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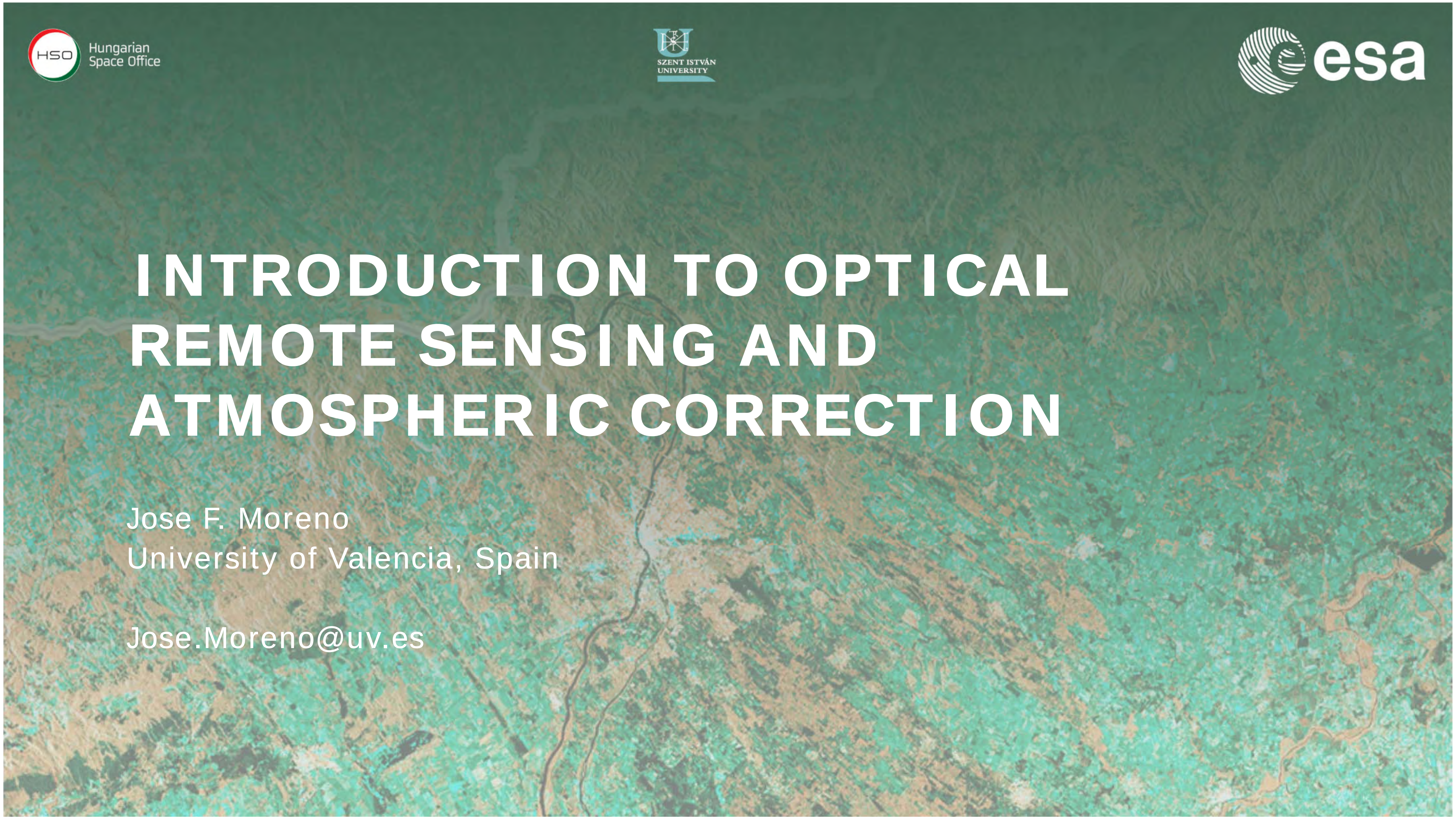
→ 7th ADVANCED TRAINING COURSE ON LAND REMOTE SENSING

4–9 September 2017 | Szent István University | Gödöllő, Hungary



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INTRODUCTION TO OPTICAL REMOTE SENSING AND ATMOSPHERIC CORRECTION

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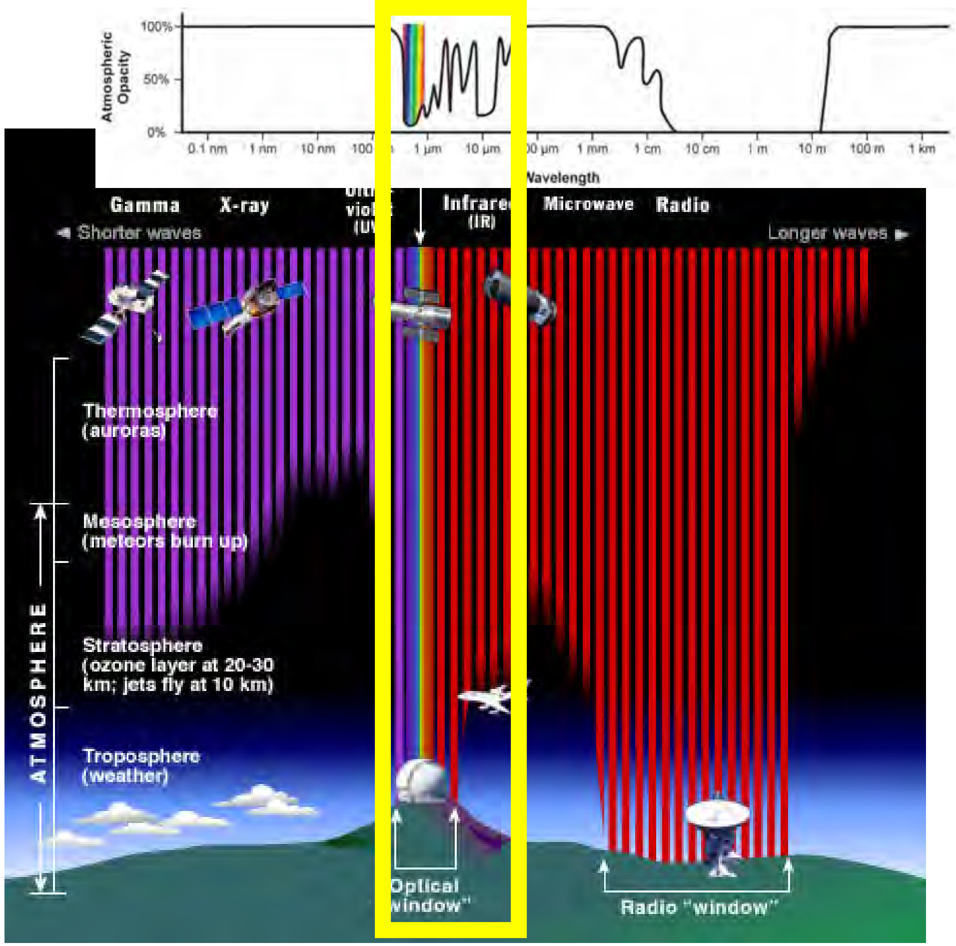
OUTLINE

- Radiometric quantities: definitions, units and physical meaning.
- The information content of optical data.
- Measurements with optical instruments: radiometric and spectral calibration and pre-processing aspects.
- Atmospheric correction of optical remote sensing data, compensation for topographic effects and BRDF normalization.
- Retrieval of information from optical data for science and applications.
- Uncertainty estimates for optical measurements and product validation.



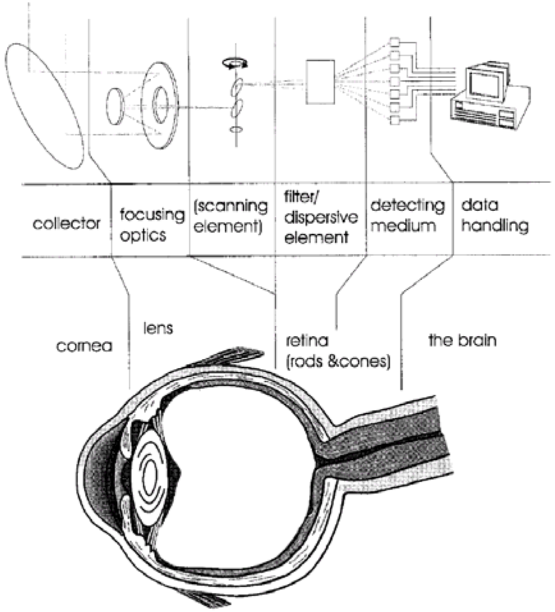
OPTICAL SYSTEMS:

-Visible

-Near infrared

-Shortwave infrared

-Thermal infrared



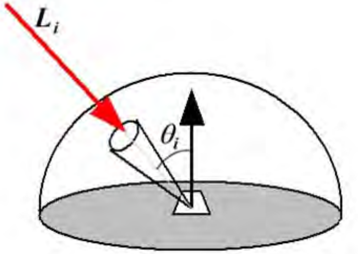
All we measure are radiances !

$$L = \frac{d^2\Phi}{d\Omega \; dS} = \frac{d^2\Phi}{d\Omega \; dA \; \cos\vartheta}$$

$W \; m^{-2} \; sr^{-1}$

$$L = \frac{d^3E}{dt \; d\Omega \; dA \; \cos\vartheta}$$

solid angle



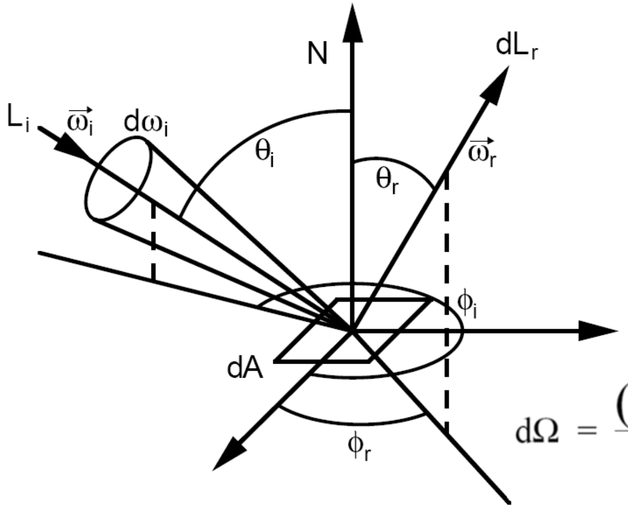
$$M = \int L \cos\vartheta \; d\Omega = \int_0^{2\pi} d\varphi \int_0^{\pi/2} d\vartheta \;\; L(\vartheta,\varphi) \sin\vartheta \cos\vartheta$$

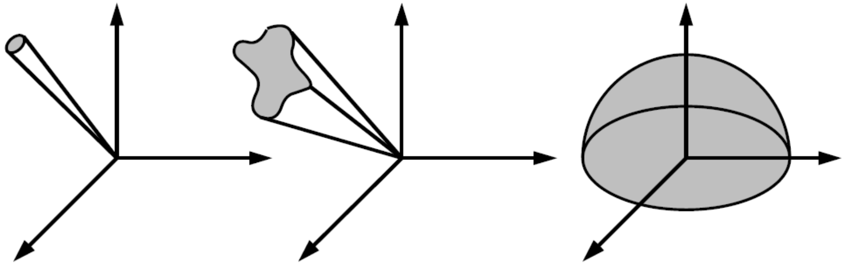
Lambertian case

$$M = \pi L$$

nine types of reflectance measurements

	Incident		
Reflected	directional	conical	hemispherical
directional	bidirectional	conical-directional	hemispherical-directional
conical	directional-conical	biconical	hemispherical-conical
hemispherical	directional-hemispherical	conical-hemispherical	bihemispherical

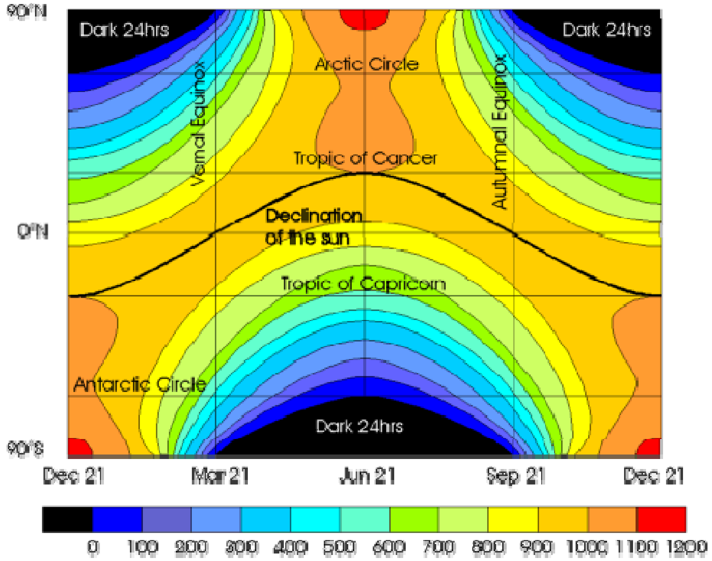




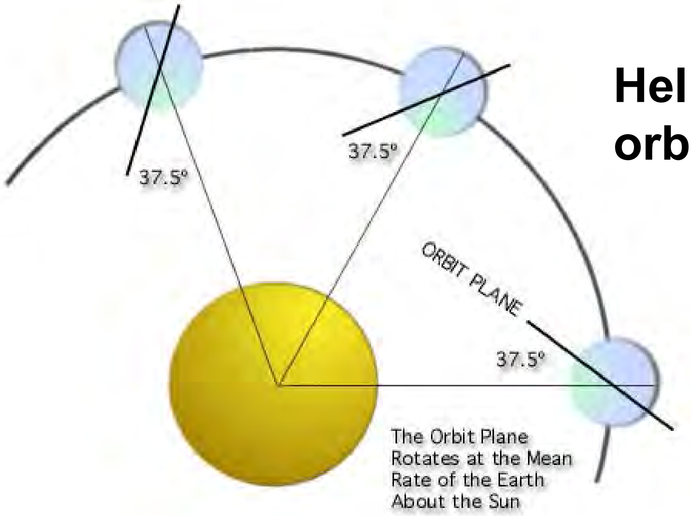
$$d\Omega = \frac{\left(r\ \mathrm{sen}\,\vartheta\ d\varphi\right)\left(r\ d\vartheta\right)}{r^2} = \mathrm{sen}\,\vartheta\ d\vartheta\ d\varphi$$

Bidirectional Reflectance Factor

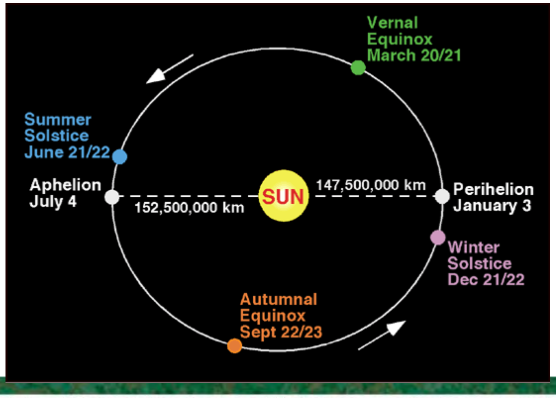
$$R(\vartheta,\varphi,\vartheta',\varphi') = \frac{dL_t^{\uparrow}}{dL_p^{\uparrow}}$$



