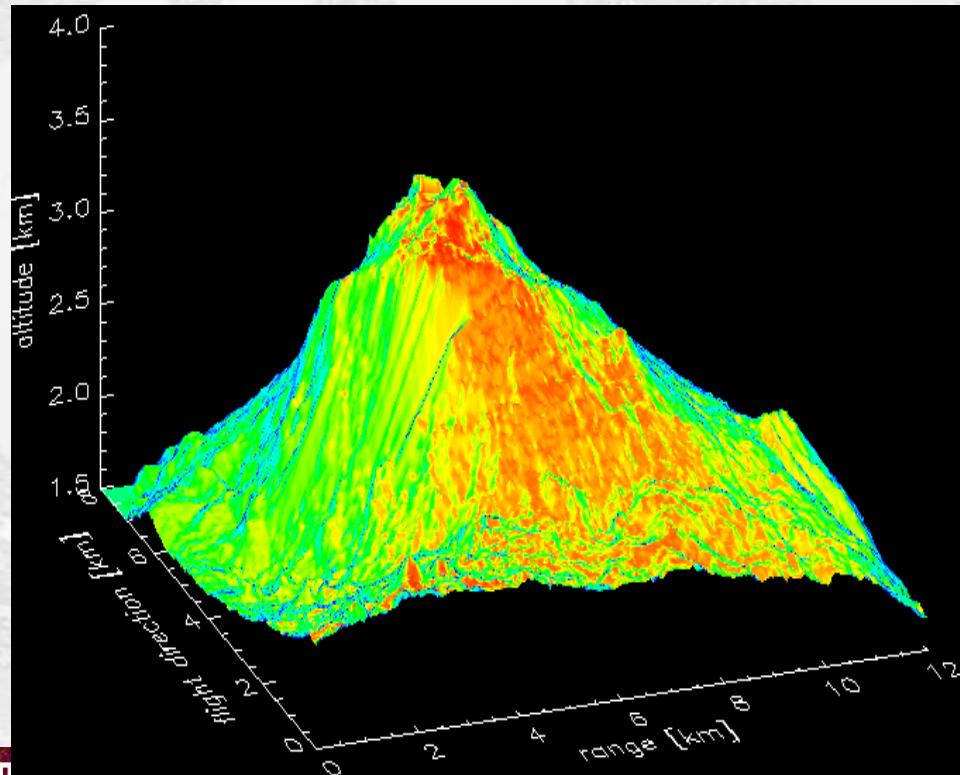
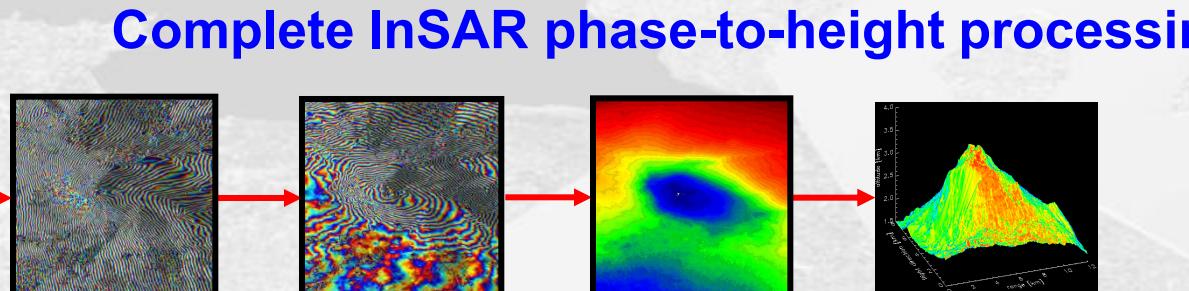
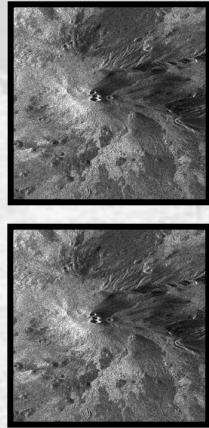


Relating In-SAR phase to height



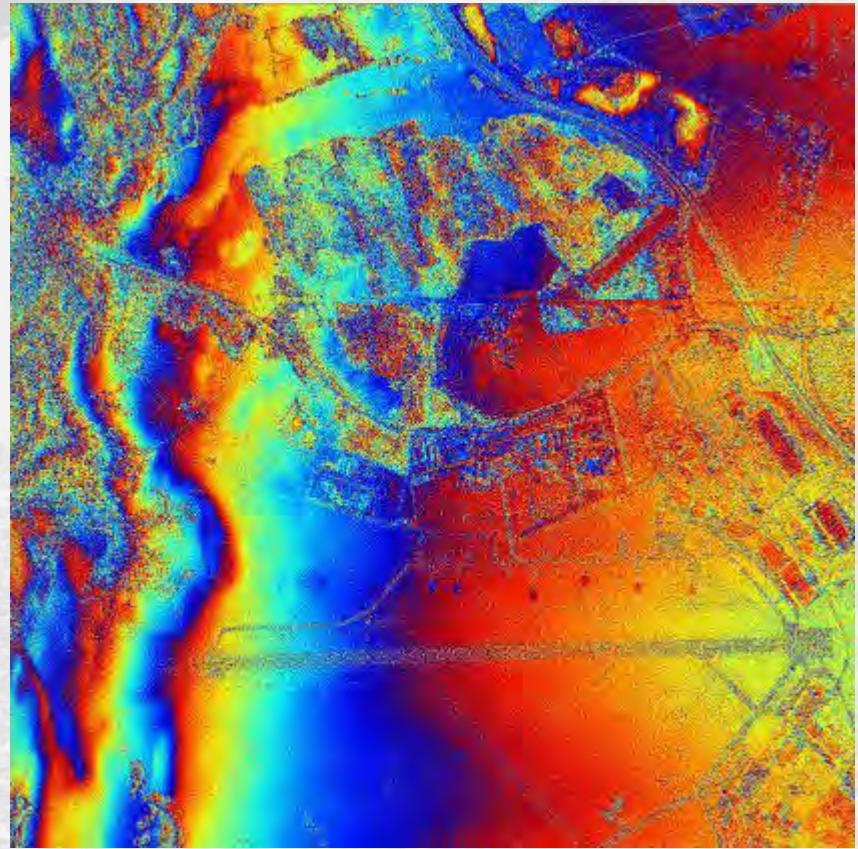
PHASE TO
HEIGHT



 γ

Interferometric coherence γ  $|\gamma|$

DLR E-SAR L Band
Pol-In SAR (1.5m x 3m) – Baseline 5m

 $\text{Arg}(\gamma)$

Interferometric coherence γ : decorrelation sources

γ fixed by a set of external sources :

System :

- Thermal or system noise : SAR amplifiers, ADC, antennas ...
- Quantization noise
- Geometric decorrelation : Baseline, squint ...
- Azimuth : Doppler decorrelation ...
- Ambiguities ...
- Processing errors : coregistration, interpolation ...

Environment :

- Random media : Surface & Volumetric media e.g. forest ...
- Temporal variations : wind, flowing or plowing, building ...

$$\gamma = \gamma_{SNR} \cdot \gamma_{quant} \cdot \gamma_{amb} \cdot \gamma_{geo} \cdot \gamma_{az} \cdot \gamma_{proc} \cdot \gamma_{media} \cdot \gamma_{temp}$$

Interferometric coherence γ : decorrelation sources

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$$\gamma = \gamma_{SNR} \cdot \gamma_{quant} \cdot \gamma_{amb} \cdot \gamma_{geo} \cdot \gamma_{az} \cdot \gamma_{proc} \cdot \gamma_{media} \cdot \gamma_{temp}$$

Interferometric coherence γ

$$\gamma = \frac{E(s_1 s_2^*)}{\sqrt{E(s_1 s_1^*) E(s_2 s_2^*)}} = \frac{E(s_1 s_2^*)}{\sqrt{I_1 I_2}} \approx \frac{E(s_1 s_2^*)}{I} = |\gamma| e^{j\phi}$$

$$\bar{I}_1 \approx \bar{I}_2 \approx \bar{I}$$

In-SAR signal formulation

$$s_1(x, r) = e^{-jkr_{01}} \int_V a_{c_1}(\vec{r}') e^{-j\vec{k}_1 \cdot (\vec{r}' - \vec{r}_0)} h(x - x', r - r') dv'$$

$$s_2(x, r) = e^{-jkr_{02}} \int_V a_{c_2}(\vec{r}') e^{-j\vec{k}_2 \cdot (\vec{r}' - \vec{r}_0)} h(x - x', r - r') dv'$$

Interferometric coherence γ

$$\gamma = \frac{E(s_1 s_2^*)}{\sqrt{E(s_1 s_1^*) E(s_2 s_2^*)}} = \frac{E(s_1 s_2^*)}{\sqrt{I_1 I_2}} \approx \frac{E(s_1 s_2^*)}{I} = |\gamma| e^{j\phi}$$

$$\bar{I}_1 \approx \bar{I}_2 \approx \bar{I}$$

Volume complex reflectivity $a_{c_i}(\vec{r})$

$$\bar{I}_i(x, r) = E(s_i(x, r)s_i^*(x, r)) = \int_V \sigma_{v_i}(\vec{r}') |h(x - x', r - r')|^2 dv'$$

$$E(s_1(x, r)s_2^*(x, r)) = \int_V \sigma_{v_e}(\vec{r}') e^{-j(\bar{k}_1 - \bar{k}_2)(r' - \bar{r}_0)} |h(x - x', r - r')|^2 dv'$$

With :

$$E(a_{c_i}(\vec{r})a_{c_i}^*(\vec{r}')) = \sigma_{v_i}(\vec{r}) \delta(\vec{r} - \vec{r}')$$

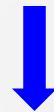
Reflectivity density

$$E(a_{c_1}(\vec{r})a_{c_2}^*(\vec{r}')) = \sigma_{v_e}(\vec{r}) \delta(\vec{r} - \vec{r}')$$

Effective reflectivity density

Interferometric coherence γ

$$\gamma = \frac{E(s_1 s_2^*)}{\sqrt{E(s_1 s_1^*) E(s_2 s_2^*)}} = \frac{E(s_1 s_2^*)}{\sqrt{I_1 I_2}} \approx \frac{E(s_1 s_2^*)}{I} = |\gamma| e^{j\phi}$$



$$\gamma = \frac{\int \sigma_{v_e}(\vec{r}') e^{-j(\vec{k}_1 - \vec{k}_2)(\vec{r}' - \vec{r}_0)} |h(x - x', r - r')|^2 dv'}{\int_V \sigma_v(\vec{r}') |h(x - x', r - r')|^2 dv'}$$

With :

$$E(a_{c_i}(\vec{r}) a_{c_i}^*(\vec{r}')) = \sigma_{v_i}(\vec{r}) \delta(\vec{r} - \vec{r}')$$

Reflectivity density

$$E(a_{c_1}(\vec{r}) a_{c_2}^*(\vec{r}')) = \sigma_{v_e}(\vec{r}) \delta(\vec{r} - \vec{r}')$$

Effective reflectivity density

Interferometric coherence γ

$$\gamma = \frac{\int_V \sigma_{v_e}(\vec{r}') e^{-j(\vec{k}_I - \vec{k}_2)(\vec{r}' - \vec{r}_0)} |h(x - x', r - r')|^2 dv'}{\int_V \sigma_v(\vec{r}') |h(x - x', r - r')|^2 dv'}$$



$$\gamma = \gamma_{temp} \cdot \gamma_{media} = \gamma_{temp} \cdot (\gamma_{x,y} \cdot \gamma_z)$$

Surface / Volume



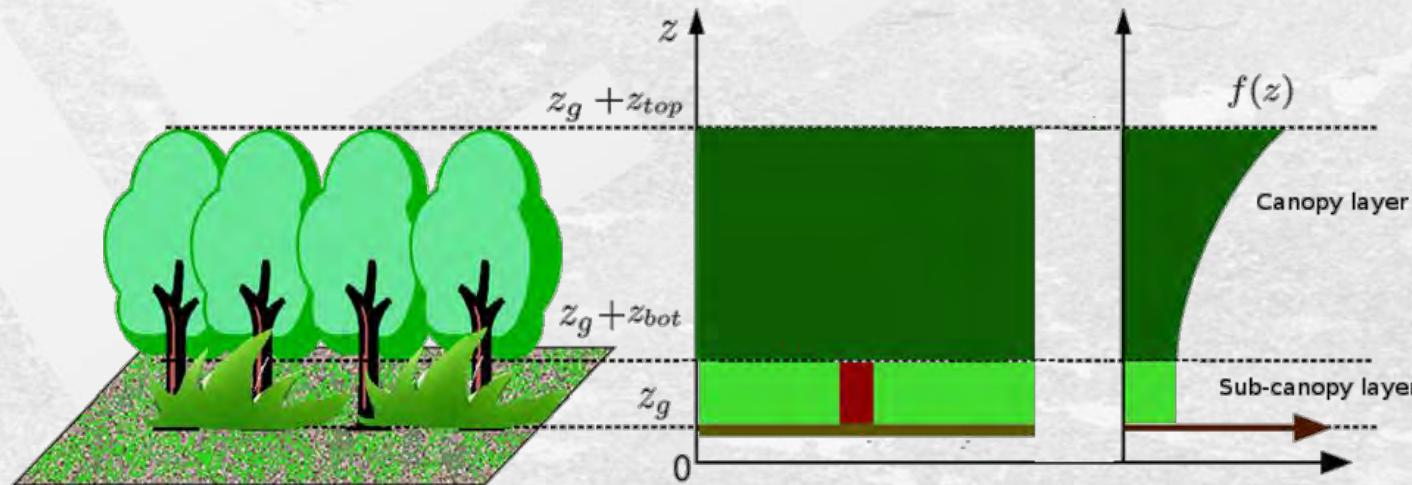
$$\gamma_{vol} = \gamma_z = \frac{\int_Z \sigma_{v_e}(\vec{r}') e^{jk_z(z - z_0)} dz'}{\int_Z \sigma_{v_e}(\vec{r}') dz'}$$

