→ 6th ESA ADVANCED TRAINING COURSE ON LAND REMOTE SENSING

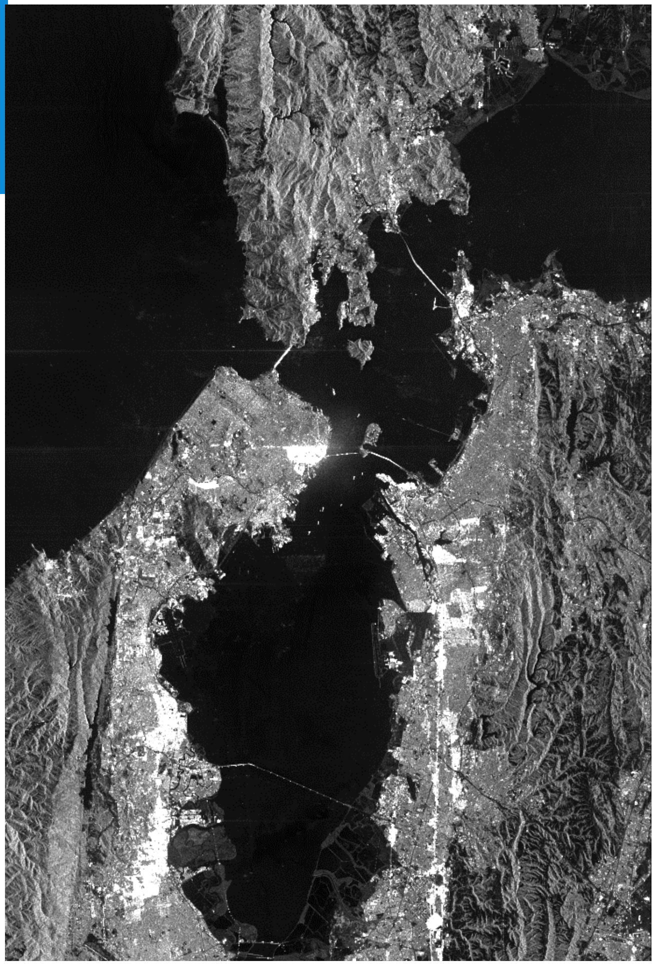
# SAR interferometry


Ramon Hanssen

Delft University of Technology (TU Delft)
The Netherlands
e-mail: r.f.hanssen@tudelft.nl / Web: www.tudelft.nl/hanssen

RECAP

# SAR SLC observations



SLC: Single-Look Complex data

•Single-look: no averaging, finest spatial resolution

•Complex: both real and imaginary (In-phase and quadrature phase) stored

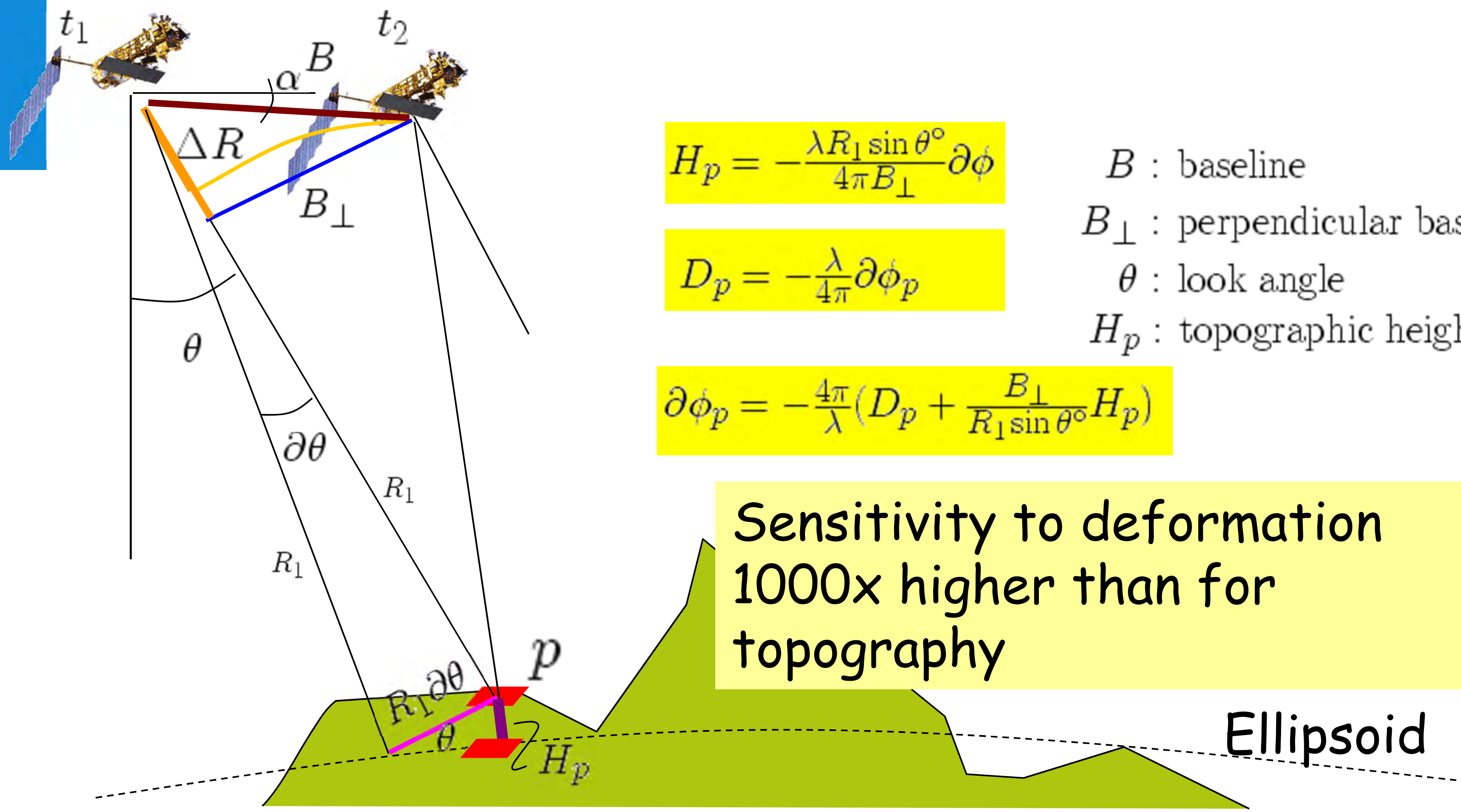
*Coherent imaging*

$$y_1 = |y_1| \exp(j\psi_1)$$

Amplitude　　Phase

Uninterpretable, due to scattering mechanism

# Topography and deformation




$$H_p = -\frac{\lambda R_1 \sin\theta^\circ}{4\pi B_\perp}\partial\phi$$

$$D_p = -\frac{\lambda}{4\pi}\partial\phi_p$$

$$\partial\phi_p = -\frac{4\pi}{\lambda}\left(D_p + \frac{B_\perp}{R_1 \sin\theta^\circ} H_p\right)$$

$B$ : baseline
$B_\perp$ : perpendicular baseline
$\theta$ : look angle
$H_p$ : topographic height

Sensitivity to deformation 1000x higher than for topography

# Model of observation equations (1)

Functional model:

$$\partial\phi_p = -\frac{4\pi}{\lambda}\left(D_p + \frac{B_\perp}{R_1 \sin\theta^\circ} H_p\right)$$

Observation

Unknowns

Rank deficiency!

Often treated opportunistically

Stochastic model:

$$Q_\phi = \sigma_\phi^2 I_n$$

Based on thermal (instrumental) noise

This is too much simplified, let's make it more realistic!

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# Model of observation equations (2)

- **Add unknown parameter:**
  - Phase ambiguity

Integer valued unknown

$$\partial\phi_p = -\frac{4\pi}{\lambda}\left(D_p + \frac{B_\perp}{R_1 \sin\theta^\circ} H_p\right) + 2\pi k$$

- **Add error signal to stochastic model:**
  - Atmosphere (troposphere, ionosphere)
  - Orbit errors
  - Decorrelation
    - Geometric
    - Temporal

Spatial varying disturbance

~trend

Pixel-based noise

Spatially ~constant

Spatially varying

Main condition for interferometry:

# Coherence!

# Phase contributions

The phase $\boldsymbol{\psi_{1,obs}}$ of a resolution cell in SAR image 1 is composed of two parts:

1. Geometric phase (dependent on distance antenna-scatterer):

$\boldsymbol{\psi_{1,geom}}$

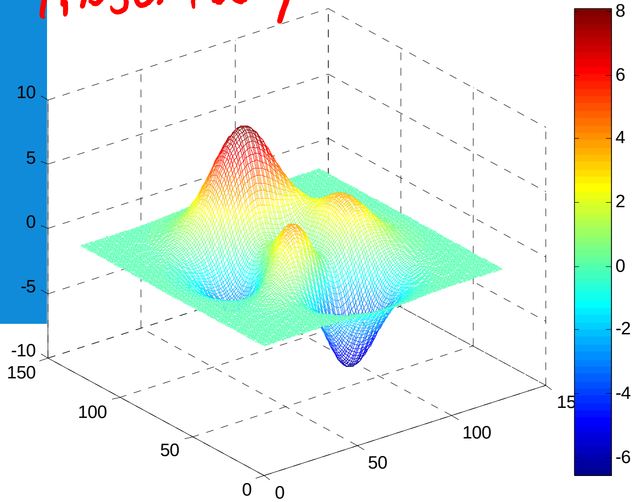
♦

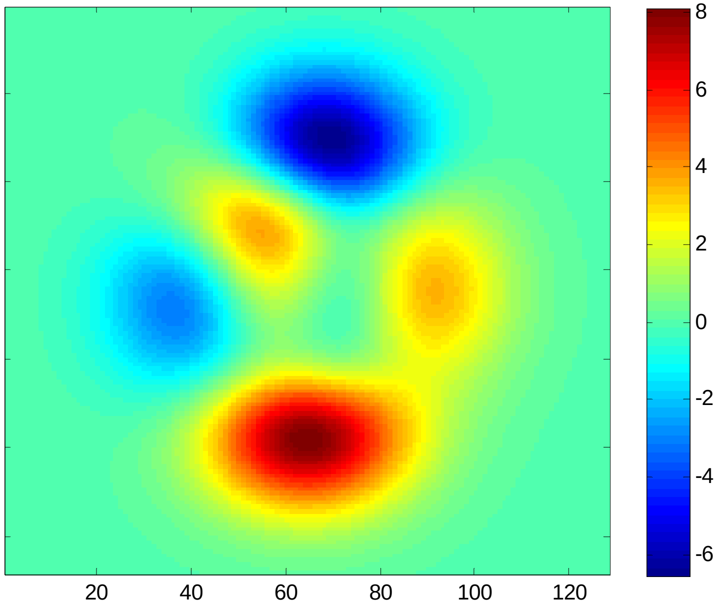
$$\psi_{1,geom} = \frac{2\pi}{\lambda} 2R = \frac{4\pi}{\lambda} R$$

# 1. Geometric Phase

Geometric phase: distance dependent.

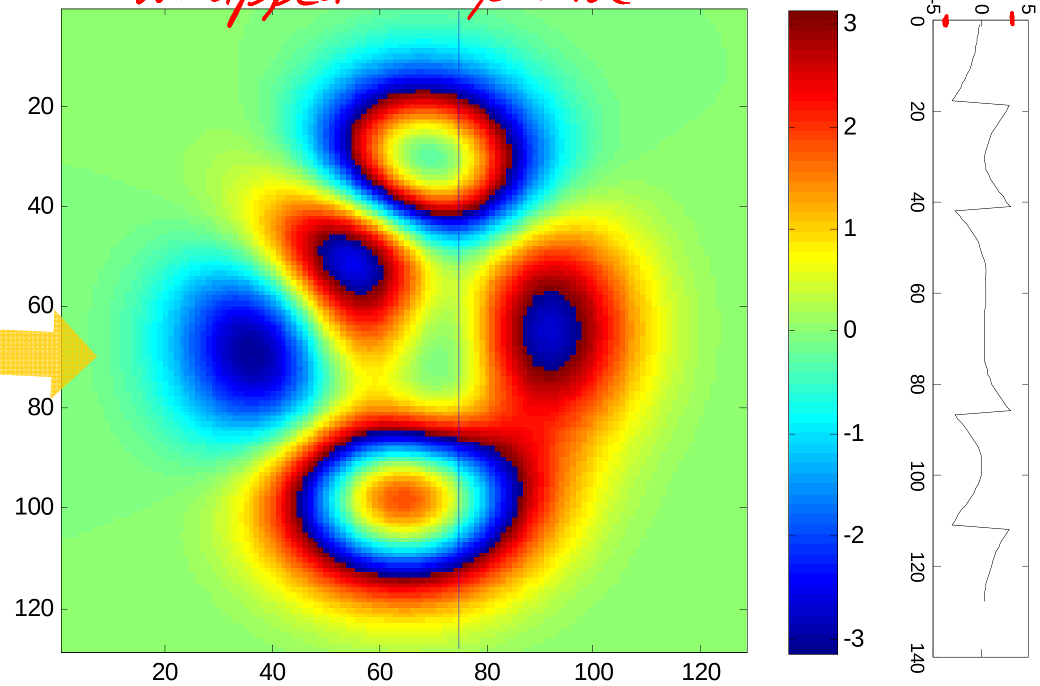
Absolute phase





"wrapped" phase

$-\pi$ $+\pi$



# Phase contributions

The phase $\psi_{1,obs}$ of a resolution cell in SAR image 1 is composed of two parts:

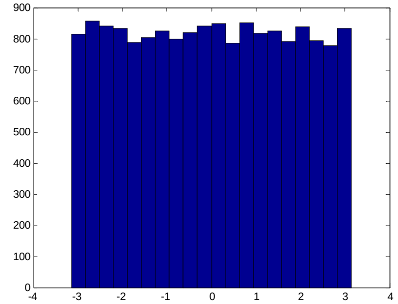
1. Geometric phase (dependent on distance antenna-scatterer):

$$\psi_{1,geom}$$

2. Scattering phase (dependent on interaction of EM wave with objects on the ground,

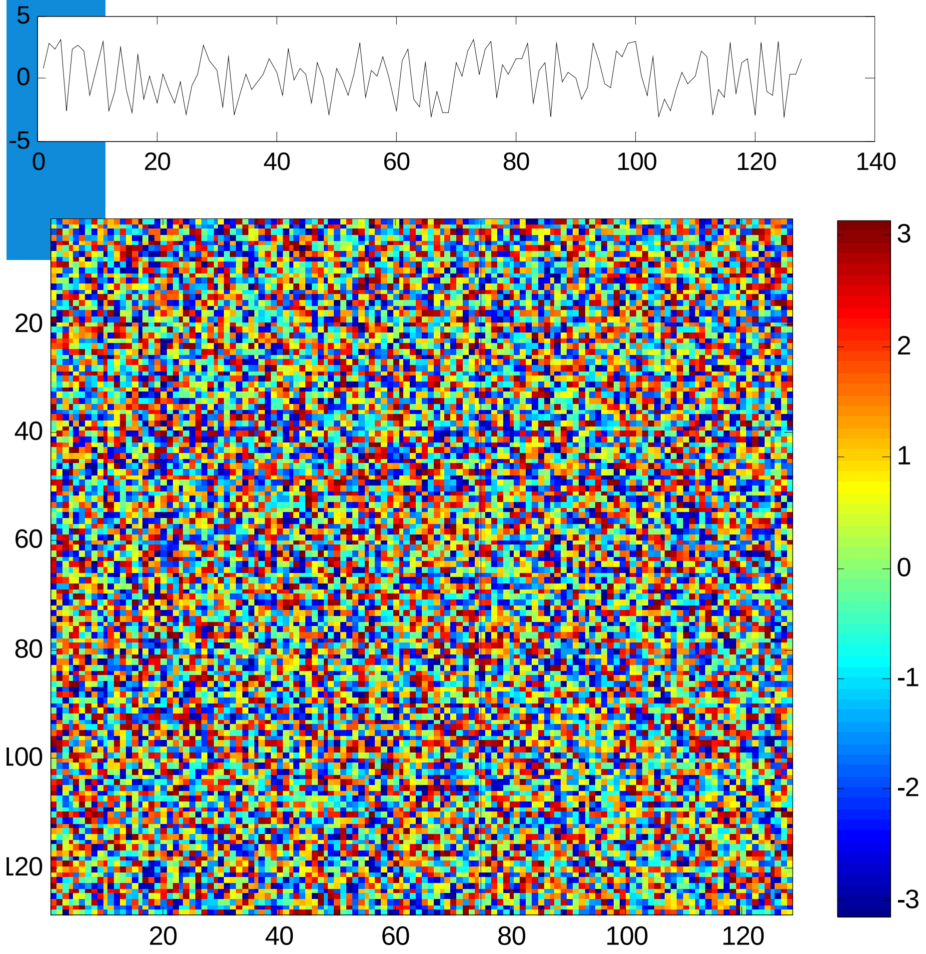
$$\psi_{1,scat}$$

This is a random number: uniform distribution

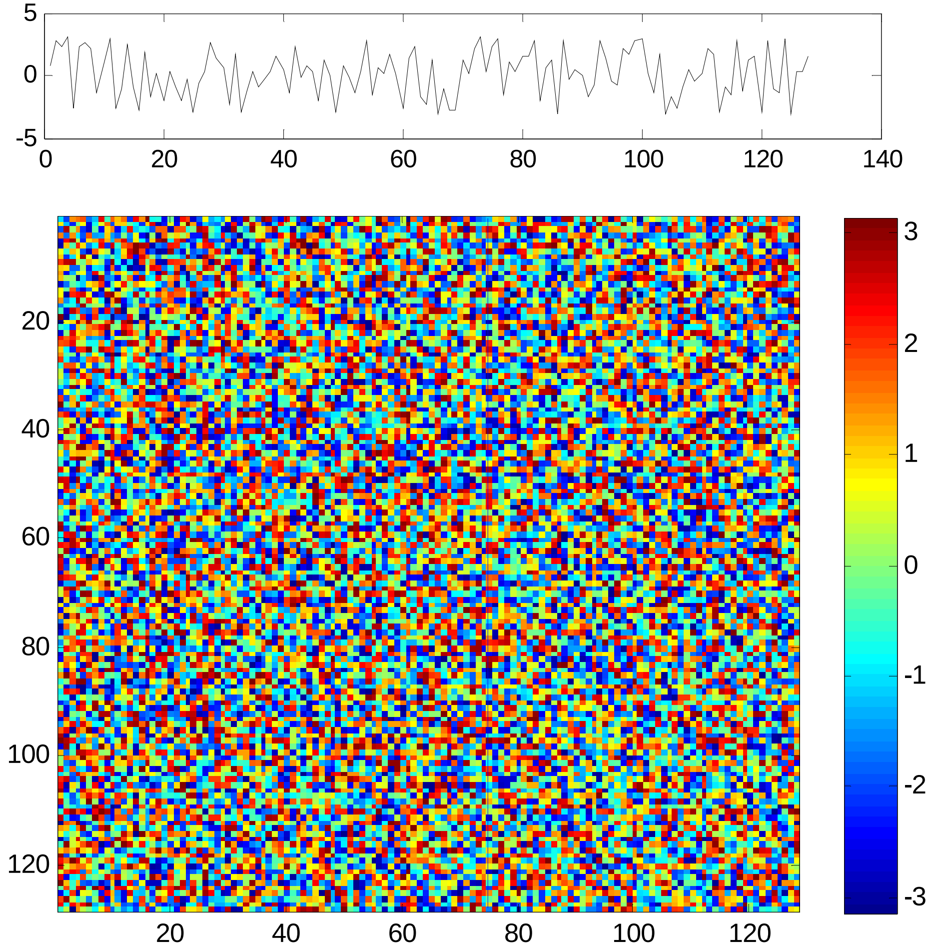



♦

# 2. Scattering Phase




$\psi_{scat,1}$ (image 1)




$\psi_{scat,2}$ (image 2)

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# Phase contributions

The phase $\psi_{1,obs}$ of a resolution cell in SAR image 1 is composed of two parts:

1. Geometric phase (dependent on distance antenna-scatterer):

$$\psi_{1,geom}$$

2. Scattering phase (dependent on interaction of EM wave with objects on the ground,

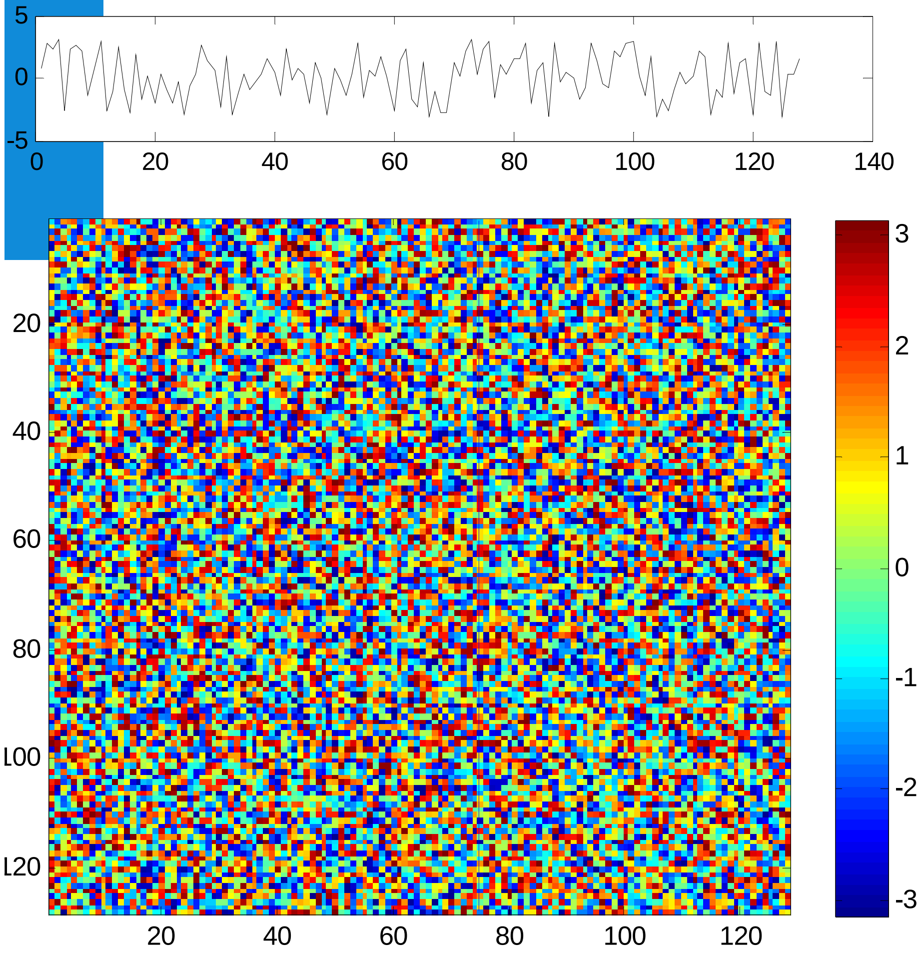
$$\psi_{1,scat}$$

The observed phase $\psi_{1,obs}$ is the **sum** of both components:

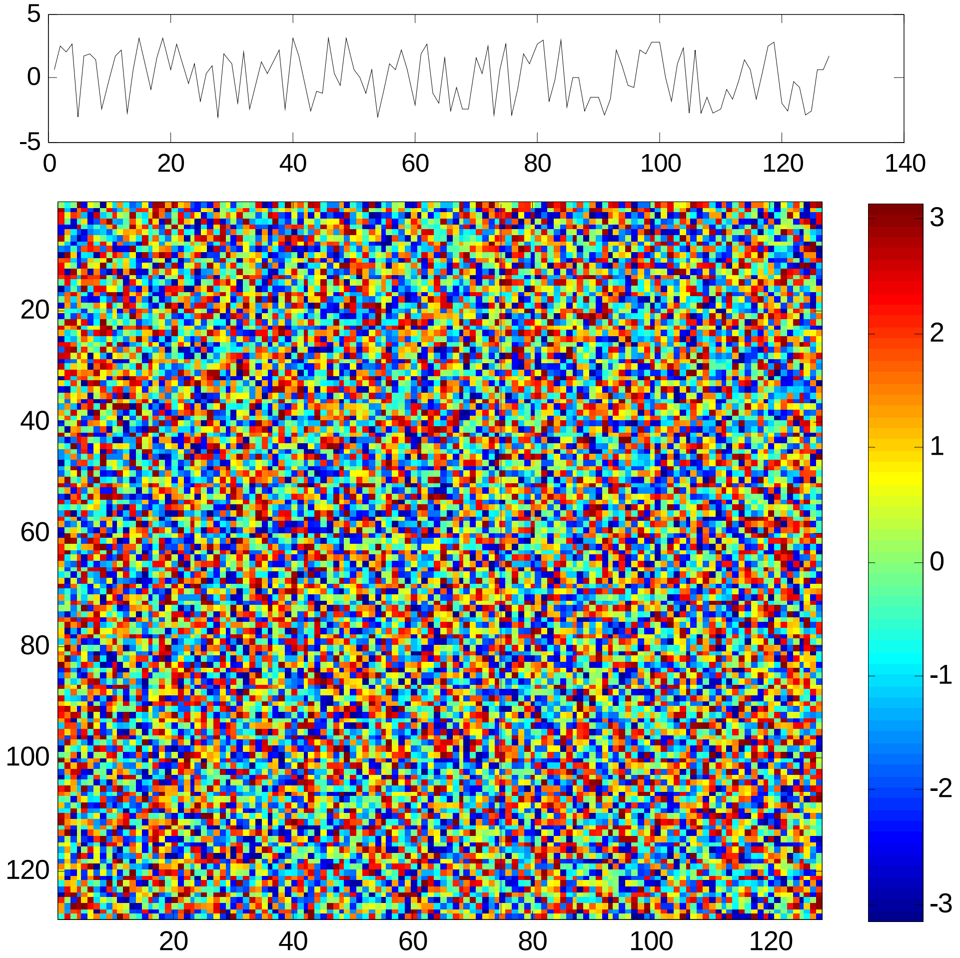
$$\psi_{1,obs} = \psi_{1,geom} + \psi_{1,scat}$$

Due to the wrapping, the observed phase has a uniform random distribution too!

# Geometric + Scattering Phase



$$\psi_{1,obs} = \psi_{1,geom} + \psi_{1,scat}$$

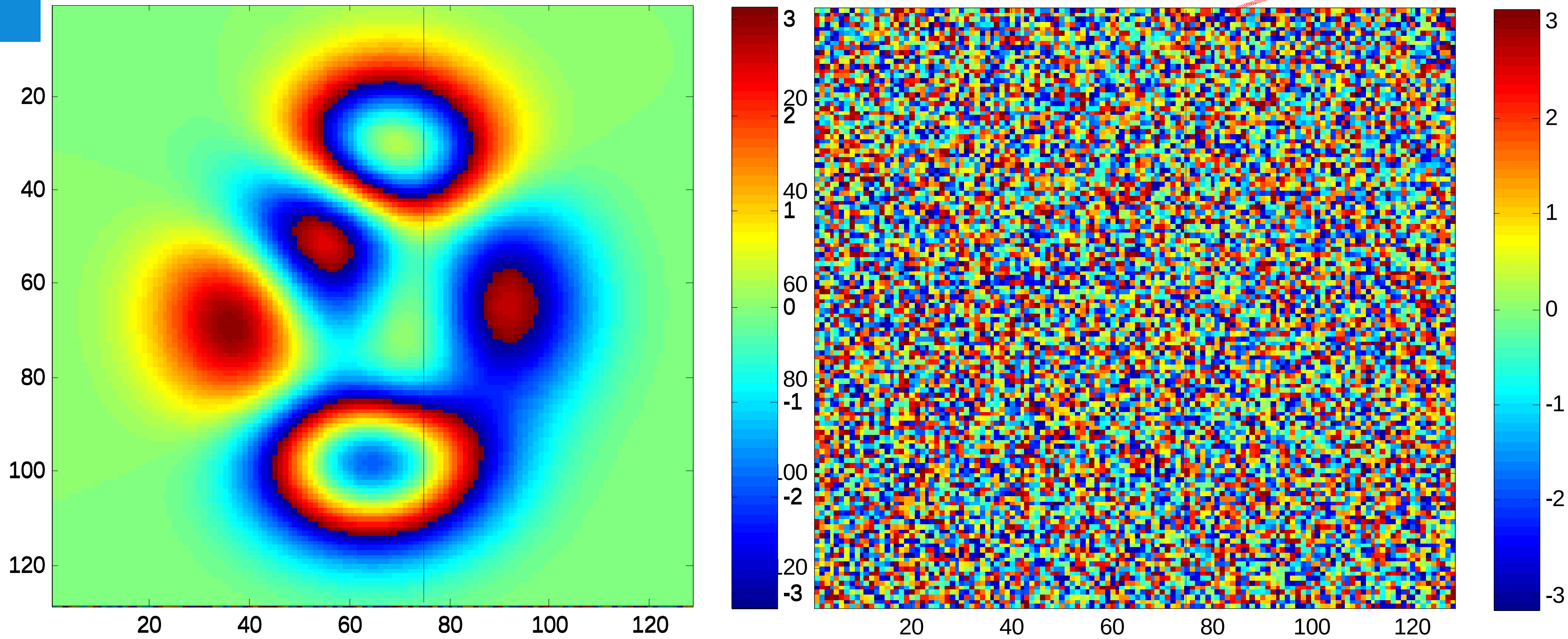


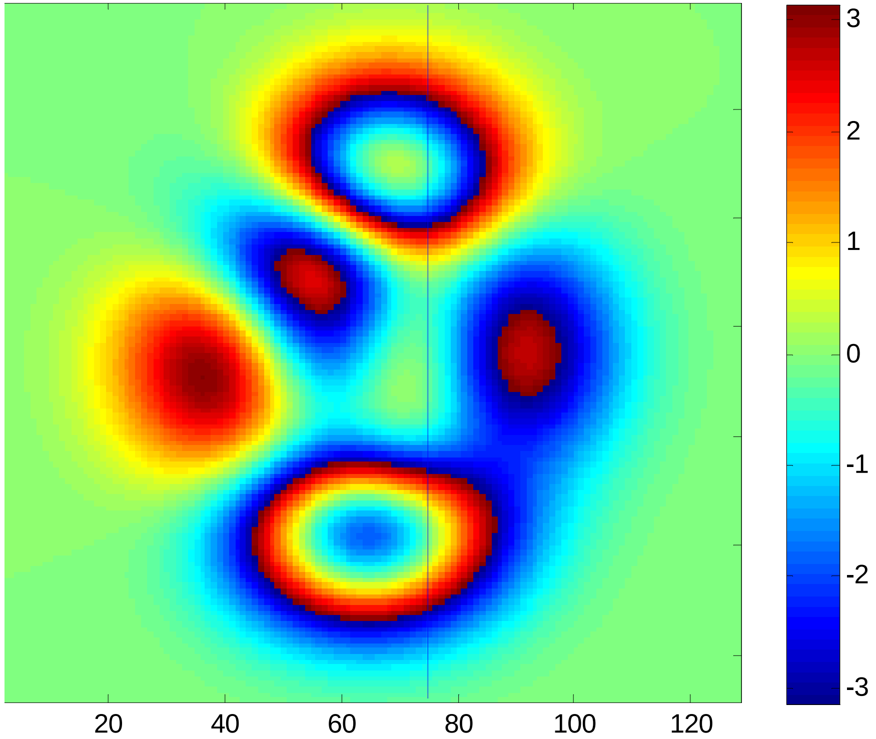
$$\psi_{2,obs} = \psi_{2,geom} + \psi_{2,scat}$$

# The interferometric combination eliminates the scattering phase

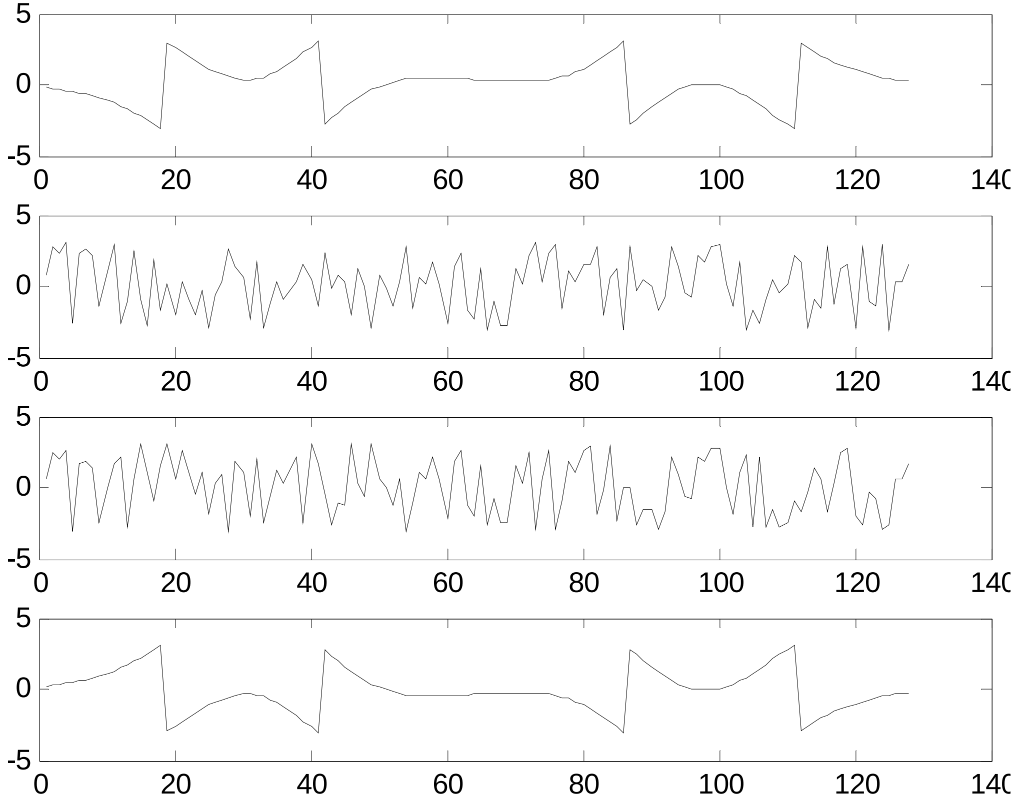
$$\psi_{1,obs} = \psi_{1,geom} + \psi_{1,scat}$$
$$\psi_{2,obs} = \psi_{2,geom} + \psi_{2,scat}$$

$$\varphi_{1,2} = \psi_{1,obs} - \psi_{2,obs} = [\psi_{1,geom} - \psi_{2,geom}] + [\psi_{1,scat} - \psi_{2,scat}]$$





