

A short course on Altimetry

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with contributions by Peter Challenor, Ian Robinson, R. Keith Raney + some other friends...



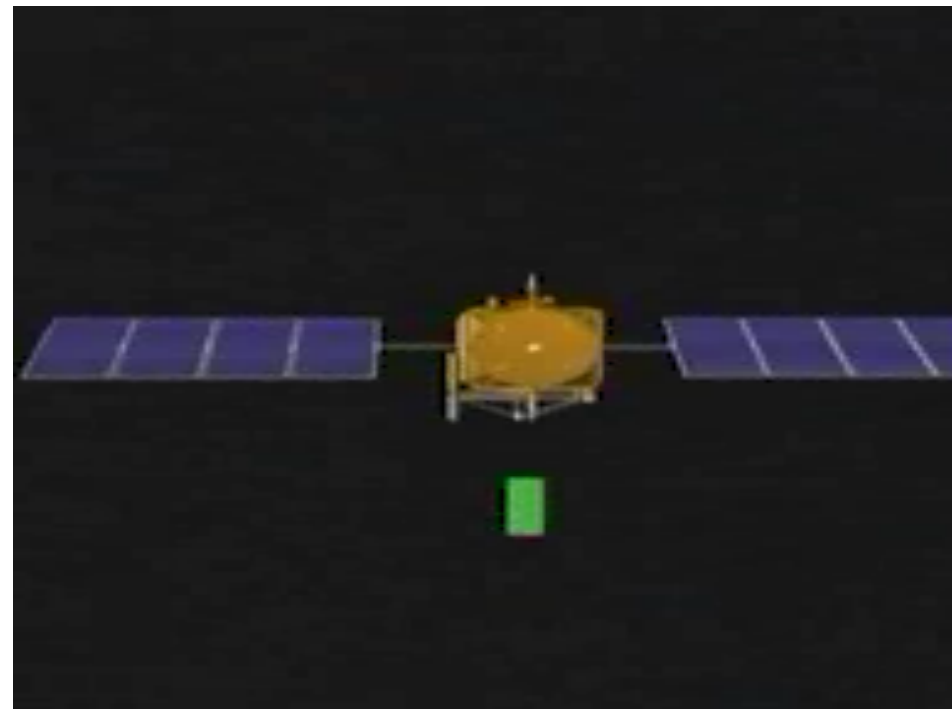
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Oceanography Centre**

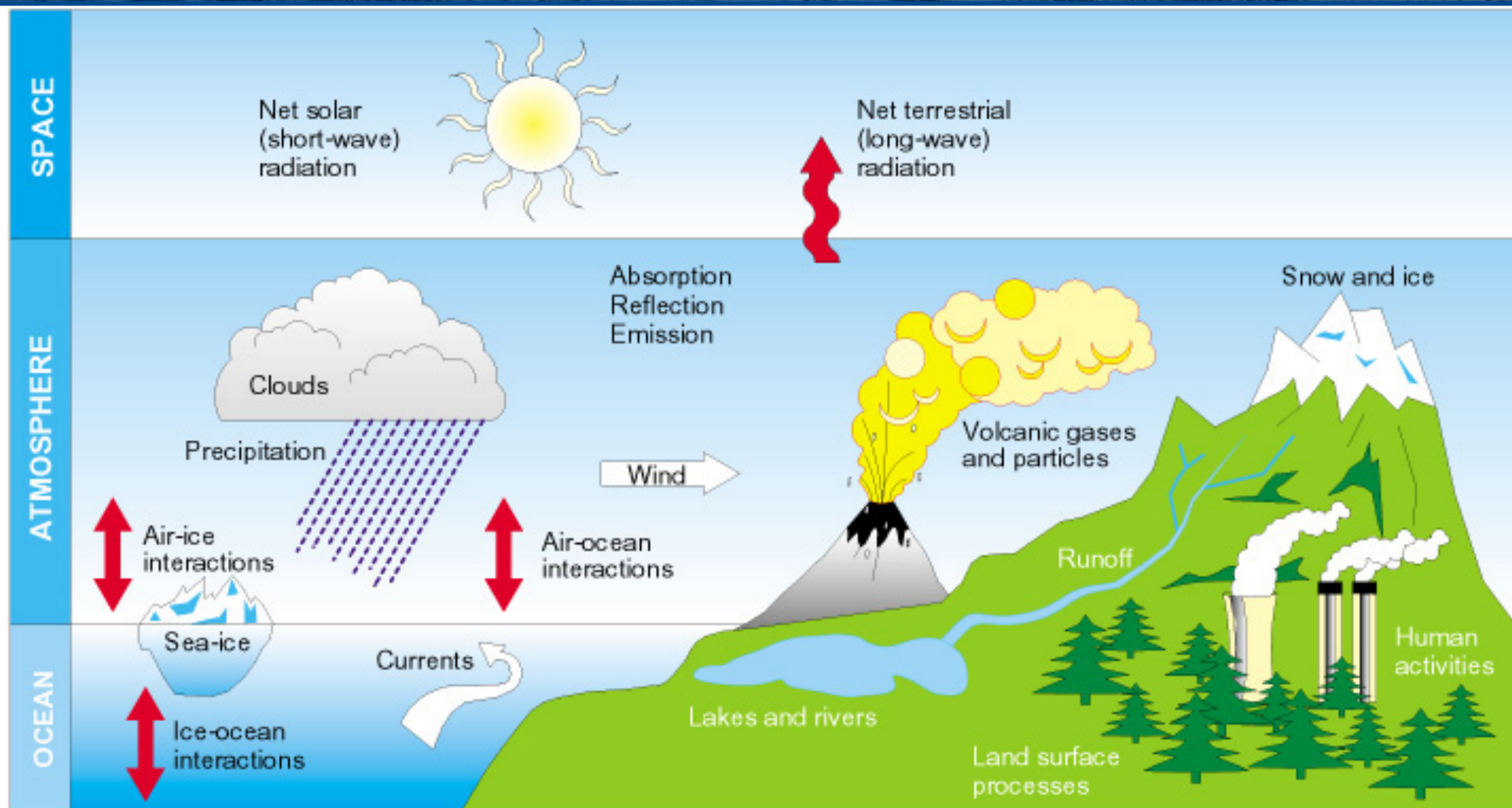
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- Rationale
 - why we need altimetry
- **A1** – Principles of altimetry
 - how it works in principle
 - New techniques
- **A2** – Altimeter Data Processing
 - From satellite height to surface height: corrections
 - (or how it is made accurate)
- **A3** – Altimetry and Oceanography: applications of altimetry over the ocean and coastal zone
 - what quantities we measure
 - how we use them!

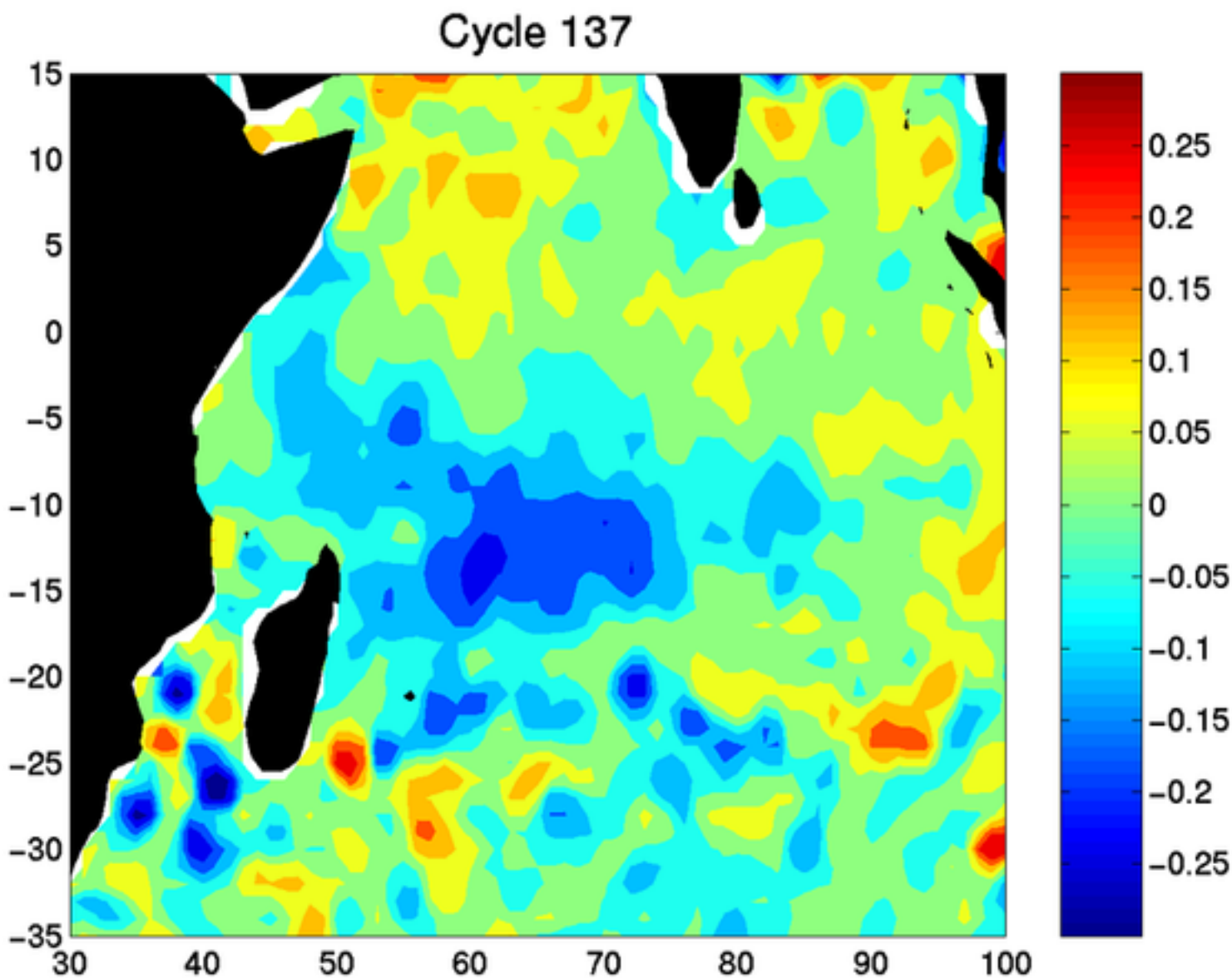
- **Climate change**
 - oceans are a very important component of the climate system
- Altimeters monitor **currents / ocean circulation**...
- ...that can be used to estimate **heat** storage and transport
- ... and to assess the interaction between **ocean and atmosphere**
- and **also sea level**, a global indicator of climate change
- We also get interesting by-products: **wind/waves, rain**





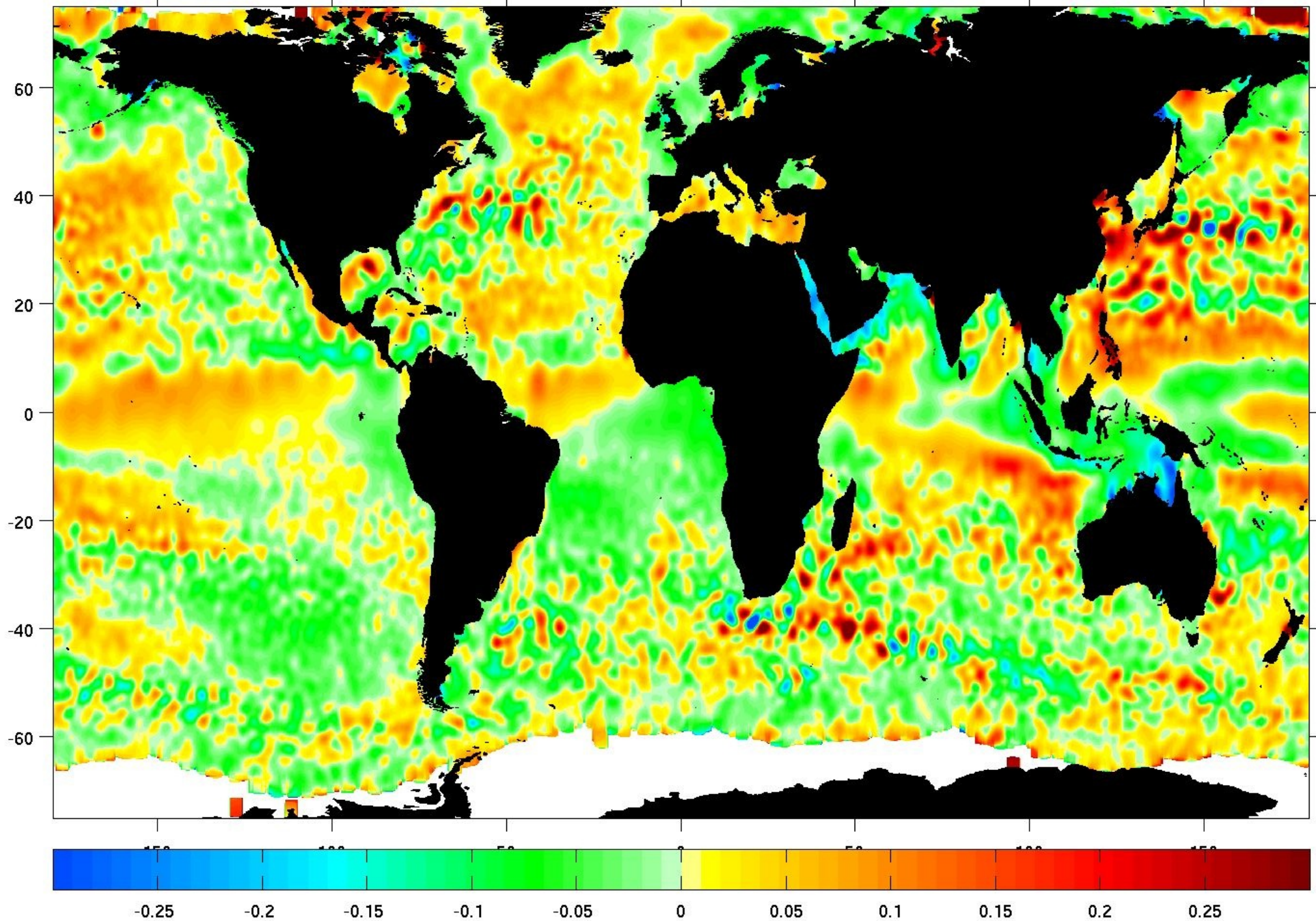
courtesy N. Noreiks, L. Bengtsson, MPI

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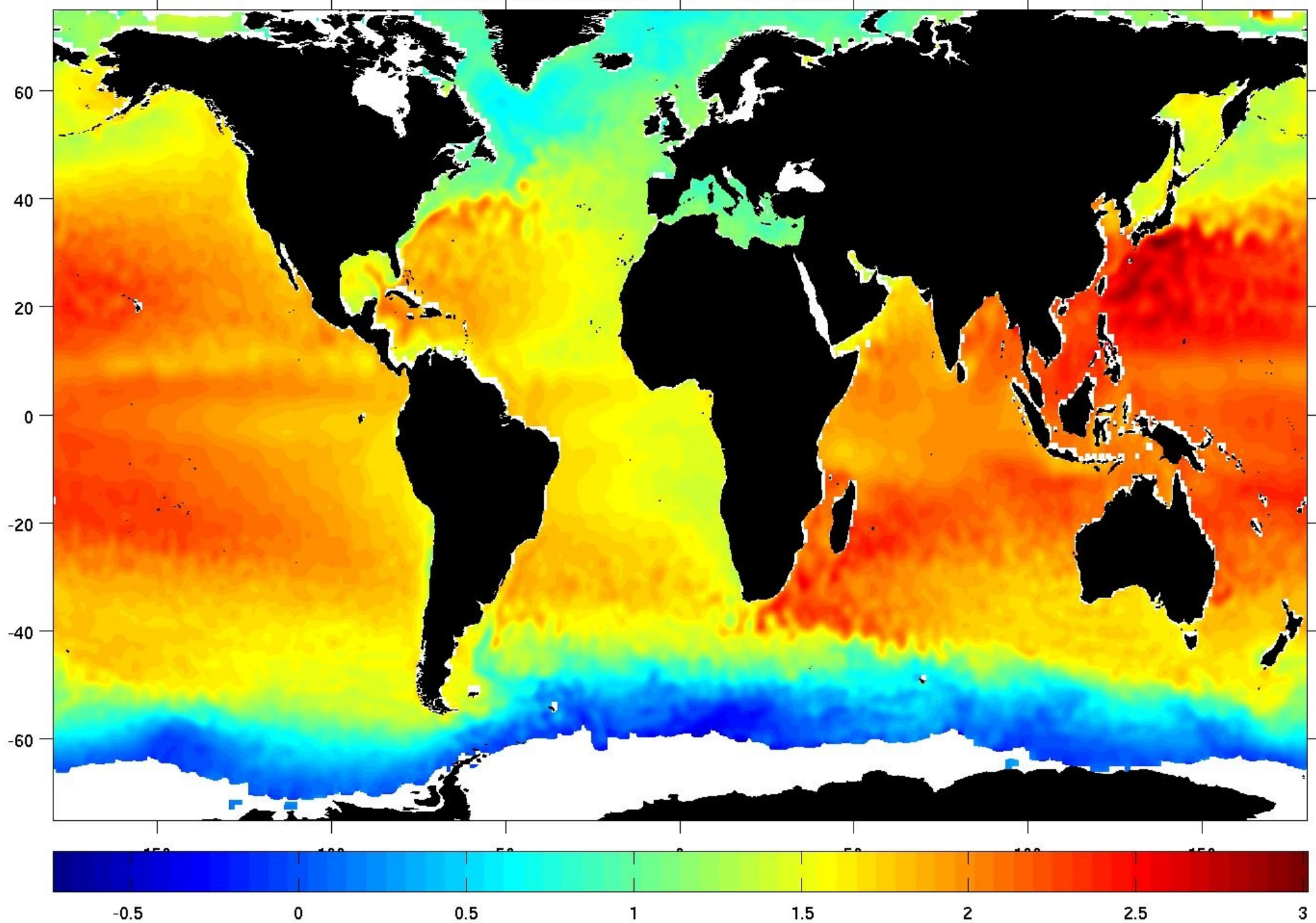
SEA LEVEL ANOMALY