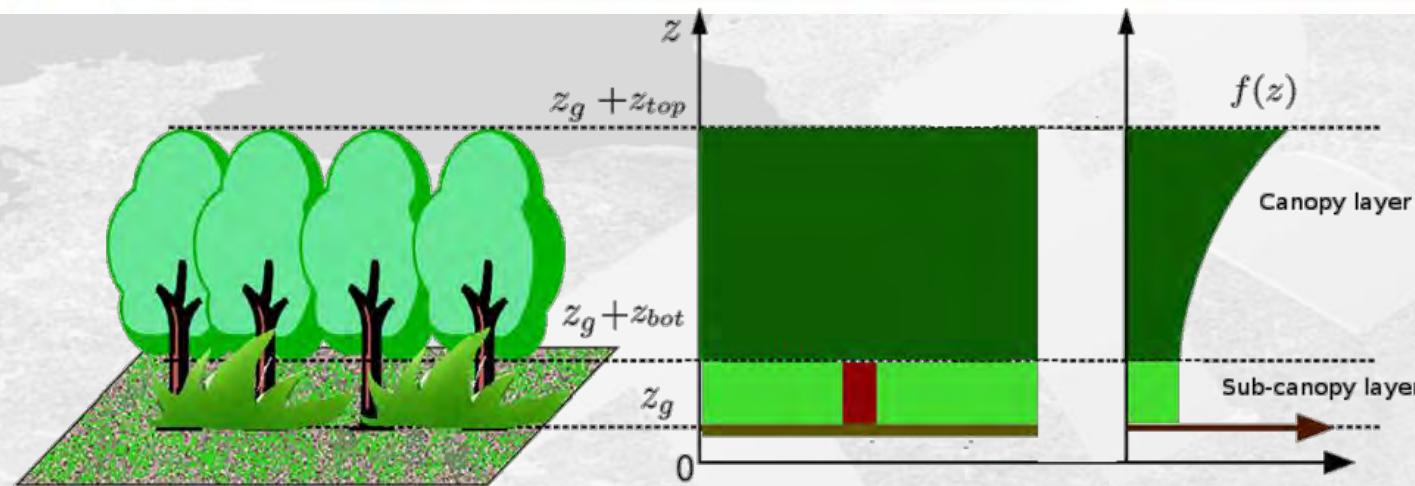
Volumetric / random media decorrelation

Volume decorrelation

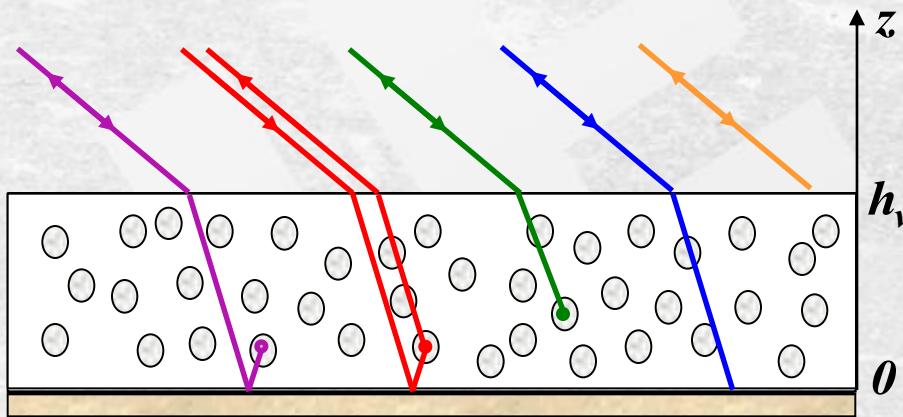
$$\gamma_z = \frac{\int \sigma_{v_e}(z) e^{j k_z (z - z_0)} dz'}{\int \sigma_{v_e}(z) dz'}$$

Decorrelation due to the vertical structure $\sigma_{v_e}(z) = A_{v_e} f(z)$

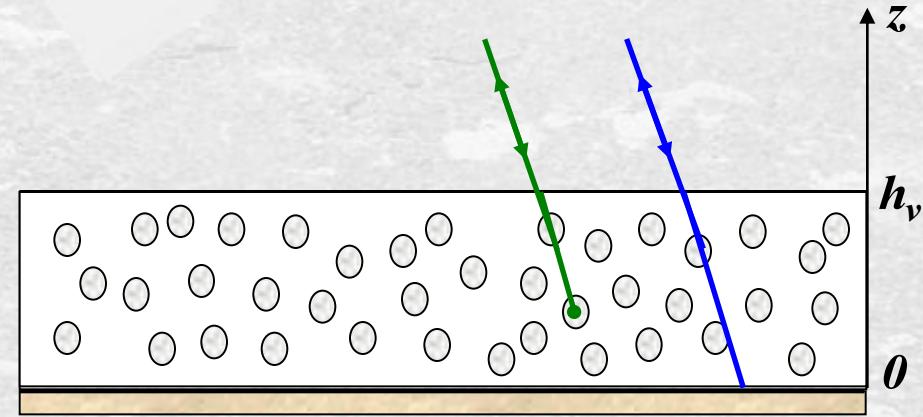




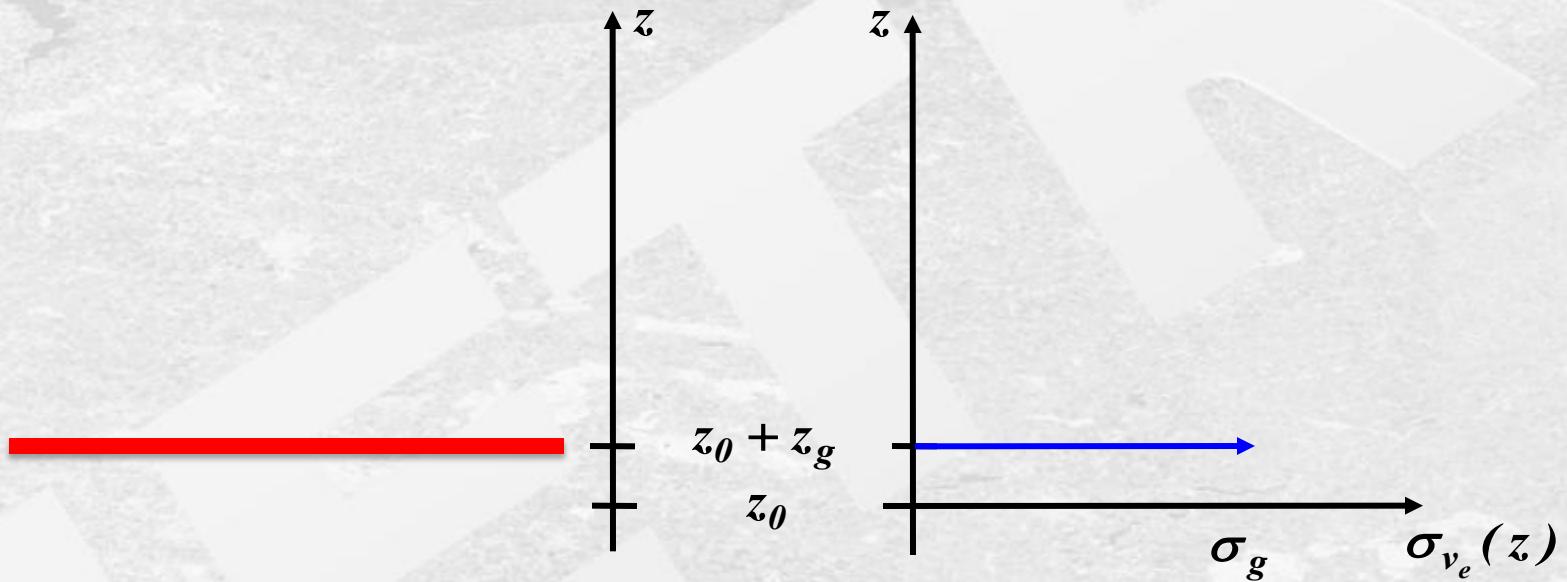
Modeling



Parameter Estimation



- 2 significant and uncorrelated mechanisms : volume + underlying ground
- low density medium = No refraction

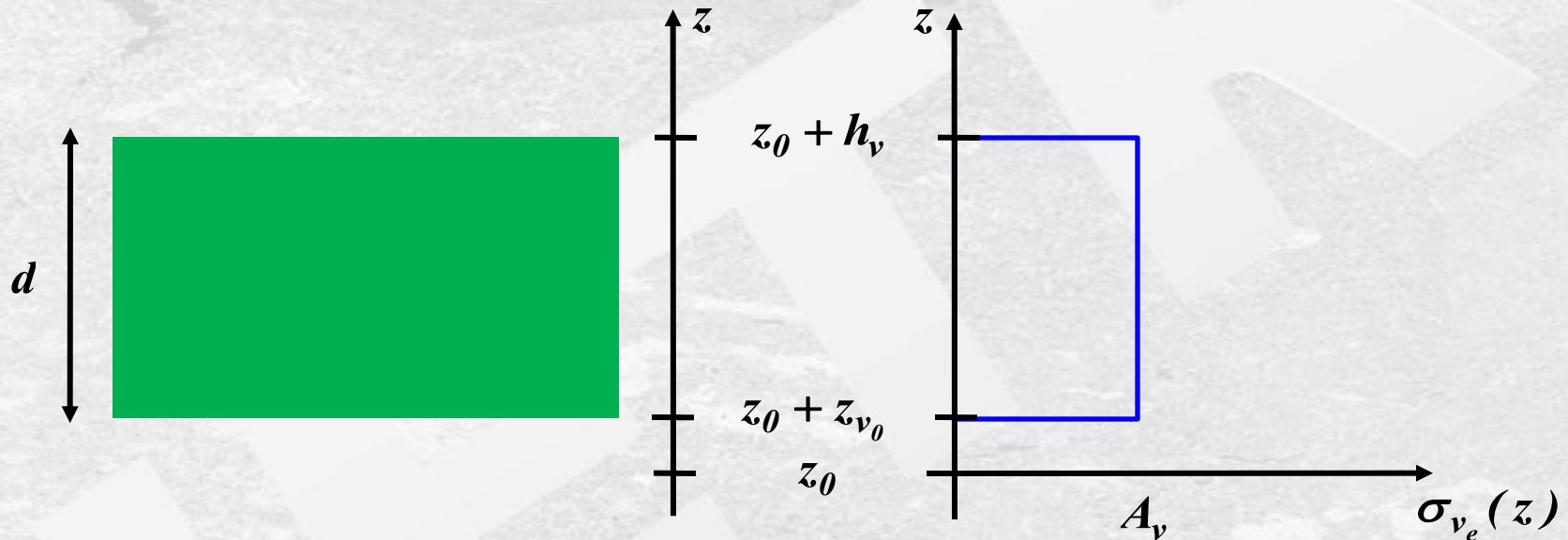
Ground only

Ground layer $\sigma_{v_e}(z) = \sigma_g \delta(z - z_g)$

No volume

$$\gamma_z = \frac{\int \sigma_{v_e}(z) e^{jk_z(z-z_0)} dz'}{\int \sigma_{v_e}(z) dz'} = e^{jk_z z_g}$$

Random volume (RV) with null extinction

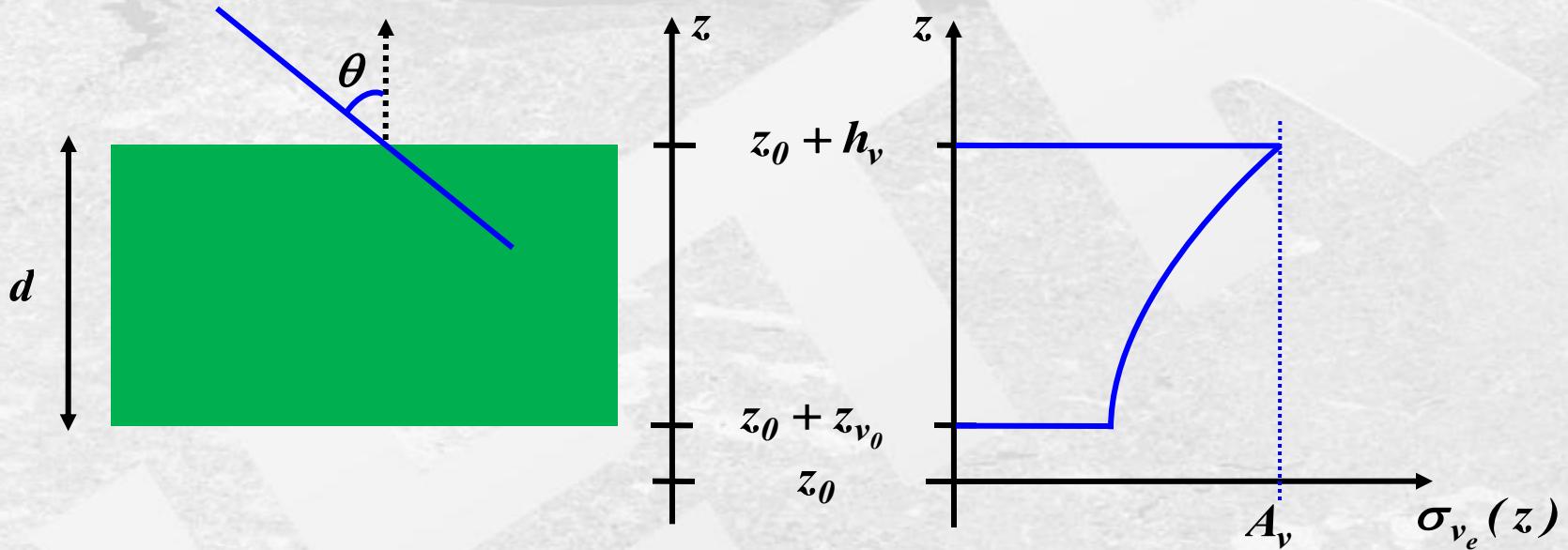


No underlying ground

Homogeneous medium

Null extinction : $\sigma_{v_e}(z) = A_v$

$$\gamma_z = \frac{\int_{z_0+z_{v_0}}^{z_0+h_v} \sigma_{v_e}(z) e^{jk_z(z-z_0)} dz'}{\int_{z_0+z_{v_0}}^{z_0+h_v} \sigma_{v_e}(z) dz'} = e^{jk_z \frac{(h_v+z_{v_0})}{2}} \text{sinc}\left(\frac{k_z d}{2}\right)$$

