esa

→ 3rd ADVANCED COURSE ON RADAR POLARIMETRY

SAR POLARIMETRY

Basics Concepts, Advanced Concepts and Applications

Eric POTTIER

19–23 January 2015 | ESA-ESRIN | Frascati (Rome), Italy

European Space Agency

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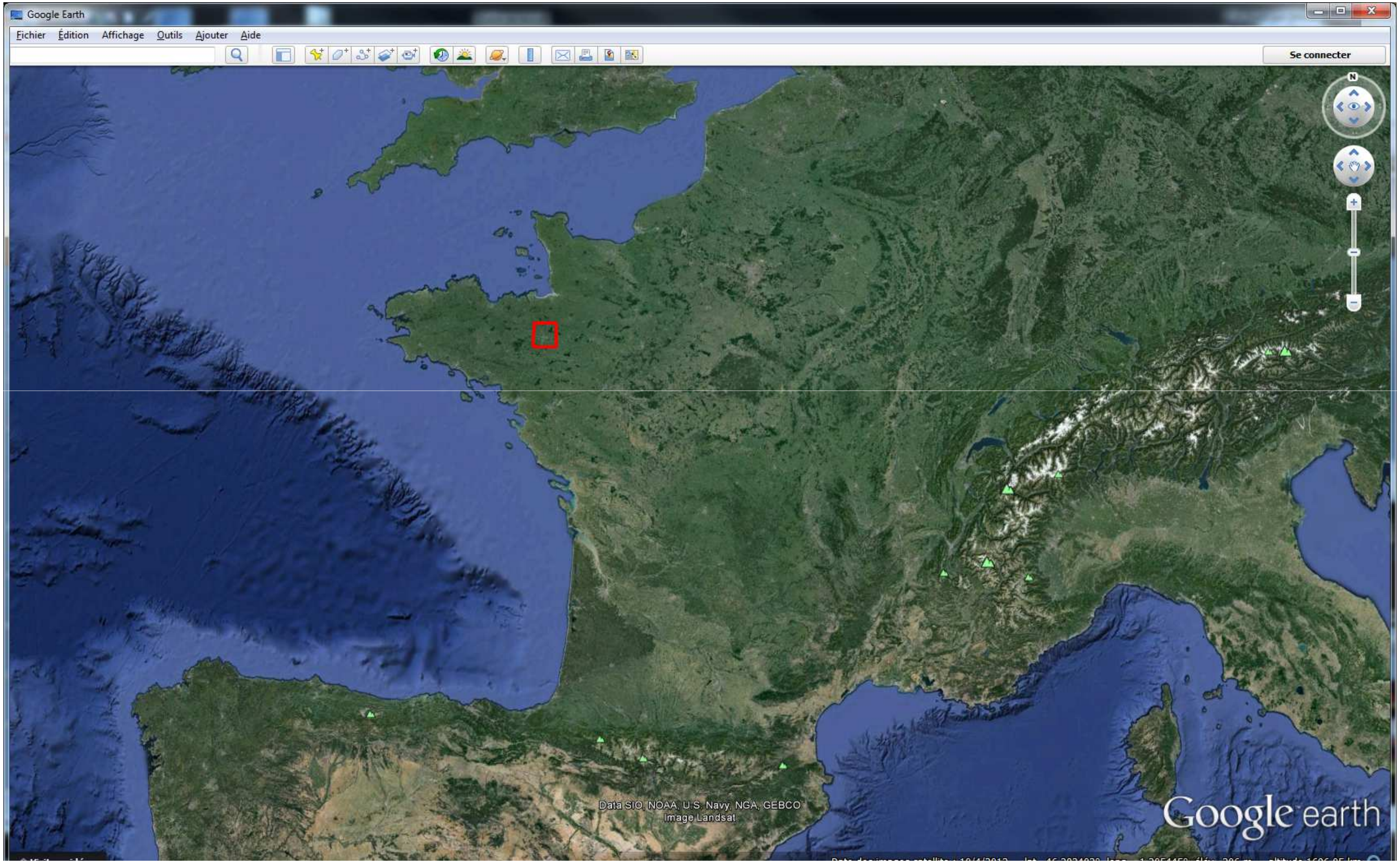
SAPHIR

SAR POLARIMETRY HOLOGRAPHY INTERFEROMETRY RADARGRAMMETRY

RADARSAT-2 (27/12/2014 – 18h14)



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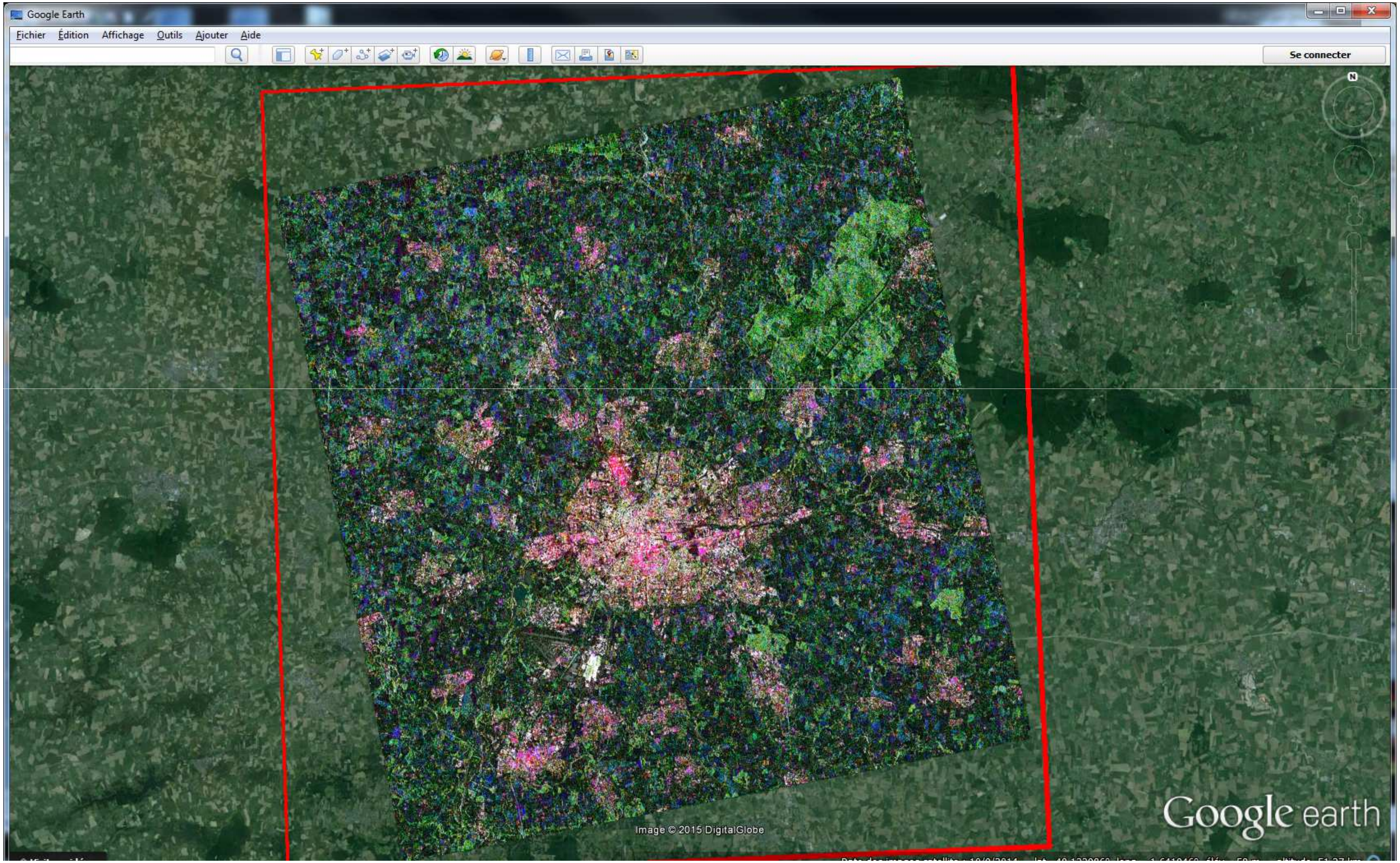
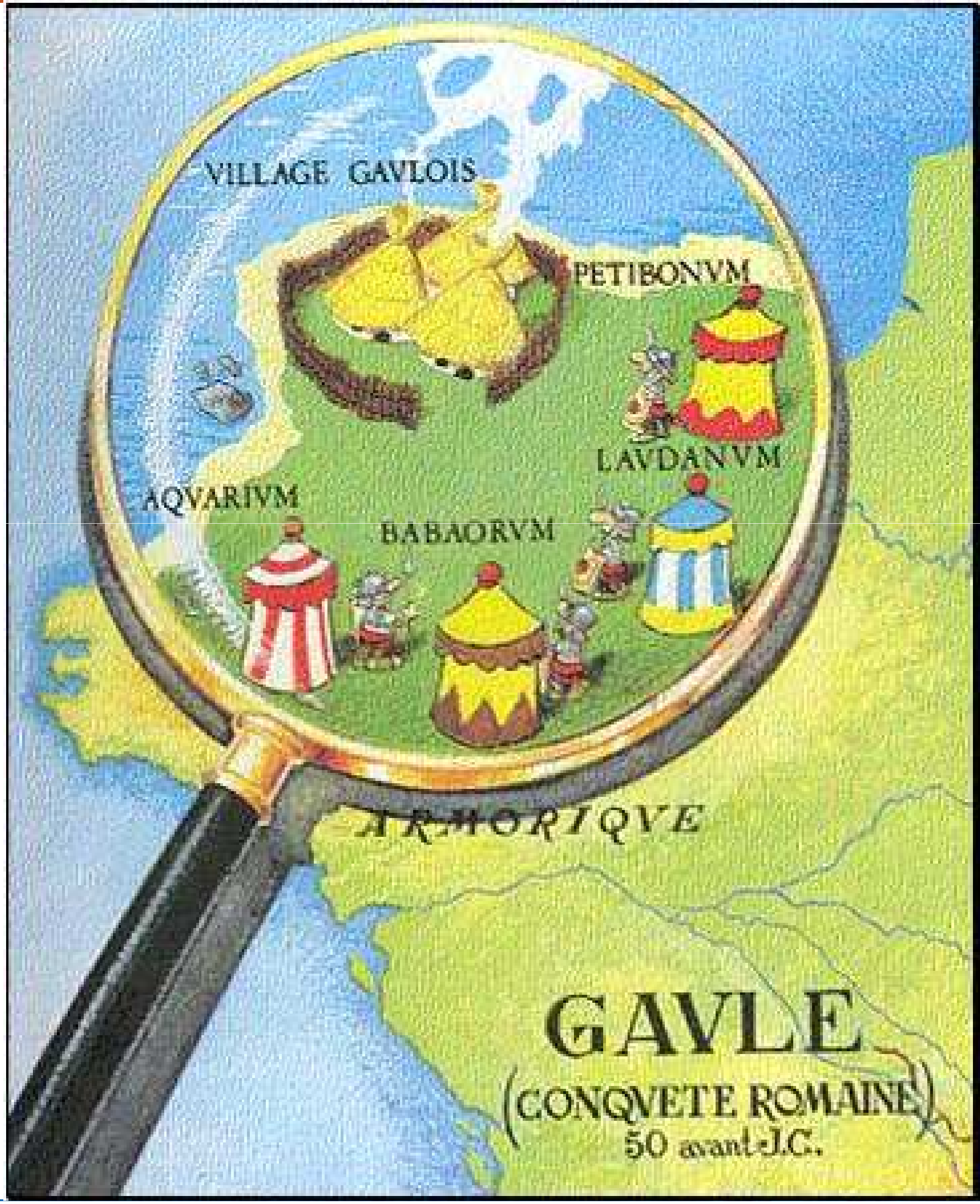
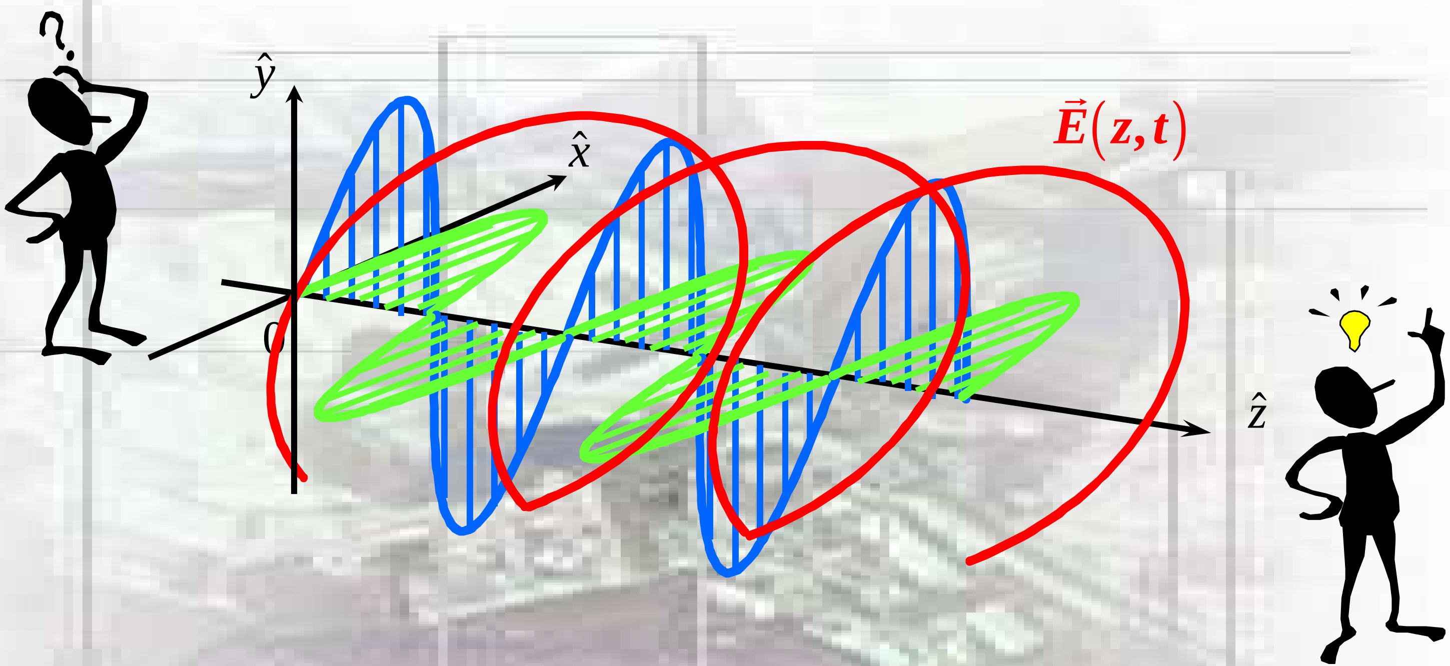
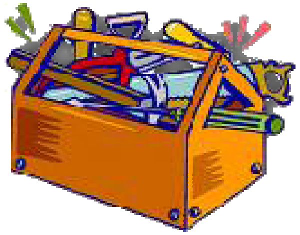


RENNES - BRITANNY





COVERED TOPICS



Objective

To provide

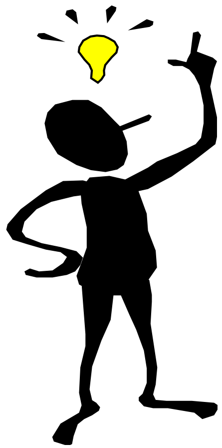
the minimum, but necessary,

amount of knowledge required

to understand

scientific works on

SAR Polarimetry (PolSAR)



Basic Concepts in PolSAR Analysis

Wave Polarimetry

- Wave Propagation
- Wave Polarisation
- Jones Vector
  - Polarisation Ratio
  - Complex Polarisation Plane
  - Orthogonal Jones Vector
  - Elliptical Basis Transformation
- Stokes Vector
  - Poincaré Sphere
  - Elliptical Basis Transformation
- Partially Polarised Waves
- Wave Polarisation Dimension

Scattering Polarimetry

- Scattering Problem
- Polarimetric Descriptors
  - Scattering / Sinclair Matrix
  - Target Vectors
  - Partially Scattering Polarimetry
  - Mueller / Kennaugh Matrix
  - Huynen Parameters
  - Coherency Matrix
  - Covariance Matrix
- Elliptical Basis Transformations
- Synthesis / Equivalence
- Polarimetric Target Dimension
  - MonostaticTarget Equations
  - Monostatic Target Diagram

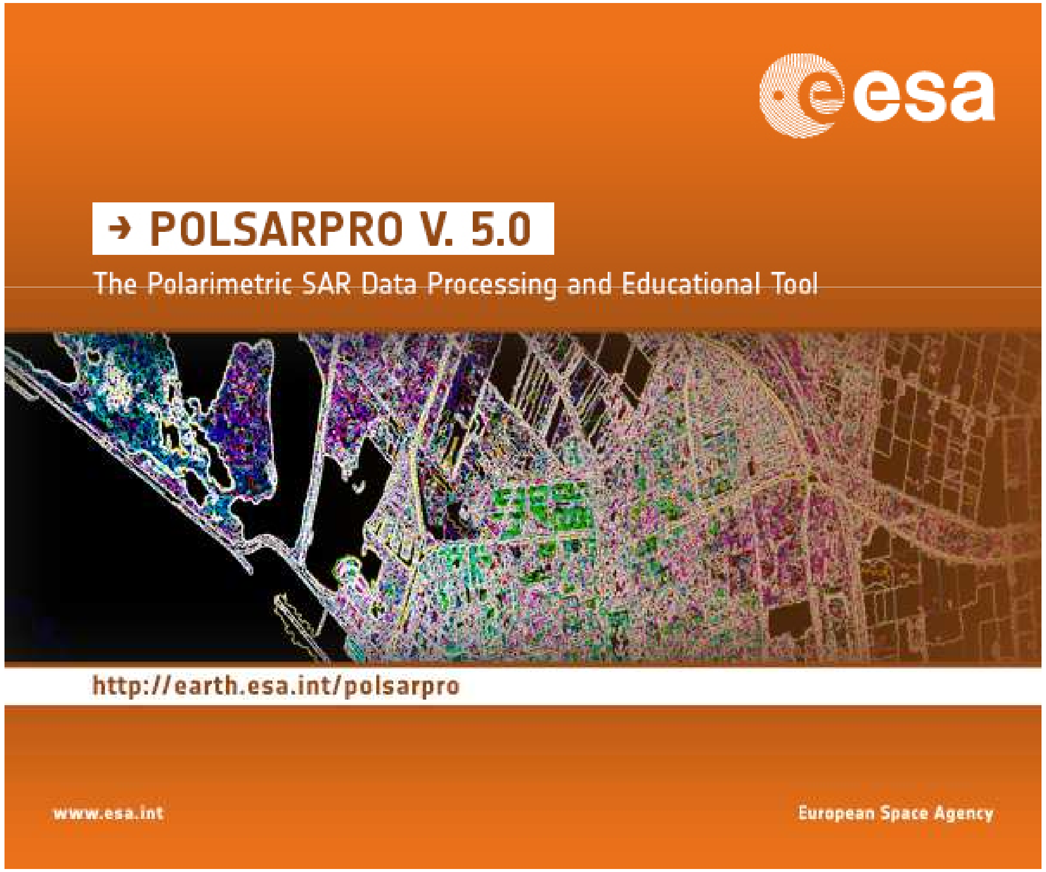
Advanced Concepts in PolSAR Analysis

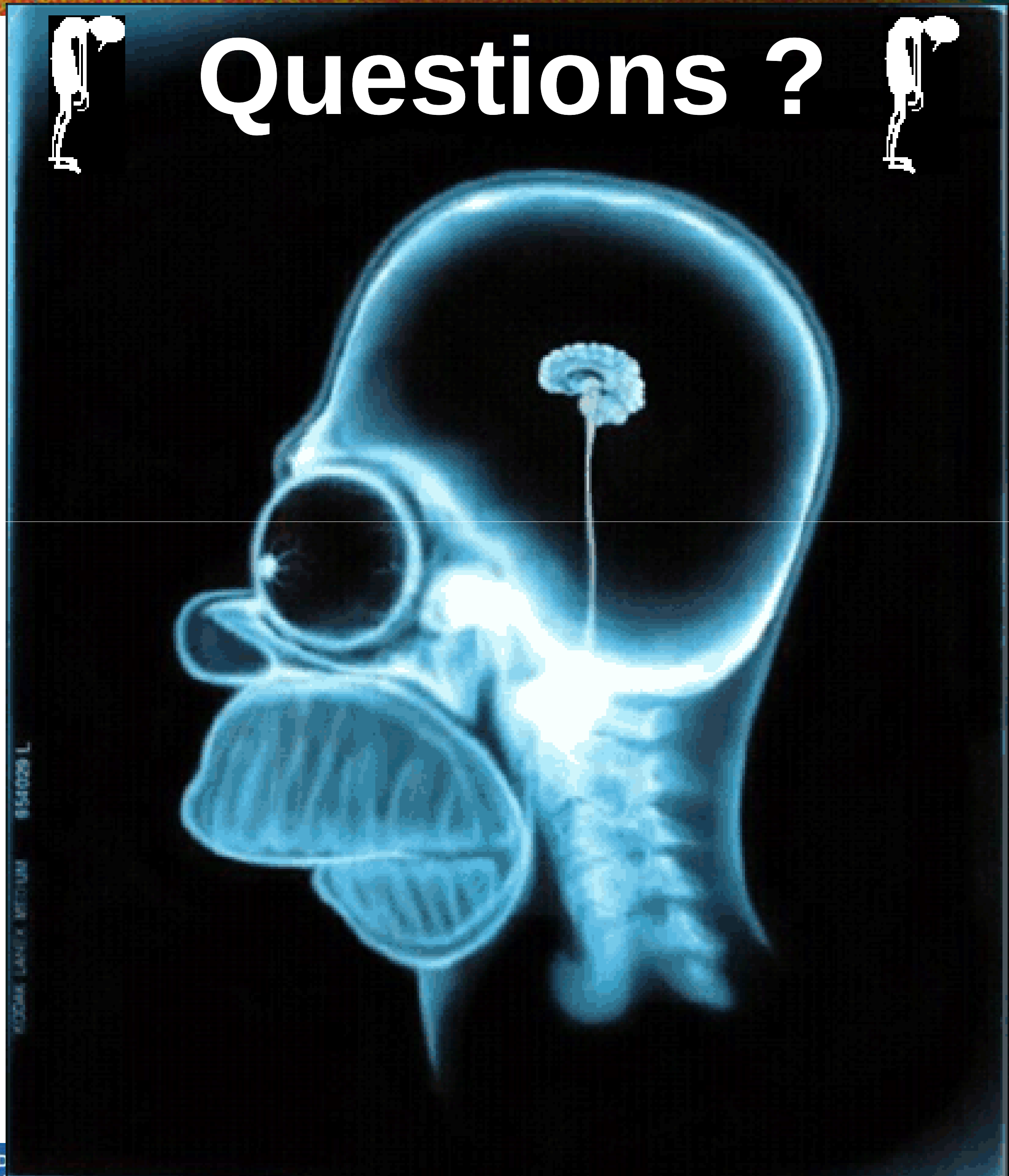
- Polarimetric Speckle Filtering
- Diffusion Symmetries
- Target Decomposition Theorems
  - Krogager Decomposition
  - Huynen / Barnes Decompositions
  - Cloude / Holm Decompositions
  - Freeman / Yamaguchi Decompositions
  - Van Zyl / Arii Decompositions
  - H / A / $\underline{\alpha}$ Decomposition
  - eigenvalues based parameters
  - TSVM Decomposition

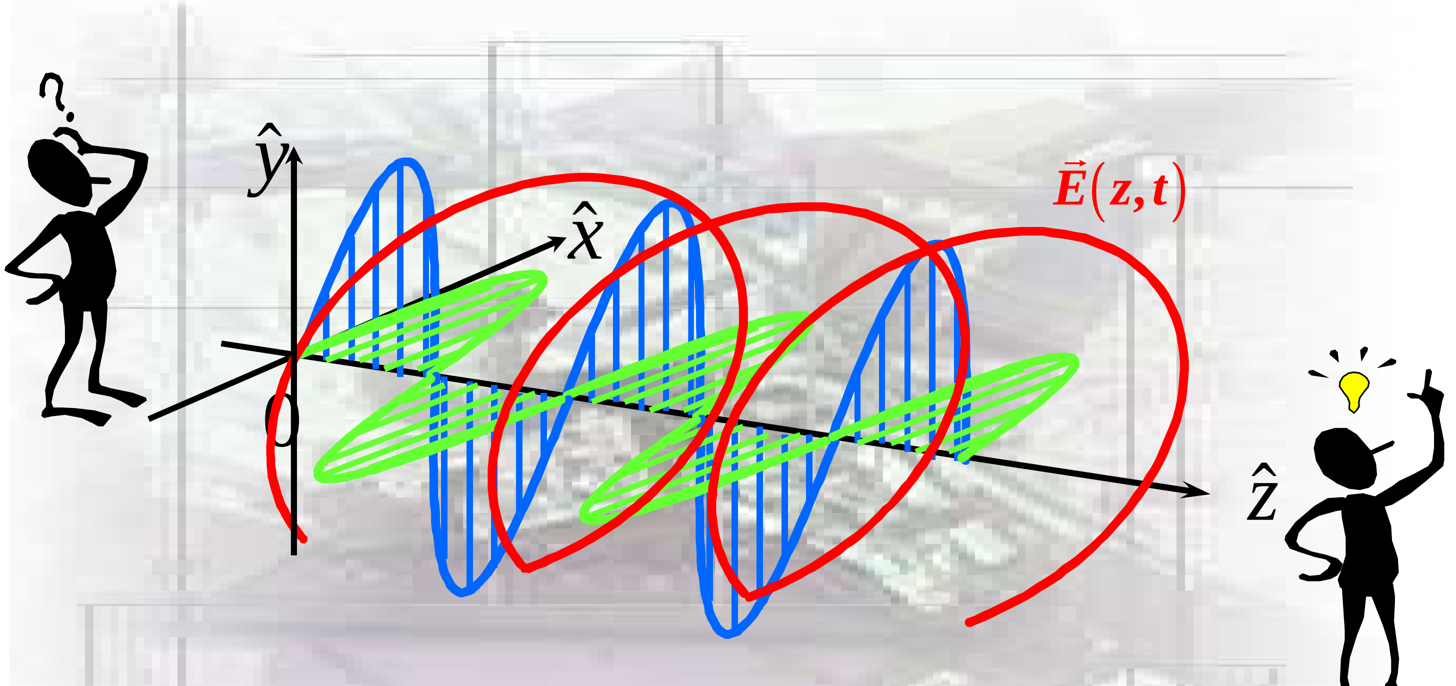
- PolSAR Image Segmentation
  - H / $\underline{\alpha}$ Unsupervised Classification
  - Wishart Classifier
  - Wishart - H / $\underline{\alpha}$ Classification
  - Wishart - H / A / $\underline{\alpha}$ Classification
  - Wishart – Freeman Classification

- Basics of Polarimetric SAR Interferometry
  - Complementarity of Polarimetry and Interferometry
  - Pol-InSAR classification of complex scenes

Practicals

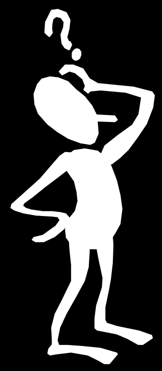

esa
→ POLSARPRO V. 5.0
The Polarimetric SAR Data Processing and Educational Tool
http://earth.esa.int/polsarpro
www.esa.int
European Space Agency


Questions ?



GENERAL INTRODUCTION

RADAR POLARIMETRY



- A bit of History
- Space-borne Polarimetric SAR Sensors
- Software / Toolbox
- Learning / Training

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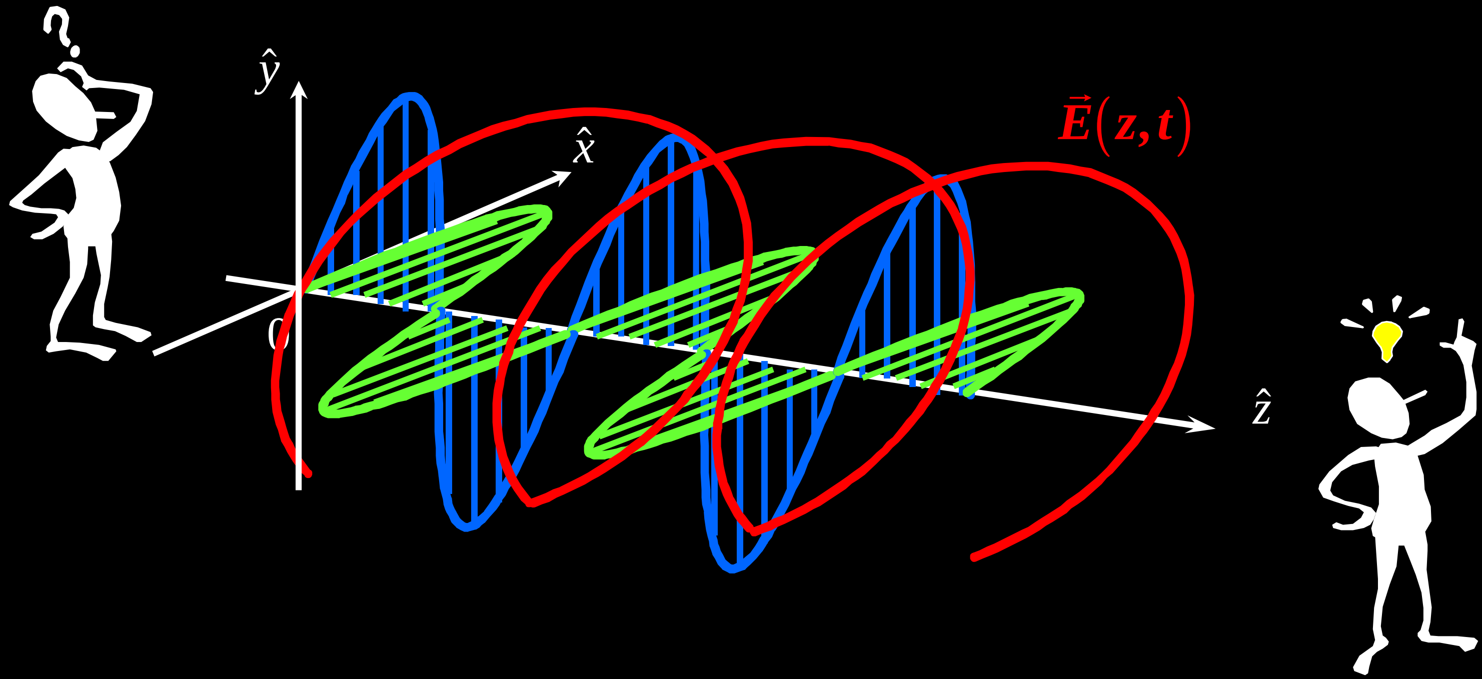
Radar Polarimetry



Radar Polarimetry (Polar : polarisation Metry: measure) is the science of acquiring, processing and analysing the polarization state of an electromagnetic field

Radar Polarimetry deals with the full vector nature of polarized electromagnetic waves

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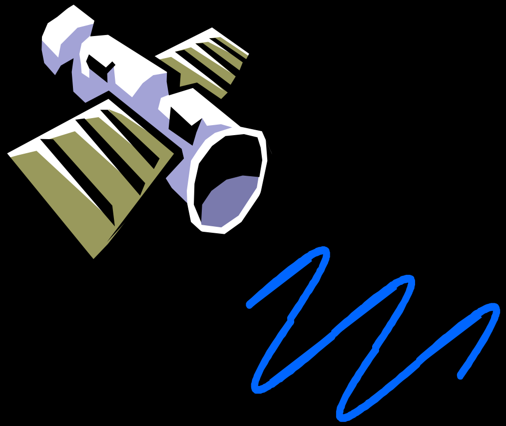
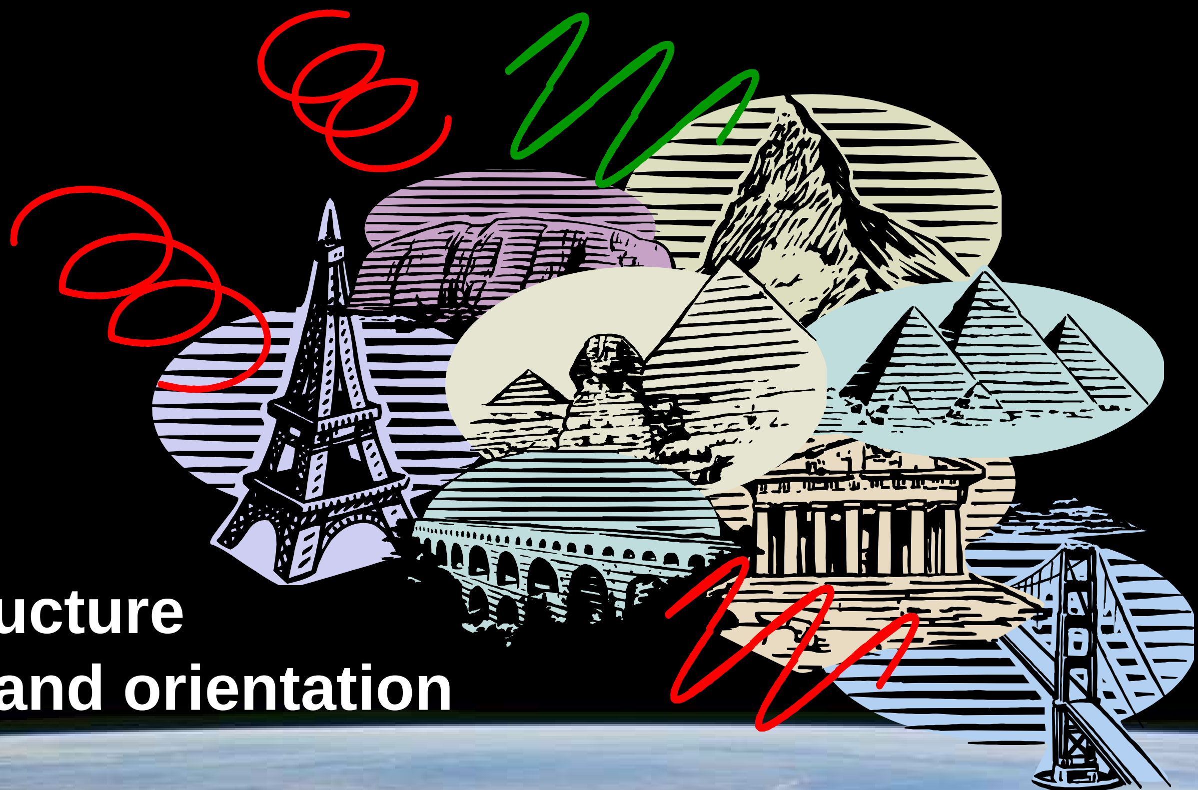
Radar Polarimetry



The POLARISATION information
Contained in the waves backscattered
from a given medium is highly related to:



its geometrical structure
reflectivity, shape and orientation

its geophysical properties such as humidity, roughness, …

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SAR Polarimetry Applications

Forest Vegetation

- Forest Height
- Forest Biomass
- Forest Structure
- Canopy Extinction
- Underlying Topography

- Forest Ecology
- Forest Management
- Ecosystem Change
- Carbon Cycle

Agriculture

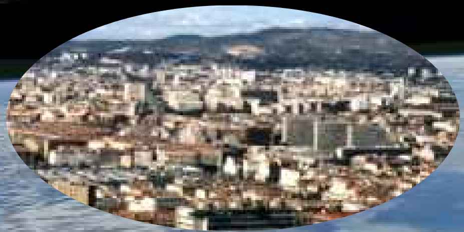
- Soil Moisture Content
- Soil roughness
- Height of Vegetation Layer
- Extinction of Vegetation Layer
- Moisture of Vegetation Layer

- Farming Management
- Water Cycle
- Desretification

Snow and Ice

- Topography
- Penetration Depth / Density
- Snow Ice Layer
- Snow Ice Extinction
- Water Equivalent

- Ecosystem Change
- Water Cycle
- Water Management



Urban Areas

- Geometric Properties
- Dielectric Properties

- Urban Monitoring

DLR Courtesy of Dr. I. Hajnsek

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A Bit Of History

Radar Polarimetry

Discovery of the Phenomena of Polarized Electromagnetic Energy

AD 1000
Use of the polarized skylight to locate a hidden sun

Crystal of calcite
Iceland Spar
Sunstone

1669
First known Quantitative work on light observation

Bartholinus

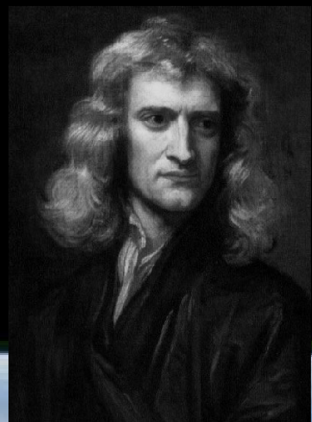
Discovery of the double refraction in calcite

1677
Wave nature of light discovery
Explanation of the double refraction

Huygens

Corpuscular model or « longitudinal » waves

1704
Corpuscular Model of light



Newton

1808
Discovery of the polarization of light (intrinsic property of light and not of crystals)

Malus
X-1795

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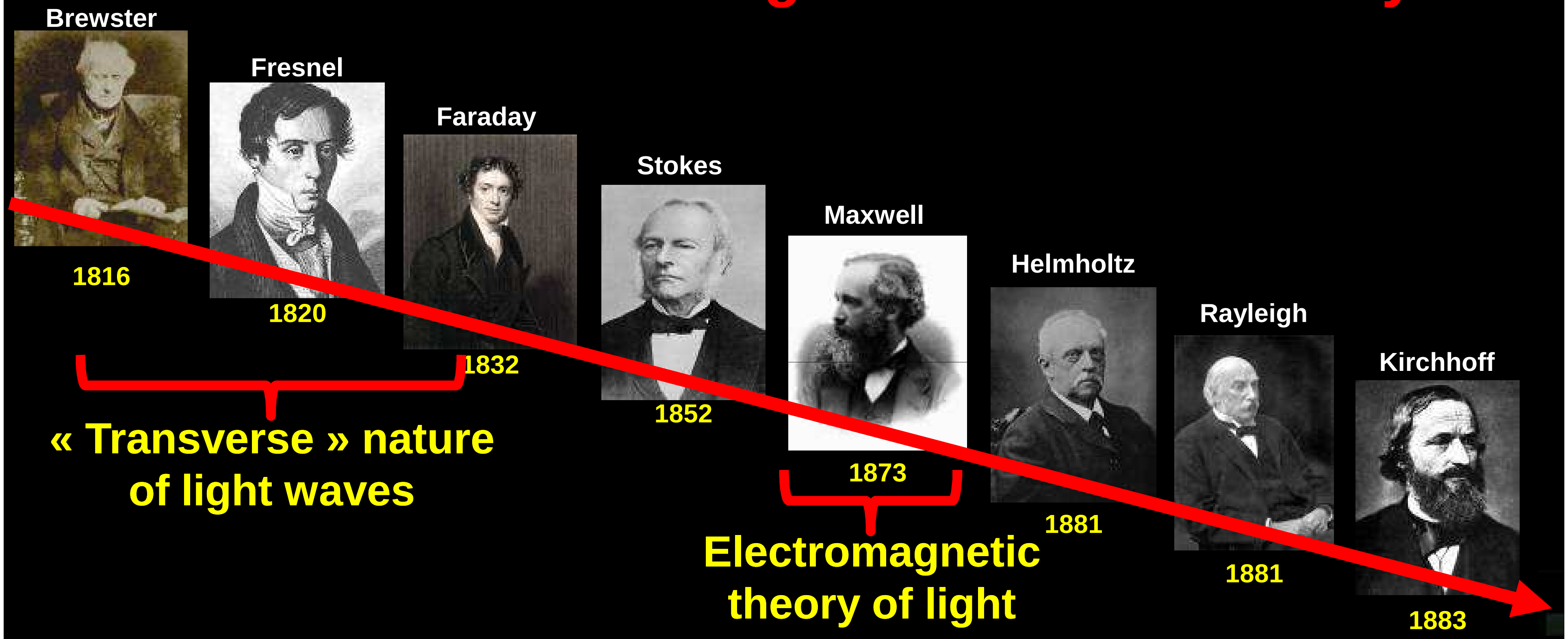
Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of Polarization leading to Radar Polarimetry