3
2
1
0
-1
-2
-3
20
40
60
80
100
120

5
0
-5
0
20
40
60
80
100
120
140

# Temporal Scattering changes

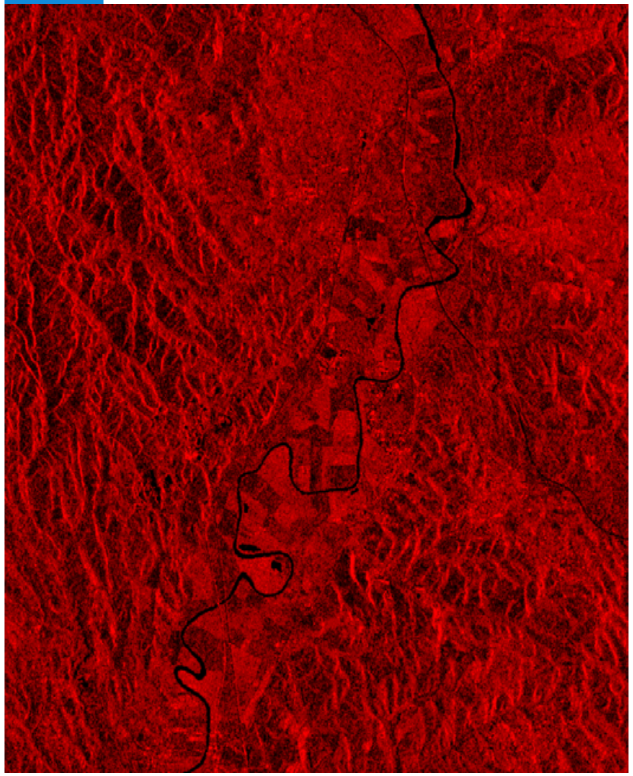
11 June 1992

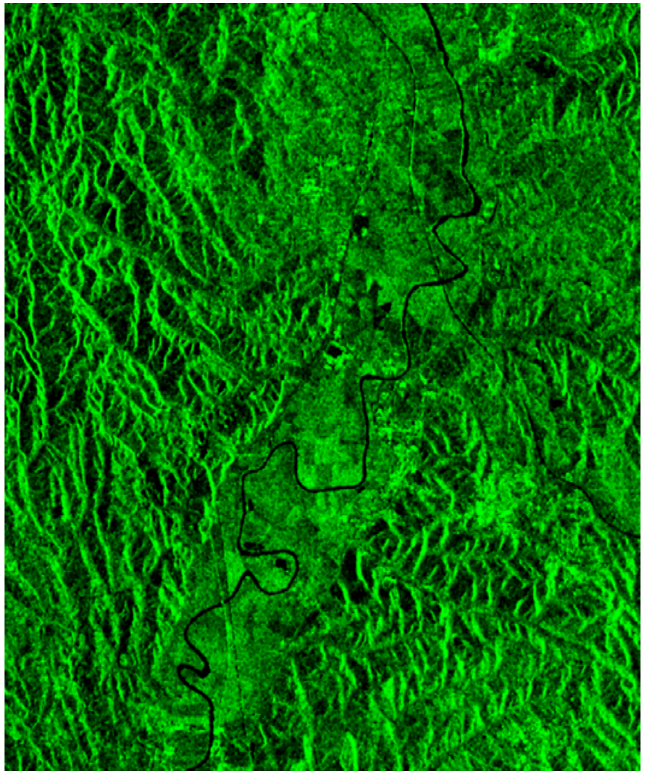
7 January 1993

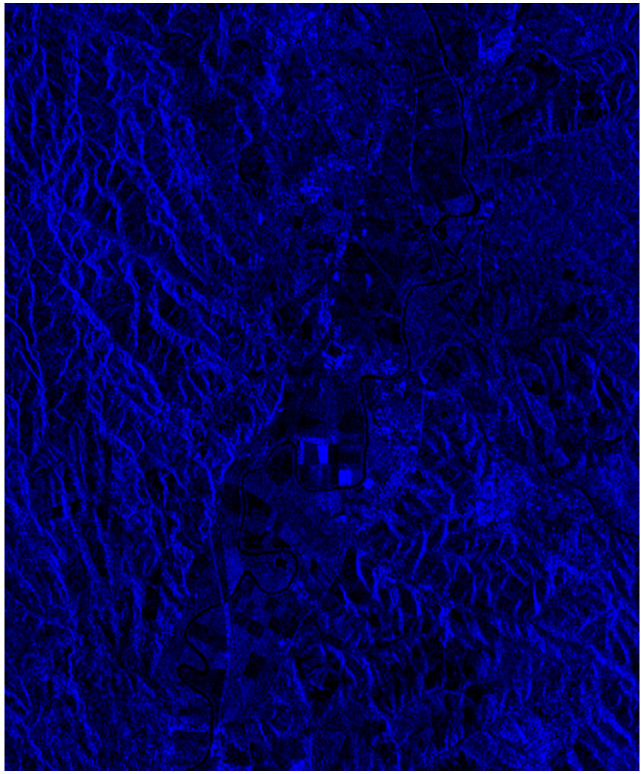
22 April 1993

Source: F. Adragna, CNES

# ERS-2 SAR in false colours (RGB)



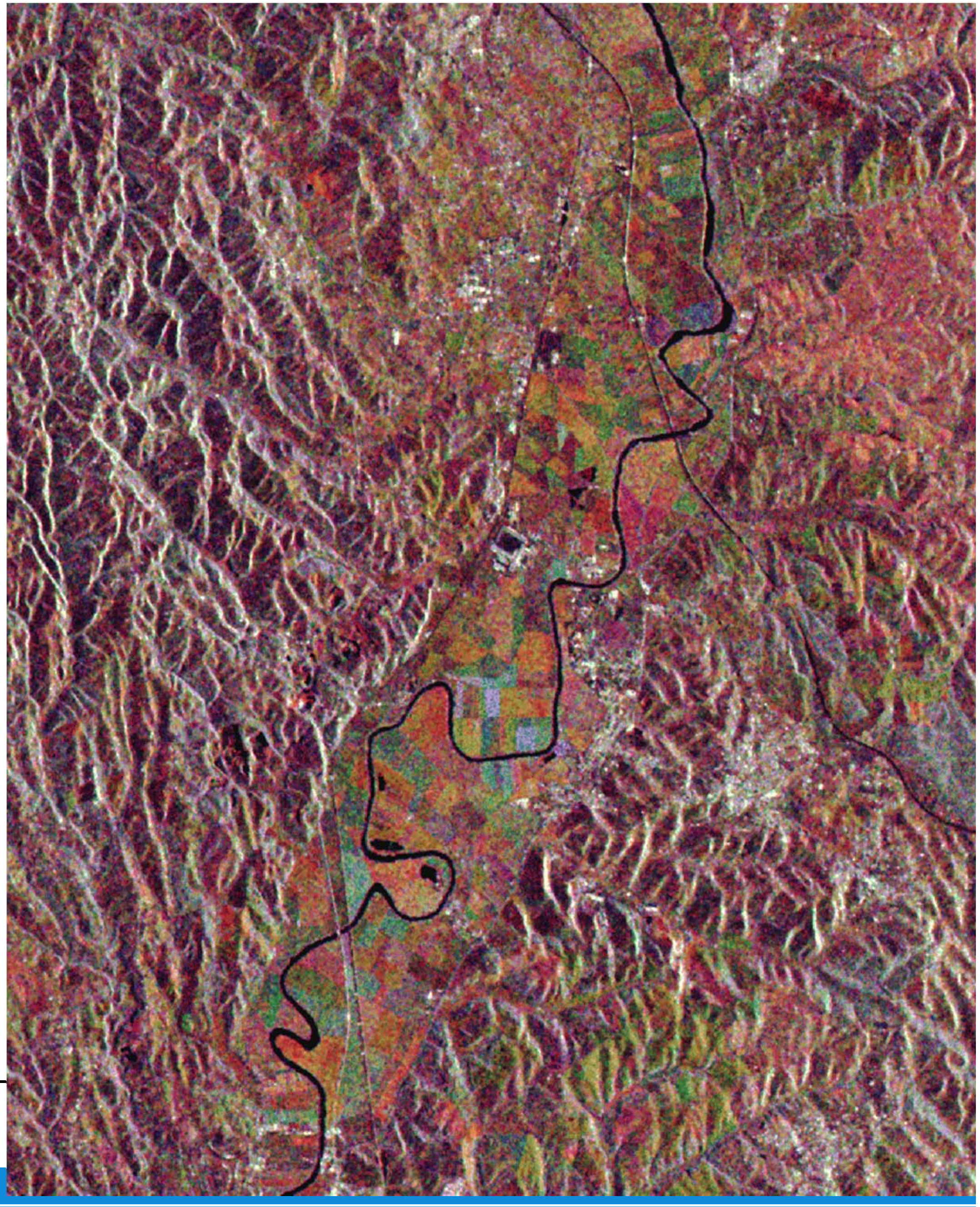




**11 June 1992** **7 January 1993** **22 April 1993**

Source: F. Adragna, CNES

# ERS-2 SAR Multitemporal Image



Source: F. Adragna, CNES

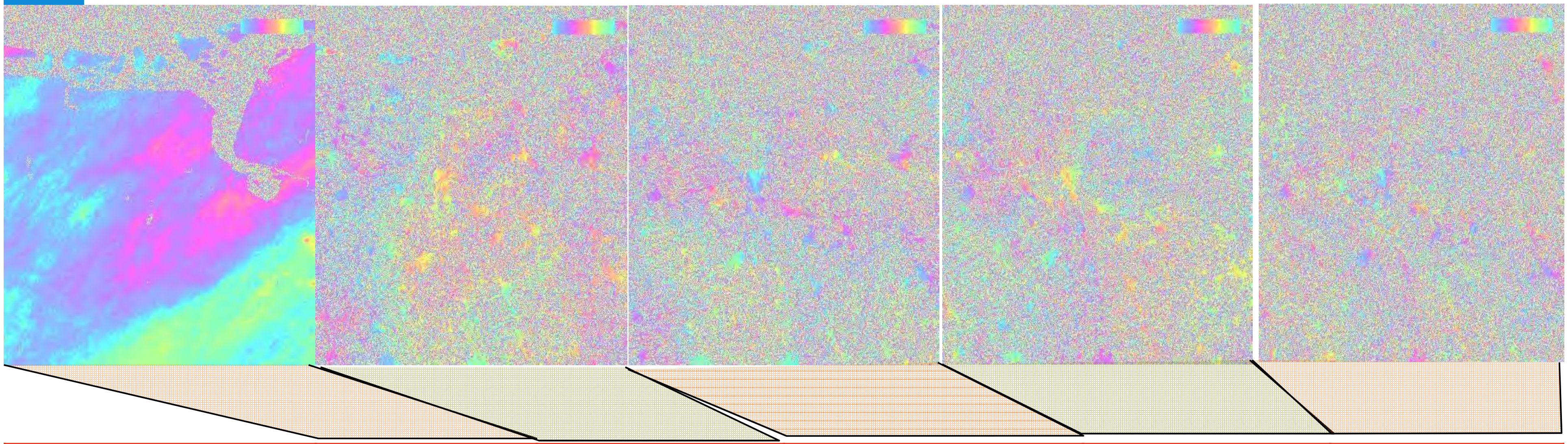
# Temporal decorrelation



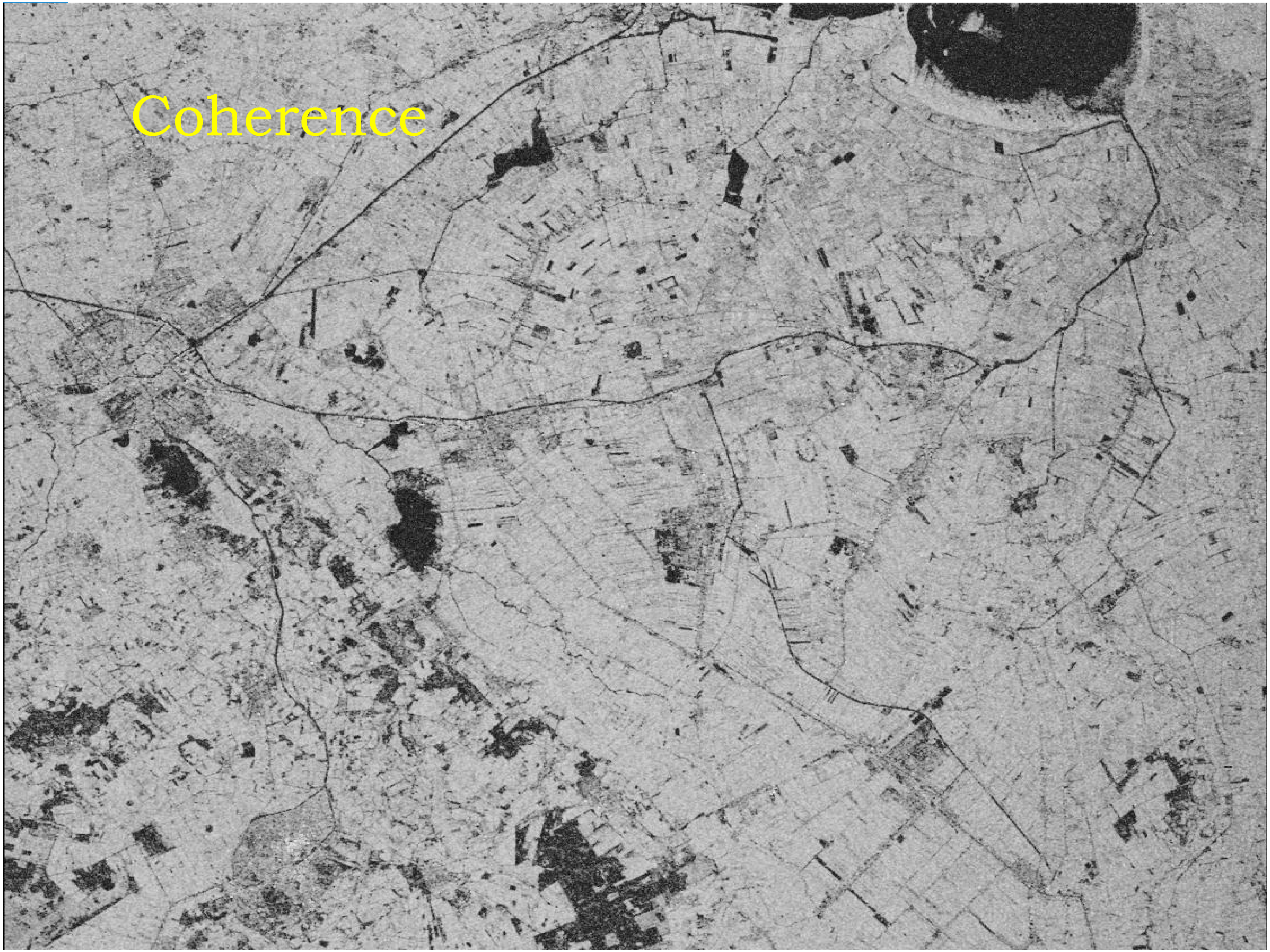


87

# Temporal decorrelation

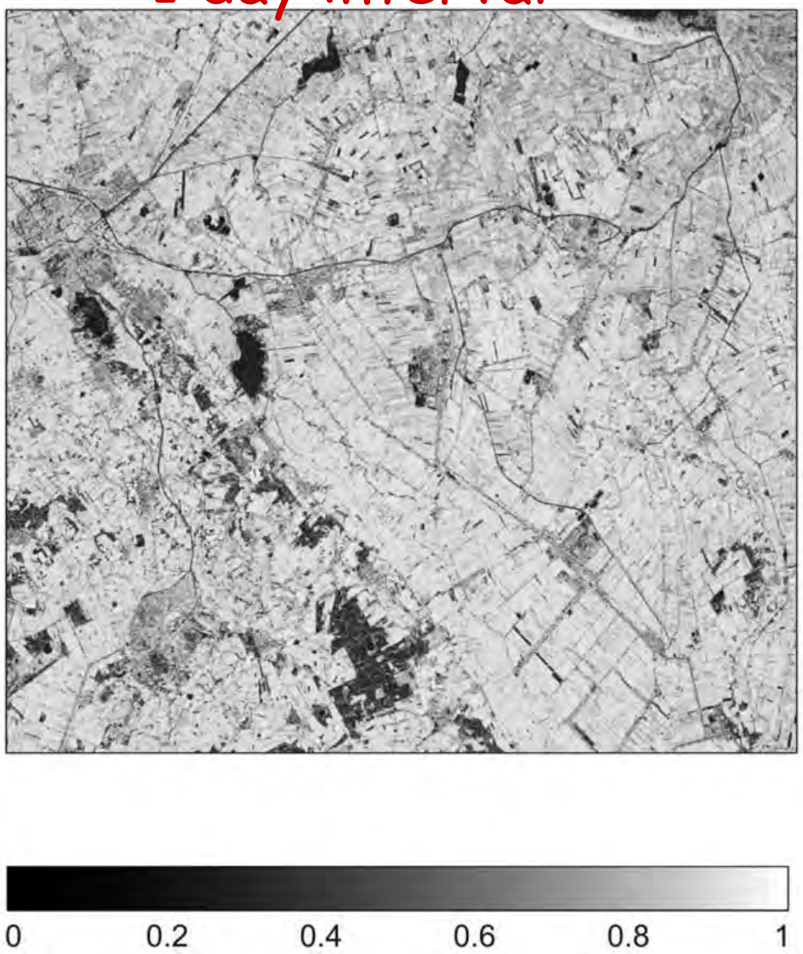


| Temporal baseline | 1 day | 1 year | 2 years | 3 years | 6 years |
|---|---|---|---|---|---|
| Perpendicular baseline (m) | 29 | 112 | 93 | 185 | 166 |

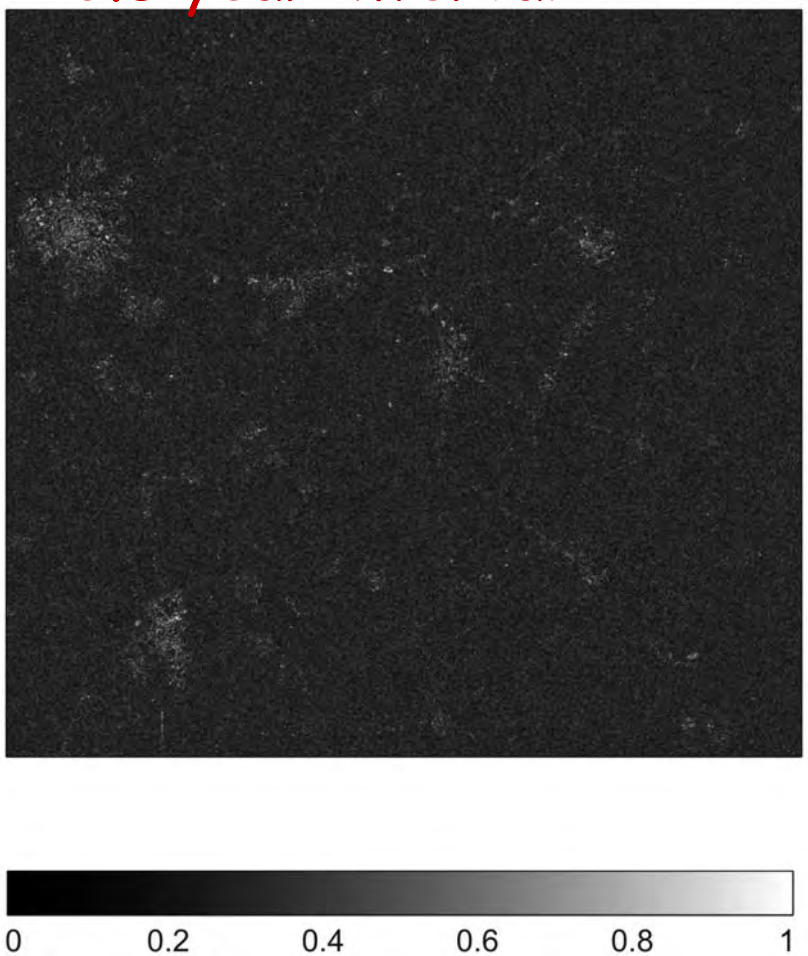

Coherence

# Coherence loss as function of time

1 day interval




3.5 year interval




Anthropogenic features remain coherent over long time intervals

# Phase variance estimation: Coherence

- Optics equivalent to correlation:

$$\gamma = \frac{E\{y_1 y_2^*\}}{\sqrt{E\{|y_1|^2\} \cdot E\{|y_2|^2\}}}$$

complex number!

- Estimation of coherence:

$$|\hat{\gamma}| = \frac{|\sum_{n=1}^{N} y_1^{(n)} y_2^{(n)}|}{\sqrt{\sum_{n=1}^{N} |y_1^{(n)}|^2 \sum_{n=1}^{N} |y_2^{(n)}|^2}}$$

# Coherence corrected for interferometric phase

Interferometric phase

- Optics equivalent to correlation:

$$\gamma = \frac{E\{y_1 y_2^*\}}{\sqrt{E\{|y_1|^2\} \cdot E\{|y_2|^2\}}} = \frac{|E\{y_1 y_2^*\}|\exp(j\phi)}{\sqrt{E\{|y_1|^2\} \cdot E\{|y_2|^2\}}}$$

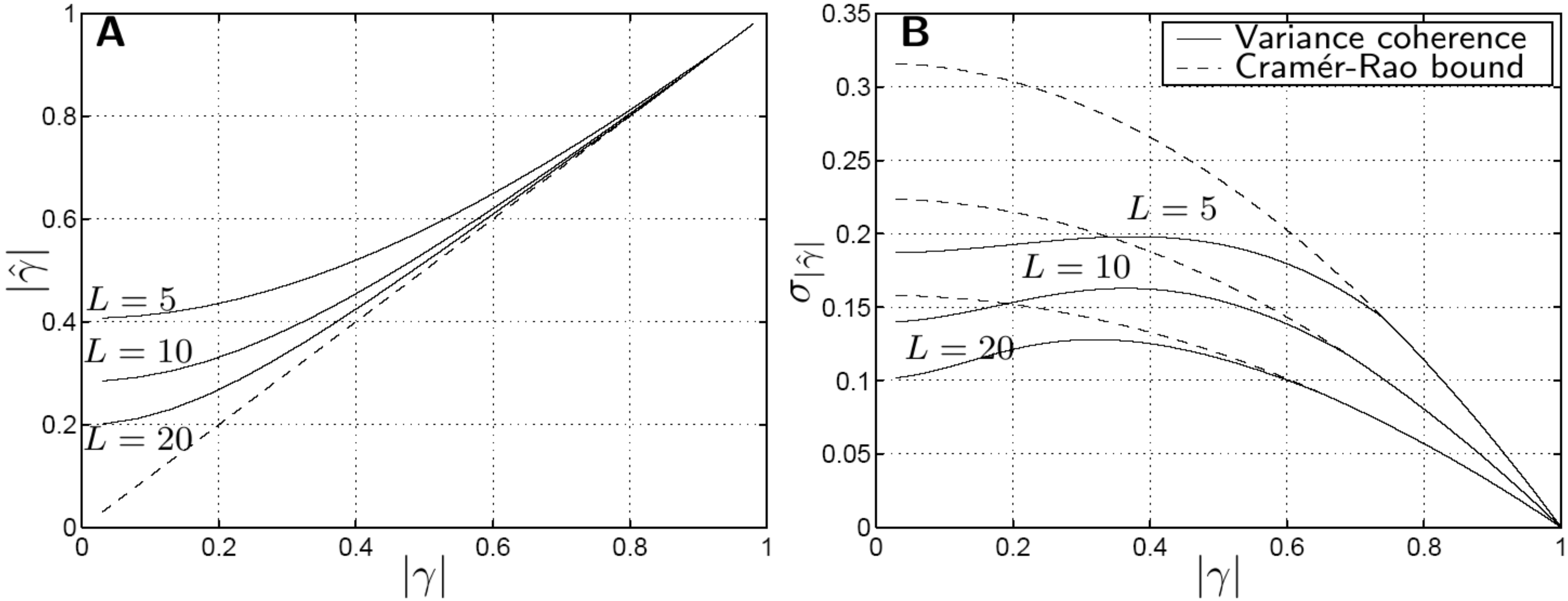
- Estimation of coherence:

Removal of the (non-ergodic) interferometric phase

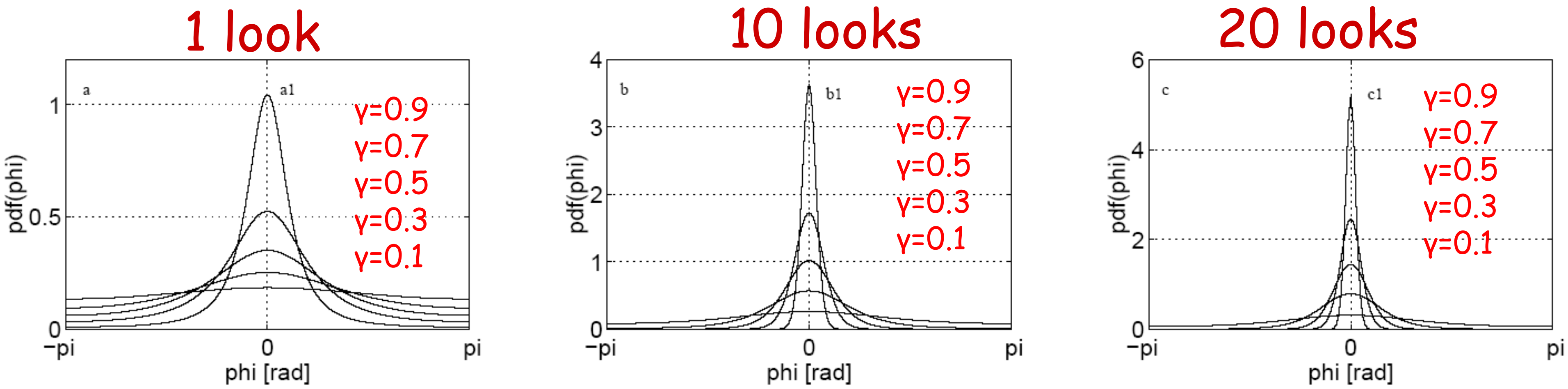
$$|\hat{\gamma}| = \frac{\left|\sum_{n=1}^{N} y_1^{(n)} y_2^{(n)} \exp(-j\phi^{(n)})\right|}{\sqrt{\sum_{n=1}^{N} |y_1^{(n)}|^2 \sum_{n=1}^{N} |y_2^{(n)}|^2}}$$

Biased estimate!
(over-estimation)

# Biased estimation



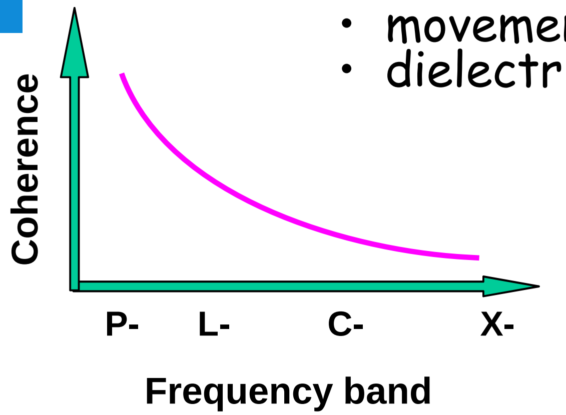
# Coherence, multilooks, and phase PDF

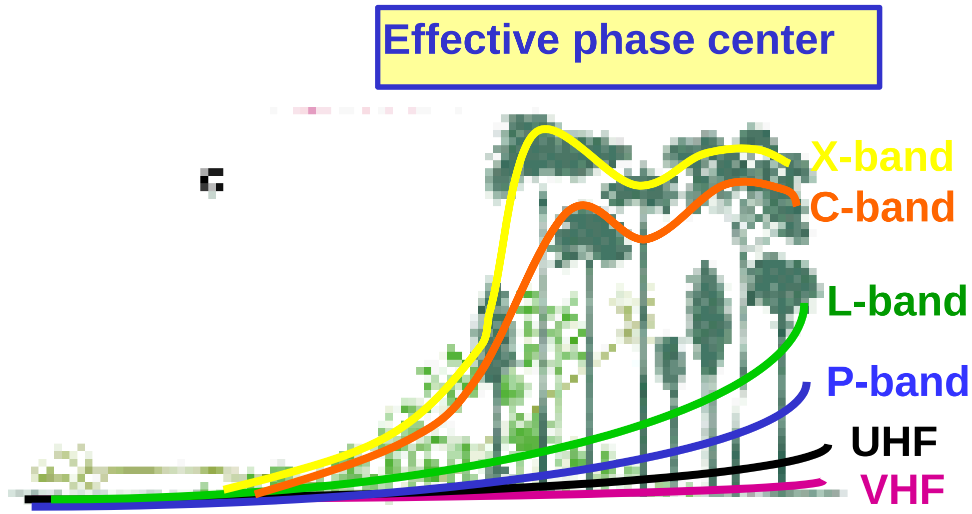


# Coherence and wavelength

Loss of correlation is due to:

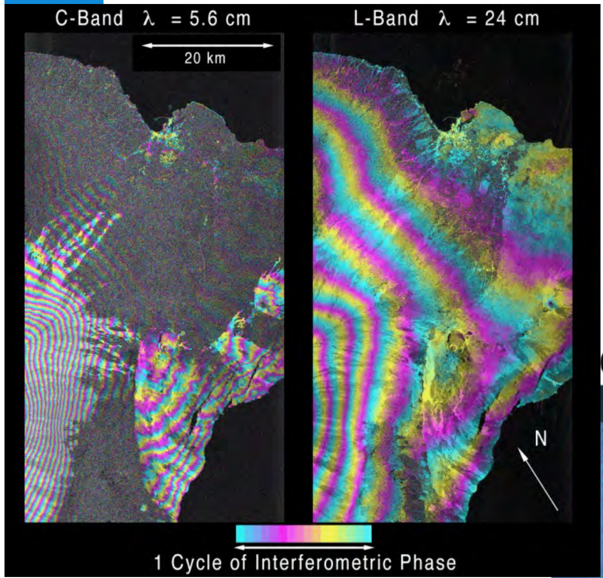
- volume of vegetation
- movement of vegetation
- dielectric change (moisture)