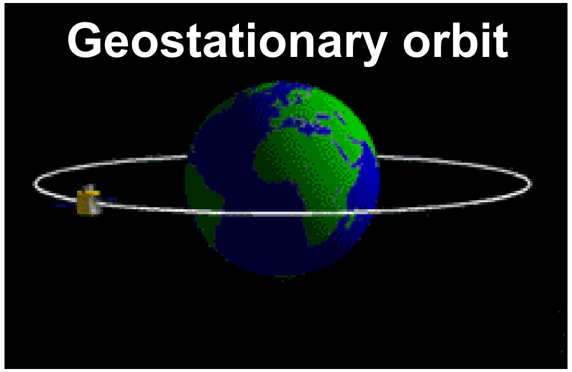
Heliosynchronous orbit

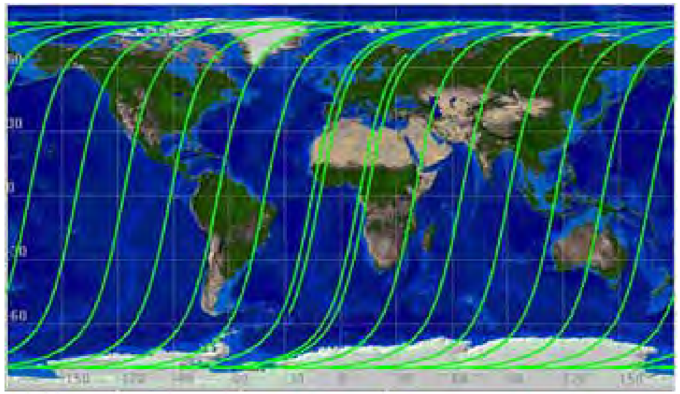


The Orbit Plane Rotates at the Mean Rate of the Earth About the Sun

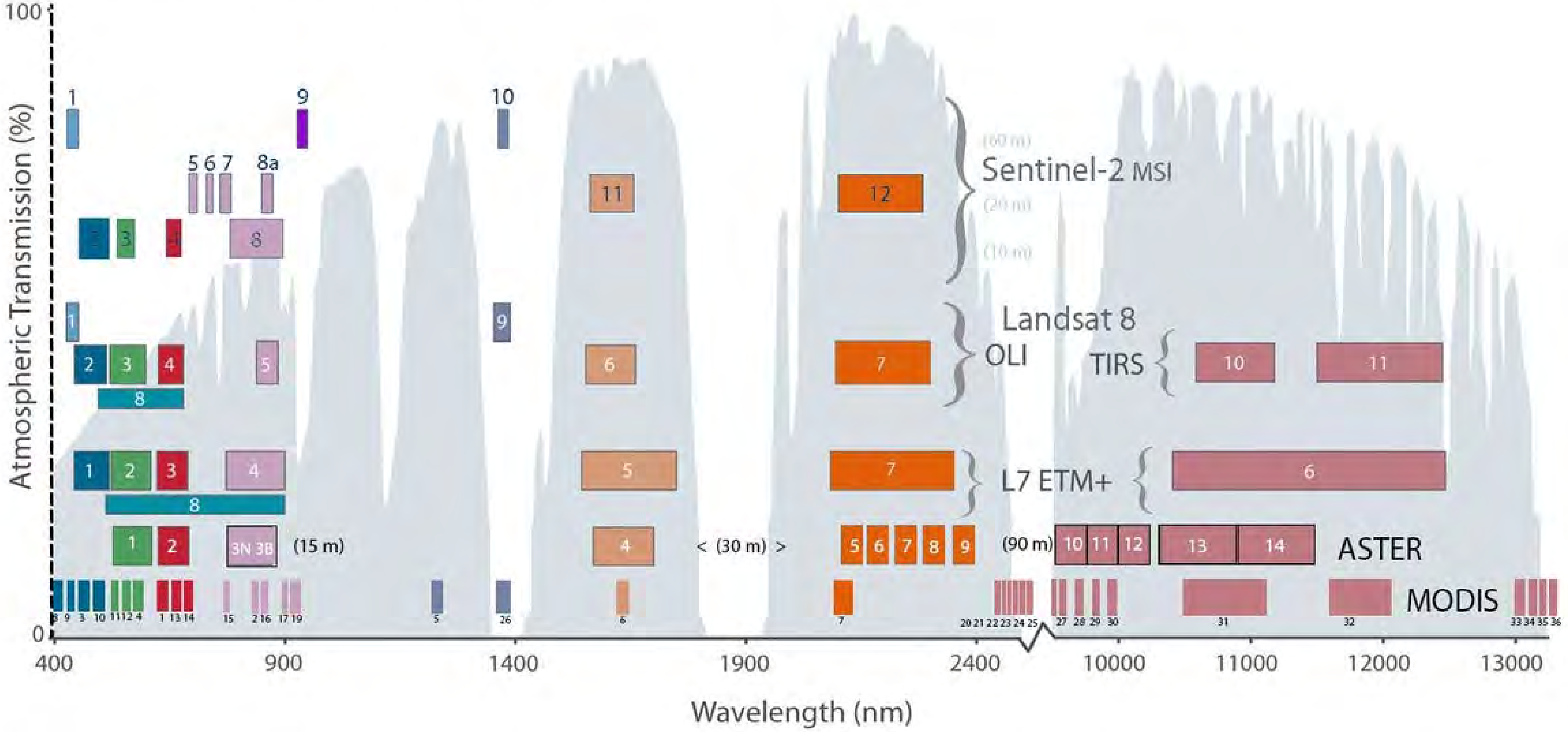


Geostationary orbit





Comparison of Landsat 7 and 8 bands with Sentinel-2



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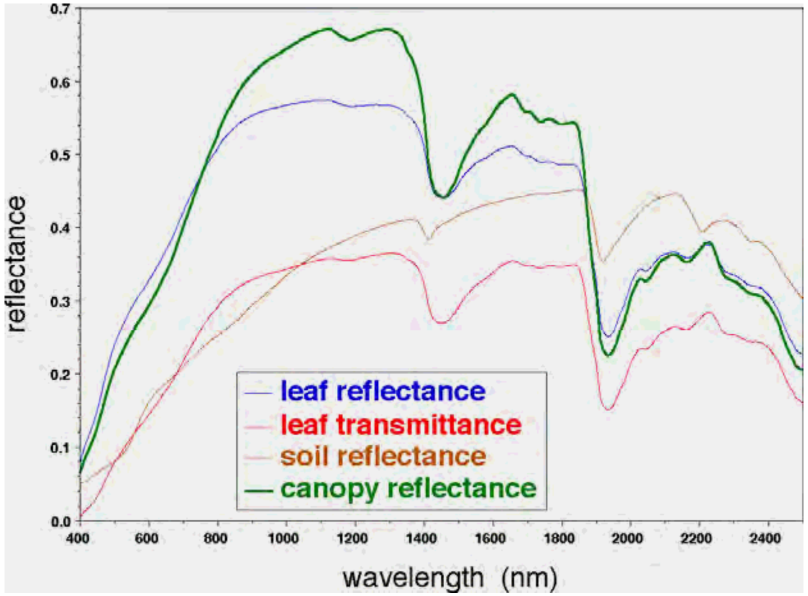
INFORMATION CONTENT OF OPTICAL DATA

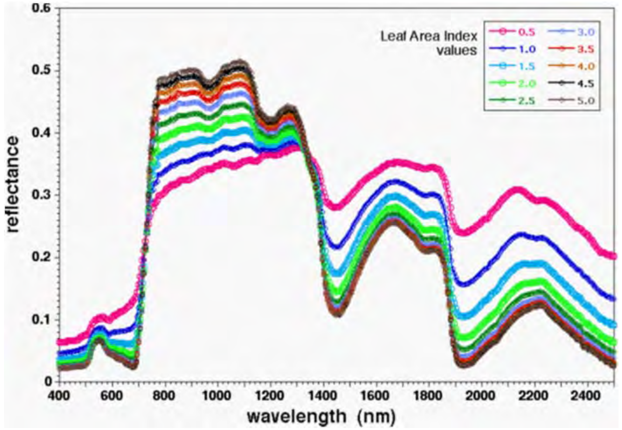
Signatures of natural targets:

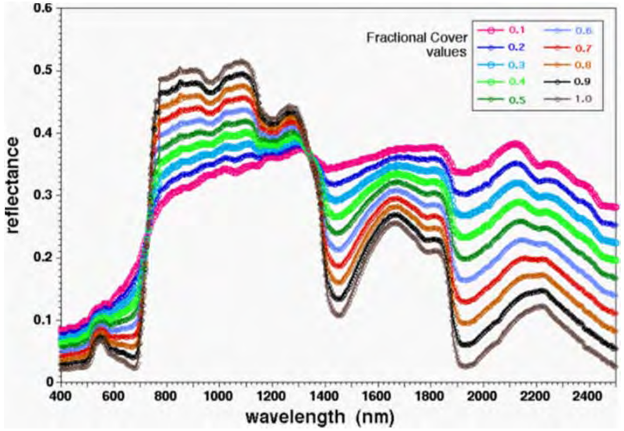
- Spectral signatures
- Angular signatures
- Spatial signatures
- Temporal signatures
- Other signatures (i.e., fluorescence, polarization, etc.)



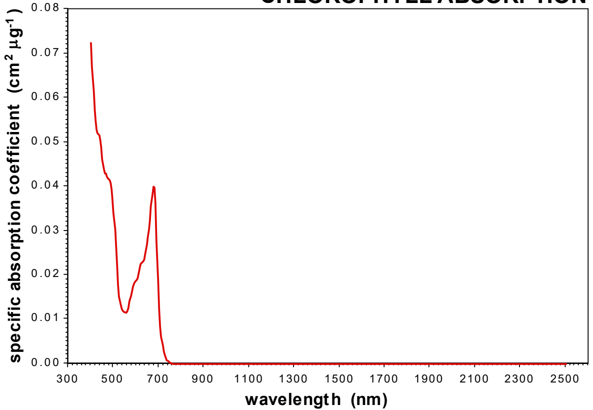
What we measure is always radiance, either reflected and / or emitted by the land surface, which variations depend on the optical properties of land targets (and illumination conditions)