Sea surface height anomaly (m), Envisat cycle 50



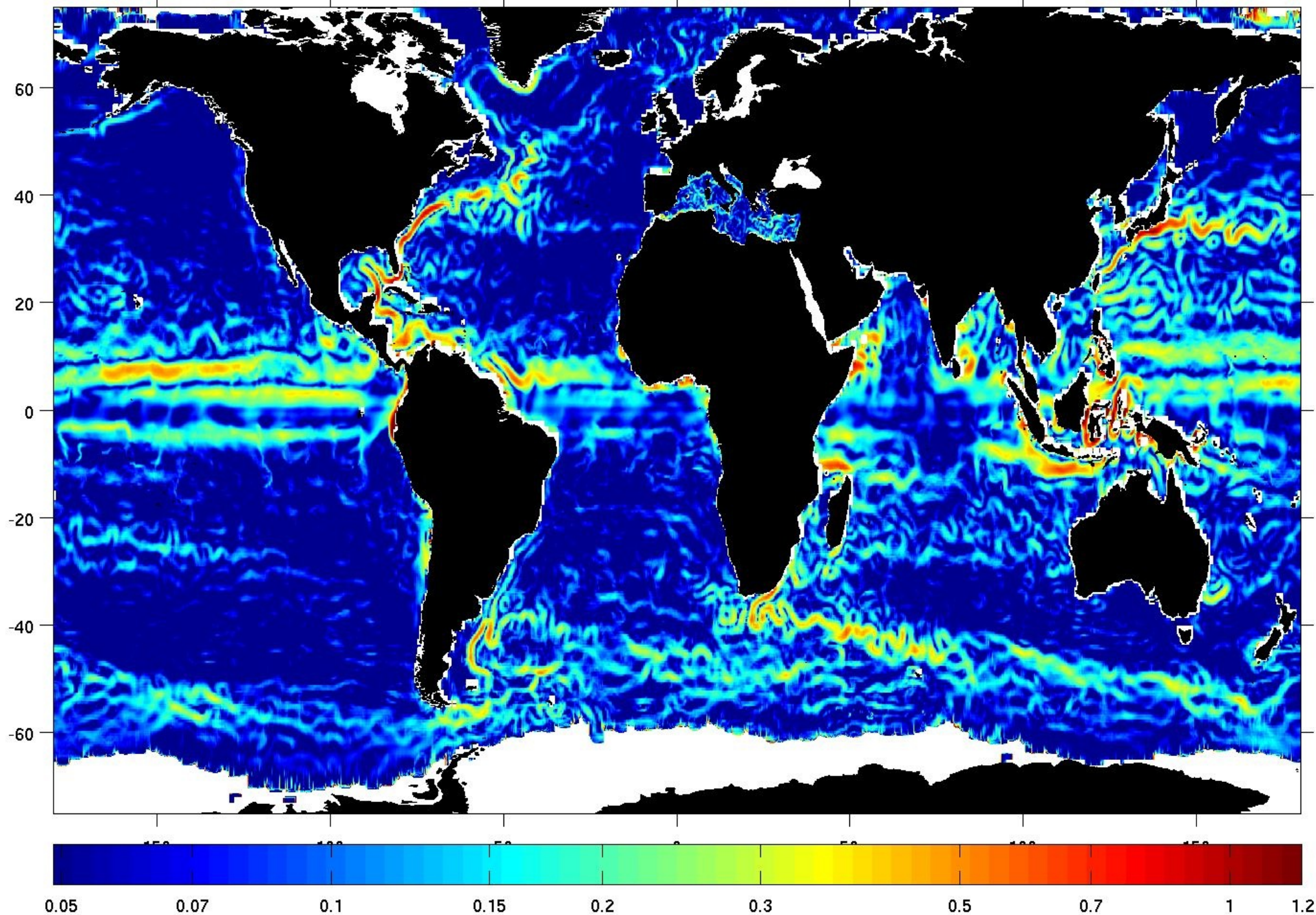
ABSOLUTE DYN TOPO

Absolute dynamic topography (m), Envisat cycle 50 + RIO05

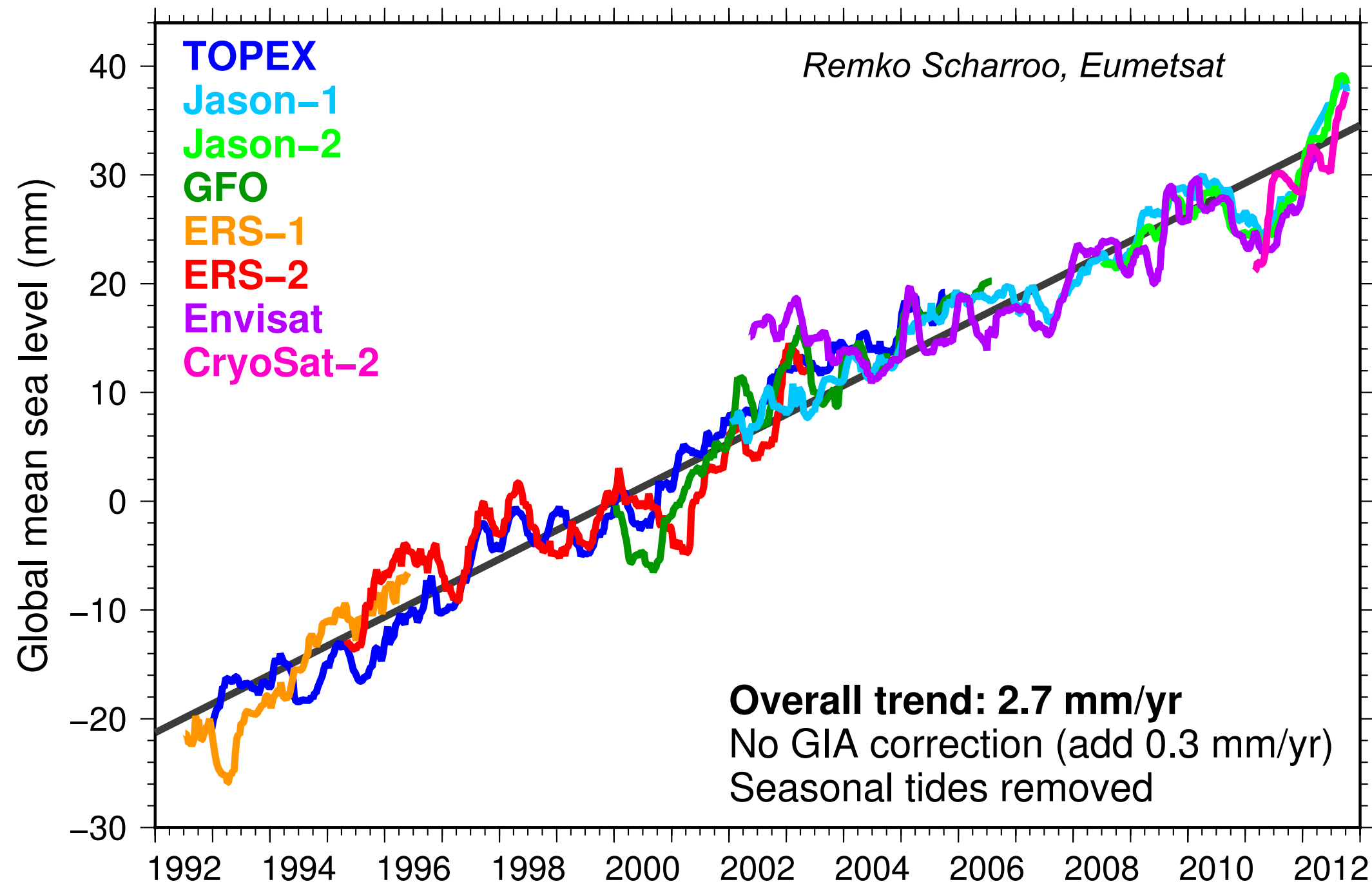


SURFACE CURRENTS

Geostrophic currents (m/s), Envisat cycle 50 + RIO05

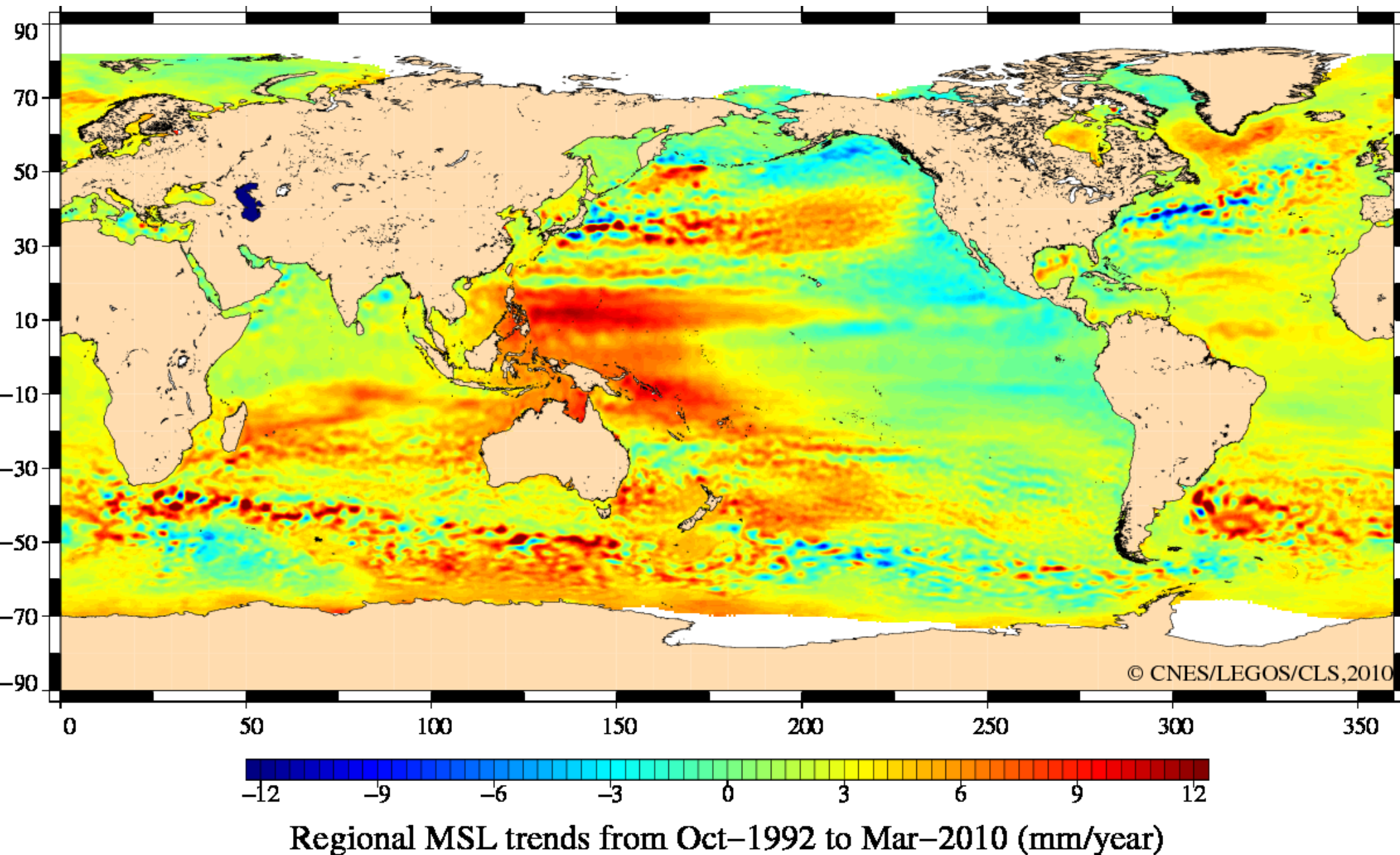


SEA LEVEL RISE - global



SEA LEVEL TRENDS - map

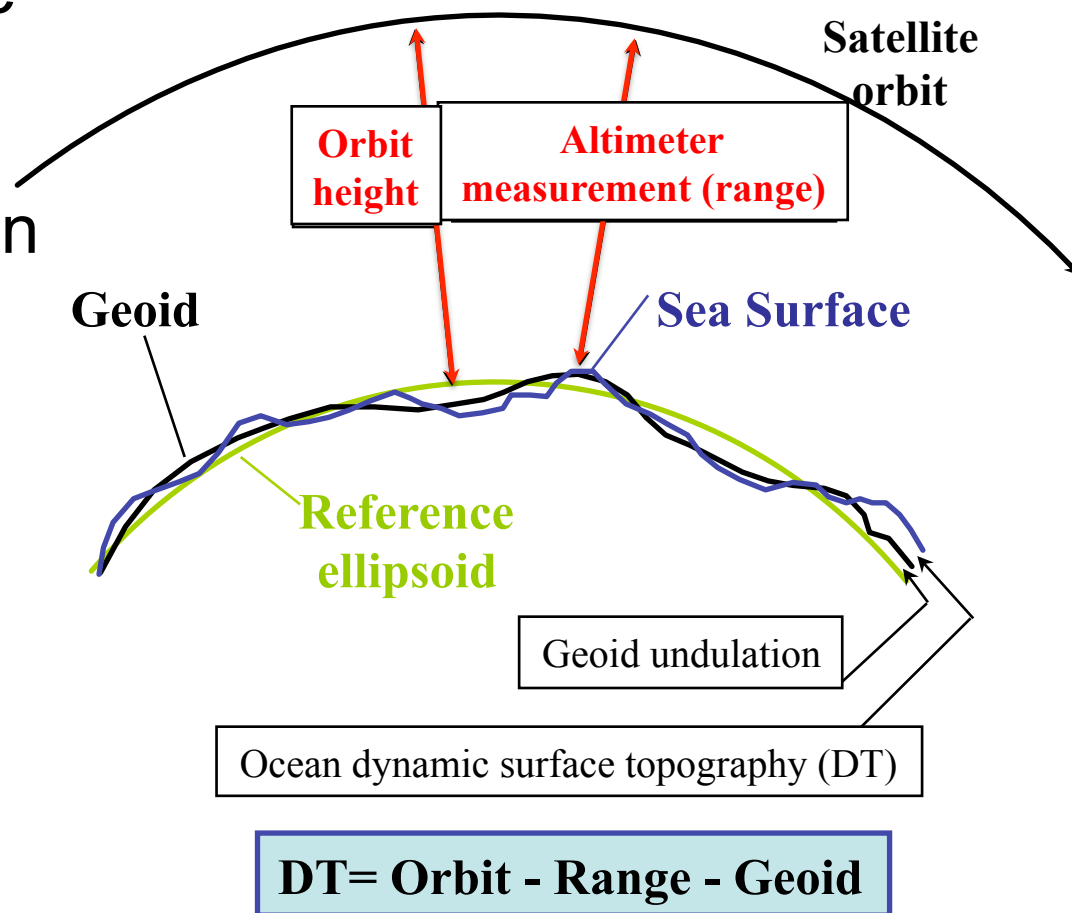
→ Sea Level component on dedicated ESA programme, the “Climate Change Initiative”