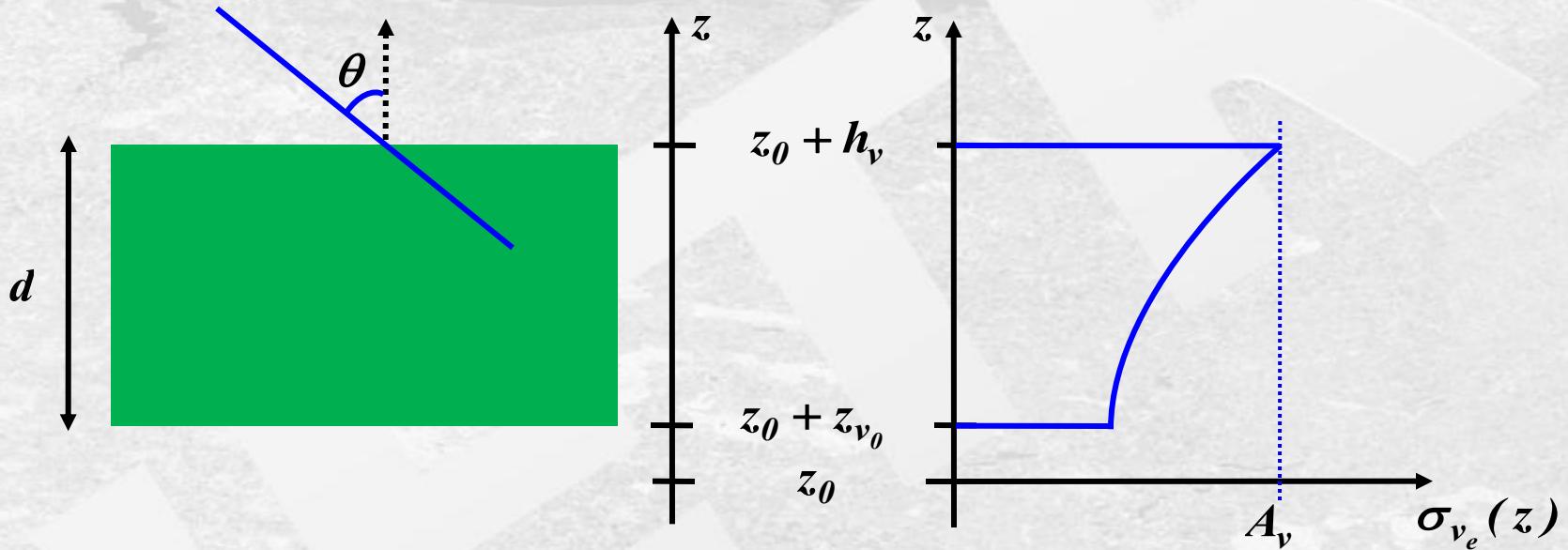
Random volume (RV) with non-null extinction

No underlying ground

Non-null extinction : κ_e

Homogeneous medium with elementary reflectivity density $\sigma_{v_e}(z) = A_v e^{\frac{2\kappa_e}{\cos(\theta)}(z - (z_0 + h_v))}$

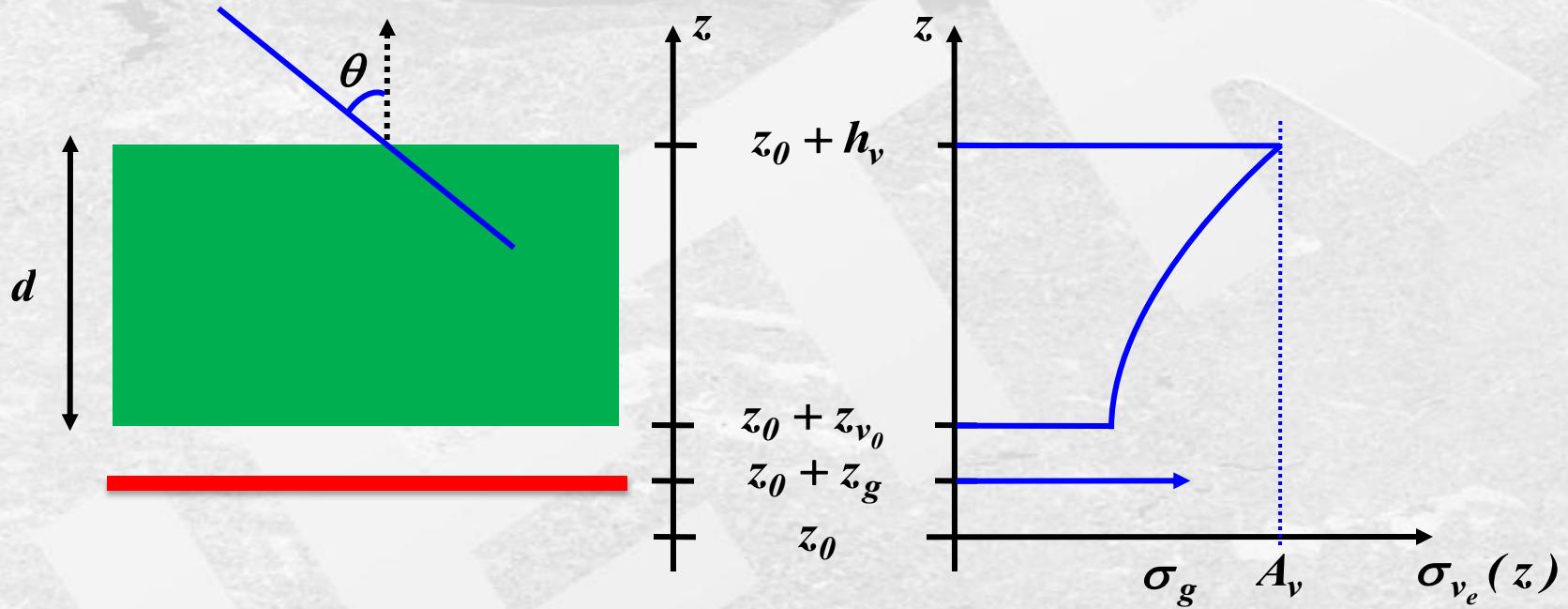
Random volume (RV) with non-null extinction



$$\gamma_z = \frac{\int_{z_0}^{z_0+h_v} \sigma_{v_e}(z) e^{jk_z(z-z_0)} dz'}{\int_{z_0}^{z_0+z_{v_0}} \sigma_{v_e}(z) dz'} = e^{jk_z z_{v_0}} \frac{p}{p_1} \left(\frac{e^{p_1 d} - 1}{e^{pd} - 1} \right)$$

$$p = \frac{2\kappa_e}{\cos(\theta)}$$

$$p_1 = \frac{2\kappa_e}{\cos(\theta)} + jk_z$$

Random volume over ground (RVoG)

Non-null extinction : κ_e

Underlying ground : $I_g = \sigma_g e^{-\frac{2\kappa_e d}{\cos(\theta)}} \delta(z - (z_0 + z_g))$

Homogeneous medium with elementary reflectivity density $\sigma_{vol}(z) = A_v e^{\frac{2\kappa_e}{\cos(\theta)}(z - (z_0 + h_v))}$

Random volume over ground (RVoG)

$$\sigma_{v_e}(z) = I_g + \sigma_{vol}(z) \Big|_{z \in [z_0 + z_{v_0} \dots z_0 + h_v]}$$

$$\gamma_z = \frac{\int \sigma_{v_e}(z) e^{jk_z(z-z_0)} dz'}{\int \sigma_{v_e}(z) dz'}$$

$$\gamma_z = \frac{\int_{z_0 + z_{v_0}}^{z_0 + h_v} \sigma_{vol}(z) e^{jk_z(z-z_0)} dz' + I_g e^{jk_z z_g}}{\int_{z_0 + z_{v_0}}^{z_0 + h_v} \sigma_{vol}(z) dz' + I_g} =$$

$$\frac{\int_{z_0 + z_{v_0}}^{z_0 + h_v} \sigma_{vol}(z) e^{jk_z(z-z_0)} dz' + I_g e^{jk_z z_g}}{I_v + I_g}$$

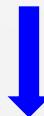
$$\gamma_z = \frac{\gamma_{vol} + \frac{I_g}{I_v} e^{jk_z z_g}}{1 + \frac{I_g}{I_v}} = e^{jk_z z_g} \frac{\tilde{\gamma}_{vol} + m}{1 + m}$$

With :

$$\gamma_{vol} = \frac{1}{I_v} \int_{z_0 + z_{v_0}}^{z_0 + h_v} \sigma_{vol}(z) e^{jk_z(z-z_0)} dz'$$

Random volume over ground (RVoG)

$$\gamma_z = e^{jk_z z_g} \frac{\tilde{\gamma}_{vol} + m}{1 + m}$$



Ground to volume intensity ratio : $m = \frac{I_g}{I_v}$

$$\tilde{\gamma}_{vol} = e^{-jk_z z_g} \gamma_{vol}$$

$$k_z = \frac{k_c B_\perp}{R_1 \sin(\theta_1)}$$

$$k_c = \frac{4\pi f_c}{c}$$

Observables (2) : γ_z

Unknowns (4) : $z_g, m, \gamma_{vol} (2)$



1 In-SAR acquisition vs. Complex RVOG structure : under-determined problem



→ another source of diversity is needed : polarization ?



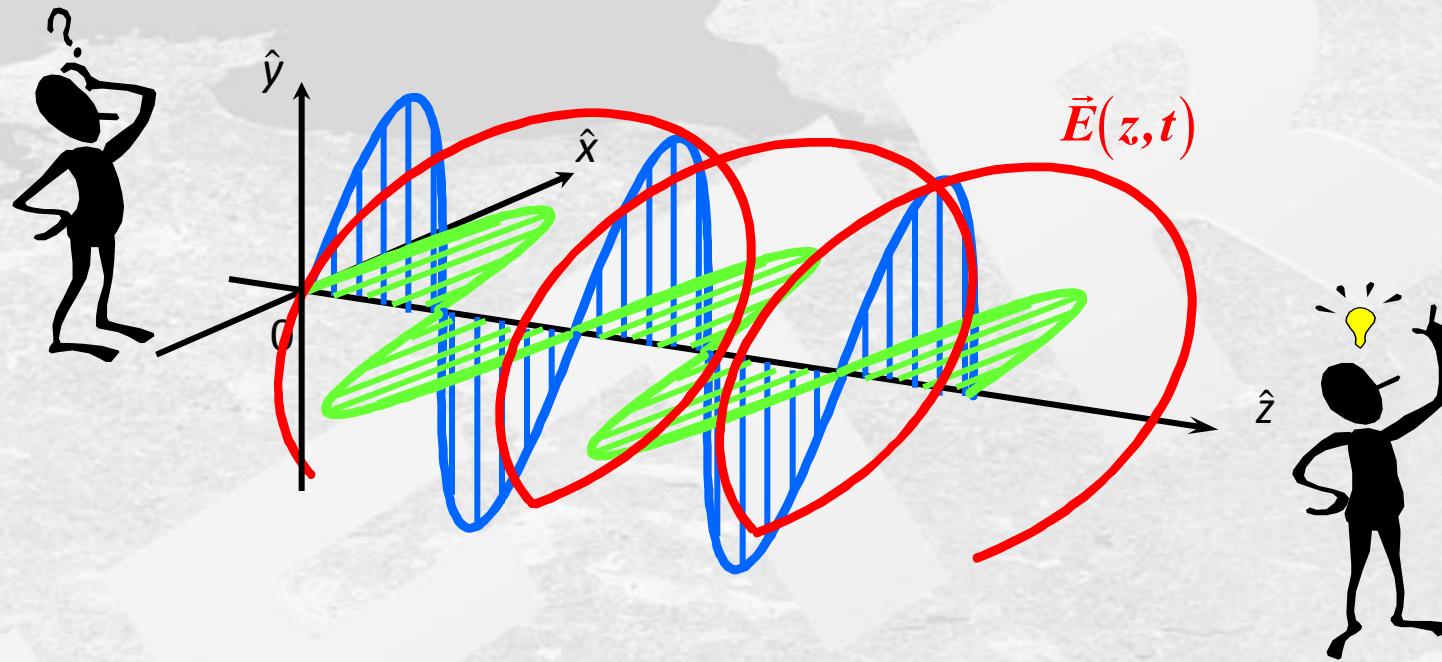
SAR

+

Polarimetry

(Pol-SAR)



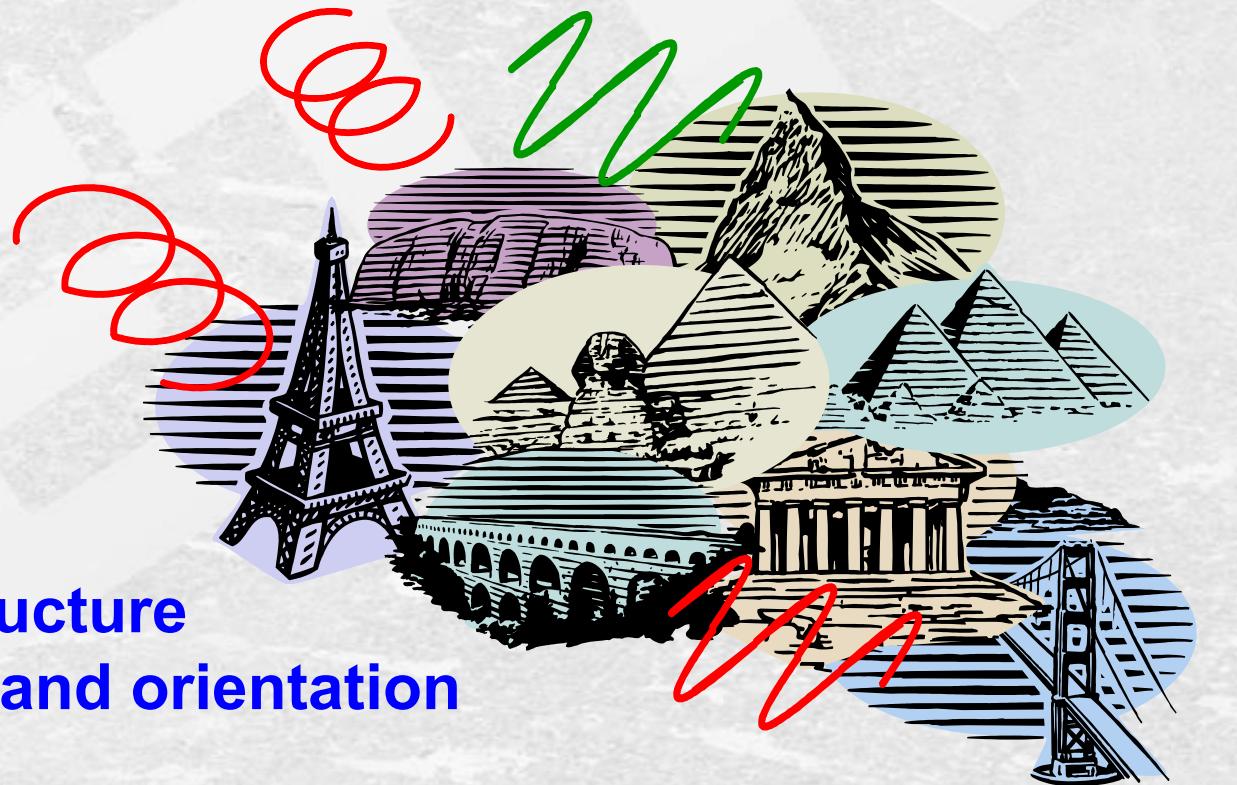


Radar Polarimetry (**Polar** : polarisation **Metry**: measure)
is the science of acquiring, processing and analysing
the polarization state of an electromagnetic field

Radar Polarimetry deals with the full vector
nature of polarized electromagnetic waves



The **POLARISATION** information
Contained in the waves backscattered
from a given medium is highly related to:



**its geometrical structure
reflectivity, shape and orientation**

its geophysical properties such as humidity, roughness, ...