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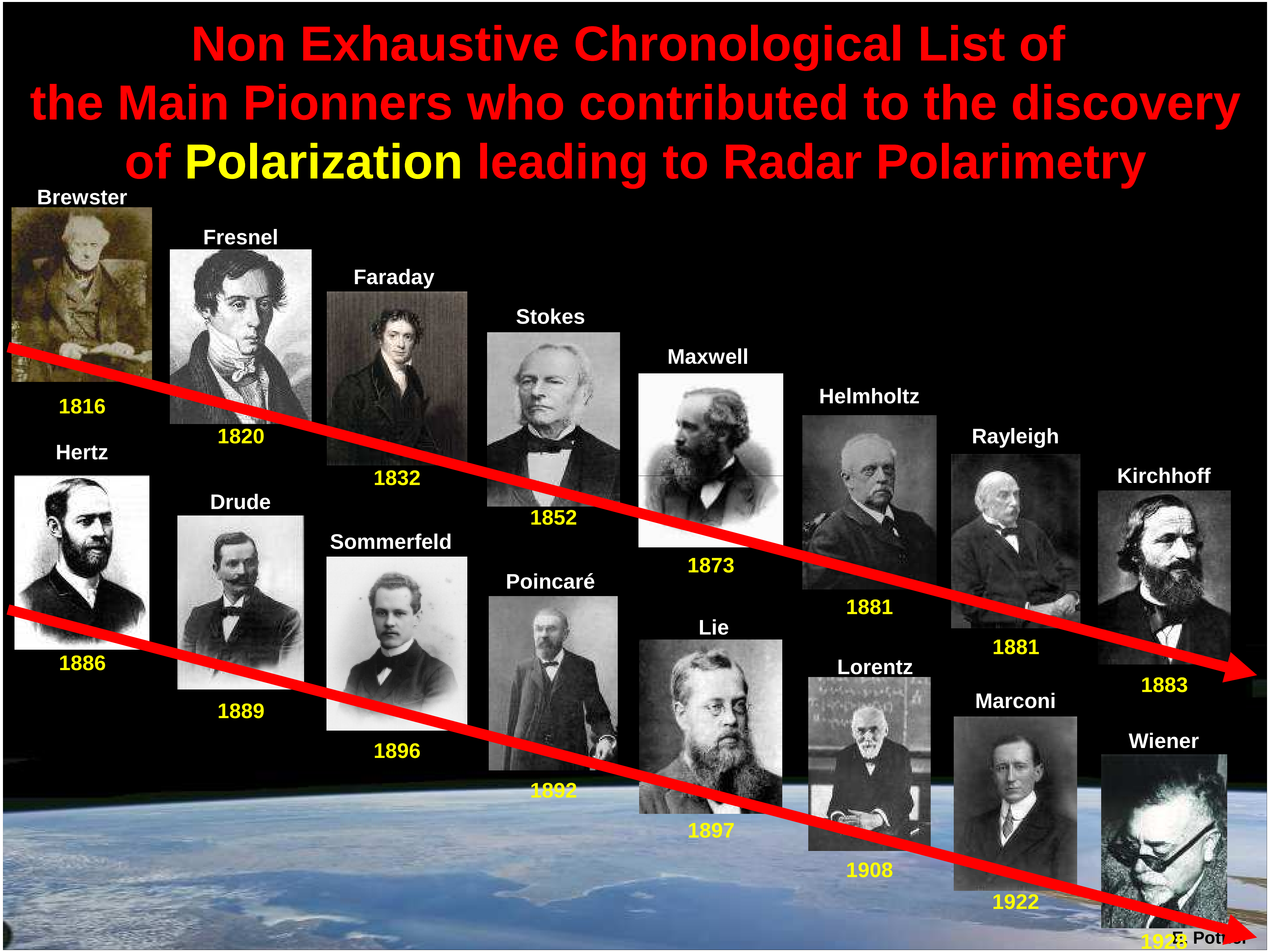
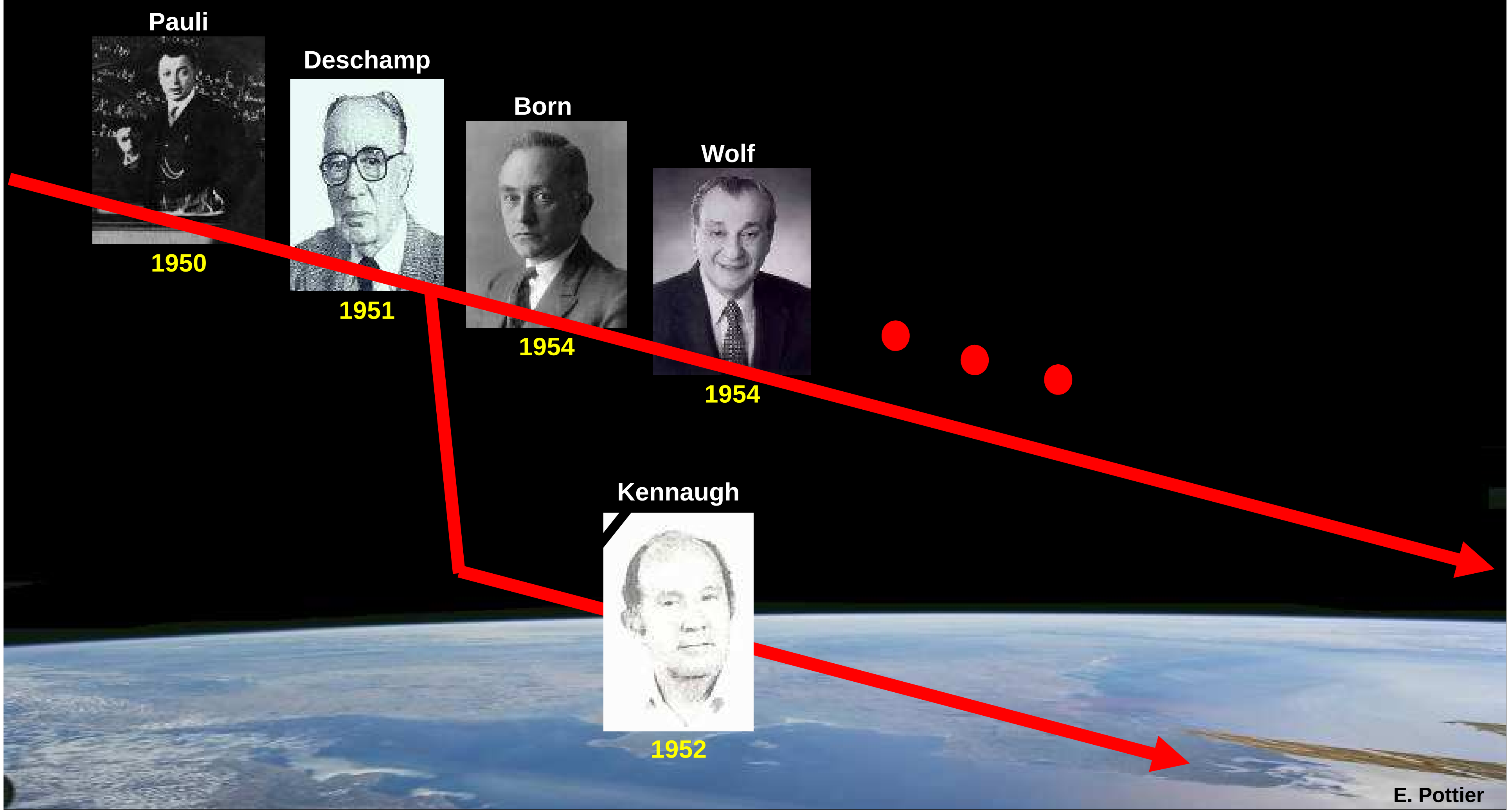

Non Exhaustive Chronological List of
the Main Pionners who contributed to the discovery
of Polarization leading to Radar Polarimetry
Brewster
1816
Fresnel
1820
Faraday
1832
Stokes
1852
Maxwell
1873
Helmholtz
1881
Rayleigh
1881
Kirchhoff
1883
Hertz
1886
Drude
1889
Sommerfeld
1896
Poincaré
1892
Lie
1897
Lorentz
1908
Marconi
1922
Wiener
1928

Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of Polarization leading to Radar Polarimetry

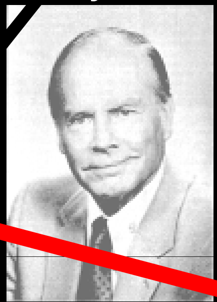


Non Exhaustive Chronological List of the Main Pionners who contributed to the discovery of Polarization leading to Radar Polarimetry

Kennaugh

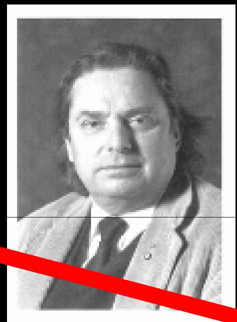
1952

Huynen



1970

W. M. Boerner



1980

The Radar Polarimetric Triptych

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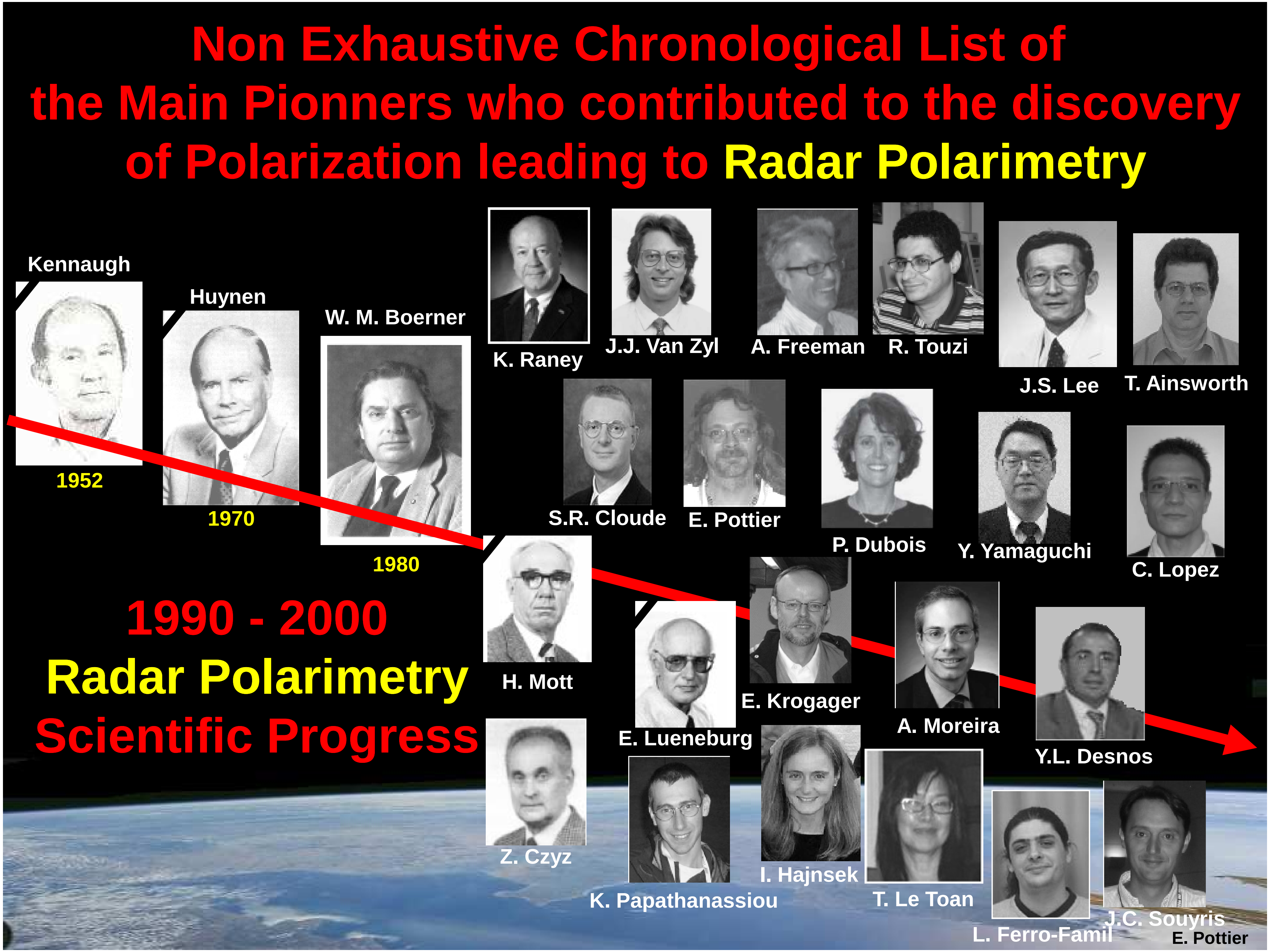
Non Exhaustive Chronological List of
the Main Pionners who contributed to the discovery
of Polarization leading to Radar Polarimetry
Kennaugh
1952
Huynen
1970
W. M. Boerner
1980
K. Raney
J.J. Van Zyl
A. Freeman
R. Touzi
J.S. Lee
T. Ainsworth
S.R. Cloude
E. Pottier
P. Dubois
Y. Yamaguchi
C. Lopez
H. Mott
E. Lueneburg
E. Krogager
A. Moreira
Y.L. Desnos
1990 - 2000
Radar Polarimetry
Scientific Progress
Z. Czyz
K. Papathanassiou
I. Hajnsek
T. Le Toan
L. Ferro-Famil
J.C. Souyris
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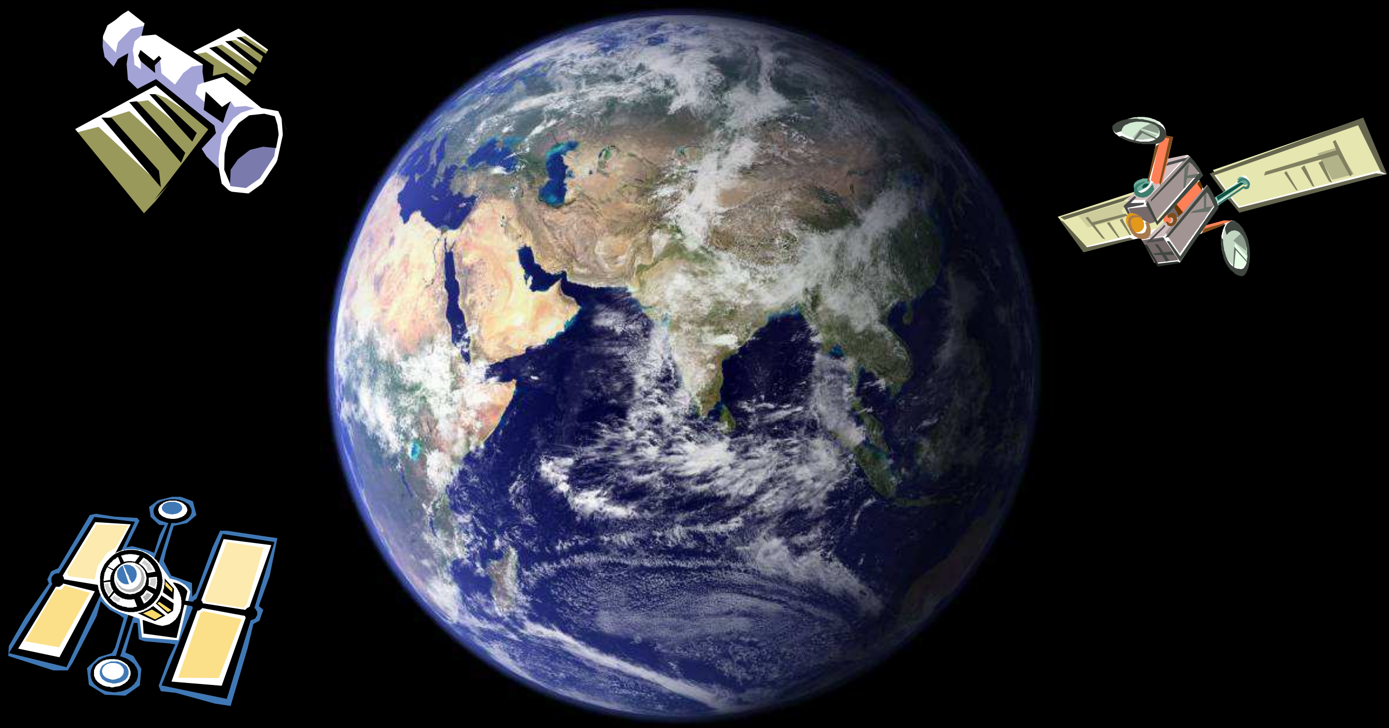
Polarimetric SAR



Spaceborne Sensors

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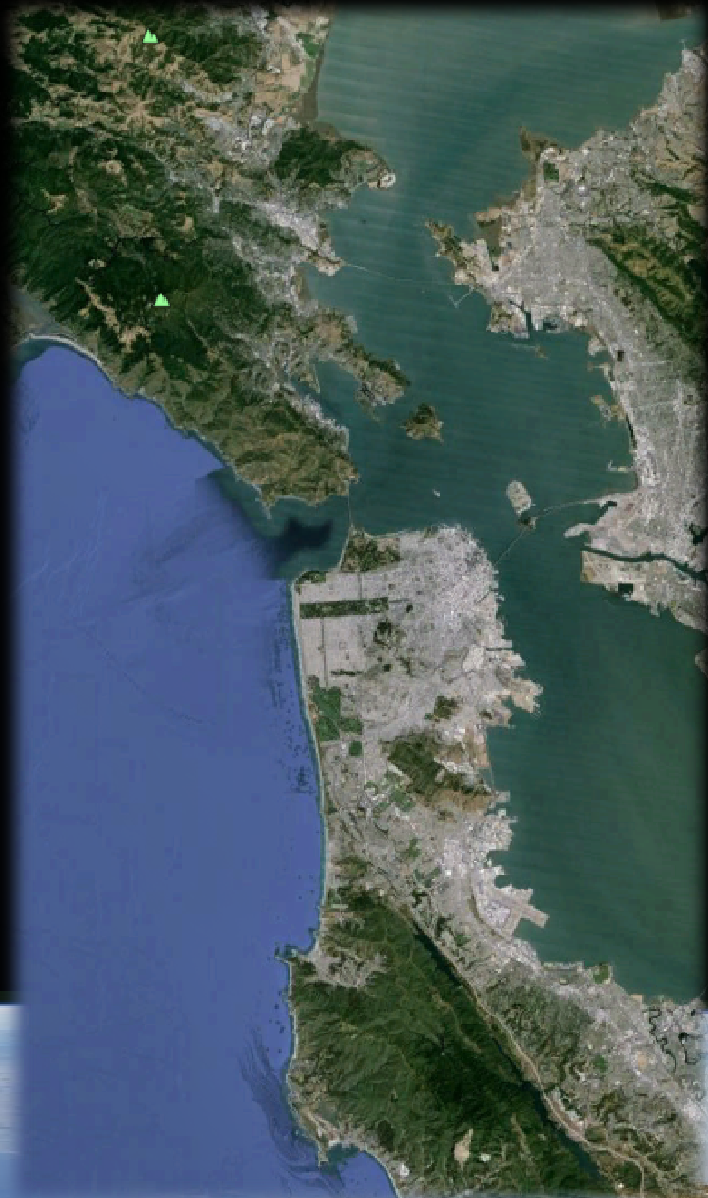
San Francisco Bay



ALOS : Advanced Land Observing Satellite
PALSAR : Phase Array L-Band SAR

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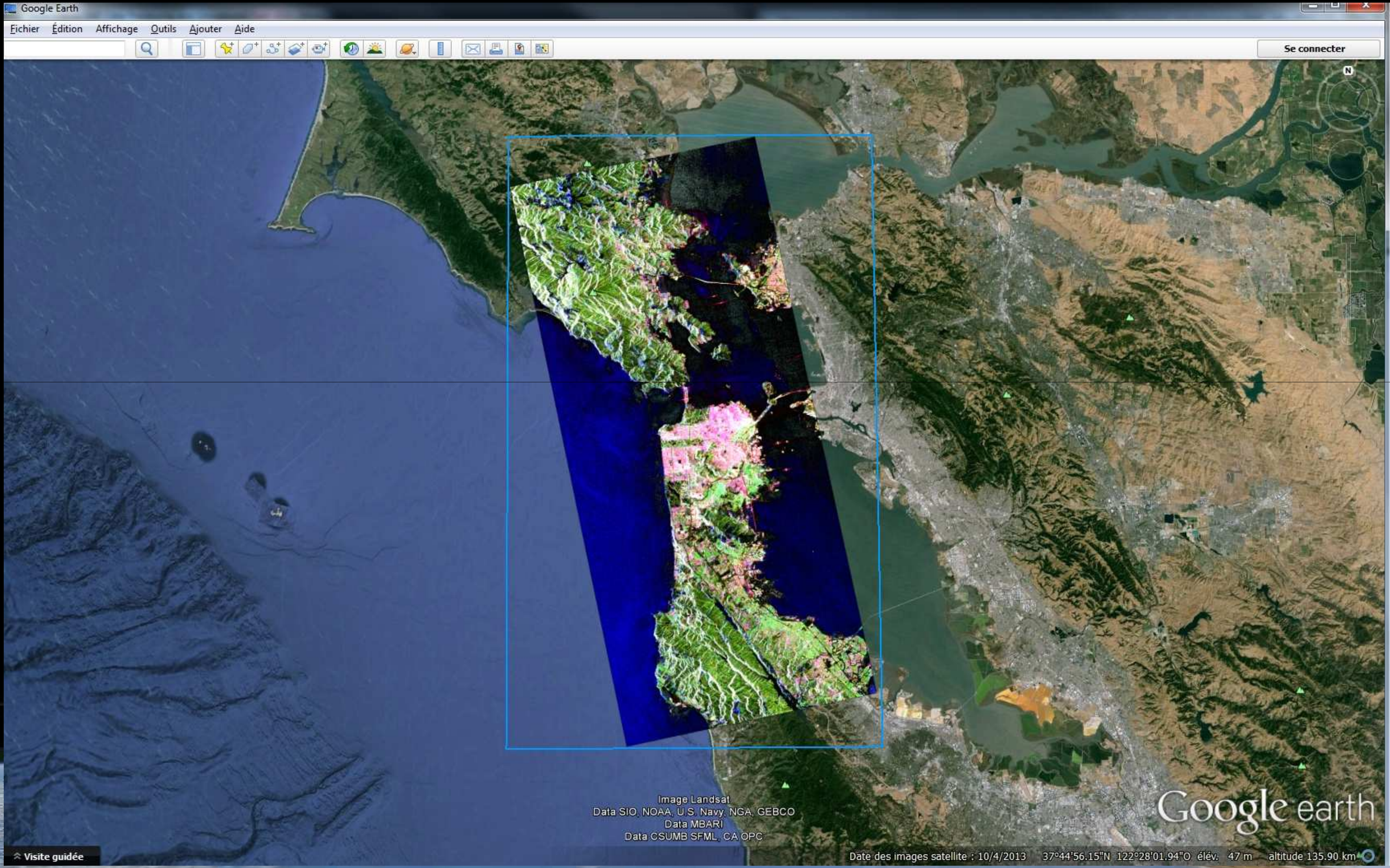
San Francisco Bay



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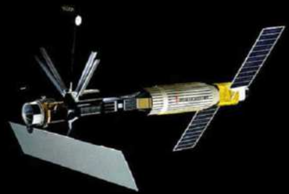
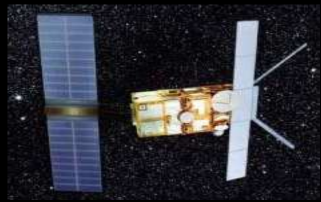
Space-borne Sensors



SEASAT
NASA/JPL (USA)
L-Band, 1978

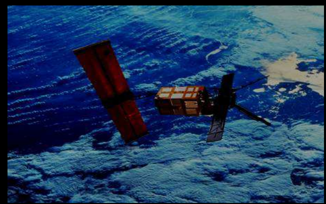


ERS-1
European Space Agency (ESA)
C-Band, 1991-2000

J-ERS-1
Japanese Space Agency (NASDA)
L-Band, 1992-1998

SIR-C/X-SAR
NASA/JPL, L- and C-Band (quad)
DLR / ASI, X-band
April and October 1994

RadarSAT-1
Canadian Space Agency (CSA)
C-Band, 1995-today



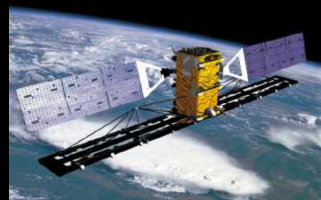
ERS-2
European Space Agency (ESA)
C-Band, 1995-today

Shuttle Radar Topography Mission (SRTM)
NASA/JPL (C-Band), DLR (X-Band)
February 2000

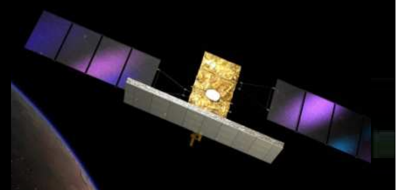
ENVISAT / ASAR
European Space Agency (ESA)
C-Band (dual), 2002-today

ALOS / PALSAR
Japanese Space Agency (JAXA)
L-Band (quad), 2006

TerraSAR-X
German Aerospace Center (DLR) / Astrium
X-Band (dual), 2007



RadarSAT-II
Canadian Space Agency (CSA)
C-Band (quad), 2007



COSMO-SkyMed
Italian Space Agency (ASI)
X-Band, 2007

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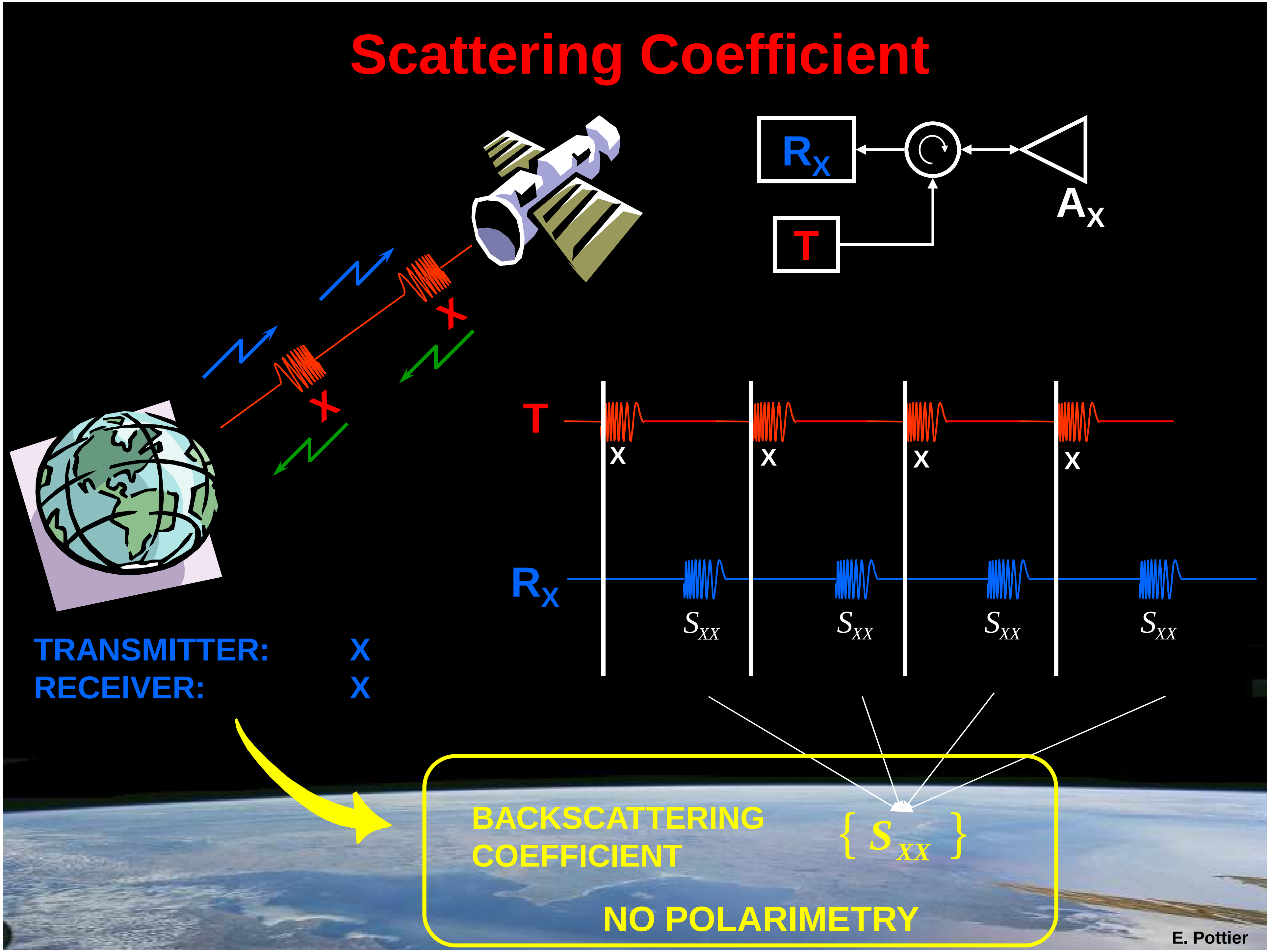
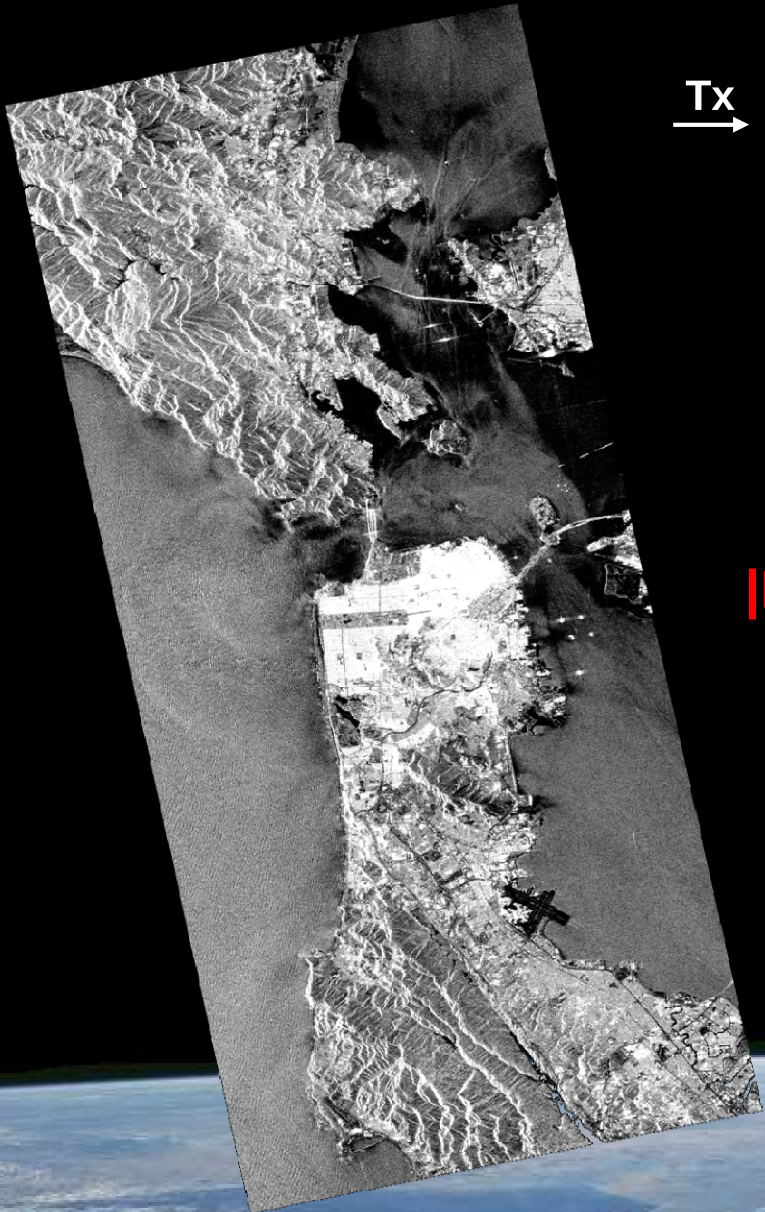
Scattering Coefficient



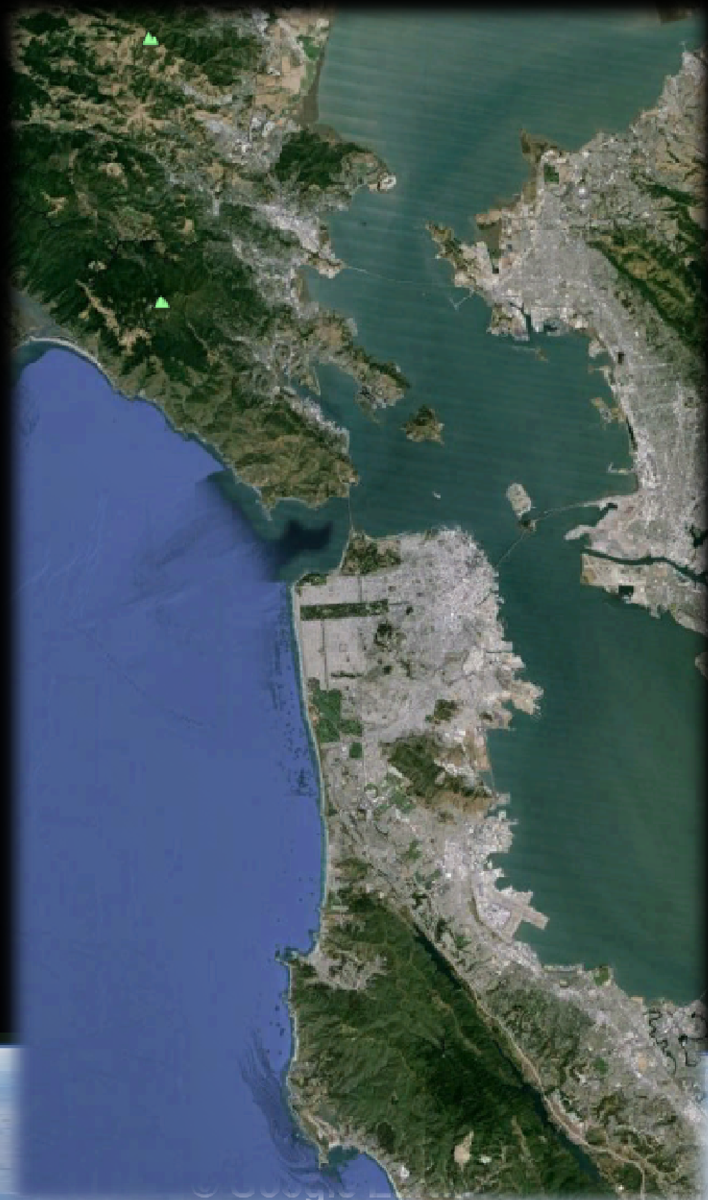
Space-borne Sensors



Tx

Rx

$|HH|_{dB}$



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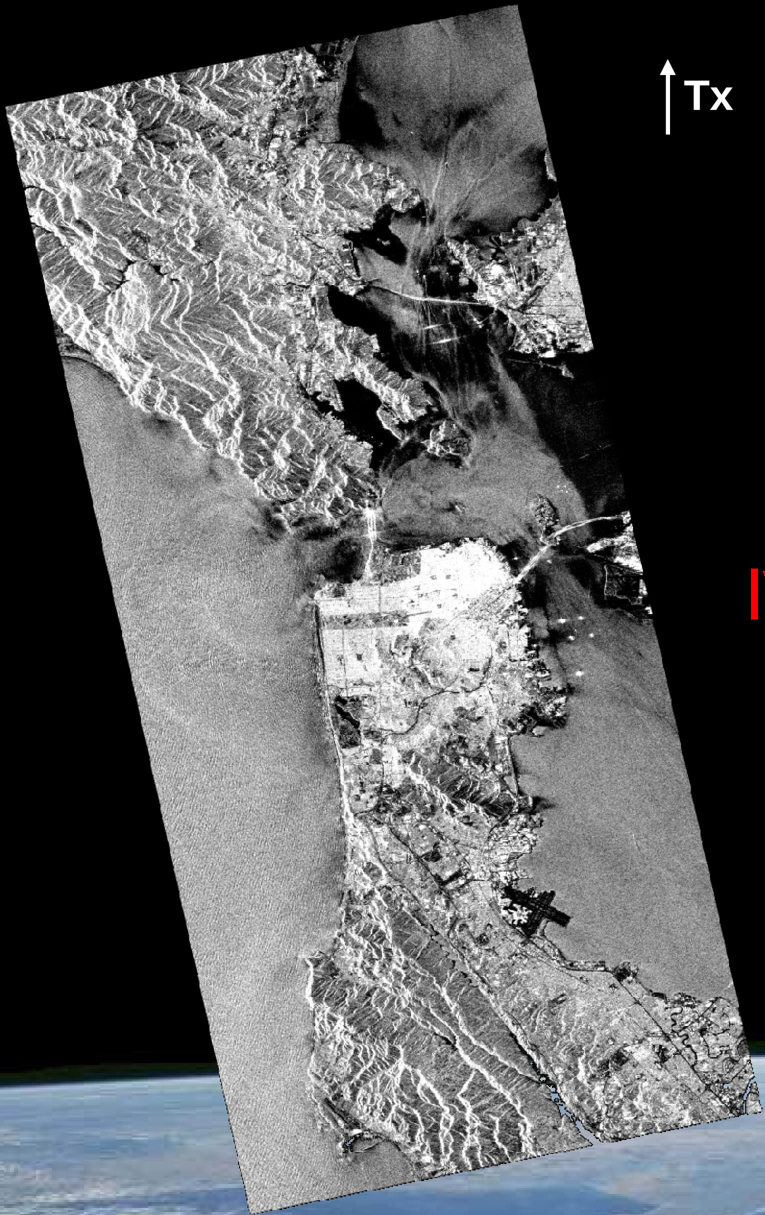
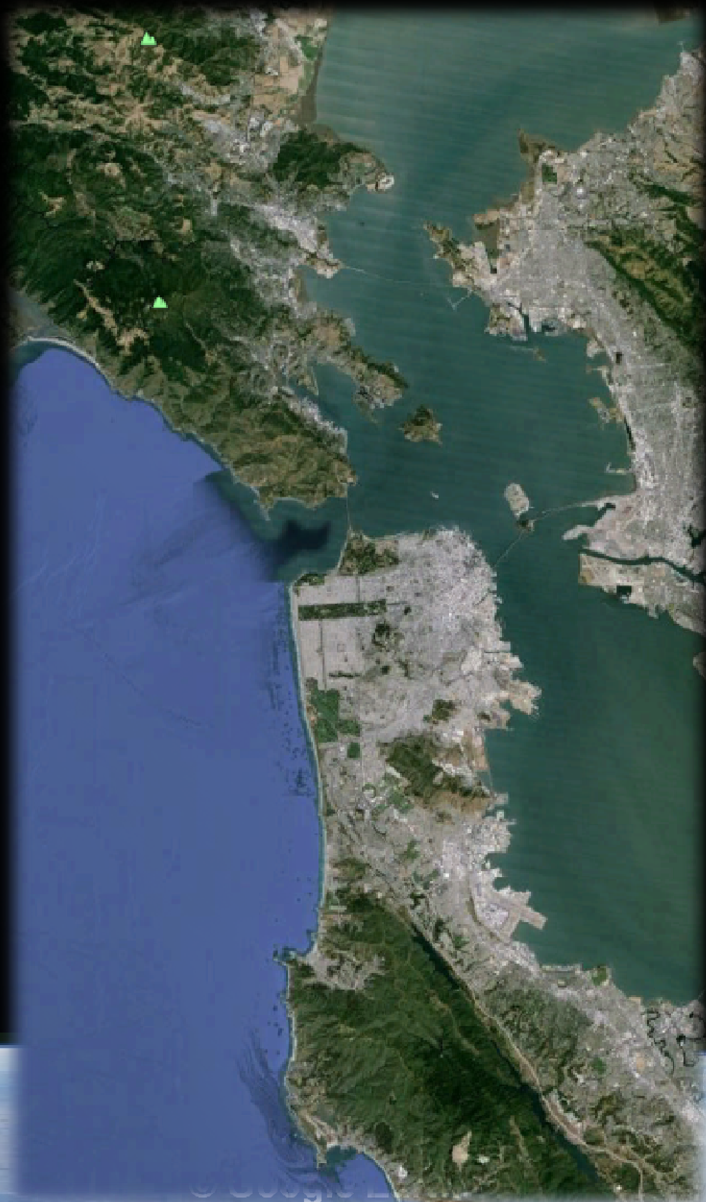
Space-borne Sensors



Tx Rx

$|VV|_{dB}$



San Francisco Bay – (L-Band)

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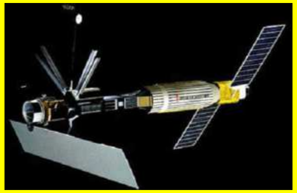
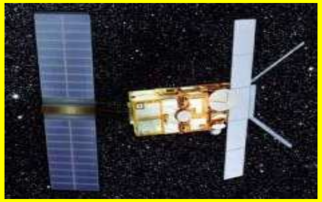
Space-borne Sensors



SEASAT
NASA/JPL (USA)
L-Band, 1978



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Japanese Space Agency (NASDA)
L-Band, 1992-1998

SIR-C/X-SAR
NASA/JPL, L- and C-Band (quad)
DLR / ASI, X-band
April and October 1994

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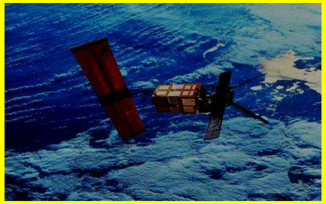
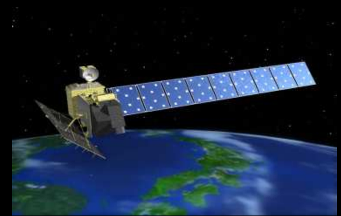
Canadian Space Agency (CSA)
C-Band, 1995-today