SPECTRAL INFORMATION

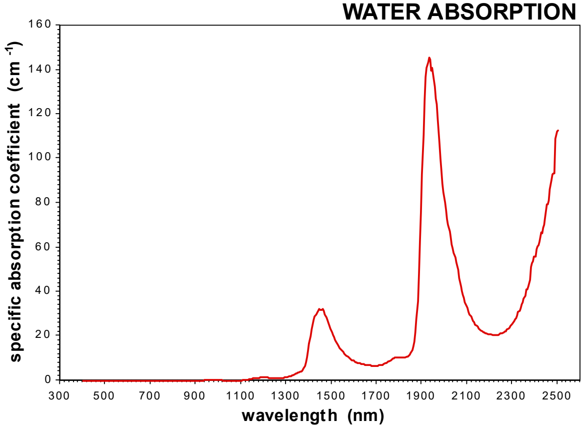


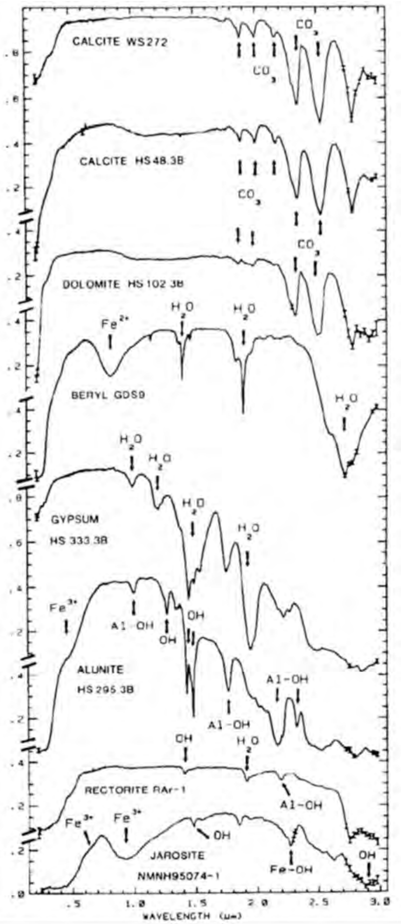


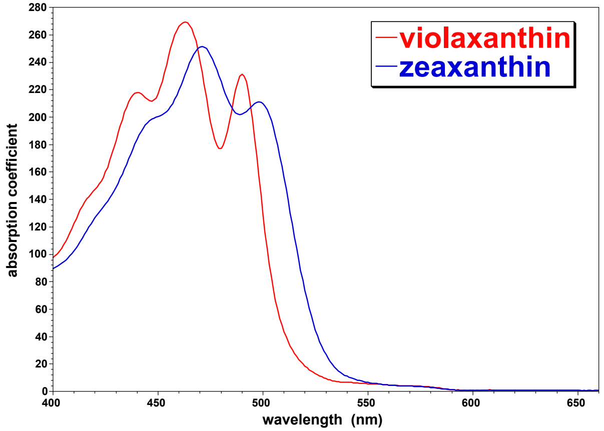
CHLOROPHYLL ABSORPTION

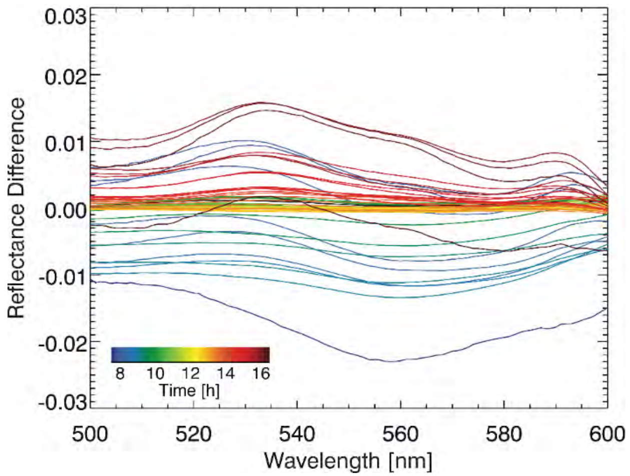


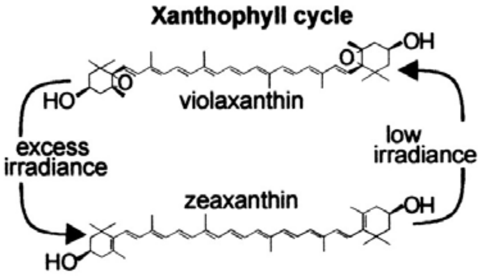
WATER ABSORPTION

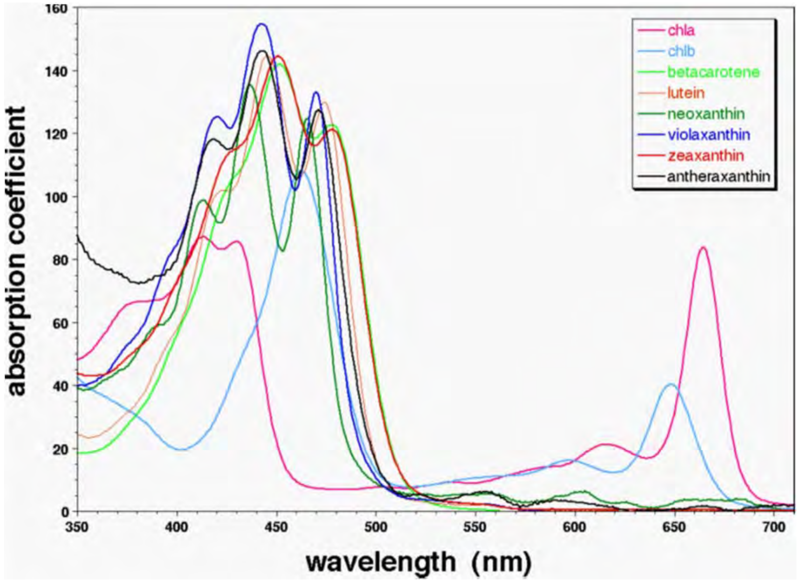




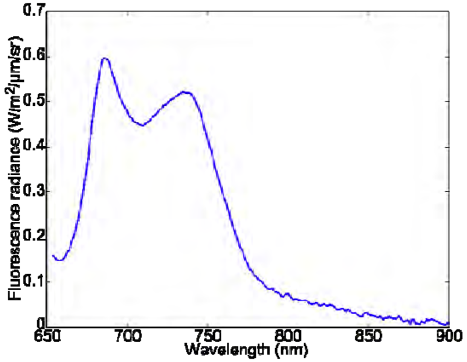


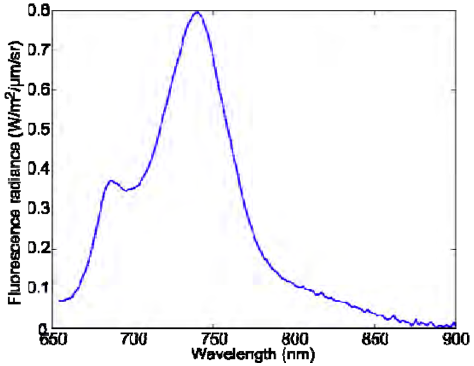




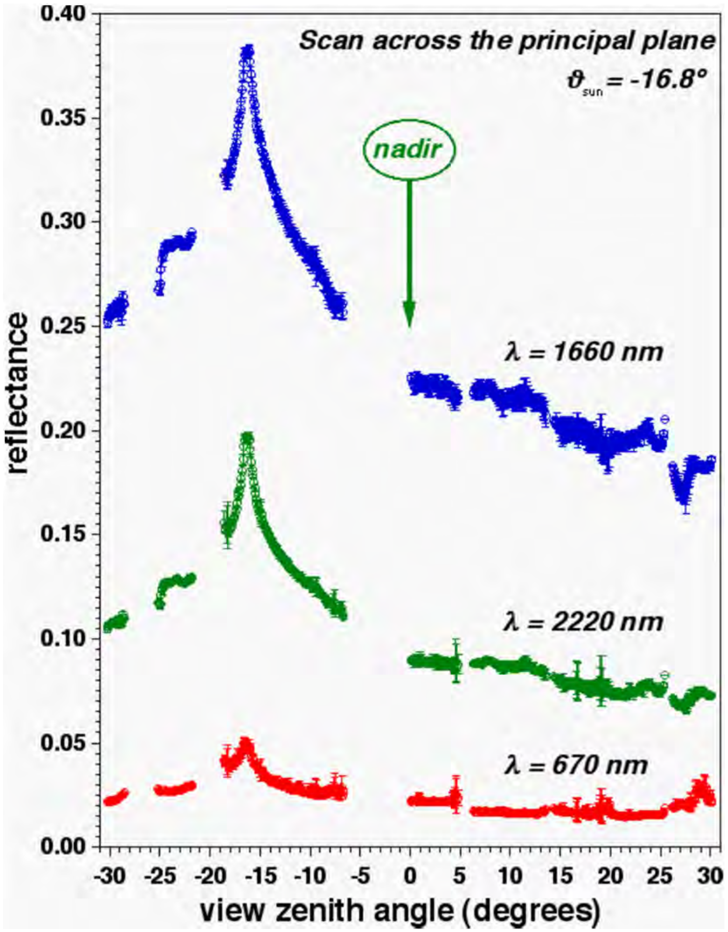


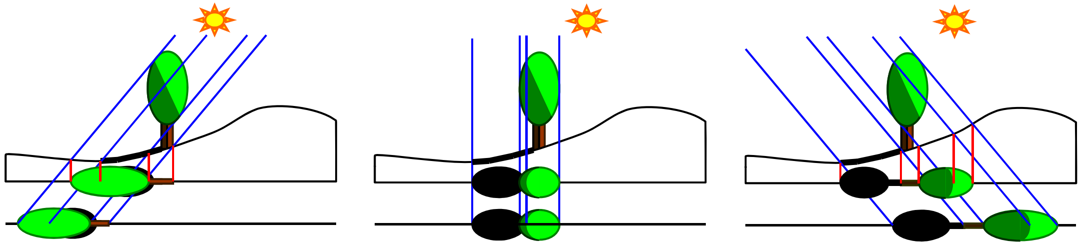
fluorescence emission

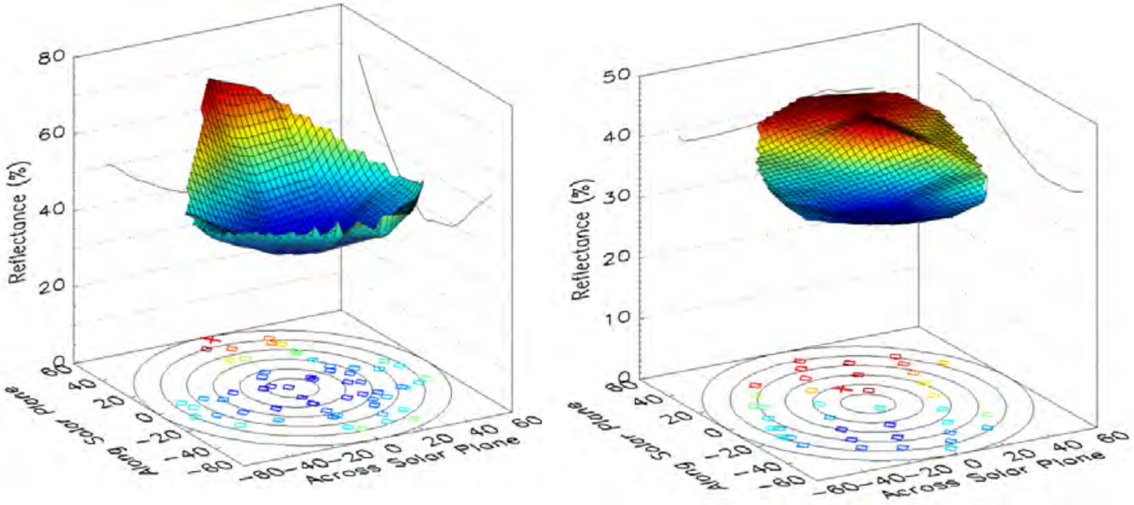




ANGULAR SIGNATURES

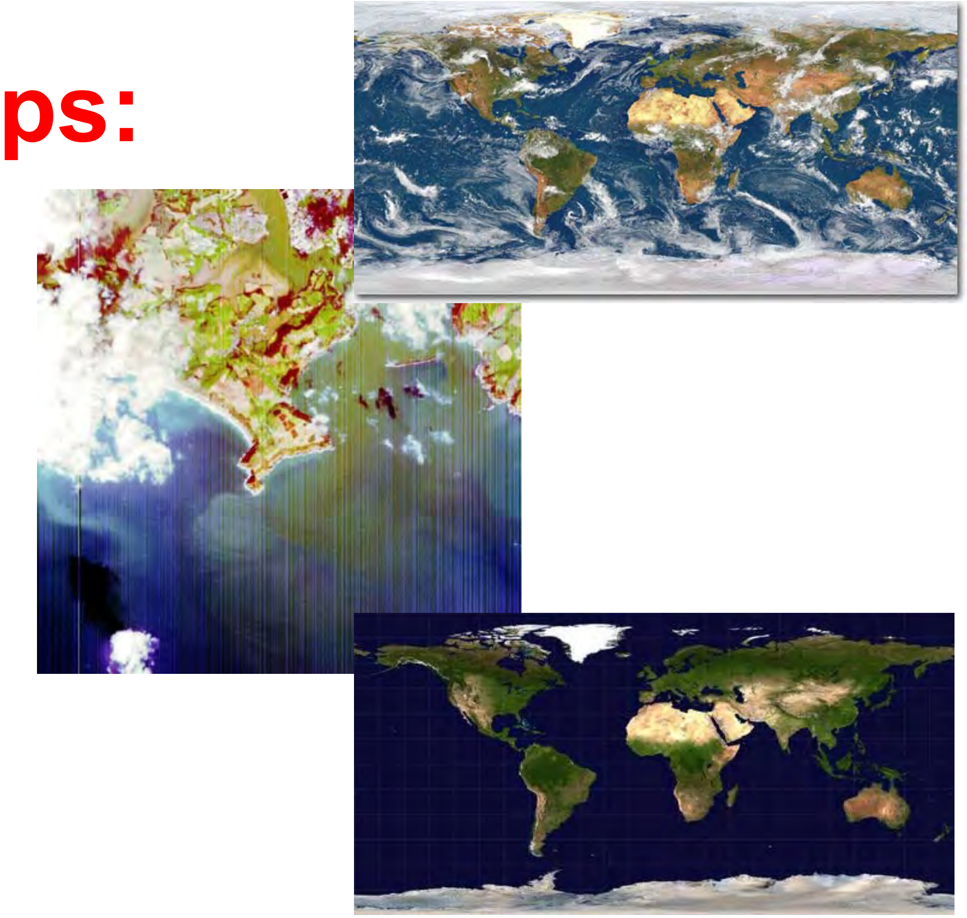




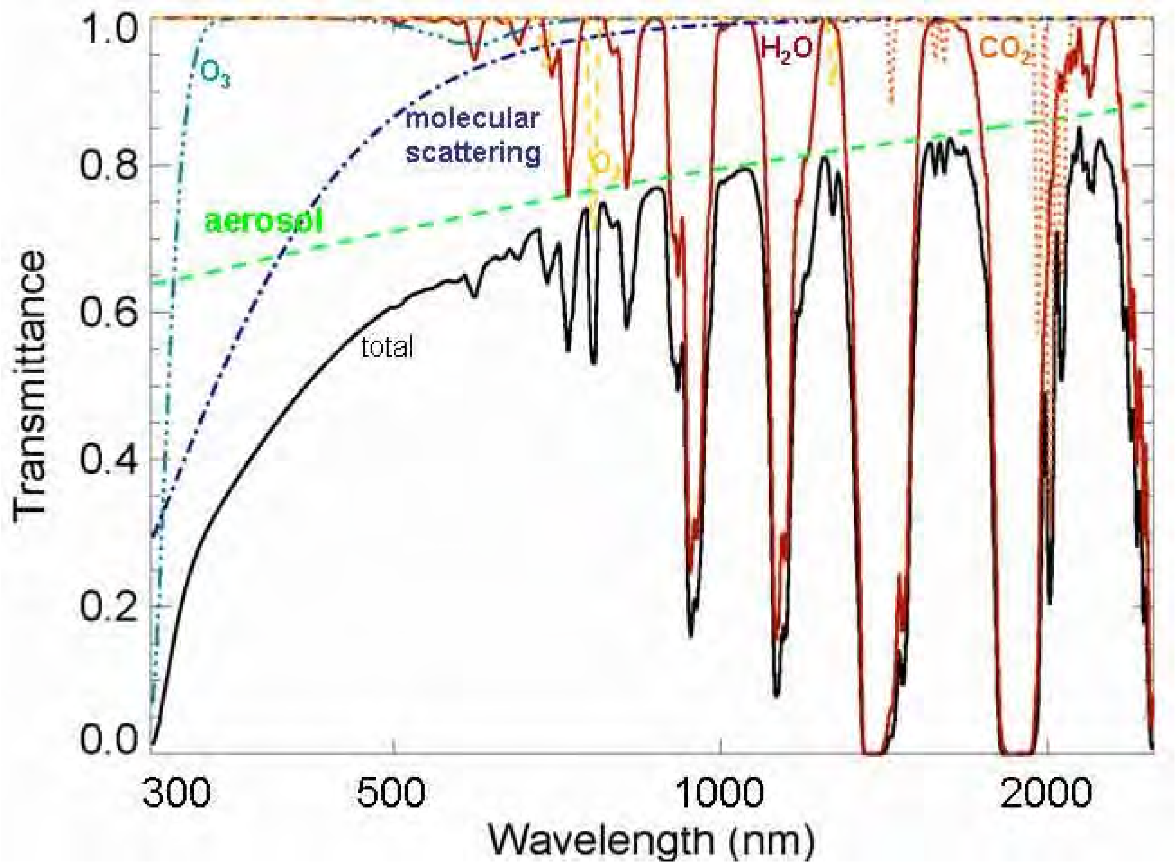


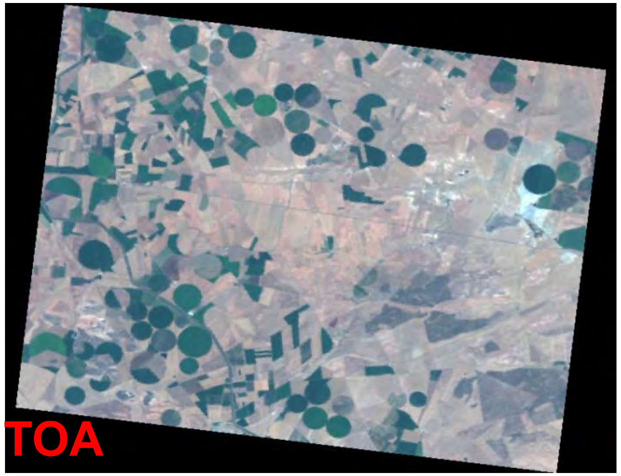
Pre-processing steps:

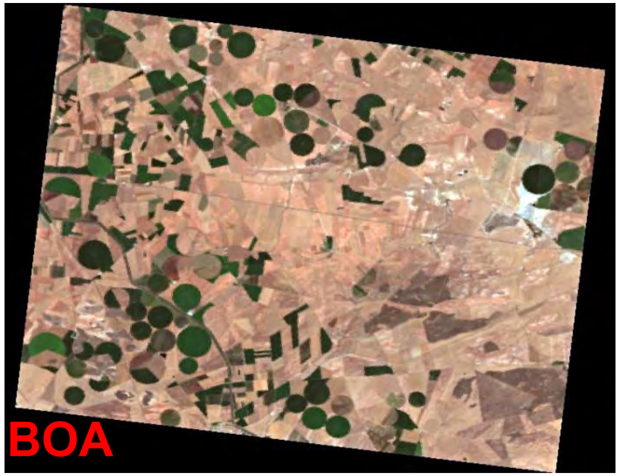
- Radiometric calibration
- Noise removal
- Cloud screening
- Geometric correction
- Atmospheric correction

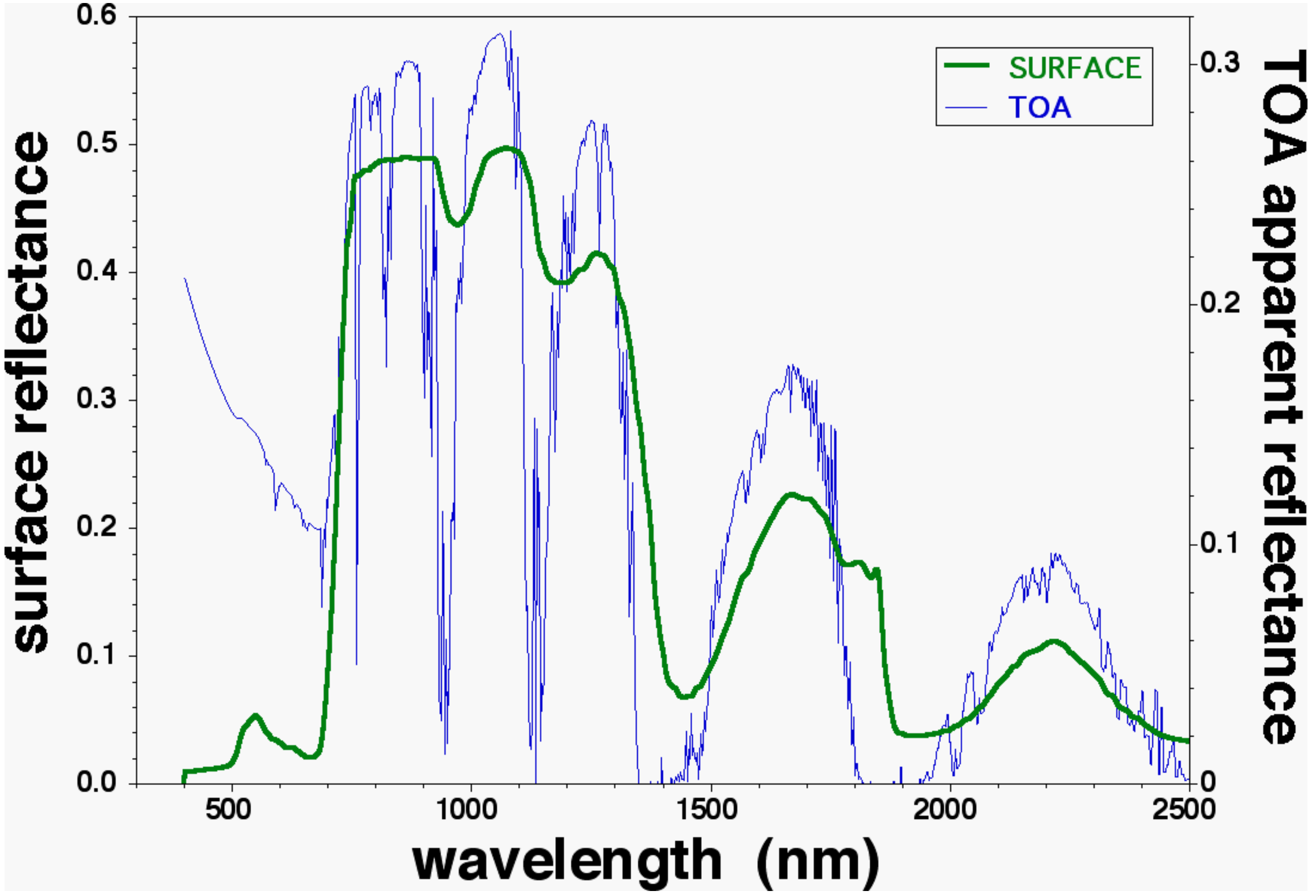


Atmospheric correction


1.0
0.8
0.6
0.4
0.2
0.0
Transmittance
300
500
1000
2000
Wavelength (nm)
O_3
molecular scattering
aerosol
total
O_2
H_2O
CO_2


TOA


BOA

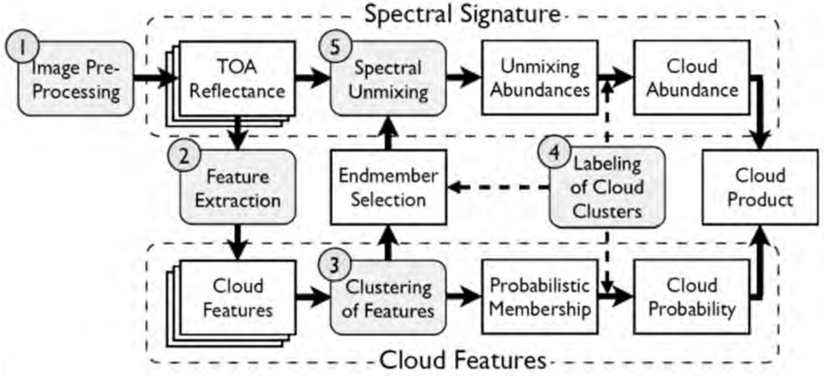


surface reflectance
0.0
0.1
0.2
0.3
0.4
0.5
0.6
TOA apparent reflectance
0
0.1
0.2
0.3
500
1000
1500
2000
2500
wavelength (nm)
SURFACE
TOA