Forest Vegetation

- Forest Height
- Forest Biomass
- Forest Structure
- Canopy Extinction
- Underlying Topography

- Forest Ecology
- Forest Management
- Ecosystem Change
- Carbon Cycle



Agriculture

- Soil Moisture Content
- Soil roughness
- Height of Vegetation Layer
- Extinction of Vegetation Layer
- Moisture of Vegetation Layer

- Farming Management
- Water Cycle
- Desretification



Snow and Ice

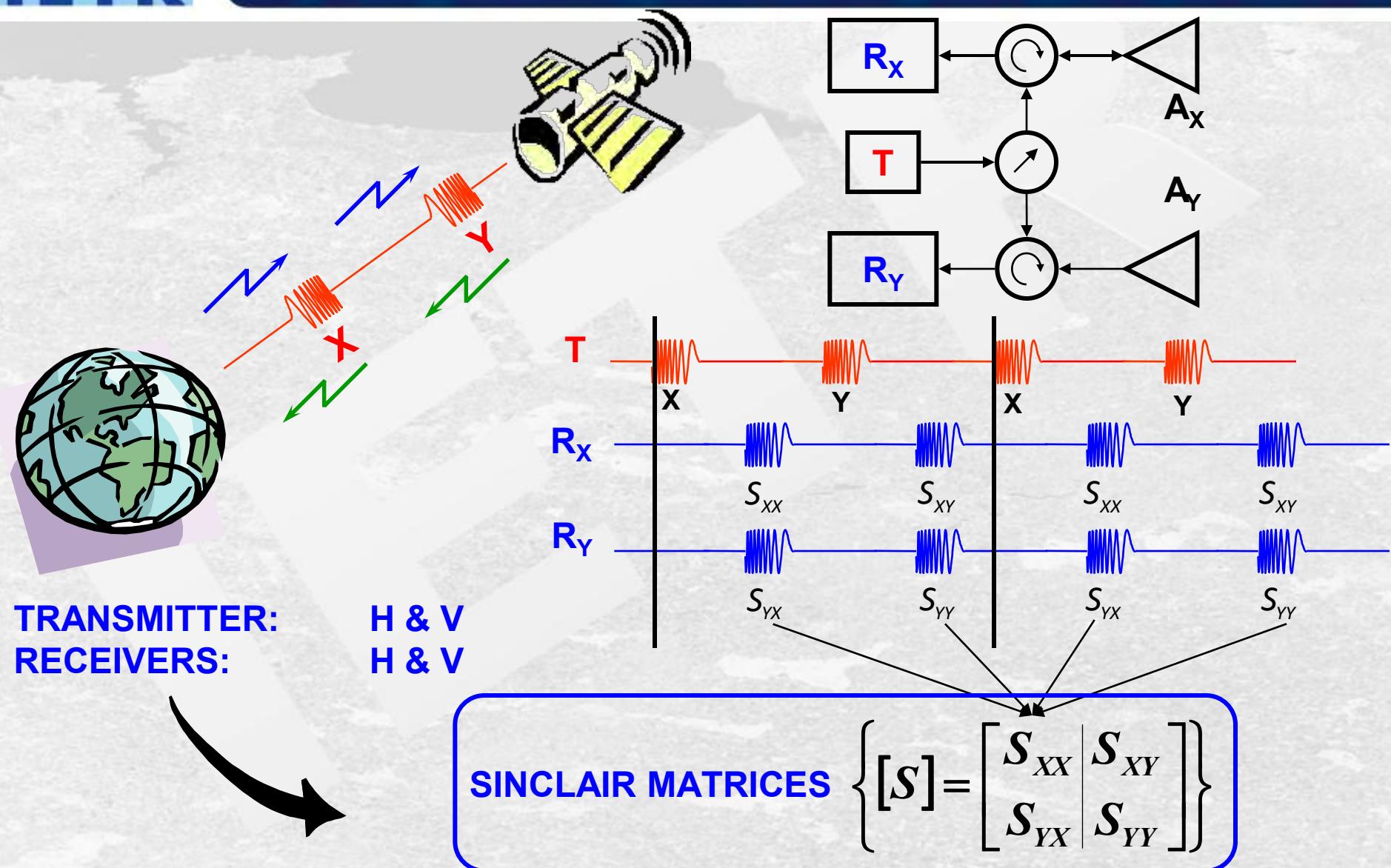
- Topography
- Penetration Depth / Density
- Snow Ice Layer
- Snow Ice Extinction
- Water Equivalent

- Ecosystem Change
- Water Cycle
- Water Management



- Geometric Properties
- Dielectric Properties

- Urban Monitoring

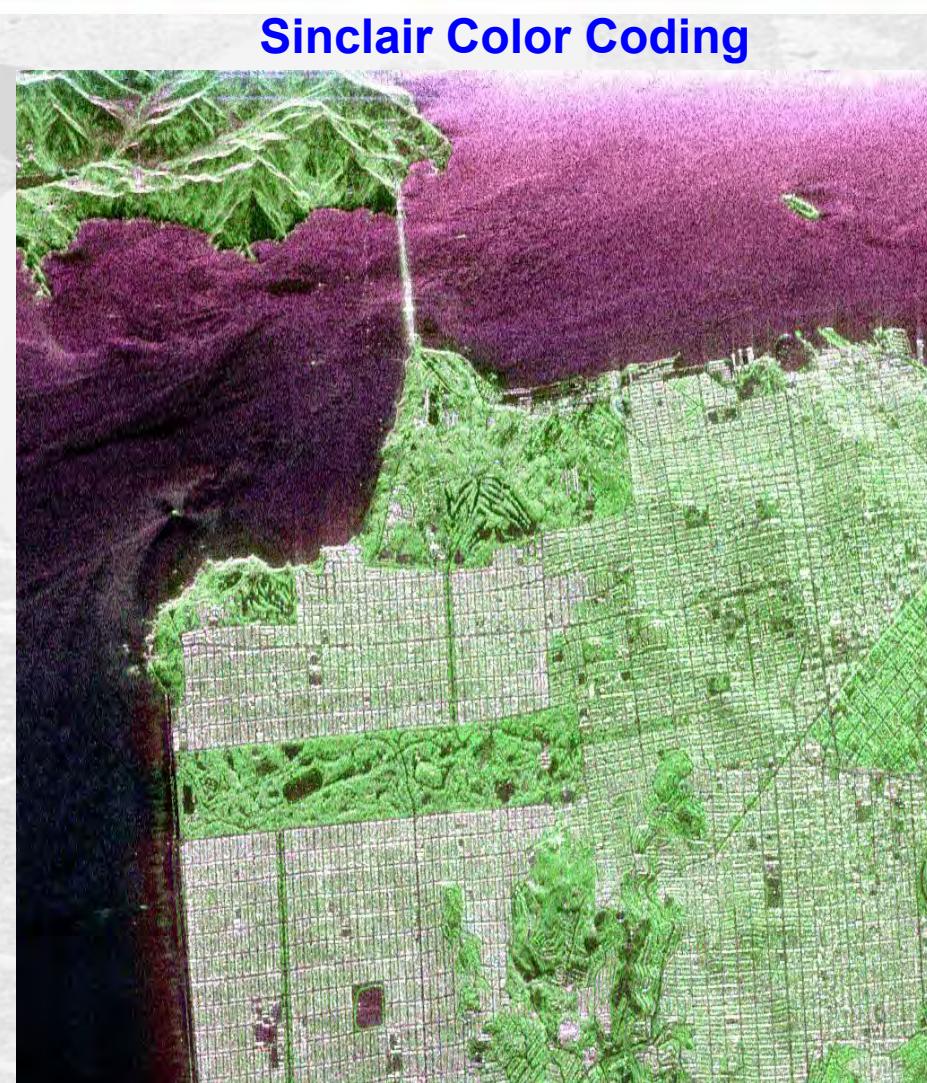


$\overrightarrow{\text{Tx}}$ $\overrightarrow{\text{Rx}}$ $\overrightarrow{\text{Tx}}$ $\overrightarrow{\text{Rx}}$ $\overrightarrow{\text{Tx}}$ $\overrightarrow{\text{Rx}}$  $|\mathbf{HH}|_{\text{dB}}$ $|\mathbf{HV}|_{\text{dB}}$ $|\mathbf{VV}|_{\text{dB}}$

-30dB -15dB 0dB



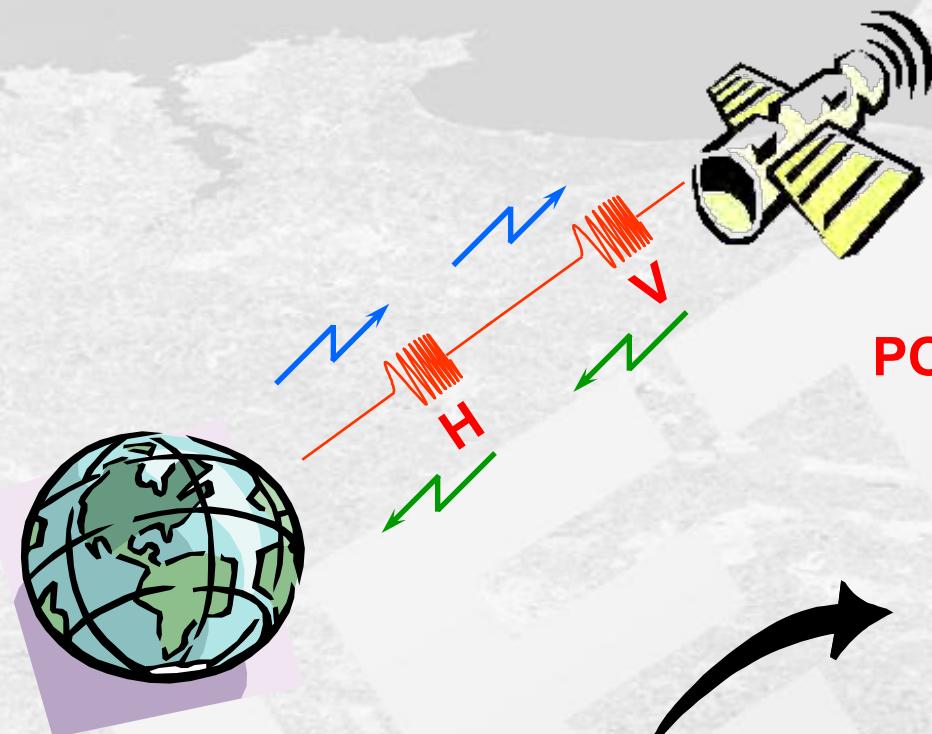
© Google Earth



$|HH|$

$|HV|$

$|VV|$

**POLARIMETRIC DESCRIPTORS**

[*S*] **SINCLAIR Matrix**

$$[S] = \begin{bmatrix} S_{HH} & | & S_{HV} \\ S_{VH} & | & S_{VV} \end{bmatrix}$$

TRANSMITTER: H & V
RECEIVERS: H & V

k Target Vector

[*T*] 3x3 COHERENCY Matrix

MONOSTATIC CASEPAULI SCATTERING VECTOR \underline{k}

$$\underline{k} = \frac{1}{\sqrt{2}} [S_{HH} + S_{VV} \quad S_{HH} - S_{VV} \quad 2S_{HV}]^T$$

COHERENCY MATRIX $[T]$

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

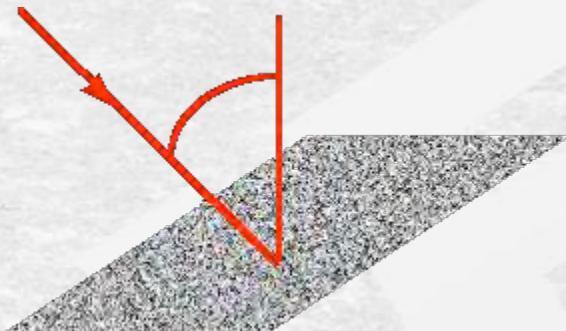
HERMITIAN MATRIX - RANK 1

A0, B0+B, B0-B : HUYNEN TARGET GENERATORS

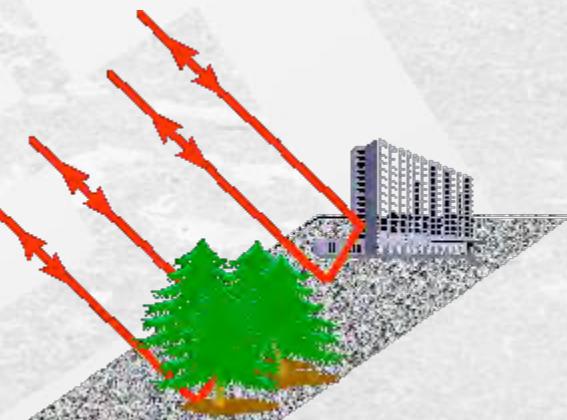
$[T]$ is closer related to Physical and Geometrical Properties of the Scattering Process, and thus allows a better and direct physical interpretation