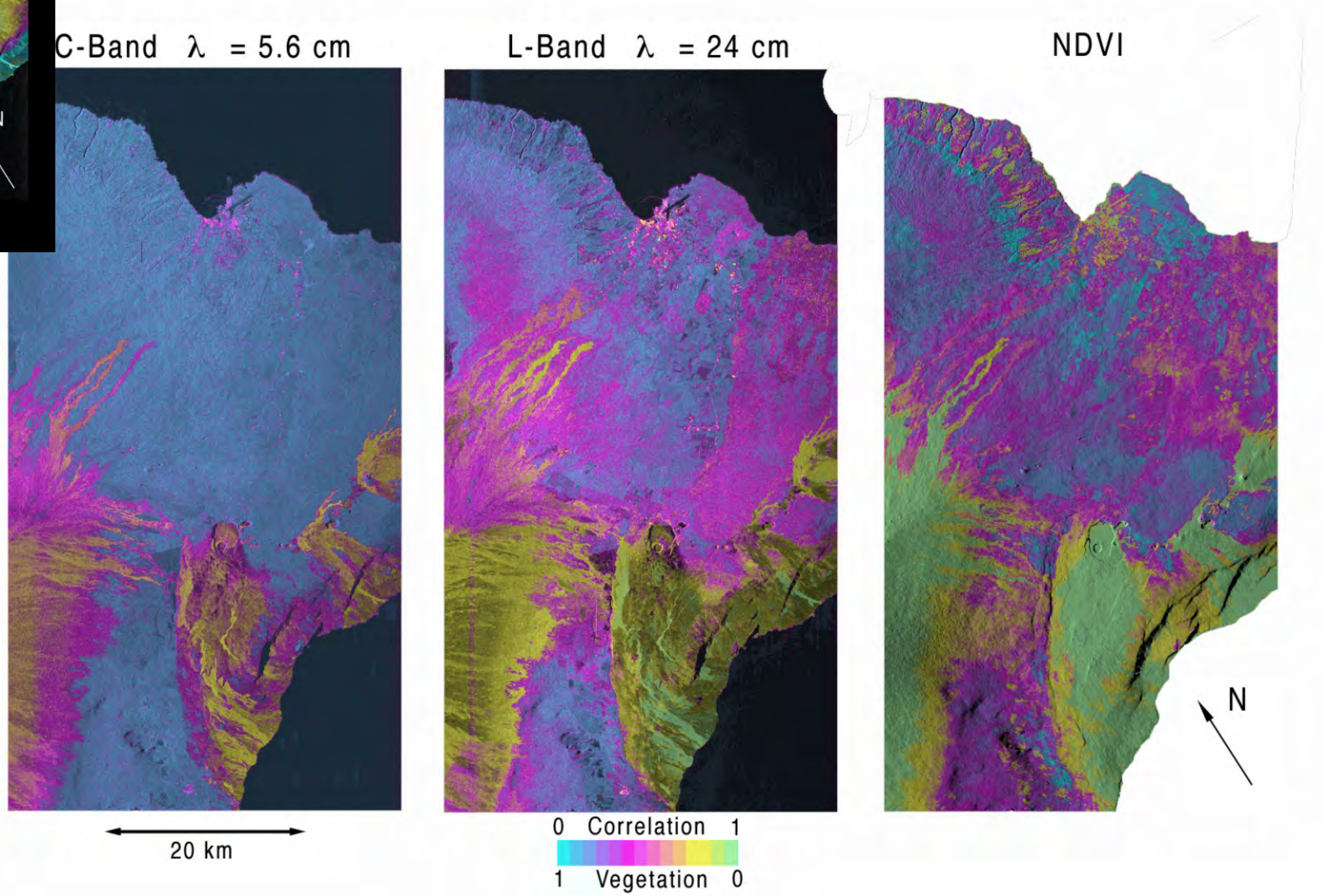




Source: H.Zebker

# Coherence as function of wavelength



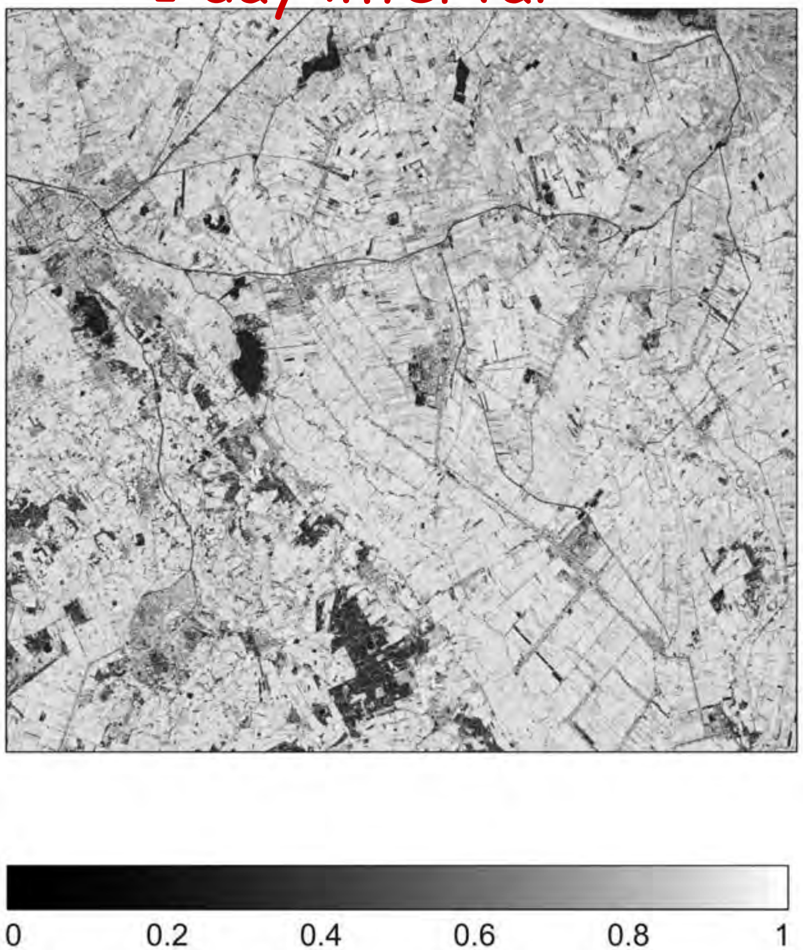

Results SIR-C mission,
Simultaneous C and L band
ΔT=6 months



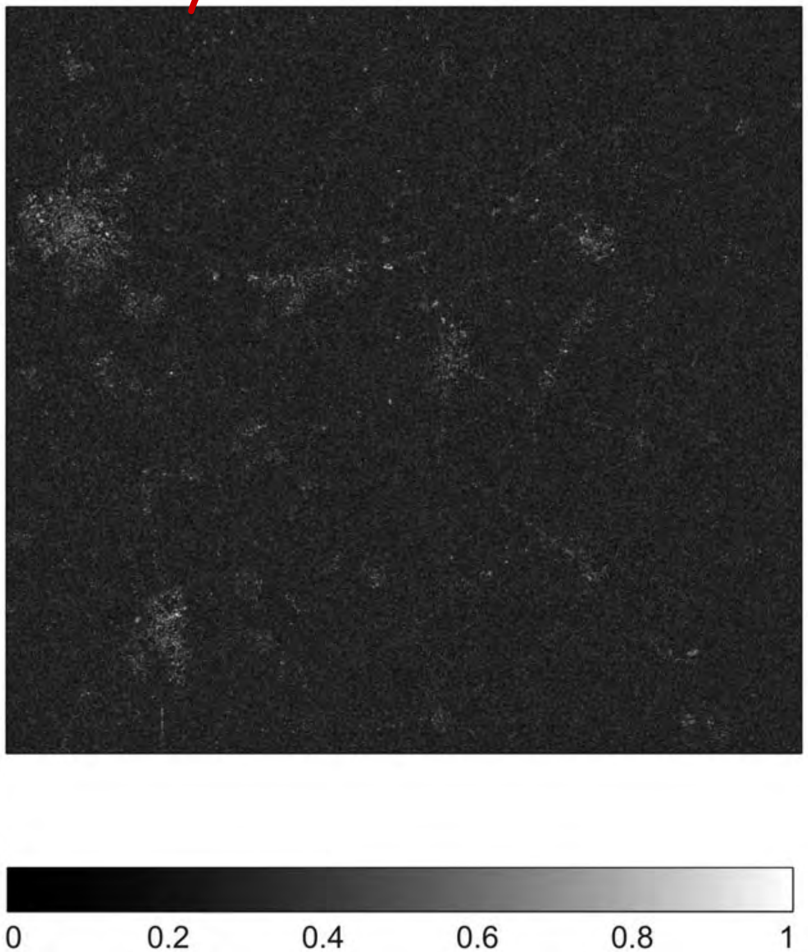

Source: H.Zebker

# Coherence loss as function of time

1 day interval



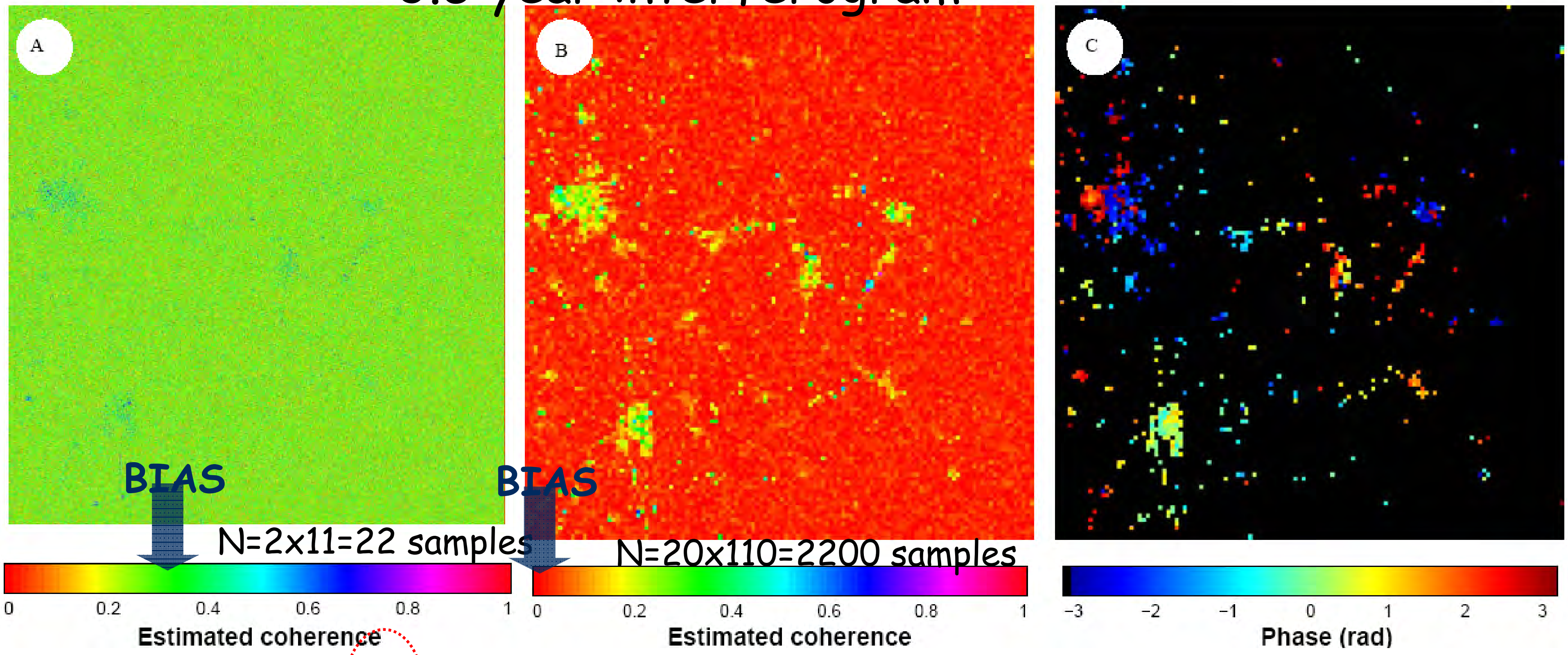

3.5 year interval




Anthropogenic features remain coherent over long time intervals

# Coherence estimation bias

3.5 year interferogram




$$|\hat{\gamma}| = \frac{\left| \sum_{n=1}^{N} y_1^{(n)} y_2^{*(n)} \right|}{\sqrt{\sum_{n=1}^{N} |y_1^{(n)}|^2 \sum_{n=1}^{N} |y_2^{(n)}|^2}}$$

Spatial coherence estimation requires large spatial window:

Trade-off resolution and precision

# Coherence vs SNR

The absolute value of the coherence can be related to the signal-to-noise ratio:

$$|\gamma| = \frac{SNR}{SNR+1} = \frac{1}{1+SNR^{-1}} = \frac{S}{S+N}$$

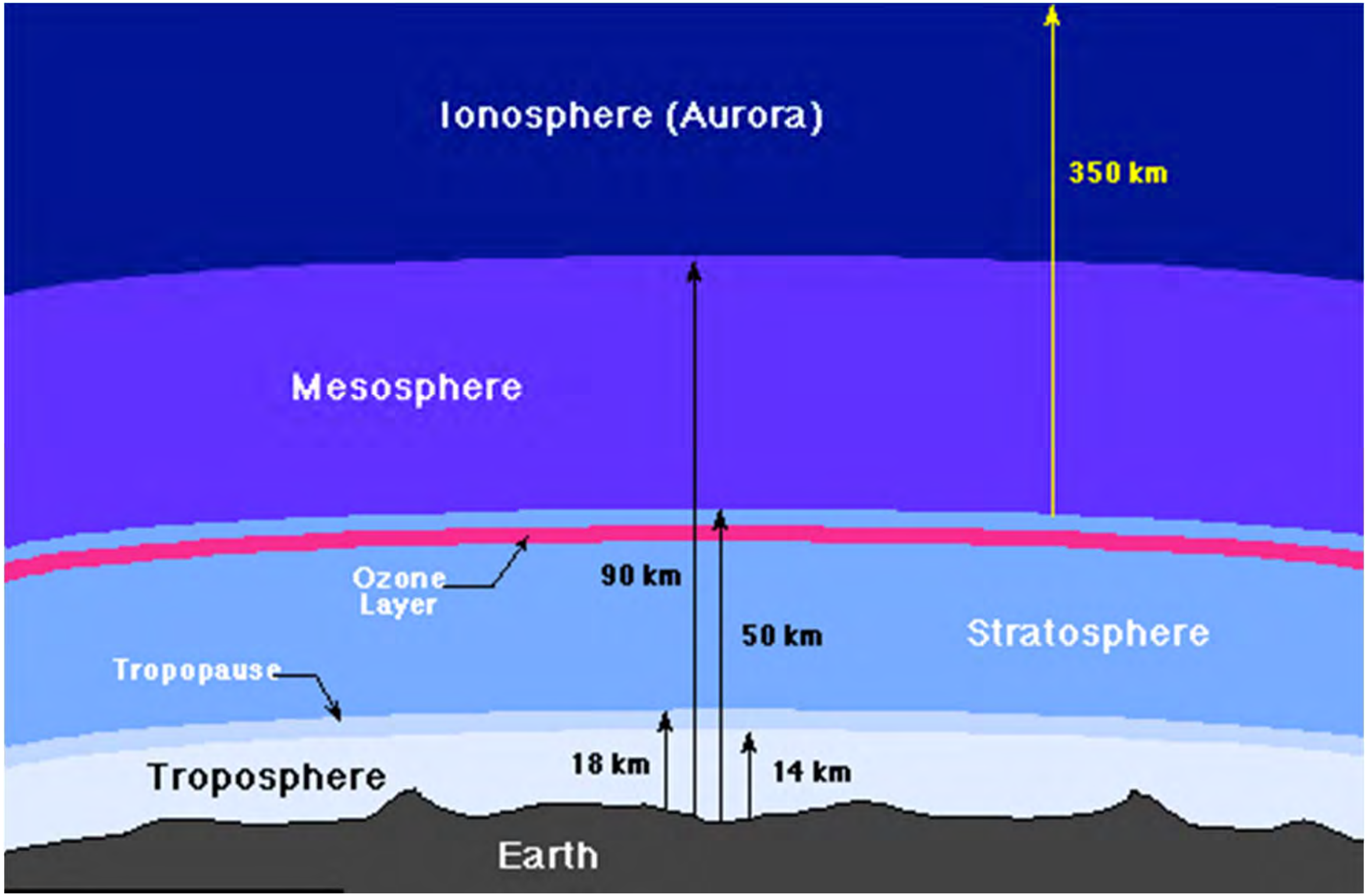
Signal to clutter ratio:

$$\sigma_\varphi = \frac{1}{\sqrt{2 \cdot SCR}}$$

# Error sources

- Decorrelation
- Atmosphere
- Orbit error
- DEM error

# Structure of Atmosphere



# Ionospheric refractivity

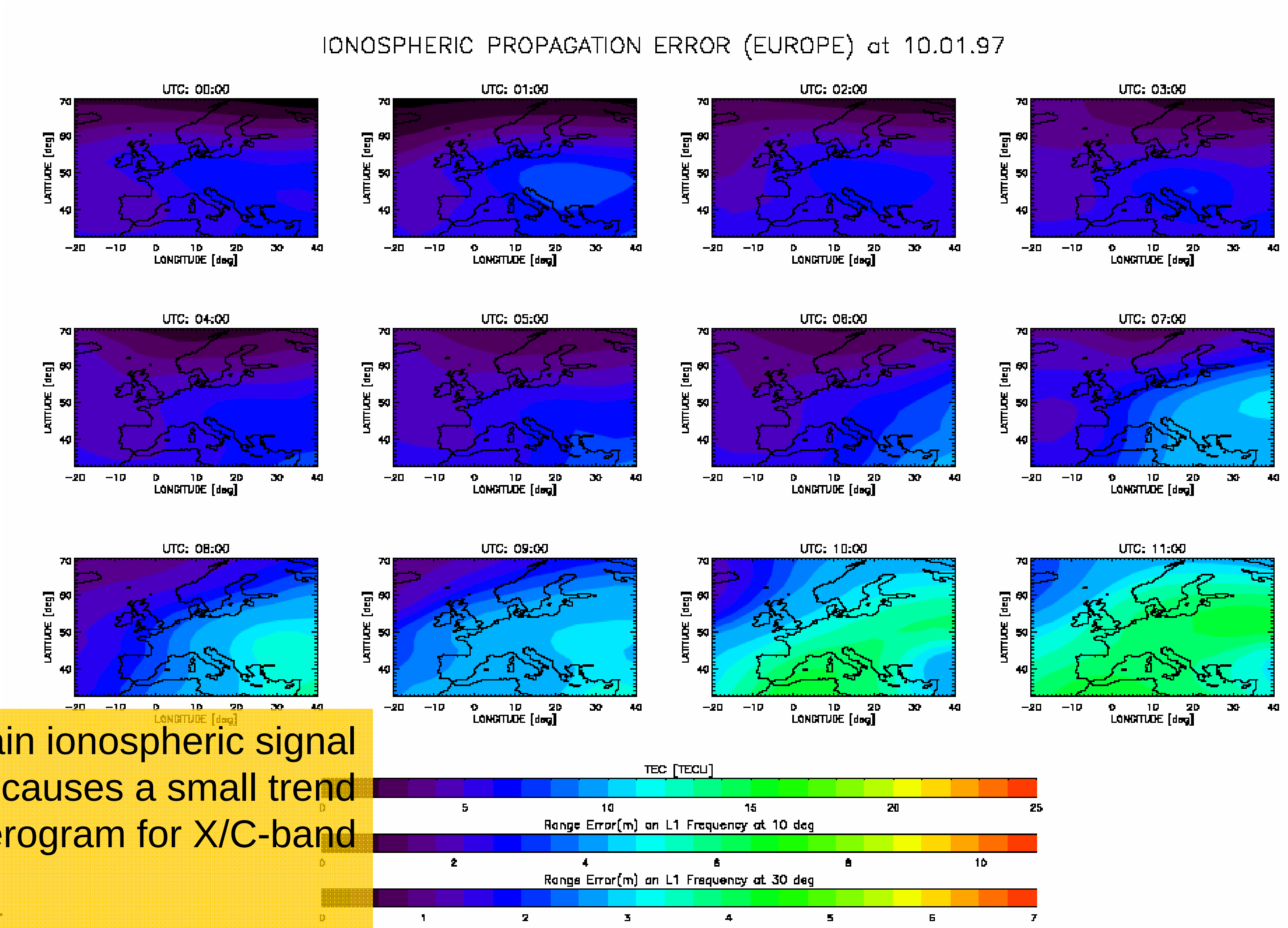
$$N(x,z,t) = -4.03 \cdot 10^{7}\, n_e / f^2$$

$n_e$ = number of electrons

$f$ = electromagnetic frequency

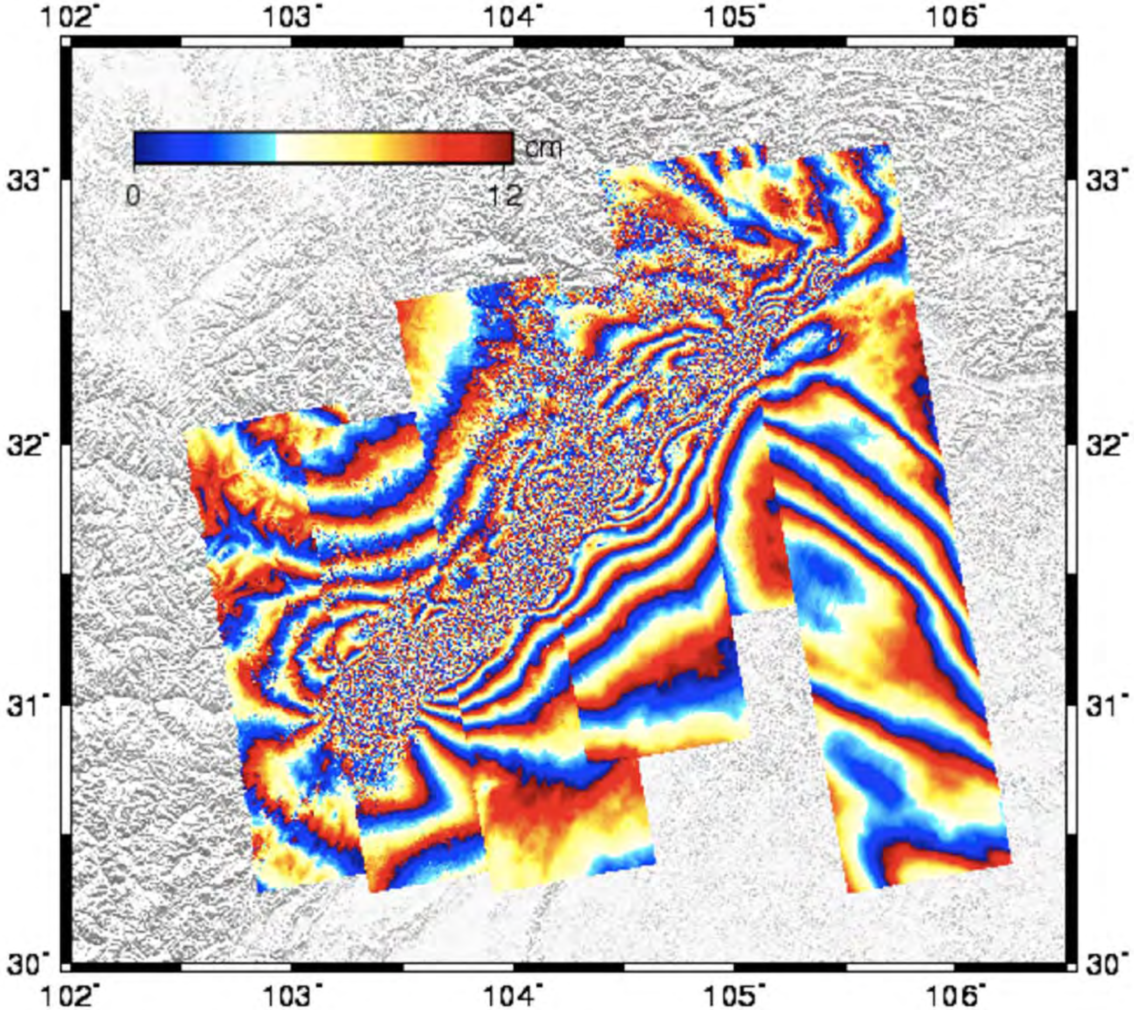
- Delay due to free electrons
- Dispersive (frequency-dependent):
  - ✓ How many times worse is L-band than C-band?