CLOUD SCREENING

Simple static thresholds over TOA reflectance and spectral slope

Classification / unmixing approaches



Spectral Signature

1 Image Pre-Processing

TOA Reflectance

5 Spectral Unmixing

Unmixing Abundances

Cloud Abundance

2 Feature Extraction

Endmember Selection

4 Labeling of Cloud Clusters

Cloud Product

Cloud Features

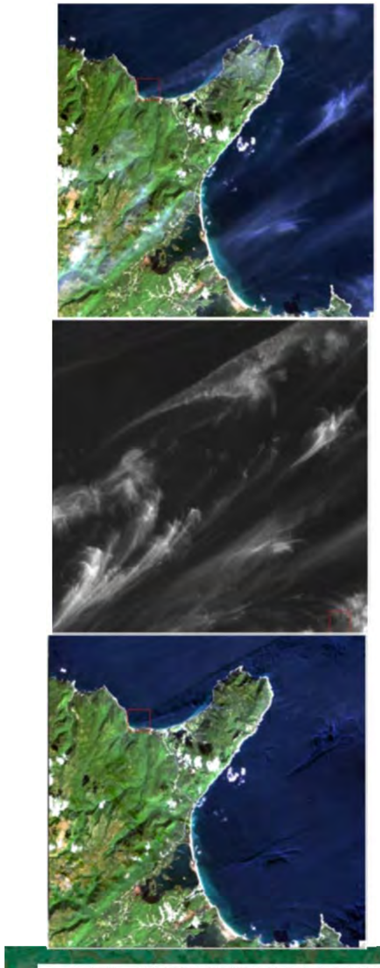
3 Clustering of Features

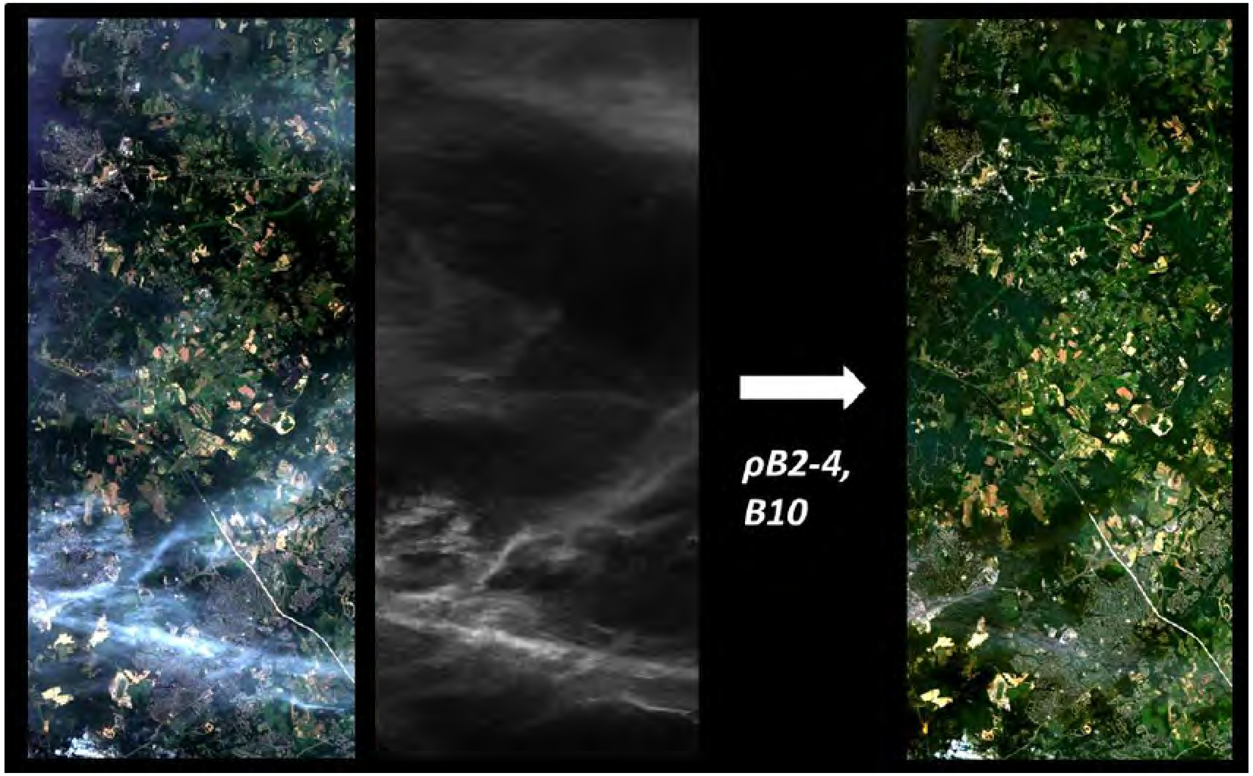
Probabilistic Membership

Cloud Probability

Cloud Features

Correction of surface reflectance for cirrus transmítance effects





TOA SIGNAL

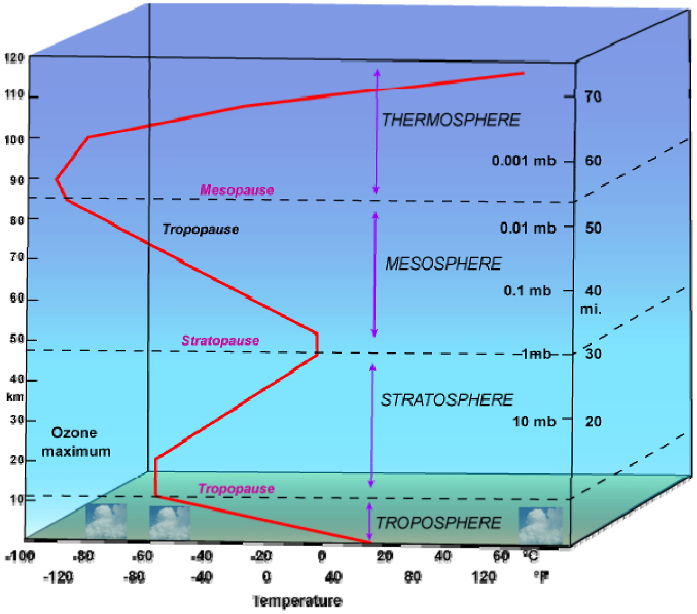
$$
\begin{aligned}
\rho_{sat} = {} & \rho_{atm} + \\
& + T^{\downarrow} \rho_s T^{\uparrow} + \\
& + T^{\downarrow} \rho_s S \rho_s T^{\uparrow} + \\
& + T^{\downarrow} \rho_s S \rho_s S \rho_s T^{\uparrow} + \\
& + T^{\downarrow} \rho_s S \rho_s S \rho_s S \rho_s T^{\uparrow} + \\
& + T^{\downarrow} \rho_s S \rho_s S \rho_s S \rho_s S \rho_s T^{\uparrow} + \\
& + \ldots
\end{aligned}
$$

$$= \rho_{atm} + T^{\downarrow}\left[\rho_s + S(\rho_s)^2 + S^2(\rho_s)^3 + S^3(\rho_s)^4 + S^4(\rho_s)^5 + \ldots\right] T^{\uparrow} =$$

$$= \rho_{atm} + T^{\downarrow}\rho_s\left[1 + S\rho_s + S^2(\rho_s)^2 + S^3(\rho_s)^3 + S^4(\rho_s)^4 + \ldots\right] T^{\uparrow} =$$

$$= \rho_{atm} + T^{\downarrow}\rho_s\left[\sum_{n=0}^{\infty}(S\rho_s)^n\right] T^{\uparrow} =$$

$$= \rho_{atm} + T^{\downarrow}\rho_s\left[\frac{1}{1 - S\rho_s}\right] T^{\uparrow}$$



$$\boxed{L_{\mathrm{TOA}} = L_0 + \frac{1}{\pi}\frac{\rho_s(E_{\mathrm{dir}}\mu_{\mathrm{il}} + E_{\mathrm{dif}})T_{\uparrow}}{1 - S\rho_s}}$$

Atmospheric correction

Flat, Lambertian and horizontally homogeneous areas:

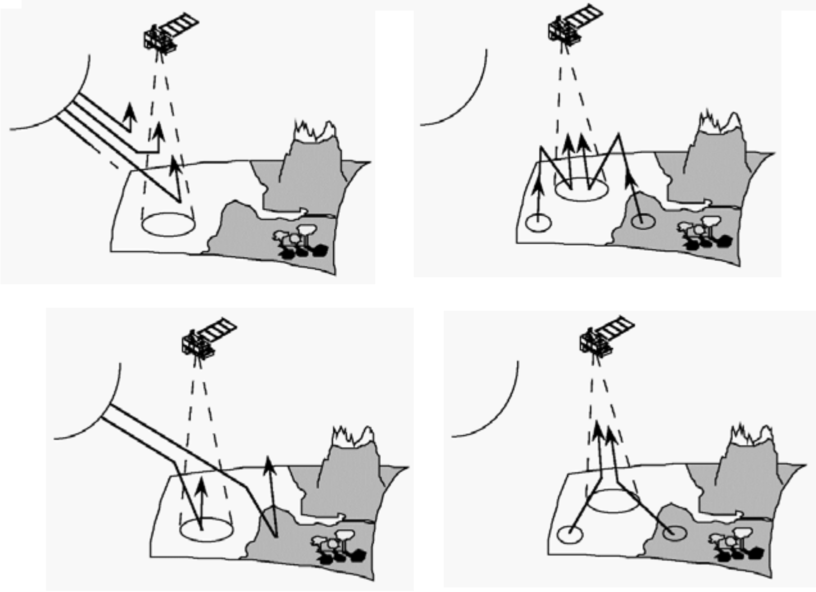
$$L_{\mathrm{TOA}} = L_0 + \frac{1}{\pi}\frac{\rho_s(E_{\mathrm{dir}}\mu_{\mathrm{il}} + E_{\mathrm{dif}})T_{\uparrow}}{1 - S\rho_s}$$

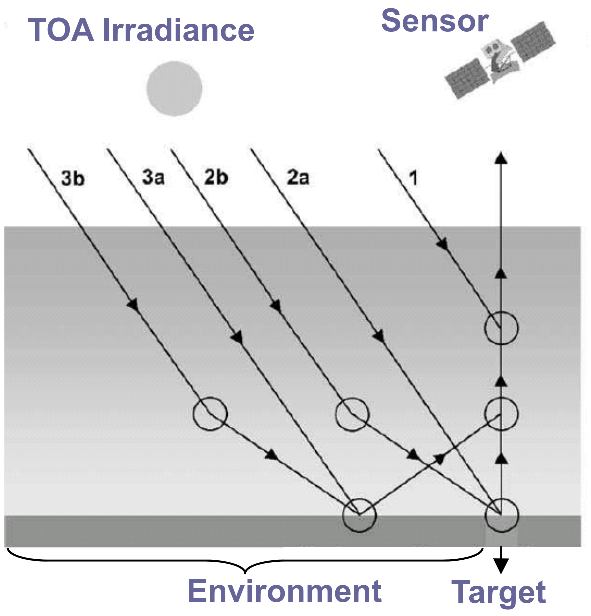
$$\boxed{\rho_s = \frac{L_{\mathrm{TOA}} - L_0}{[(E_{\mathrm{dir}}\mu_{\mathrm{il}} + E_{\mathrm{dif}})\frac{T_{\uparrow}}{\pi}] + S[L_{\mathrm{TOA}} - L_0]}}$$

Analytical inversion possible in this case !

MULTIPLE CONTRIBUTIONS TO THE SIGNAL

$$\rho'(\theta_s,\theta_v,\phi_v) = t_g(\theta_s,\theta_v)\,\{\rho_a(\theta_s,\theta_v,\phi_v) + \frac{T(\theta_s)}{1-<\rho(M)>S}[\rho_c(M)e^{-\tau/\mu_v} + <\rho(M)>t_d(\theta_v)]\}$$





6S formulation

INVERSION OF SURFACE REFLECTANCE

Inhomogeneous flat Lambertian areas:

$$\rho' = A + \frac{B\,\rho_c + C\,\langle\rho\rangle}{1 - S\,\langle\rho\rangle}$$

$$\boxed{\rho_c = \frac{\left(\frac{\rho' - A}{B}\right) - \frac{C}{B + C}\left(\frac{\langle\rho'\rangle - A}{B}\right)}{1 + S\,\frac{B}{B + C}\left(\frac{\langle\rho'\rangle - A}{B}\right)}}$$

Non-Lambertian areas with topographic structure:

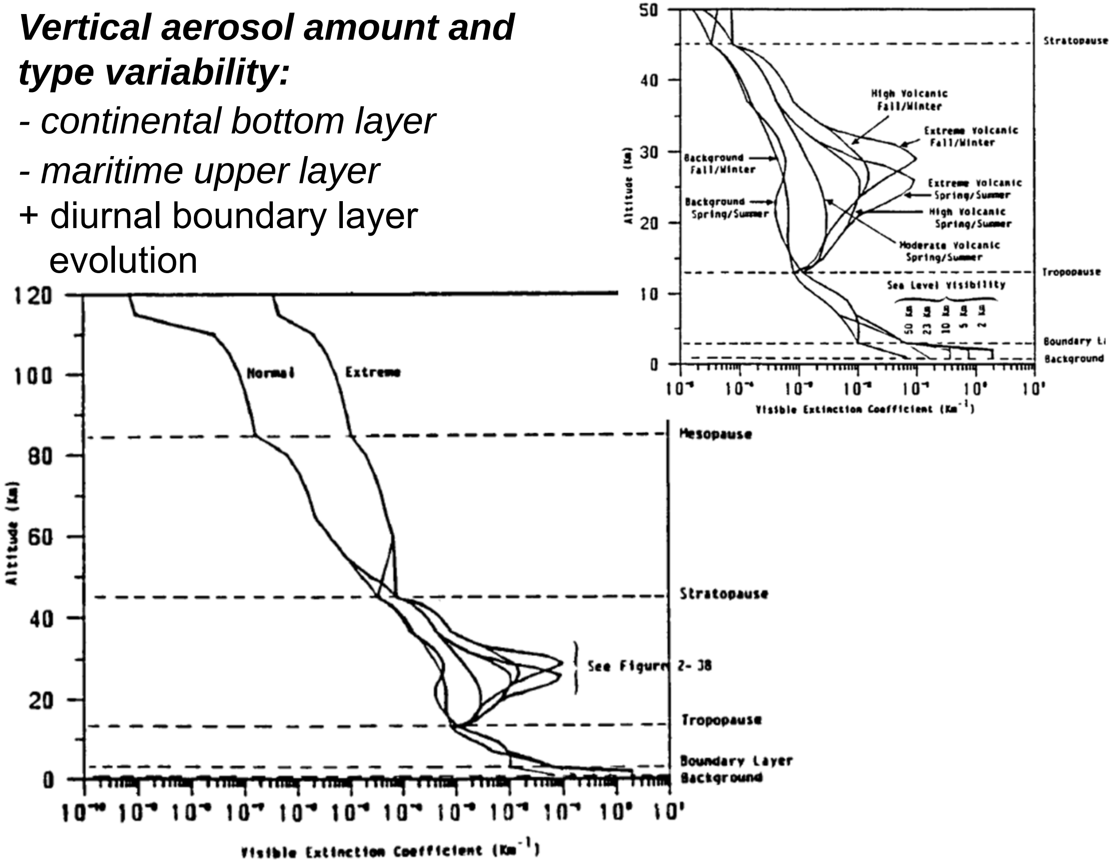
- no analytic inversion under approximations
- decoupling 'effective' reflectances and 'effective‘ geometric terms required for environment
- multistep numerical procedure required for inversion
- multiple reflection terms only significant for high reflectance surroundings

COUPLING OF AEROSOLS AND WATER VAPOUR VERTICAL STRUCTURE

Vertical aerosol amount and type variability:

- *continental bottom layer*
- *maritime upper layer*

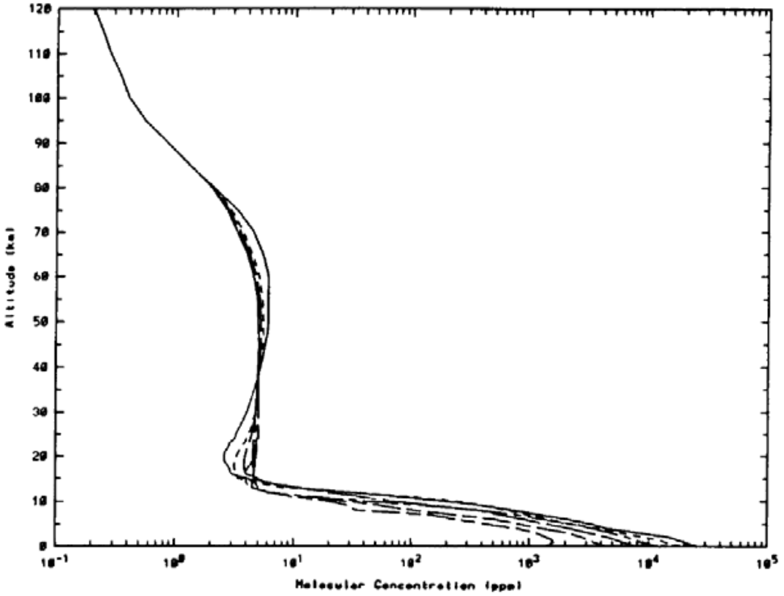
+ diurnal boundary layer evolution



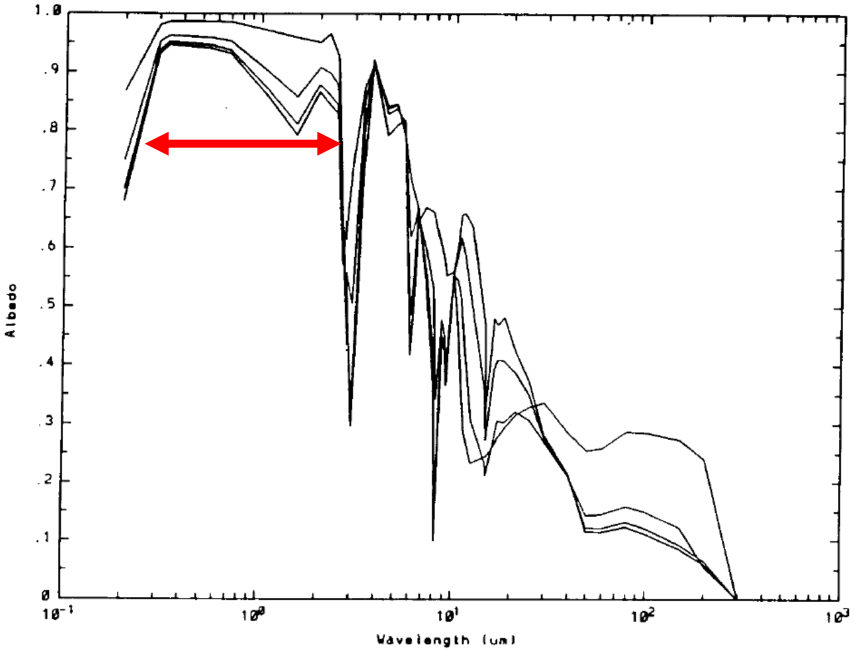
High spatial and temporal variability:

- *atmospheric circulation*
- *topography*

+ turbulent structure



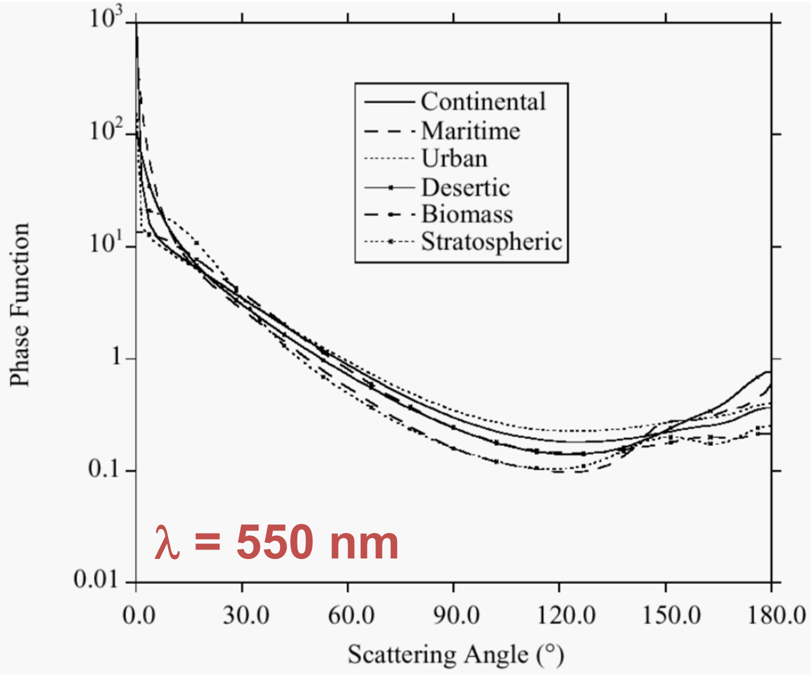
VARIABILITY IN AEROSOLS EFFECTS



spectral scattering albedo

rural aerosol model

(for 0, 70, 80 and 99% humidity)

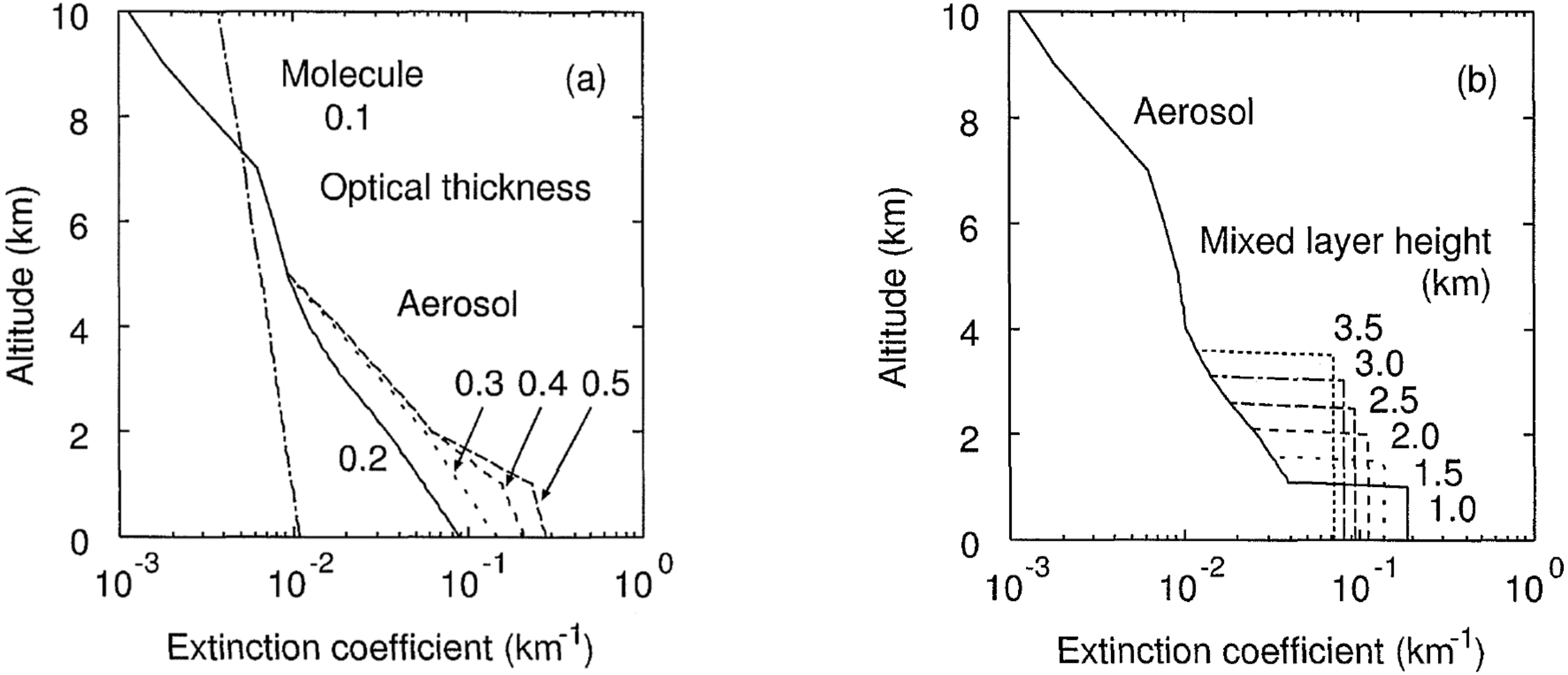


Aerosol type phase function

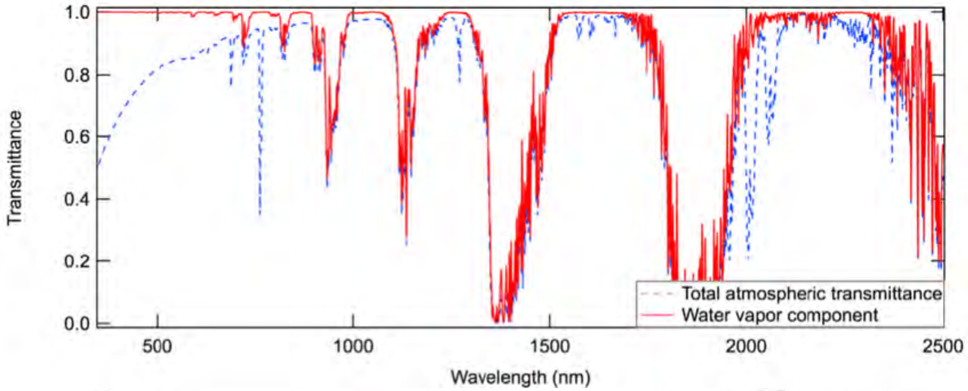
(angular variability

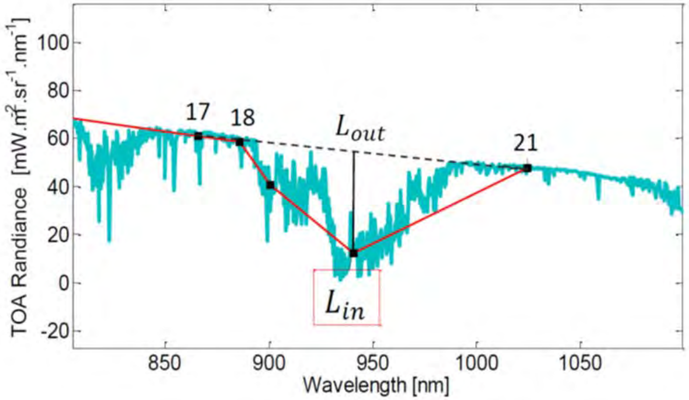
coupled to vertical structure)

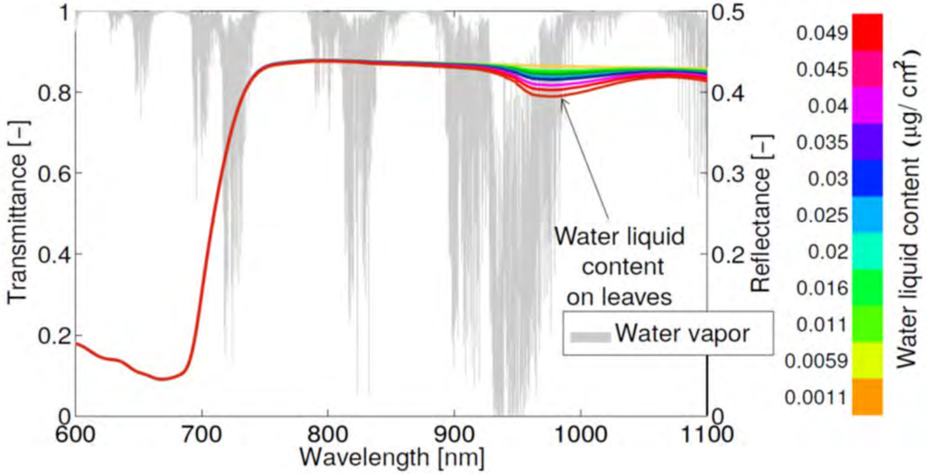
VARYING VERTICAL STRUCTURE OF AEROSOLS

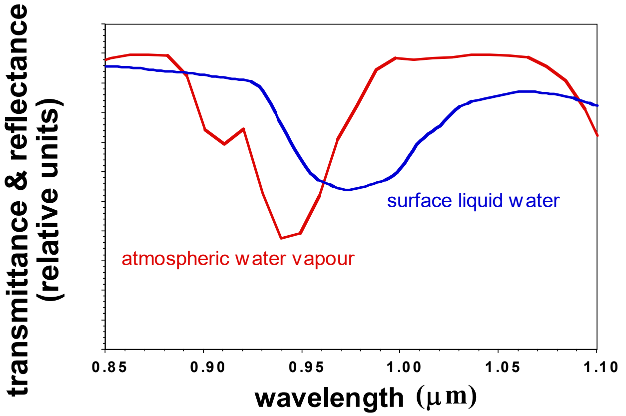


WATER VAPOR EFFECTS IN ATMOSPHERIC CORRECTIONS



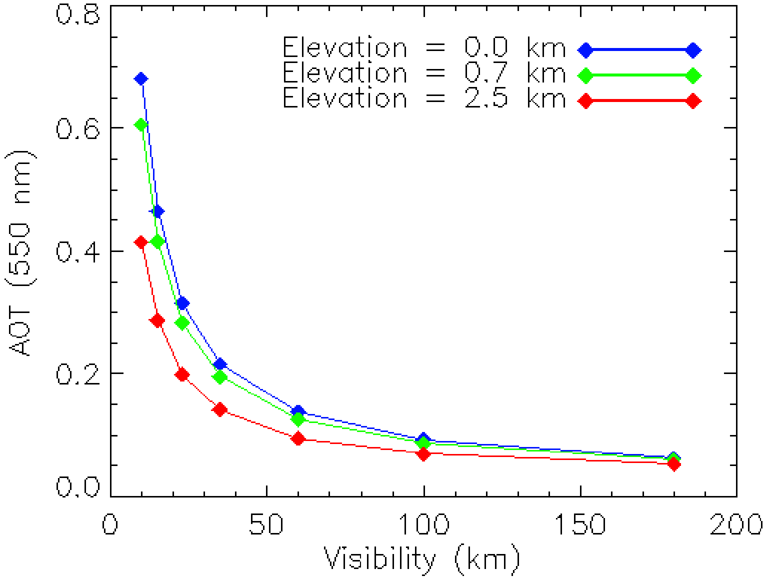


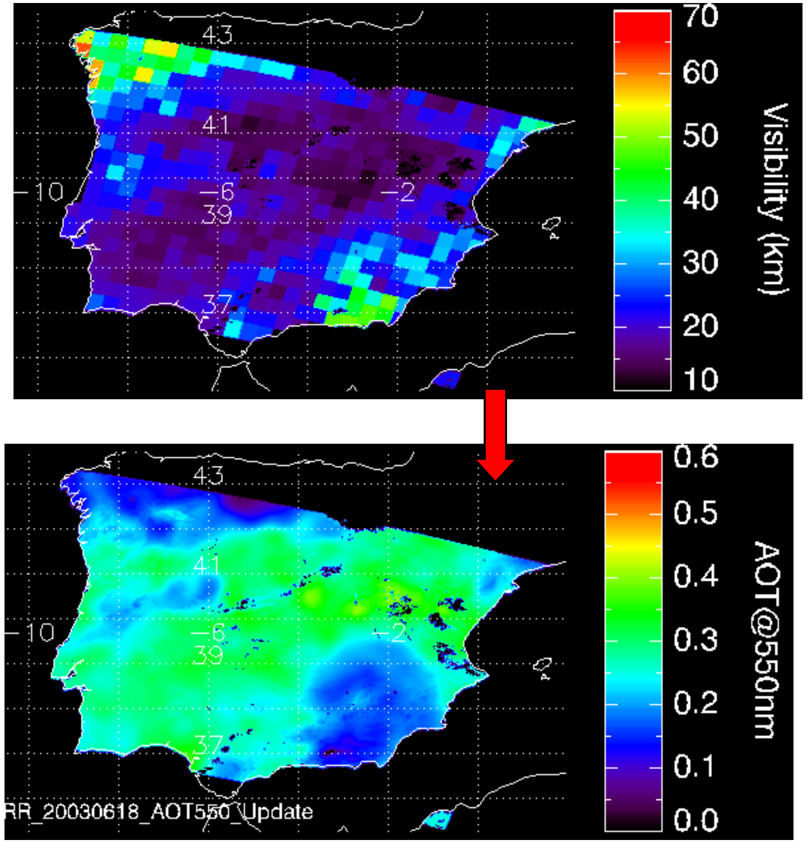




Aerosols (AOT) retrieval

- Retrieval of AOT for each cell
- Filling-in empty cells.
- Conversion from VIS to AOT at 550nm





AOT retrieval

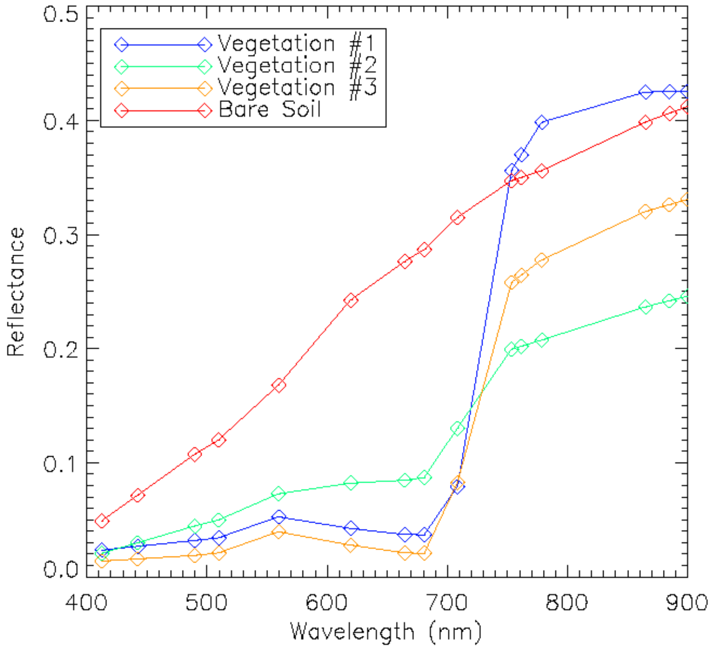
- Surface reflectance is given by the linear combination of 2 endmembers of typical vegetation and bare soil spectra:

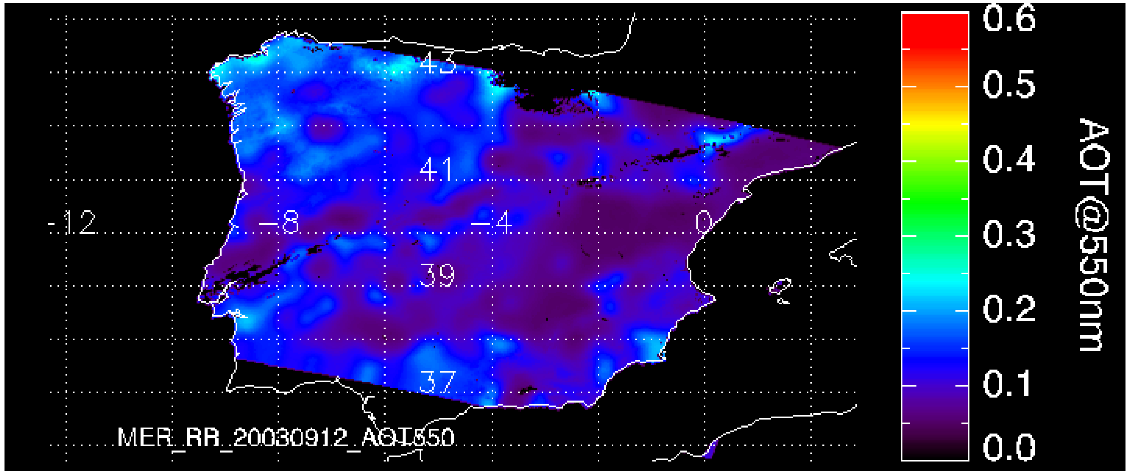
$$\rho_s = C_v \rho_{veg} + C_s \rho_{soil} \qquad C_v, C_s > 0$$

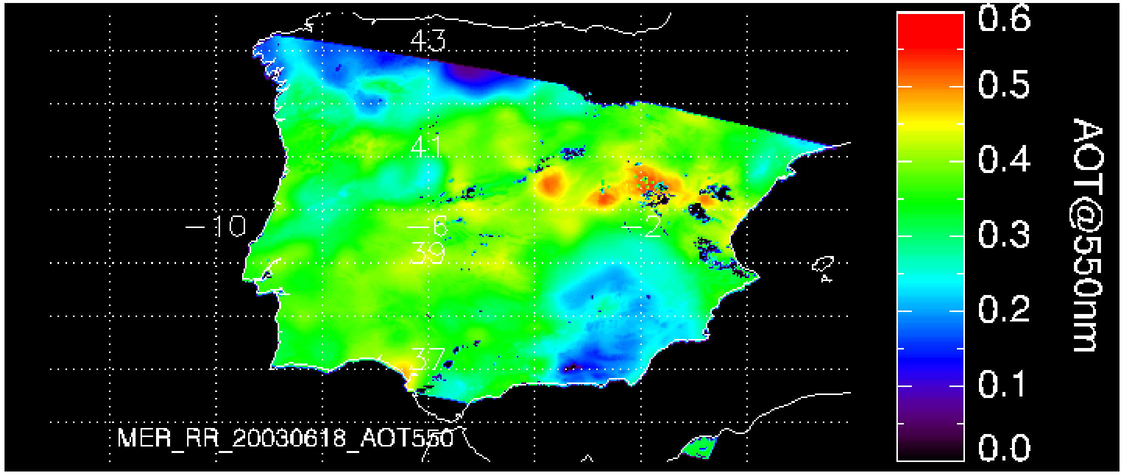
- Vegetation endmember is varied to account for different vegetation types
- Merit Function:

$$\delta^2 = \sum_{\mathrm{pix}=1}^{5} \omega_{\mathrm{pix}} \sum_{\lambda_i} \frac{1}{\lambda_i^2} \left[L^{\mathrm{SIM}}|_{\mathrm{pix},\lambda_i} - L^{\mathrm{SEN}}|_{\mathrm{pix},\lambda_i} \right]^2$$

- VIS + 5(C_v,C_s) free parameters
- Numerical inversion (minimization)

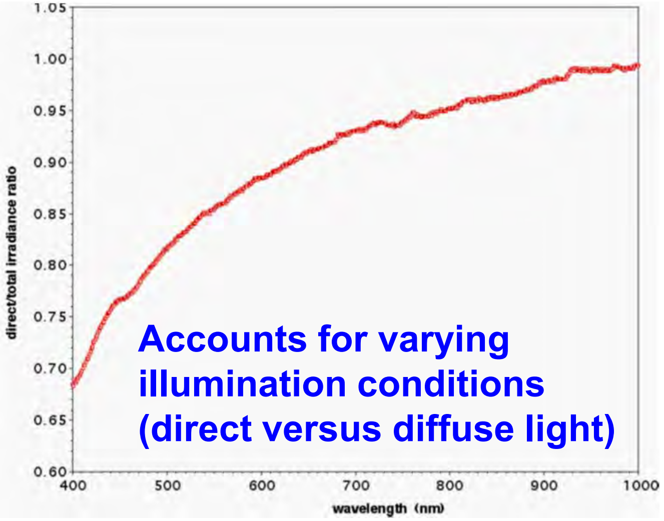






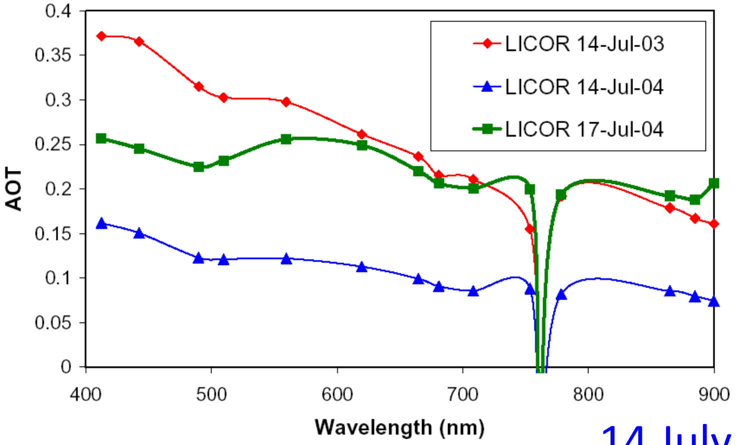
AOT estimation from actual images (MERIS data)

Very large variability in aerosols content

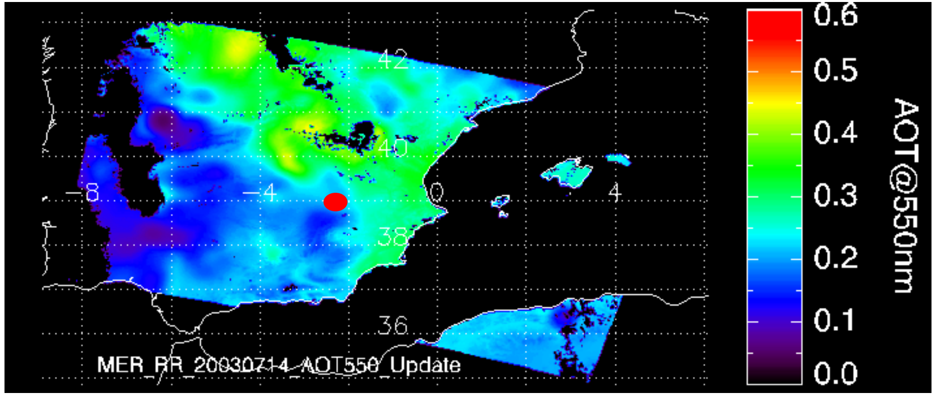


Accounts for varying illumination conditions (direct versus diffuse light)

VALIDATION OF AEROSOL RETRIEVALS

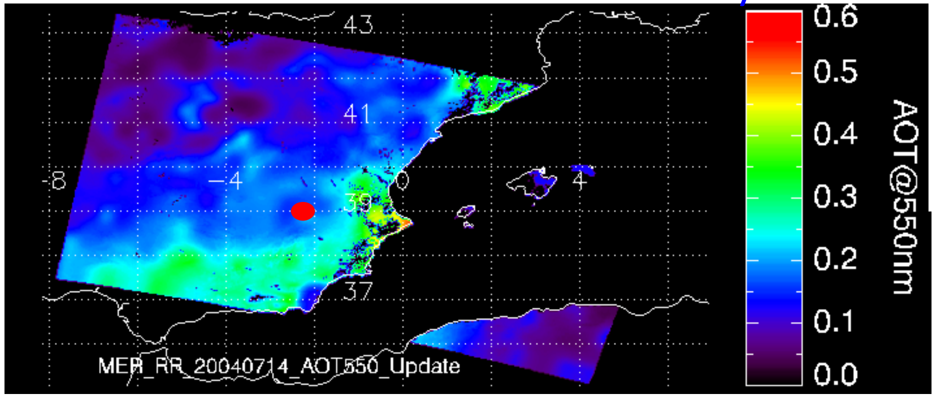


14 July 2003

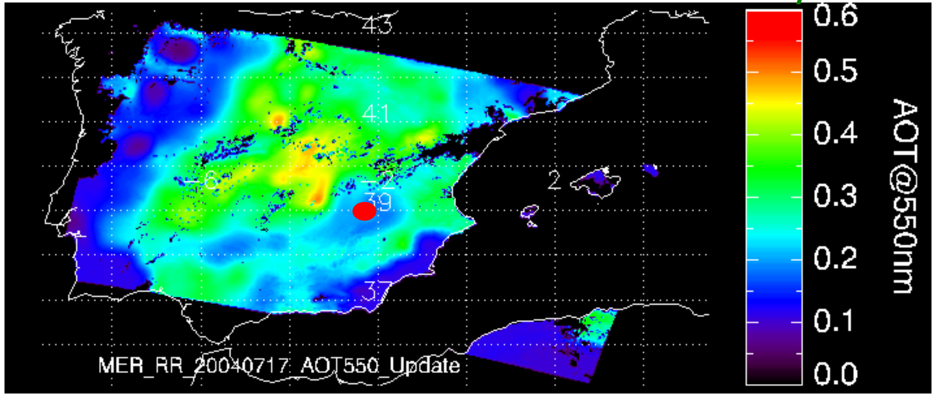


SPARC Campaigns, Barrax (Spain)

14 July 2004



17 July 2004



MODELING OF ADJACENCY EFFECTS IN THE DEFINITION OF SPATIALLY AVERAGED 'ENVIRONMENT' REFLECTANCES

$$<\rho(\vartheta_s,\phi_s;\vartheta_v,\phi_v)>= \frac{1}{\overline{T}^{\uparrow}(\vartheta)} \int_0^{2\pi} d\phi' \int_0^{\pi/2} d\vartheta' \int_0^{2\pi} d\psi \int_0^{\propto} dr \;\; \rho(r,\psi;\vartheta_s,\phi_s;\vartheta',\phi')\; T^{\uparrow *}(r,\psi;\vartheta',\phi';\vartheta_v,\phi_v)$$

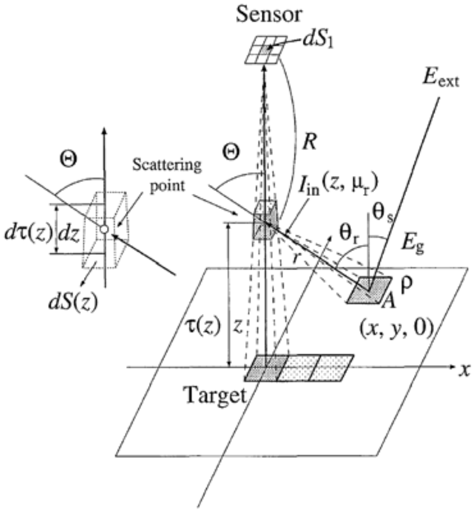
ρ = type of reflectance (direct-direct, diffuse-direct, etc.)

$<\rho>$= corresponding average for each reflectance type

$\rho(r,\psi;\vartheta_s,\phi_s;\vartheta',\phi')$ = reflectance at the point of (cilindrical) coordinates (r,ψ) around the central target being at the origin

$T^{\uparrow *}(r,\psi;\vartheta',\phi';\vartheta_v,\phi_v)$ = contribution of point (r,ψ) to the transmission function $T^{\uparrow}(\vartheta',\phi';\vartheta_v,\phi_v)$ at the observation point

$$\overline{T}^{\uparrow}(\vartheta) = \int_0^{2\pi} d\phi' \int_0^{\pi/2} d\vartheta' \;\; T^{\uparrow}(\vartheta',\phi';\vartheta_v,\phi_v)$$



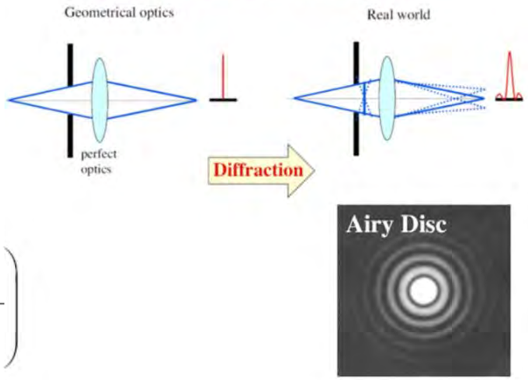
Dealing with atmospheric adjacency effects
in an accurate way
requires quite complicated
numerical computations !!!

Effective atmospheric Point Spread Function (PSF)

$$PSF(x,y) = \frac{h_r}{4\pi\, K \cos \vartheta_r} \sum_{k=1}^{K} \kappa_s^{(k)}\, P(\xi_P^k)\, \frac{\Delta x\, \Delta y \cos \vartheta_k}{\pi\, |\vec{r}_k|^2} \exp\left[-\kappa_t^{(k)} \left\{ |\vec{r}_k| + \left(1 - \frac{k}{K}\right) \frac{h_r}{\cos \vartheta_r} \right\} \right]$$

$$PSF = \left|FT\left(P(x,y)\right)\right|^2$$

Point Spread Function (PSF)



h_r = height of the sensor

ϑ_r = sensor zenith angle

$\kappa_s^{(k)}$ = scattering coefficient (in m^{-1})

$\kappa_a^{(k)}$ = absorption coefficient

$\kappa_t^{(k)} = \kappa_s^{(k)} + \kappa_a^{(k)}$

$\alpha^{(k)} = \kappa_s^{(k)} / \kappa_t^{(k)}$

$P(\xi_P^k)$ = scattering phase function

ξ_P^k = phase angle (introduces azimuthal dependence)

$$\xi_P^k = \cos^{-1}\left(\frac{\left(\vec{P}_0 - \vec{P}_k\right) \cdot \vec{r}_k}{|\vec{P}_0 - \vec{P}_k|\ |\vec{r}_k|} \right)$$

$\vec{P}_0$ = position of the central 'target' on the surface

$\vec{P}$ = position of each surface element

$\vec{P}_k$ = generic point along the line-of-sight

$\vec{r}_k = \vec{P} - \vec{P}_k$

Modulation Transfer Function (MTF)

$$MTF(f_x, f_y) = \mathrm{Re}\left[FT\left(PSF(x,y)\right)\right]$$

Effects introduced by topography:

A - Vertical geometric distorsion (horizontal displacement due to relief)

B - Variation of atmospheric (optical) properties with height

C - Relative changes in slope and orientation of surface introduce variations in illumination conditions:

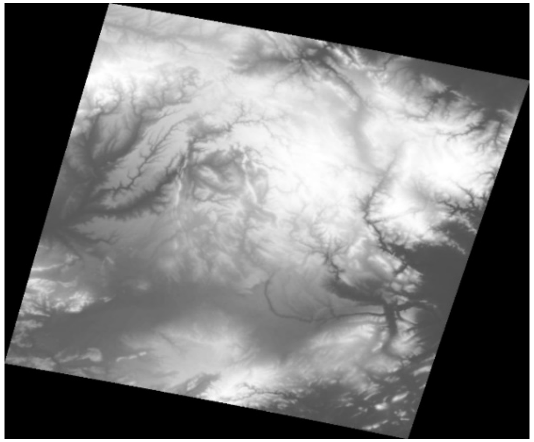
Direct irradiance:
- illuminated areas
- self-shadowed areas
- cast-shadowed areas

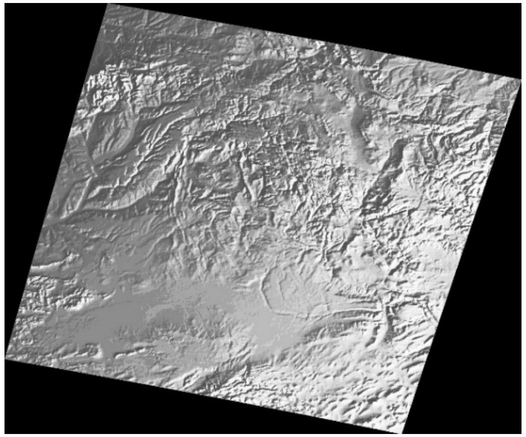
Diffuse irradiance:
- directional distribution
- modeling of sky view factors

Surface reflectance model:
- non-Lambertian effects
- modeling of direct/diffuse components