PHYSICAL INTERPRETATION

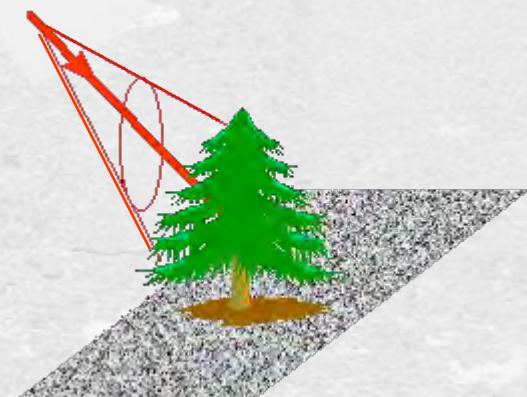
SINGLE BOUNCE
SCATTERING
(ROUGH SURFACE)



DOUBLE BOUNCE
SCATTERING



VOLUME
SCATTERING



$$T_{11} = 2A_\theta = |S_{HH} + S_{VV}|^2$$

$$T_{33} = B_\theta - B = 2|S_{HV}|^2$$

$$T_{22} = B_\theta + B = |S_{HH} - S_{VV}|^2$$



$|HH+VV|_{\text{dB}}$



$|HV|_{\text{dB}}$

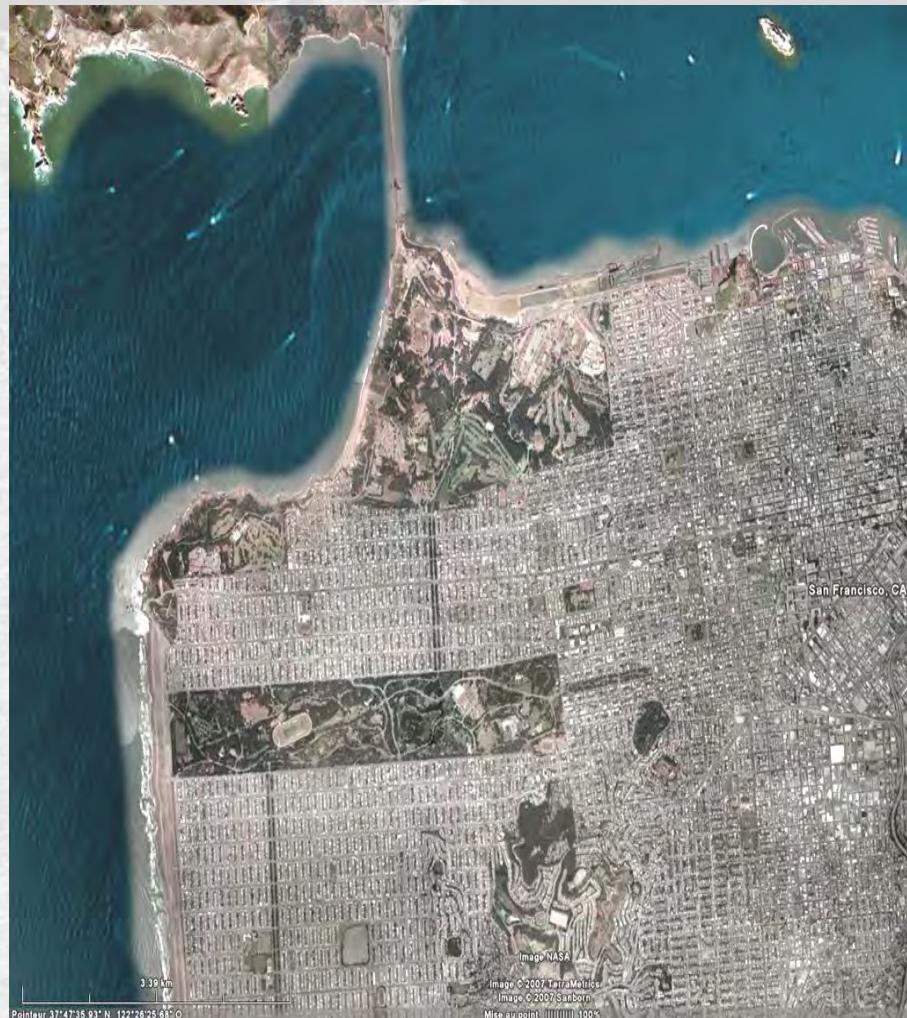


-15dB

0dB



$|HH-VV|_{\text{dB}}$



© Google Earth

(H,V) POLARISATION BASIS



|HH+VV|

|HV|

|HH-VV|

SAR



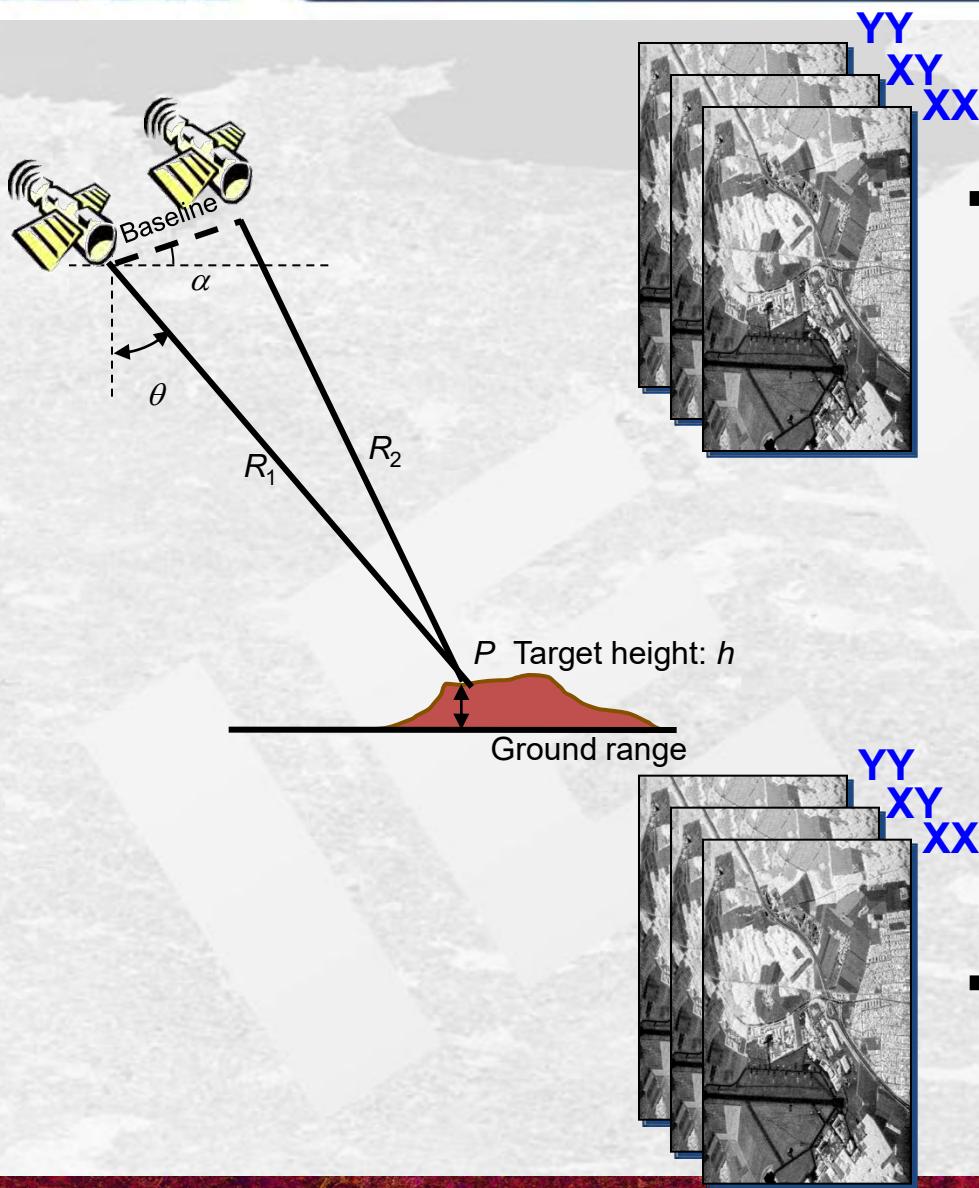
Polarimetry



Interferometry



(Pol-InSAR)



$$\underline{k}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH_1} + S_{VV_1} \\ S_{HH_1} - S_{VV_1} \\ 2S_{HV_1} \end{bmatrix}$$

$$\underline{k} = \begin{bmatrix} \underline{k}_1 \\ \underline{k}_2 \end{bmatrix}$$

**POLARIMETRIC
INTERFEROMETRIC
TARGET VECTOR**

$$\underline{k}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH_2} + S_{VV_2} \\ S_{HH_2} - S_{VV_2} \\ 2S_{HV_2} \end{bmatrix}$$

$$\underline{\underline{k}} = \begin{bmatrix} \underline{k}_1 \\ \underline{k}_2 \end{bmatrix}$$

POLARIMETRIC
INTERFEROMETRIC
TARGET VECTOR



$$\langle [T_6] \rangle = \left\langle \underline{\underline{k}} \cdot \underline{\underline{k}}^{T^*} \right\rangle = \begin{bmatrix} \left\langle \underline{k}_1 \cdot \underline{k}_1^{T^*} \right\rangle & \left\langle \underline{k}_1 \cdot \underline{k}_2^{T^*} \right\rangle \\ \left\langle \underline{k}_2 \cdot \underline{k}_1^{T^*} \right\rangle & \left\langle \underline{k}_2 \cdot \underline{k}_2^{T^*} \right\rangle \end{bmatrix} = \begin{bmatrix} \langle [T_1] \rangle & \langle [\Omega_{12}] \rangle \\ \langle [\Omega_{12}]^{T^*} \rangle & \langle [T_2] \rangle \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC COHERENCY MATRIX (6x6)

$\langle [T_1] \rangle$ HERMITIAN POLARIMETRIC COHERENCY MATRIX (3x3)

$\langle [T_2] \rangle$ HERMITIAN POLARIMETRIC COHERENCY MATRIX (3x3)

$\langle [\Omega_{12}] \rangle$ NON HERMITIAN POLARIMETRIC INTER-COHERENCY MATRIX (3x3)

POLSAR IMAGES

$$I_1 = \underline{w}_1^{T^*} \cdot \underline{k}_1 \quad \text{and} \quad I_2 = \underline{w}_2^{T^*} \cdot \underline{k}_2$$

With: $(\underline{w}_1, \underline{w}_2)$ complex Unitary Vectors

$$\gamma(\underline{w}_1, \underline{w}_2) = \frac{\langle I_1 I_2^* \rangle}{\sqrt{\langle I_1 I_1^* \rangle \langle I_2 I_2^* \rangle}} = \frac{\langle \underline{w}_1 [\Omega_{12}] \underline{w}_2^{T^*} \rangle}{\sqrt{\langle \underline{w}_1 [T_1] \underline{w}_1^{T^*} \rangle \langle \underline{w}_2 [T_2] \underline{w}_2^{T^*} \rangle}}$$

COMPLEX POLARIMETRIC INTERFEROMETRIC COHERENCE **$\arg(\gamma)$ INTERFEROMETRIC PHASE** **$|\gamma|$ CROSS-CORRELATION COEFFICIENT**



$$\gamma_{HH_1-HH_2}$$

$$\underline{w}_1 = \underline{w}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

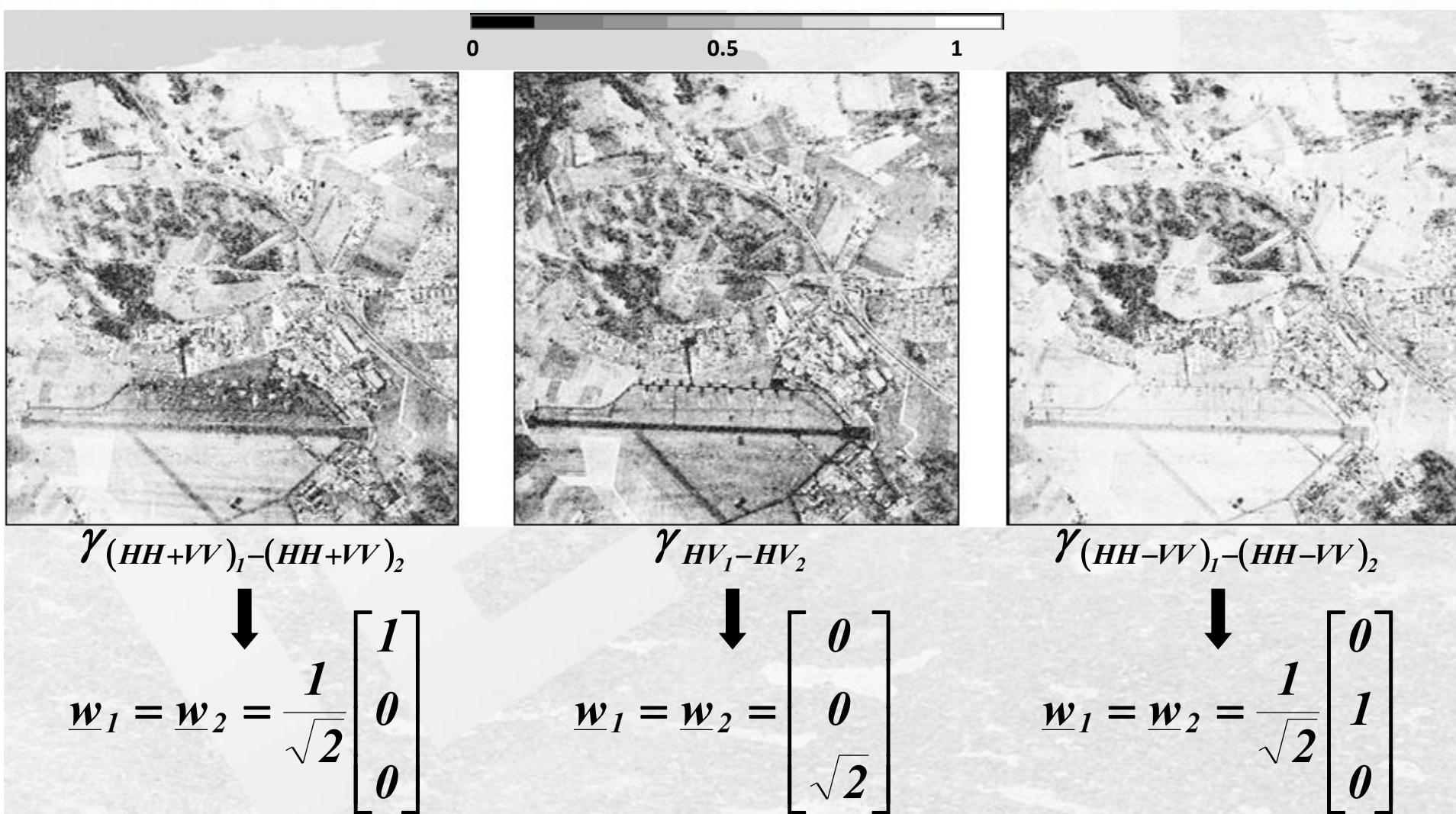
$$\gamma_{HV_1-HV_2}$$

$$\underline{w}_1 = \underline{w}_2 = \begin{bmatrix} 0 \\ 0 \\ \sqrt{2} \end{bmatrix}$$

$$\gamma_{VV_1-VV_2}$$

$$\underline{w}_1 = \underline{w}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

if $\underline{w}_1 = \underline{w}_2 \Rightarrow \gamma = \gamma_{INT}$ if $\underline{w}_1 \neq \underline{w}_2 \Rightarrow \gamma = \gamma_{INT} \cdot \gamma_{POL}$