ERS-2
European Space Agency (ESA)
C-Band, 1995-today

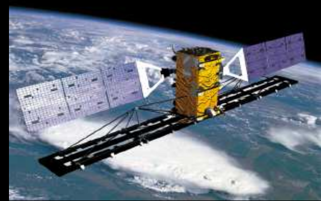
Shuttle Radar Topography Mission (SRTM)
NASA/JPL (C-Band), DLR (X-Band)
February 2000

ENVISAT / ASAR
European Space Agency (ESA)
C-Band (dual), 2002-today

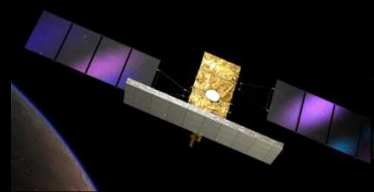


ALOS / PALSAR
Japanese Space Agency (JAXA)
L-Band (quad), 2006

TerraSAR-X
German Aerospace Center (DLR) / Astirum
X-Band (dual), 2007



RadarSAT-II
Canadian Space Agency (CSA)
C-Band (quad), 2007



COSMO-SkyMed
Italian Space Agency (ASI)
X-Band, 2007

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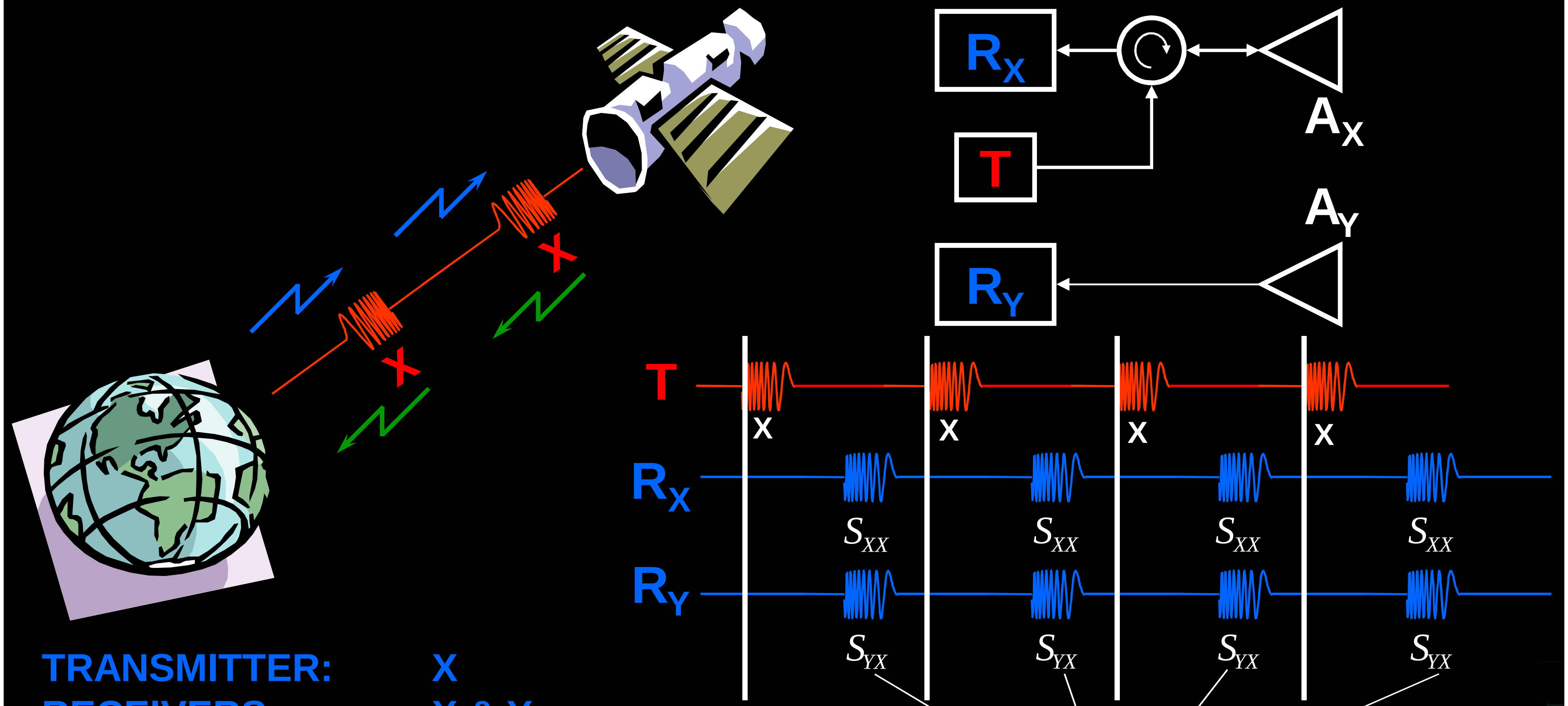
Wave Polarimetry



TRANSMITTER: X

RECEIVERS: X & Y

JONES VECTORS $\left\{ \underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix} \right\}$

WAVE POLARIMETRY

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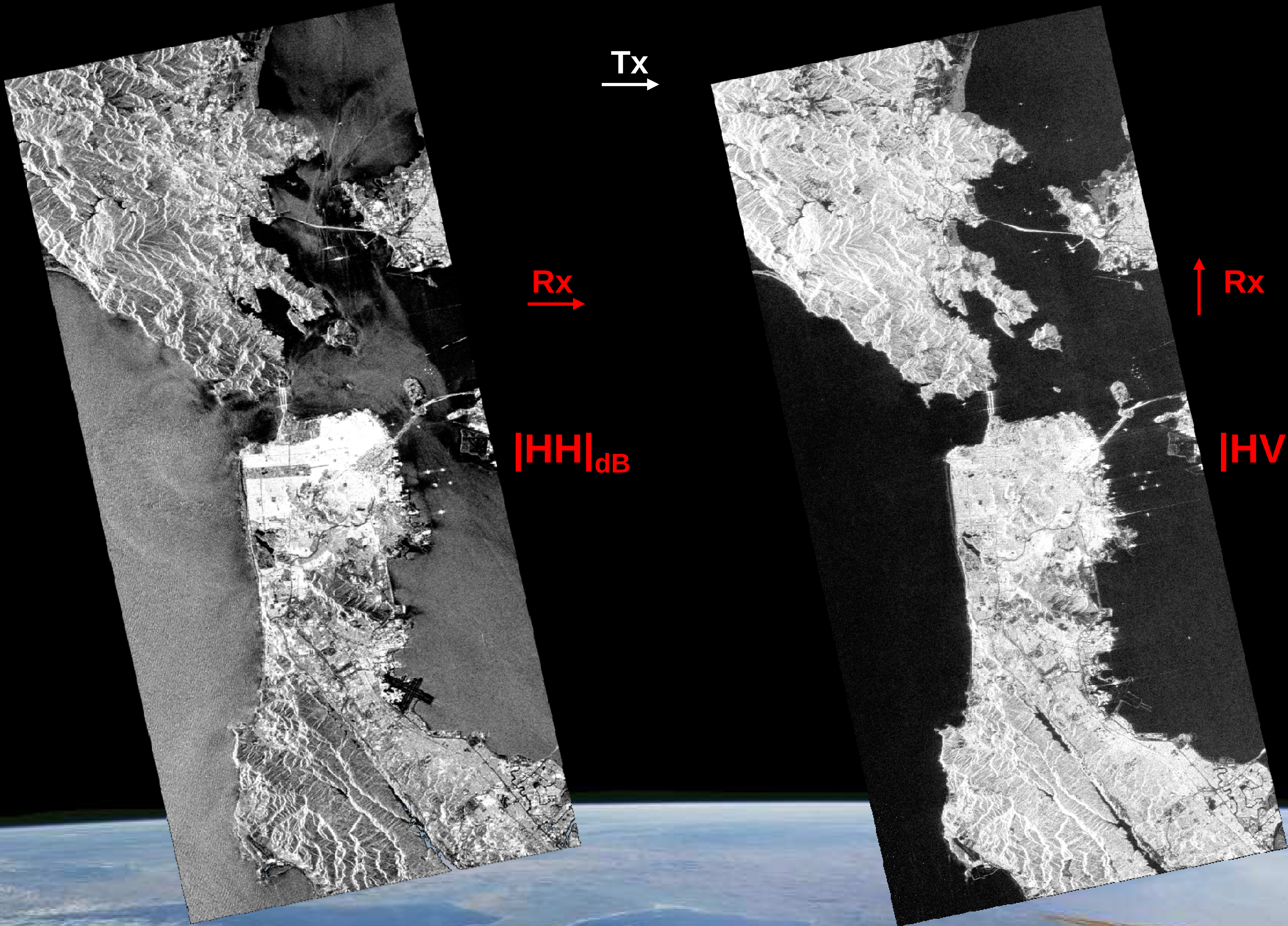
Space-borne Sensors



San Francisco Bay – (L-Band)

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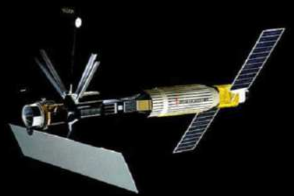
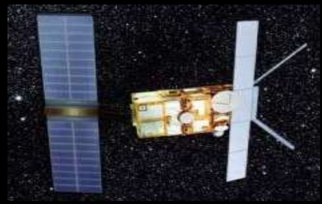
Space-borne PolSAR Sensors



SEASAT
NASA/JPL (USA)
L-Band, 1978



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DLR / ASI, X-band
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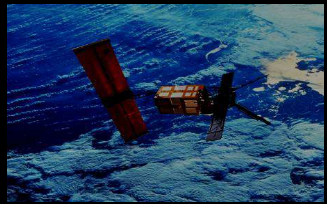

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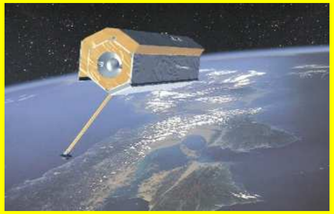
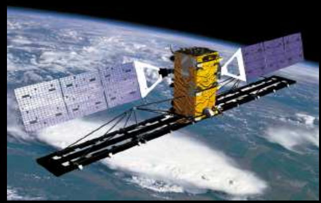
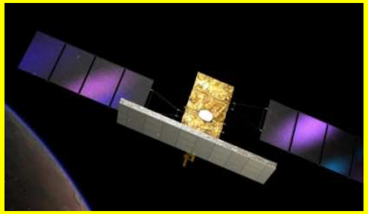


TerraSAR-X
German Aerospace Center (DLR) / Astrium
X-Band (dual), 2007



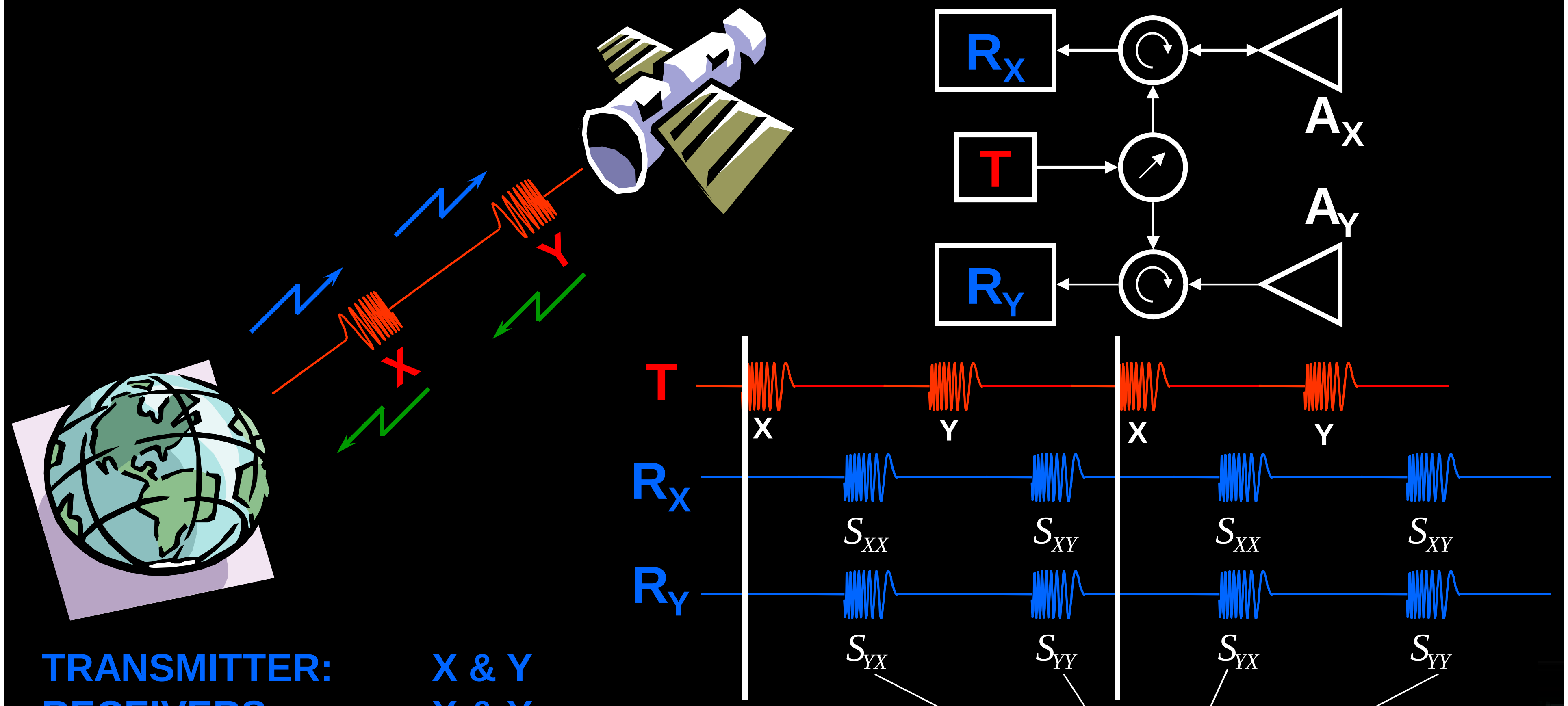
RadarSAT-II
Canadian Space Agency (CSA)
C-Band (quad), 2007



COSMO-SkyMed
Italian Space Agency (ASI)
X-Band, 2007

E. Pottier

Scattering Polarimetry



TRANSMITTER: X & Y
RECEIVERS: X & Y

SINCLAIR MATRICES

$$\left\{ [S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \right\}$$

SCATTERING POLARIMETRY

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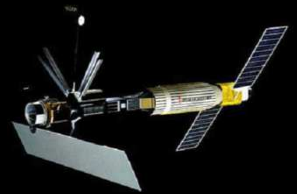
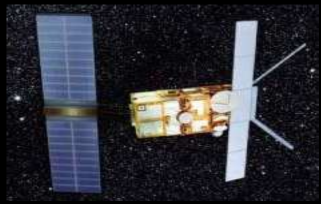
Space-borne PolSAR Sensors

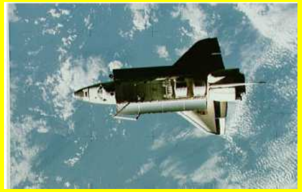


SEASAT
NASA/JPL (USA)
L-Band, 1978



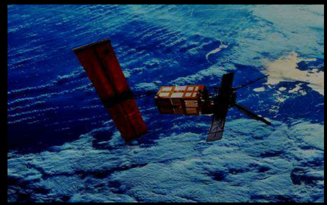
ERS-1
European Space Agency (ESA)
C-Band, 1991-2000

J-ERS-1
Japanese Space Agency (NASDA)
L-Band, 1992-1998



SIR-C/X-SAR
NASA/JPL, L- and C-Band (quad)
DLR / ASI, X-band
April and October 1994

RadarSAT-1
Canadian Space Agency (CSA)
C-Band, 1995-today

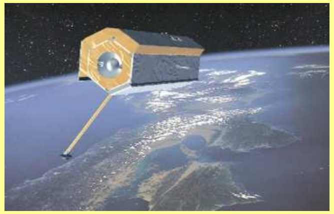


ERS-2
European Space Agency (ESA)
C-Band, 1995-today

Shuttle Radar Topography Mission (SRTM)
NASA/JPL (C-Band), DLR (X-Band)
February 2000

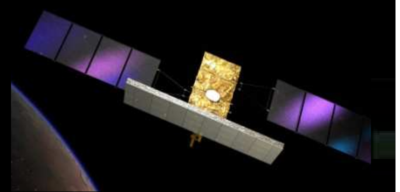
ENVISAT / ASAR
European Space Agency (ESA)
C-Band (dual), 2002-today

ALOS / PALSAR
Japanese Space Agency (JAXA)
L-Band (quad), 2006



TerraSAR-X
German Aerospace Center (DLR) / Astirum
X-Band (quad), 2007

RadarSAT-II
Canadian Space Agency (CSA)
C-Band (quad), 2007



COSMO-SkyMed
Italian Space Agency (ASI)
X-Band, 2007

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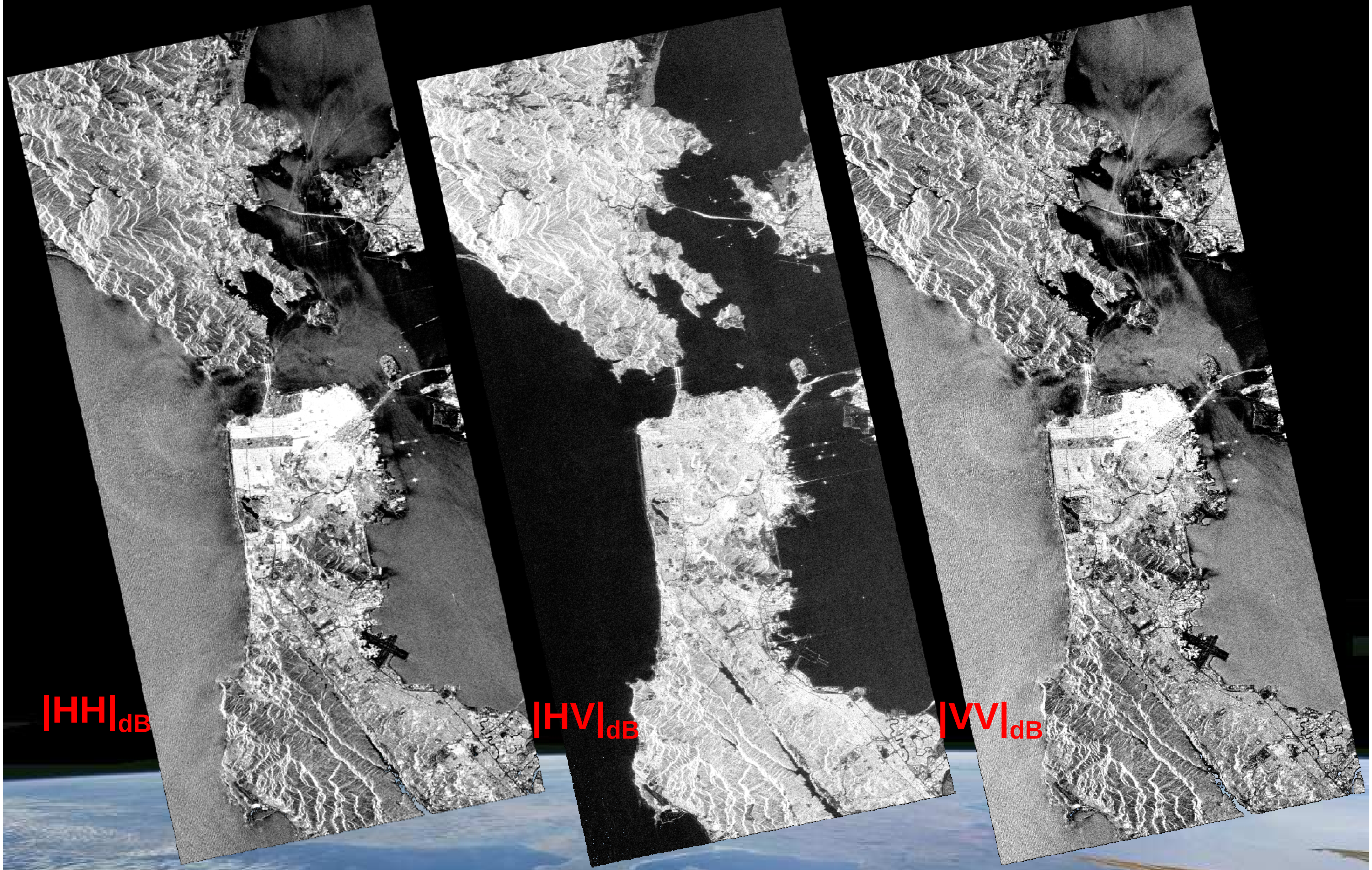
Space-borne Sensors



San Francisco Bay – (L-Band)

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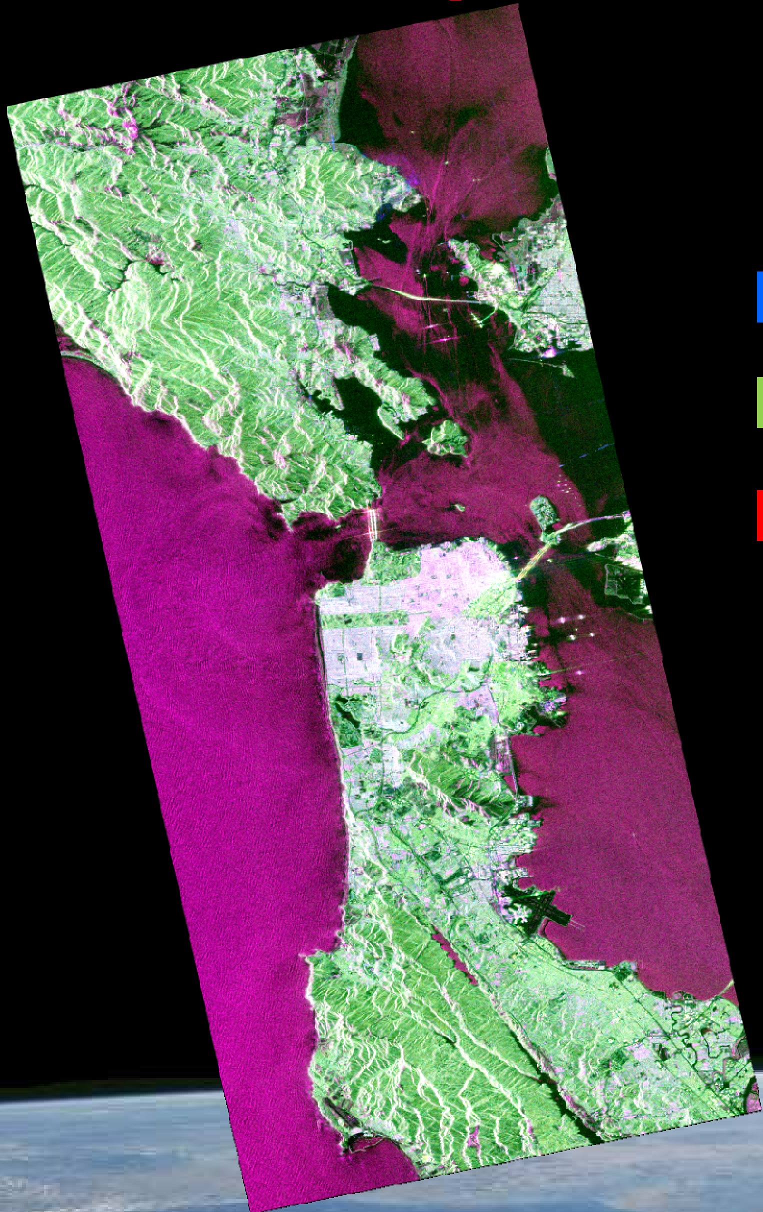
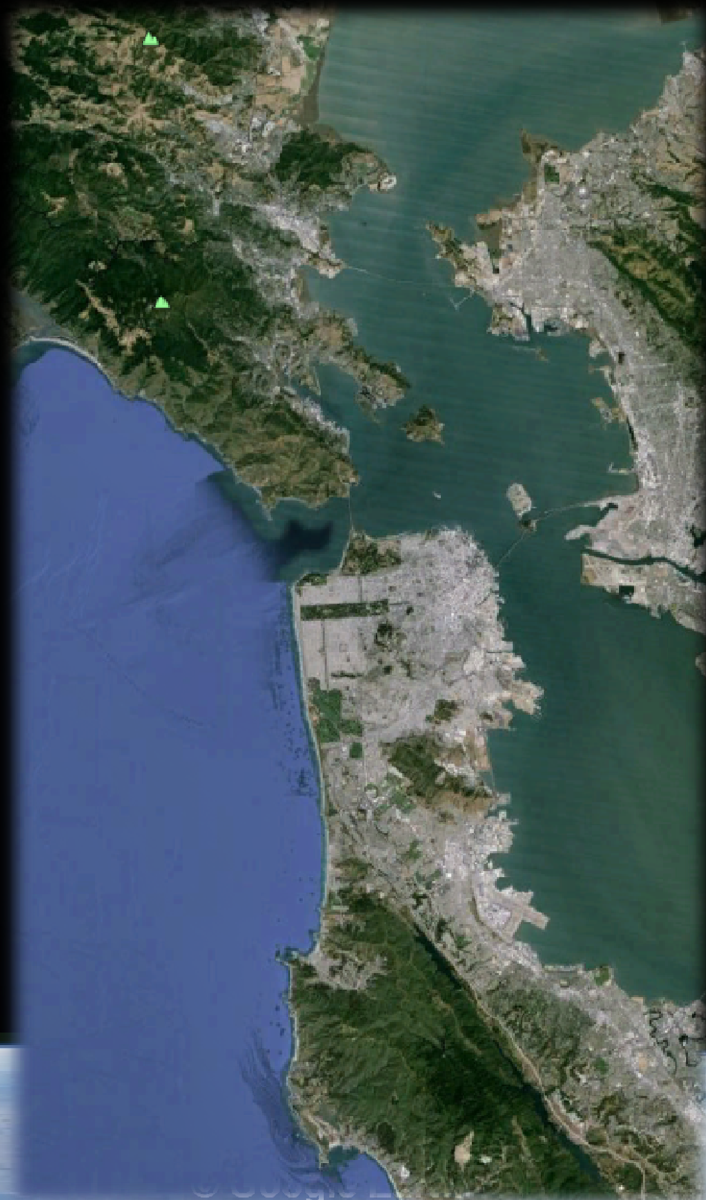
Space-borne Sensors



$|HH|_{dB}$

$|HV|_{dB}$

$|VV|_{dB}$



San Francisco Bay – (L-Band)

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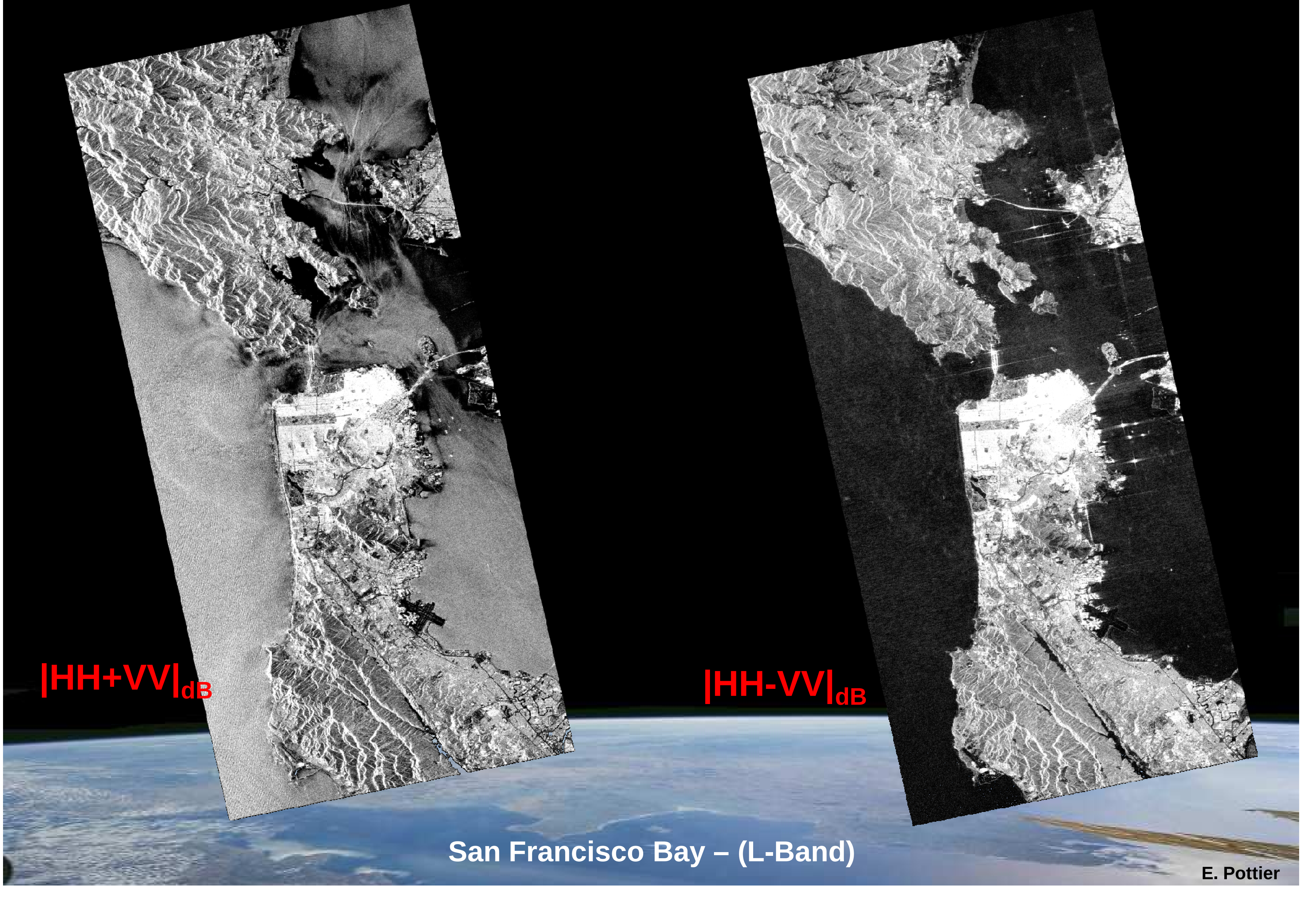
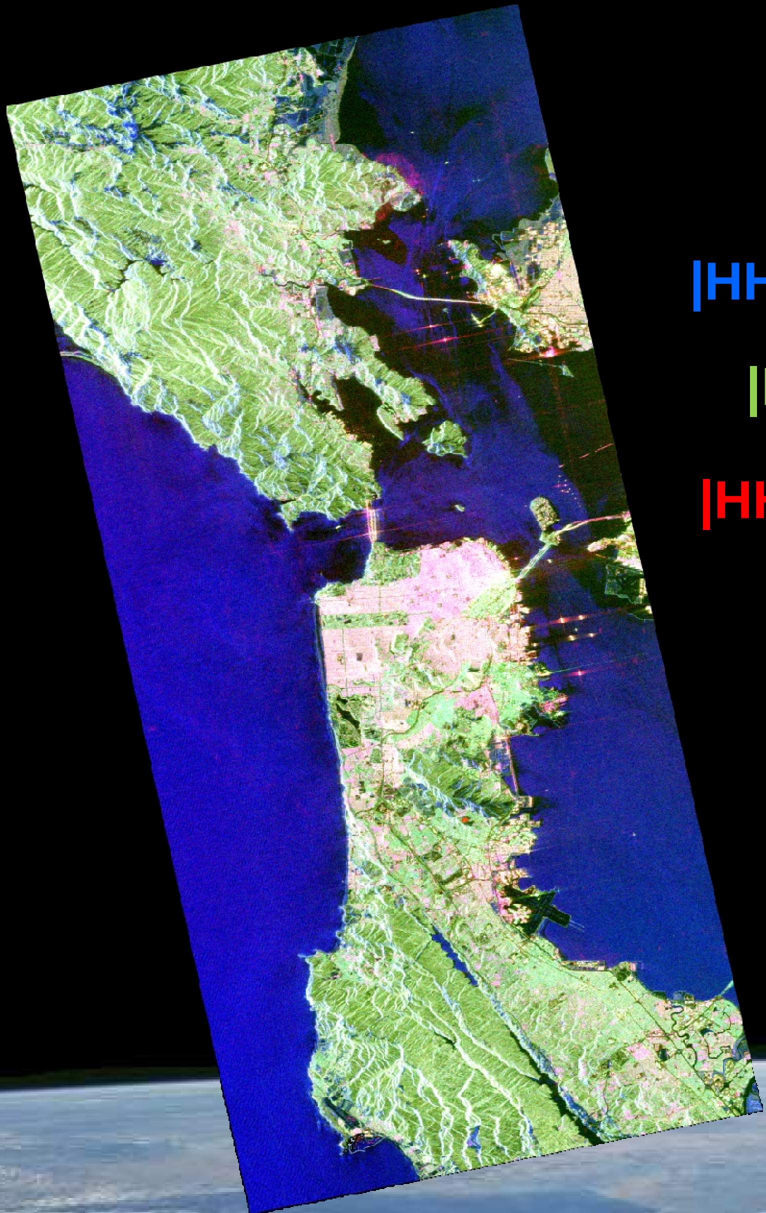
Space-borne Sensors



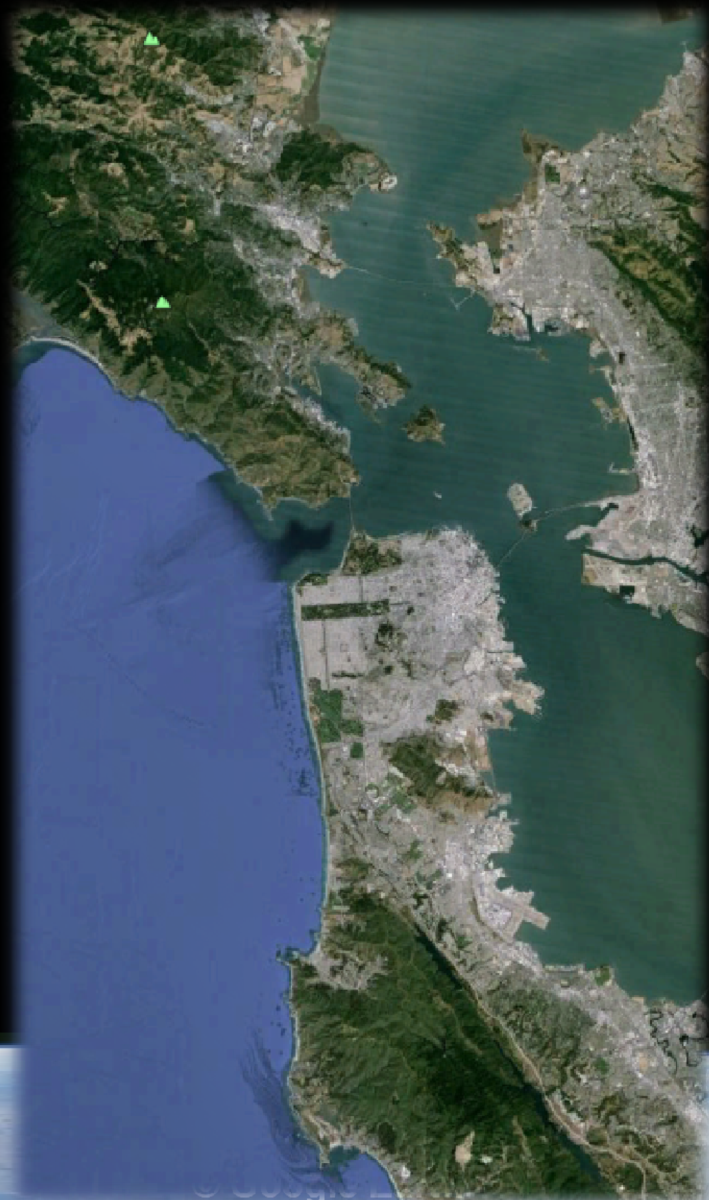
Space-borne Sensors



$|HH+VV|_{dB}$

$|HV|_{dB}$

$|HH-VV|_{dB}$



San Francisco Bay – (L-Band)

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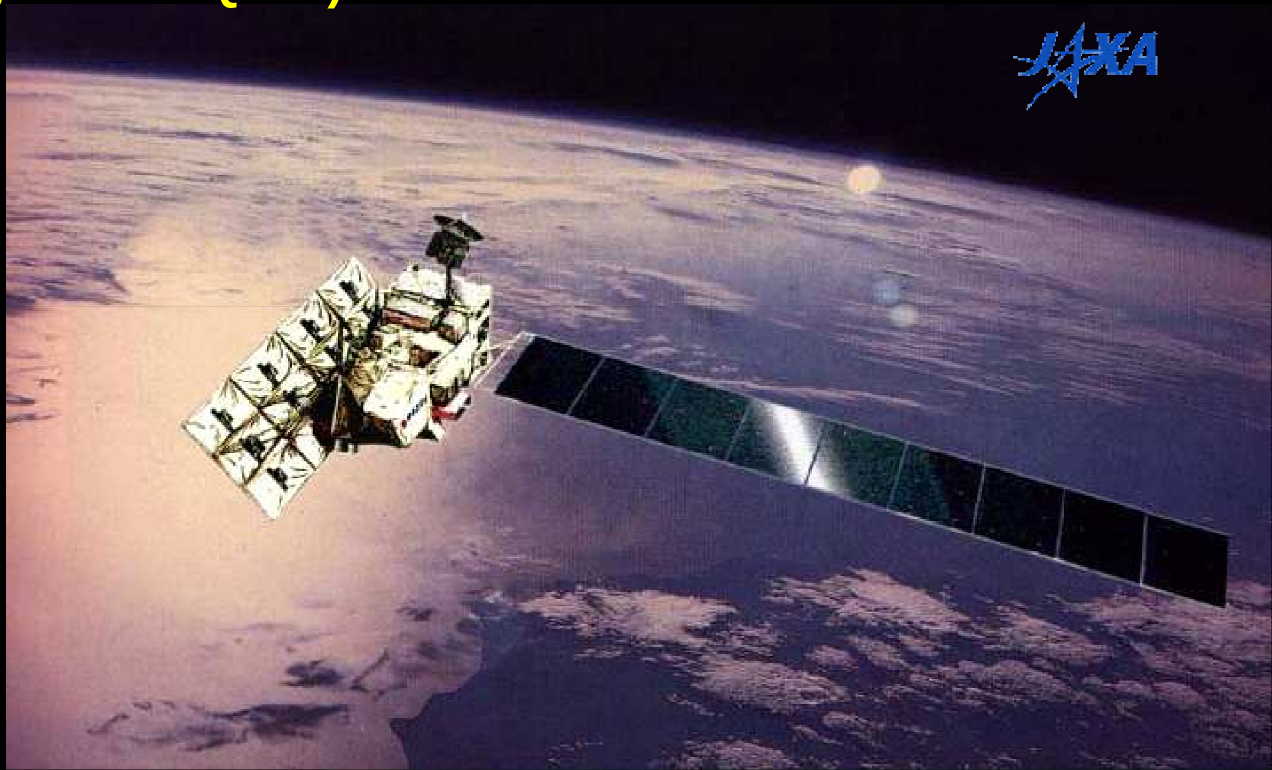
Space-borne PolSAR Sensors

ALOS - PALSAR

January 2006
L-Band (Sngl / Twin / Quad)



ALOS : Advanced Land Observing Satellite
PALSAR : Phase Array L-Band SAR

Space-borne PolSAR Sensors

TerraSAR - X

June 2007

X-Band (Sngl / Twin / Quad ?)