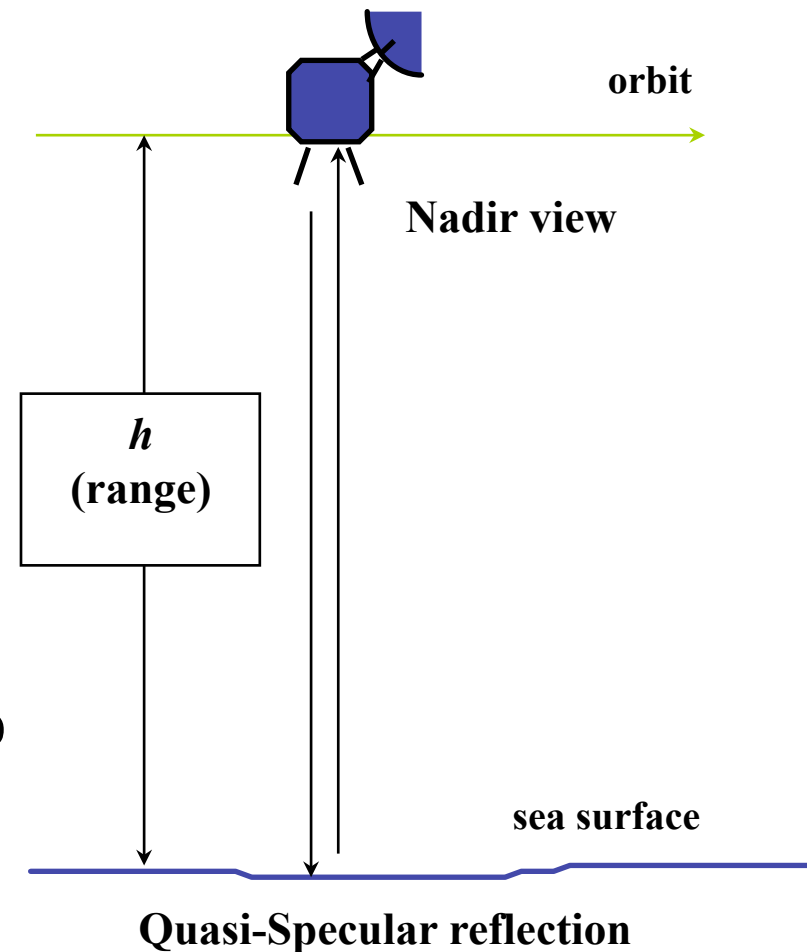
P. Cipollini, H. Snaith – A short course on Altimetry

Altimetry 1 – principles & instruments

- The altimeter is a radar at vertical incidence
- The signal returning to the satellite is from quasi-specular reflection
- Measure distance between satellite and sea (**range**)
- Determine position of satellite (precise **orbit**)
- Hence determine **height** of sea surface
- Oceanographers require height relative to **geoid**

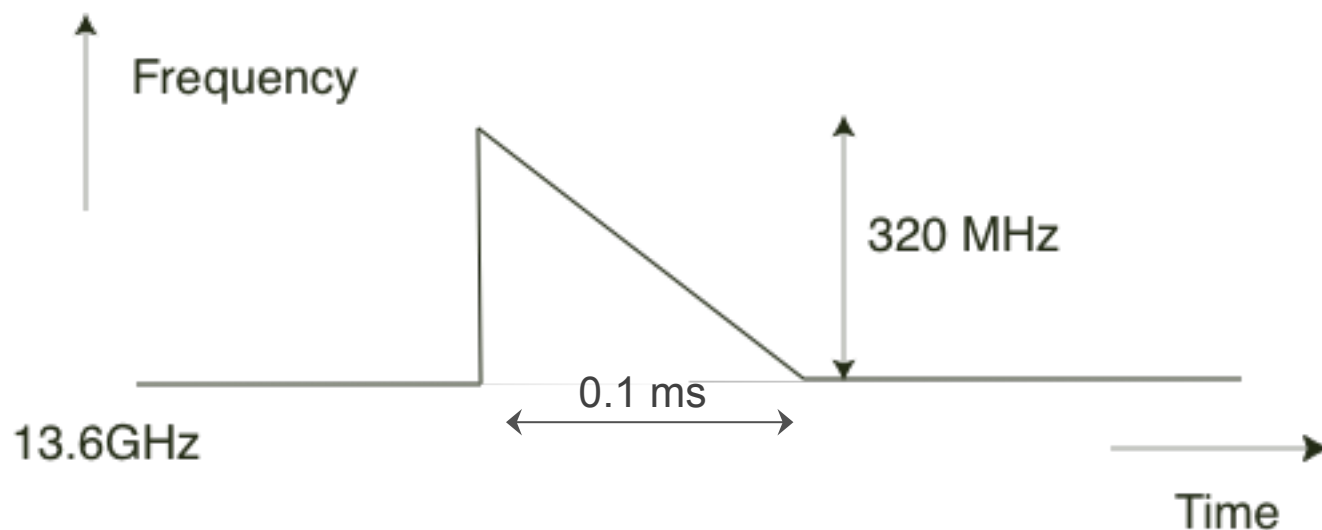


- Measure travel time, T , from emit to return
- $h = cT/2$ ($c \approx 3 \times 10^8$ m/s)
- **Resolution to ~5 cm** would need a single very narrow pulse of 3×10^{-10} s (0.3 nanoseconds)
- 0.3ns... and we would still need to pack in this pulse enough energy to give a discernible return...
- ...technically impossible!



- So we have to use two tricks:

1. **chirp** pulse compression

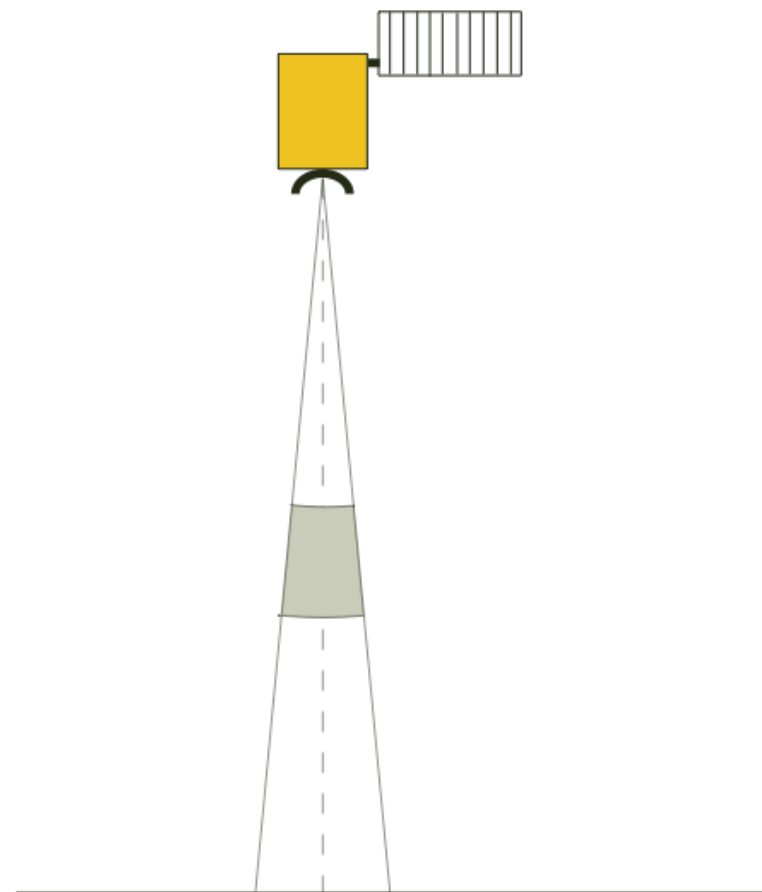


2. **average many pulses (typically ~1000)**

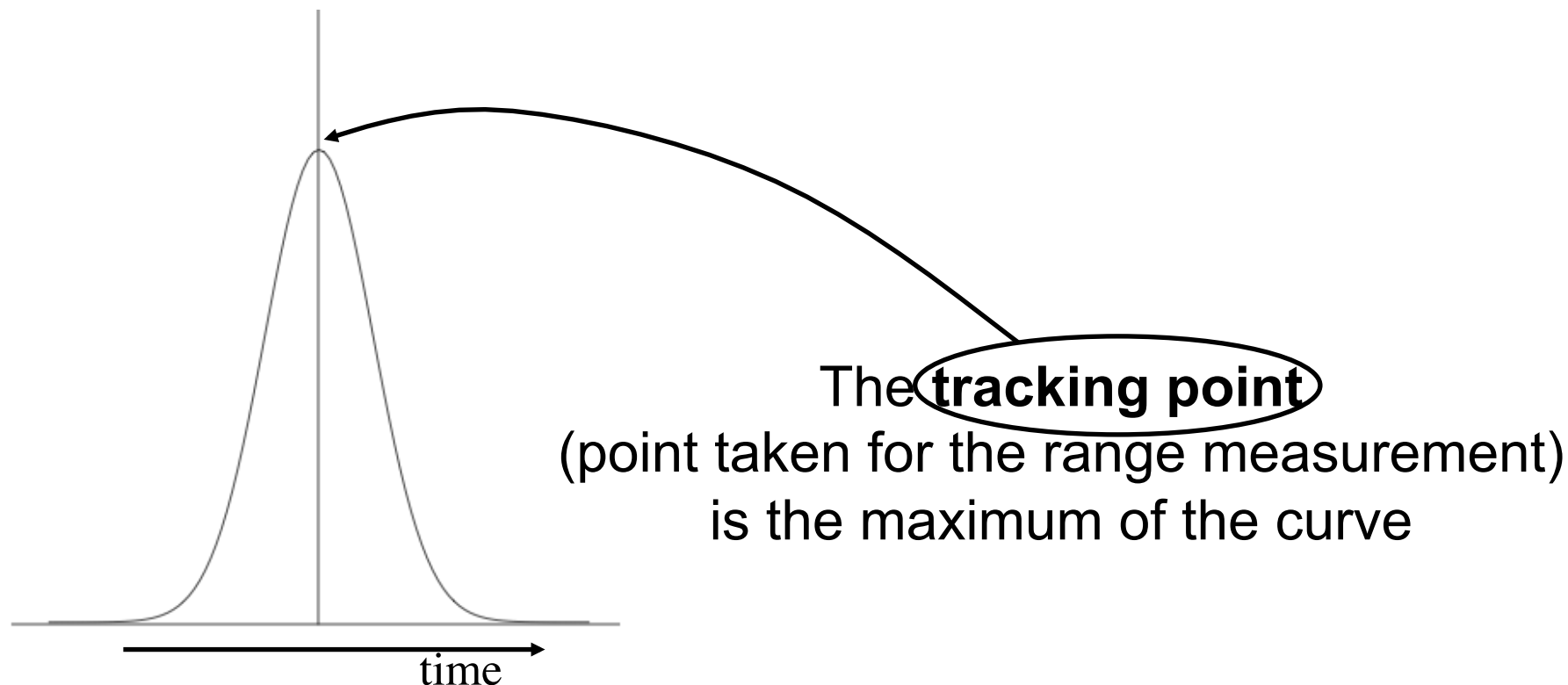
- It is also necessary to apply a number of corrections for atmospheric and surface effects

- In principle here are two types of altimeter:
 - beam-limited
 - pulse-limited

- Return pulse is dictated by the width of the beam
- but...IMPRACTICAL IN SPACE, for two reasons
 1. Narrow beams require large antennas: 5 m in Ku-band (13.5 GHz) for a 5-km footprint
 2. highly sensitive to mispointing of the platform

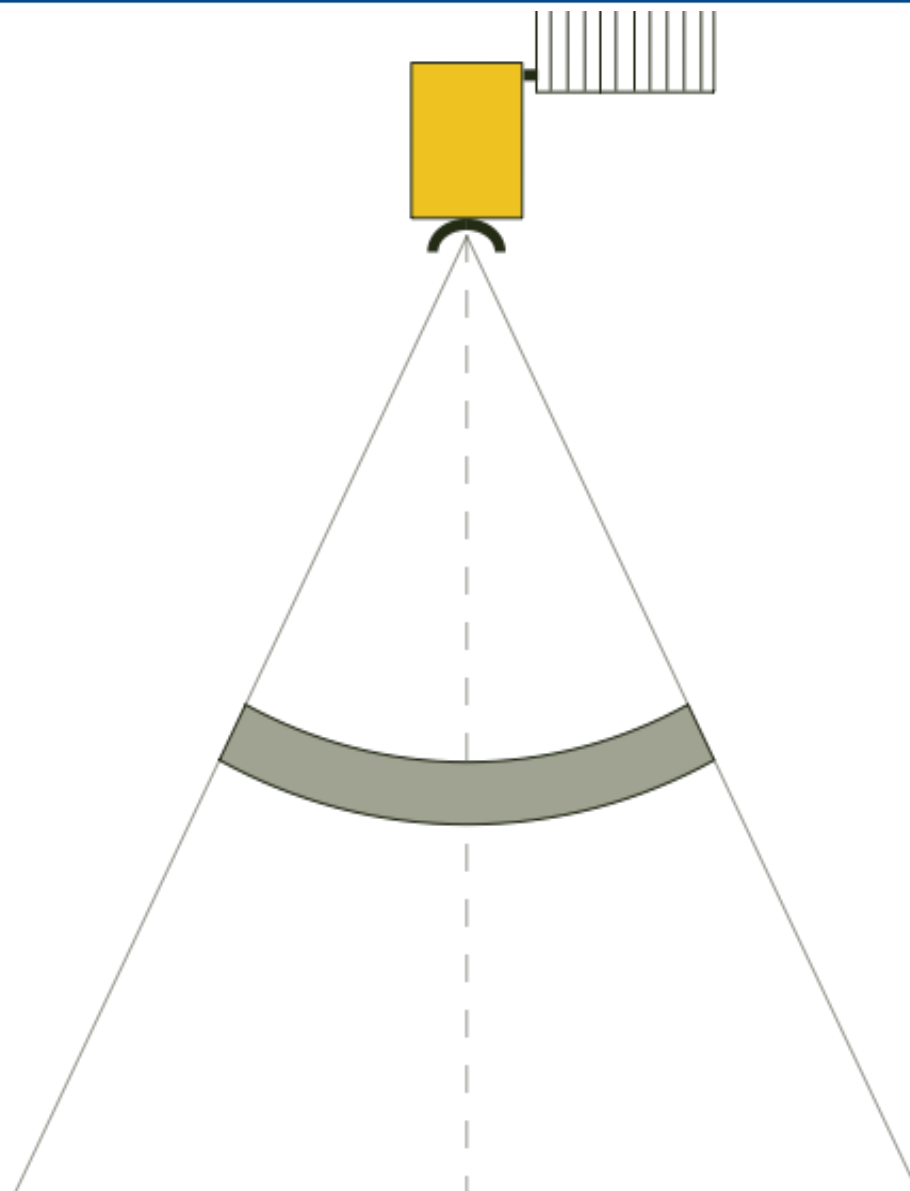


- A plot of return power versus time for a beam-limited altimeter looks like the *heights* of the specular points, i.e. the probability density function (pdf) of the specular scatterers