# Ionospheric Delay




Main ionospheric signal usually causes a small trend in interferogram for X/C-band

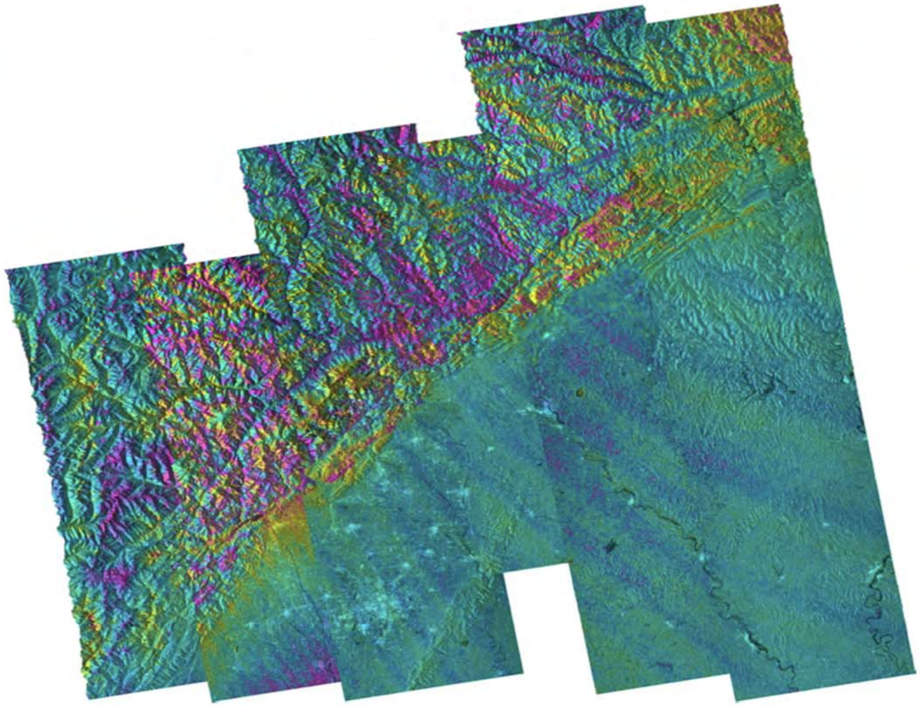
# Wenchuan Earthquake, L-Band



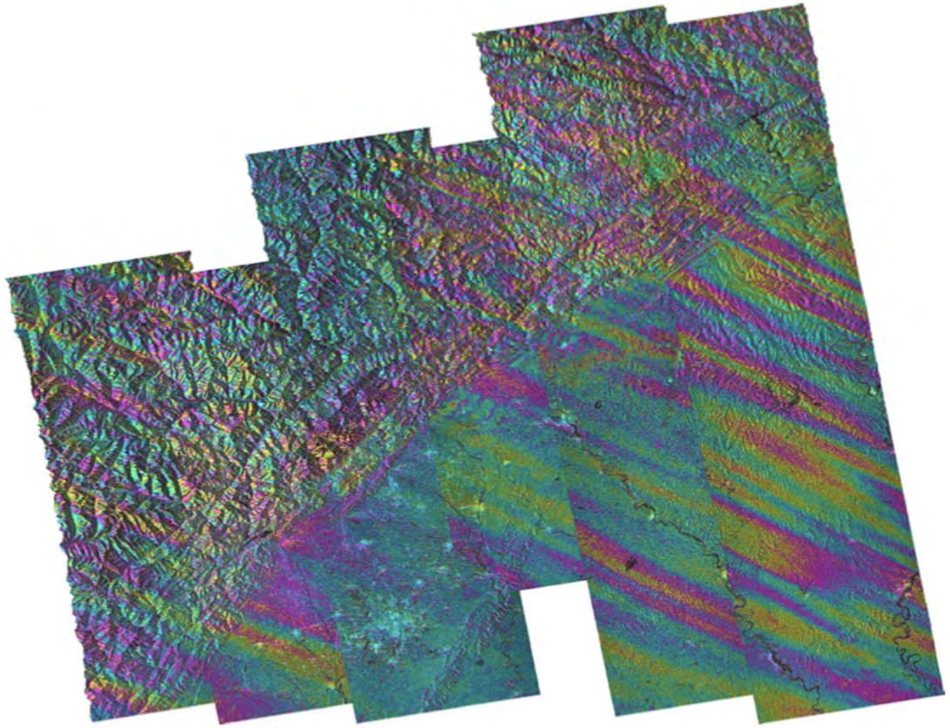

Ionospheric fringes?

Ding et al, ALOS Symposium, 2008

# Coregistration Offsets



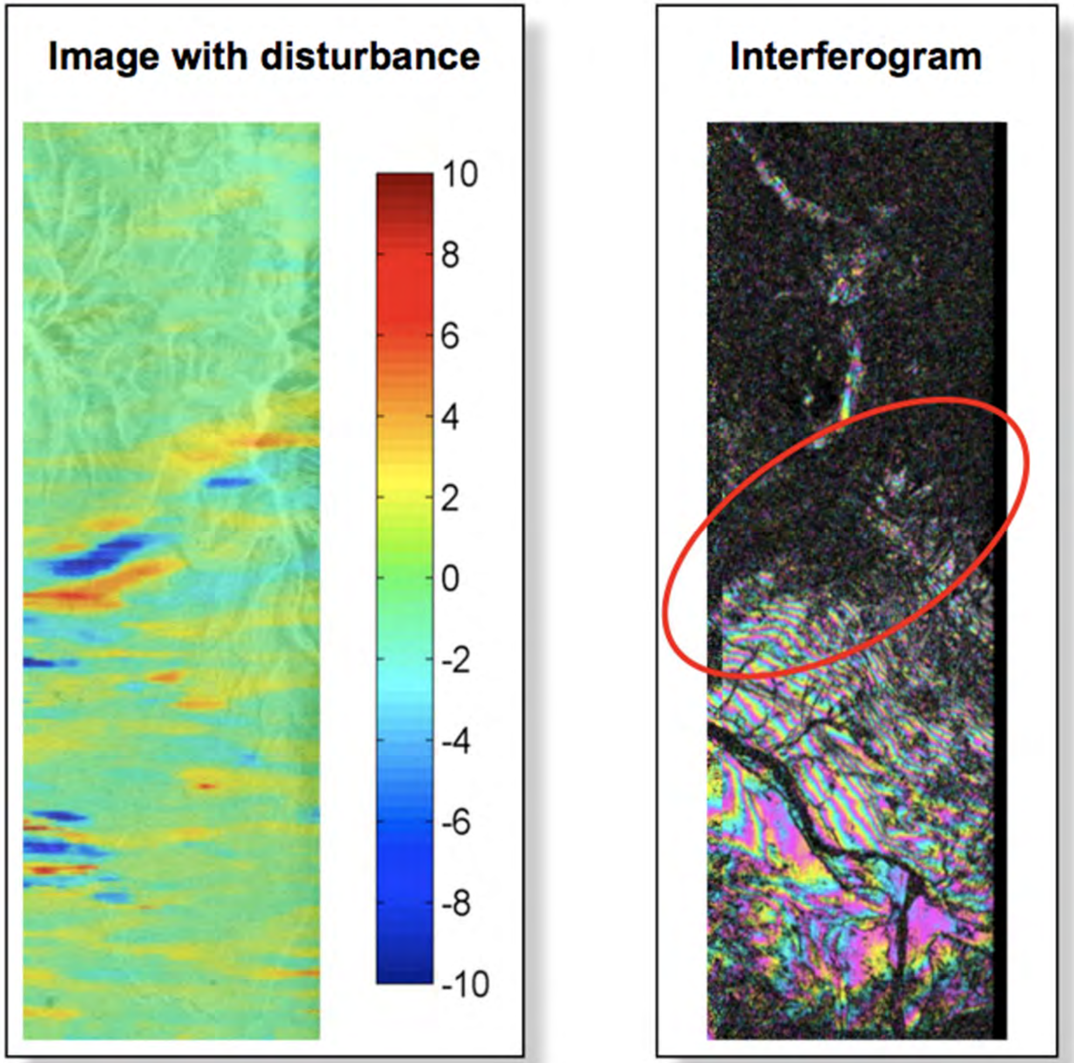
Range offsets



Azimuth offsets

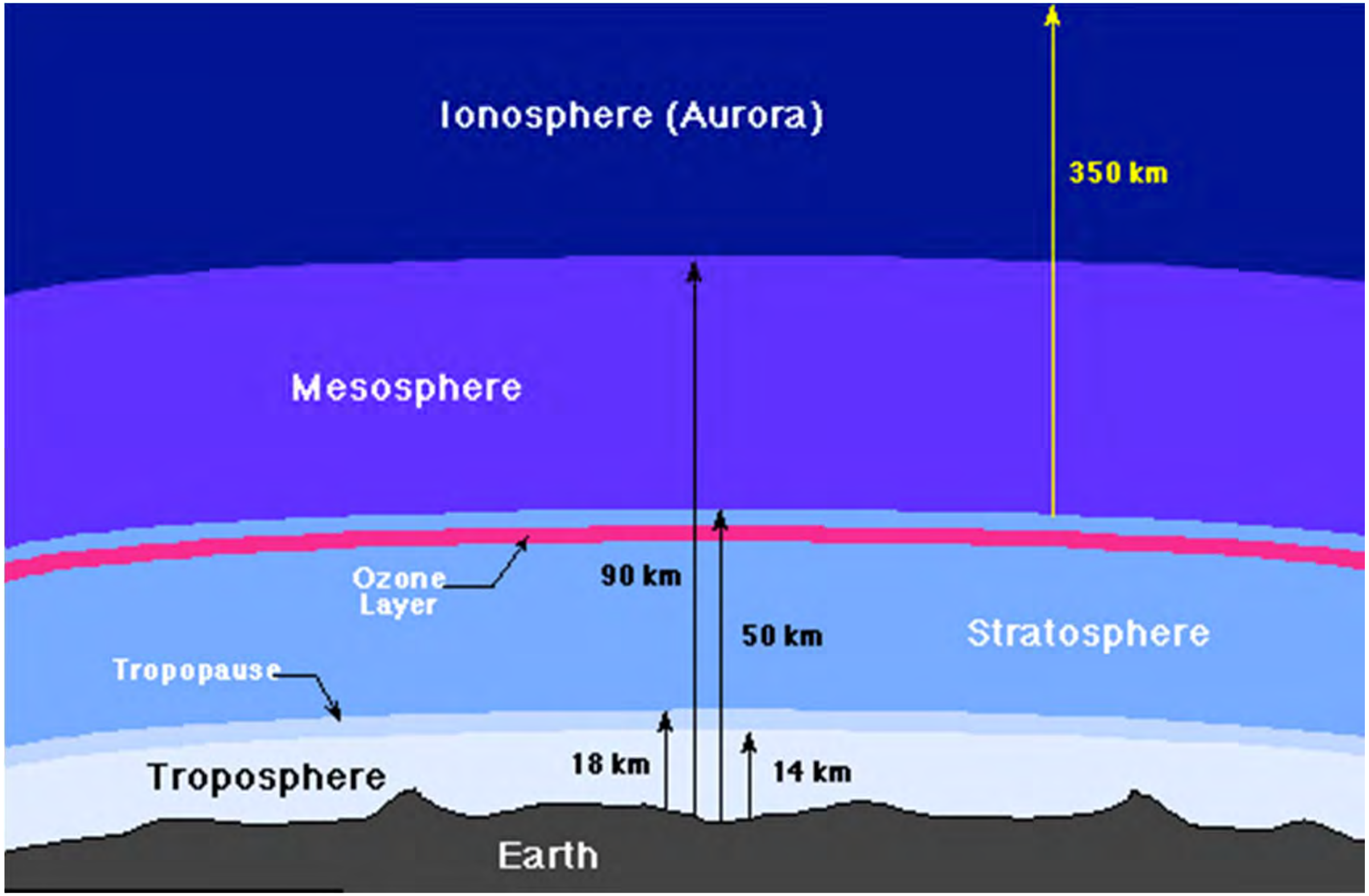
Ding et al, ALOS Symposium, 2008

# High-Latitude Azimuth offsets




Meyer and Nicoll,
Fringe 2007

# Structure of Atmosphere



# Tropospheric Refractivity

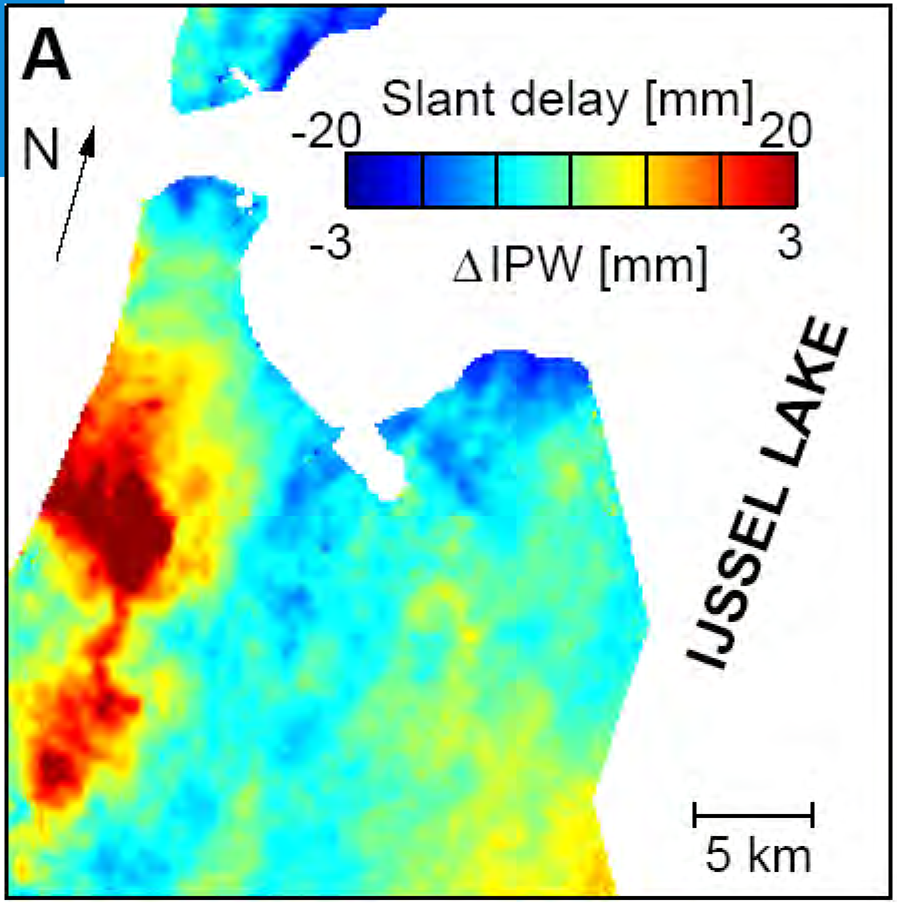
$$N(x,z,t) = k_1 \frac{P}{T} + \left( k_2^{'} \frac{e}{T} + k_3 \frac{e}{T^2} \right) + 1.4W$$

Most spatial variability

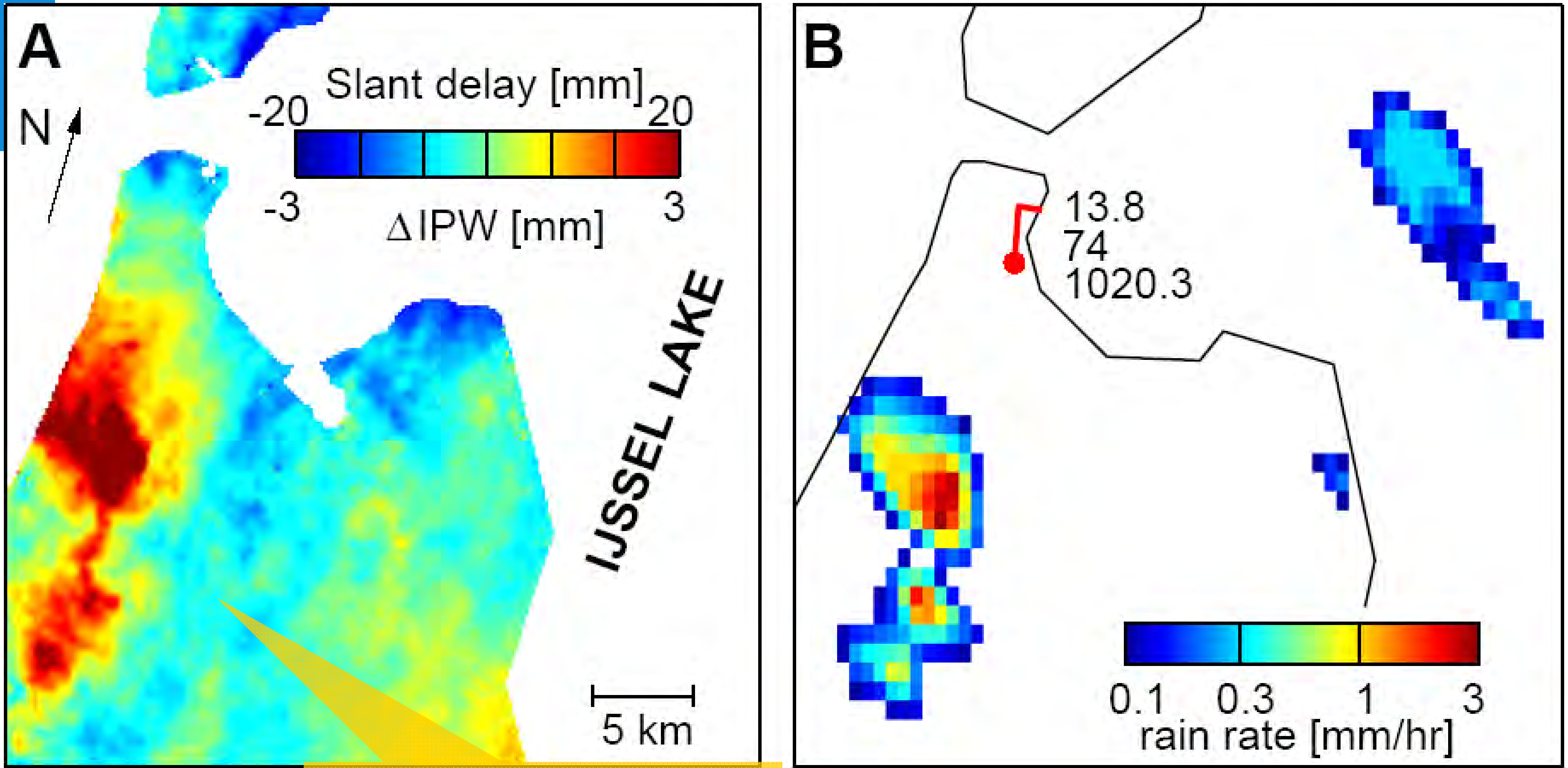
- P=Pressure
- T=Temperature
- e=Partial water vapour pressure
- W=Liquid water

- Hydrostatic term from surface measurements
- Wet delay term (sensitivity 4-20 times higher for WV than for T)
- Liquid term limited (<5%)