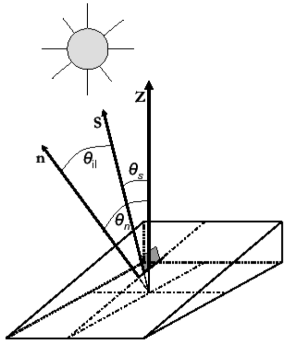
D - Adjacency effects (additional contributions)

E - Additional multiple reflections





$$L_{\text{TOA}} = \enclose{circle}{L_0} + \frac{1}{\pi}\frac{\rho_s(E_{\text{dir}}\enclose{circle}{\mu_{\text{il}}} + \enclose{circle}{E_{\text{dif}}})T\uparrow}{1 - S\rho_s}$$



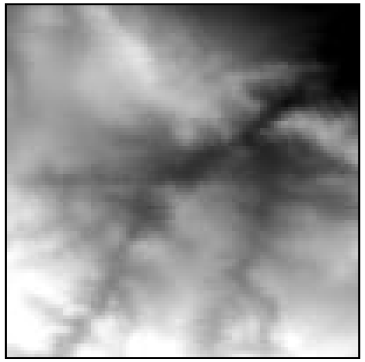
$$\mu_{\text{il}} = \mathbf{n}\cdot\mathbf{S}$$

Change in direct radiation

$$E^t_{\text{dif}}(x,y,z) = E_{\text{dif}}(z)\left[t_{\text{dir}}(z)\mu_{\text{il}}(x,y) + [1 - t_{\text{dir}}(z)\mu_s]\frac{1+\mu_n(x,y)}{2}\right]$$

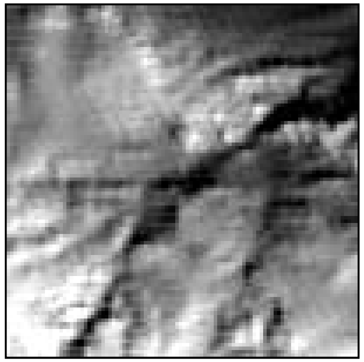
Change in diffuse radiation

DEM



L_0

$E_{dir}\cdot\mu_s$



E_{dif}

SURFACE REFLECTANCE

Heterogeneous, non-Lambertian and topographically structured surfaces

$$\rho' = \rho_{atm} + T^{\downarrow}_{dir}(\vartheta_s)\left[\rho_c\,\Theta\,\frac{\cos\vartheta_i}{\cos\vartheta_s}\right]T^{\uparrow}_{dir}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dir}(\vartheta_s)\left[\ <\overset{*}{\rho}>\ \right]T^{\uparrow}_{dif}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dif}(\vartheta_s)\left[\bar{\rho}_c^{\,cis}\,\frac{\cos\vartheta_i}{\cos\vartheta_s}\,V_d^{cis}\,F_d^{cis}\right]T^{\uparrow}_{dir}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dif}(\vartheta_s)\left[\bar{\rho}_c^{\,iso}\,V_d^{iso}\,F_d^{iso}\right]T^{\uparrow}_{dir}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dif}(\vartheta_s)\left[\bar{\rho}_c^{\,hor}\,V_d^{hor}\,F_d^{hor}\right]T^{\uparrow}_{dir}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dif}(\vartheta_s)\left[\overline{<\overset{*}{\rho}>}^{\,cis}<V_d^{cis}>F_d^{cis}\right]T^{\uparrow}_{dif}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dif}(\vartheta_s)\left[\overline{<\overset{*}{\rho}>}^{\,iso}<V_d^{iso}>F_d^{iso}\right]T^{\uparrow}_{dif}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dif}(\vartheta_s)\left[\overline{<\overset{*}{\rho}>}^{\,hor}<V_d^{hor}>F_d^{hor}\right]T^{\uparrow}_{dif}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dir}(\vartheta_s)\left[<\hat{\rho}_{slp}>P^{slp}\,V^{slp}\,\bar{\rho}_c^{\,slp}\right]T^{\uparrow}_{dir}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dif}(\vartheta_s)\left[<\hat{\rho}_{slp}>^{cis}\frac{<\cos\vartheta_i>_{slp}}{\cos\vartheta_s}<V_d^{cis}>_{slp}F_d^{cis}\,V^{slp}\,\bar{\rho}_c^{\,slp}\right]T^{\uparrow}_{dir}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dif}(\vartheta_s)\left[<\hat{\rho}_{slp}>^{iso}<V_d^{iso}>_{slp}F_d^{iso}\,V^{slp}\,\bar{\rho}_c^{\,slp}\right]T^{\uparrow}_{dir}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dir}(\vartheta_s)\left[<\overset{*}{\overline{\rho}}_{slp}>P^{slp}<V^{slp}>\overline{<\overset{*}{\rho}>}^{\,slp}\right]T^{\uparrow}_{dif}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dif}(\vartheta_s)\left[<\overset{*}{\overline{\rho}}_{slp}>^{cis}\frac{<\cos\vartheta_i>_{slp}}{\cos\vartheta_s}<V_d^{cis}>_{slp}F_d^{cis}<V^{slp}>\overline{<\overset{*}{\rho}>}^{\,slp}\right]T^{\uparrow}_{dif}(\vartheta_v) +$$

$$+\ T^{\downarrow}_{dif}(\vartheta_s)\left[<\overset{*}{\overline{\rho}}_{slp}>^{iso}<V_d^{iso}>_{slp}F_d^{iso}<V^{slp}>\overline{<\overset{*}{\rho}>}^{\,slp}\right]T^{\uparrow}_{dif}(\vartheta_v) +$$

$$+\left[\left(T^{\downarrow}_{dir}(\vartheta_s)<\rho>+T^{\downarrow}_{dif}(\vartheta_s)\left\{\overline{<\rho>}^{\,cis}<V_d^{cis}>F_d^{cis}+\overline{<\rho>}^{\,iso}<V_d^{iso}>F_d^{iso}+\right.\right.\right.$$

$$\left.\left.+\ \overline{<\rho>}^{\,hor}<V_d^{hor}>F_d^{hor}\right\}\right)\frac{S}{1-S\,\overline{<\rho>}^{\,iso}}\left(T^{\uparrow}_{dir}(\vartheta_v)\bar{\rho}_c^{\,iso}+T^{\uparrow}_{dif}(\vartheta_v)\overline{<\overset{*}{\rho}>}^{\,iso}\right)\Bigg] +$$

$$\left[\left(T^{\downarrow}_{dir}(\vartheta_s)<\overline{\rho}_{slp}>+T^{\downarrow}_{dif}(\vartheta_s)\left\{<\overline{\rho}_{slp}>^{cis}<V_d^{cis}>_{slp}F_d^{cis}+<\overline{\rho}_{slp}>^{iso}<V_d^{iso}>_{slp}F_d^{iso}\right\}\right)\right.$$

$$\left.\frac{S}{1-S\,\overline{<\rho>}^{\,iso}}\left(T^{\uparrow}_{dir}(\vartheta_v)\bar{\rho}_c^{\,iso}+T^{\uparrow}_{dif}(\vartheta_v)\overline{<\overset{*}{\rho}>}^{\,iso}\right)\right]$$

SIMPLE SEMI-EMPIRICAL FORMULATIONS OF SURFACE BIDIRECTIONAL REFLECTANCE MODELS USED FOR ATMOSPHERIC/TOPOGRAPHIC NORMALIZATION

$$\rho = \rho_0 \frac{k+1}{2} \left[\cos \vartheta_s \cos \vartheta_v\right]^{k-1}$$

Minnaert model

ρ_0 = surface albedo

k = Minnaert parameter (=1 for Lambertian case)

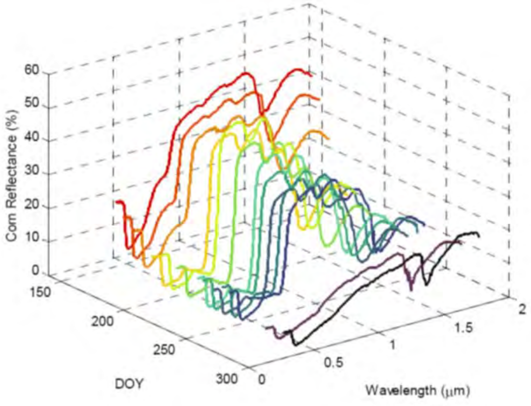
$$\rho = \rho_0 \frac{k+1}{2} \left(\vec{n} \cdot \vec{s}\right)^{k-1} \left(\vec{n} \cdot \vec{v}\right)^{k-1}$$

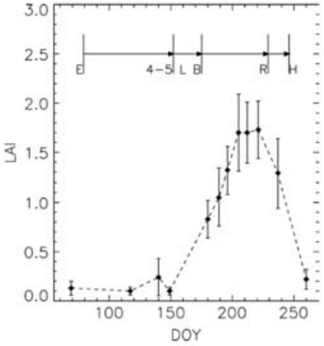
$$\left(\vec{n} \cdot \vec{s}\right) \left(\vec{n} \cdot \vec{v}\right) = \frac{1}{2} \left[\left(\vec{v} \cdot \vec{s}\right) + \left(\vec{v} \cdot \vec{p}\right)\right]$$

$$\rho = \rho_0 \frac{k+1}{4} \left[\left(\vec{v} \cdot \vec{s}\right) + \left(\vec{v} \cdot \vec{p}\right)\right]^{k-1}$$

Model generalization

OUTPUT OF THE ATMOSPHERIC/TOPOGRAPHIC CORRECTION FOR QUANTITATIVE COMPARISONS IN MULTITEMPORAL STUDIES





a.- reflectance

$$\rho(\vartheta_s,\phi_s;\vartheta_v,\phi_v)$$

- no comparison is possible among different dates
- no comparison is possible among different points of an image

b.- spectral albedo

$$\alpha(\vartheta_s,\phi_s) = \int\int_\Omega d\Omega\ \rho(\vartheta_s,\phi_s;\vartheta_v,\phi_v)$$

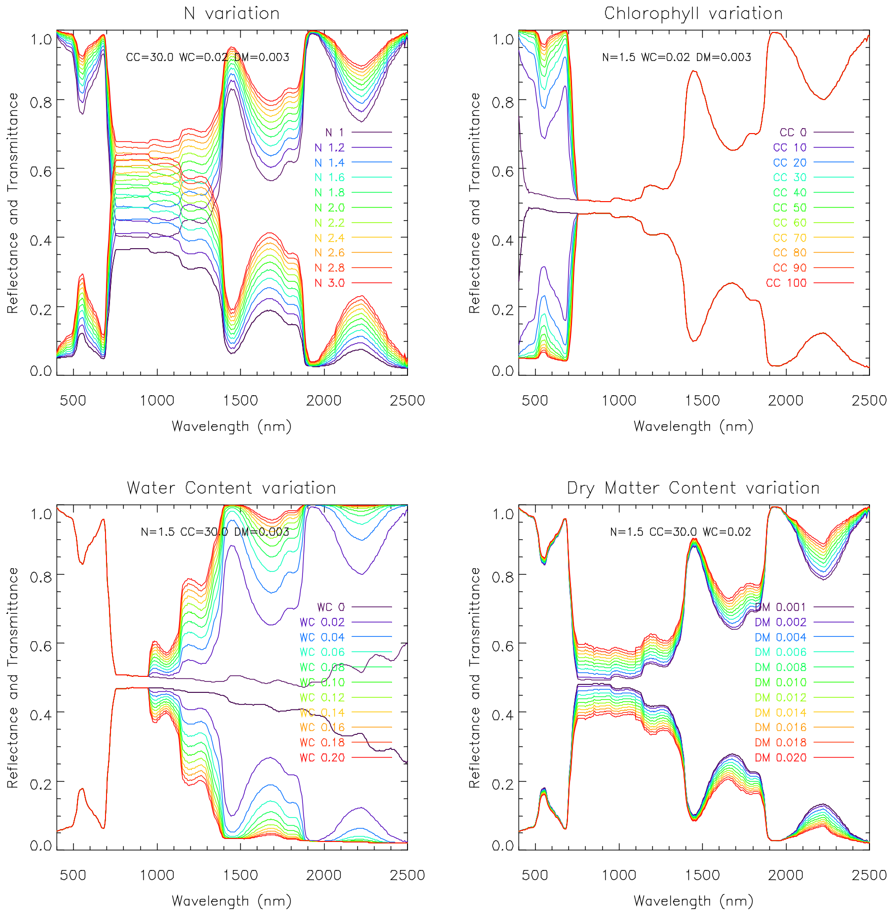
- comparison is possible within an image but not among different dates
- results are model-dependent (!)

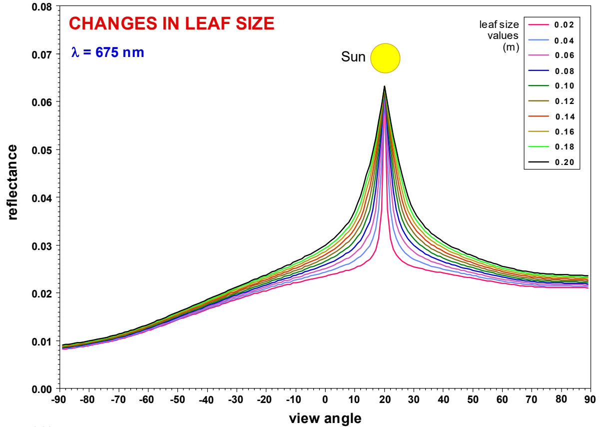
c.- 'normalized' spectral albedo

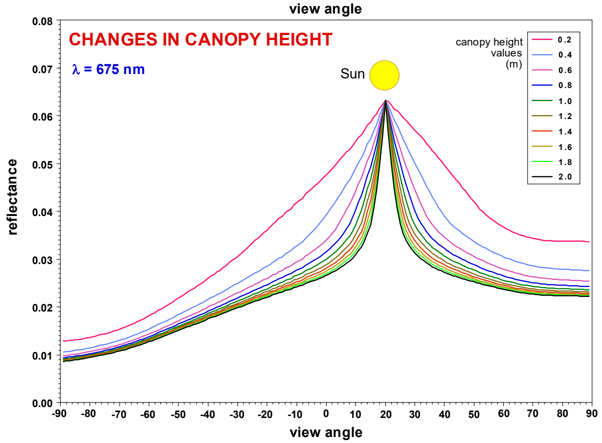
$$\alpha(\vartheta_0,\phi_0) = \alpha(\vartheta_s,\phi_s)\Big|_{(\vartheta_s=\vartheta_0,\phi_s=\phi_0)}$$

- comparison is possible within an image and among different dates
- strongly model-dependent (!)

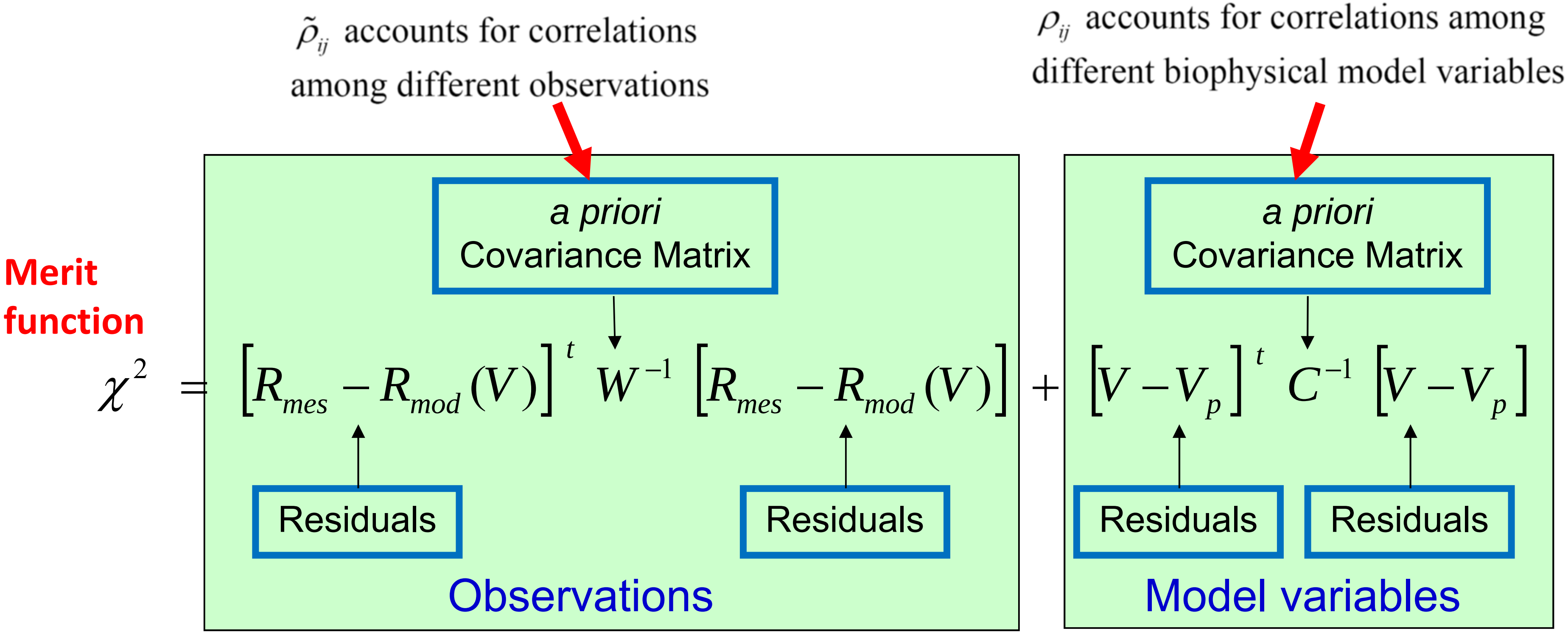
OPTICAL MODELS







MODEL INVERSION: minimization of a ‘merit function’



$\tilde{\rho}_{ij}$ accounts for correlations among different observations

ρ_{ij} accounts for correlations among different biophysical model variables

a priori Covariance Matrix

a priori Covariance Matrix

Merit function

$$\chi^2 = \left[R_{mes} - R_{mod}(V)\right]^t W^{-1} \left[R_{mes} - R_{mod}(V)\right] + \left[V - V_p\right]^t C^{-1} \left[V - V_p\right]$$

Residuals

Residuals

Residuals

Residuals

Observations

Model variables

Use of constrained minimization procedures that guarantee the minimal variation of model variables to produce the same output, and a robust initialization procedure of such variables (consistency even if model has global bias).