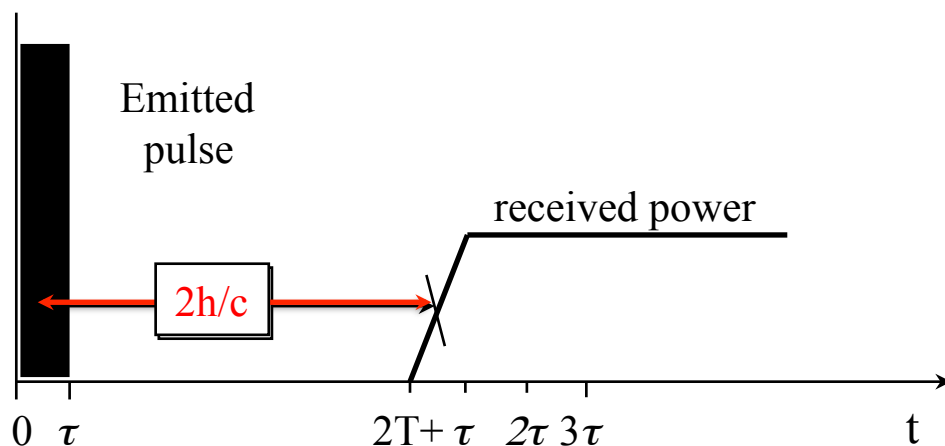
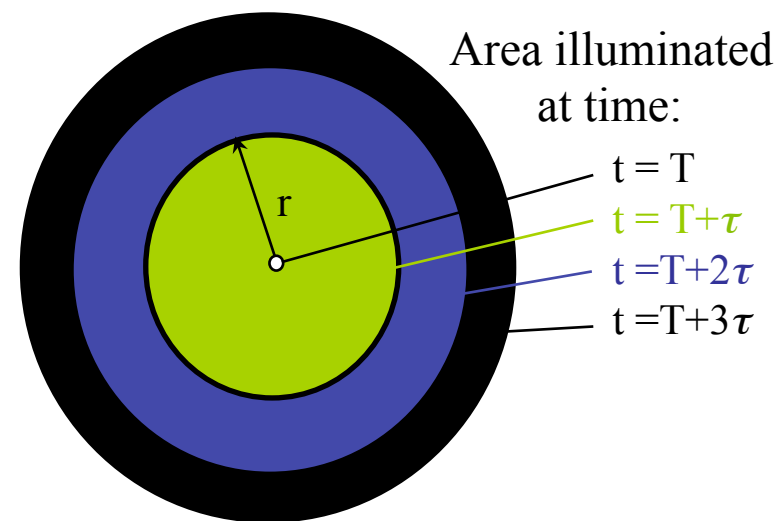
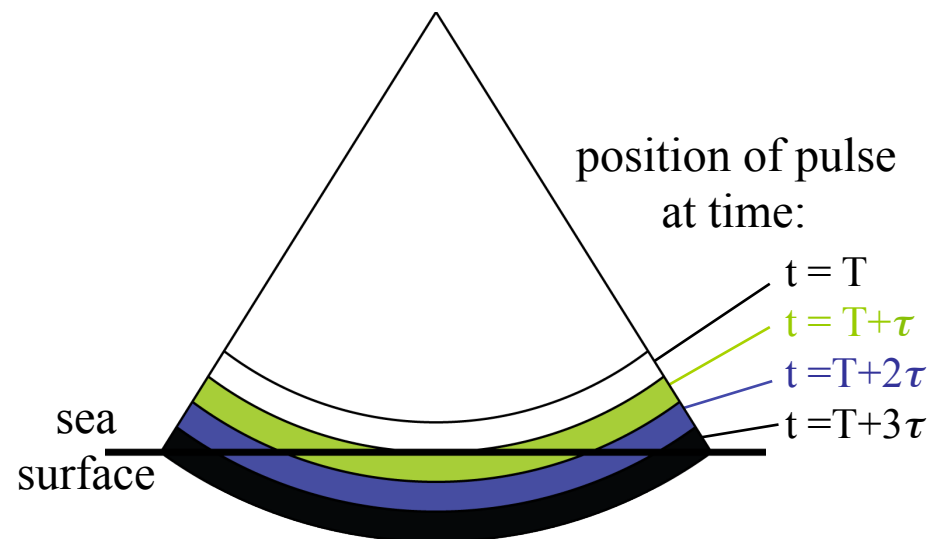


- Narrow beams require very large antennae and are impractical in space
 - For a **5 km** footprint a beam width of about **0.3°** is required.
 - For a 13.6 GHz altimeter this would imply a **5 m** antenna.
- Even more important: **highly sensitivity to mispointing**, which affects both amplitude and measured range
- SAR-altimeter missions like ESA's CryoSat (launched Apr 2010) and Sentinel-3 use synthetic aperture techniques (delay-Doppler Altimeter) that “can be seen as” a beam-limited instrument in the along-track direction.

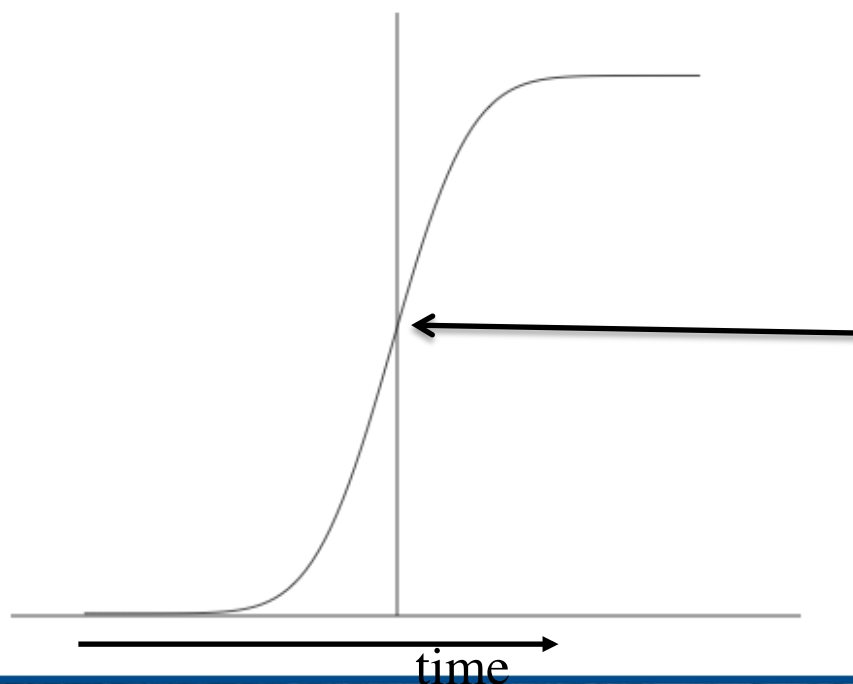
- In a pulse-limited altimeter the shape of the return is dictated by the length (width) of the pulse



- Full illumination when rear of pulse reaches the sea – then area illuminated stays constant
- Area illuminated has radius $r = \sqrt{(2hc\tau)}$
- Measure interval between mid-pulse emission and time to reach half full height



- A plot of return power versus time (i.e. at *waveform*) for a pulse-limited altimeter looks like the *integral* of the heights of the specular points, i.e. the cumulative distribution function (cdf) of the specular scatterers



The tracking point is the half power point of the curve

- All the microwave altimeters flown in space to date, including the very successful TOPEX/Jason1/Jason2 and ERS1/ERS2/Envisat series, are pulse-limited except....
- ... laser altimeters to measure the ice topography (like GLAS on ICESAT) are beam-limited
- ...and the newest SAR altimeters are beam-limited in one direction (along-track) and pulse-limited in the other (across-track)
- To understand the basics of altimetry we will focus on the pulse-limited design
 - BUT, we will see more on SAR altimetry later, as this is the technology demonstrated by CryoSat and used by future missions such as Sentinel-3

- We send out a thin shell of radar energy which is reflected back from the sea surface
- The power in the returned signal is detected by a number of gates (bins) each at a slightly different time

Shell of energy from the pulse

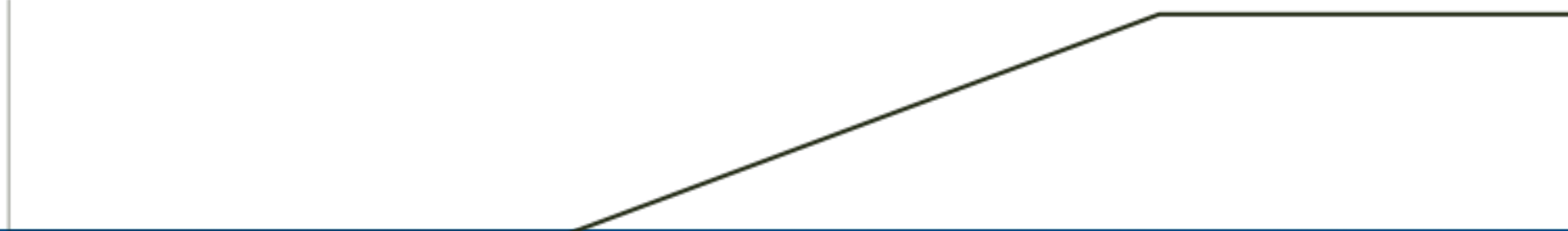
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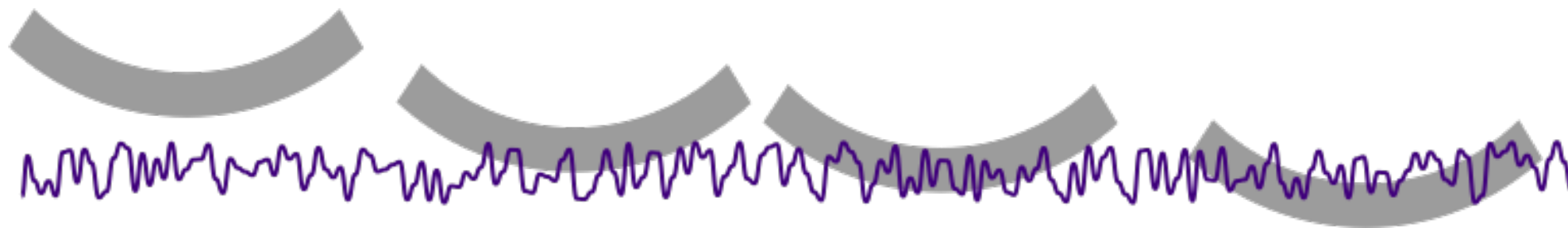
Sea Surface



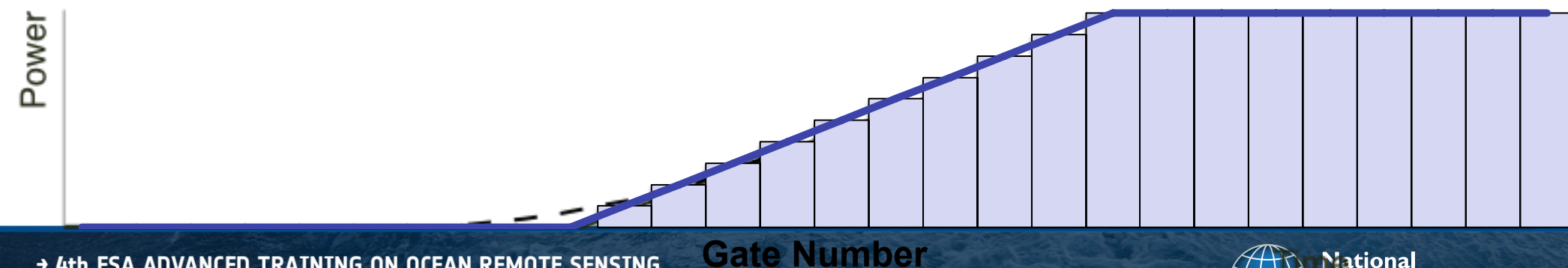
Power



If we add waves ...



Sea Surface



- The total area illuminated is related to the significant wave height noted as SWH [or H_s] ($SWH \approx 4 \times \text{std of the height distribution}$)
- The formula is

$$\frac{\pi R_0 (c\tau + 2H_s)}{1 + R_0/R_E}$$

Where

c is the speed of light

τ is the pulse length

H_s significant wave height

R_0 the altitude of the satellite

R_E the radius of the Earth

Diameters of the effective footprint

H_s (m)	ERS-1/2, ENVISAT Effective footprint (km) (800 km altitude)	TOPEX, Jason-1/2 Effective footprint (km) (1335 km altitude)
0	1.6	2.0
1	2.9	3.6
3	4.4	5.5
5	5.6	6.9
10	7.7	9.6
15	9.4	11.7
20	10.8	13.4

From Chelton et al (1989)

- Assume that the sea surface is a perfectly conducting rough mirror which reflects only at specular points, i.e. those points where the radar beam is reflected directly back to the satellite

- Under these assumptions the return power is given by a three fold convolution

$$P_r(t) = P_{FS}(t) * P_{PT}(t) * P_H(-z)$$

Where

$P_r(t)$ is the returned power

$P_{FS}(t)$ is the flat surface response

$P_{PT}(t)$ is the point target response

$P_H(-z)$ is the pdf of specular points on the sea surface

- The Flat surface response function is the response you would get from reflecting the radar pulse from a flat surface.
- It looks like

$$P_{FS}(t) = U(t - t_0) \cdot G(t)$$

Where

$U(t)$ is the Heaviside function

$U(t) = 0$ for $t < 0$; $U(t) = 1$ otherwise

$G(t)$ is the two way antenna gain pattern

- The point target response (PTR) function is the shape of the transmitted pulse
- Its true shape is given by

$$P_{PT}(t) = \left[\frac{\sin\left(\frac{\pi t}{\tau}\right)}{\frac{\pi t}{\tau}} \right]^2$$

- For the Brown model we approximate this with a Gaussian.

$$P_r(t) = P_{FS}(0) \eta P_T \sqrt{2\pi} \frac{\sigma_p}{2} \left[1 + \operatorname{erf} \left\{ \frac{(t - t_0)}{\sqrt{2}\sigma_c} \right\} \right] \quad \text{for } t < t_0$$

$$P_r(t) = P_{FS}(t - t_0) \eta P_T \sqrt{2\pi} \frac{\sigma_p}{2} \left[1 + \operatorname{erf} \left\{ \frac{(t - t_0)}{\sqrt{2}\sigma_c} \right\} \right] \quad \text{for } t \geq t_0$$

$$\sigma_c = \sqrt{\sigma_p^2 + \frac{4\sigma_s^2}{c^2}}$$

$$\sigma_s \approx \frac{SWH}{4}$$

$$P_{FS}(t) = \frac{G_0^2 \lambda_R^2 c \sigma_0}{4(4\pi)^2 L_p h^3} \exp \left\{ -\frac{4}{\gamma} \sin^2 \xi - \frac{4ct}{\gamma h} \cos 2\xi \right\} I_0 \left(\frac{4}{\gamma} \sqrt{\frac{ct}{h}} \sin 2\xi \right)$$

where

$$\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} dx$$

(compare this with the Normal cumulative distribution function)

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^t e^{-\frac{x^2}{2}} dx$$

$$\Phi(x) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x}{\sqrt{2}} \right) \right]$$

$I_0()$ is a modified Bessel function of the first kind

- **SWH** - significant wave height
- **t_0** - the **time** for the radar signal to reach the Earth and return to the satellite
 - we then convert into **range** and finally into **height** – see in the next slides
- **σ_0** - the **radar backscatter coefficient**
 - note this is set by the **roughness** at **scales comparable with radar wavelength**, i.e. cm, therefore it is (in some way) related to wind
- sometimes **mispointing angle** **ξ** can be also estimated from the waveforms