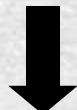


$$\gamma(\underline{w}_1, \underline{w}_2) = \frac{\langle \mathbf{I}_1 \mathbf{I}_2^* \rangle}{\sqrt{\langle \mathbf{I}_1 \mathbf{I}_1^* \rangle \langle \mathbf{I}_2 \mathbf{I}_2^* \rangle}} = \frac{\langle \underline{w}_1 [\Omega_{12}] \underline{w}_2^{T*} \rangle}{\sqrt{\langle \underline{w}_1 [T_1] \underline{w}_1^{T*} \rangle \langle \underline{w}_2 [T_2] \underline{w}_2^{T*} \rangle}}$$

COMPLEX POLARIMETRIC INTERFEROMETRIC COHERENCE



QUESTION: WHICH POLARISATION COMBINATION LEADS TO THE MAXIMUM POSSIBLE INTERFEROMETRIC COHERENCE ?



POLARIMETRIC INTERFEROMETRIC COHERENCE OPTIMISATION PROCEDURE

S.R CLOUDE – K. PAPATHANASSIOU (1999)

POLARIMETRIC INTERFEROMETRIC COHERENCE OPTIMISATION PROCEDURE

S.R CLOUDE – K. PAPATHANASSIOU (1999)

$$\gamma(\underline{w}_1, \underline{w}_2) = \frac{\langle \mathbf{I}_1 \mathbf{I}_2^* \rangle}{\sqrt{\langle \mathbf{I}_1 \mathbf{I}_1^* \rangle \langle \mathbf{I}_2 \mathbf{I}_2^* \rangle}} = \frac{\langle \underline{w}_1 [\Omega_{12}] \underline{w}_2^{T*} \rangle}{\sqrt{\langle \underline{w}_1 [T_1] \underline{w}_1^{T*} \rangle \langle \underline{w}_2 [T_2] \underline{w}_2^{T*} \rangle}}$$

↓

Optimum Coherence set (3x3 eigenvector problem) :

$$(\underline{w}_{opt_1}, \underline{w}_{opt_2}) = \underset{(\underline{w}_1, \underline{w}_2)}{\arg \max} (\gamma(\underline{w}_1, \underline{w}_2))^2$$

→ $\frac{\partial |\gamma(\underline{w}_1, \underline{w}_2)|^2}{\partial \underline{w}_1} = \frac{\partial |\gamma(\underline{w}_1, \underline{w}_2)|^2}{\partial \underline{w}_2} = 0$

POLARIMETRIC INTERFEROMETRIC COHERENCE OPTIMISATION PROCEDURE

S.R CLOUDE – K. PAPATHANASSIOU (1999)

$$\frac{\partial |\gamma(\underline{w}_1, \underline{w}_2)|^2}{\partial \underline{w}_1} = \frac{\partial |\gamma(\underline{w}_1, \underline{w}_2)|^2}{\partial \underline{w}_2} = 0$$



$$[T_1]^{-1} [\Omega_{12}] [T_2]^{-1} [\Omega_{12}]^{T^*} \underline{w}_{opt_1} = |\gamma_{opt}|^2 \underline{w}_{opt_1}$$

$$[T_2]^{-1} [\Omega_{12}]^{T^*} [T_1]^{-1} [\Omega_{12}] \underline{w}_{opt_2} = |\gamma_{opt}|^2 \underline{w}_{opt_2}$$

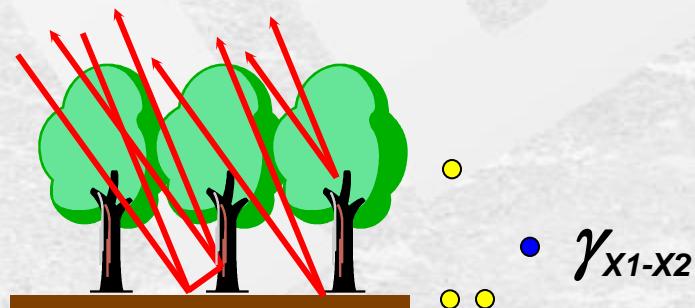


3 Real Eigenvalues (Optimum Coherence Values) : $\gamma_{opt_1} \geq \gamma_{opt_2} \geq \gamma_{opt_3} \geq 0$

3 Pairs of Eigenvectors (Optimum Scattering Mechanisms) :

$$\{\underline{w}_{opt_{1-1}}, \underline{w}_{opt_{2-1}}\}, \{\underline{w}_{opt_{1-2}}, \underline{w}_{opt_{2-2}}\}, \{\underline{w}_{opt_{1-3}}, \underline{w}_{opt_{2-3}}\}$$

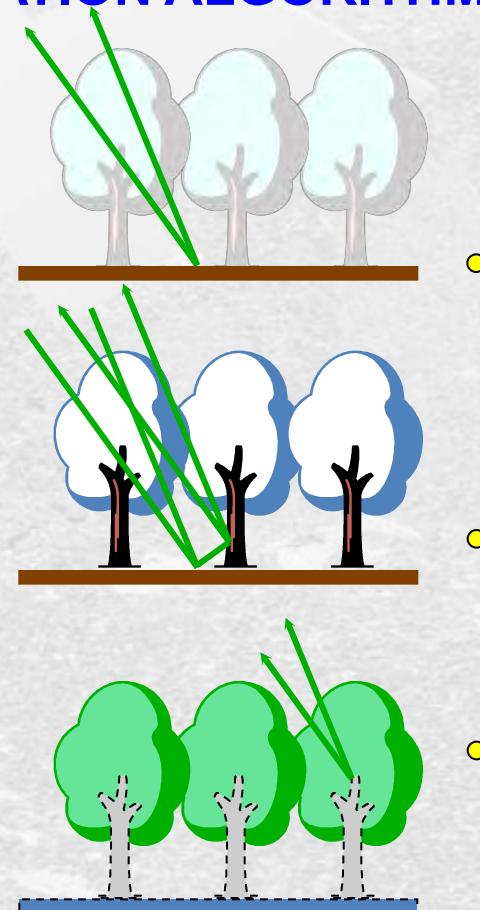
PHYSICAL INTERPRETATION OF POLARIMETRIC INTERFEROMETRIC COHERENCES OPTIMISATION ALGORITHM



HH+VV

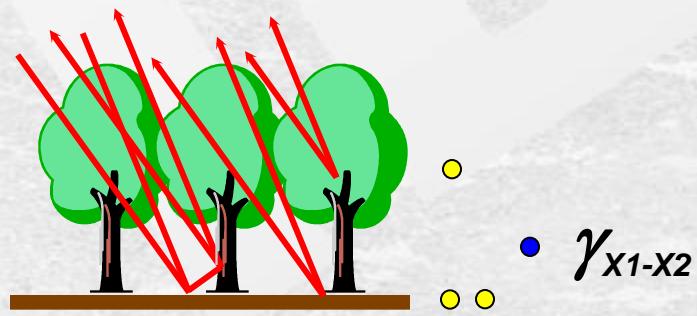
HH-VV

2HV



IN A PERFECT WORLD

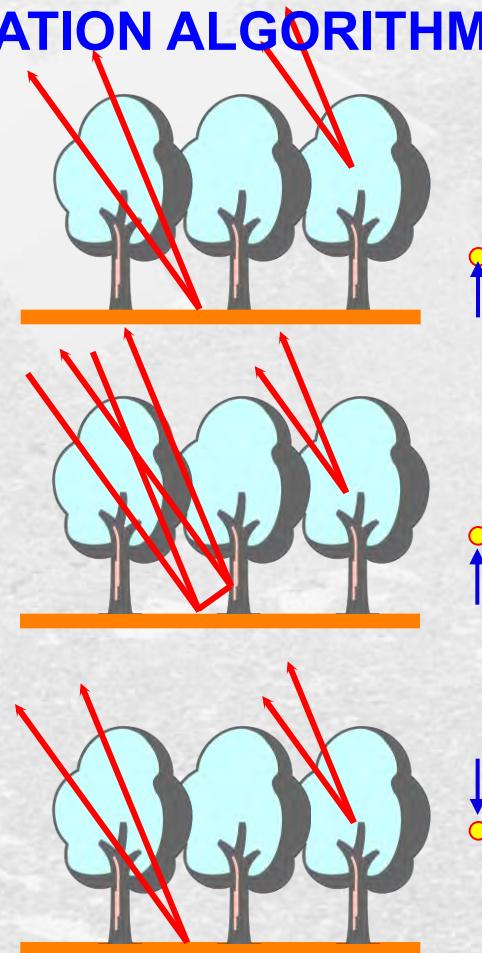
PHYSICAL INTERPRETATION OF POLARIMETRIC INTERFEROMETRIC COHERENCES OPTIMISATION ALGORITHM



$HH+VV$

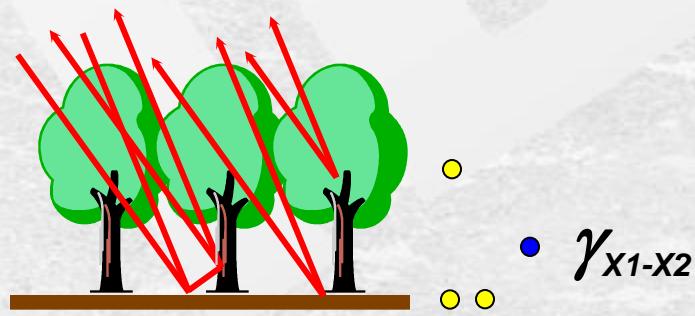
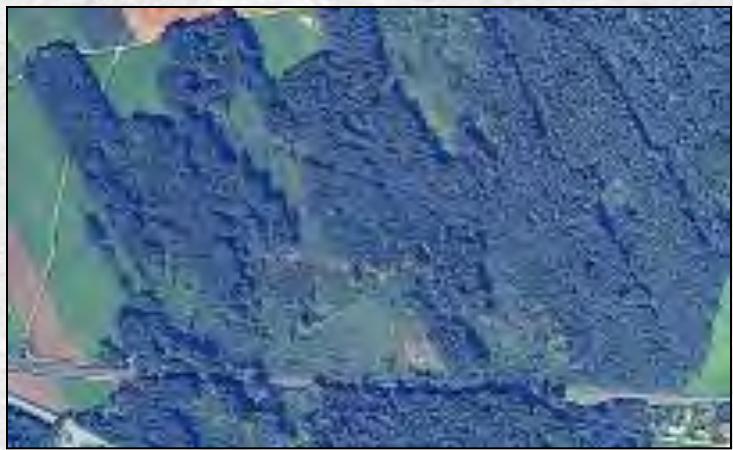
$HH-VV$

$2HV$



IN A REAL WORLD

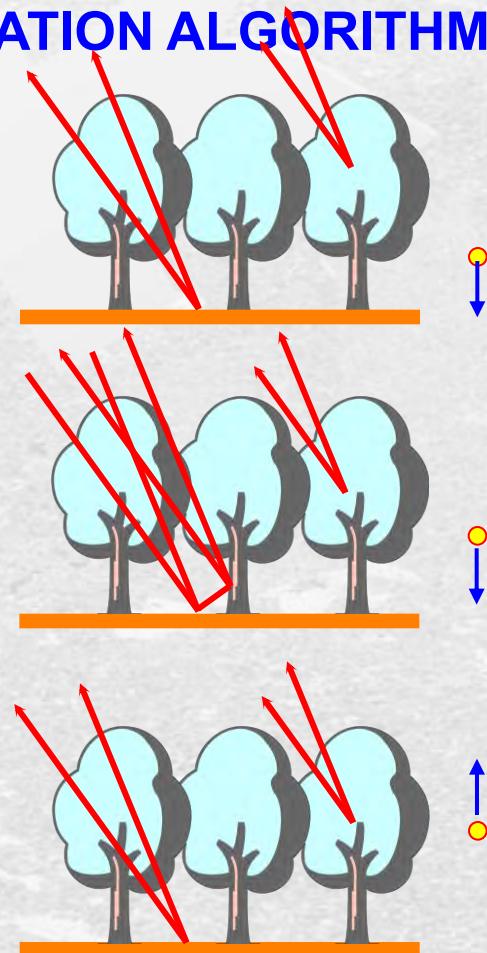
PHYSICAL INTERPRETATION OF POLARIMETRIC INTERFEROMETRIC COHERENCES OPTIMISATION ALGORITHM