# Water vapor

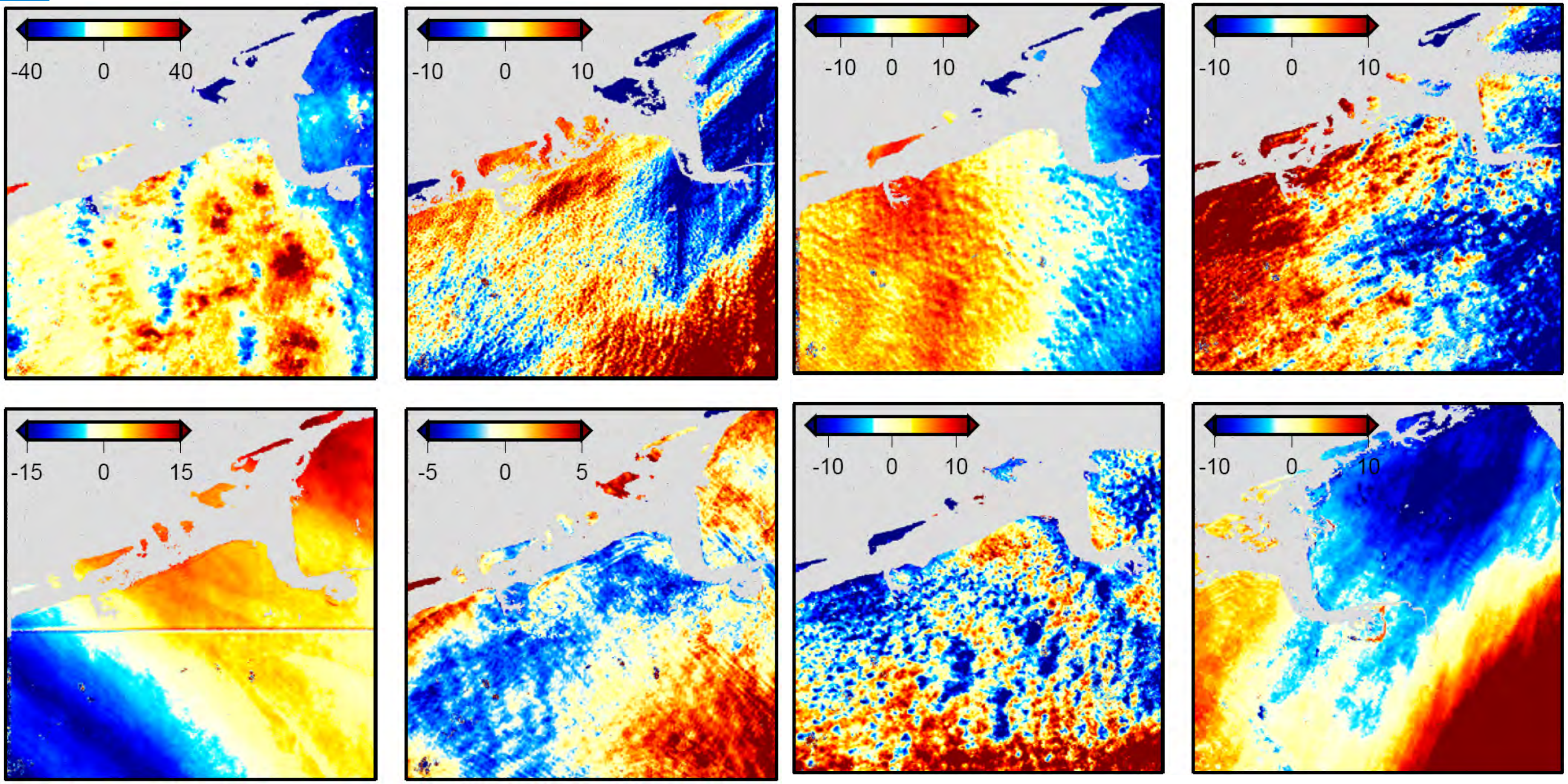


# Spatial variability of water vapour




260 million liter excess water in cloud system

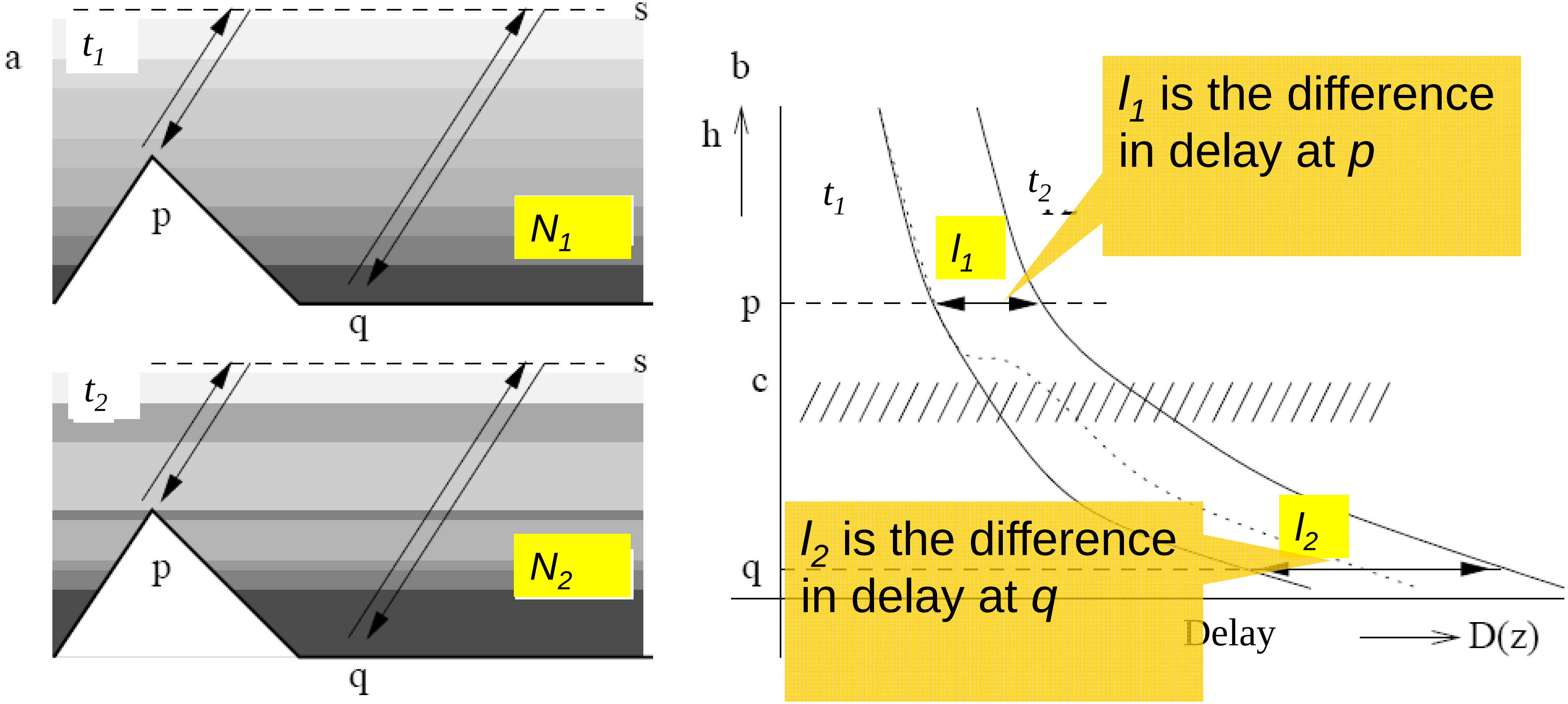
# Tropospheric signal



# Temporal variability in water vapour

In the presence of topography, changes in the refractivity between the two acquisitions will cause an interferometric phase even if no spatial variability

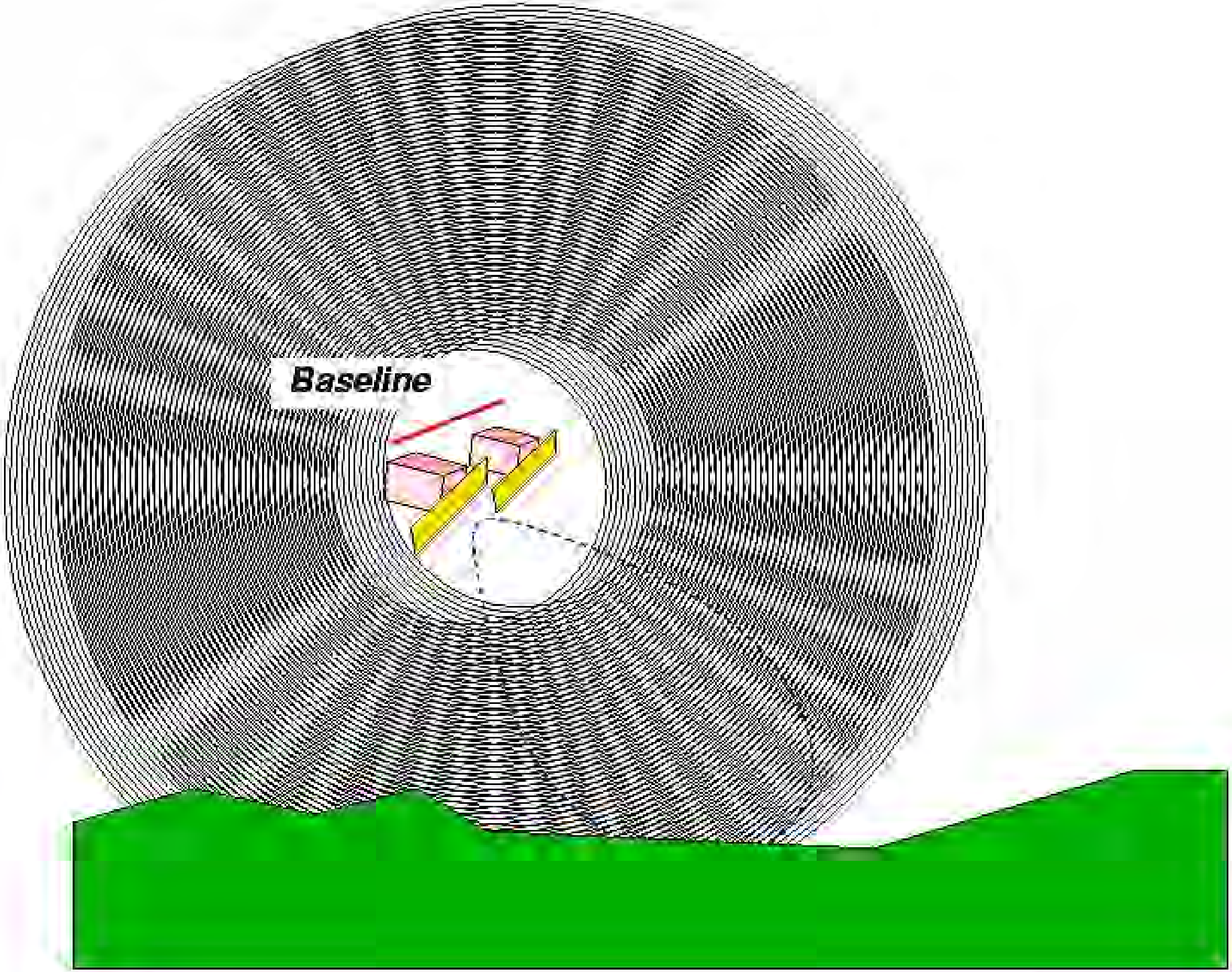
# Change in refractivity profile

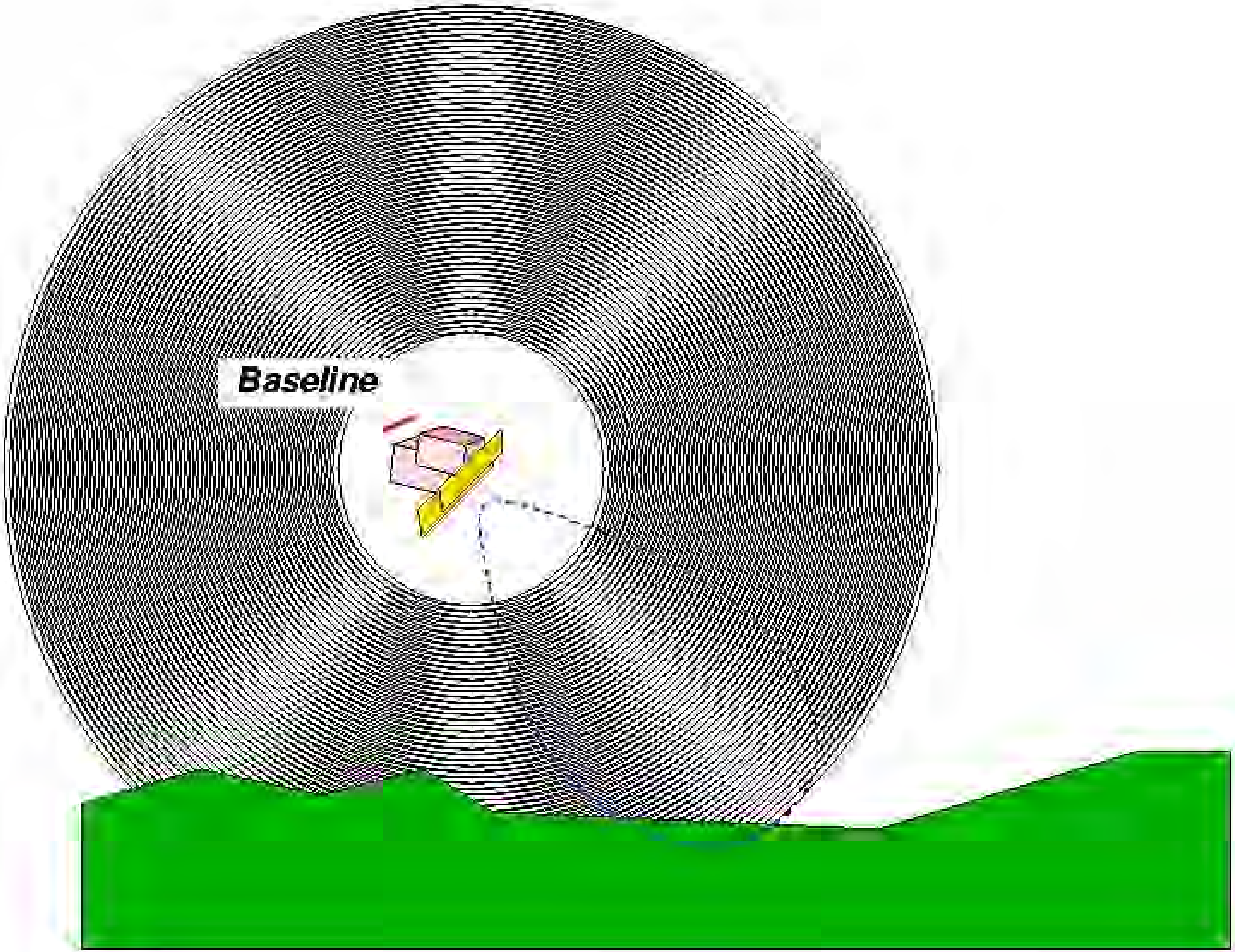



✓ A difference between $l_2$ and $l_1$ will result in a phase offset between *p* and *q*

# Error sources

- Decorrelation
- Atmosphere
- Orbit error
- DEM error


Baseline

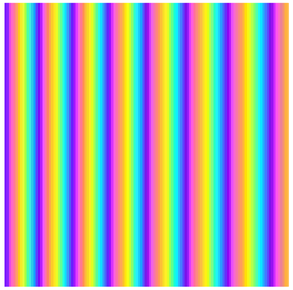

Baseline

# Orbit Error Components

$dB_{\parallel}$

$dB_{\perp}$

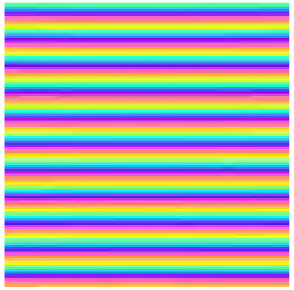
(nearly linear)



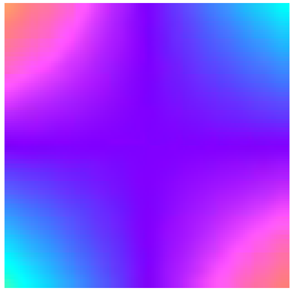
2 m

$d\frac{\partial B_{\parallel}}{\partial t}$

(linear)