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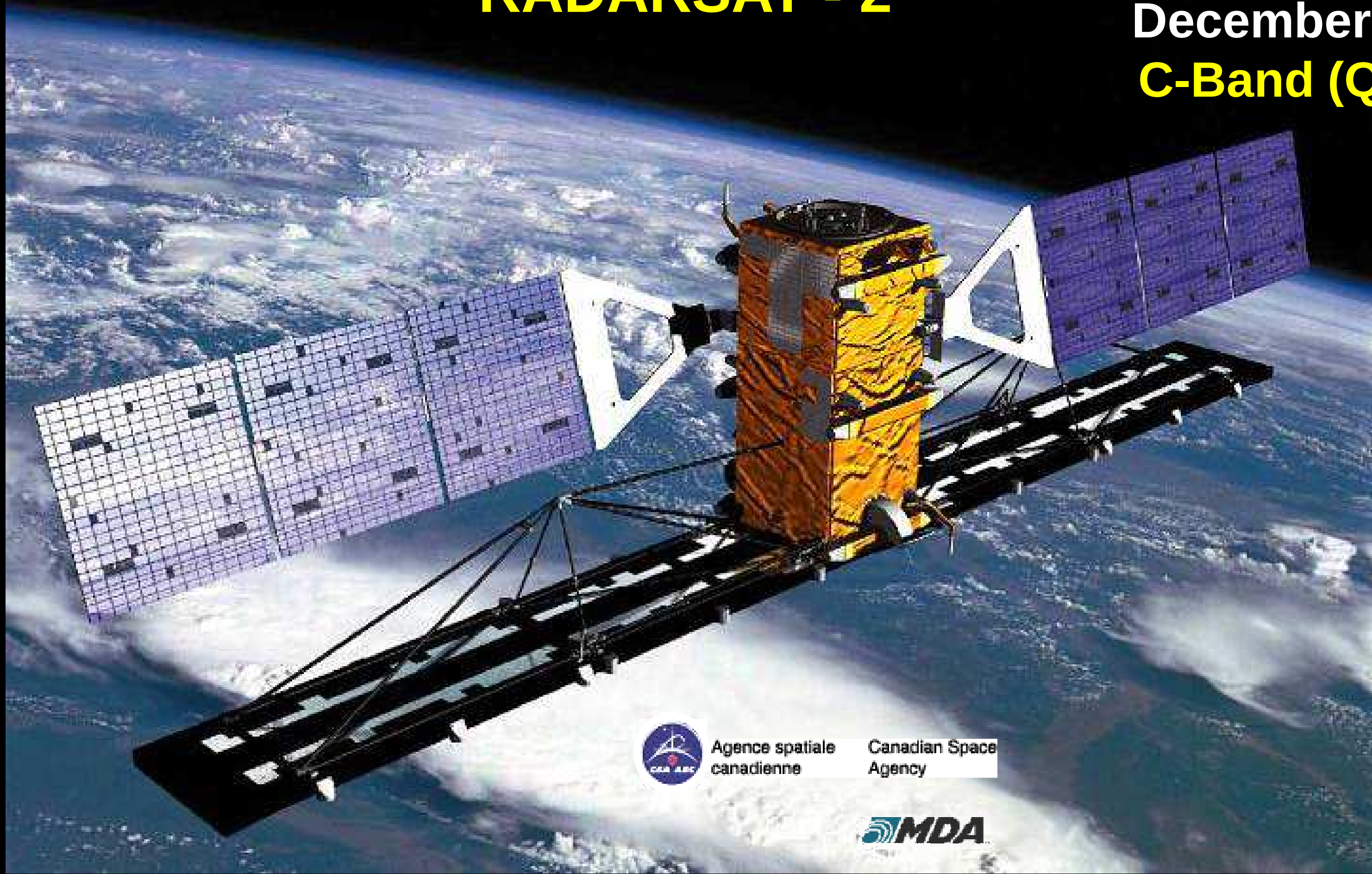
Space-borne PolSAR Sensors

RADARSAT - 2

December 2007
C-Band (Quad)



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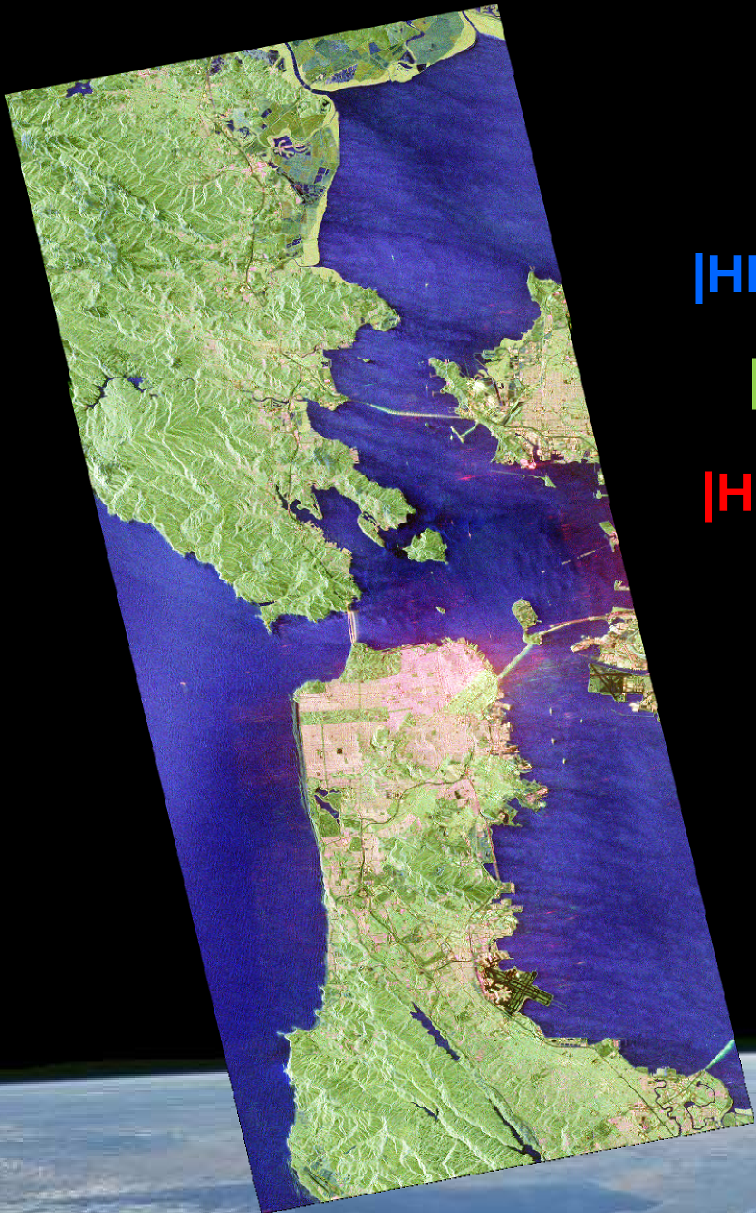
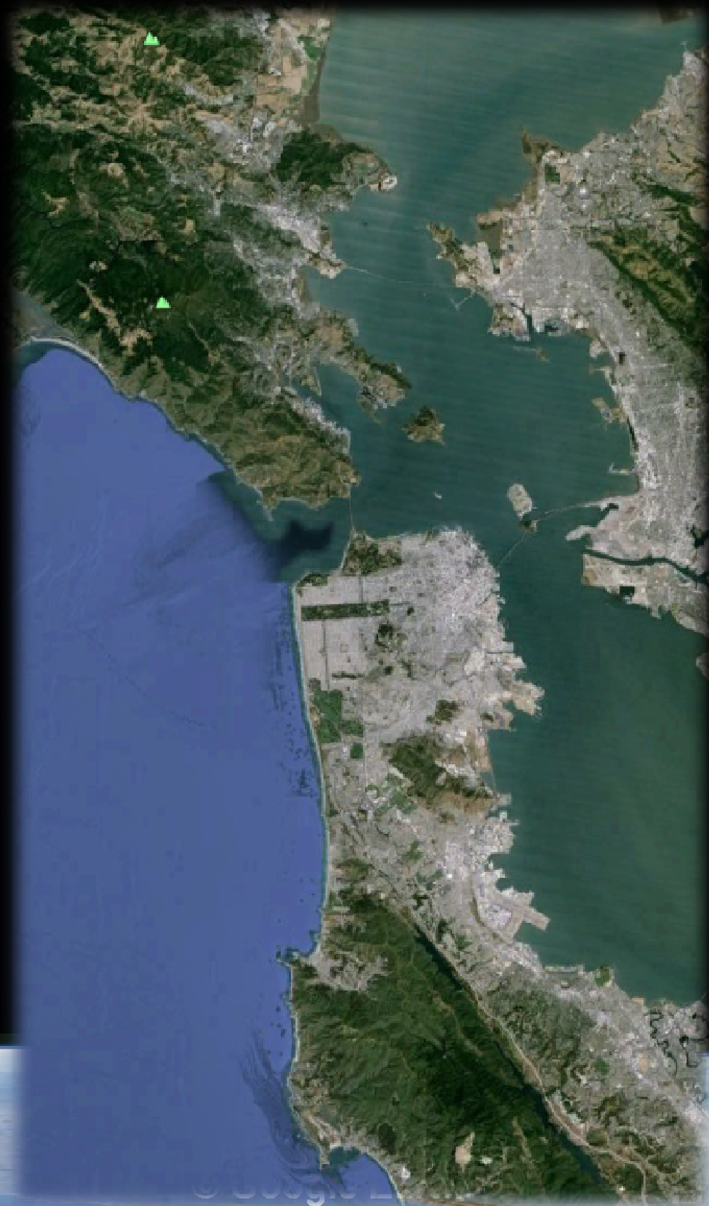
Space-borne Sensors



$|HH+VV|_{dB}$

$|HV|_{dB}$

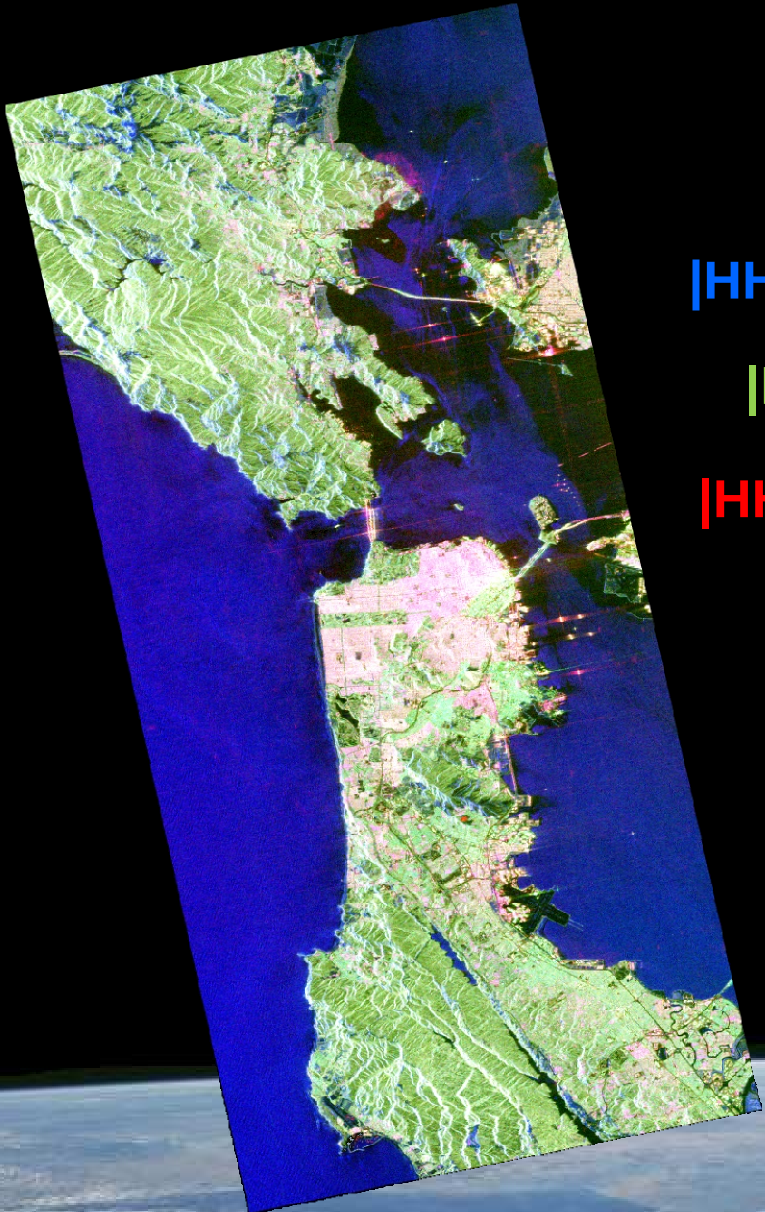
$|HH-VV|_{dB}$



San Francisco Bay – (C-Band)

E. Pottier

Space-borne Sensors



$|HH+VV|_{dB}$

$|HV|_{dB}$

$|HH-VV|_{dB}$

San Francisco Bay – (L-Band and C-Band)

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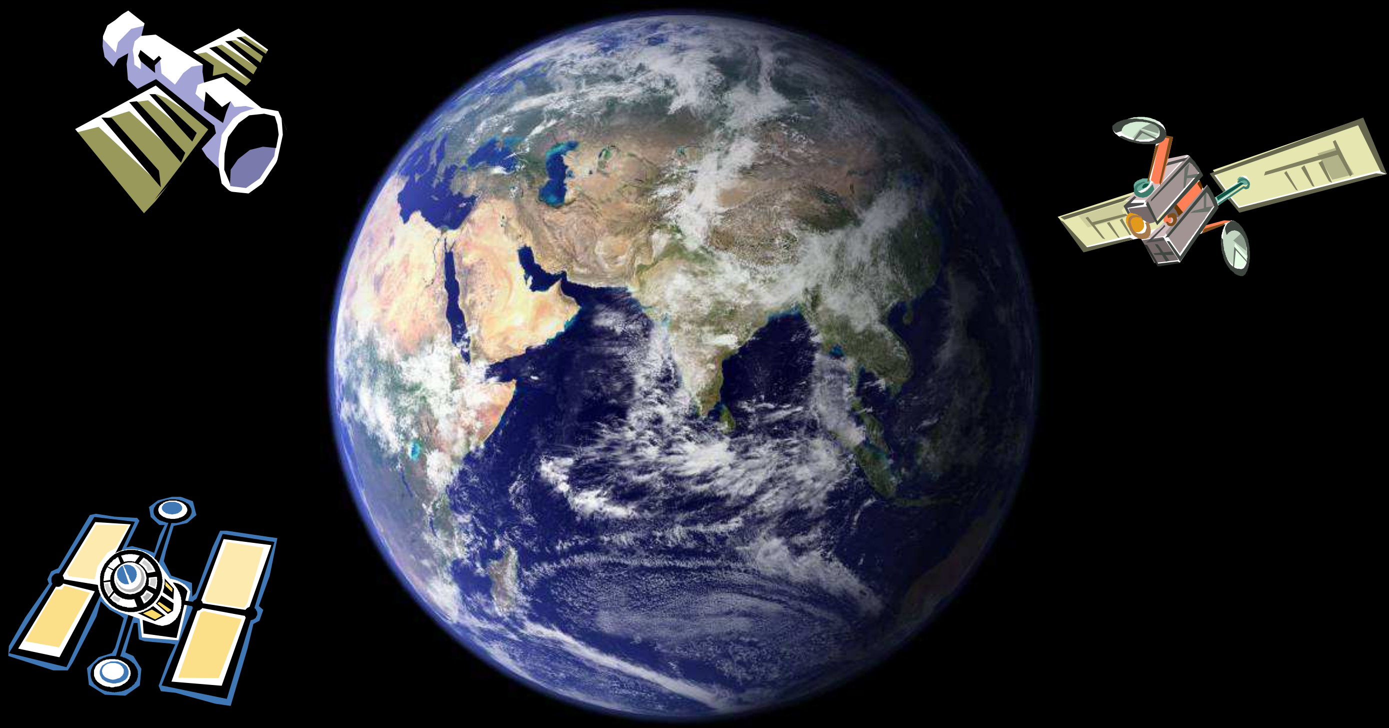
What About The Future ?



From Tomorrow …

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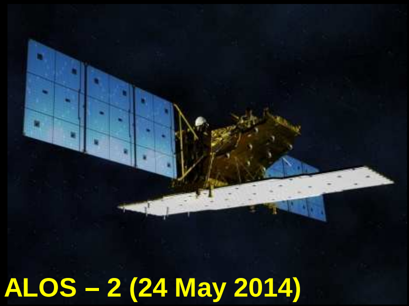
New and Future Space-Borne PolSAR Pol-InSAR Sensors

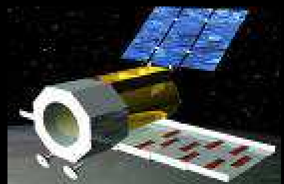
RISAT (April 2012)



ALOS – 2 (24 May 2014)

Sentinel – 1

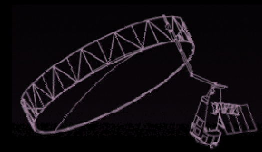
1A : 3 April 2014
1B : 2016

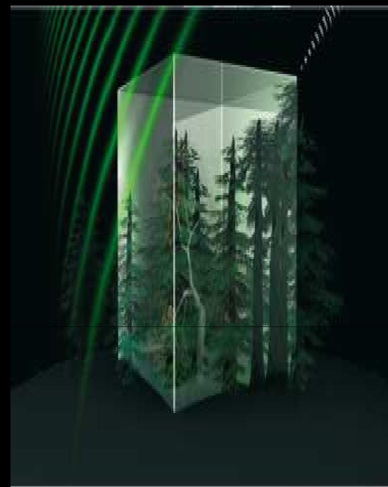


SAOCOM
1A : 2015
1B : 2016

RCM (R1 : 2016, R2-R3 :2017)

TanDEM–L
DLR - JAXA





BIOMASS (2019)

……

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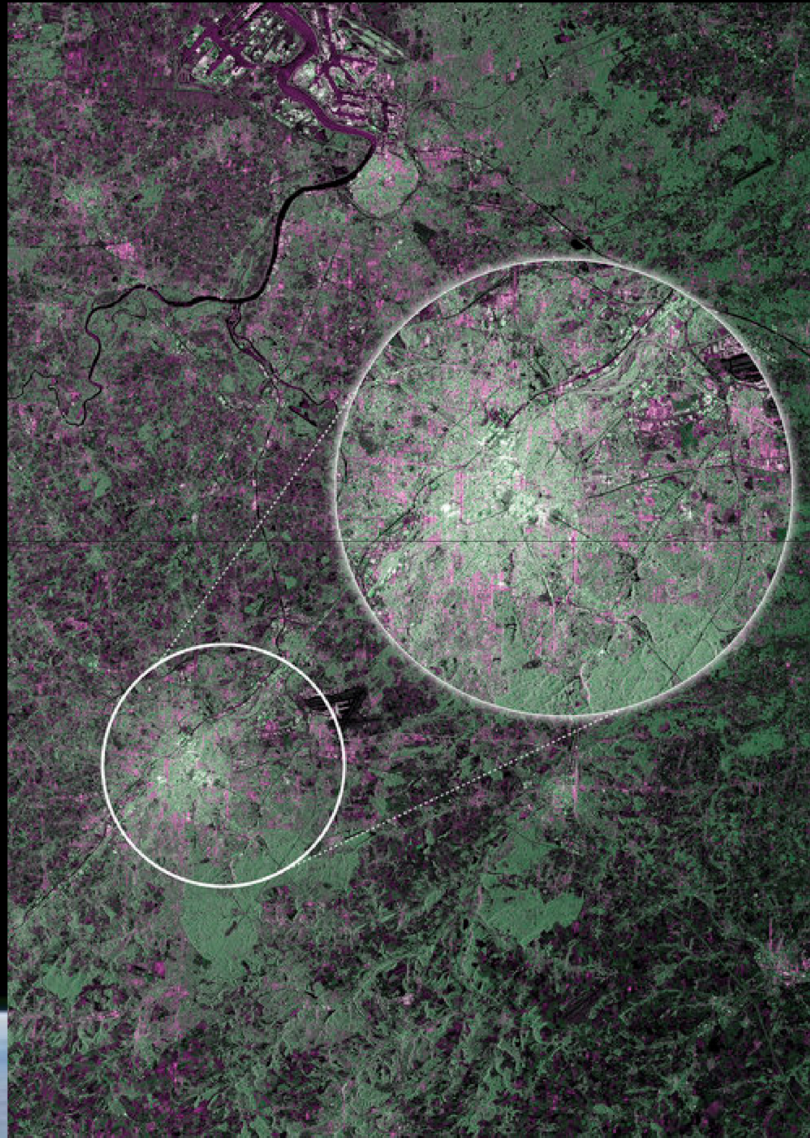
Space-borne PolSAR Sensors

SENTINEL – 1A

April 2014
C-Band (Dual)



Brussels – 12 April 2014

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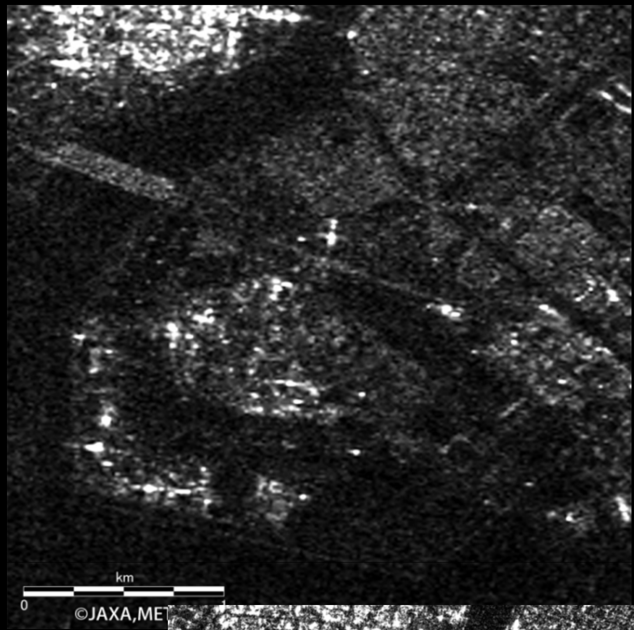
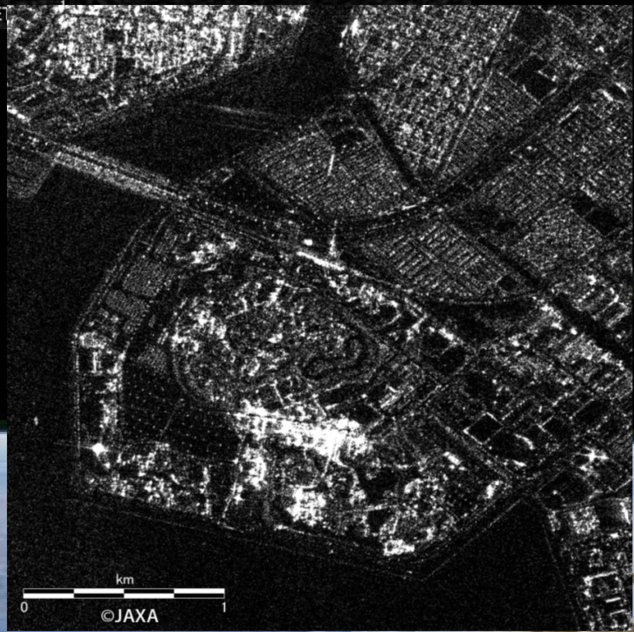
Space-borne PolSAR Sensors

ALOS - 2

May 2014
L-Band (Quad)





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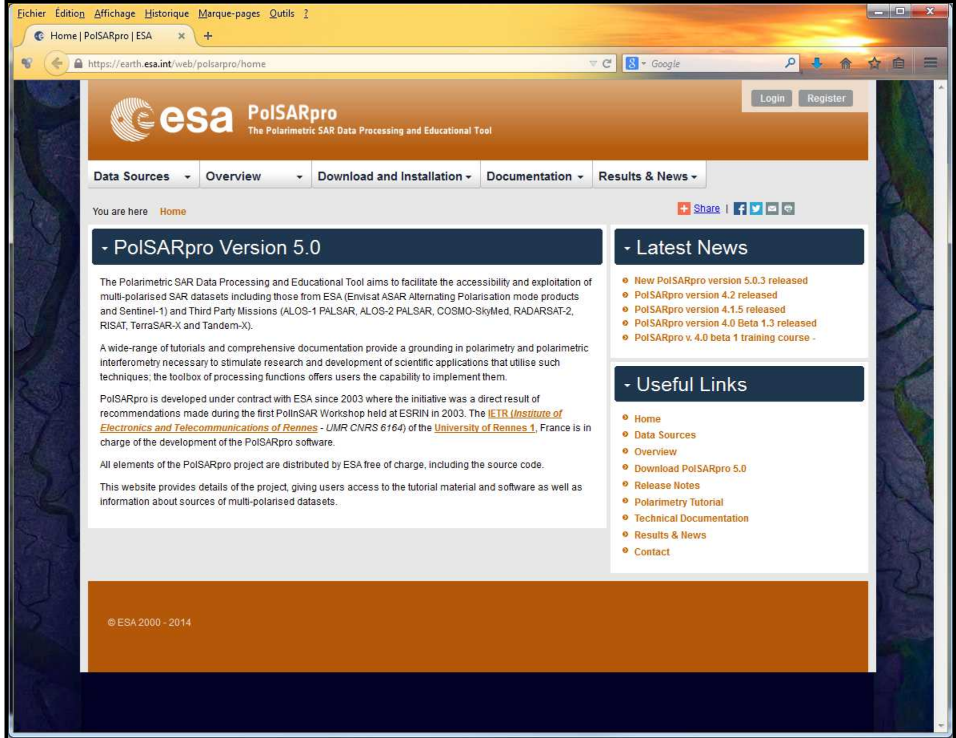
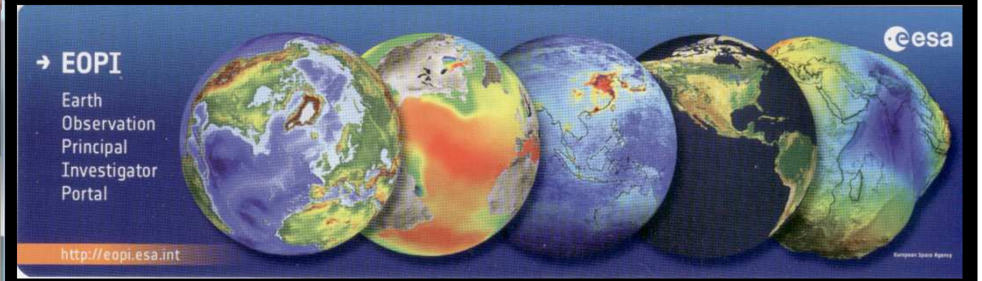
What About

Software / Toolbox ?

PolSARpro v5.0

The Polarimetric SAR Data Processing and Educational Tool v5.0





https://earth.esa.int/web/polsarpro

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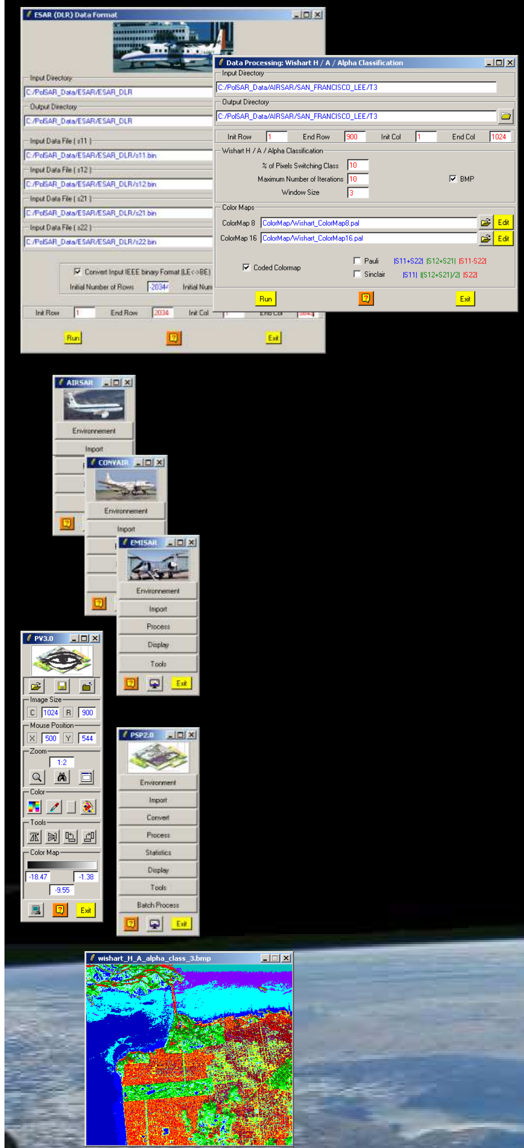
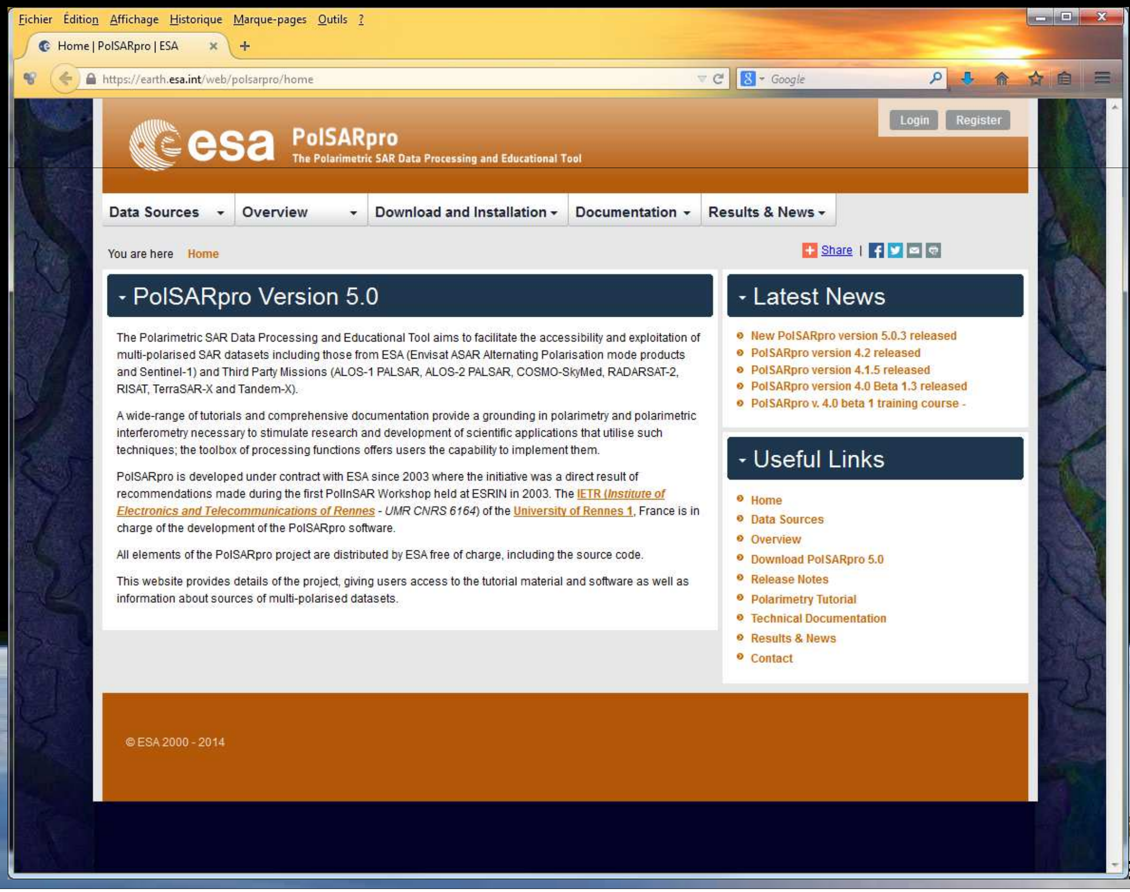
PolSARpro v5.0

OPEN SOURCE DEVELOPMENT

The Tool is free download on the Internet from the ESA Web Portal (Earthnet) at :
https://earth.esa.int/web/polsarpro





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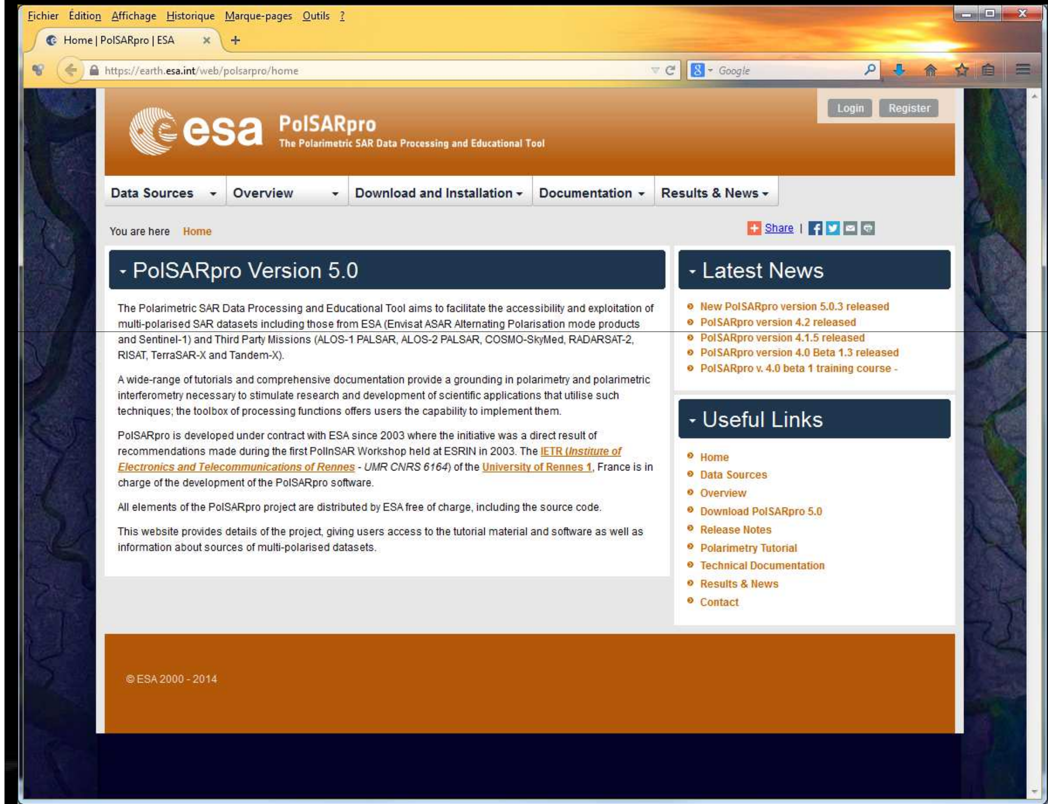
PolSARpro v5.0

http://earth.esa.int/web/polsarpro



The Web Site provides

- **Details of the project**
- **Access to the tutorial and software**
- **Information about status of the development**
- **Demonstration Sample Datasets**

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Learning / Training

Next P.I Generations

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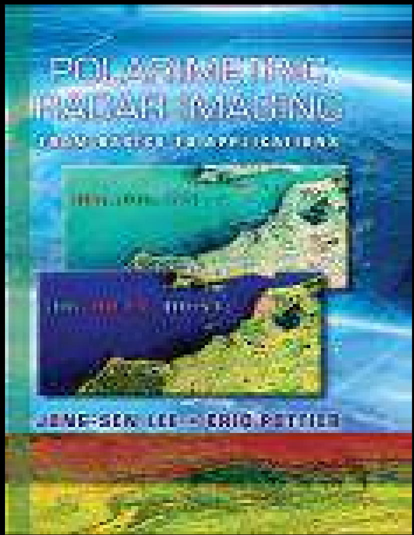
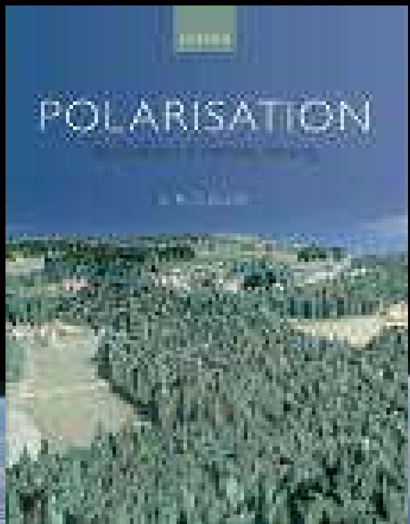
Books On Polarimetric Radar
SAR, Polarimetric Interferometry

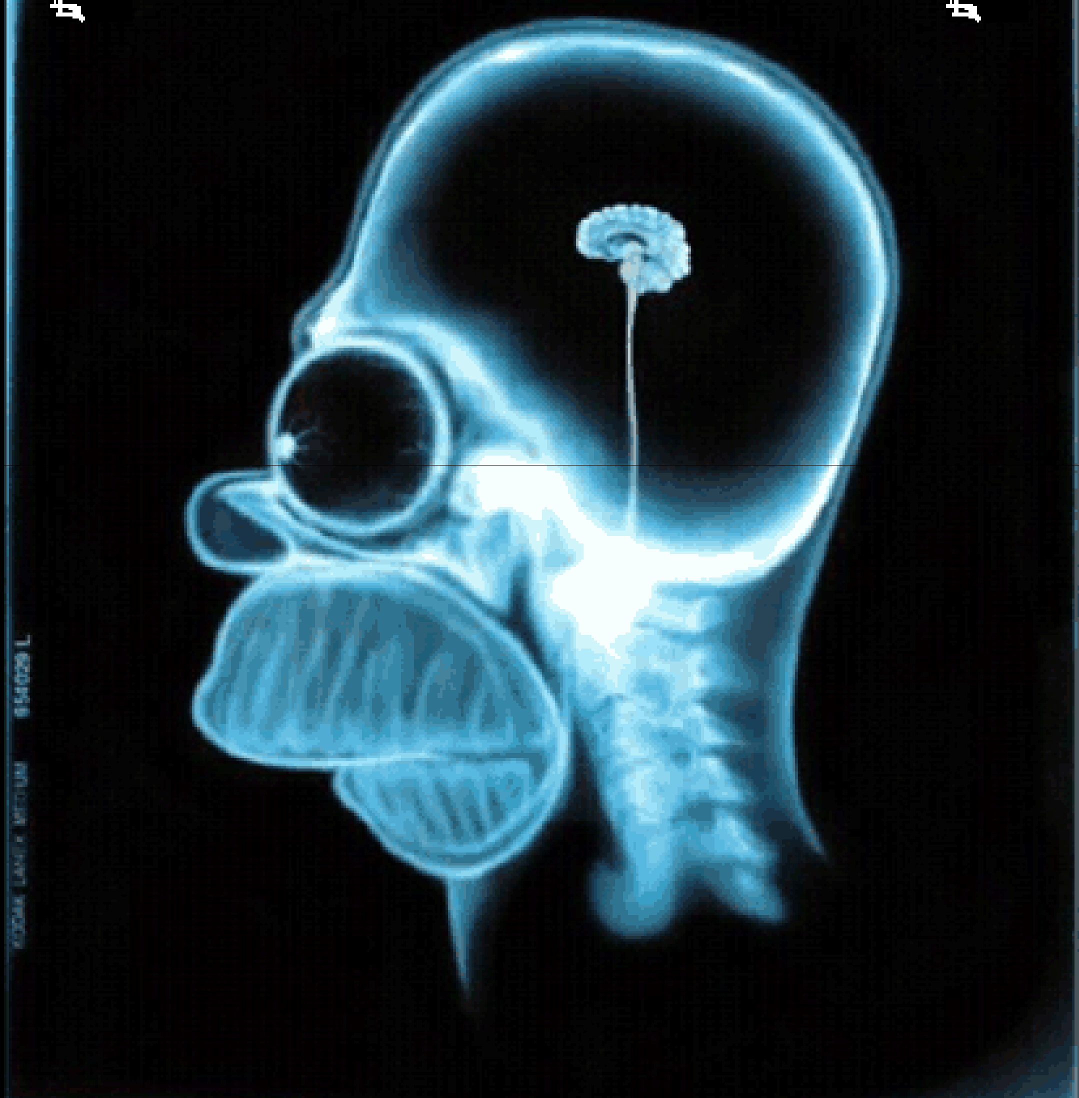


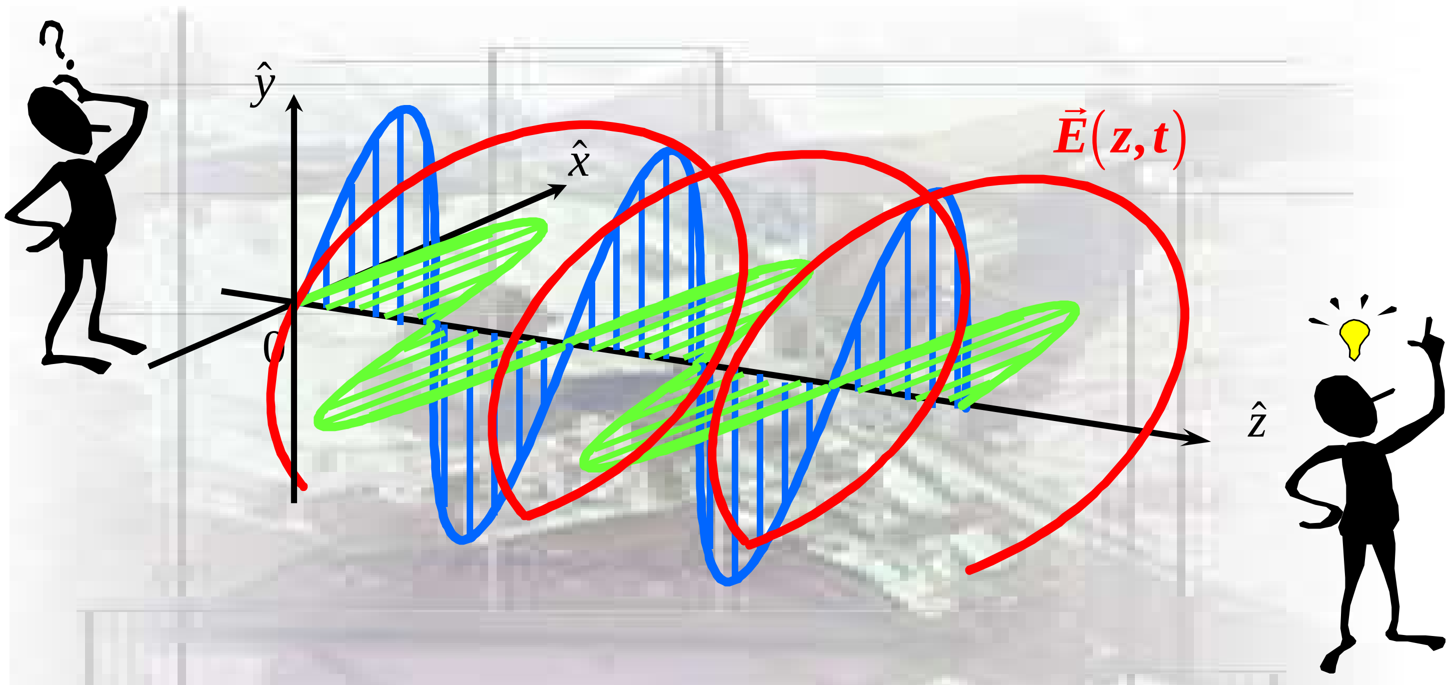
Polarimetric Radar Imaging: From basics to applications
Jong-Sen LEE – Eric POTTIER
CRC Press; 1st ed., February 2009, pp 422
ISBN: 978-1420054972



Polarisation: Applications in Remote Sensing
Shane R. CLOUDE
Oxford University Press, October 2009, pp 352
ISBN: 978-0199569731

Questions ?