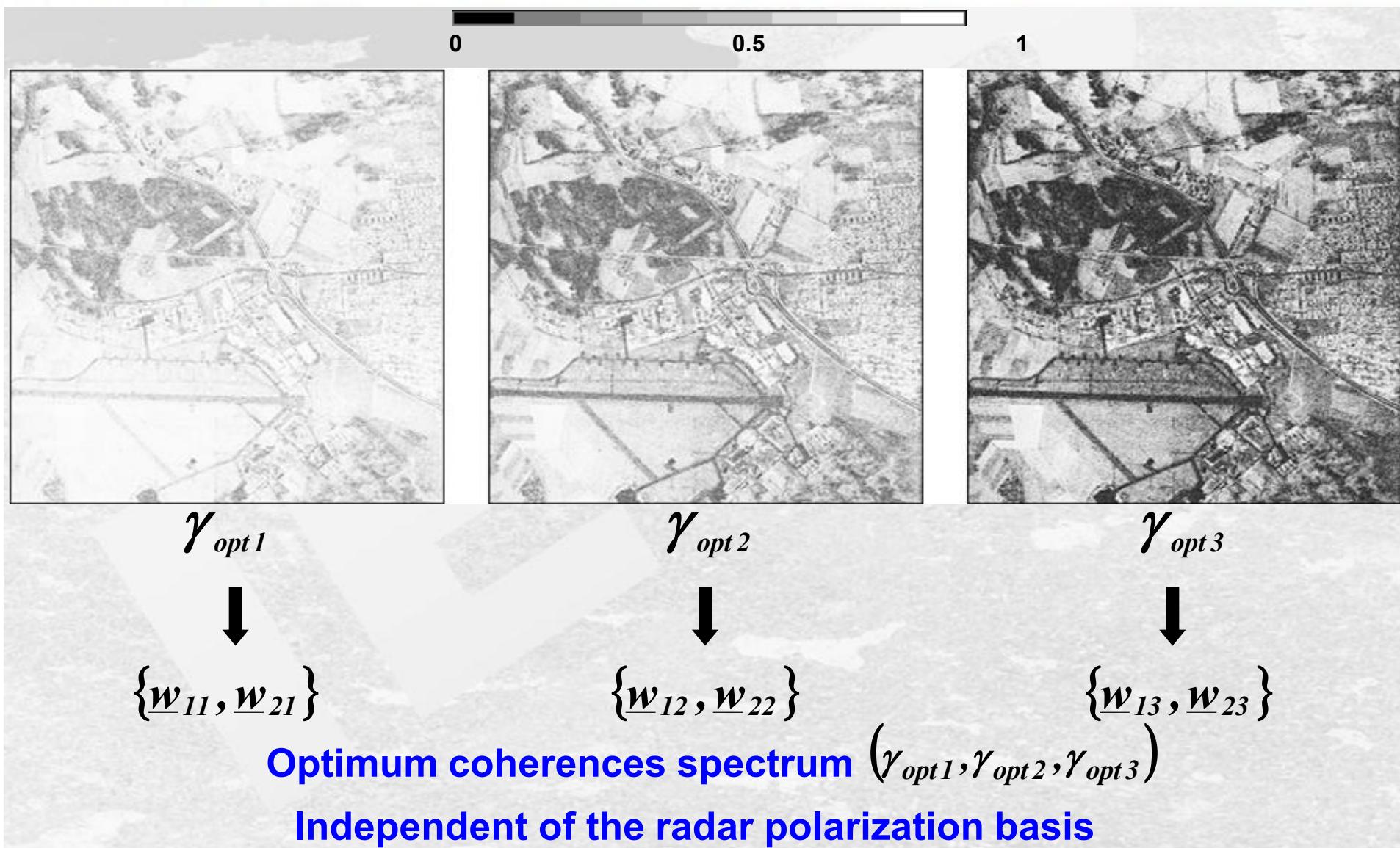
$$\{\underline{w}_{11}, \underline{w}_{21}\}$$

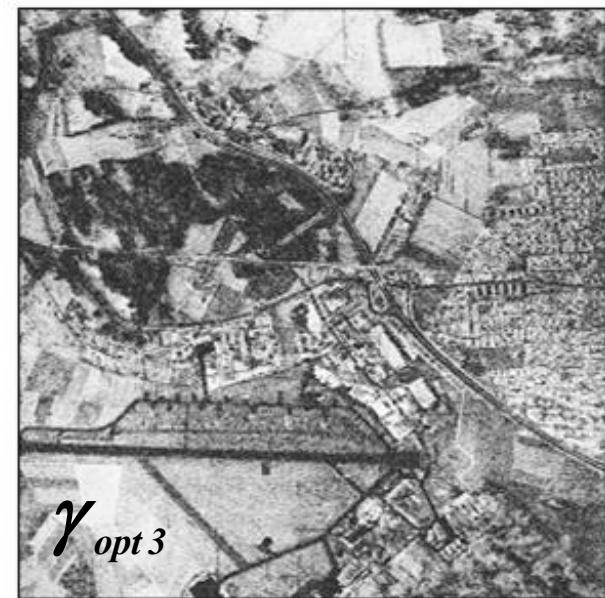
$$\{\underline{w}_{12}, \underline{w}_{22}\}$$

$$\{\underline{w}_{13}, \underline{w}_{23}\}$$



IN AN OPTIMISED WORLD





Importance: Key parameter for forest management worldwide

Height as a product itself

- Phase of stand development
- Spatial height distribution (risk assessment, diversity)
- Management with esthetical protection goals
- Change of topography by forests (water runoff, skidding, roads)

Height as an input parameter

- Wood volume / forest biomass
- Site Index (with age and species information)

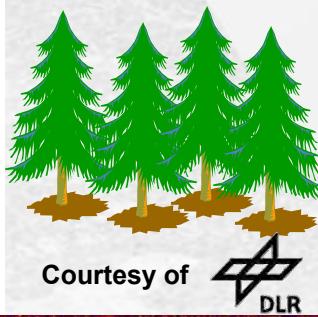
Height and density determine the microclimate and ecological processes within the forest

Height is dependent on and therefore reflects the site conditions

Courtesy of  DLR

Which Height does POLinSAR measure ? H100 (forest standard)

- Most characteristic height in a forest
- Formed by the crown of the trees exposed to the sun light
- Typically concentrate most of the forest biomass
- Simple to measure: it is the intuitive forest height, measurement of few representative tree heights
- Sufficient for a good estimation

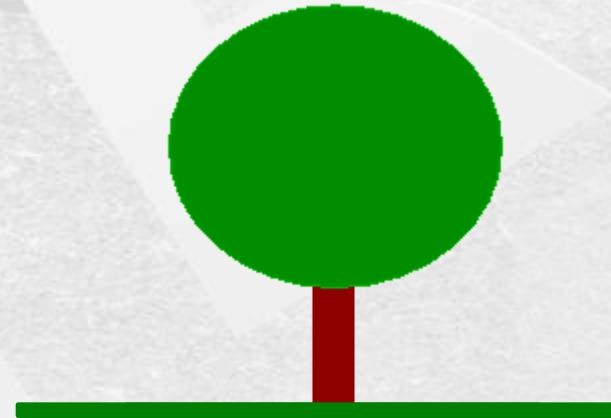
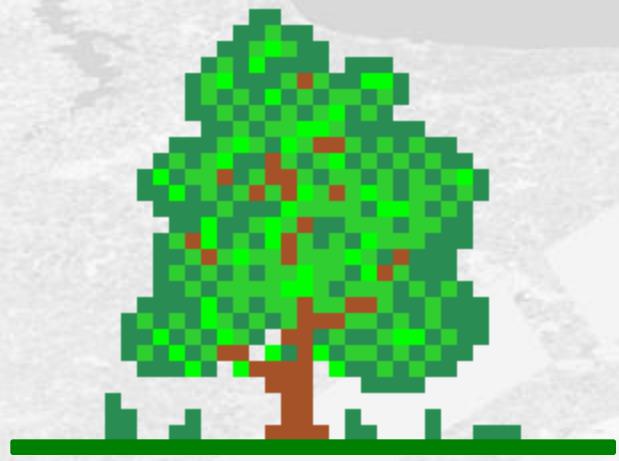


Forest Stand Type I
Homogeneous stands
Typical for managed forests



Forest Stand Type II
Very heterogeneously structured stands
Large height variations on short distance
Typical for natural uneven-aged forest

MODELING AND PARAMETER ESTIMATION

Modeling

Establishment of scattering model [M]

$$\begin{bmatrix} \text{Radar} \\ \text{Observables} \end{bmatrix} = [M] \begin{bmatrix} \text{Scatterer} \\ \text{Parameters} \end{bmatrix}$$

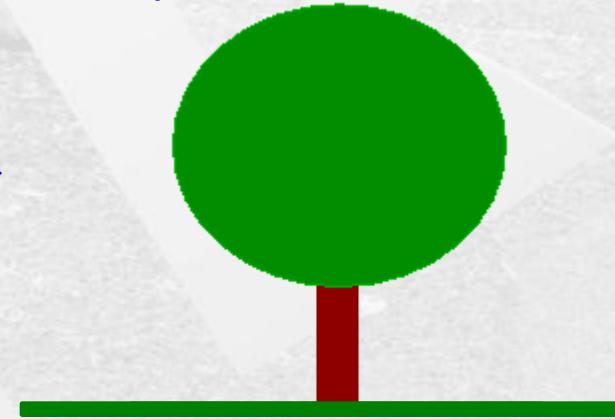
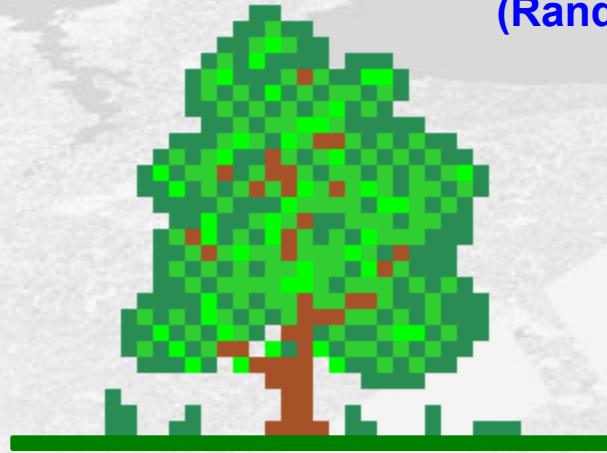
Parameter Estimation

Inversion of the scattering model [M]

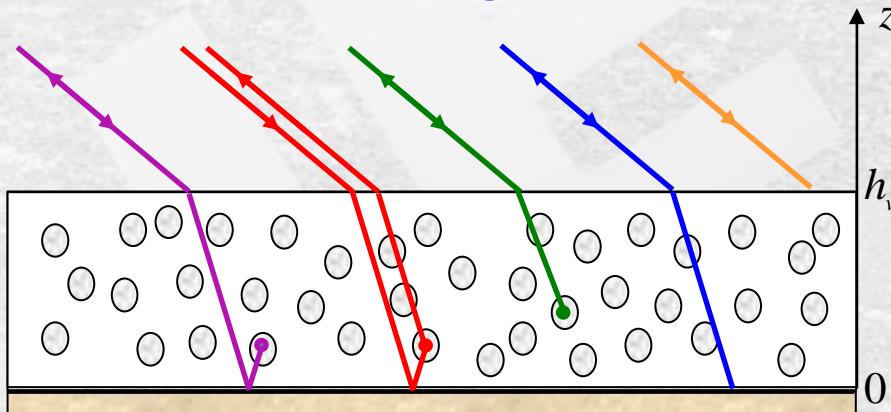
$$\begin{bmatrix} \text{Scatterer} \\ \text{Parameters} \end{bmatrix} = [M]^{-1} \begin{bmatrix} \text{Radar} \\ \text{Observables} \end{bmatrix}$$

Requirements on [M]: 1. Correctness in Interpretation and prediction of the observables
2. Simplicity in terms of parameters in order to be determined

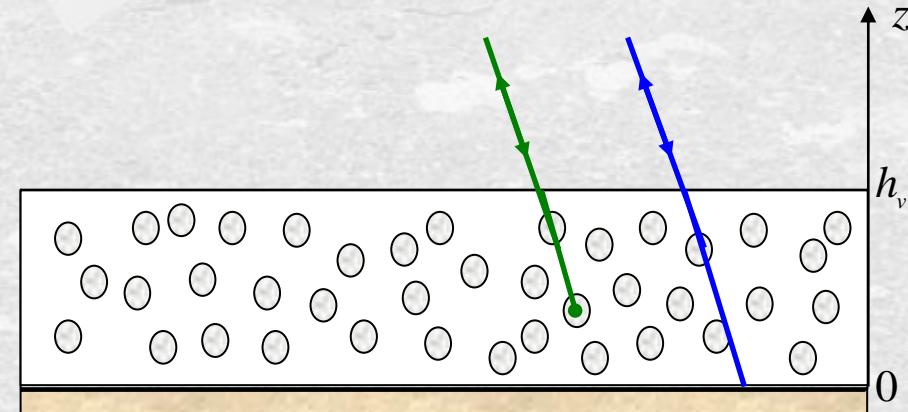
RVOG COHERENCE MODEL
(Random Volume Over Ground)



Modeling



Parameter Estimation



Simplifications : Only 2 significant mechanisms Low density medium \Rightarrow No refraction

Interferometric coherence γ : decorrelation sources

γ fixed by a set of external sources :

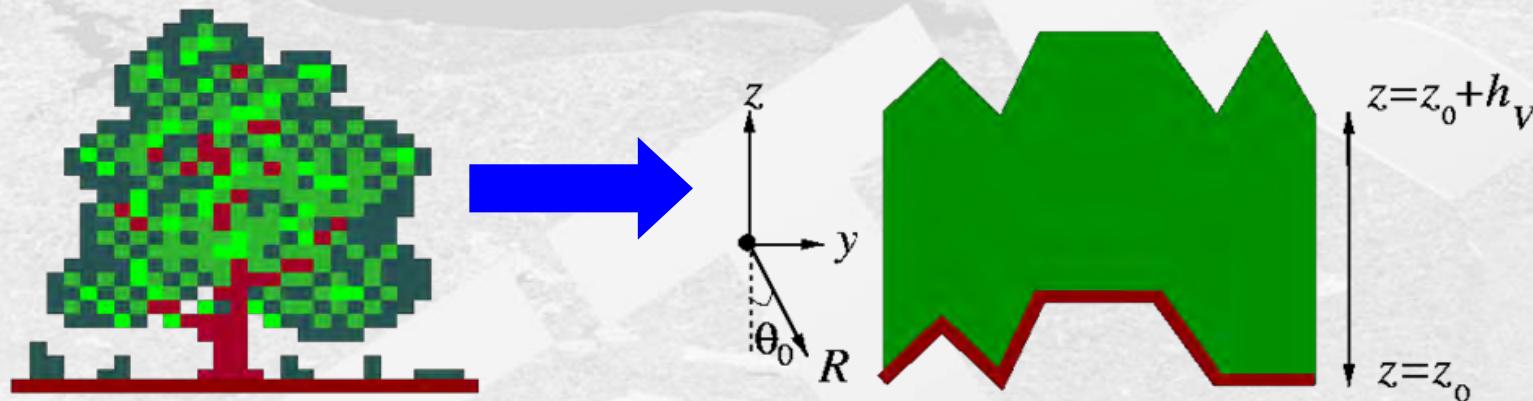
System :

- Thermal or system noise : SAR amplifiers, ADC, antennas ...
- Quantization noise
- Geometric decorrelation : Baseline, squint ...
- Azimuth : Doppler decorrelation ...
- Ambiguities ...
- Processing errors : coregistration, interpolation ...