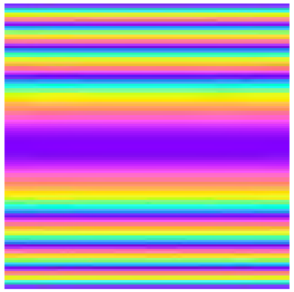


$d\frac{\partial B_{\perp}}{\partial t}$



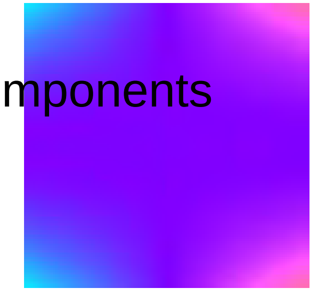
2 cm/s

$d\frac{\partial^2 B_{\parallel}}{\partial t^2}$



D omponents

$d\frac{\partial^2 B_{\perp}}{\partial t^2}$

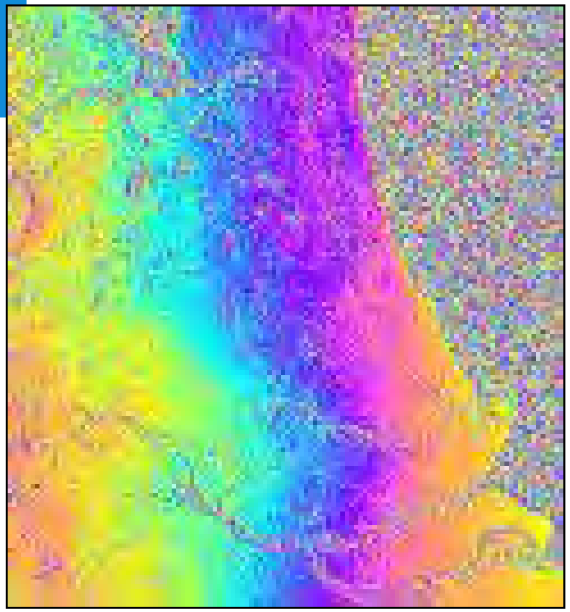


2 mm/s²

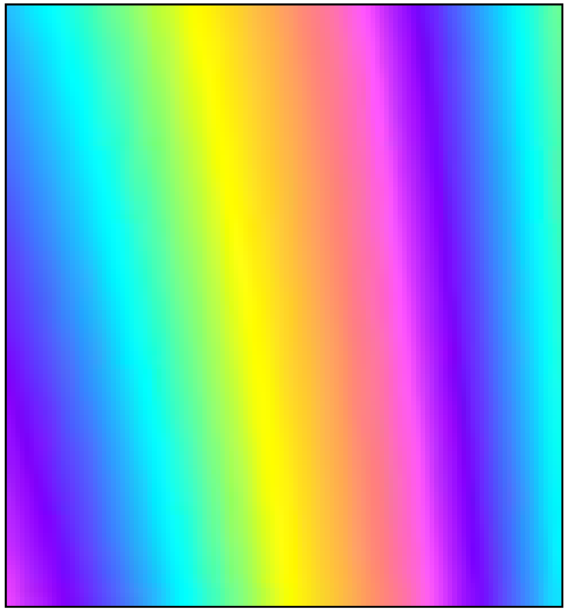
Courtesy Herman Baehr

TU Delft

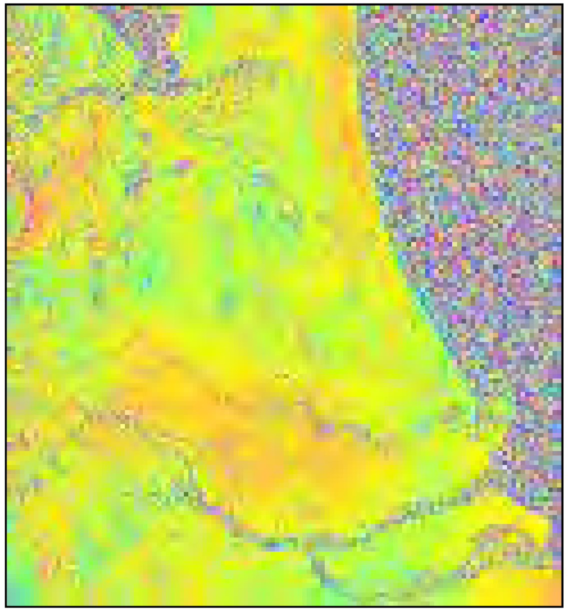
# Orbit Error Correction



Interferogram



Orbit Correction Estimate



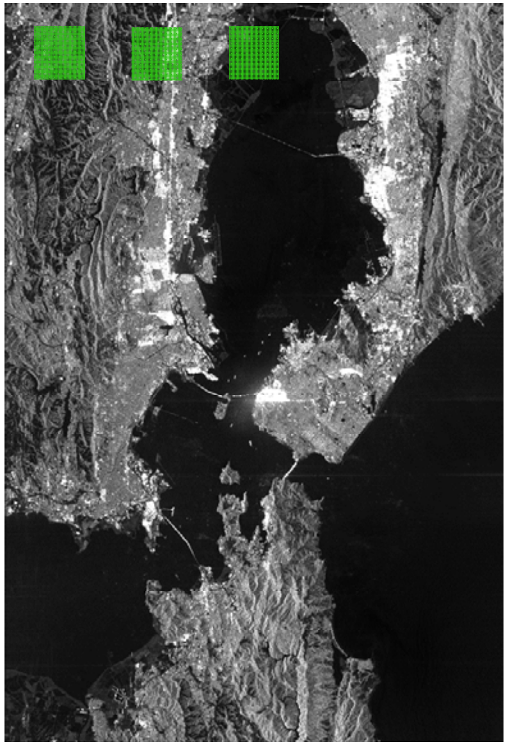
Remaining Phase

Courtesy Herman Baehr

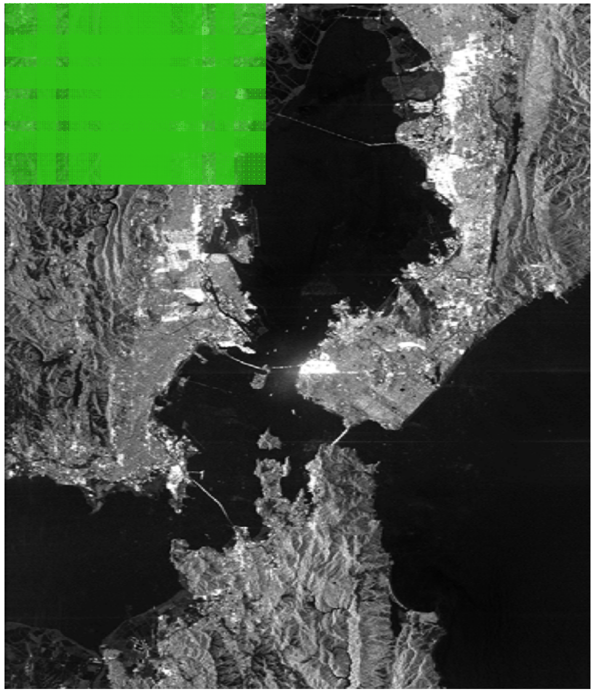
# InSAR data processing

# Coregistration

- **Sampling is different for the two acquisitions**



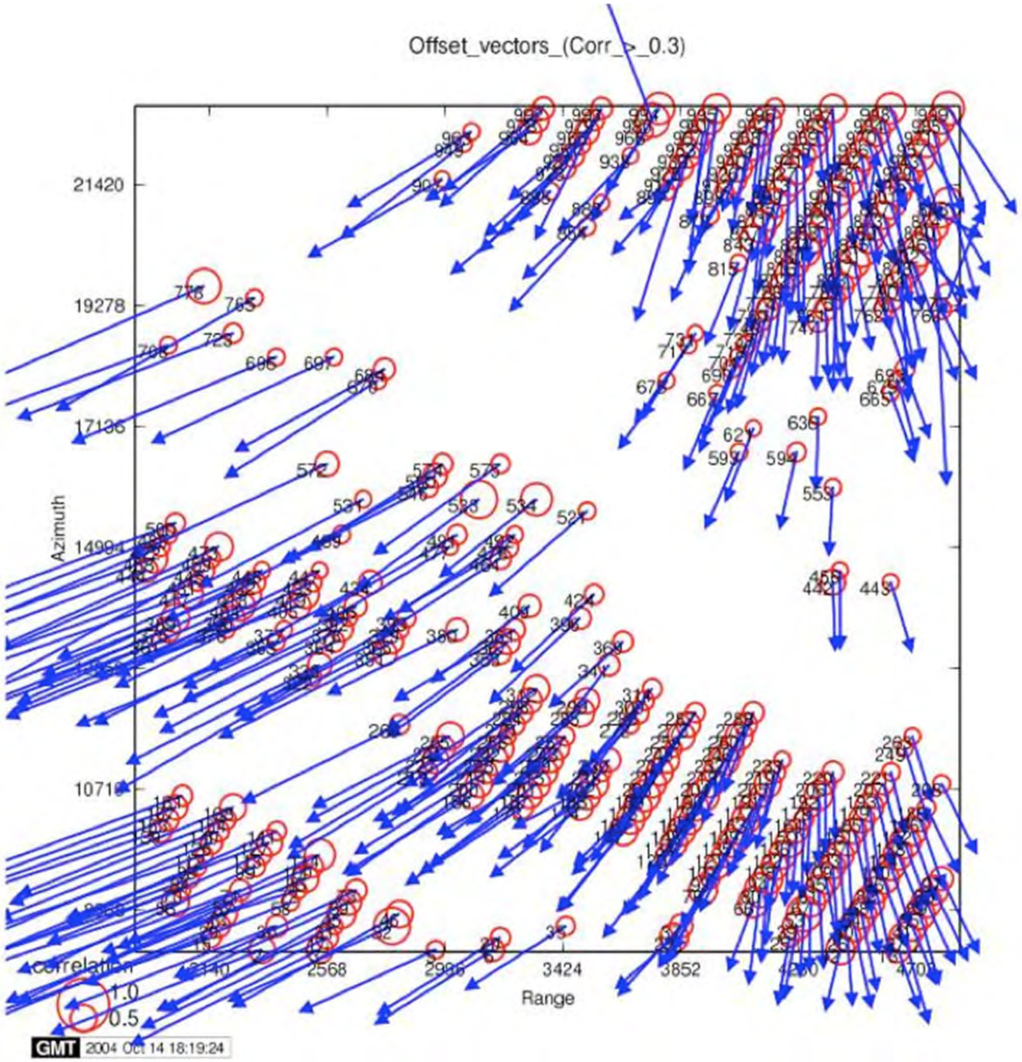
Master



Slave

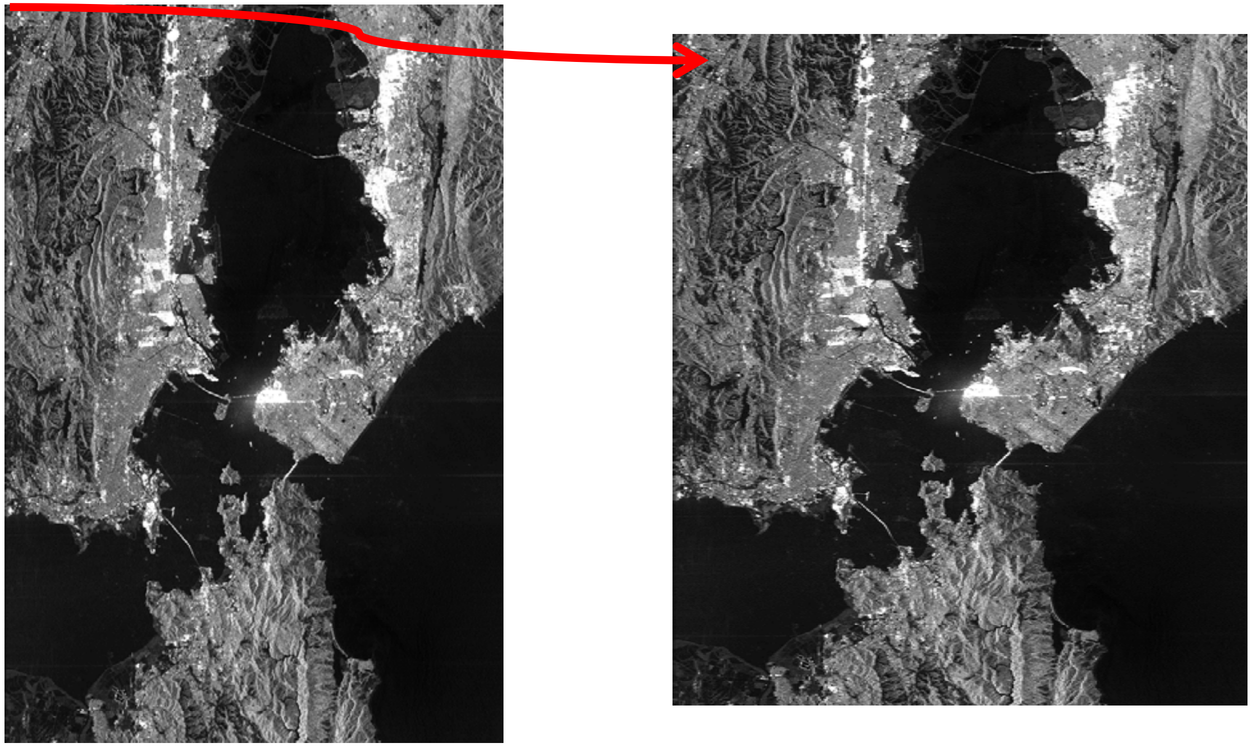
- **Use amplitude cross-correlation**

# Master-Slave Offsets




Fit a polynomial,
and remove outliers

# Resampling

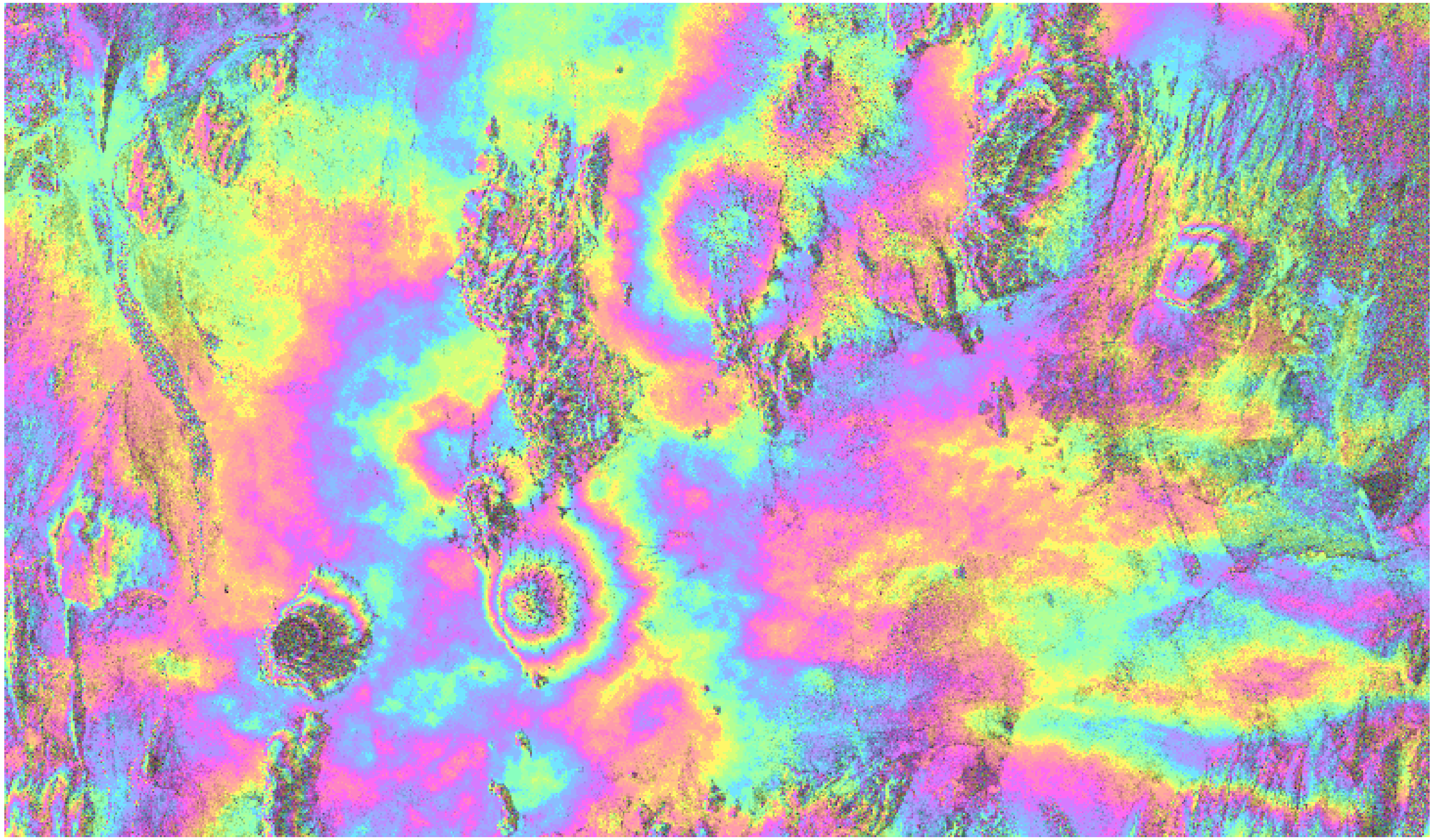


- Use polynomial to calculate position of each master pixel in slave
- Interpolate value in slave

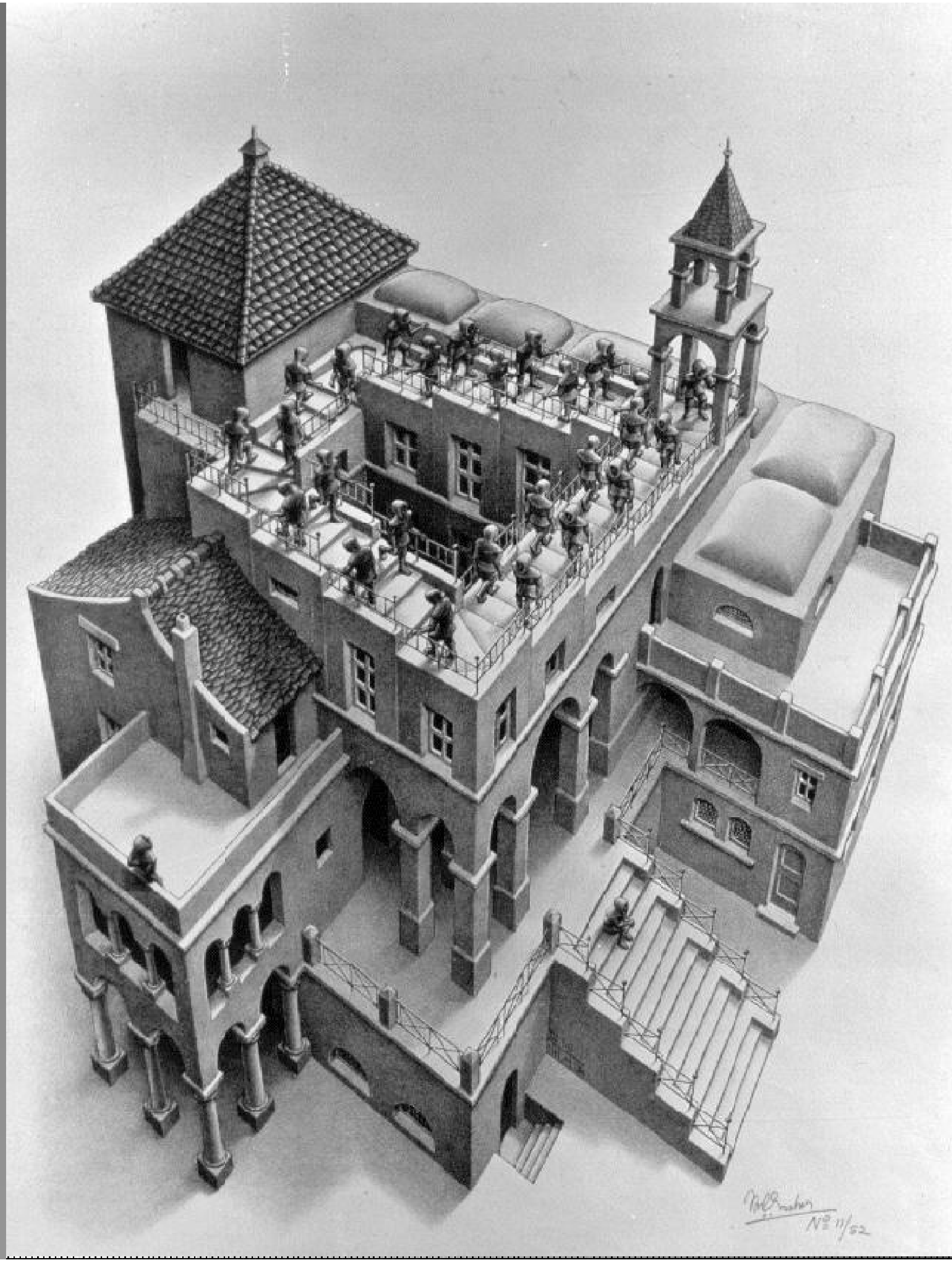
TU Delft

# Phase ambiguity estimation

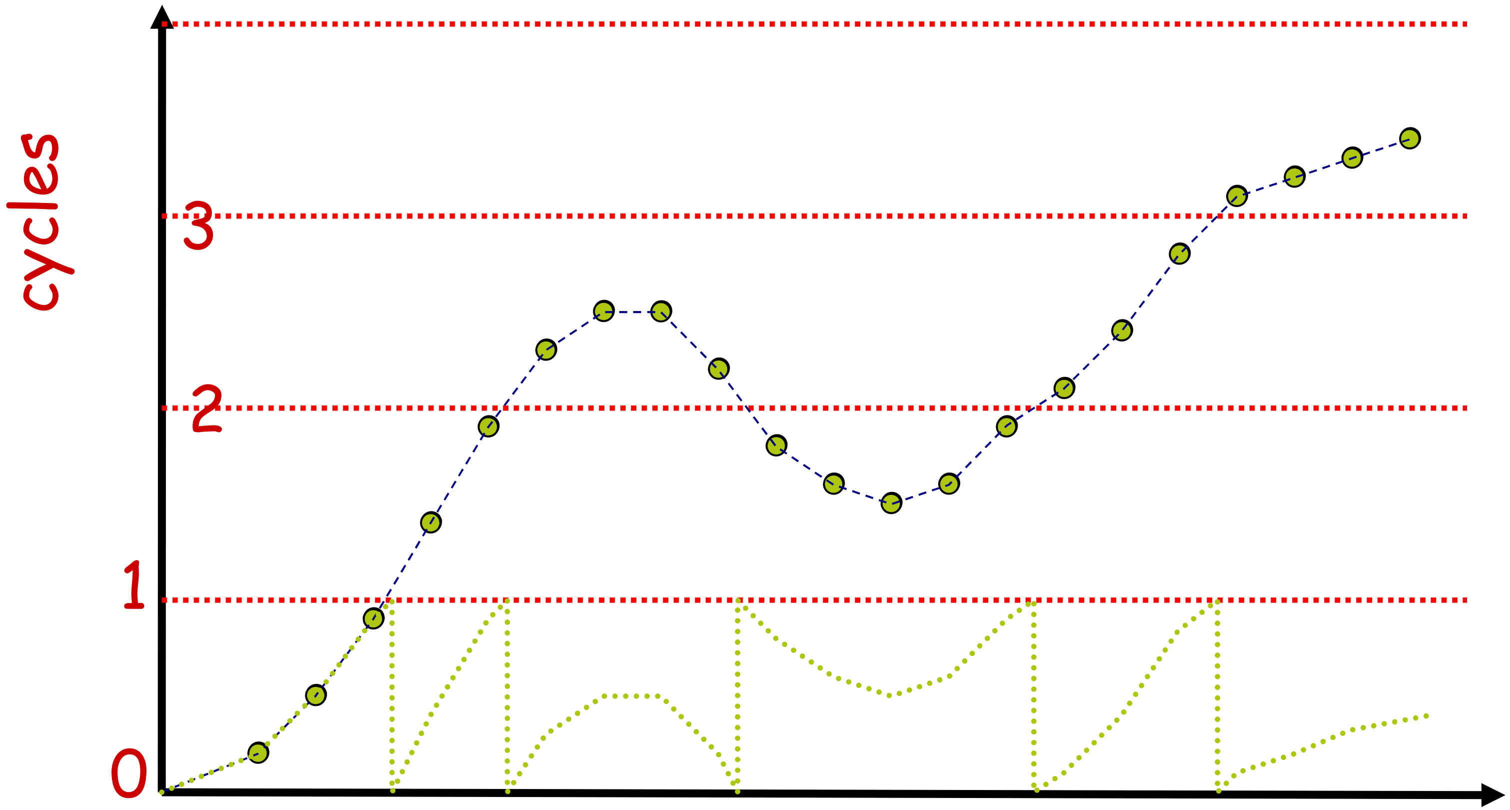
(AKA Phase unwrapping. Essentially means counting fringes)



September 17, 2015



# Unwrapping Phase Images




- Unwrapping involves finding (integer) *shift* values for each point.
- Unwrapping is simple in one dimension
  - One path through data
  - Use local gradient to estimate shift.
- For 2D images, the problem is more difficult (NP-hard)
  - Many paths through the data
  - Shifts along all paths must be consistent

# General approach

- Strictly: phase unwrapping is ill-posed problem (not possible to obtain unique solution)
- Heuristic approach: assumption of Nyquist criterion:
  sampling rate is high enough to avoid aliasing
- In other words:
  True (unwrapped) phase values of neighboring pixels assumed to lie with one-half cycle

Heuristic approach: A method based on empirical information that has no explicit rationalization

# Forward problem

- Define the Wrapping operator:

$$\psi = W\{\phi\} = \mathrm{mod}\{\phi + \pi, 2\pi\} - \pi \qquad \in [-\pi, \pi).$$

# Inverse problem

- Main condition for wrapped phase gradients:

$$|\Delta\psi(x)| = |\,\psi(x+1) - \psi(x)\,| < \pi$$

- Phase unwrapping is the integration of phase gradients

# One-dimensional example

Nyquist criterion: phase differences between adjacent samples are element of [-0.5, 0.5) cycles

Wrapped data (modulo 1 cycle):

$\psi(x)$: 0.5 0.75 0.0 0.25 0.5 0.75 0.0 0.25

Gradient:

$\Delta\psi(x)$: 0.25 -0.75 0.25 0.25 0.25 -0.75 0.25

Wrapped differences of wrapped phases:

+1 +1

$\overset{\triangle}{\nabla}\psi(x)=W\{\Delta\psi(x)\}$: 0.25 0.25 0.25 0.25 0.25 0.25 0.25

$\nabla\varphi = \overset{\triangle}{\nabla}\psi(x)$

Integration:

$\varphi(x)$: 0.5 0.75 1.0 1.25 1.5 1.75 2.0 2.25

Possible solution that violates Nyquist criterion:

-0.5 -0.25 1.0 1.25 1.5 1.75 2.0 2.25

+1.25 !