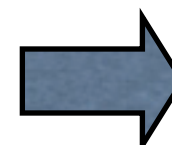
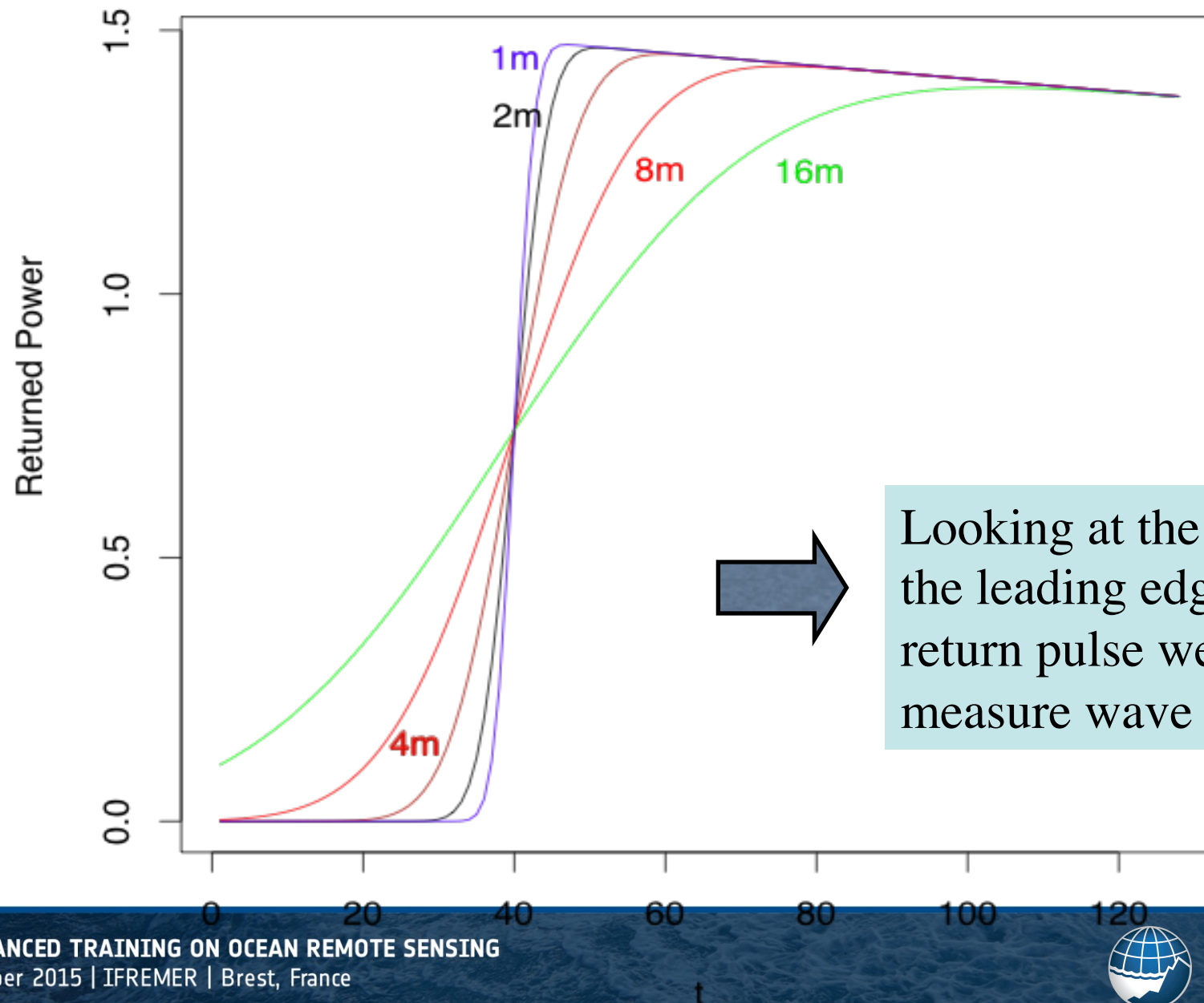
$$P_r(t) = P_{FS}(0) \eta P_T \sqrt{2\pi} \frac{\sigma_p}{2} \left[1 + \operatorname{erf} \left\{ \frac{(t - t_0)}{\sqrt{2}\sigma_c} \right\} \right] \quad \text{for } t < t_0$$

$$P_r(t) = P_{FS}(t - t_0) \eta P_T \sqrt{2\pi} \frac{\sigma_p}{2} \left[1 + \operatorname{erf} \left\{ \frac{(t - t_0)}{\sqrt{2}\sigma_c} \right\} \right] \quad \text{for } t \geq t_0$$

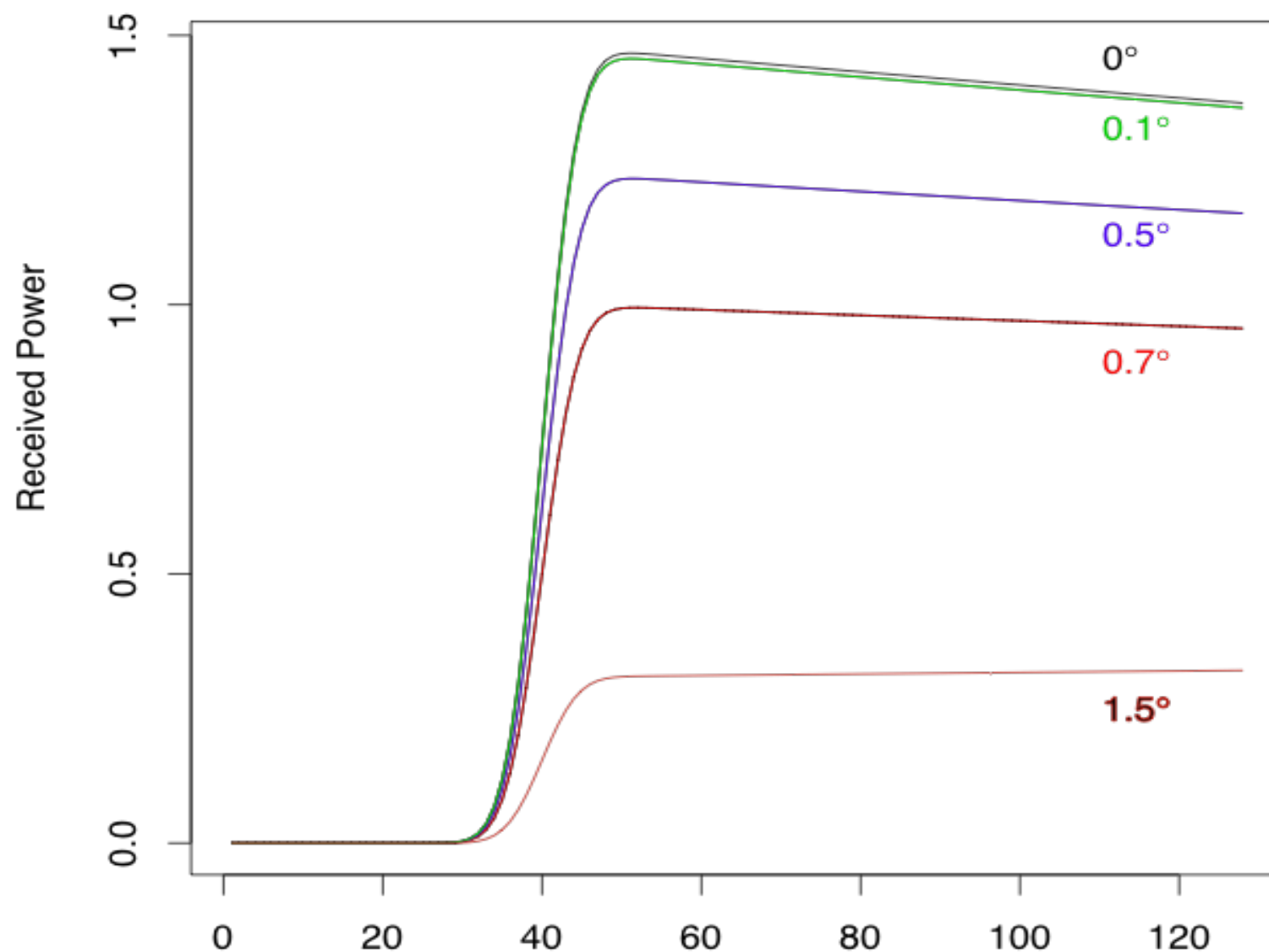
$$\sigma_c = \sqrt{\sigma_p^2 + \frac{4\sigma_s^2}{c^2}} \quad \sigma_s \approx \frac{SWH}{4}$$

$$P_{FS}(t) = \frac{G_0^2 \lambda_R^2 c \sigma_0}{4(4\pi)^2 L_p h^3} \exp \left\{ -\frac{4}{\gamma} \sin^2 \xi - \frac{4ct}{\gamma h} \cos 2\xi \right\} I_0 \left(\frac{4}{\gamma} \sqrt{\frac{ct}{h}} \sin 2\xi \right)$$

- λ_R is the radar wavelength
- L_p is the two way propagation loss
- h is the satellite altitude (nominal)
- G_0 is the antenna gain
- γ is the antenna beam width
- σ_p is the pulse width
- η is the pulse compression ratio
- P_T is the peak power
- ξ (as we said) is the mispointing angle



Looking at the slope of the leading edge of the return pulse we can measure wave height!

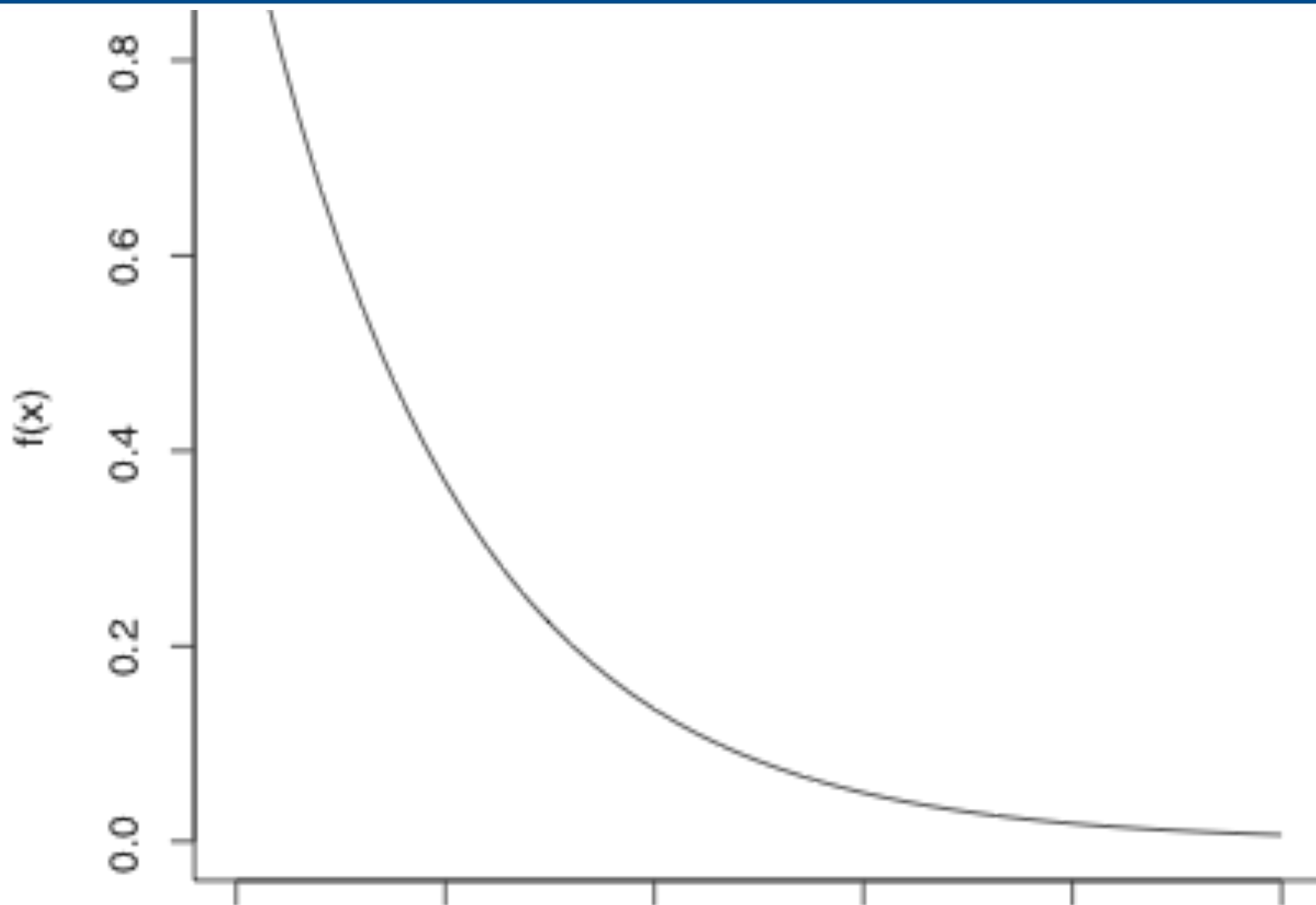


- If we simply use the altimeter as a detector we will still have a signal - known as the **thermal noise**.
- The noise on the signal is known as **fading noise**
- It is sometimes assumed to be constant, sometimes its mean is measured
- For most altimeters the noise on the signal is independent in each gate and has a negative exponential distribution.