Definition of the merit function

Merit function as a least-squares estimator (LSE):

$$D[P,Q] = \sum_{\lambda_i=1}^{\lambda_n} (p(\lambda_i) - q(\lambda_i))^2$$

- favorable properties if their underlying assumptions are true (i.e., Gaussian noise)
- misleading results if those assumptions are violated

$$D(P,Q) = \sum_{\lambda_1=1}^{\lambda_n} |p(\lambda_l) - q(\lambda_l)|$$

Estimates with robust regression methods can be more stable with respect to anomalous errors.

$$D(P,Q) = \sum_{\lambda_1=1}^{\lambda_n} \frac{(p(\lambda_l) - q(\lambda_l))^2}{(1 + (p(\lambda_l) - q(\lambda_l))^2)}$$

Geman - McClure function

’divergence measures’ merit function: based on the minimization of distances between two probability distributions

$$D[P,Q] = \sum_{\lambda_1=1}^{\lambda_n} p(\lambda_l) ln \left(\frac{p(\lambda_l)}{q(\lambda_l)} \right)$$

Kullback Leibler divergence

$$D[P,Q] = \sum_{\lambda_1=1}^{\lambda_n} \frac{(q(\lambda_l) - p(\lambda_l))^2}{p(\lambda_l)}$$

Pearson chi-square

$$D[P,Q] = \sum_{\lambda_1=1}^{\lambda_n} (p(\lambda_l) - q(\lambda_l)) (ln(p(\lambda_l)) - ln(q(\lambda_l)))$$

Jeffreys-Kullback-Leibler

$$D[P,Q] = \sum_{\lambda_1=1}^{\lambda_n} p(\lambda_l) ln \left(\frac{2p(\lambda_l)}{p(\lambda_l) + q(\lambda_l)} \right)$$

K-divergence

Non-linear general fit to a function of *N* variables

$$\boxed{\chi^2(\mathbf{p}) = \frac{1}{2}\left(\mathbf{y} - \hat{\mathbf{y}}(\mathbf{p})\right)^T \mathbf{W}\left(\mathbf{y} - \hat{\mathbf{y}}(\mathbf{p})\right)}$$

Levenberg-Marquardt approach

$$\left[\mathbf{J}^T\mathbf{W}\mathbf{J} + \lambda\,\mathrm{diag}\left(\mathbf{J}^T\mathbf{W}\mathbf{J}\right)\right]\mathbf{h}_{LM} = \mathbf{J}^T\mathbf{W}\left(\mathbf{y} -\right.$$

$$\mathbf{J} = \begin{pmatrix} \frac{\partial\hat{y}(\mathbf{x}_1,\mathbf{p})}{\partial p_1} & \frac{\partial\hat{y}(\mathbf{x}_1,\mathbf{p})}{\partial p_2} & \cdots & \frac{\partial\hat{y}(\mathbf{x}_1,\mathbf{p})}{\partial p_M} \\ \frac{\partial\hat{y}(\mathbf{x}_2,\mathbf{p})}{\partial p_1} & \frac{\partial\hat{y}(\mathbf{x}_2,\mathbf{p})}{\partial p_2} & \cdots & \frac{\partial\hat{y}(\mathbf{x}_2,\mathbf{p})}{\partial p_M} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial\hat{y}(\mathbf{x}_N,\mathbf{p})}{\partial p_1} & \frac{\partial\hat{y}(\mathbf{x}_N,\mathbf{p})}{\partial p_2} & \cdots & \frac{\partial\hat{y}(\mathbf{x}_N,\mathbf{p})}{\partial p_M} \end{pmatrix}$$

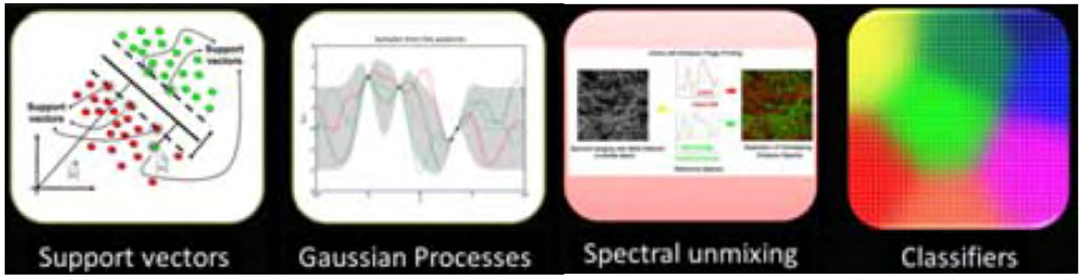
$\mathbf{W}$ = matrix of weigths for each point
(can be based on variance)

NOTE: there are many other approaches (i.e, Nelder-Mead method do not need derivatives)

ALTERNATIVE “REGRESSION” METHODS:

- Numerical inversion methods are computationally expensive (and subject to unstable results)
- Functional approximations often used as practical solution:
  - (a) Empirical approaches based on regression using many EO data points and field measurements (incomplete / biased sampling in most cases)
  - (b) Alternative (or complement) use of forward model outputs to produce a simple mathematical relationship which is then used for retrievals (complete sampling)

Signal decompositon or multiple linear/non-linear regression approaches (parametric or non-parametric):

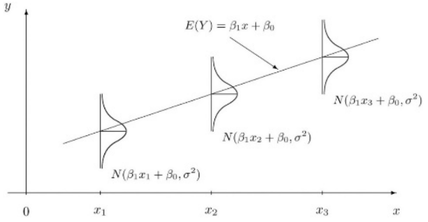


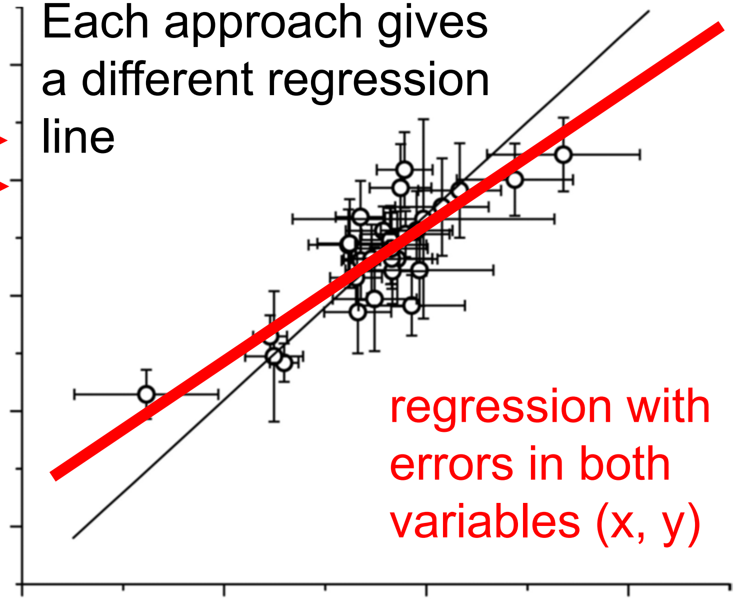
- Neural Networks, Partial least squares regression, Kernel regression, Multivariate adaptive regression, Stepwise regression, Segmented regression
- Spectral unmixing, Principal components / SVD decompositions
- Support Vector Machines, Gaussian processes, ...

RETRIEVALS BASED ON REGRESSION TECNIQUES

$$\chi^2(a,b) = \sum_{i=1}^{N} \frac{(y_i - a - bx_i)^2}{\sigma_{y_i}^2 + b^2\sigma_{x_i}^2}$$

$$\chi^2(a,b) = \sum_{i=1}^{N} \left[\left(\frac{x_i - \hat{x}_i}{\sigma_{x_i}} \right)^2 + \left(\frac{y_i - a - b\hat{x}_i}{\sigma_{y_i}} \right)^2 \right]$$





Each approach gives a different regression line

regression with errors in both variables (x, y)

- **Handling of outliers**

  “robust” regression approaches

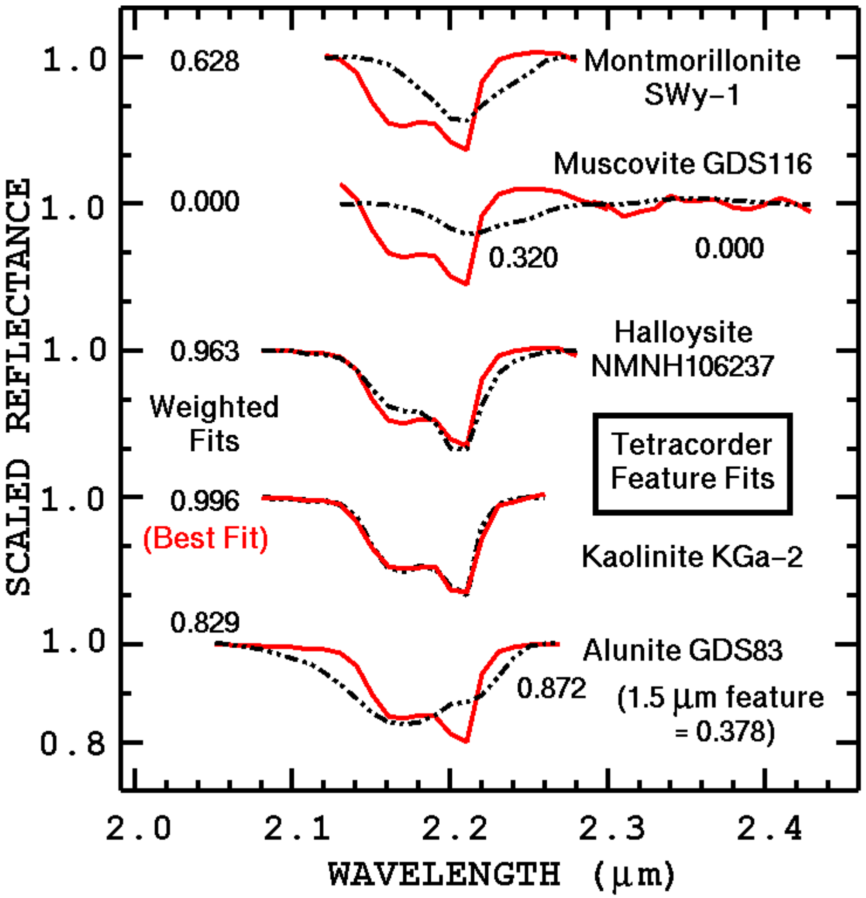
- **Adjusted regression**

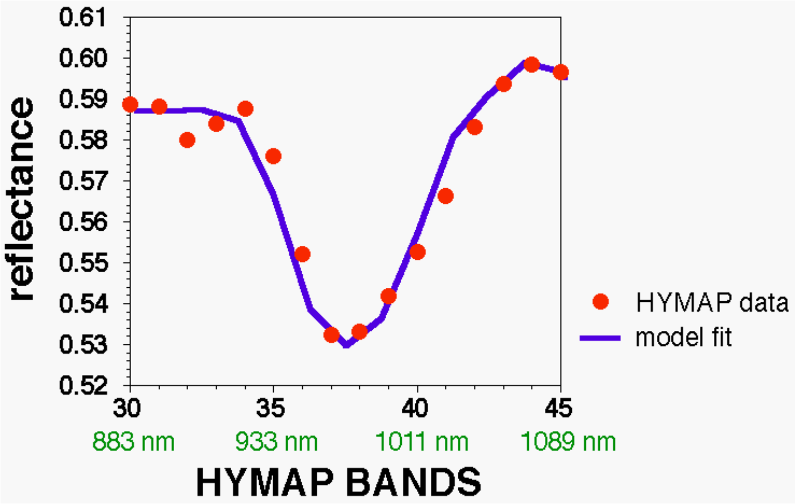
$$R_a^2 = \frac{(n-1)R^2 - k}{n-k-1} \xrightarrow[n\to\infty]{} R^2$$

n = number of points

k = number of independent variables

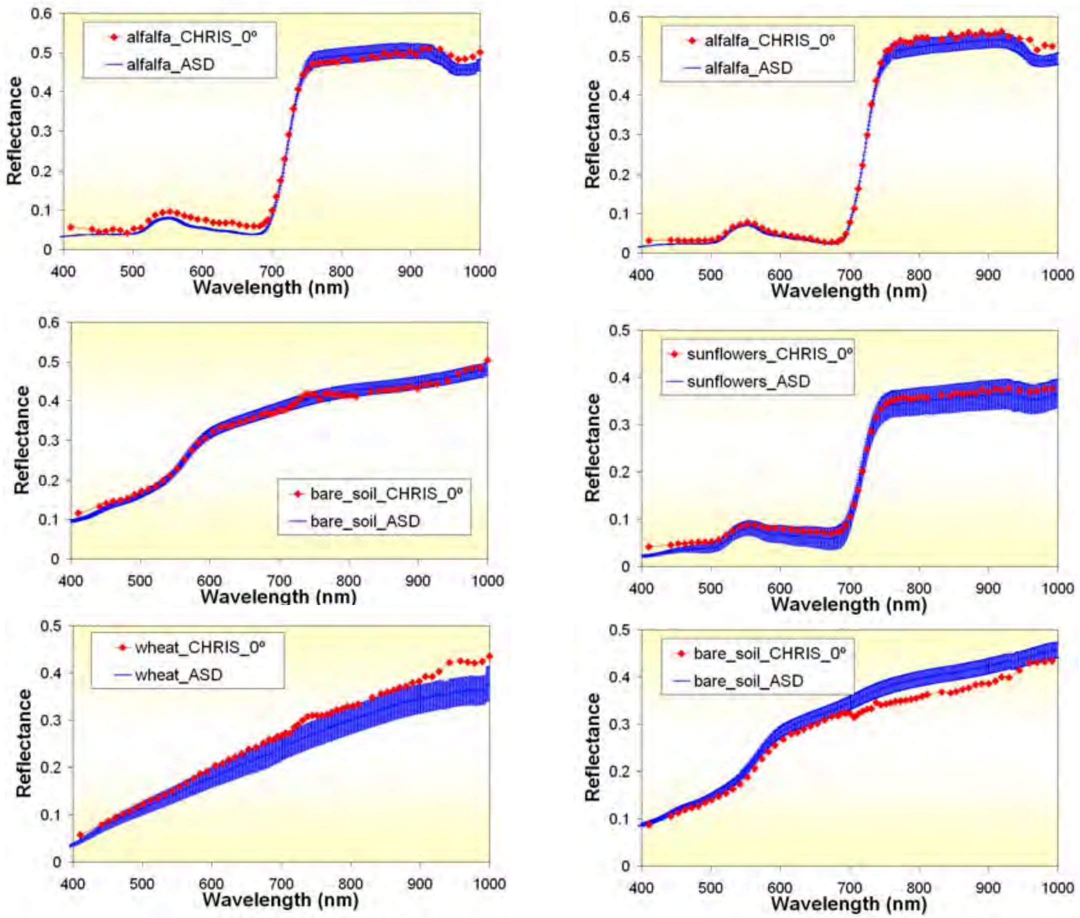
SPECTRAL FITTING METHODS





Spectral fitting methods are especially useful because we can use the well-known shape of spectral features.

Requires rather high spectral resolution.



Comparison between surface reflectance retrievals from actual satellite data (CHRIS/PROBA) and simultaneous measurements of reflectance at the surface over soil and vegetation targets

Atmospheric correction gives proper surface reflectance !

VALIDATION OF DERIVED PRODUCTS

- Statistical representativity of measurements used for validation (spatial sampling)
- Statistical extrapolation of results (sample versus population)
- Adaptation of validation methodology for each biophysical parameter retrieval
- Examination of results in view of the expected limitations
- Adaptability to the application
- Critical review of actual achievements
  - Always provide and error estimate (for a given confidence level) for each information retrieved.
  - If posible, decouple the error estimate between bias and random contributions

Great expectation from Sentinels !