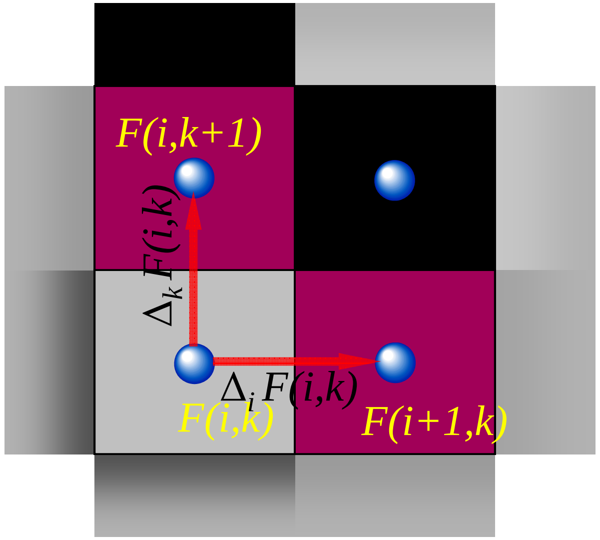
- Key to phase unwrapping:
  - not: directly estimating unwrapped phase, but...
  - Estimating the phase differences between them (phase gradients)

- Problems occur due to additive phase noise (decorrelation) or high spatial frequency phase variation

# 2D phase unwrapping

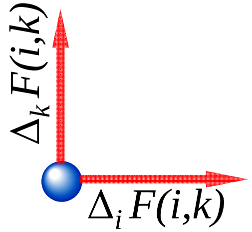
define discrete equivalents to partial derivatives of a function $F$ as

$$\Delta_i F(i,k) = F(i+1,k) - F(i,k)$$
$$\Delta_k F(i,k) = F(i,k+1) - F(i,k)$$




and compact them into *gradient* notation:

$$\nabla F(i,k) = \begin{pmatrix} \Delta_i F(i,k) \\ \Delta_k F(i,k) \end{pmatrix}.$$



Suppose 2D vector field $\boldsymbol{A} = \begin{pmatrix} A_i \\ A_k \end{pmatrix}$

Definition of *curl :*

$$\begin{pmatrix} \Delta_i \\ \Delta_k \end{pmatrix} \times \begin{pmatrix} A_i(i,k) \\ A_k(i,k) \end{pmatrix} = \Delta_i A_k(i,k) - \Delta_k A_i(i,k)$$
$$= \left[A_k(i+1,k) - A_k(i,k)\right] - \left[A_i(i,k+1) - A_i(i,k)\right]$$
$$= A_k(i+1,k) - A_k(i,k) - A_i(i,k+1) + A_i(i,k)$$

Suppose 2D vector field $\boldsymbol{A} = \begin{pmatrix} A_i \\ A_k \end{pmatrix}$

$$\nabla F(i,k) = \begin{pmatrix} \Delta_i F(i,k) \\ \Delta_k F(i,k) \end{pmatrix}$$

Definition of *curl :*

$$\begin{pmatrix} \Delta_i \\ \Delta_k \end{pmatrix} \times \begin{pmatrix} A_i(i,k) \\ A_k(i,k) \end{pmatrix} = \Delta_i A_k(i,k) - \Delta_k A_i(i,k)$$
$$= \left[A_k(i+1,k) - A_k(i,k)\right] - \left[A_i(i,k+1) - A_i(i,k)\right]$$
$$= A_k(i+1,k) - A_k(i,k) - A_i(i,k+1) + A_i(i,k)$$

Assume that $\boldsymbol{A} = \nabla F$

From vector analysis (and potential field theory) it is known that the curl of a gradient field is equal to zero. The gradient field is therefore a **conservative field**.

$$\nabla \times \nabla F = 0$$

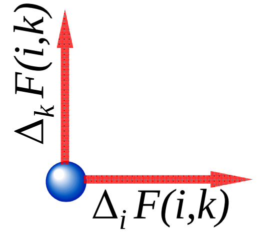
$$\nabla \times \nabla F =$$

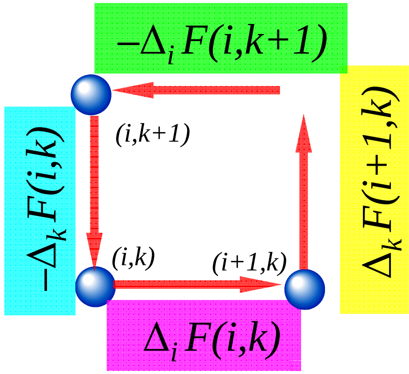
$$\begin{pmatrix} \Delta_i \\ \Delta_k \end{pmatrix} \times \begin{pmatrix} \Delta_i F(i,k) \\ \Delta_k F(i,k) \end{pmatrix} = \Delta_i \Delta_k F(i,k) - \Delta_k \Delta_i F(i,k)$$

$$= [\Delta_k F(i+1,k) - \Delta_k F(i,k)] - [\Delta_i F(i,k+1) - \Delta_i F(i,k)]$$

$$= \Delta_k F(i+1,k) - \Delta_k F(i,k) - \Delta_i F(i,k+1) + \Delta_i F(i,k) = 0$$

$\nabla F$ :




$-\Delta_i F(i,k+1) = -[F(i+1,k+1) - F(i,k+1)]$

$\Delta_k F(i+1,k) = [F(i+1,k+1) - F(i+1,k)]$

$\Delta_i F(i,k) = F(i+1,k) - F(i,k)$

$-\Delta_k F(i,k) = -[F(i,k+1) - F(i.k)]$

Curl of vector gradient of scalar potential $F$ is identically zero.

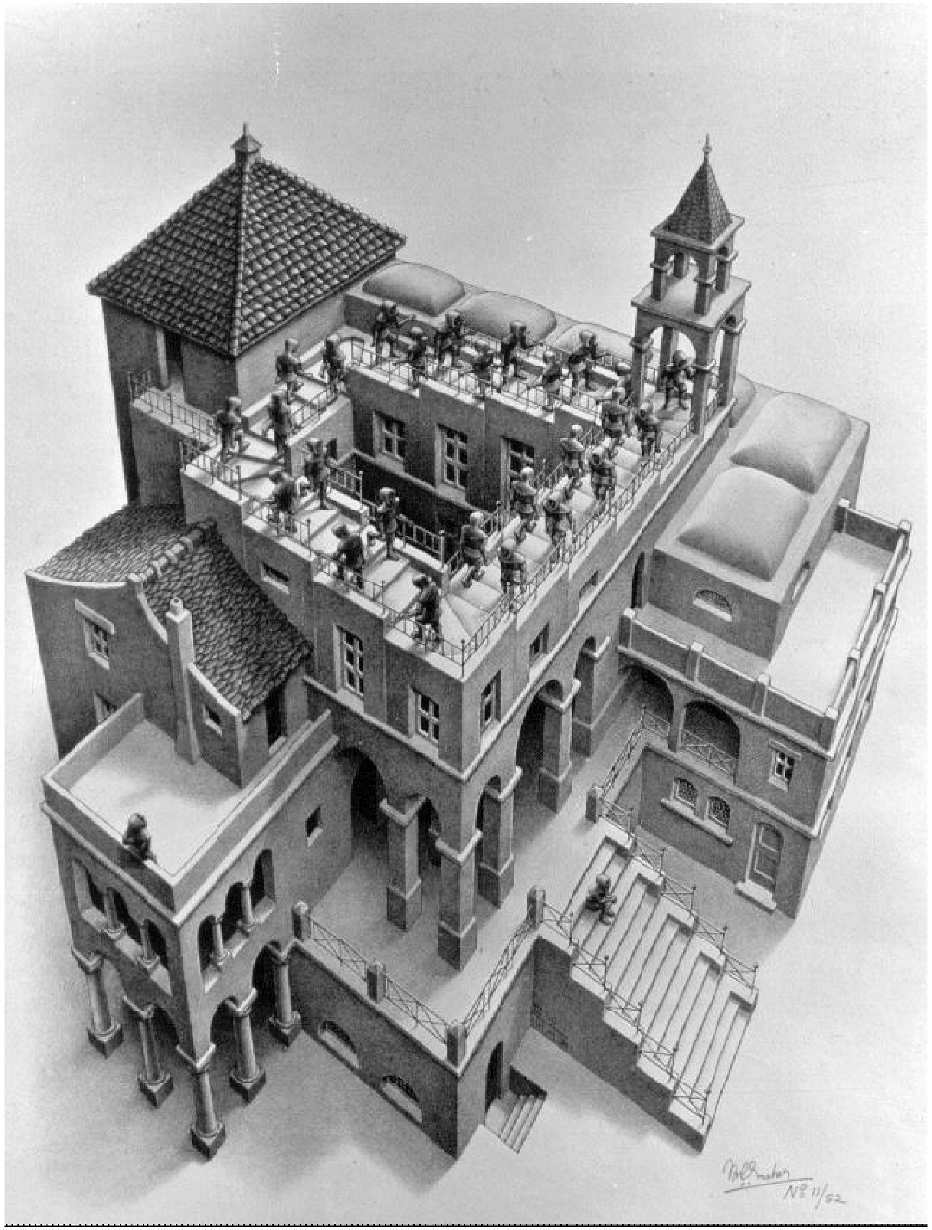
- In phase unwrapping: the vector gradient field of the *unwrapped phase* is necessarily zero (every closed loop integral is zero)

$$\nabla \times \nabla F = 0$$

- The unwrapped phase field is thus completely specified, up to an additive constant

- However, the vector gradient field of the wrapped phase can be non-conservative (closed-loop integrals can give non-zero results)

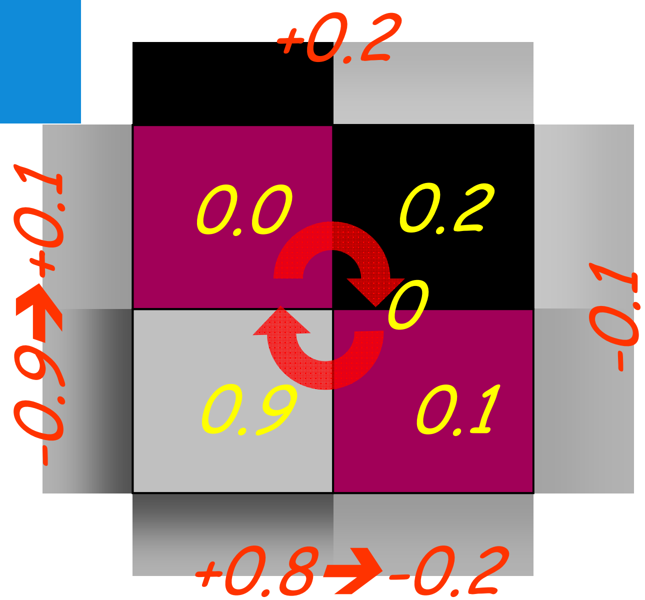
$$\nabla \times \hat{\nabla} \psi \neq 0$$

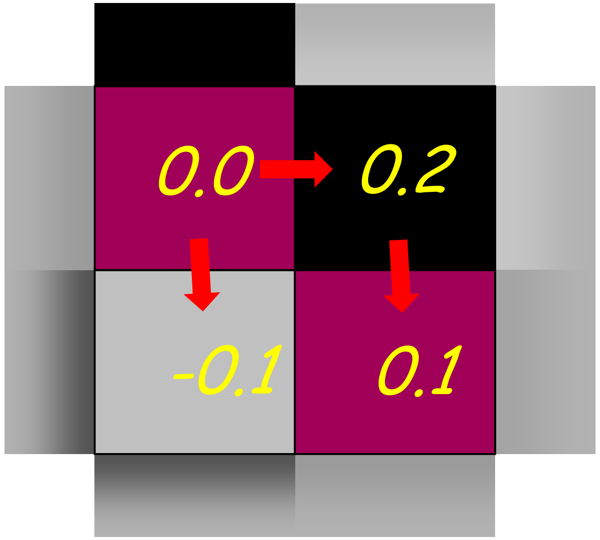


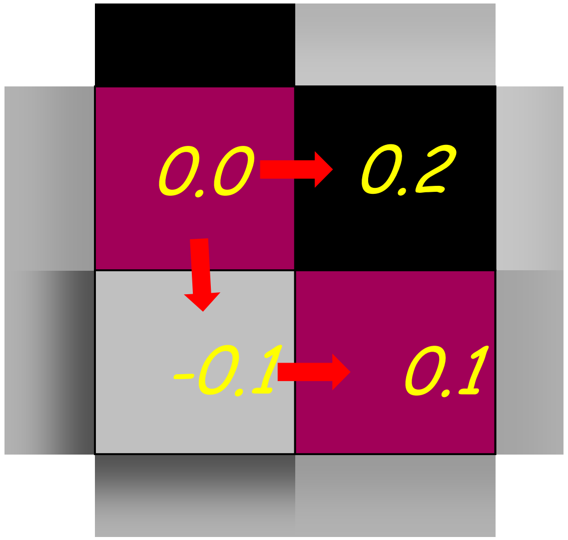
**Ascending and descending, by M.C. Escher**

e.g. a true gradient outside [−π,+π) interval will be wrapped into it

# Example



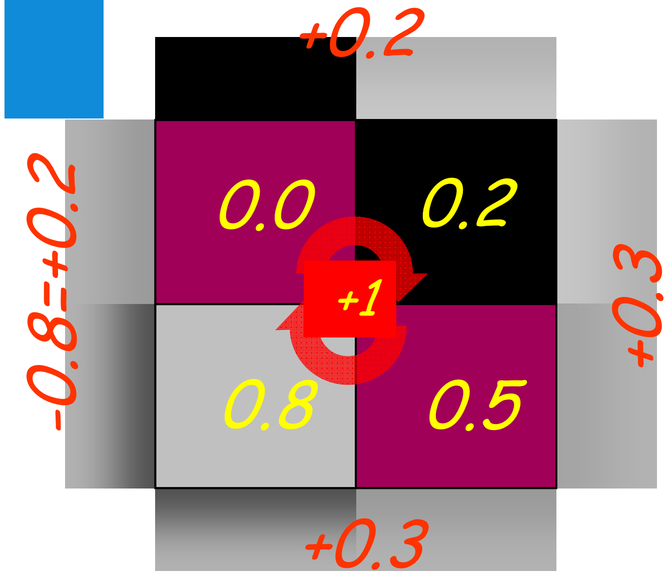



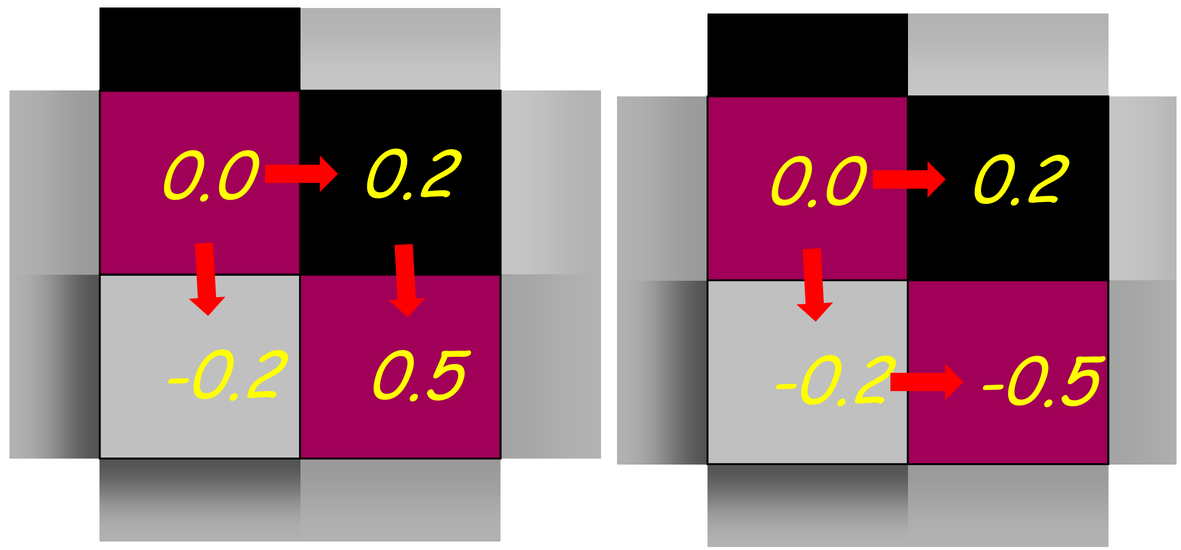

Neutral

Result is path independent

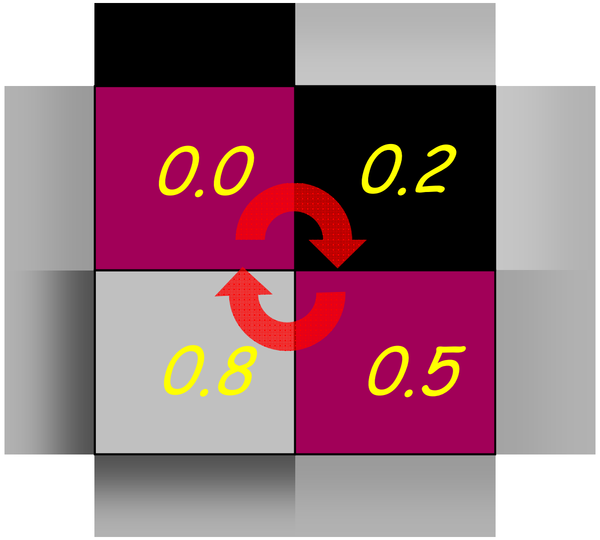
# Example residue



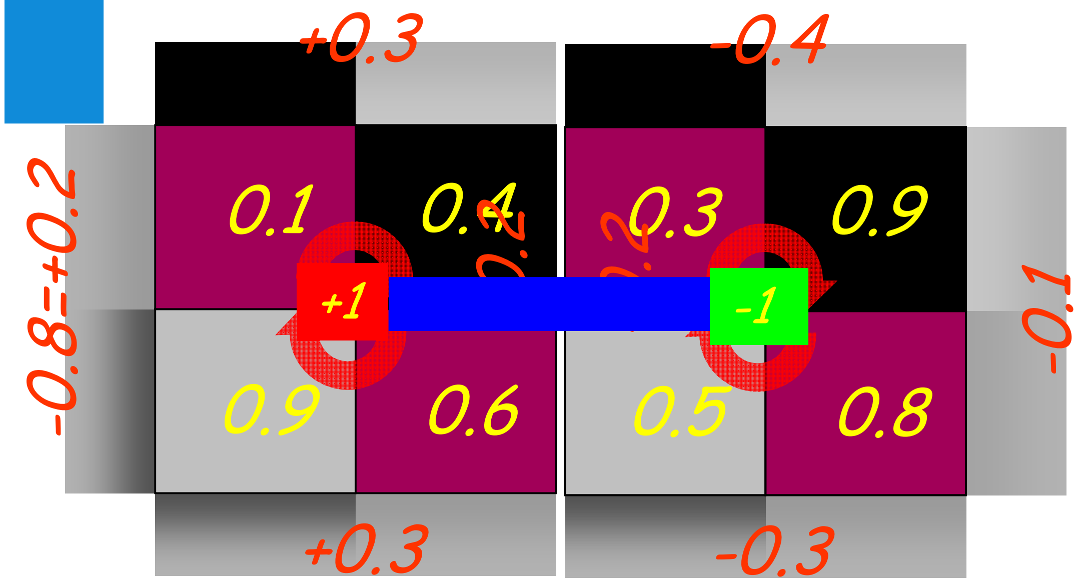

Positive residue




Result is path dependent


0.0
0.2
0.8
0.5

# Example branch cut




Unloaded residue pair

Positive residue Negative residue

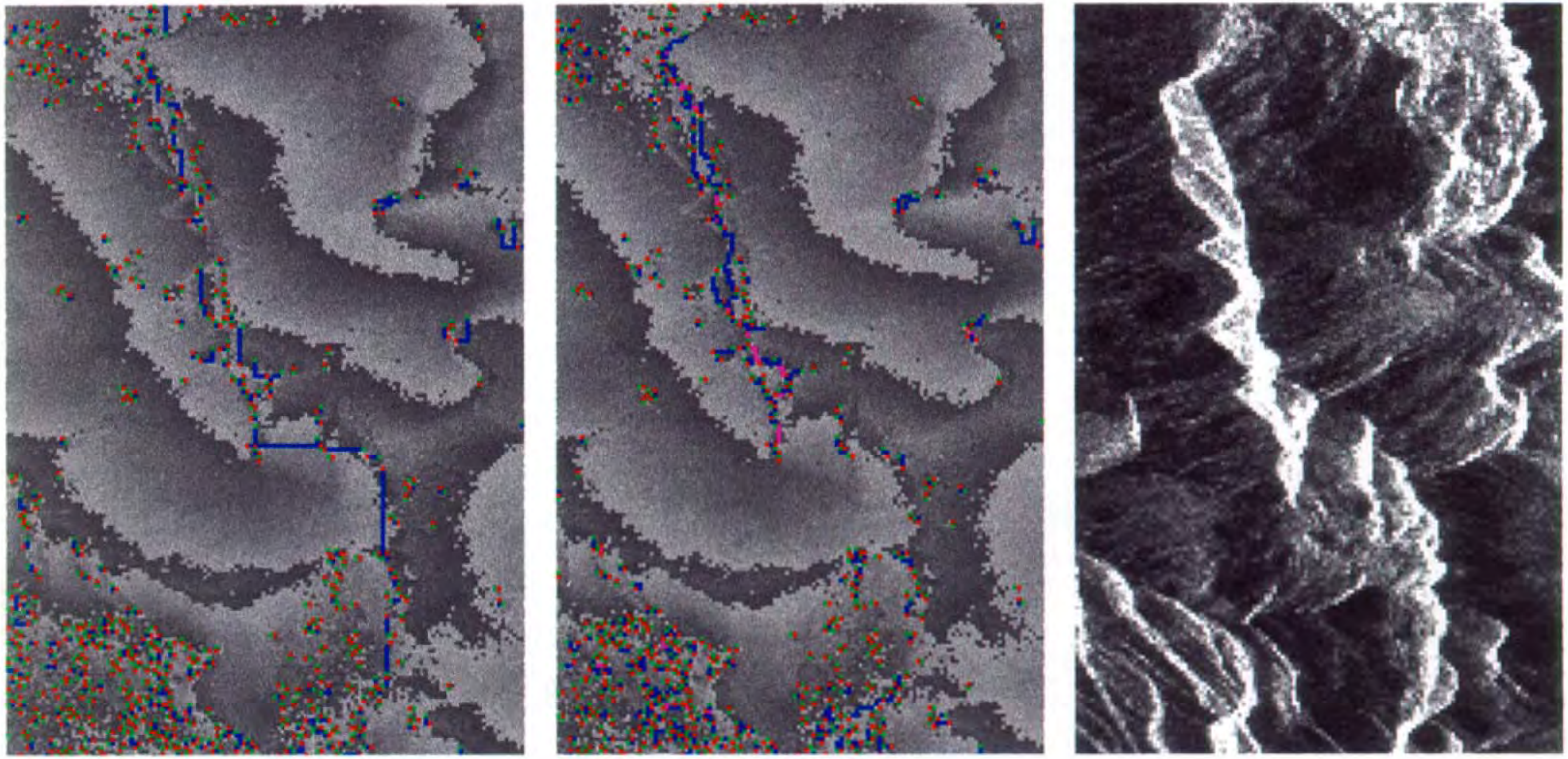


**Figure 11.** Wrapped phase (grey), residues (green, red), and branch-cuts (blue, multiple cuts: pink) found by a minimum cost flow algorithm. Left: minimization of total branch-cut length (constant costs); note the unrealistic long straight branch-cuts. Centre: minimization of a cost function derived from the phase gradient and its variance; the branch-cuts are guided along the ridges of the mountains (visible as bright areas in the intensity SAR image of the same area (right)).