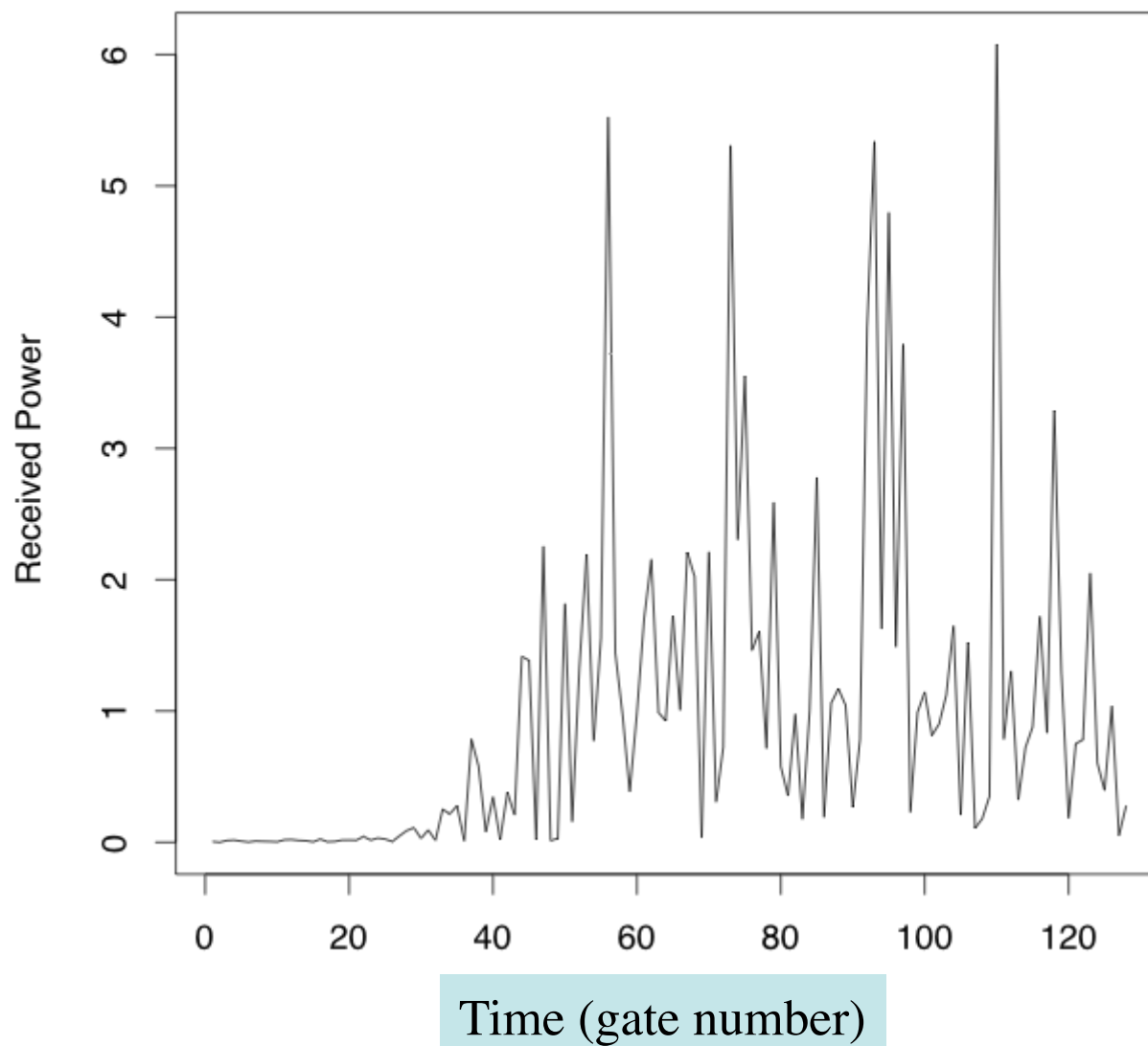
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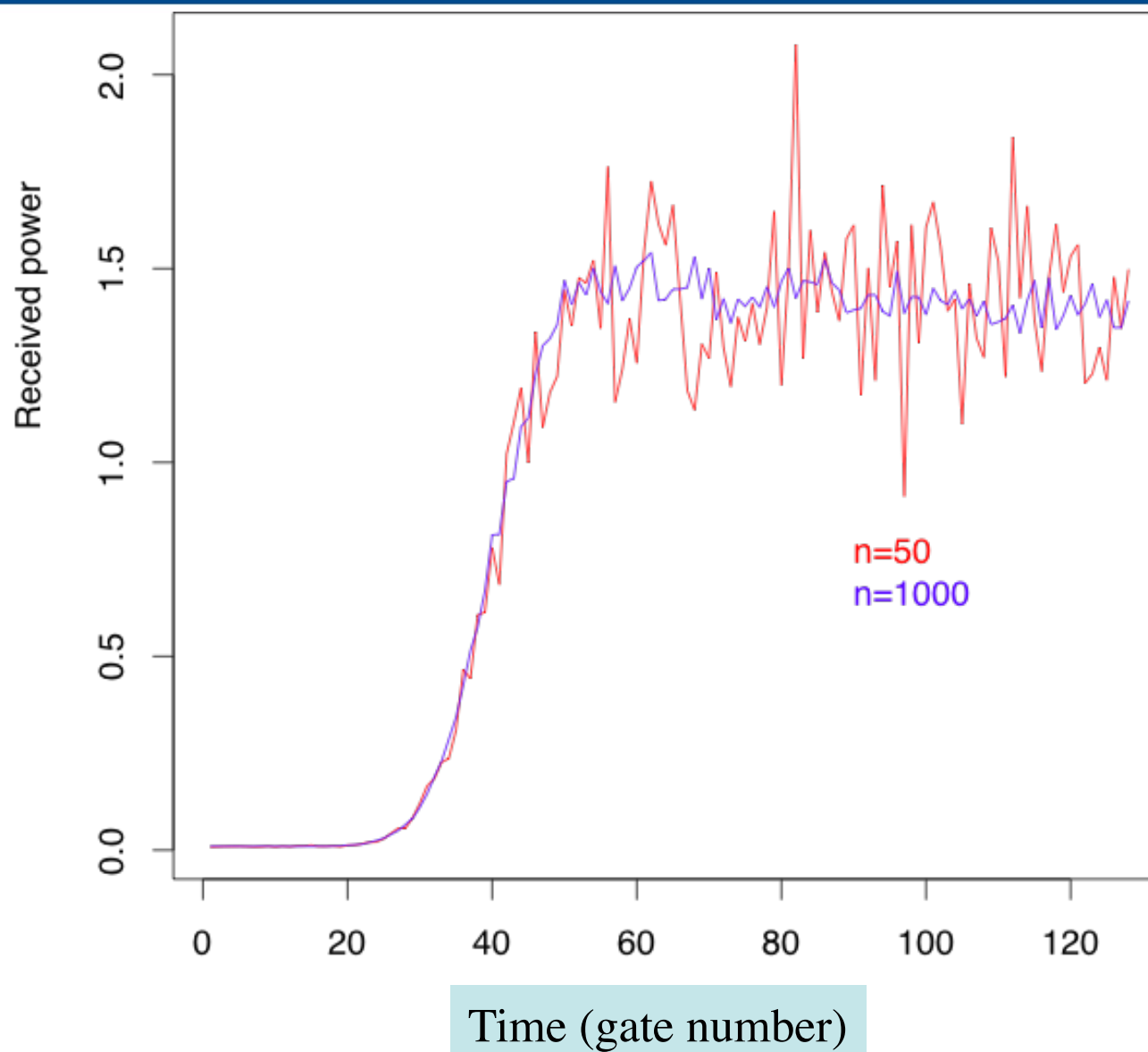
$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \quad 0 < x < \infty$$

- Mean = θ
- Variance = θ^2

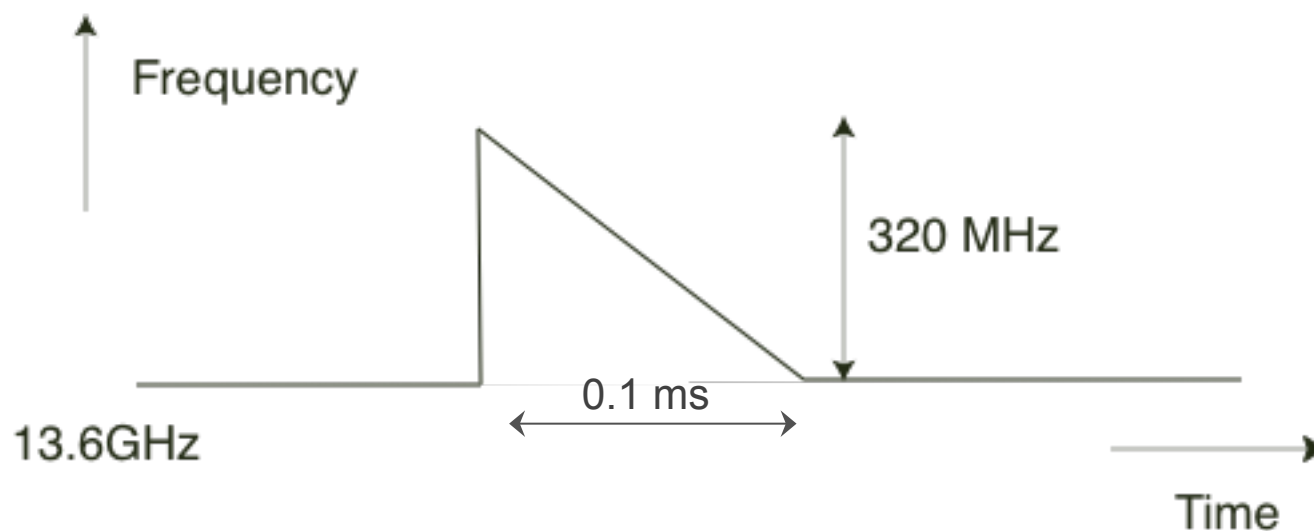


- For a negative exponential distribution the variance is equal to the square of the mean.
Thus **the individual pulses are very noisy!**
- ⇒ **We need a lot of averaging to achieve good Signal to Noise Ratio**
- The pulse repetition frequency is thousands per second
 - 1020 for ERS-1/2, 1800 for Jason & Envisat, 4500 for Topex
- Usually data are transmitted to the ground at ~20Hz and then averaged to ~1 Hz



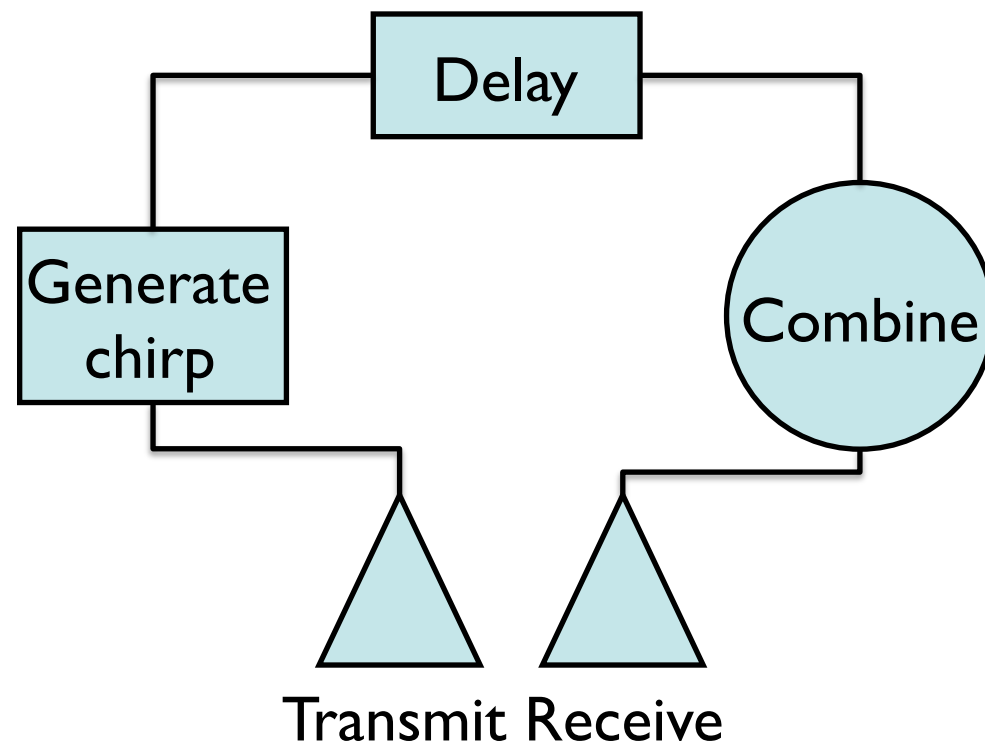


- It is very difficult (if not impossible) to generate a single-frequency pulse of length 3 ns
- It **is** possible to do something very similar in the frequency domain using a chirp: modulating the frequency of the carrier wave in a linear way



- The equivalent pulse width = $1/\text{chirp bandwidth}$

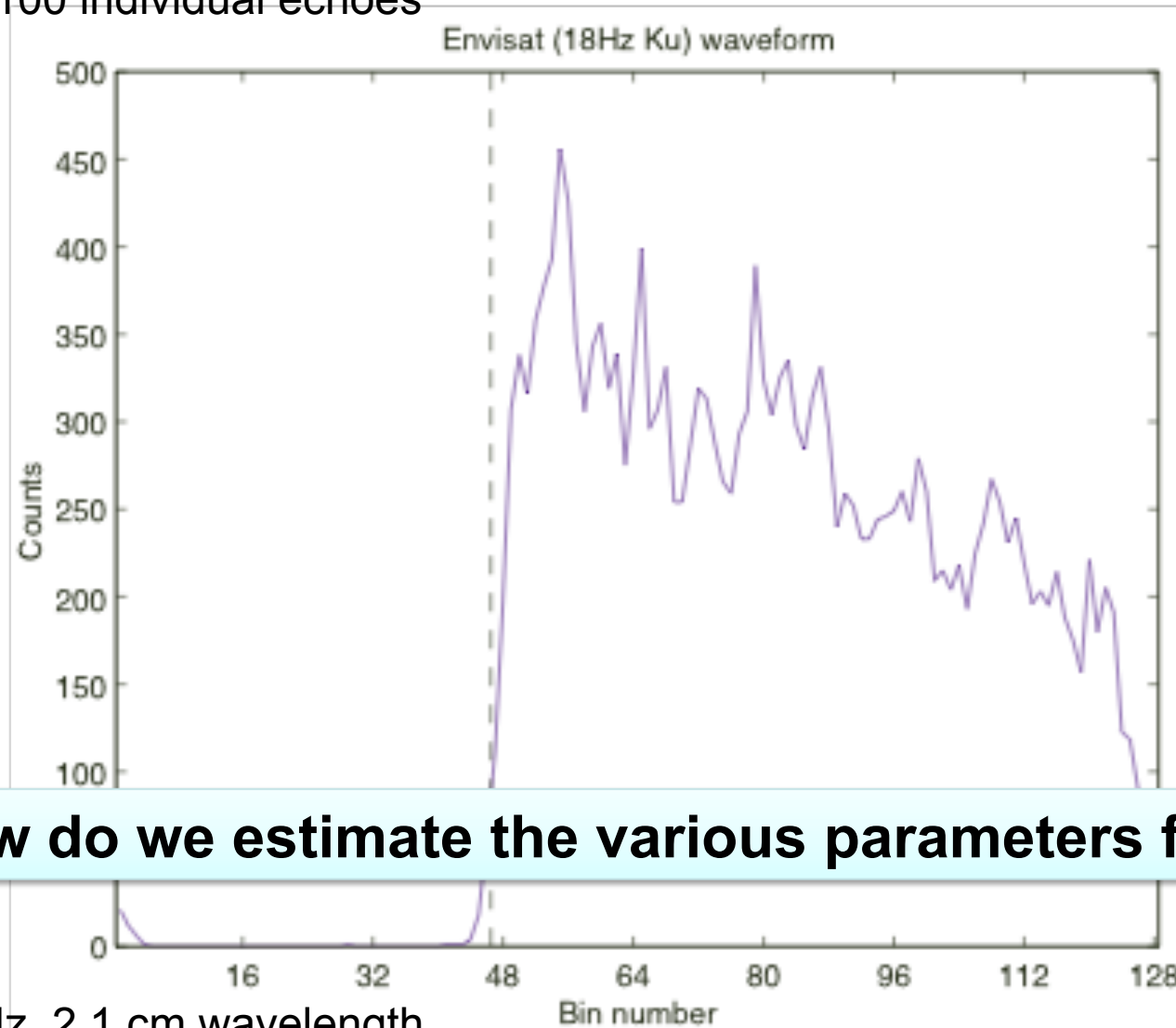
- A chirp is generated
- Two copies are taken
- The first is transmitted
- The second is delayed so it can be matched with the reflected pulse



- The two chirps are mixed.
- A point above the sea surface gives returns at frequency lower than would be expected and vice versa
- So a ‘Brown’ return is received but with frequency rather than time along the x axis

A real waveform - from the RA-2 altimeter on ESA's Envisat

From averaging 100 individual echoes



How do we estimate the various parameters from this?