DATASETS

AIRSAR

DC8

P, L, C-Band (Quad)

© Google Earth

SAN FRANCISCO BAY

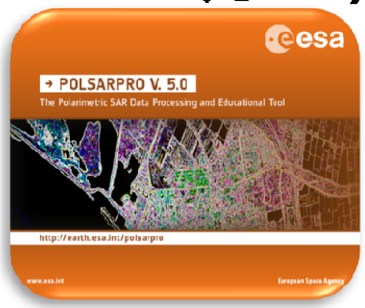
© Google Earth

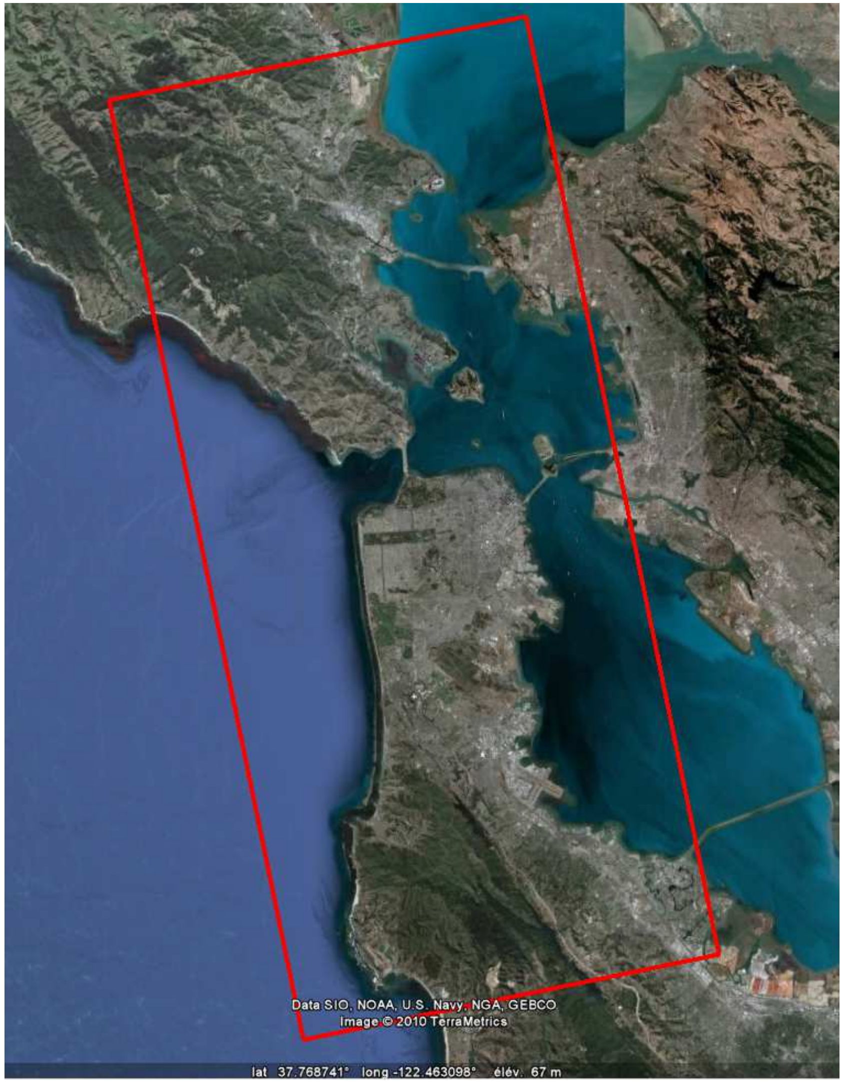
$\|HH+VV\|$	$\|HV\|$	$\|HH-VV\|$
$T_{11}=2A_0$	$T_{33}=B_0-B$	$T_{22}=B_0+B$

SAN FRANCISCO BAY

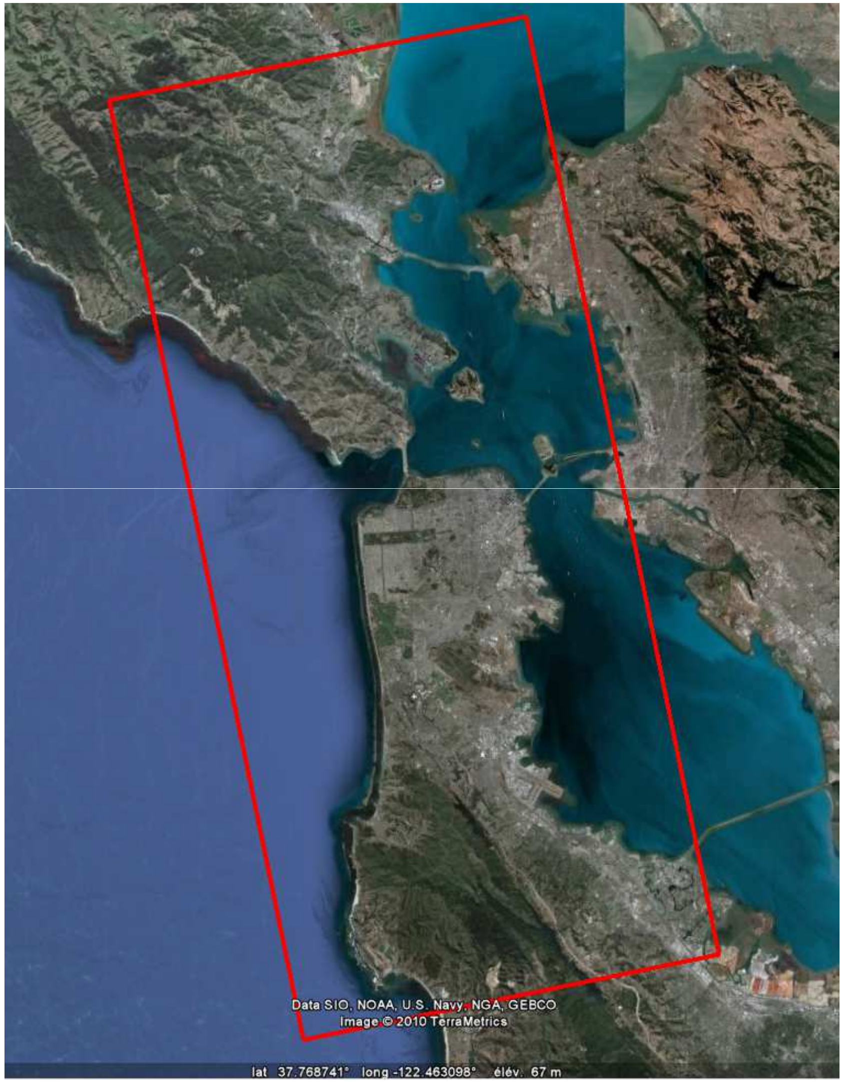
ALOS - PALSAR

L-Band (Quad)

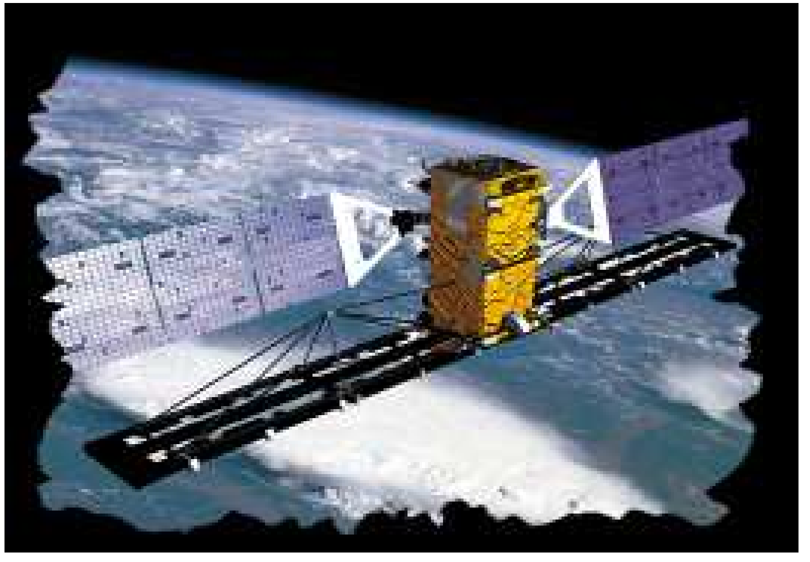




© Google Earth



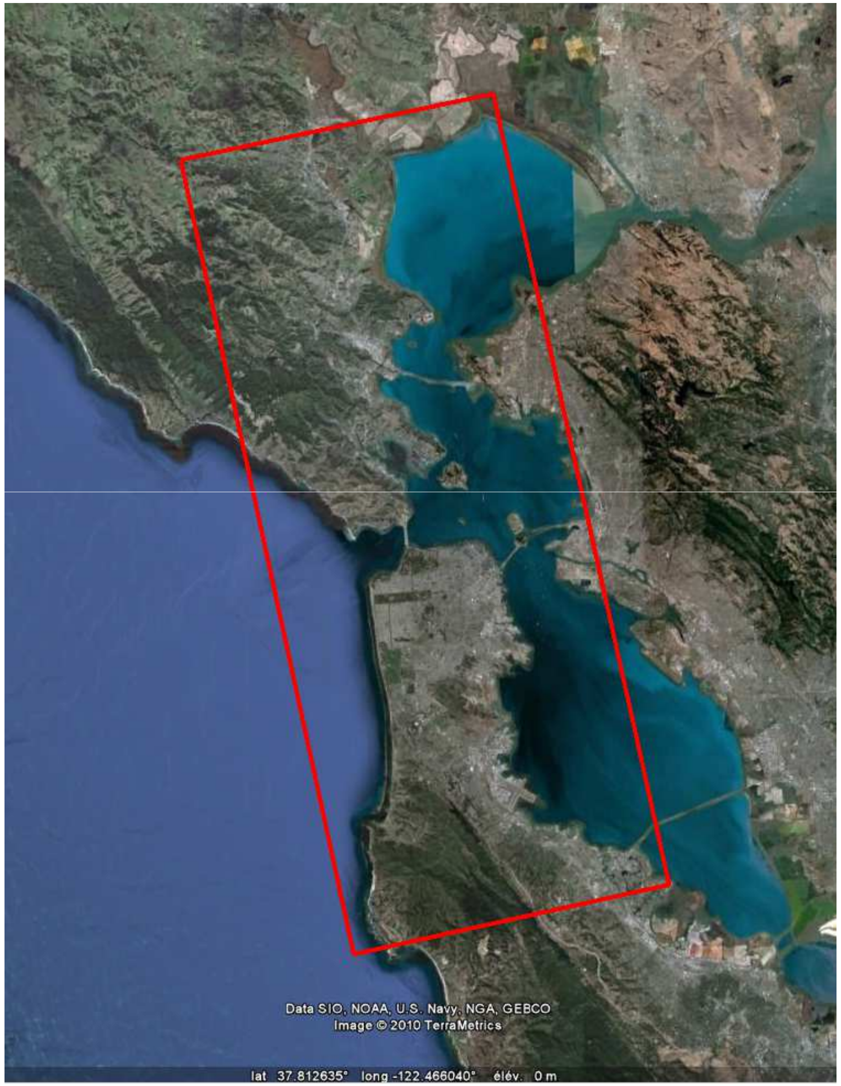
Courtesy of Dr Don Artwood (ASF)

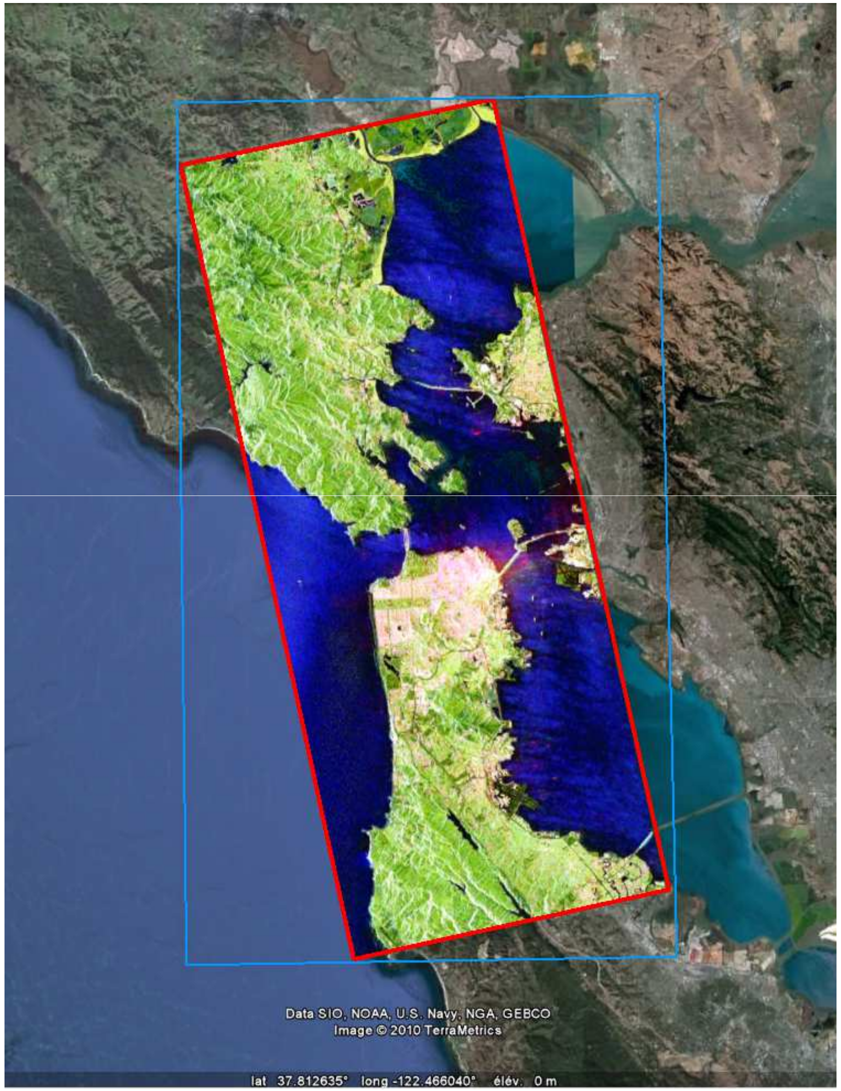


RadarSAT-2

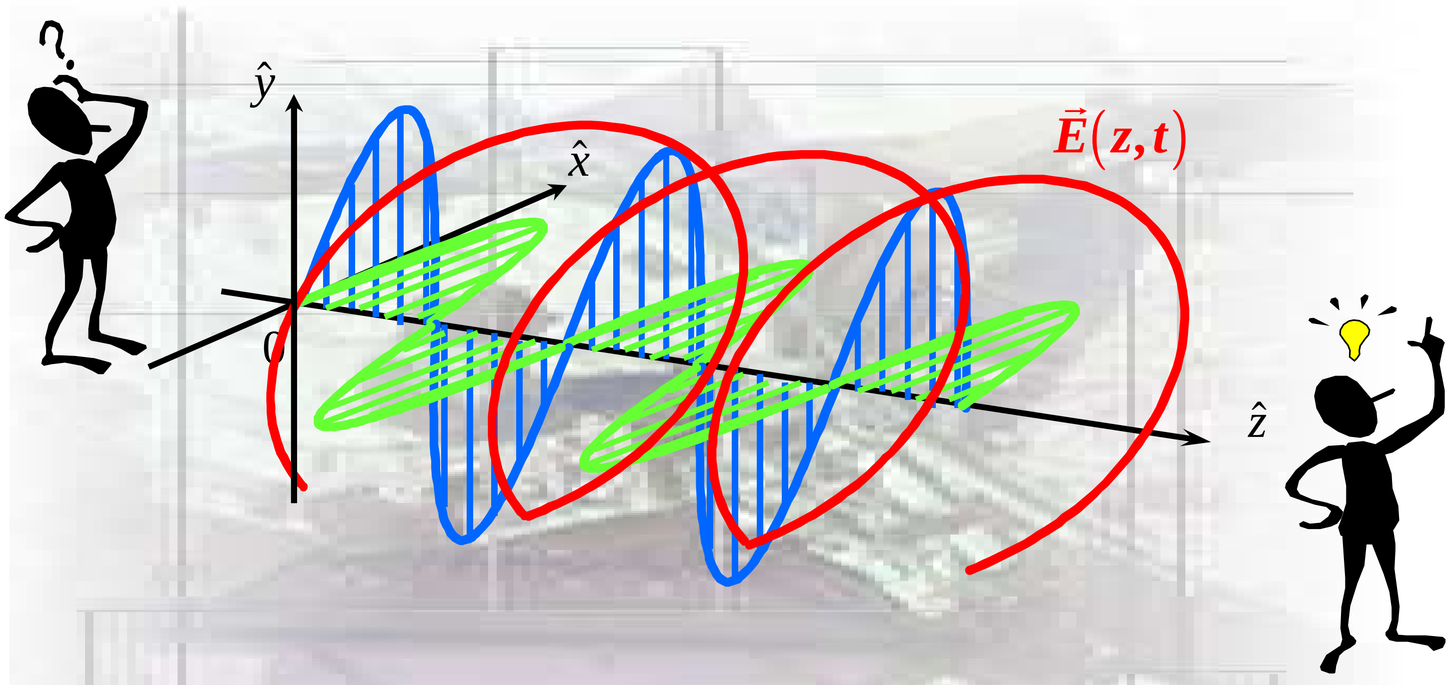
C-Band (Quad)

© Google Earth

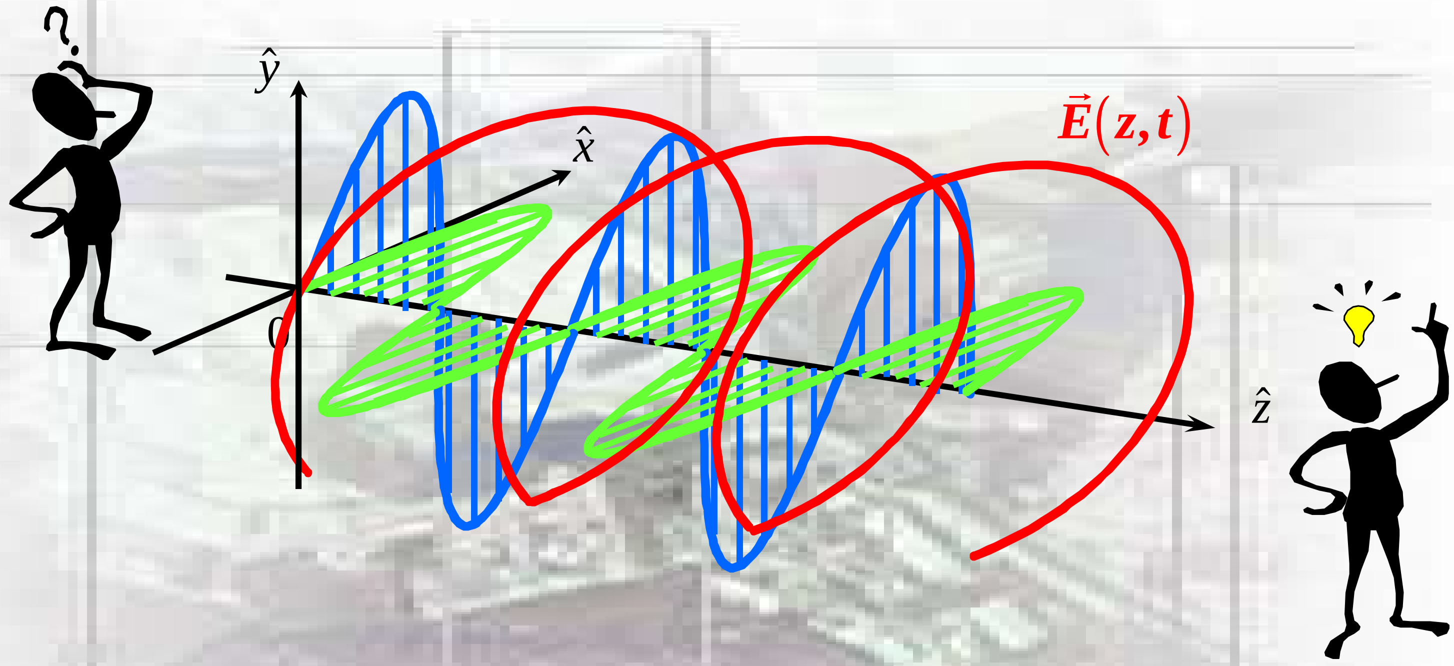




Courtesy of Dr Gordon Staples (MDA)



BASIC CONCEPTS



WAVE POLARIMETRY

REAL ELECTRIC FIELD VECTOR $\vec{E}(z,t)$

MAXWELL EQUATIONS

MAXWELL – FARADAY EQUATION $\nabla \wedge \vec{E}(z,t) = -\dfrac{\partial \vec{B}(z,t)}{\partial t}$

MAXWELL – AMPERE EQUATION $\nabla \wedge \vec{H}(z,t) = \vec{J}_T(z,t)$

GAUSS THEOREM $\nabla \cdot \vec{D}(z,t) = \rho(z,t)$

$$\nabla \cdot \vec{B}(z,t) = 0$$

$$\vec{J}_T(z,t) = \vec{J}_C(z,t) + \frac{\partial \vec{D}(z,t)}{\partial t}$$

$$\vec{J}_C(z,t) = \sigma \vec{E}(z,t)$$

$$\vec{B}(z,t) = \mu \vec{H}(z,t)$$

$$\vec{D}(z,t) = \varepsilon \vec{E}(z,t)$$

σ (Conductivity)
μ (Permeability)
ε (Permittivity)

$$\nabla \wedge \left(\nabla \wedge \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla \cdot \left(\nabla \vec{A}\right)$$

PROPAGATION EQUATION

$$\nabla^2 \vec{E}(z,t) - \mu\varepsilon \frac{\partial^2 \vec{E}(z,t)}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}(z,t)}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial \rho(z,t)}{\partial t}$$

HELMHOLTZ PROPAGATION EQUATION

$$\nabla^2 \vec{E}(z,t) - \mu\varepsilon \frac{\partial^2 \vec{E}(z,t)}{\partial t^2} = 0$$

Source Free, Linear, Homogeneous, Isotropic, Dielectric and lossless Medium

COMPLEX ELECTRIC FIELD VECTOR $\underline{E}(z)$ With: $\vec{E}(z,t) = \Re\left(\underline{E}(z)e^{j\omega t}\right)$

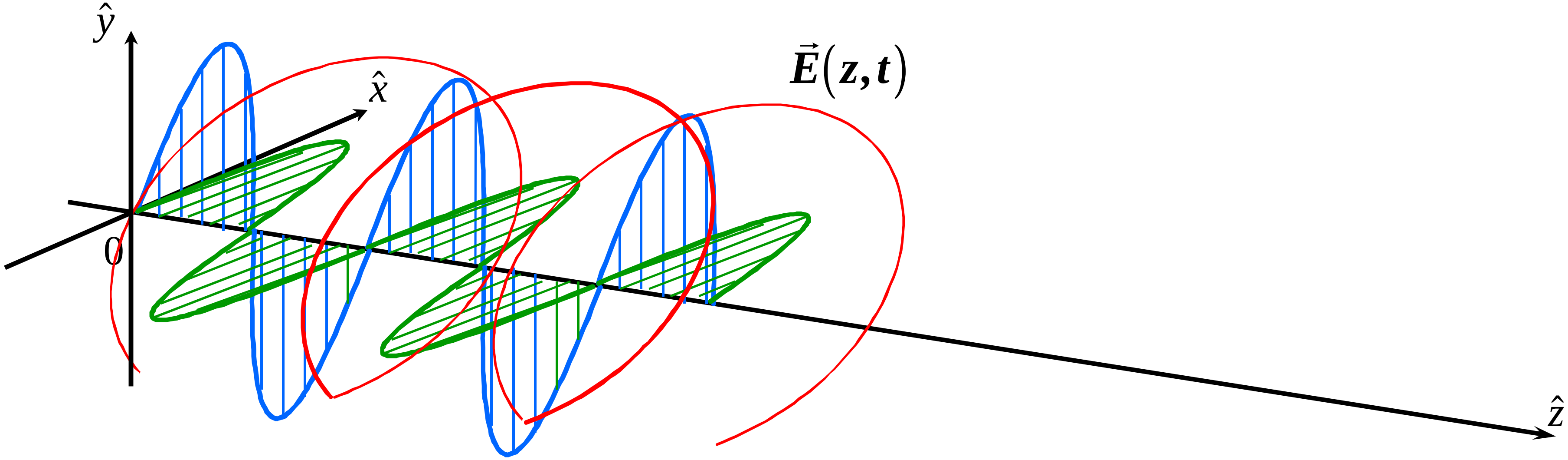
HELMHOLTZ PROPAGATION EQUATION

$$\nabla^2 \underline{E}(z) + \underline{k}^2 \underline{E}(z) = 0$$

SOLUTION: $\underline{E}(z) = \underline{E}e^{-jkz}$ With: $\underline{E} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} E_{ox}e^{j\delta_x} \\ E_{oy}e^{j\delta_y} \\ E_{oz}e^{j\delta_z} \end{bmatrix}$

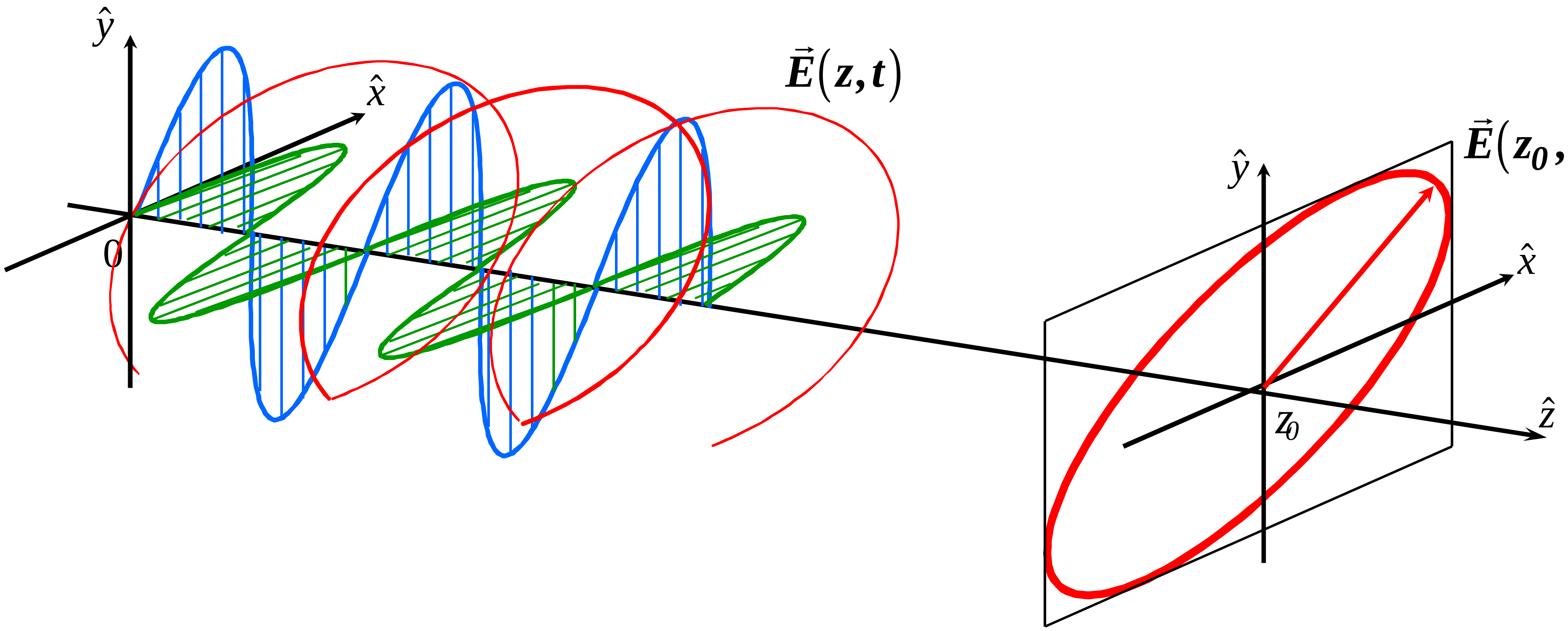
SINUSOIDAL PLANE WAVE

$$\nabla \cdot \vec{E}(z,t) = 0 \quad \Rightarrow \quad \frac{\partial E_z}{\partial z} = 0$$



REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t) = \begin{cases} E_x = E_{0x} \cos(\omega t - kz - \delta_x) \\ E_y = E_{0y} \cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$

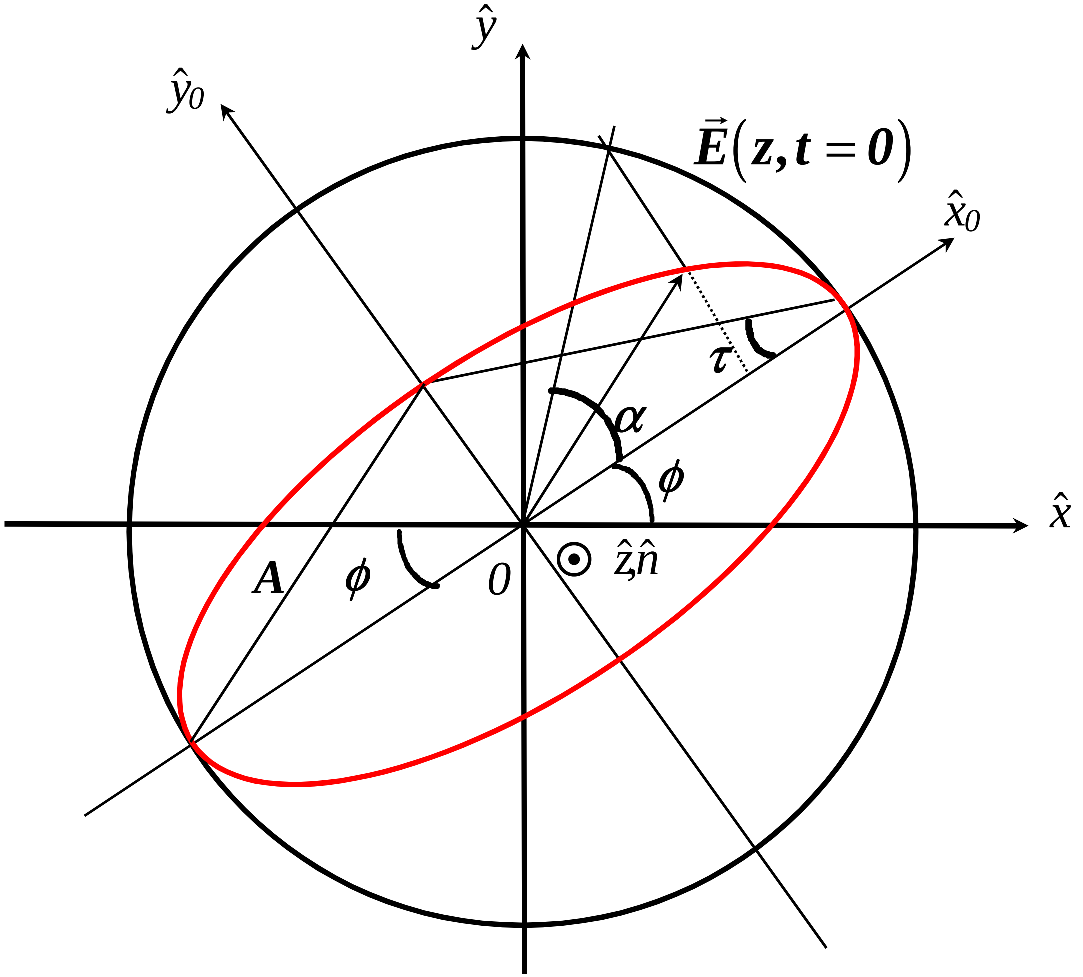


THE REAL ELECTRIC FIELD VECTOR MOVES IN TIME ALONG AN ELLIPSE

$$\left(\frac{E_x}{E_{0x}}\right)^2 - 2\frac{E_x E_y}{E_{0x}E_{0y}}\cos(\delta) + \left(\frac{E_y}{E_{0y}}\right)^2 = \sin^2(\delta)$$

With: $\delta = \delta_y - \delta_x$

POLARISATION ELLIPSE



A : WAVE AMPLITUDE

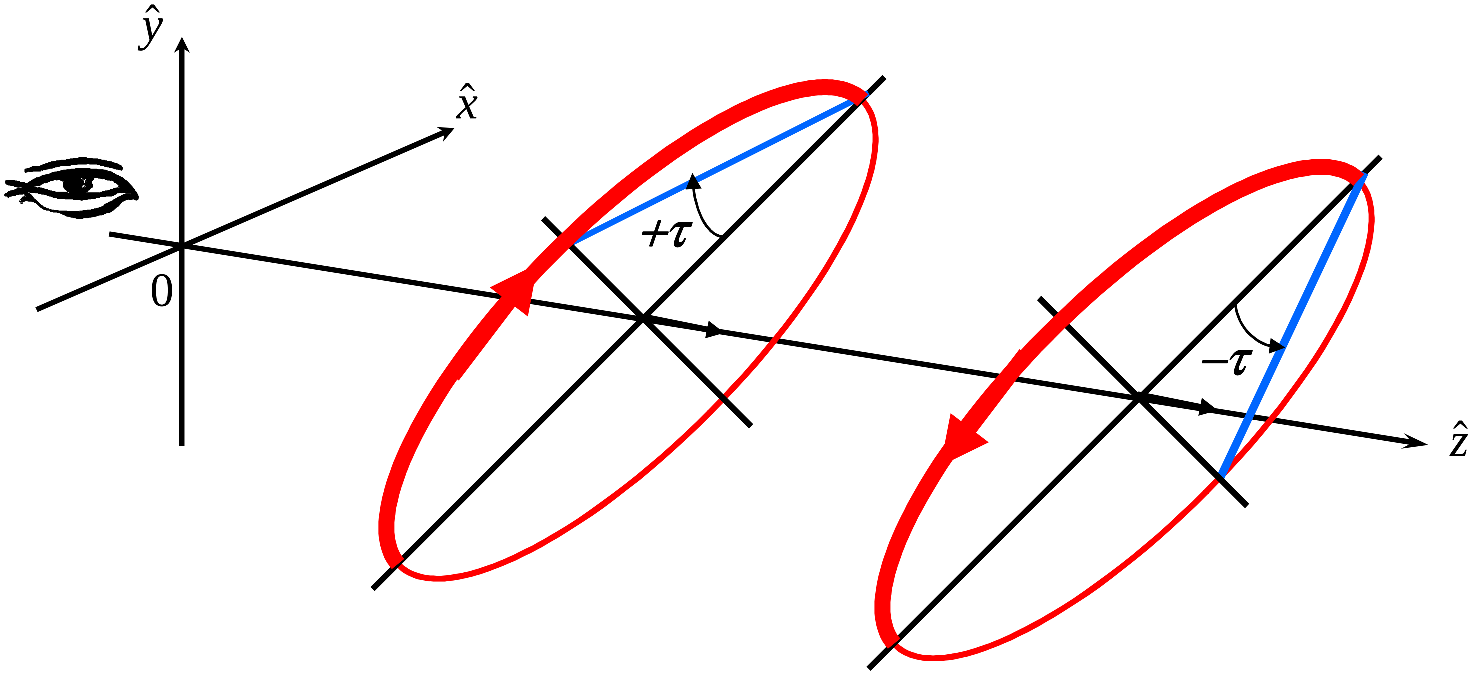
α : ABSOLUTE PHASE

ϕ : ORIENTATION ANGLE $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$

τ : ELLIPTICITY ANGLE $0 \leq \tau \leq \frac{\pi}{4}$

POLARISATION HANDENESS

ROTATION SENSE: LOOKING INTO THE DIRECTION OF THE WAVE PROPAGATION



ANTI-CLOCKWISE ROTATION | **CLOCKWISE ROTATION**

LEFT HANDED POLARISATION → **ELLIPTICITY ANGLE : $\tau > 0$**

RIGHT HANDED POLARISATION → **ELLIPTICITY ANGLE : $\tau < 0$**

$$-\frac{\pi}{4} \leq \tau \leq \frac{\pi}{4}$$

JONES VECTOR

REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t) = \begin{cases} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$

PHASOR = JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix}$$

With: $\vec{E}(z,t) = \Re\left(\underline{E} e^{j(\omega t - kz)}\right)$

GEOMETRICAL PARAMETERS

ABSOLUTE PHASE

$$\alpha = \delta_x$$

AMPLITUDE

$$A = \sqrt{E_{0x}^2 + E_{0y}^2}$$

ORIENTATION ANGLE

$$\tan 2\phi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta$$

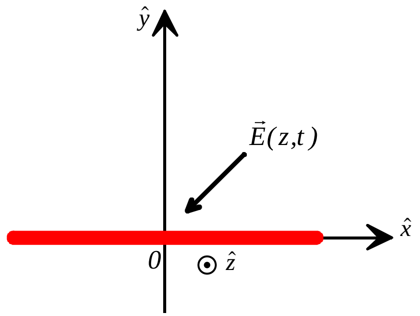
ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta$$