Environment :

- Random media : Surface & Volumetric media e.g. forest ...
- Temporal variations : wind, flowing or plowing, building ...

$$\gamma = \gamma_{SNR} \cdot \gamma_{quant} \cdot \gamma_{amb} \cdot \gamma_{geo} \cdot \gamma_{az} \cdot \gamma_{proc} \cdot \gamma_{surf} \cdot \gamma_{vol} \cdot \gamma_{temp}$$

**RVOG COHERENCE MODEL
(Random Volume Over Ground)****2 Layer Combined Surface and random Volume Scattering**

$$\gamma_z(\underline{w}) = e^{j\phi_0} \frac{\tilde{\gamma}_{vol} + m(\underline{w})}{1 + m(\underline{w})}$$

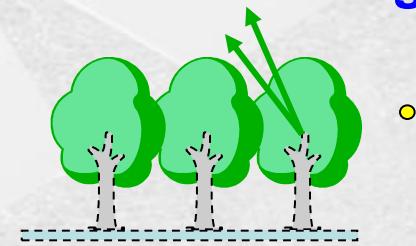
$$m(\underline{w}) = \frac{\text{Surface Scattering Contribution}}{\text{Volume Scattering Contribution}} \quad \text{G / V ratio}$$

B. Treuhaft (2000), S.R. Cloude (2003)**POLARIZATION DEPENDENT**

\underline{w}_v Polarisation Channel corresponding to Volume Scattering

$$\gamma_z(\underline{w}_v) \underset{m \rightarrow 0}{\mapsto} e^{j\phi_0} \tilde{\gamma}_{vol}$$

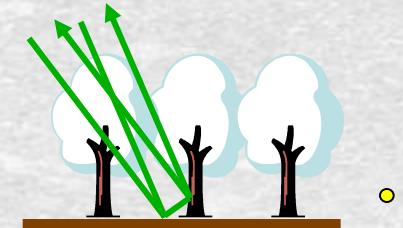
2HV

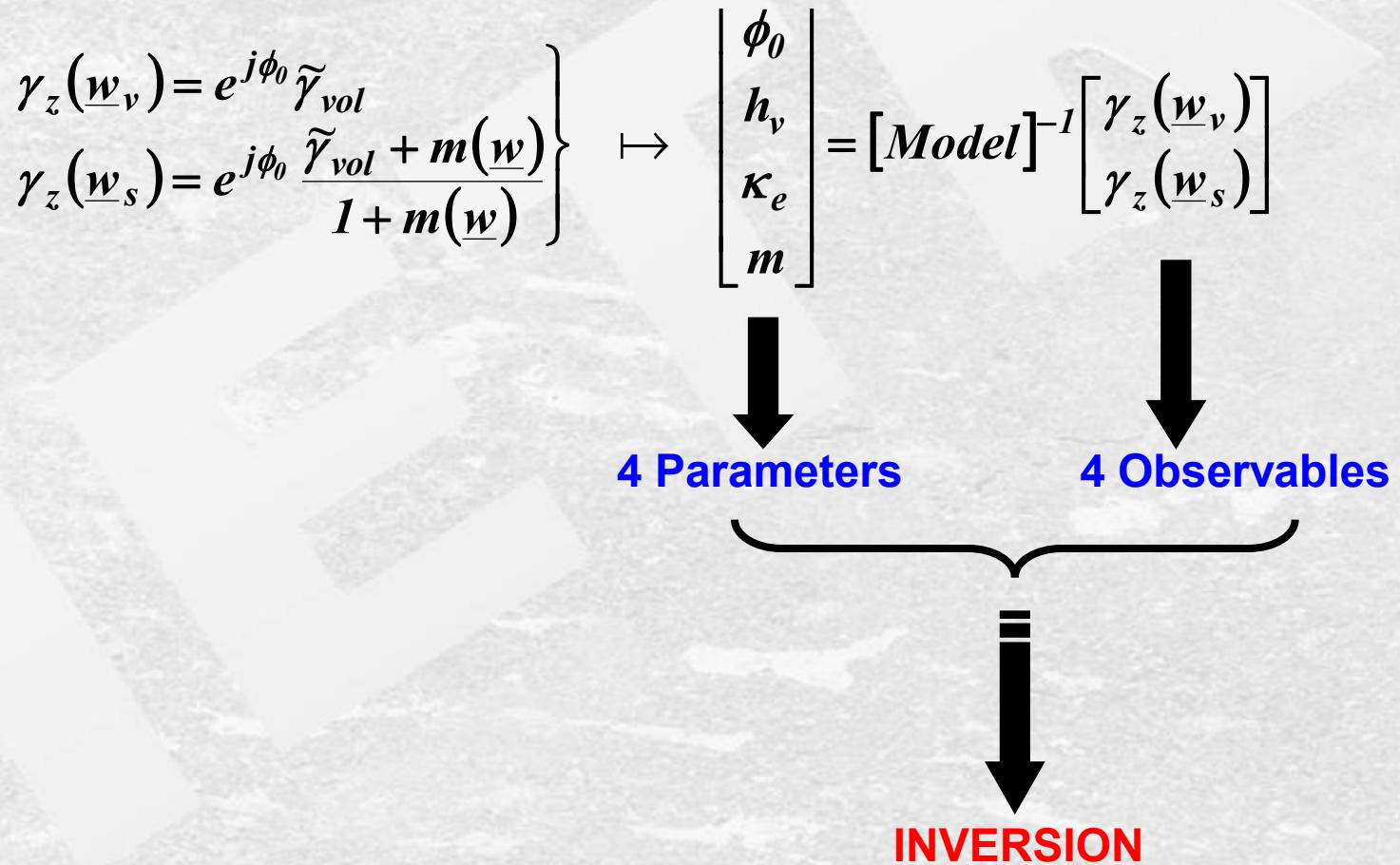


\underline{w}_s Polarisation Channel corresponding to Surface Scattering

$$\gamma_z(\underline{w}_s) = e^{j\phi_0} \frac{\tilde{\gamma}_{vol} + m(\underline{w})}{1 + m(\underline{w})} \underset{m \rightarrow \infty}{\mapsto} e^{j\phi_0}$$

HH-VV





DEM Differencing Algorithm

$$\left. \begin{array}{l} \gamma_z(\underline{w}_v) = e^{j\phi_0} \tilde{\gamma}_{vol} \\ \gamma_z(\underline{w}_s) \mapsto e^{j\phi_0} \end{array} \right\} \mapsto \gamma_z(\underline{w}_v) = \gamma_z(\underline{w}_s) \tilde{\gamma}_{vol} \approx \gamma_z(\underline{w}_s) \alpha e^{jk_z h_v}$$



$$h_v \approx \frac{\arg[\gamma_z(\underline{w}_v)] - \arg[\gamma_z(\underline{w}_s)]}{k_z}$$

Coherence Amplitude Inversion Procedure

Assumption : Only volume scattering is present

$$\gamma_z(\underline{w}_v) = e^{j\phi_0} \tilde{\gamma}_{vol} \quad \mapsto \quad |\gamma_z(\underline{w}_v)| = |\tilde{\gamma}_{vol}|$$



$$\arg \min_d \left\| |\gamma_z(\underline{w}_v)| - \left| \frac{p}{p_1} \left(\frac{e^{p_1 d} - 1}{e^{pd} - 1} \right) \right| \right\|$$

1-D search procedure with Look-Up-Table (LUT)

Topographic Phase Estimation

$$\left. \begin{array}{l} \gamma_z(\underline{w}_v) = e^{j\phi_0} \tilde{\gamma}_{vol} \\ \gamma_z(\underline{w}_s) = e^{j\phi_0} \frac{\tilde{\gamma}_{vol} + m(\underline{w})}{1 + m(\underline{w})} \end{array} \right\} \mapsto e^{j\phi_0} = \frac{\gamma_z(\underline{w}_s) - \gamma_z(\underline{w}_v)(1 - L)}{L}$$

With : $L = \frac{m(\underline{w}_s)}{1 + m(\underline{w}_s)}$



$$\hat{\phi}_0 = \arg[\gamma_z(\underline{w}_s) - \gamma_z(\underline{w}_v)(1 - L)]$$

Estimation of L :

$$e^{j\phi_0} = \frac{\gamma_z(\underline{w}_s) - \gamma_z(\underline{w}_v)(1-L)}{L}$$



$$\left| \frac{\gamma_z(\underline{w}_s) - \gamma_z(\underline{w}_v)(1-L)}{L} \right|^2 = 1 \quad \Rightarrow \quad AL^2 + BL + C = 0$$

With : $A = |\gamma_z(\underline{w}_v)|^2 - 1$ $B = 2\Re\{(\gamma_z(\underline{w}_s) - \gamma_z(\underline{w}_v))\gamma_z^*(\underline{w}_s)\}$

$$C = |\gamma_z(\underline{w}_s) - \gamma_z(\underline{w}_v)|^2$$

$$\rightarrow L = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

Topographic Phase Estimation

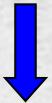
$$\{\gamma_z(\underline{w}_s), \gamma_z(\underline{w}_v)\} \rightarrow A = |\gamma_z(\underline{w}_v)|^2 - 1$$

$$B = 2\Re\{\gamma_z(\underline{w}_s) - \gamma_z(\underline{w}_v)\}\gamma_z^*(\underline{w}_s)$$

$$C = |\gamma_z(\underline{w}_s) - \gamma_z(\underline{w}_v)|^2$$



$$L = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

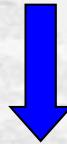


$$\hat{\phi}_0 = \arg[\gamma_z(\underline{w}_s) - \gamma_z(\underline{w}_v)(1 - L)]$$

Random Vegetation Over Ground (RVoG) inversion procedure

$$\arg \min_{d, \kappa_e} \left\| \gamma_z(\underline{w}_v) - e^{j\hat{\phi}_0} \frac{p}{p_1} \left(\frac{e^{p_1 d} - 1}{e^{pd} - 1} \right) \right\|$$

Expensive 2-D search procedure



$$d \approx \frac{\arg[\gamma_z(\underline{w}_v)] - \hat{\phi}_0}{k_z} + \varepsilon \frac{2 \operatorname{sinc}^{-1}(|\gamma_z(\underline{w}_v)|)}{k_z}$$

$$0.3 \leq \varepsilon \leq 0.5$$

Suitable
Compromise

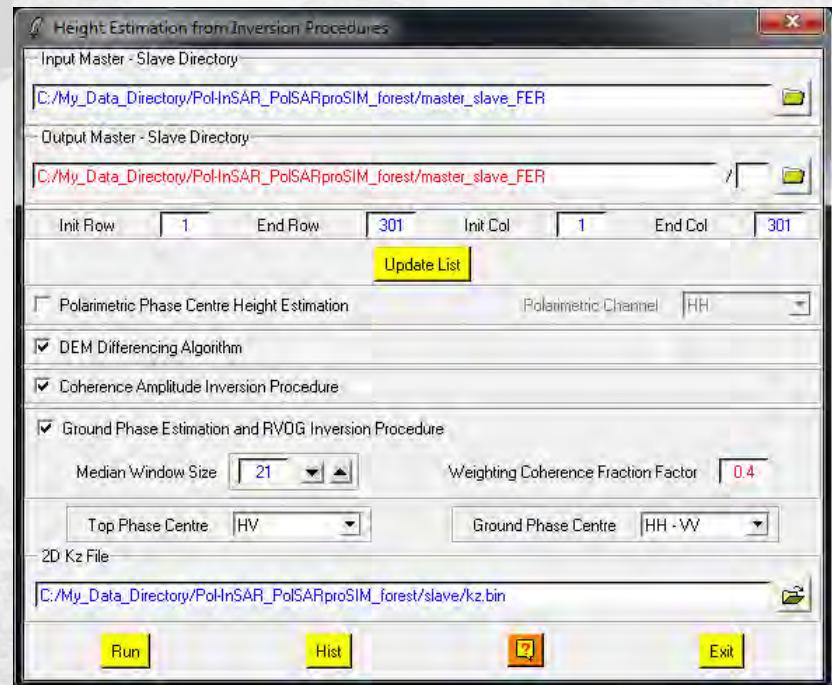
$$d \approx \underbrace{\frac{\arg[\gamma_z(\underline{w}_v)] - \hat{\phi}_0}{k_z}}_{\text{DEM Differencing Inversion}} + \varepsilon \underbrace{\frac{2 \operatorname{sinc}^{-1}(|\gamma_z(\underline{w}_v)|)}{k_z}}_{\text{Coherence Amplitude Inversion}}$$


DEM Differencing
Inversion

Coherence Amplitude
Inversion



Complex Coherence Estimation



Height estimation Inversion procedures

Questions ?