Ku band, 13.5 GHz, 2.1 cm wavelength

= fitting the waveforms with a waveform model (Brown or other),
therefore estimating the parameters

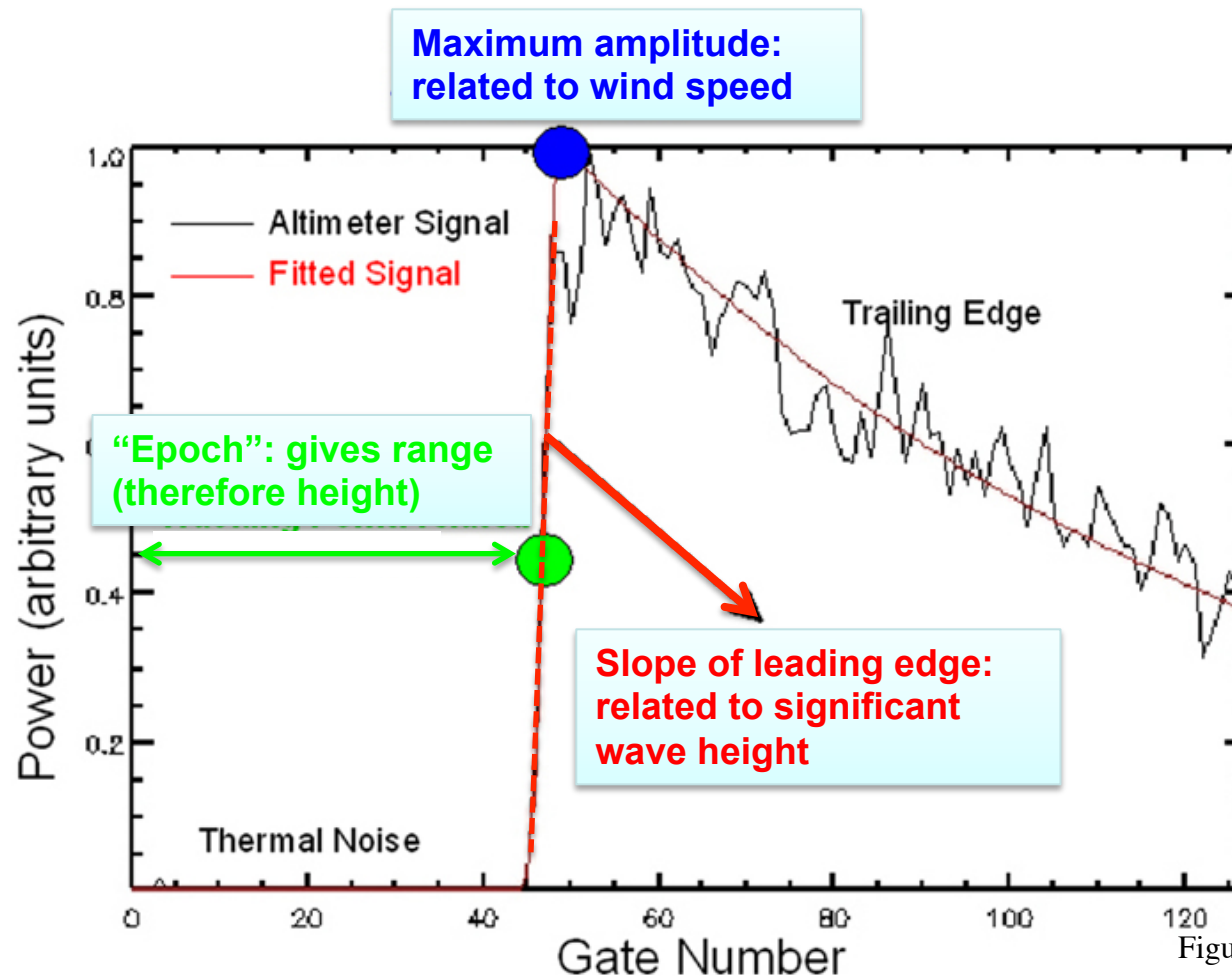
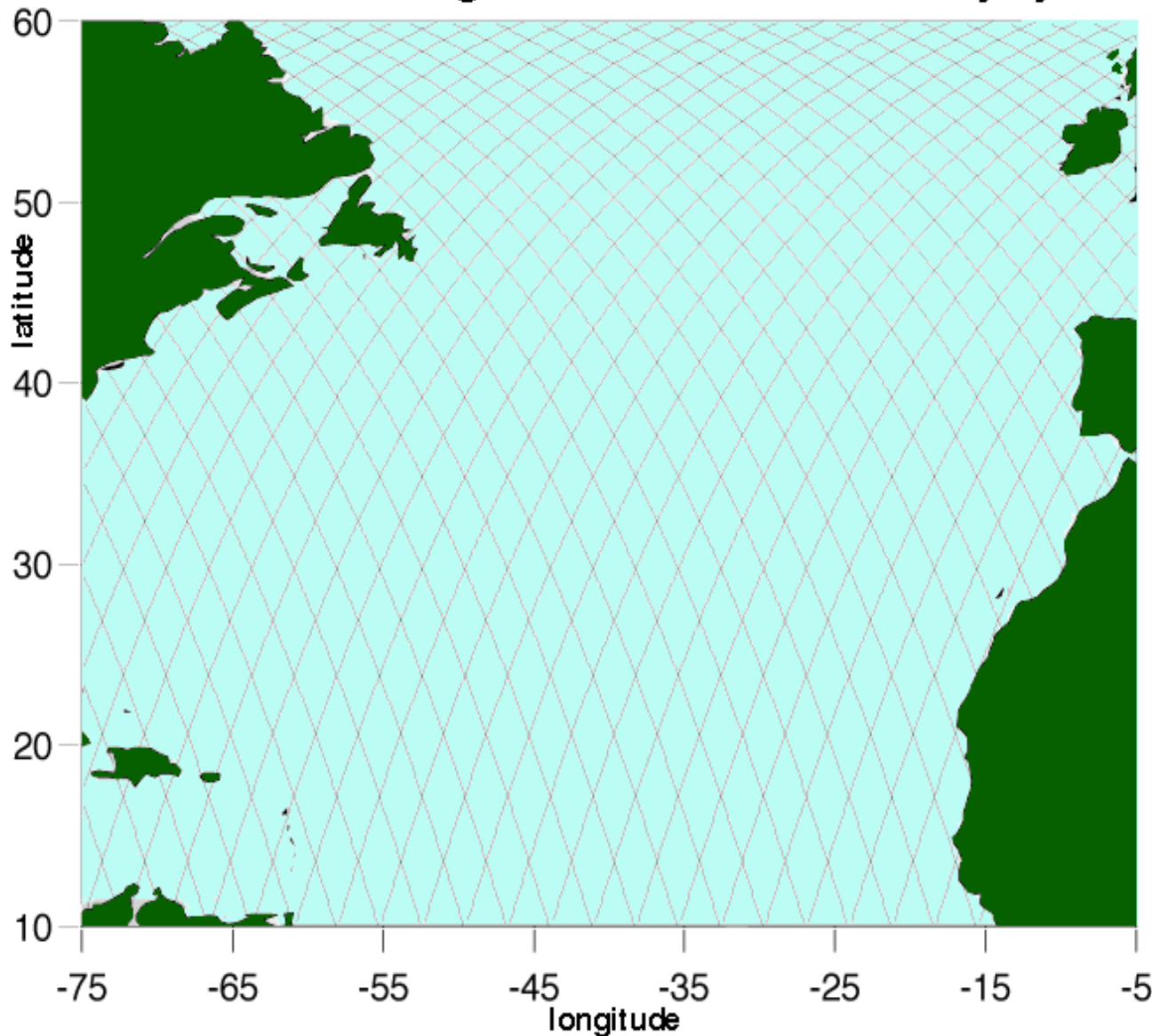


Figure from J Gomez-Enri et al. (2009)

Height	inclination	accuracy	repeat period
GEOS-3 (04/75 – 12/78)			
845 km	115 deg	0.5 m	-
Seasat (06/78 – 09/78)			
800 km	108 deg	0.10 m	3 days
Geosat (03/85 – 09/89)			
785.5 km	108.1 deg	0.10 m	17.5 days
ERS-1 (07/91 – 03/2000); ERS-2 (04/95 – 09/2011)			
785 km	98.5 deg	0.05 m	35 days
TOPEX/Poseidon (09/92 – 10/2005); Jason-1 (12/01 – 06/2013); Jason-2 (06/08 – present)			
1336 km	66 deg	0.02 m	9.92 days
Geosat follow-on (GFO) (02/98 – 09/2008)			
800 km	108 deg	0.10 m	17.5 days
Envisat (03/02 – 04/12)			
785 km	98.5 deg	0.03 m	35 days
CryoSat-2 (04/10 – present) [delay-Doppler]			
717 km	92 deg	0.05 m	369 days (30d sub-cycle)
SARAL/AltiKa (02/13 – present) [Ka-band]			
785 km	98.5 deg	0.02 m	35 days

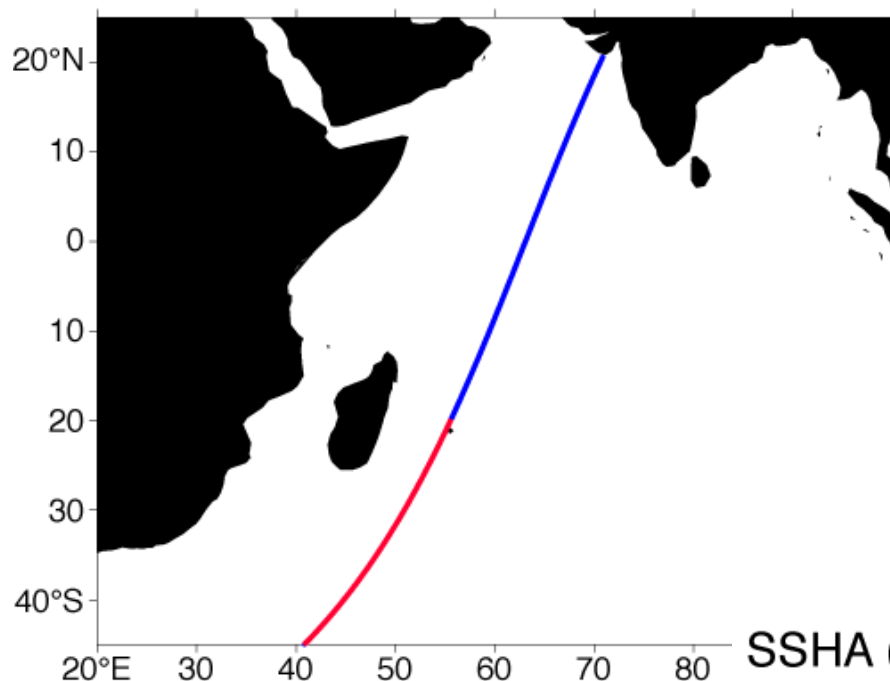
TOPEX/POSEIDON ground tracks over a 10-day cycle



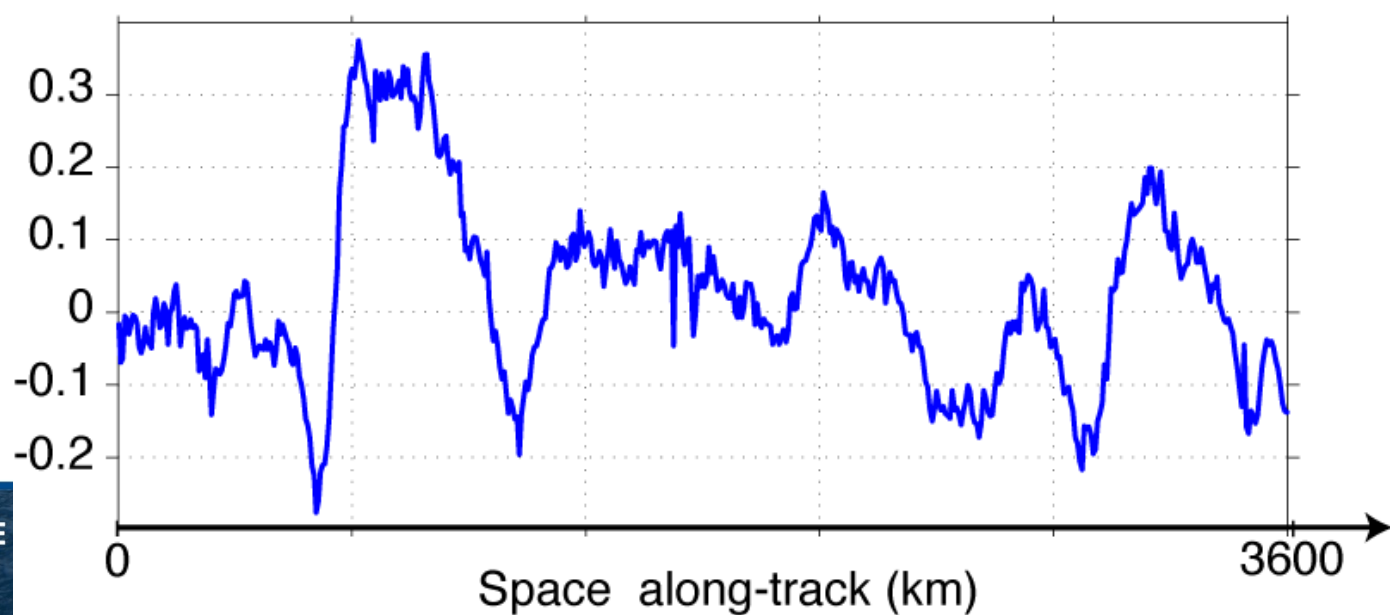
Example: Sea Surface Height along the ground track of a satellite altimeter