POLARISATION HANDENESS: *Sign(τ)*

JONES VECTOR

HORIZONTAL POLARISATION STATE

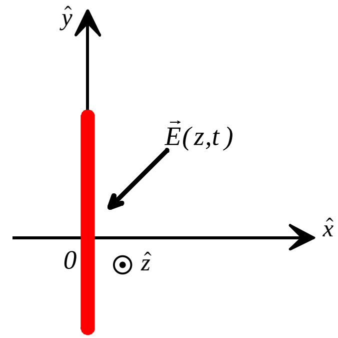


$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\phi = 0$$

$$\tau = 0$$

VERTICAL POLARISATION STATE

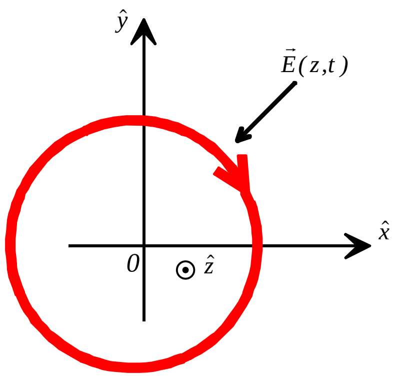


$$\underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\phi = \frac{\pi}{2}$$

$$\tau = 0$$

LEFT CIRCULAR POLARISATION STATE

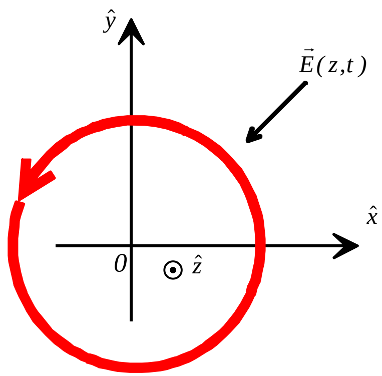


$$\underline{LC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}$$

$$-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2}$$

$$\tau = +\frac{\pi}{4}$$

RIGHT CIRCULAR POLARISATION STATE



$$\underline{RC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

$$-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2}$$

$$\tau = -\frac{\pi}{4}$$

Special Unitary Matrices Group and Jones Vector

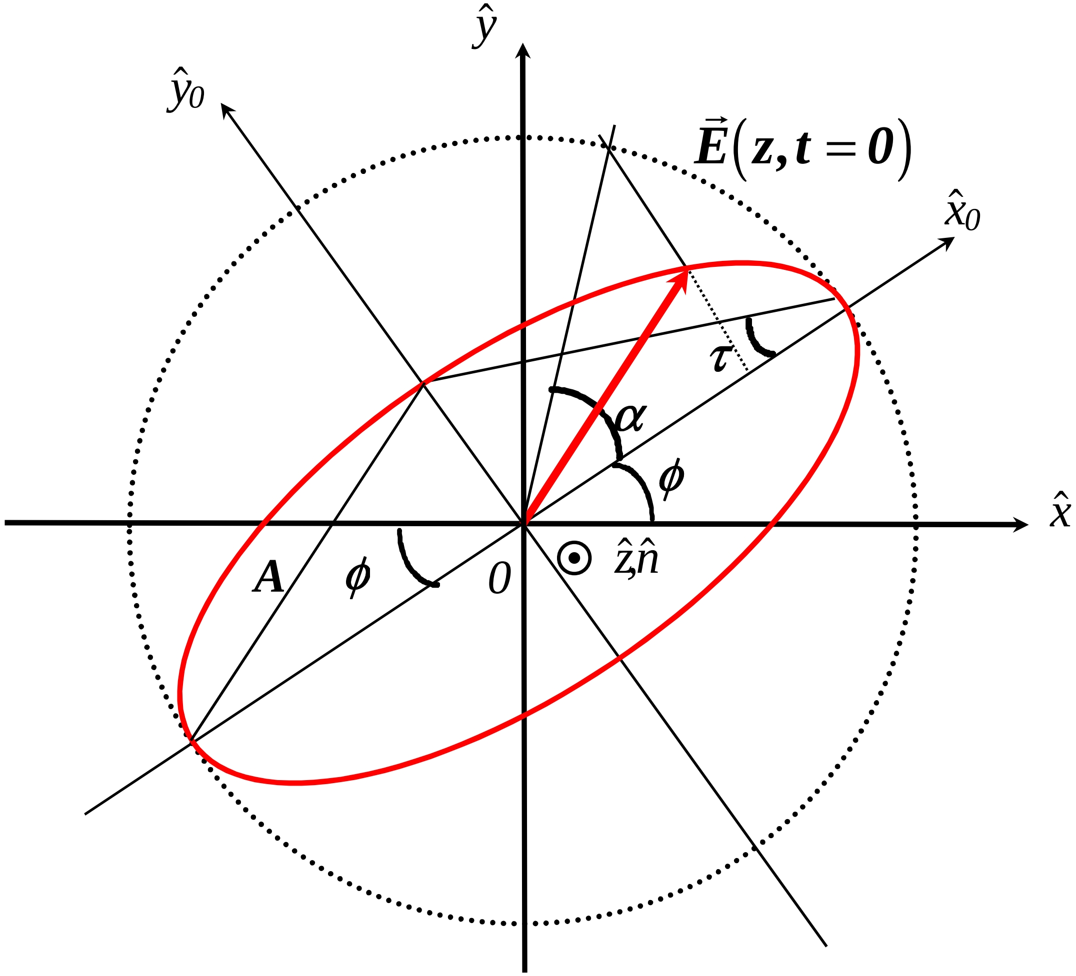
$$\underline{E}_{(\hat{x},\hat{y})} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix} = Ae^{j\alpha} \begin{bmatrix} \cos(\phi)\cos(\tau) - j\sin(\phi)\sin(\tau) \\ \sin(\phi)\cos(\tau) + j\cos(\phi)\sin(\tau) \end{bmatrix}$$

$$\Downarrow$$

$$\underline{E}_{(\hat{x},\hat{y})} = Ae^{j\alpha} \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) \\ j\sin(\tau) \end{bmatrix}$$

$$\Downarrow$$

$$\underline{E}_{(\hat{x},\hat{y})} = A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{ox} e^{j\delta_x} \\ E_{oy} e^{j\delta_y} \end{bmatrix}$$

$$= A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} \hat{u}_x$$

↓

POLARISATION ALGEBRA

NORM OF A JONES VECTOR $\left\| \underline{E} \right\| = \sqrt{E_{0x}^2 + E_{0y}^2}$

SCALAR PRODUCT $\langle \underline{A}, \underline{B} \rangle = \underline{A}^{T*} \underline{B}$

ORTHOGONALITY $\langle \underline{A}, \underline{A}_\perp \rangle = 0$

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

↓

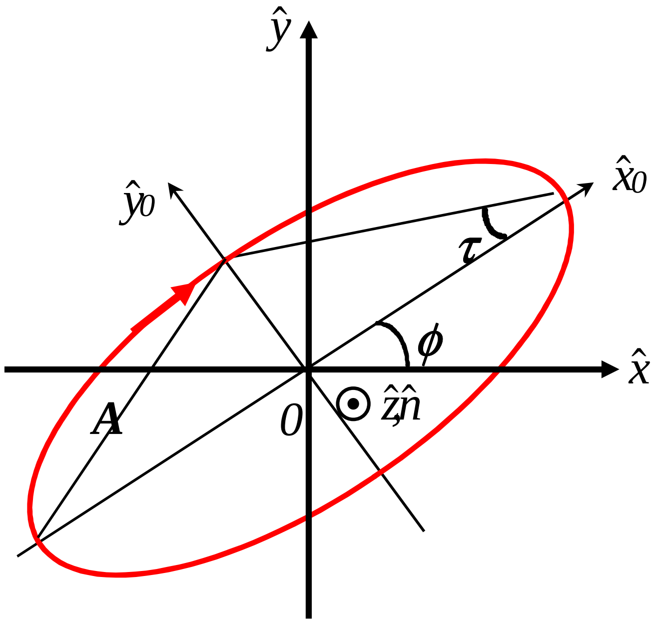
ORTHOGONAL JONES VECTOR

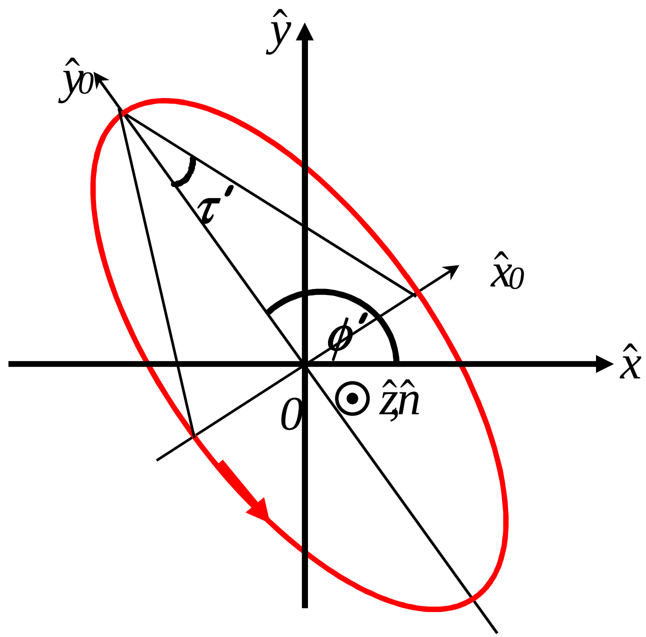
$$\underline{E}_\perp = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_y$$

$$= A\begin{bmatrix} -\sin(\phi) & -\cos(\phi) \\ \cos(\phi) & -\sin(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

↓

$$\underline{E}_\perp = A\begin{bmatrix} \cos(\phi+\frac{\pi}{2}) & -\sin(\phi+\frac{\pi}{2}) \\ \sin(\phi+\frac{\pi}{2}) & \cos(\phi+\frac{\pi}{2}) \end{bmatrix}\begin{bmatrix} \cos(-\tau) & j\sin(-\tau) \\ j\sin(-\tau) & \cos(-\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$





ORTHOGONALITY CONDITIONS

$$(\phi,\tau) \mapsto \begin{cases} \phi' = \phi + \frac{\pi}{2} \\ \tau' = -\tau \end{cases}$$

➡ CHANGE OF POLARISATION HANDENESS

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR

$$\underline{E}_\perp = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_y$$

↓

$$[\underline{E}, \underline{E}_\perp] = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}[\hat{u}_x, \hat{u}_y]$$

↓

ELLIPTICAL BASIS TRANSFORMATION

ELLIPTICAL BASIS TRANSFORMATION

ORTHOGONAL JONES VECTORS

$$[\underline{E},\underline{E}_{\perp}] = A\begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}[\hat{u}_x,\hat{u}_y]$$

$$\Downarrow$$

SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

$$[U(\phi,\tau,\alpha)] = \begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad\qquad [U_2(\tau)] \qquad\qquad [U_2(\alpha)]$$

$[U_2][U_2]^{T*} = [I_{D2}]$ CONSERVATION OF THE WAVE ENERGY

$det([U_2]) = +1$ ENSURES THE CORRECT PHASE DEFINITION

ELLIPTICAL BASIS TRANSFORMATION

SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

$$[U(\phi,\tau,\alpha)] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

↓

ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\begin{aligned} \left[U_{(\underline{A},\underline{A}_\perp)\mapsto(\underline{B},\underline{B}_\perp)}\right] &= [U(\phi,\tau,\alpha)]^{-1} \\ &= \begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix} \end{aligned}$$

FROM LINEAR TO CIRCULAR BASIS

$$\underline{E} = E_H\,\underline{H} + E_V\,\underline{V} = E_{LC}\,\underline{LC} + E_{RC}\,\underline{RC}$$

With:

$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \underline{LC} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ j \end{bmatrix} \quad \underline{RC} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -j \end{bmatrix}$$

⬇

$$[\underline{LC}, \underline{RC}] = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}$$

ELLIPTICAL BASIS TRANSFORMATION

FROM LINEAR TO CIRCULAR BASIS

$$\underline{E} = E_H \underline{H} + E_V \underline{V} = E_{LC} \underline{LC} + E_{RC} \underline{RC}$$

With:

$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \underline{LC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix} \quad \underline{RC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

⬇

$$\left[\underline{LC}, \underline{RC}\right] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} \quad \notin SU(2)$$

Ernst LÜNEBURG
(PIERS95 - Pasadena)

ELLIPTICAL BASIS TRANSFORMATION

FROM LINEAR TO CIRCULAR BASIS

$$\underline{E} = E_H \underline{H} + E_V \underline{V} = E_{LC} \underline{LC} + E_{LC_\perp} \underline{LC}_\perp$$

With:

$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \underline{LC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix} \quad \underline{LC}_\perp = \frac{1}{\sqrt{2}} \begin{bmatrix} j \\ 1 \end{bmatrix}$$

⬇

$$\left[\underline{LC}, \underline{LC}_\perp\right] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \qquad \in SU(2)$$

ELLIPTICAL BASIS TRANSFORMATION

FROM LINEAR TO CIRCULAR BASIS

$$\underline{E} = E_H \underline{H} + E_V \underline{V} = E_{LC} \underline{LC} + E_{LC_\perp} \underline{LC}_\perp$$

⬇

$$[\underline{LC}, \underline{LC}_\perp] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} = \left[U\left(0, \frac{\pi}{4}\right)\right] \Rightarrow \left[U_{(\underline{H}, \underline{V}) \mapsto (\underline{LC}, \underline{LC}_\perp)}\right] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}$$

⬇

$$\begin{bmatrix} E_{LC} \\ E_{LC_\perp} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix} \begin{bmatrix} E_H \\ E_V \end{bmatrix}$$

STOKES VECTOR

REAL REPRESENTATION OF THE POLARISATION STATE OF A MONOCHROMATIC WAVE

$$\underline{E} \cdot \underline{E}^{T*} = \begin{bmatrix} E_x E_x^* & E_x E_y^* \\ E_y E_x^* & E_y E_y^* \end{bmatrix}$$

PAULI MATRICES GROUP

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}$$

$$\underline{E} \cdot \underline{E}^{T*} = \frac{1}{2}\{g_0\sigma_0 + g_1\sigma_1 + g_2\sigma_2 + g_3\sigma_3\} = \frac{1}{2}\begin{bmatrix} g_0 + g_1 & g_2 - jg_3 \\ g_2 + jg_3 & g_0 - g_1 \end{bmatrix}$$

$\{ g_0, g_1, g_2, g_3 \}$ **STOKES PARAMETERS**

STOKES VECTOR

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix}$$

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 = |E_x|^2 + |E_y|^2 \\ g_1 = |E_x|^2 - |E_y|^2 \\ g_2 = 2\Re\left(E_x E_y^*\right) \\ g_3 = -2\Im\left(E_x E_y^*\right) \end{bmatrix}$$

WAVE POLARISATION STATE ESTIMATION FROM INTENSITIES MEASUREMENTS

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 = E_{0x}^2 + E_{0y}^2 \\ g_1 = E_{0x}^2 - E_{0y}^2 \\ g_2 = 2E_{0x}E_{0y}\cos(\delta) \\ g_3 = 2E_{0x}E_{0y}\sin(\delta) \end{bmatrix} = \begin{bmatrix} g_0 = A^2 \\ g_1 = A^2\cos 2\phi \cos 2\tau \\ g_2 = A^2 \sin 2\phi \cos 2\tau \\ g_3 = A^2 \sin 2\tau \end{bmatrix}$$

GEOMETRICAL PARAMETERS

ORIENTATION ANGLE

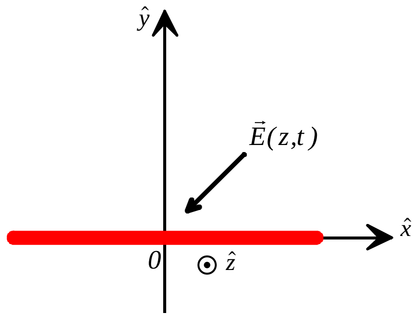
$$\tan 2\phi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta = \frac{g_2}{g_1}$$

ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta = \frac{g_3}{g_0}$$

STOKES VECTOR

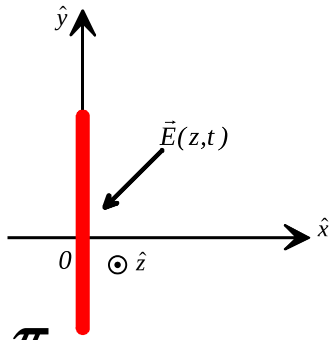
HORIZONTAL POLARISATION STATE



$$\phi = 0 \qquad \tau = 0$$

$$\underline{g}_{H} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

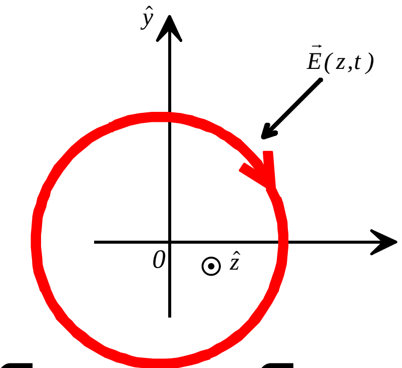
VERTICAL POLARISATION STATE



$$\phi = \frac{\pi}{2} \qquad \tau = 0$$

$$\underline{g}_{V} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

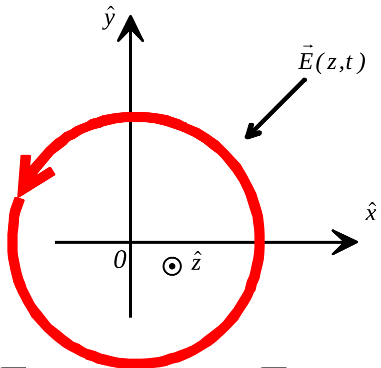
LEFT CIRCULAR POLARISATION STATE



$$\underline{g}_{LC} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$-\frac{\pi}{2} \le \phi \le +\frac{\pi}{2} \qquad \tau = +\frac{\pi}{4}$$

RIGHT CIRCULAR POLARISATION STATE



$$\underline{g}_{RC} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$-\frac{\pi}{2} \le \phi \le +\frac{\pi}{2} \qquad \tau = -\frac{\pi}{4}$$

O(4) UNITARY ROTATION ROUP

JONES VECTOR

$$\underline{E} = A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} \hat{u}_x$$

$[U_2(\phi)]$ $[U_2(\tau)]$ $[U_2(\alpha)]$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2} Tr\left([U_2(\theta)]^{T*} \sigma_p [U_2(\theta)] \sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

STOKES VECTOR

$$\underline{g}_{\underline{E}} = A^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix} \underline{g}_{\hat{u}_x}$$

$[O_3(2\phi)]$ $[O_3(2\tau)]$ $[O_3(2\alpha)]$

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2} Tr\left([U_2(\theta)]^{T*} \sigma_p [U_2(\theta)] \sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

O(3) UNITARY GROUP

$$\begin{bmatrix} \cos 2\phi & -\sin 2\phi & 0 \\ \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 2\tau & 0 & -\sin 2\tau \\ 0 & 1 & 0 \\ \sin 2\tau & 0 & \cos 2\tau \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\alpha & -\sin 2\alpha \\ 0 & \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$[O_3(2\phi)] \qquad [O_3(2\tau)] \qquad [O_3(2\alpha)]$$

STOKES VECTOR

$$\underline{g}_E = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} |E_x|^2 + |E_y|^2 \\ |E_x|^2 - |E_y|^2 \\ 2\Re(E_x E_y^*) \\ -2\Im(E_x E_y^*) \end{bmatrix} = \begin{bmatrix} E_{0x}^2 + E_{0y}^2 \\ E_{0x}^2 - E_{0y}^2 \\ 2E_{0x}E_{0y}\cos(\delta) \\ 2E_{0x}E_{0y}\sin(\delta) \end{bmatrix} = \begin{bmatrix} A^2 \\ A^2\cos 2\phi\cos 2\tau \\ A^2\sin 2\phi\cos 2\tau \\ A^2\sin 2\tau \end{bmatrix}$$

$\{g_0\}$ TOTAL WAVE INTENSITY

$\{g_1, g_2, g_3\}$ POLARISED WAVE INTENSITIES

↓

$$g_0^2 = g_1^2 + g_2^2 + g_3^2$$

WAVE FULLY POLARISED

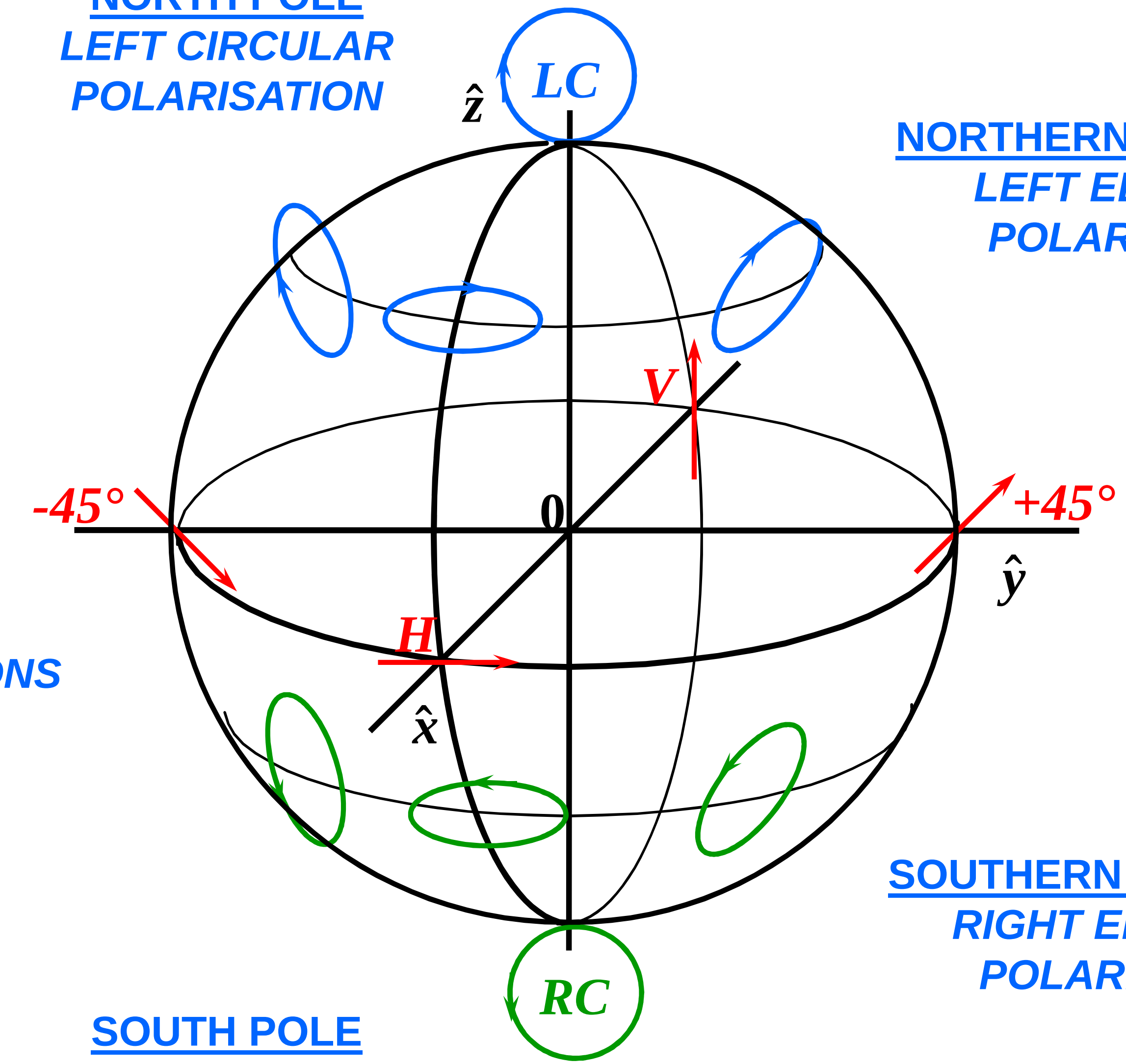
$\{g_1, g_2, g_3\}$ Spherical Coordinates of a point P on a sphere with radius g_0

POINCARE SPHERE

NORTH POLE
LEFT CIRCULAR POLARISATION

NORTHERN HEMISPHERE
LEFT ELLIPTICAL POLARISATIONS

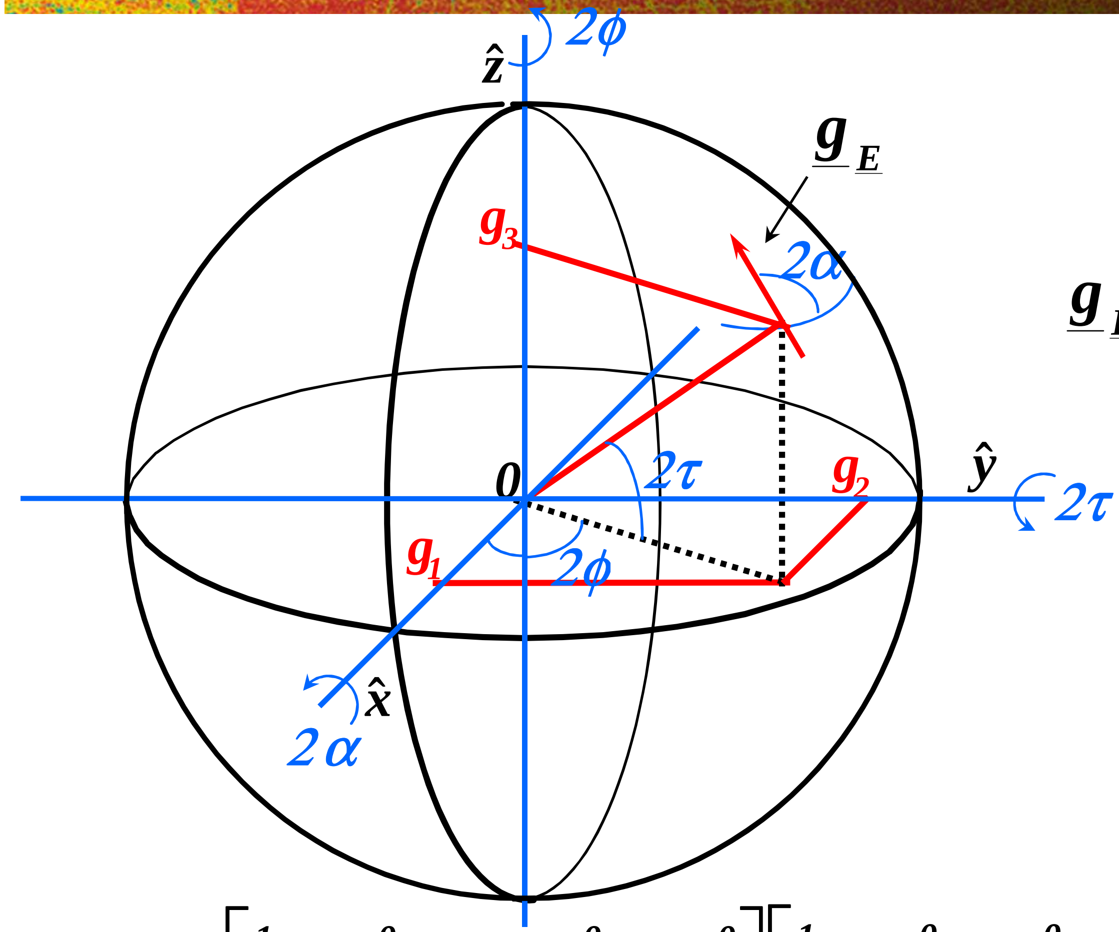
EQUATOR
LINEAR POLARISATIONS



SOUTHERN HEMISPHERE
RIGHT ELLIPTICAL POLARISATIONS

SOUTH POLE
RIGHT CIRCULAR POLARISATION

POINCARE SPHERE



$$\underline{g}_E = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A^2 \\ A^2 \cos 2\phi \cos 2\tau \\ A^2 \sin 2\phi \cos 2\tau \\ A^2 \sin 2\tau \end{bmatrix}$$

$$\underline{g}_E = A^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix} \underline{g}_{\hat{u},}$$

$[O_3(2\phi)]$ $[O_3(2\tau)]$ $[O_3(2\alpha)]$

POINCARE SPHERE

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

ORTHOGONAL JONES VECTOR

$$\underline{E}_\perp = \begin{bmatrix} E'_x \\ E'_y \end{bmatrix}$$

ORTHOGONALITY CONDITIONS

$$(\phi, \tau) \mapsto \begin{cases} \phi' = \phi + \frac{\pi}{2} \\ \tau' = -\tau \end{cases}$$

STOKES VECTOR

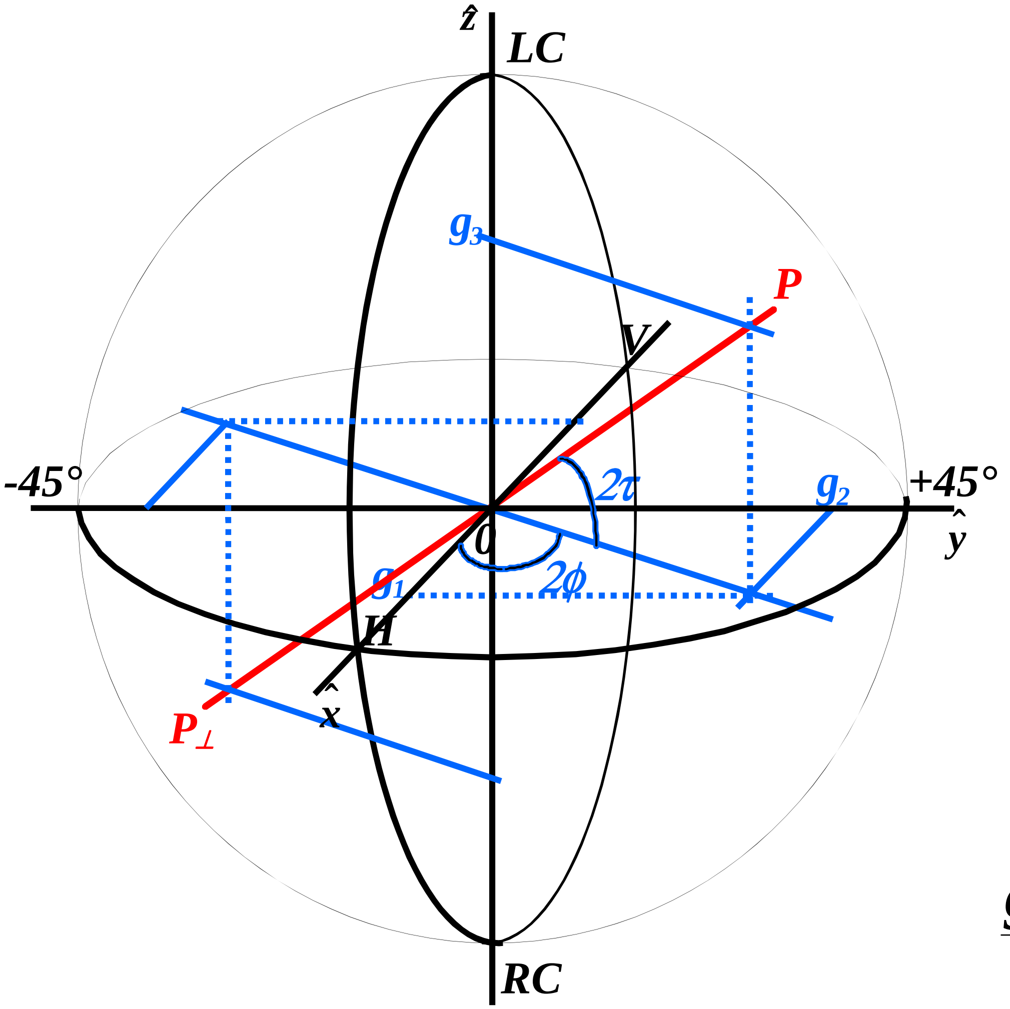
$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ A\cos 2\phi \cos 2\tau \\ A\sin 2\phi \cos 2\tau \\ A\sin 2\tau \end{bmatrix}$$

ORTHOGONAL STOKES VECTOR

$$\underline{g}_{\underline{E}_\perp} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ -A\cos 2\phi \cos 2\tau \\ -A\sin 2\phi \cos 2\tau \\ -A\sin 2\tau \end{bmatrix}$$

ORTHOGONALITY = ANTIPODALITY

POINCARE SPHERE



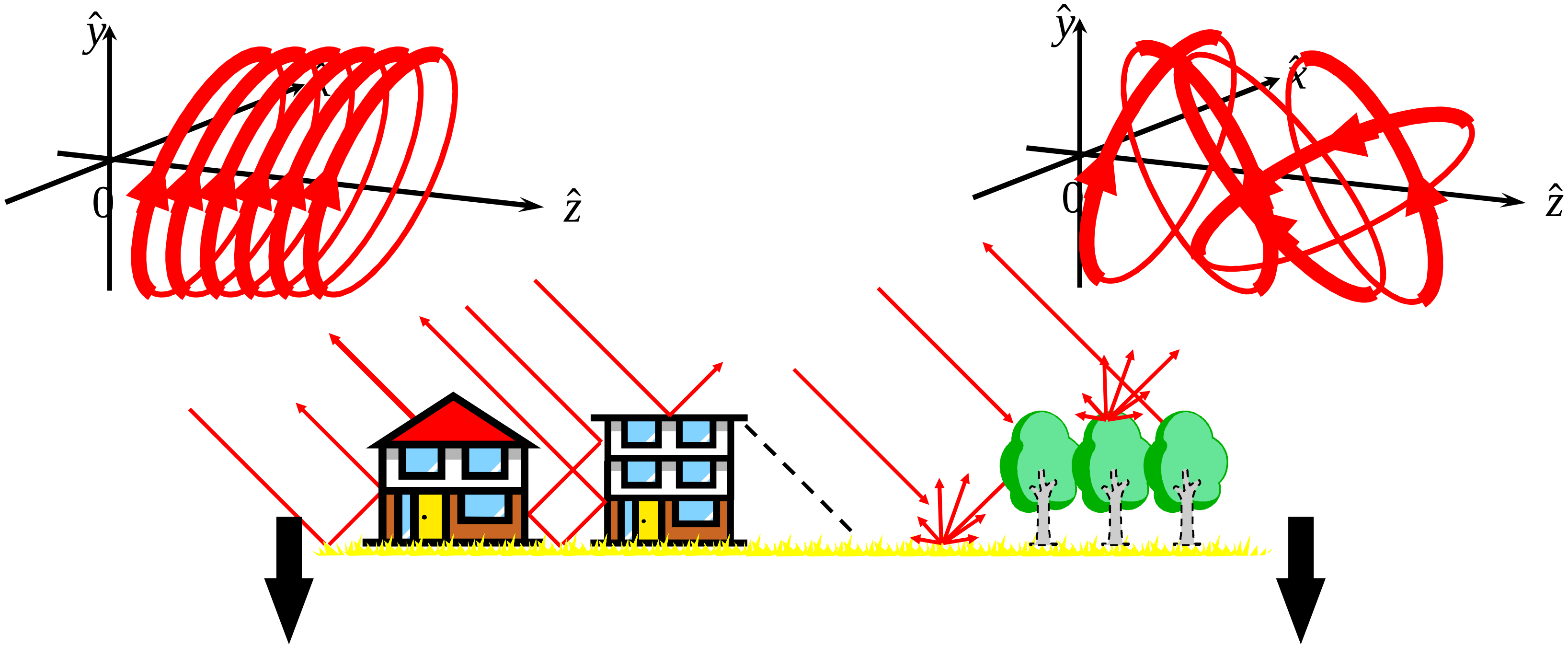
STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ A\cos 2\phi \cos 2\tau \\ A\sin 2\phi \cos 2\tau \\ A\sin 2\tau \end{bmatrix}$$

ORTHOGONAL STOKES VECTOR

$$\underline{g}_{\underline{E}_\perp} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ -A\cos 2\phi \cos 2\tau \\ -A\sin 2\phi \cos 2\tau \\ -A\sin 2\tau \end{bmatrix}$$

ORTHOGONALITY = ANTIPODALITY



DETERMINISTIC SCATTERING

COMPLETELY POLARISED WAVE

RANDOM SCATTERING

PARTIALLY POLARISED WAVE

Polarisation Ellipse varies in time
Amplitude, Phase: Random processes

STATISTICAL DESCRIPTION

JONES VECTORS $\{\underline{E}\}$

⬇

WAVE COVARIANCE MATRIX

$$\langle [J] \rangle = \langle \underline{E}\,\underline{E}^{T*} \rangle = \begin{bmatrix} \langle |E_x|^2 \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle |E_y|^2 \rangle \end{bmatrix}$$

⬍

$$\langle [J] \rangle = \frac{1}{2} \begin{bmatrix} \langle g_0 \rangle + \langle g_1 \rangle & \langle g_2 \rangle - j\langle g_3 \rangle \\ \langle g_2 \rangle + j\langle g_3 \rangle & \langle g_0 \rangle - \langle g_1 \rangle \end{bmatrix}$$

$$\langle g_0 \rangle^2 \geq \langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2$$

PARTIALLY POLARISED WAVES

EIGENVALUES DECOMPOSITION

$$\langle[J]\rangle = [U_2]\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}[U_2]^{-1} = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} + \lambda_2 \underline{u}_2 \underline{u}_2^{T*}$$

- 2 ORTHOGONAL EIGENVECTORS $[U_2] = [\underline{u}_1, \underline{u}_2]$

- 2 REAL EIGENVALUES

$$\lambda_1 = \frac{1}{2}\left\{\langle g_0\rangle + \sqrt{\langle g_1\rangle^2 + \langle g_2\rangle^2 + \langle g_3\rangle^2}\right\}$$

$$\lambda_2 = \frac{1}{2}\left\{\langle g_0\rangle - \sqrt{\langle g_1\rangle^2 + \langle g_2\rangle^2 + \langle g_3\rangle^2}\right\}$$

PARTIALLY POLARISED WAVES DESCRIPTORS

Degree of Polarisation

$$DoP = \frac{\sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2}}{\langle g_0 \rangle} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \left(1 - \frac{4 \det([J])}{Trace^2([J])} \right)$$

Polarised Wave Power / Total Wave Power

Anisotropy

$$0 \leq DoP \leq 1$$

Wave Entropy

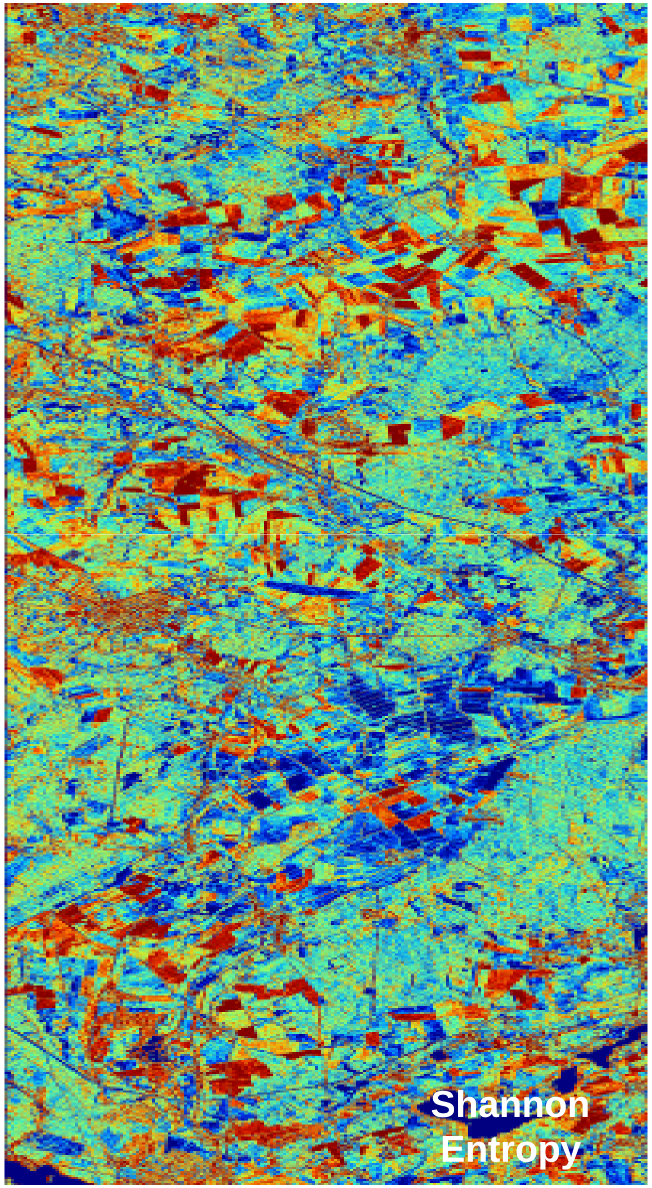
$$0 \leq H \leq 1$$

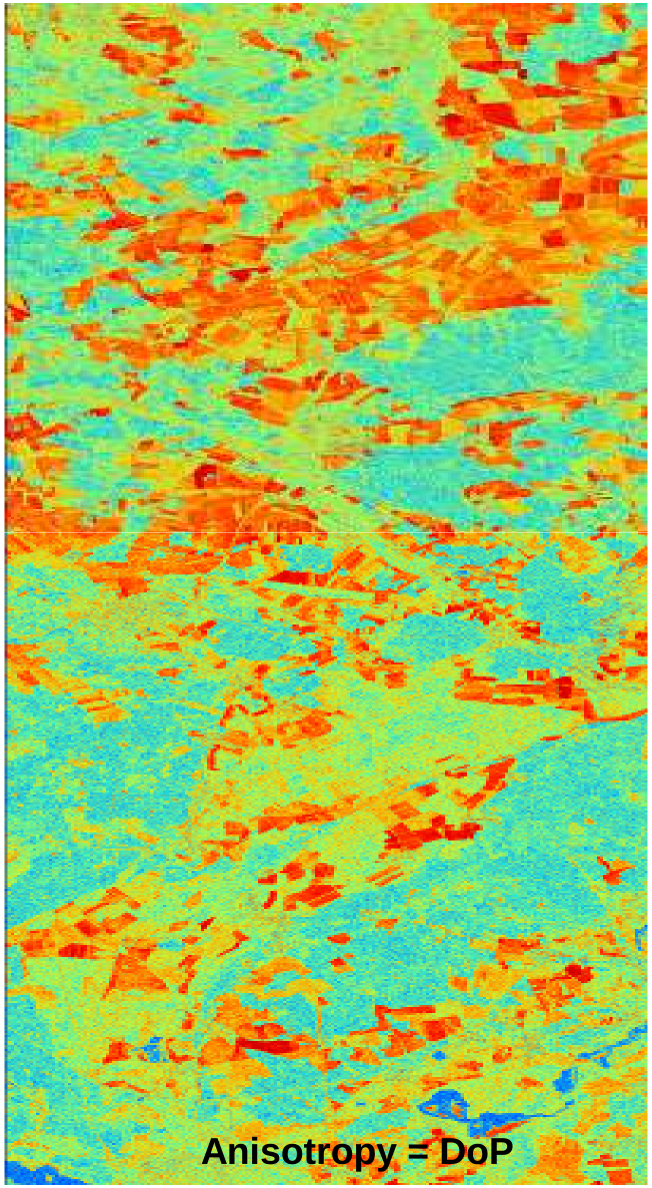
$$H = -\sum_{i=1}^{i=2} p_i \log_2(p_i)$$

With: $p_i = \frac{\lambda_i}{\lambda_1 + \lambda_2}$

Degree of randomness, statistical disorder

PARTIALLY POLARISED WAVES




Anisotropy = DoP

MONOCHROMATIC PLANE WAVES

COMPLEX DOMAIN

JONES VECTOR $\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$

REAL DOMAIN

STOKES VECTOR $\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix}$

PLANE WAVES FULLY DESCRIBED
BY 3 INDEPENDANT PARAMETERS

- $E_{0x}, E_{0x}, \delta = \delta_y - \delta_x$
- (A, ϕ, τ) or (A, γ, δ)
- $\{g_1, g_2, g_3\}$

WAVE POLARIMETRIC DIMENSION = 3

PARTIALLY POLARISED PLANE WAVES

COMPLEX DOMAIN

COVARIANCE MATRIX $\langle [J] \rangle = \left\langle \underline{E}\,\underline{E}^{T*} \right\rangle$