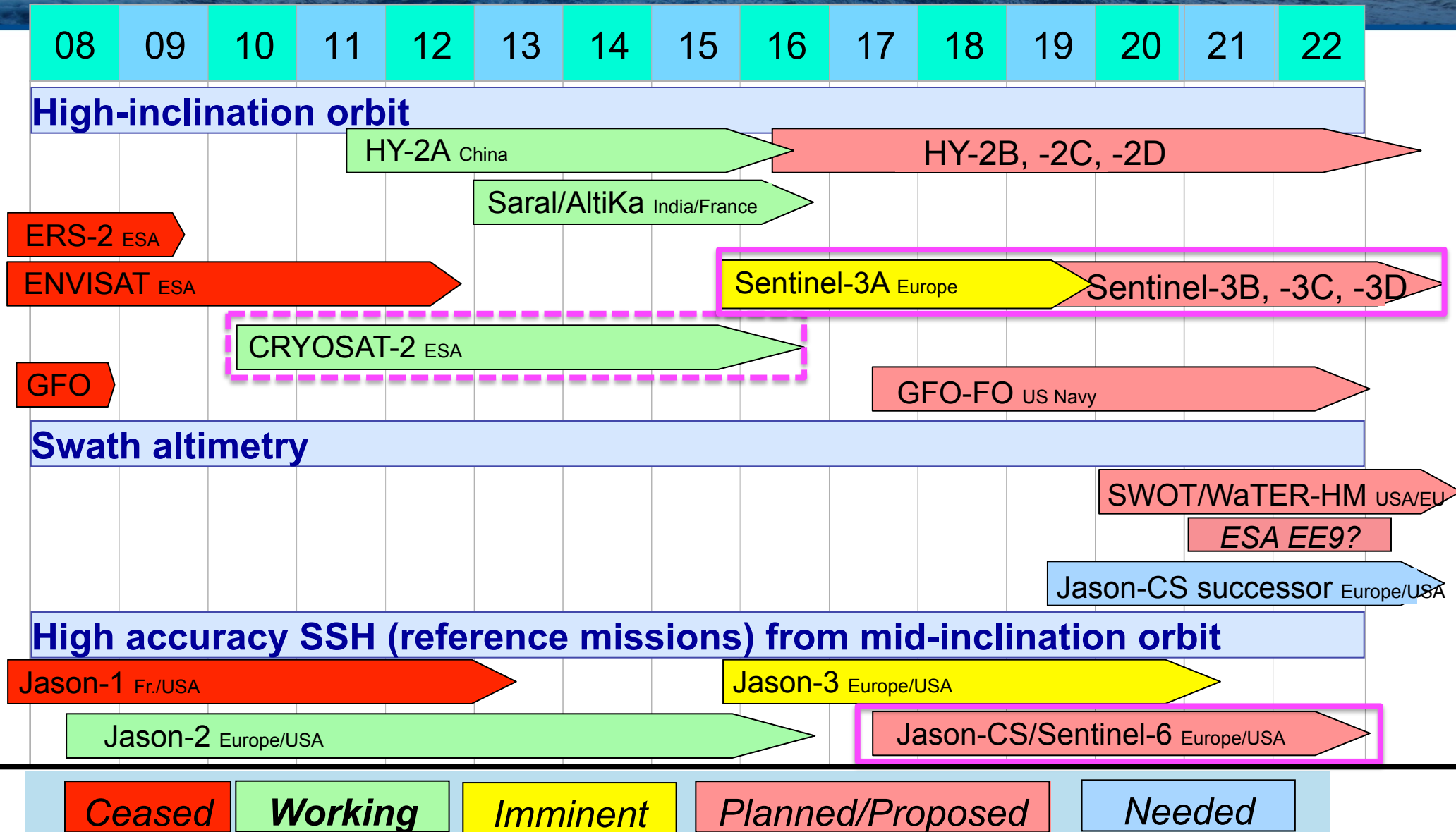


TOPEX/POSEIDON pass 029



SSHA (m) along TOPEX/POSEIDON cycle 350 pass 29 16/03/02



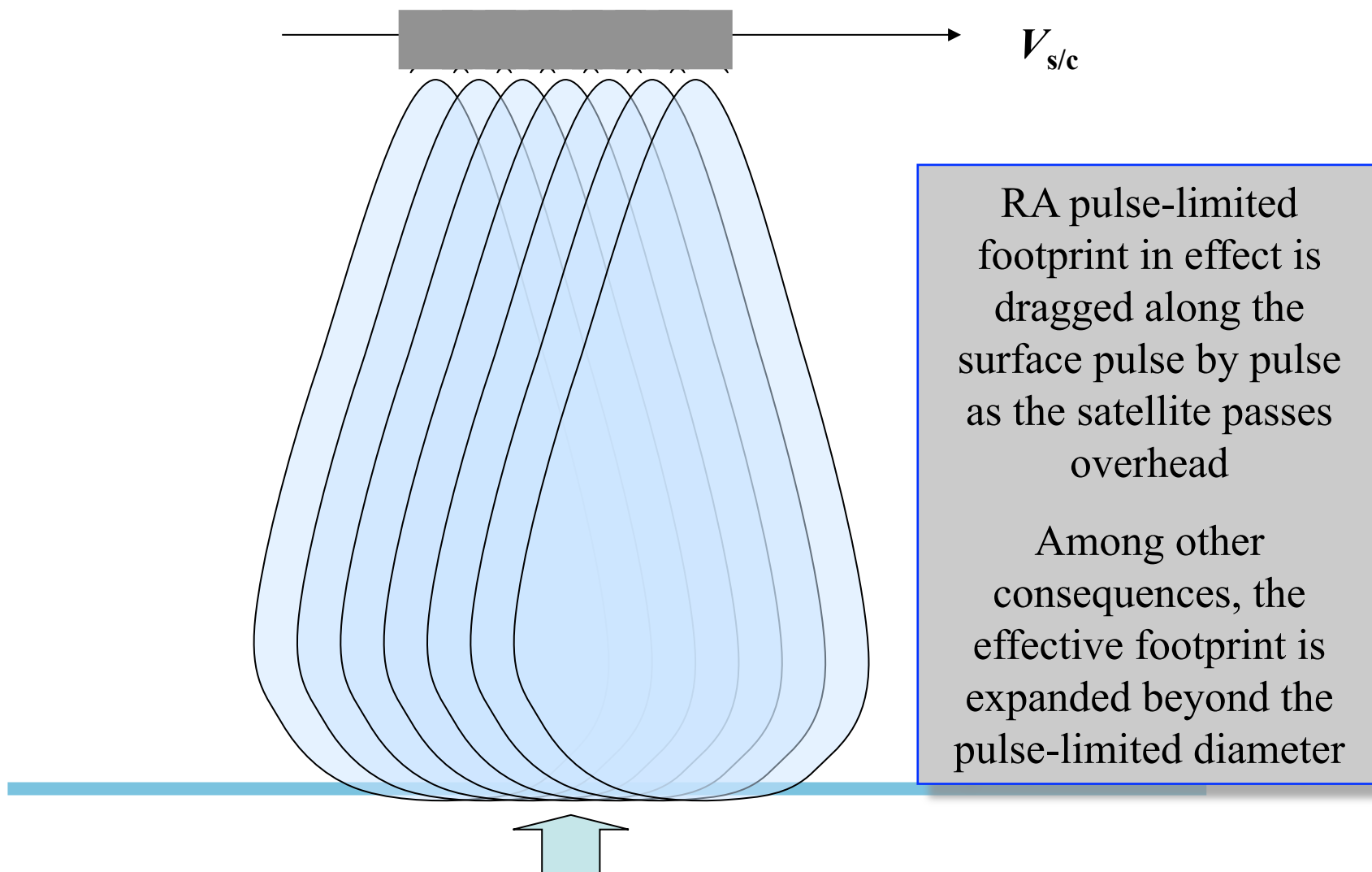


Adapted from CNES, 2009, with acknowledgement

SAR mode



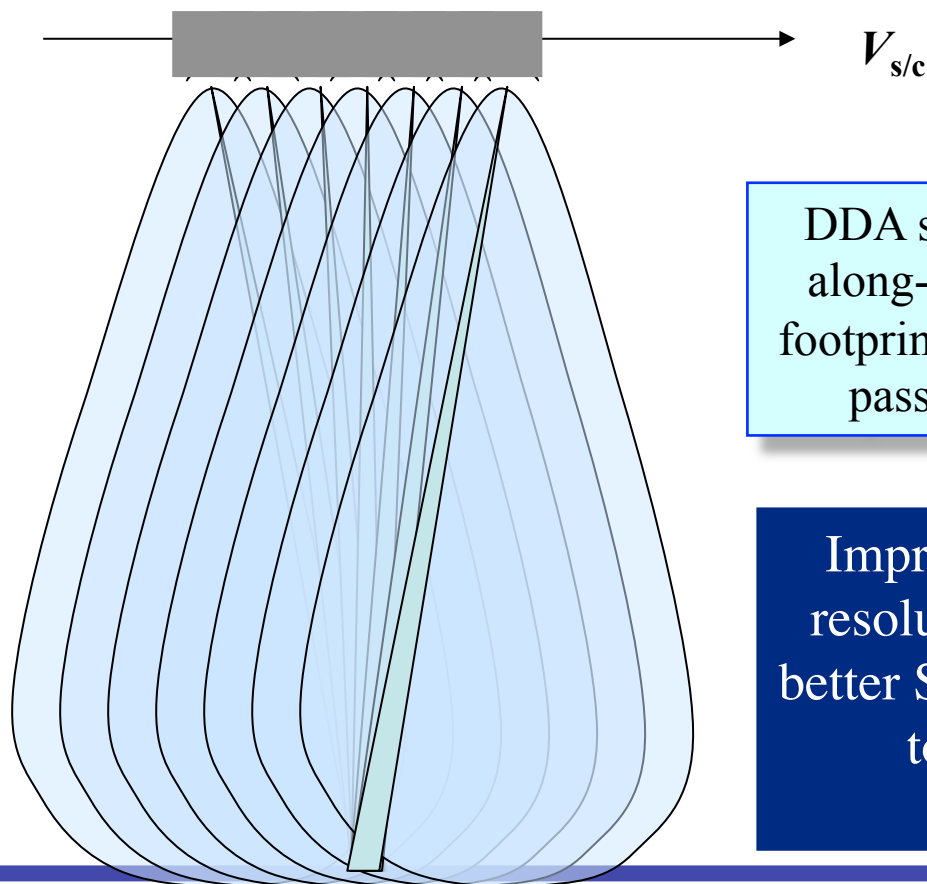
- ESA mission; launched **8 April 2010**
- LEO, non sun-synchronous
 - 369 days repeat (30d sub-cycle)
 - Mean altitude: 717 km
 - Inclination: 92°
- Prime payload: SIRAL
 - SAR/Interferometric Radar Altimeter (**delay/Doppler**)
 - Modes: Low-Res / SAR / SARIn
- Ku-band only; no radiometer
- Primary mission objective is observing the cryosphere, but **very successful on oceans too!**



Delay-Doppler Altimetry (aka SAR altimetry)



R.K. Raney, *IEEE*
TGARS, 1998

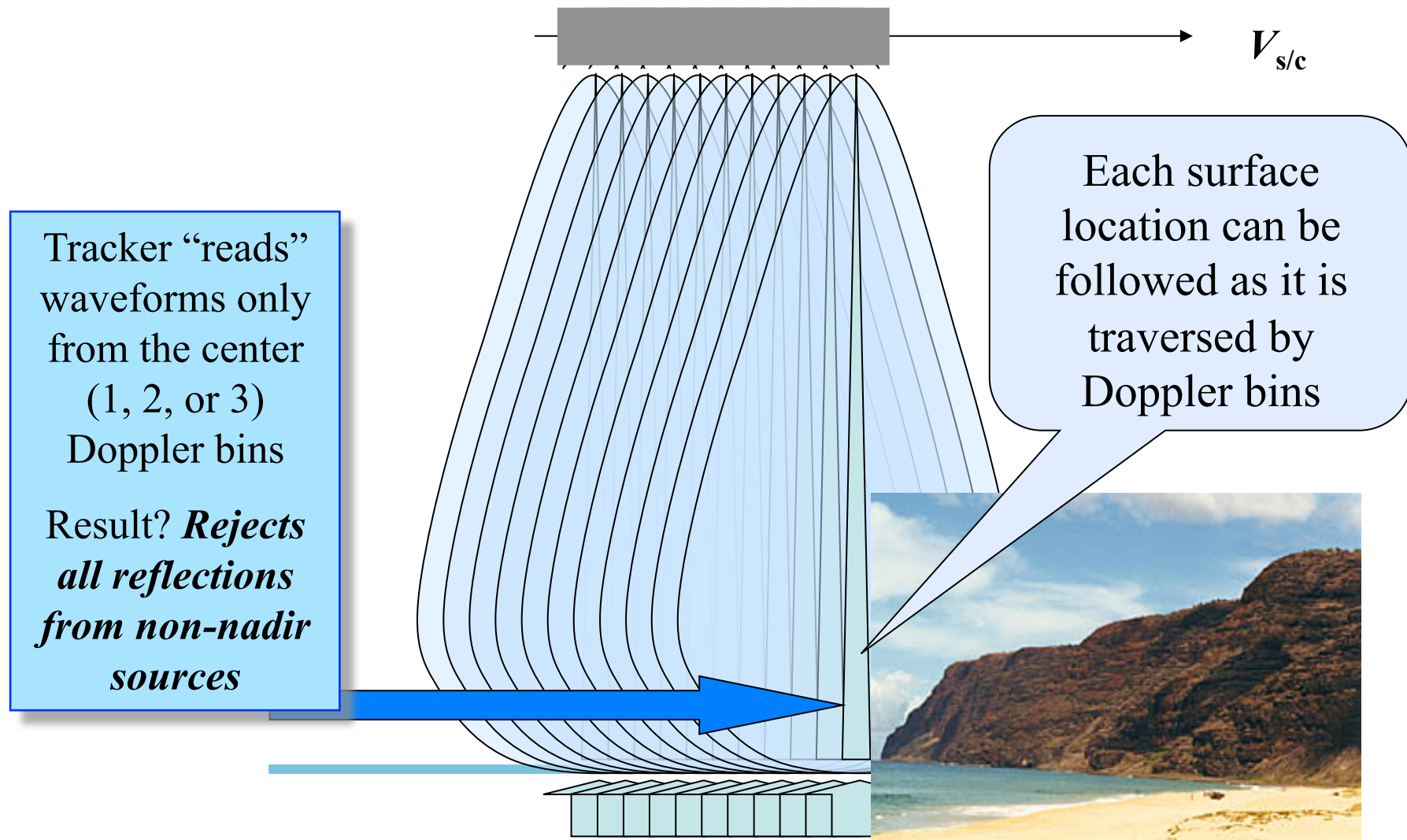


DDA spotlights each
along-track resolved
footprint as the satellite
passes overhead

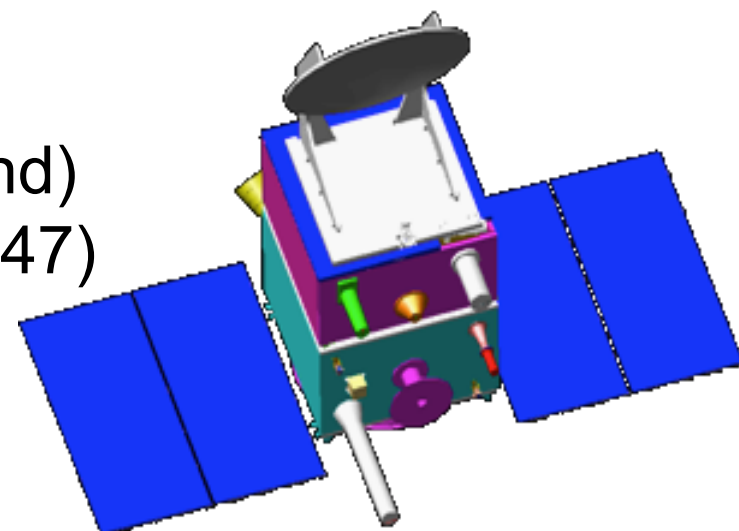
Improved along-track
resolution, higher PRF,
better S/N, less sensitivity
to sea state,...

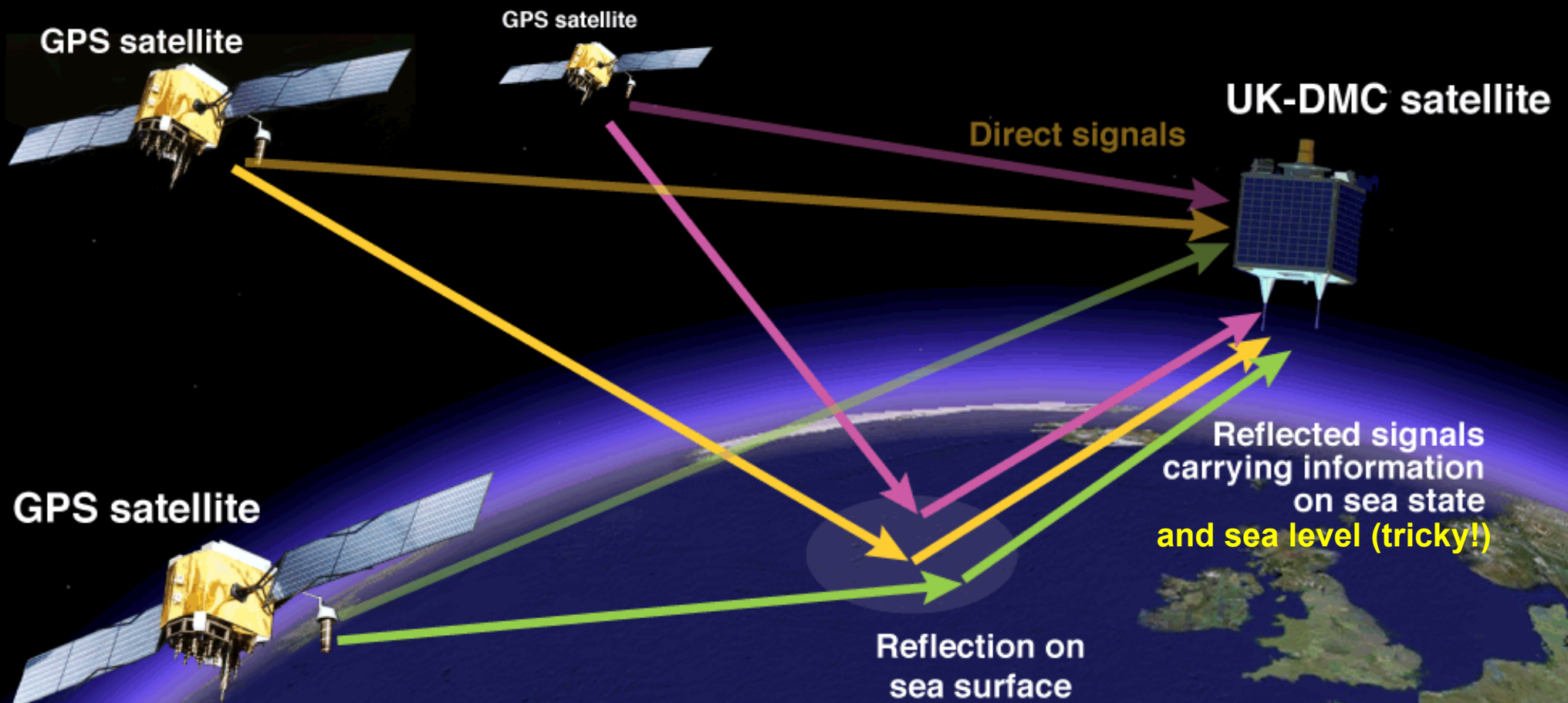


DDA (SAR-mode) Footprint Characteristic



- Satellite: Indian Space Research Organization (ISRO)
 - carrying **AltiKa** altimeter by CNES
 - Ka-band 0.84 cm (viz 2.2 cm at Ku-band)
 - Bandwidth (480 MHz) \Rightarrow 0.31 ρ (viz 0.47)
 - Otherwise “conventional” RA
 - PRF \sim 4 kHz (viz 2 kHz at Ku-band)
 - Full waveform mode
- payload includes dual-frequency radiometer
- Sun-synchronous, 35-day repeat cycle (same as ERS/Envisat)
- Navigation and control: DEM and DORIS
- Launched February 2013





GNSS (GPS/Galileo) Reflectometry

HOW GNSS-R WORKS