REAL DOMAIN

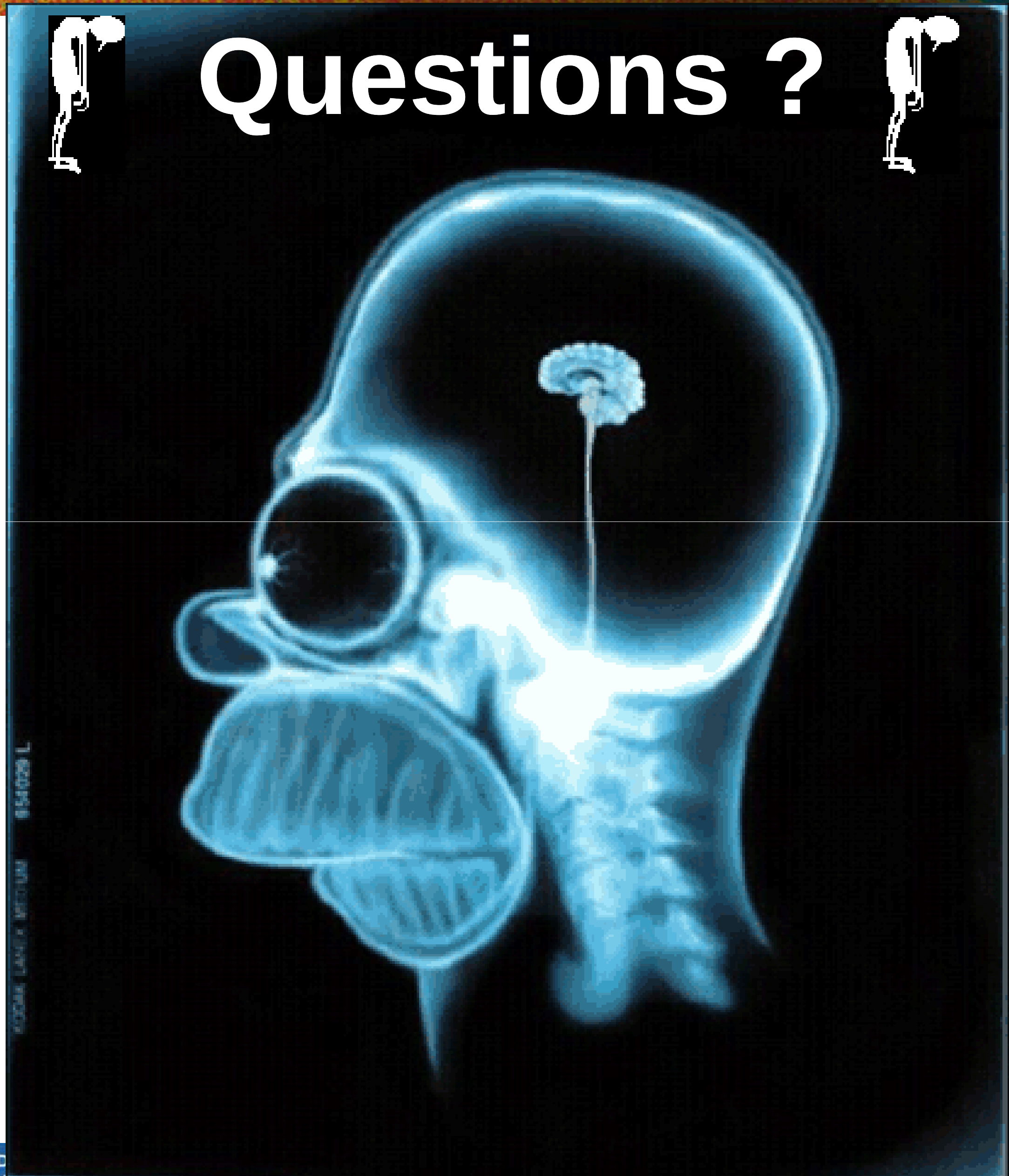
STOKES VECTOR $\langle \underline{g}_{\underline{E}} \rangle = \begin{bmatrix} \langle g_0 \rangle \\ \langle g_1 \rangle \\ \langle g_2 \rangle \\ \langle g_3 \rangle \end{bmatrix}$

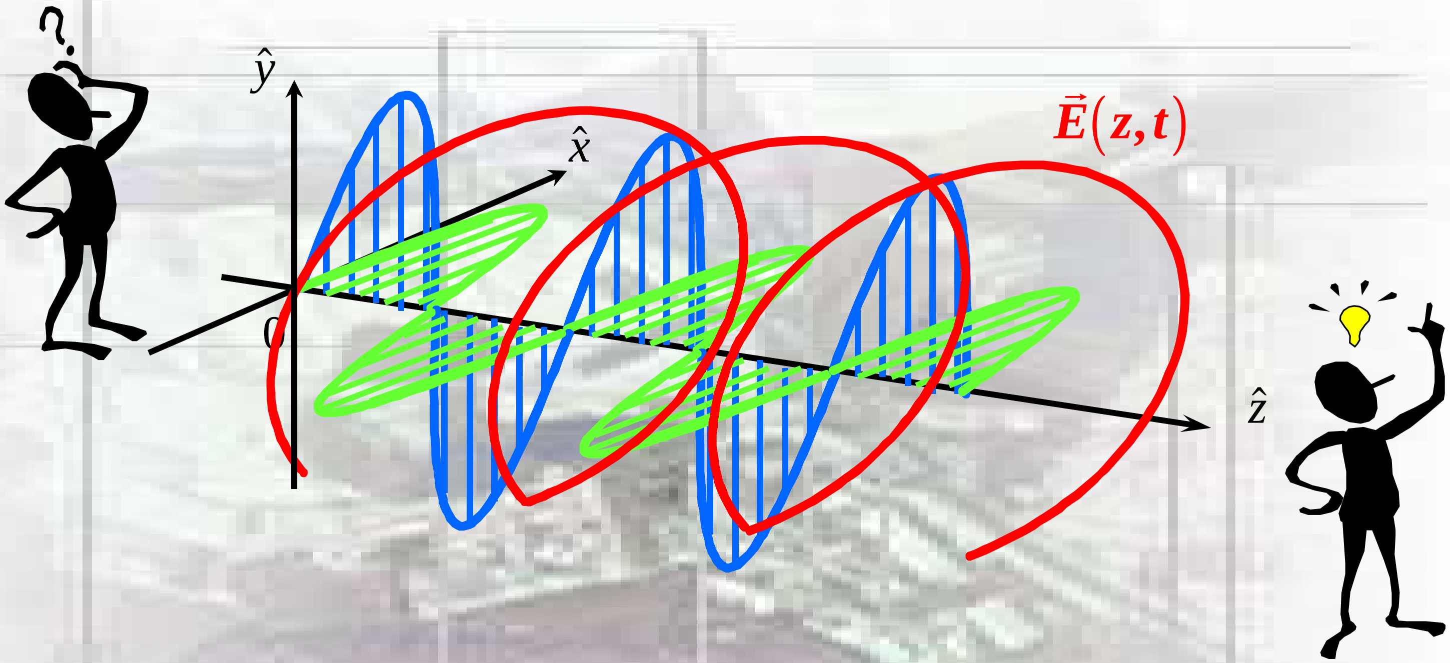
PLANE WAVES FULLY DESCRIBED
BY 4 INDEPENDANT PARAMETERS

- $\left\langle |E_x|^2 \right\rangle, \left\langle E_x E_y^* \right\rangle, \left\langle E_y E_x^* \right\rangle, \left\langle |E_y|^2 \right\rangle$
- $\left\{ \langle g_0 \rangle, \langle g_1 \rangle, \langle g_2 \rangle, \langle g_3 \rangle \right\}$

WAVE POLARIMETRIC DIMENSION = 4

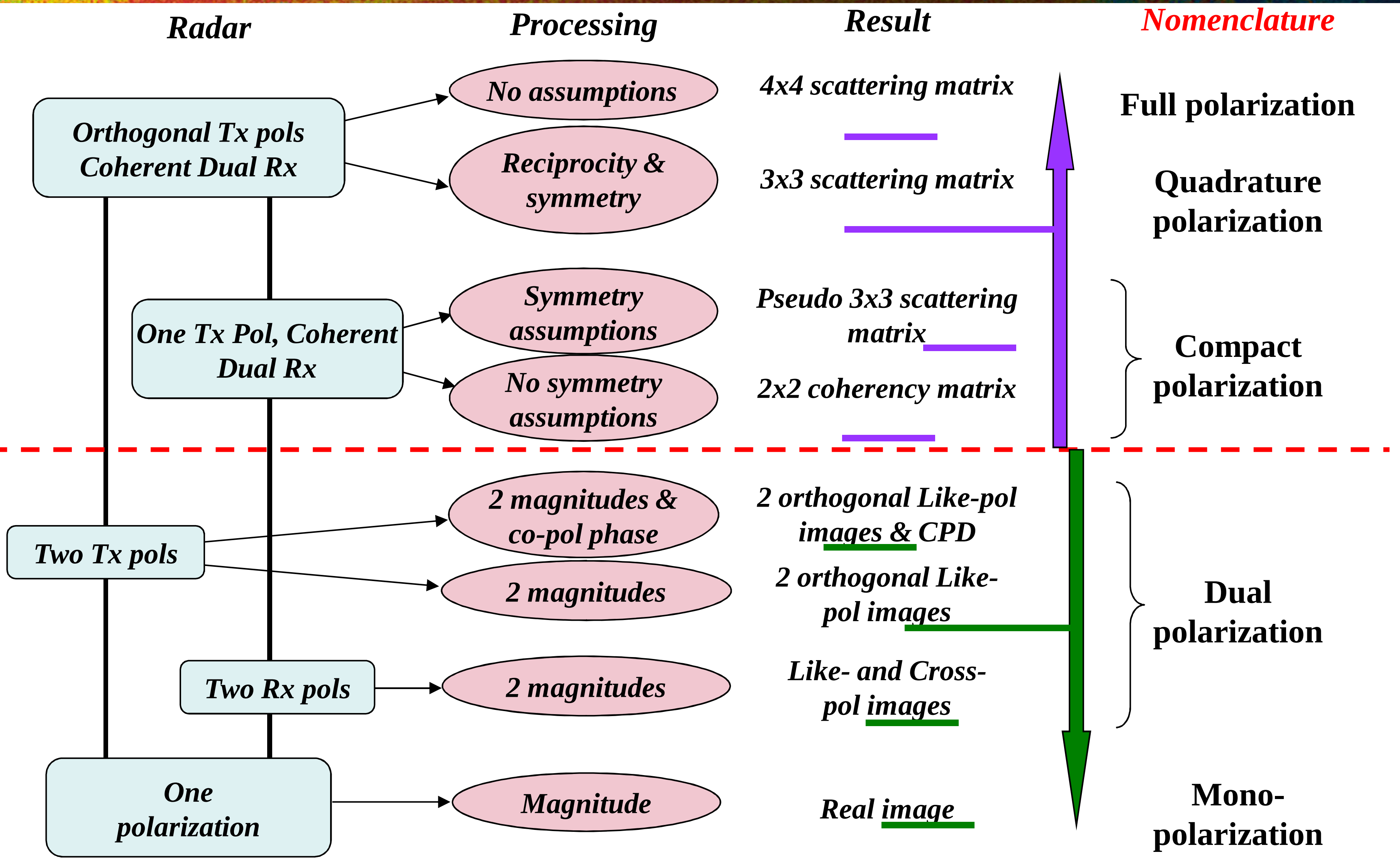
Questions ?





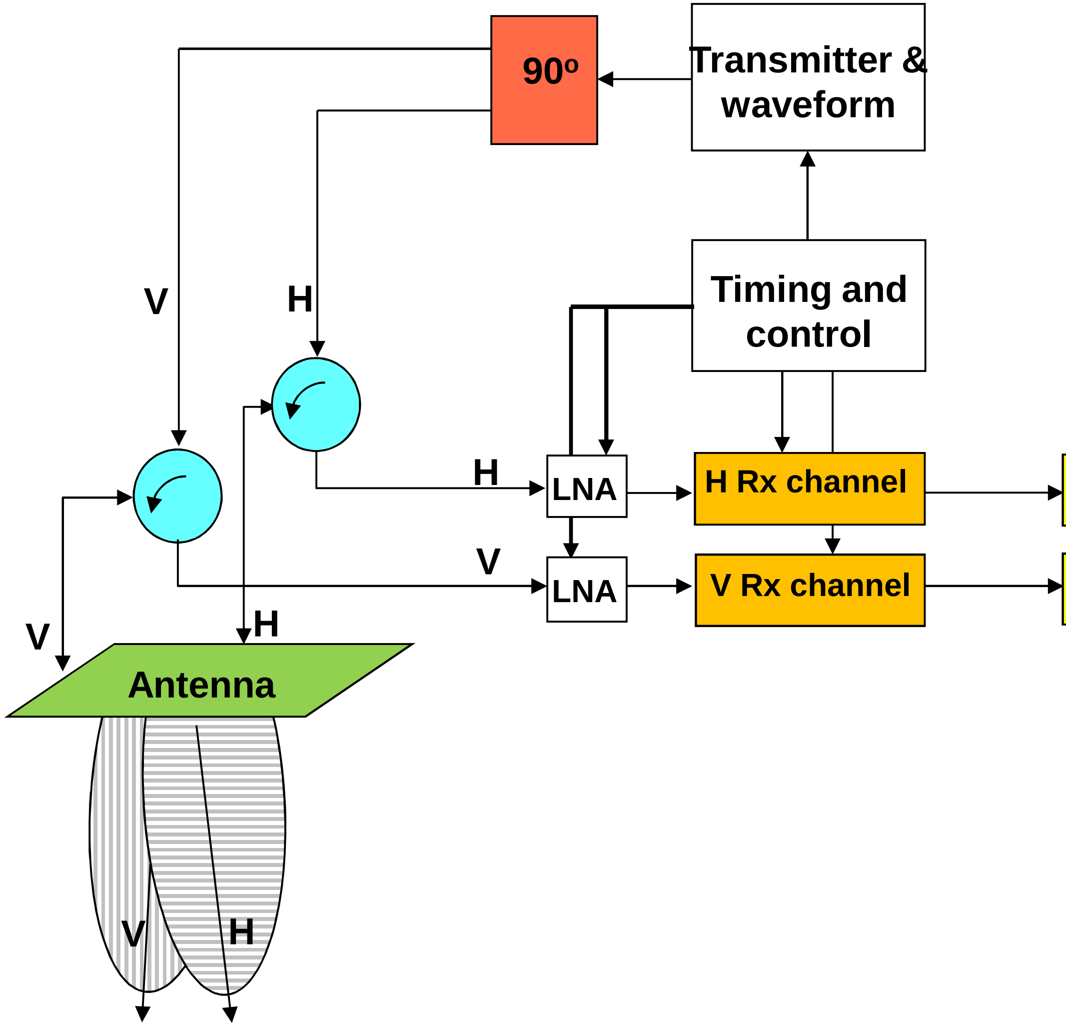
COMPACT / HYBRID POLARIMETRY

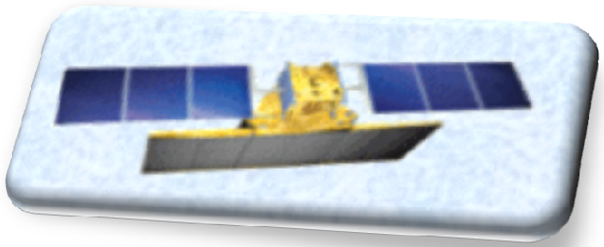
POLARIMETRIC RADARS: AN OVERVIEW



Courtesy of Dr. R. K. Raney

SYSTEM ARCHITECTURE





$$\underline{E} = \begin{bmatrix} E_{HR} \\ E_{VR} \end{bmatrix}$$

Courtesy of Dr. R. K. Raney

COMPACT / HYBRID POLARIMETRY

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_{HH} \\ E_{VH} \end{bmatrix}$$

STOKES VECTOR

$$\underline{g}_E = \begin{bmatrix} g_0 = |E_{HH}|^2 + |E_{VH}|^2 \\ g_1 = |E_{HH}|^2 - |E_{VH}|^2 \\ g_2 = 2\Re\left(E_{HH}E_{VH}^*\right) \\ g_3 = -2\Im\left(E_{HH}E_{VH}^*\right) \end{bmatrix}$$

DUAL – POL MODE

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_{HR} = E_{HH} - jE_{HV} \\ E_{VR} = E_{VH} - jE_{VV} \end{bmatrix}$$

STOKES VECTOR

$$\underline{g}_E = \begin{bmatrix} g_0 = |E_{HR}|^2 + |E_{VR}|^2 \\ g_1 = |E_{HR}|^2 - |E_{VR}|^2 \\ g_2 = 2\Re\left(E_{HR}E_{VR}^*\right) \\ g_3 = -2\Im\left(E_{HR}E_{VR}^*\right) \end{bmatrix}$$

COMPACT – POL MODE

COMPACT / HYBRID POLARIMETRY

WAVE COVARIANCE MATRIX

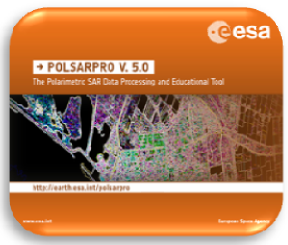
$$\langle [J] \rangle = \langle \underline{E}\,\underline{E}^{T*} \rangle = \begin{bmatrix} \langle |E_{HR}|^2 \rangle & \langle E_{HR} E_{VR}^{*} \rangle \\ \langle E_{VR} E_{HR}^{*} \rangle & \langle |E_{VR}|^2 \rangle \end{bmatrix}$$

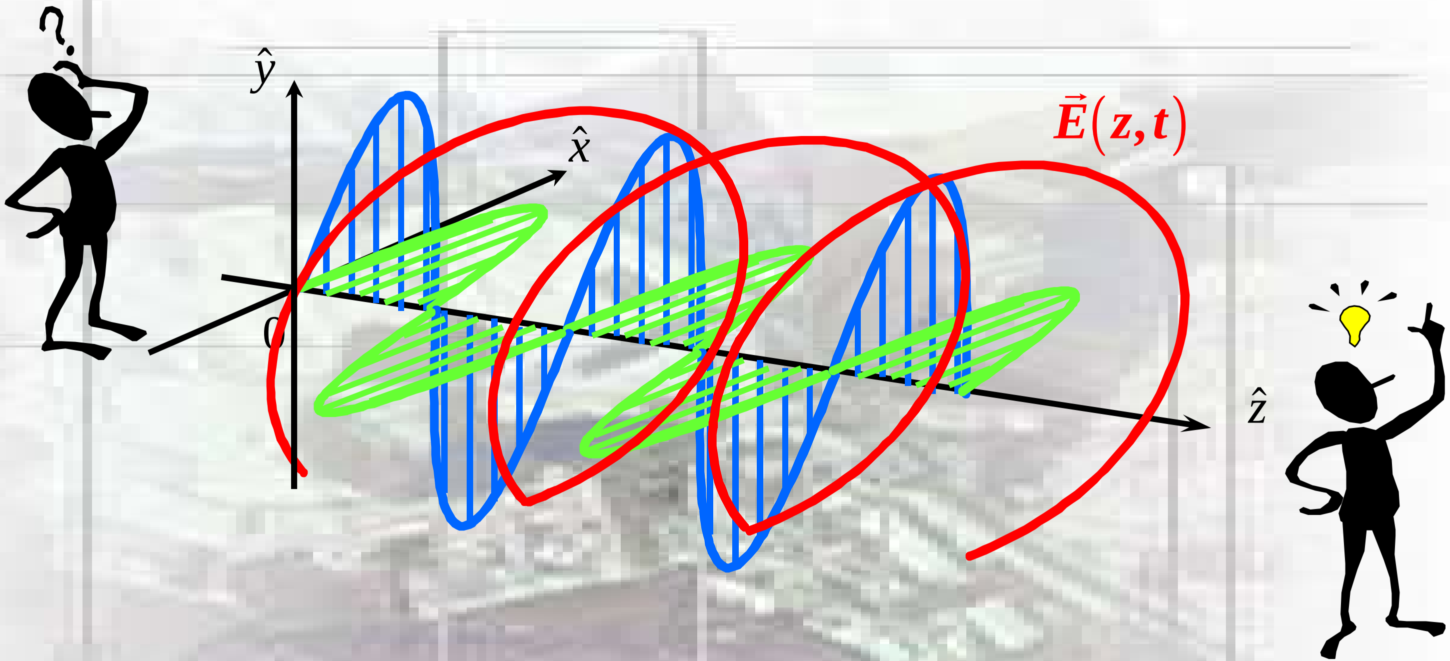
➡ **Degree of Polarisation, Wave Entropy,**

Polarimetric parameters : **(m,δ) and (m,ϕ) : K.R.Raney**
$(m_s, \alpha_s, m_v, \alpha, \tau, \delta)$: S.R. Cloude

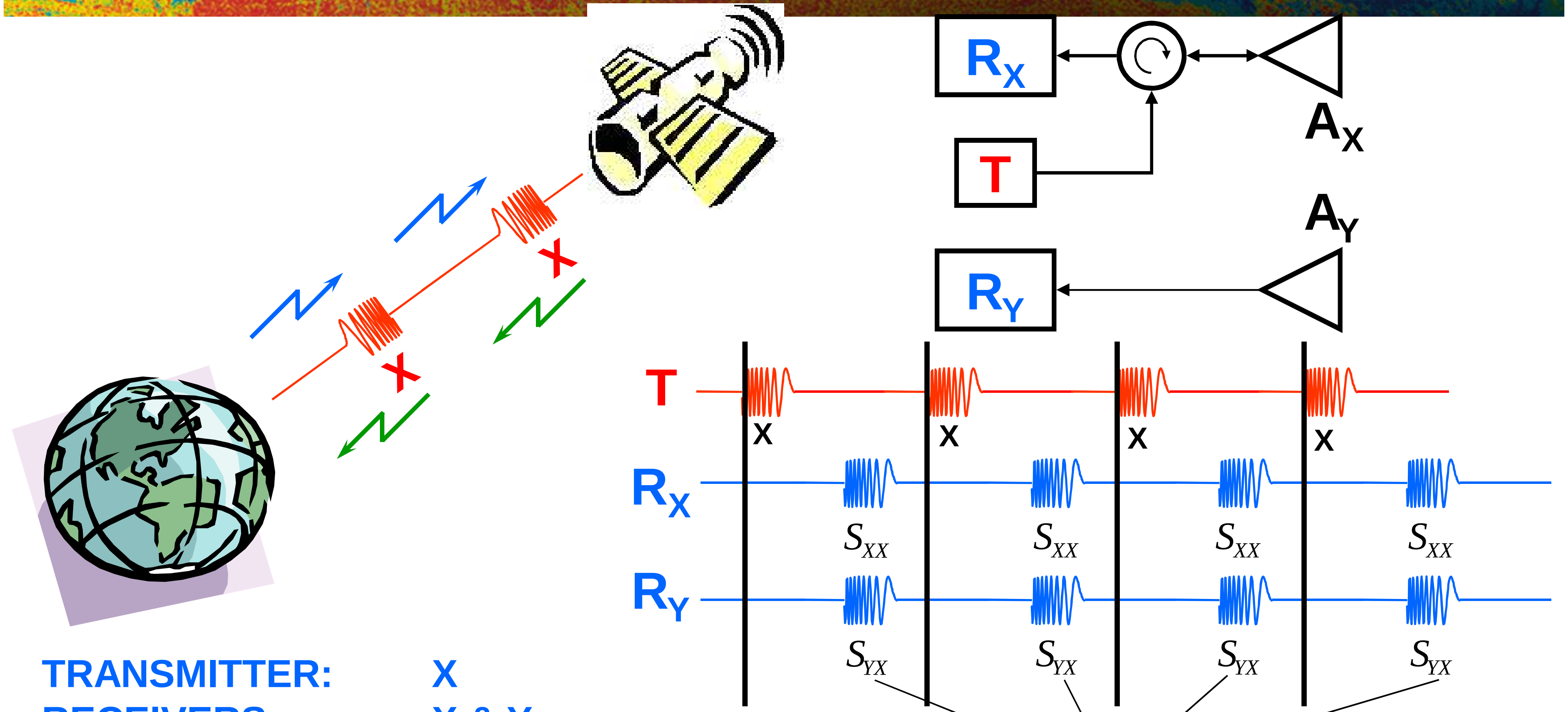
S.R. Cloude, D. Goodenough, H. Chen, « *Compact Decomposition Theory* »
IEEE Geoscience and Remote Sensing Letters, vol. 9 (1), pp 28-32, jan 2012

K. Raney, D. Goodenough, H. Chen, « *Compact Decomposition Theory* »
IEEE Geoscience and Remote Sensing Letters, vol. 9 (1), pp 28-32, jan 2012





SCATTERING POLARIMETRY

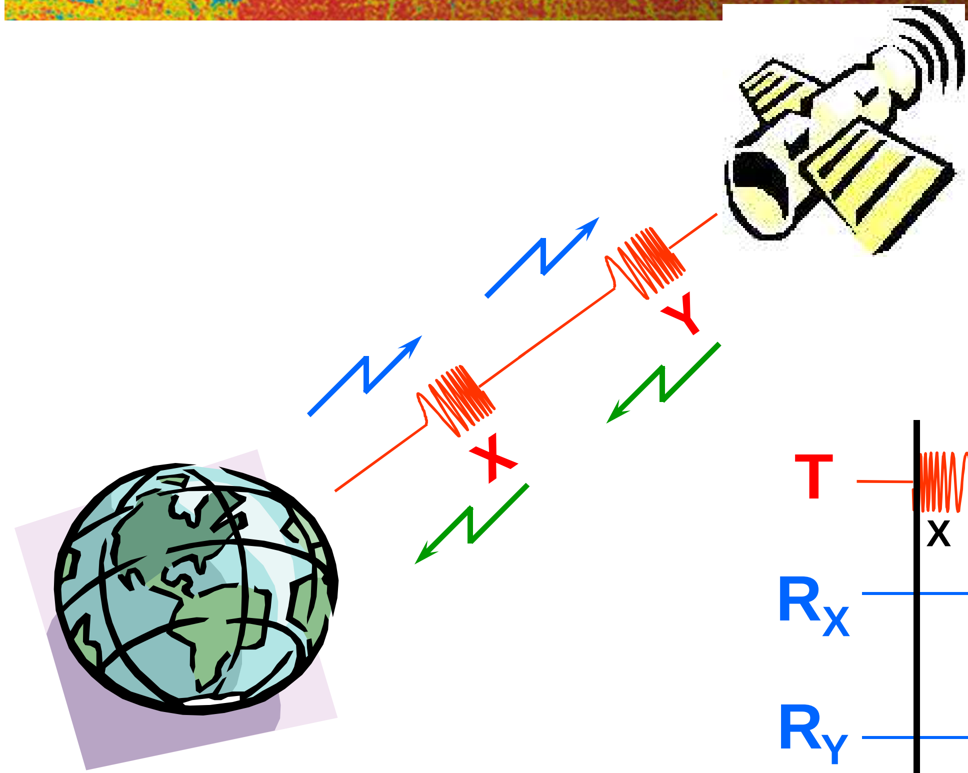


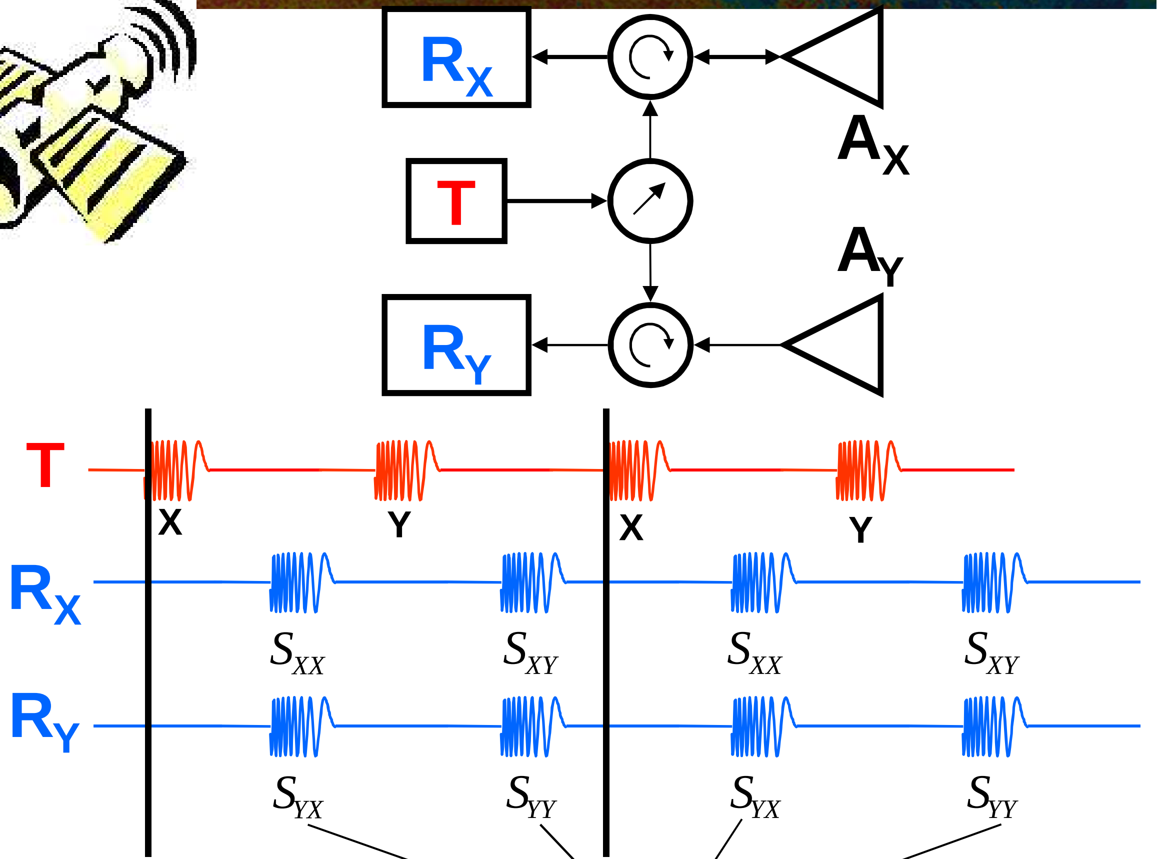
TRANSMITTER: X

RECEIVERS: X & Y

JONES VECTORS $\left\{\underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix}\right\}$

WAVE POLARIMETRY



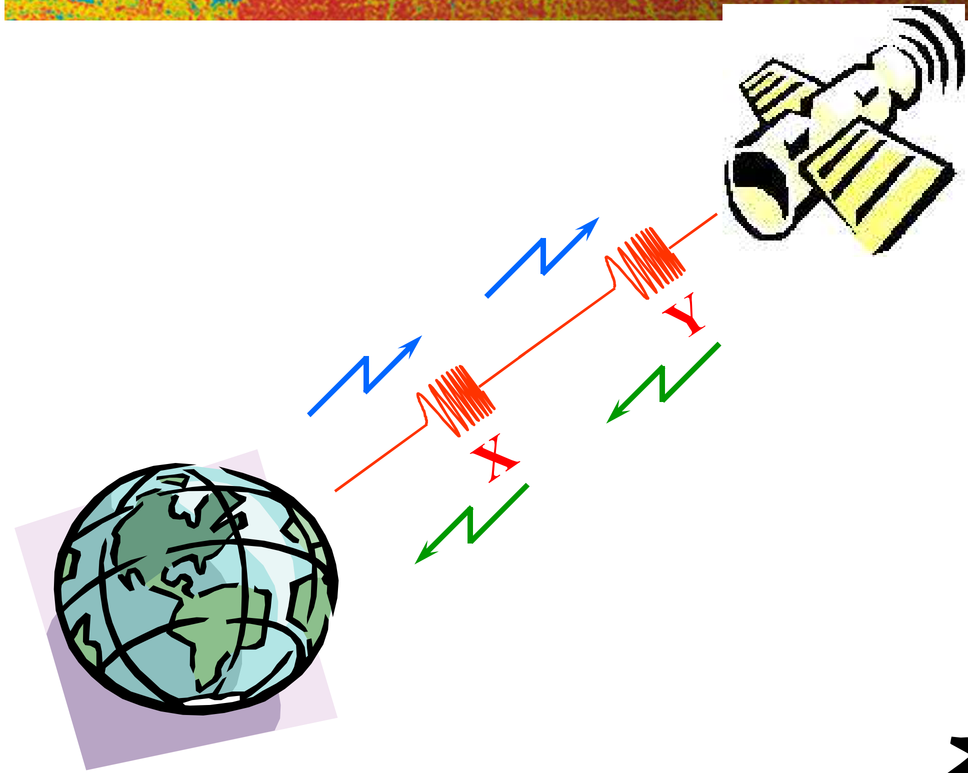


TRANSMITTER: X & Y
RECEIVERS: X & Y

SINCLAIR MATRICES

$$\left\{ [S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \right\}$$

SCATTERING POLARIMETRY



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
<u>k</u>, <u>Ω</u>	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

BISTATIC CASE

SCATTERING MATRIX or JONES MATRIX

$$\begin{bmatrix} E_X^s \\ E_Y^s \end{bmatrix} = \frac{e^{jkr}}{r} \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \begin{bmatrix} E_X^i \\ E_Y^i \end{bmatrix}$$

DEFINED IN THE LOCAL COORDINATES SYSTEM

$[S]$ IS INDEPENDENT OF THE POLARISATION STATE OF THE INCIDENCE WAVE

$[S]$ IS DEPENDENT ON THE FREQUENCY AND THE GEOMETRICAL AND ELECTRICAL PROPERTIES OF THE SCATTERER

TOTAL SCATTERED POWER

$$Span([S]) = Trace\left([S][S]^{T*}\right) = |S_{XX}|^2 + |S_{XY}|^2 + |S_{YX}|^2 + |S_{YY}|^2$$

SCATTERING MATRIX

$$[S]=\frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}=\frac{e^{jkr}}{r}\begin{bmatrix} |S_{XX}|e^{j\varphi_{XX}} & |S_{XY}|e^{j\varphi_{XY}} \\ |S_{YX}|e^{j\varphi_{YX}} & |S_{YY}|e^{j\varphi_{YY}} \end{bmatrix}$$

ABSOLUTE SCATTERING MATRIX

$$[S]=\frac{e^{jkr}e^{j\phi_{XX}}}{r}\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} \\ |S_{YX}|e^{j(\varphi_{YX}-\varphi_{XX})} & |S_{YY}|e^{j(\varphi_{YY}-\varphi_{XX})} \end{bmatrix}$$

Absolute Phase Factor

RELATIVE SCATTERING MATRIX
Seven Parameters: 4 Amplitudes and 3 Phases

SCATTERER POLARIMETRIC DIMENSION = 7

BACKSCATTERING MATRIX

$$[S] = \frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix} = \frac{e^{jkr}}{r}\begin{bmatrix} |S_{XX}|e^{j\varphi_{XX}} & |S_{XY}|e^{j\varphi_{XY}} \\ |S_{XY}|e^{j\varphi_{XY}} & |S_{YY}|e^{j\varphi_{YY}} \end{bmatrix}$$

ABSOLUTE BACKSCATTERING MATRIX

$$[S] = \frac{e^{jkr}e^{j\phi_{XX}}}{r}\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} \\ |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} & |S_{YY}|e^{j(\varphi_{YY}-\varphi_{XX})} \end{bmatrix}$$

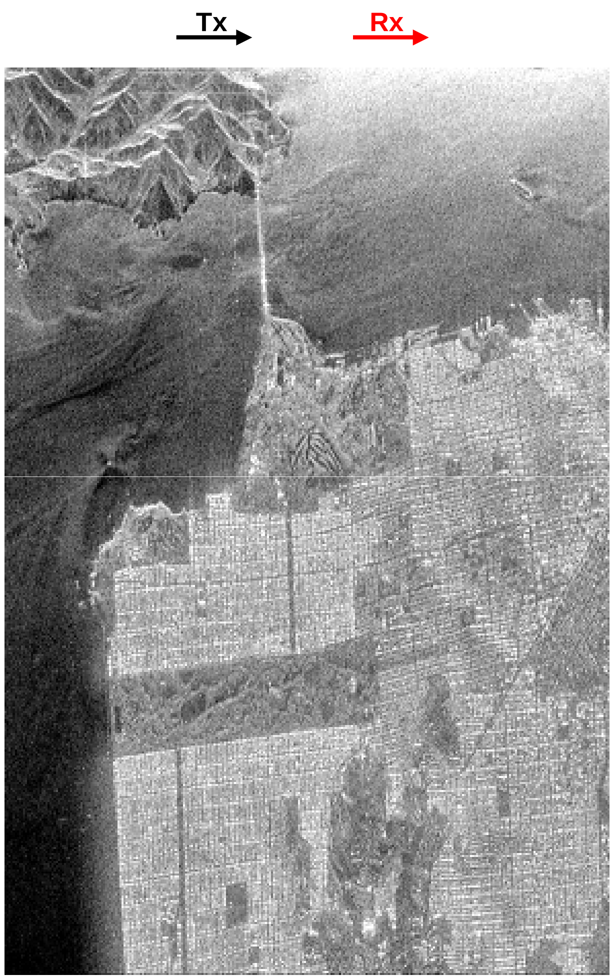
Absolute Phase Factor

RELATIVE BACKSCATTERING MATRIX

Five Parameters: 3 Amplitudes and 2 Phases

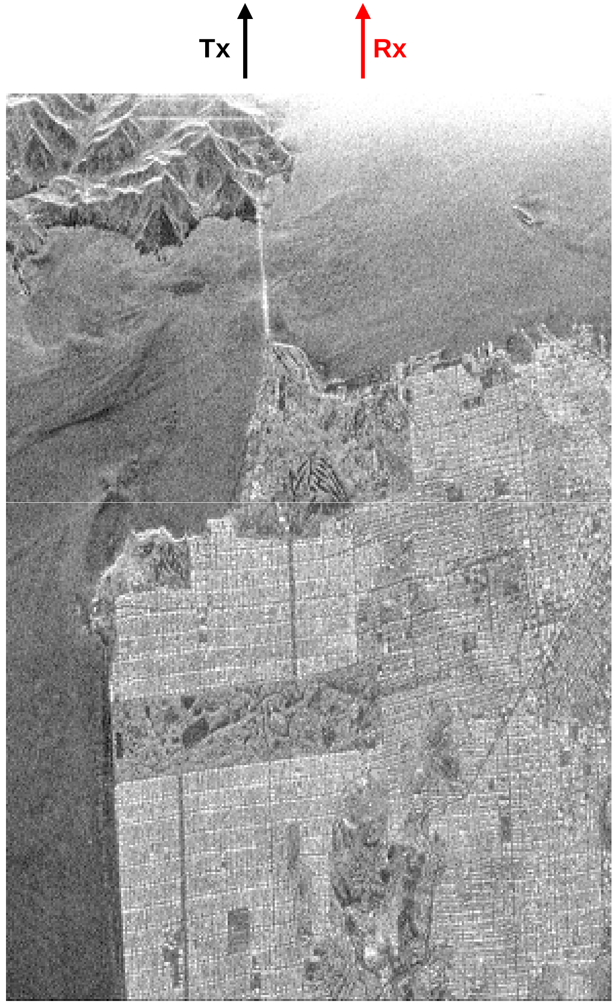
SCATTERER POLARIMETRIC DIMENSION = 5

SCATTERING POLARIMETRY



$|HH|_{dB}$

$|HV|_{dB}$



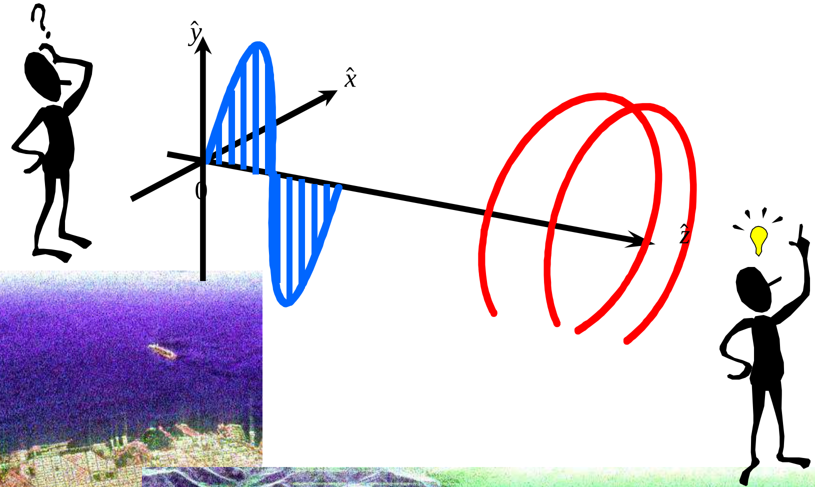
$|VV|_{dB}$

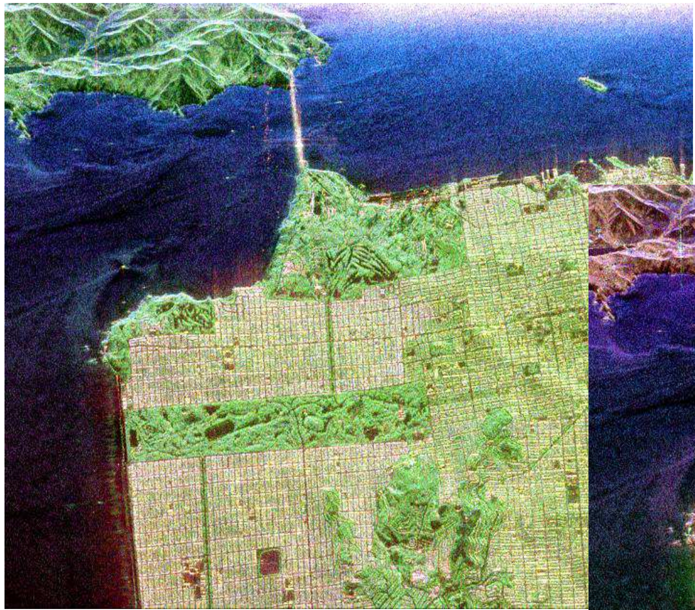
Sinclair Color Coding

© Google Earth

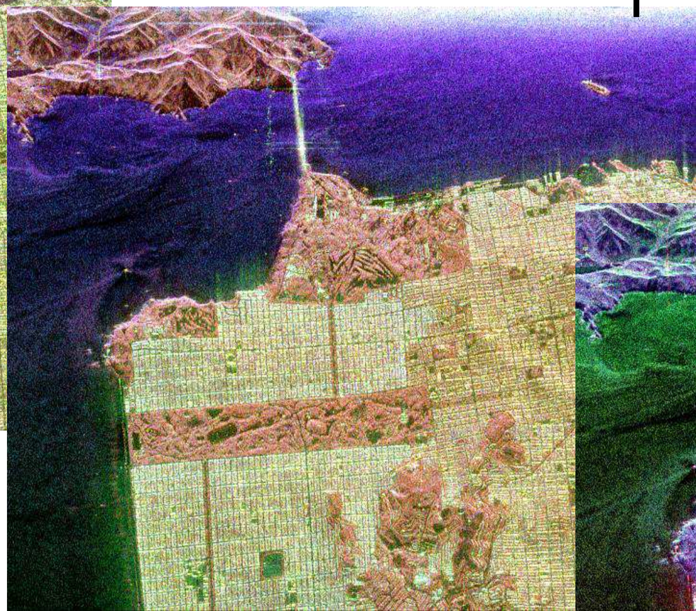
|HH| |HV| |VV|

ELLIPTICAL BASIS TRANSFORMATION

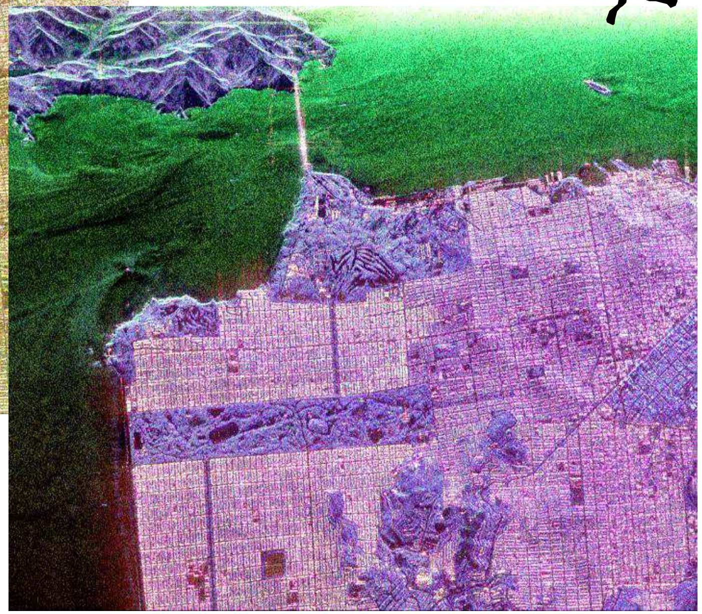




Pauli Color Coding (H,V)



Pauli Color Coding (+45,-45)



Pauli Color Coding (L,R)

Ernst LÜNEBURG
(PIERS95 - Pasadena)

ELLIPTICAL BASIS TRANSFORMATION EXPRESSED IN THE ORIENTED ANTENNA COORDINATES SYSTEM

EMISSION:

$$\underline{\boldsymbol{E}}^{i}_{(A,A_\perp)} = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]\underline{\boldsymbol{E}}^{i}_{(B,B_\perp)}$$

SU(2) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

RECEPTION: IN THE BSA CONVENTION $\underline{\boldsymbol{E}}^{s}_{(A,A_\perp)}$ PROPAGATES IN THE: $\hat{\boldsymbol{p}}_s = -\hat{\boldsymbol{p}}_i$

⬇

$\left(\underline{\boldsymbol{E}}^{s}_{(A,A_\perp)}\right)^*$ PROPAGATES IN THE: $\hat{\boldsymbol{p}}_s = \hat{\boldsymbol{p}}_i$

Time reversal = Complex conjugation

⬇

$$\left(\underline{\boldsymbol{E}}^{s}_{(A,A_\perp)}\right)^* = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]\left(\underline{\boldsymbol{E}}^{s}_{(B,B_\perp)}\right)^*$$

ELLIPTICAL BASIS TRANSFORMATION

$$\underline{E}^{s}_{(A,A_\perp)} = \left[S_{(A,A_\perp)}\right] \underline{E}^{i}_{(A,A_\perp)}$$

$$\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^{*} \underline{E}^{s}_{(B,B_\perp)} = \left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right] \underline{E}^{i}_{(B,B_\perp)}$$

$$\underline{E}^{s}_{(B,B_\perp)} = \left(\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^{*}\right)^{-1} \left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right] \underline{E}^{i}_{(B,B_\perp)}$$

⬇

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^{T} \left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

$$\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

SU(2) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

ELLIPTICAL BASIS TRANSFORMATION

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T \left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

$$\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

SU(2) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\begin{aligned}\left[U_{(\underline{A},\underline{A}_\perp)\mapsto(\underline{B},\underline{B}_\perp)}\right] &= \left[U(\phi,\tau,\alpha)\right]^{-1} \\ &= \underbrace{\begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix}}_{[U_2(-\alpha)]} \underbrace{\begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix}}_{[U_2(-\tau)]} \underbrace{\begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix}}_{[U_2(-\phi)]}\end{aligned}$$

ELLIPTICAL BASIS TRANSFORMATION

FROBENIUS NORM OF $\left[S_{(A,A_\perp)}\right]$

$$Span\left(\left[S_{(A,A_\perp)}\right]\right) = Trace\left(\left[S_{(A,A_\perp)}\right]\left[S_{(A,A_\perp)}\right]^{T*}\right) = |S_{AA}|^2 + 2|S_{AA_\perp}|^2 + |S_{A_\perp A_\perp}|^2$$

FROBENIUS NORM OF $\left[S_{(B,B_\perp)}\right]$

$$Span\left(\left[S_{(B,B_\perp)}\right]\right) = Trace\left(\left[S_{(B,B_\perp)}\right]\left[S_{(B,B_\perp)}\right]^{T*}\right) = |S_{BB}|^2 + 2|S_{BB_\perp}|^2 + |S_{B_\perp B_\perp}|^2$$

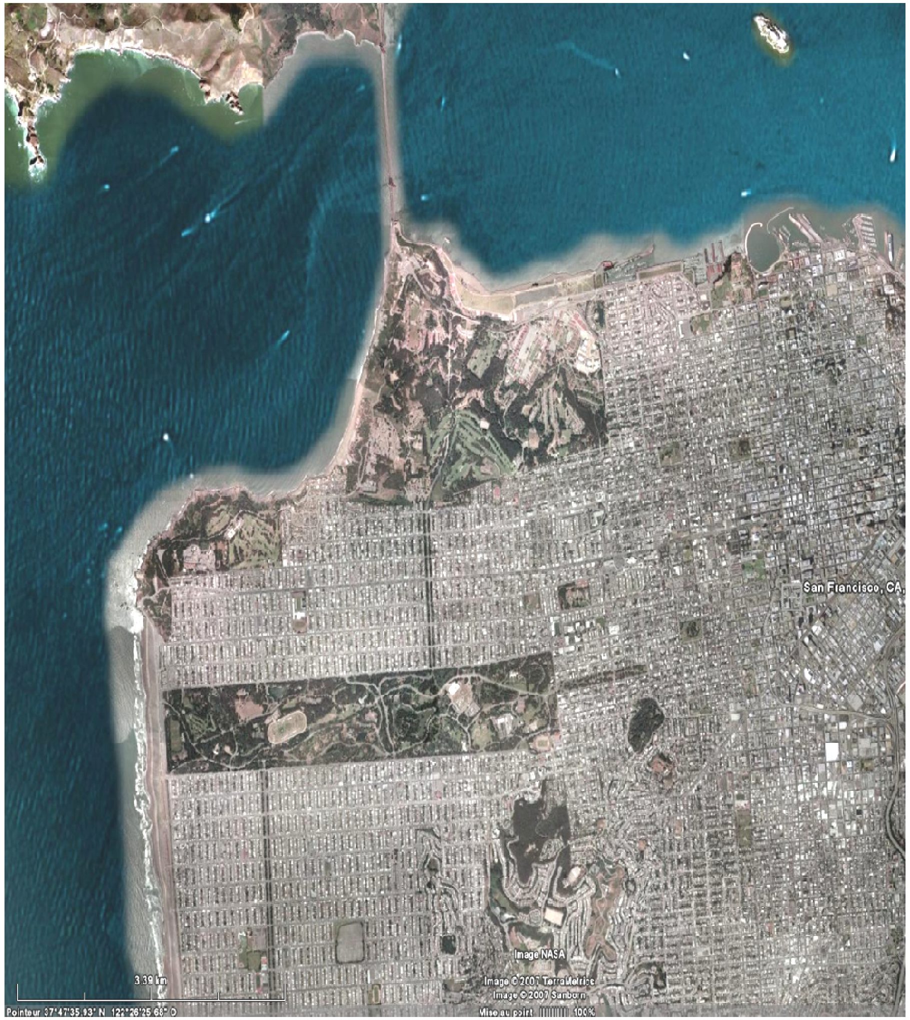
SPECIAL UNITARY SU(2) GROUP $\quad [U][U]^{T*} = [I_{D2}] \quad det([U]) = +1$

⬇

$$Span\left(\left[S_{(A,A_\perp)}\right]\right) = Span\left(\left[S_{(B,B_\perp)}\right]\right)$$

FROBENIUS NORM OF A SCATTERING MATRIX
IS INVARIANT UNDER BASIS ELLIPTICAL TRANSFORMATION

(H,V) POLARISATION BASIS



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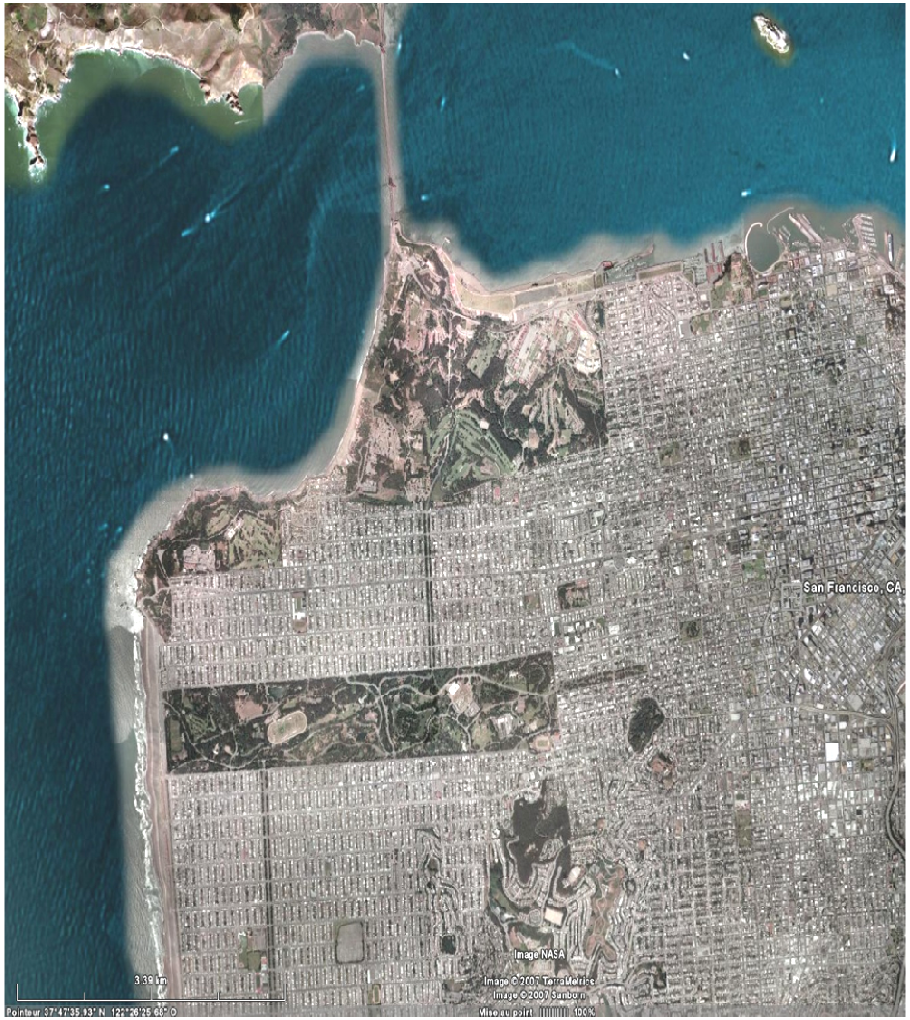
|HH+VV| |HV | |HH-VV|



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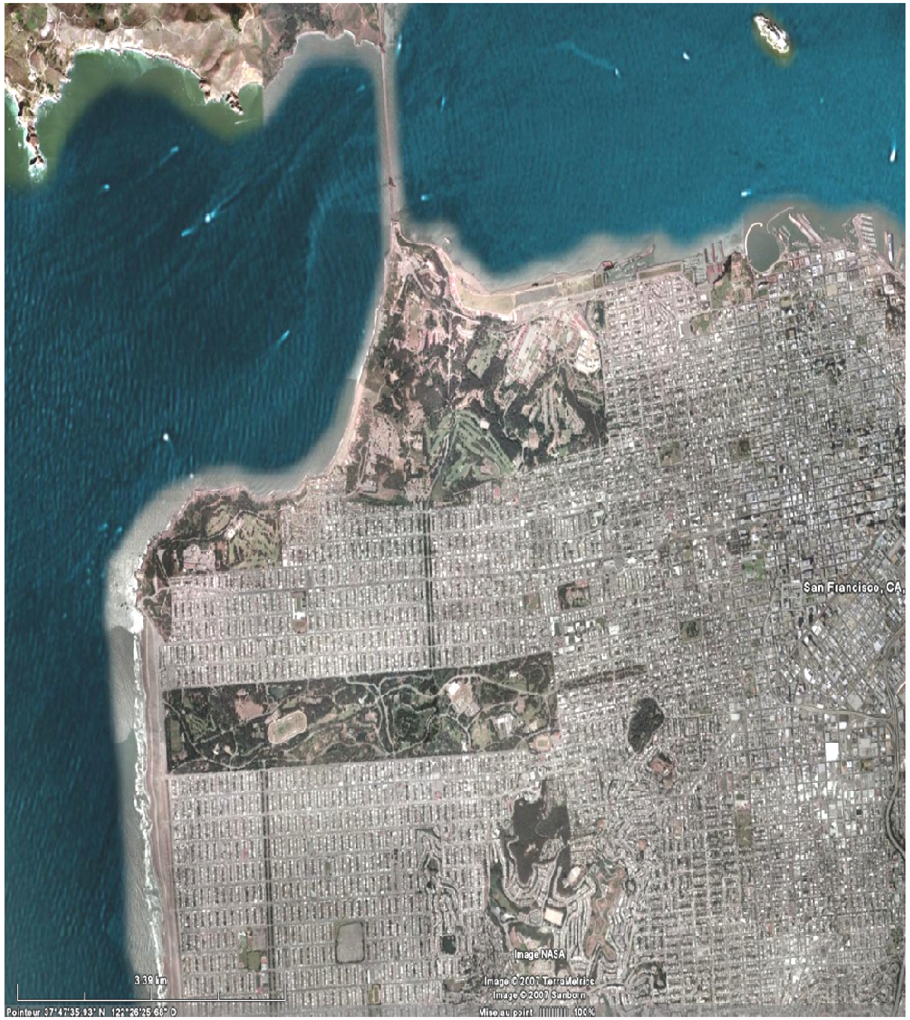
(+45°,-45°) POLARISATION BASIS

|AA+BB| |AB | |AA-BB|

With: A=Linear +45°, B=Linear –45°



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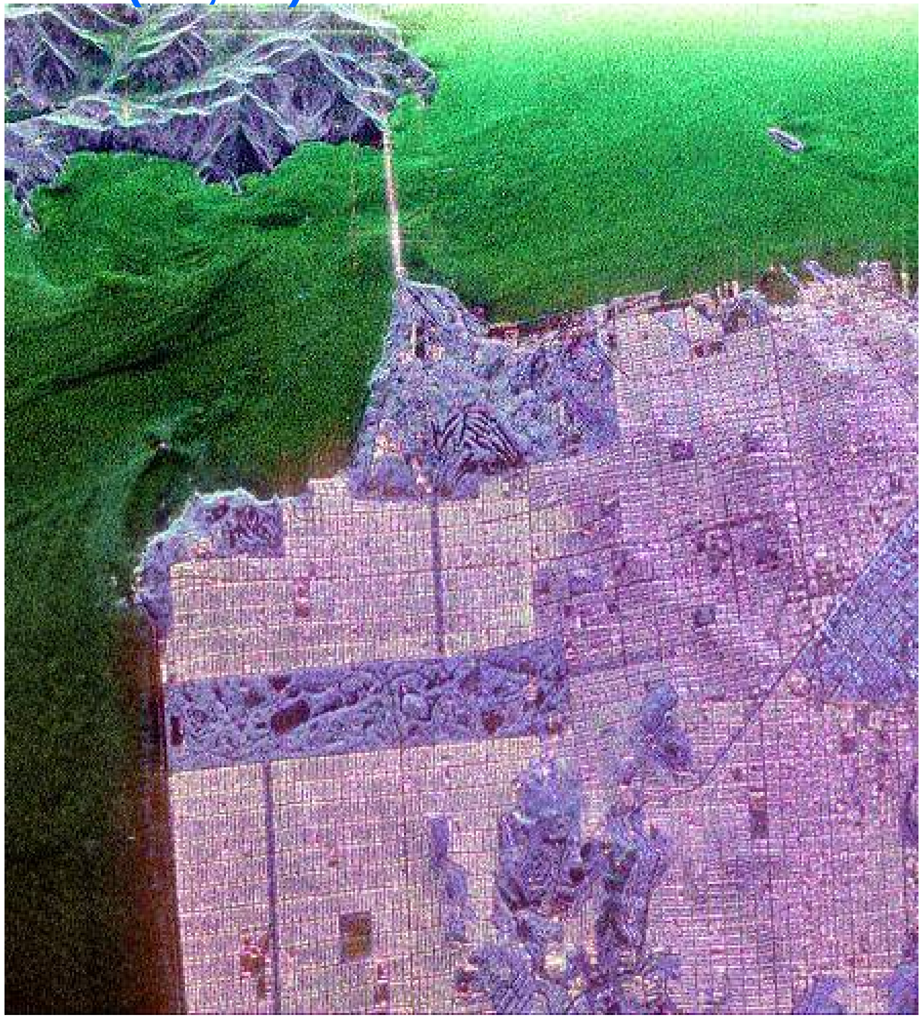
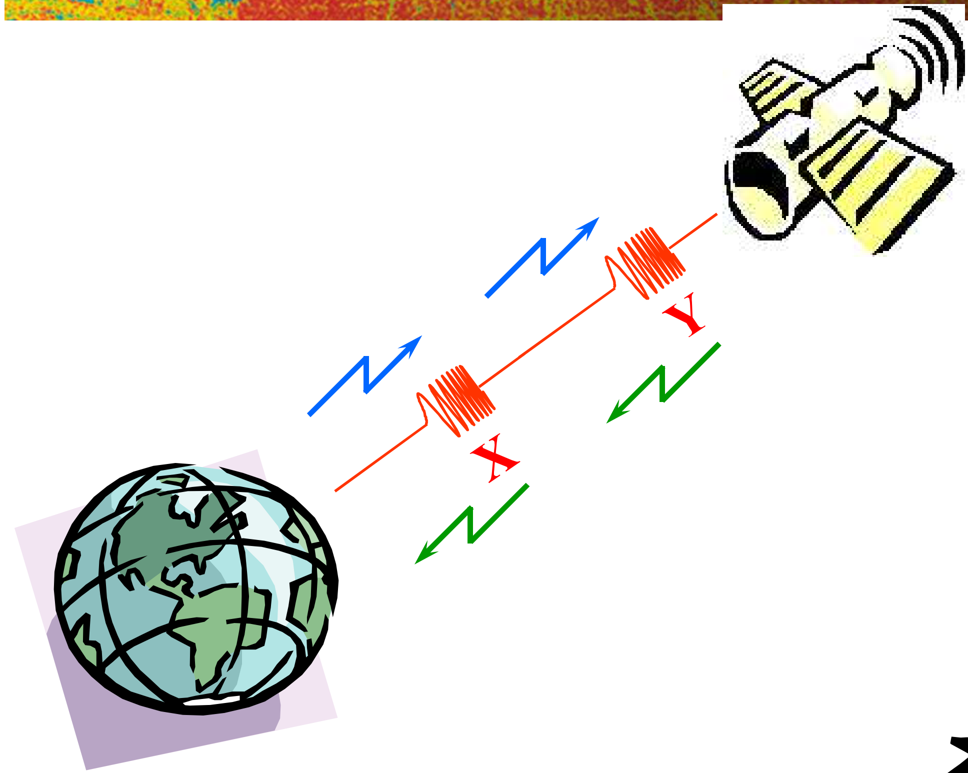
(LC,RC) POLARISATION BASIS



|LL+RR| |LR | |LL-RR|



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

- [S] SINCLAIR Matrix
- **k, Ω Target Vectors**
- [K] KENNAUGH Matrix
- [T] Coherency Matrix
- [C] Covariance Matrix

VECTORIAL FORMULATION OF THE SCATTERING PROBLEM

SCATTERING MATRIX

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

SCATTERING VECTOR

$$\vec{S} := V([S]) = \frac{1}{2} Trace([S][\Psi]) = \begin{bmatrix} S1 \\ S2 \\ S3 \\ S4 \end{bmatrix} \in C_4$$

With: $V([S])$ MATRIX VECTORISATION OPERATOR

$[\Psi]$ SET OF ORTHOGONAL 2x2 MATRICES

FROBENIOUS NORM OF $\vec{S}$

$$\|\vec{S}\|^2 = \vec{S}^{T*} \cdot \vec{S} = |S_1|^2 + |S_2|^2 + |S_3|^2 + |S_4|^2$$

$$= Span([S]) = |S_{XX}|^2 + |S_{YX}|^2 + |S_{XY}|^2 + |S_{YY}|^2$$

PAULI SCATTERING VECTOR

$$\underline{k} = V([S]) = \frac{1}{2} Trace([S][\psi_P])$$

SET OF 2x2 COMPLEX MATRICES
FROM THE PAULI MATRICES GROUP

$$[\psi_P] = \left\{ \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix} \right\}$$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad S_{XY} + S_{YX} \quad j(S_{XY} - S_{YX})\right]^T$$

Advantage: Closer related to physical properties of the scatterer

Note: Also known as $\underline{k}_{4P}$

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega} = V([S]) = \frac{1}{2} Trace([S][\psi_L])$

SET OF 2x2 COMPLEX MATRICES
FROM THE LEXICOGRAPHIC MATRICES GROUP

$$[\psi_L] = \left\{ 2\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

⬇

$$\underline{\Omega} = [S_{XX} \quad S_{XY} \quad S_{YX} \quad S_{YY}]^T$$

Advantage: **Directly related to the system measurables**

Note: **Also known as $\underline{k}_{4L}$**

SCATTERING VECTOR TRANSFORMATIONS

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ S_{XY} + S_{YX} \\ j(S_{XY} - S_{YX}) \end{bmatrix}$$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ S_{XY} \\ S_{YX} \\ S_{YY} \end{bmatrix}$$

UNITARY TRANSFORMATION

$$\underline{k} = [D_4]\underline{\Omega} \quad \text{and} \quad \underline{\Omega} = [D_4]^{-1}\underline{k} = [D_4]^{T*}\underline{k}$$

WHERE $[D_4]$ IS A SU(4) MATRIX
IN ORDER TO PRESERVE THE NORM
OF THE SCATTERING VECTOR

$$[D_4] = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix}$$

MONOSTATIC CASE

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ S_{XY} + S_{YX} \\ j\left(S_{XY} - S_{YX}\right) \end{bmatrix} \Rightarrow \underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ 2S_{XY} \end{bmatrix}$$

Note: Also known as $\underline{k}_{3P}$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ S_{XY} \\ S_{YX} \\ S_{YY} \end{bmatrix} \Rightarrow \underline{\Omega} = \begin{bmatrix} S_{XX} \\ \sqrt{2}S_{XY} \\ S_{YY} \end{bmatrix}$$

Note: Also known as $\underline{k}_{3L}$

SCATTERING VECTOR TRANSFORMATIONS

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ 2S_{XY} \end{bmatrix}$$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ \sqrt{2}S_{XY} \\ S_{YY} \end{bmatrix}$$

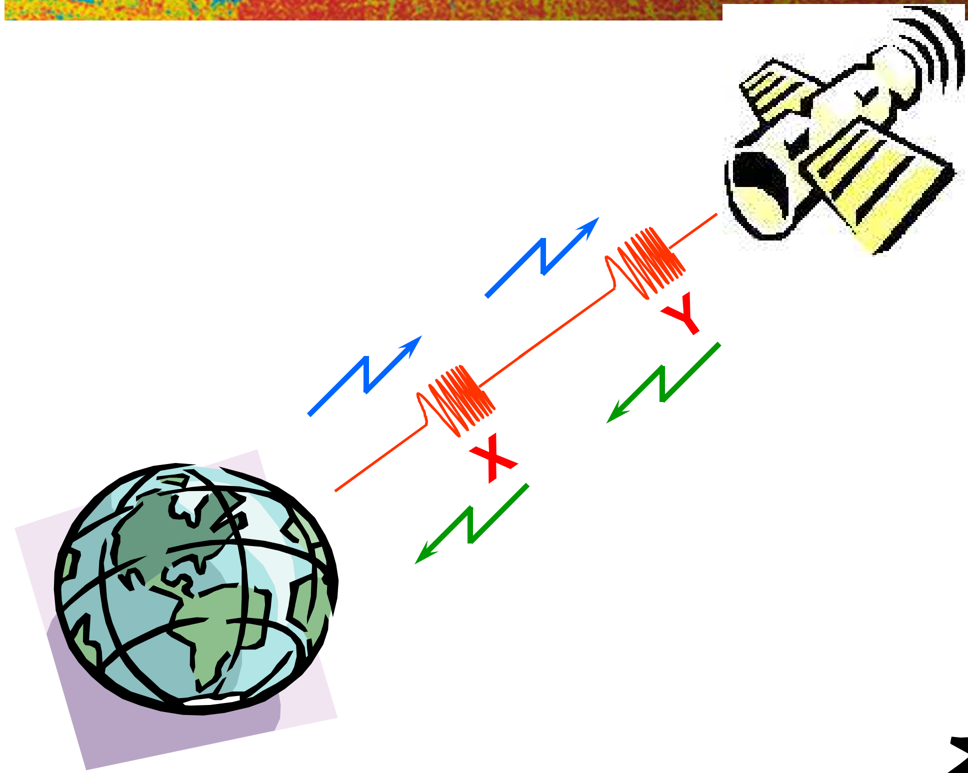
UNITARY TRANSFORMATION

$$\underline{k} = [D_3]\underline{\Omega} \quad \text{and} \quad \underline{\Omega} = [D_3]^{-1}\underline{k} = [D_3]^T\underline{k}$$

WHERE $[D_3]$ IS A SU(3) MATRIX IN ORDER TO PRESERVE THE NORM OF THE SCATTERING VECTOR

$$[D_3] = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \end{bmatrix}$$

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

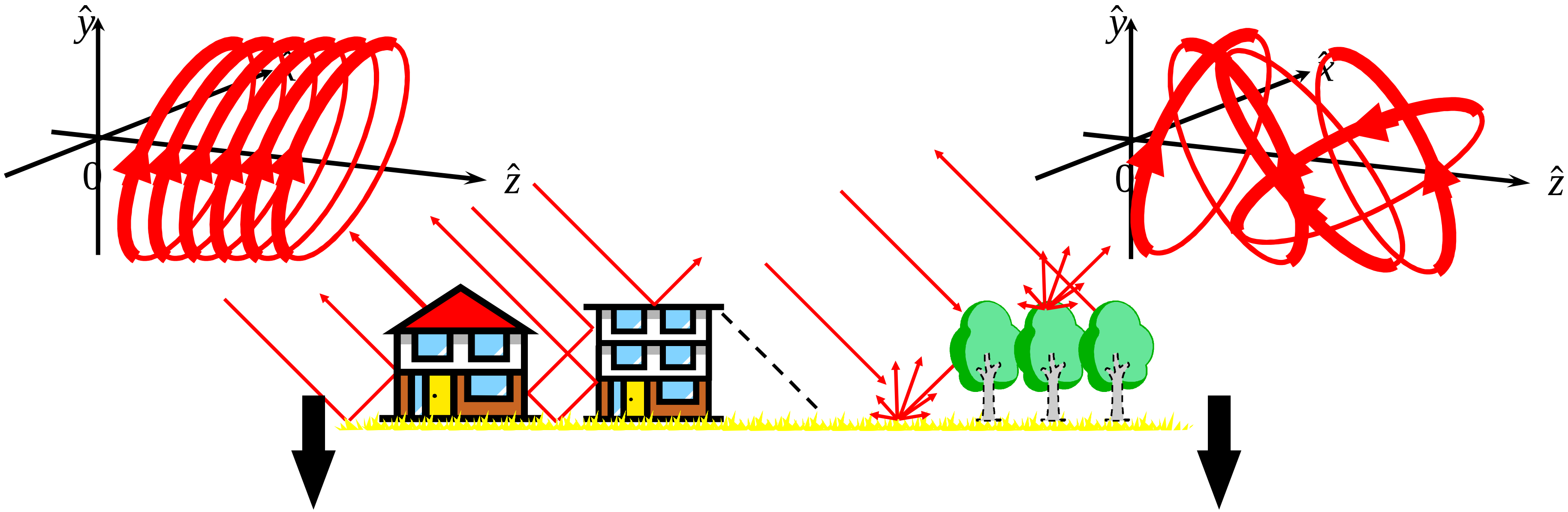
THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
$\underline{k}$, $\underline{\Omega}$	Target Vectors
[K]	**KENNAUGH Matrix**
[T]	Coherency Matrix
[C]	Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

DETERMINISTIC SCATTERERS | PARTIAL SCATTERERS



DETERMINISTIC SCATTERING → **COMPLETELY POLARISED SCATTERING** → **COMPLETELY DESCRIBED BY $[S]$**

RANDOM SCATTERING (Variation in Time / Space) → **PARTIALLY POLARISED SCATTERING** → **CAN NOT BE DESCRIBED BY $[S]$**

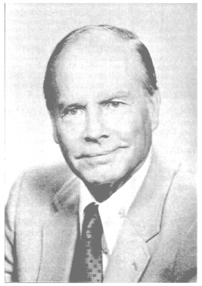
→ **STATISTICAL DESCRIPTION**

MONOSTATIC CASE

SINCLAIR MATRIX : $\underline{E}_s = [S]\underline{E}_i$

KENNAUGH MATRIX : $\underline{g}_{\underline{E}_s} = [K]\underline{g}_{\underline{E}_i}$

$$[K] = \frac{1}{2}\left([V]^T\left[[S]\otimes[S]^*\right][V]\right) \quad [V] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -j \\ 0 & 0 & 1 & +j \\ 1 & -1 & 0 & 0 \end{bmatrix}$$



HUYNEN PARAMETERS

$$[K] = \begin{bmatrix} A_0 + B_0 & C & H & F \\ C & A_0 + B & E & G \\ H & E & A_0 - B & D \\ F & G & D & -A_0 + B_0 \end{bmatrix}$$

PHYSICAL INTERPRETATION

MAN-MADE TARGET DECOMPOSITION
IDENTIFICATION and ANALYSIS

« *PHENOMENOLOGICAL THEORY OF RADAR TARGETS* » (1970)

A0 : GENERATOR OF TARGET SYMMETRY

B0+B : GENERATOR OF TARGET NON-SYMMETRY

B0-B : GENERATOR OF TARGET IRREGULARITY

C : GENERATOR OF TARGET GLOBAL SHAPE (LINEAR)

D : GENERATOR OF TARGET LOCAL SHAPE (CURVATURE)

E : GENERATOR OF TARGET LOCAL TWIST (TORSION)

F : GENERATOR OF TARGET GLOBAL TWIST (HELICITY)

G : GENERATOR OF TARGET LOCAL COUPLING (GLUE)

H : GENERATOR OF TARGET GLOBAL COUPLING (ORIENTATION)

STOKES VECTOR

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

$[U_2(\phi)]$ $[U_2(\tau)]$ $[U_2(\alpha)]$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2}Tr\left([U_2(\theta)]^{T*}\sigma_p[U_2(\theta)]\sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

STOKES VECTOR

$$\underline{g}_{\underline{E}} = A^2\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}\underline{g}_{\hat{u}_x}$$

$[O_3(2\phi)]$ $[O_3(2\tau)]$ $[O_3(2\alpha)]$

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$\underline{E}^s_{(A,A_\perp)} = \left[S_{(A,A_\perp)}\right]\underline{E}^i_{(A,A_\perp)} \qquad \underline{E}^s_{(B,B_\perp)} = \left[S_{(B,B_\perp)}\right]\underline{E}^i_{(B,B_\perp)}$$

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T\left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

KENNAUGH MATRIX

$$\underline{g}_{\underline{E}^s_{(A,A_\perp)}} = \left[K_{(A,A_\perp)}\right]\underline{g}_{\underline{E}^i_{(A,A_\perp)}} \qquad \underline{g}_{\underline{E}^s_{(B,B_\perp)}} = \left[K_{(B,B_\perp)}\right]\underline{g}_{\underline{E}^i_{(B,B_\perp)}}$$

$$\left[K_{(B,B_\perp)}\right] = \begin{bmatrix} 1 & \underline{0} \\ \underline{0} & O_{3(A,A_\perp)\mapsto(B,B_\perp)} \end{bmatrix}\left[K_{(A,A_\perp)}\right]\begin{bmatrix} 1 & \underline{0} \\ \underline{0} & O_{3(A,A_\perp)\mapsto(B,B_\perp)} \end{bmatrix}^{-1}$$

SIMILARITY TRANSFORMATION

$\left[O_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]$ O(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2} Tr\left([U_2(\theta)]^{T*} \sigma_p [U_2(\theta)] \sigma_q\right)$$

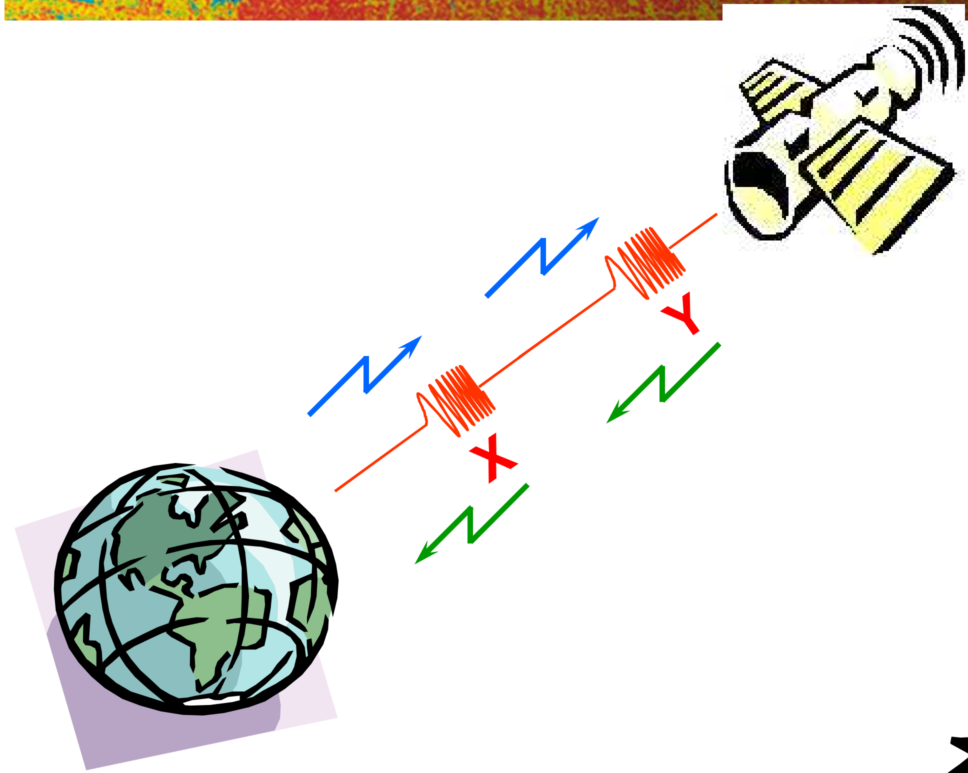
(σ_p, σ_q) : Pauli Matrices

O(3) UNITARY GROUP

$$\begin{bmatrix} \cos 2\phi & -\sin 2\phi & 0 \\ \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 2\tau & 0 & -\sin 2\tau \\ 0 & 1 & 0 \\ \sin 2\tau & 0 & \cos 2\tau \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\alpha & -\sin 2\alpha \\ 0 & \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$[O_3(2\phi)] \qquad [O_3(2\tau)] \qquad [O_3(2\alpha)]$$

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

- [S] SINCLAIR Matrix
- k, Ω Target Vectors
- [K] KENNAUGH Matrix
- **[T] Coherency Matrix**
- [C] Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

BISTATIC CASE

PAULI SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad S_{XY} + S_{YX} \quad j\left(S_{XY} - S_{YX}\right)\right]^T$$

↓

COHERENCY MATRIX $[T]$

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG & L - jK \\ C + jD & B_0 + B & E + jF & M - jN \\ H - jG & E - jF & B_0 - B & J + jI \\ L + jK & M + jN & J - jI & 2A \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

COHERENCY MATRIX

MONOSTATIC CASE

PAULI SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

⬇

COHERENCY MATRIX $[T]$

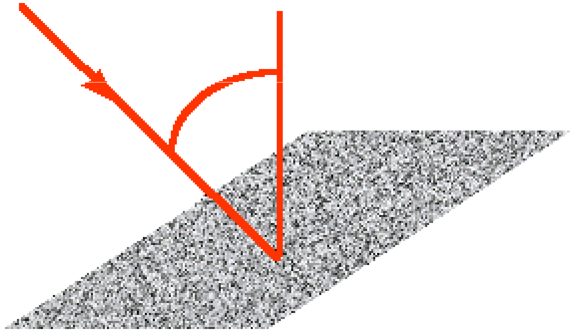
$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

HERMITIAN MATRIX - RANK 1

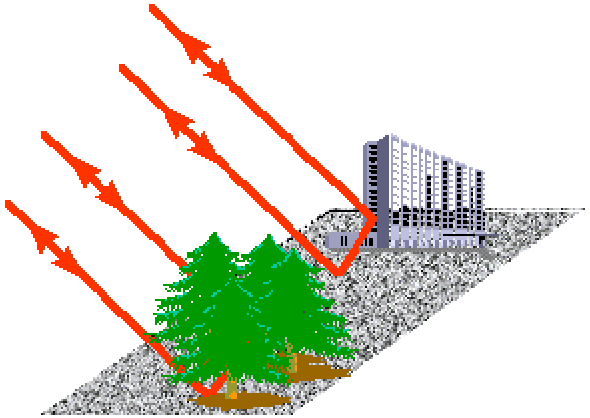
A0, B0+B, B0-B : HUYNEN TARGET GENERATORS

PHYSICAL INTERPRETATION

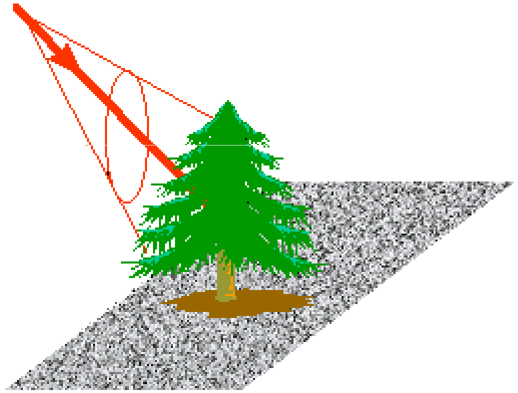
SINGLE BOUNCE SCATTERING (ROUGH SURFACE)



DOUBLE BOUNCE SCATTERING



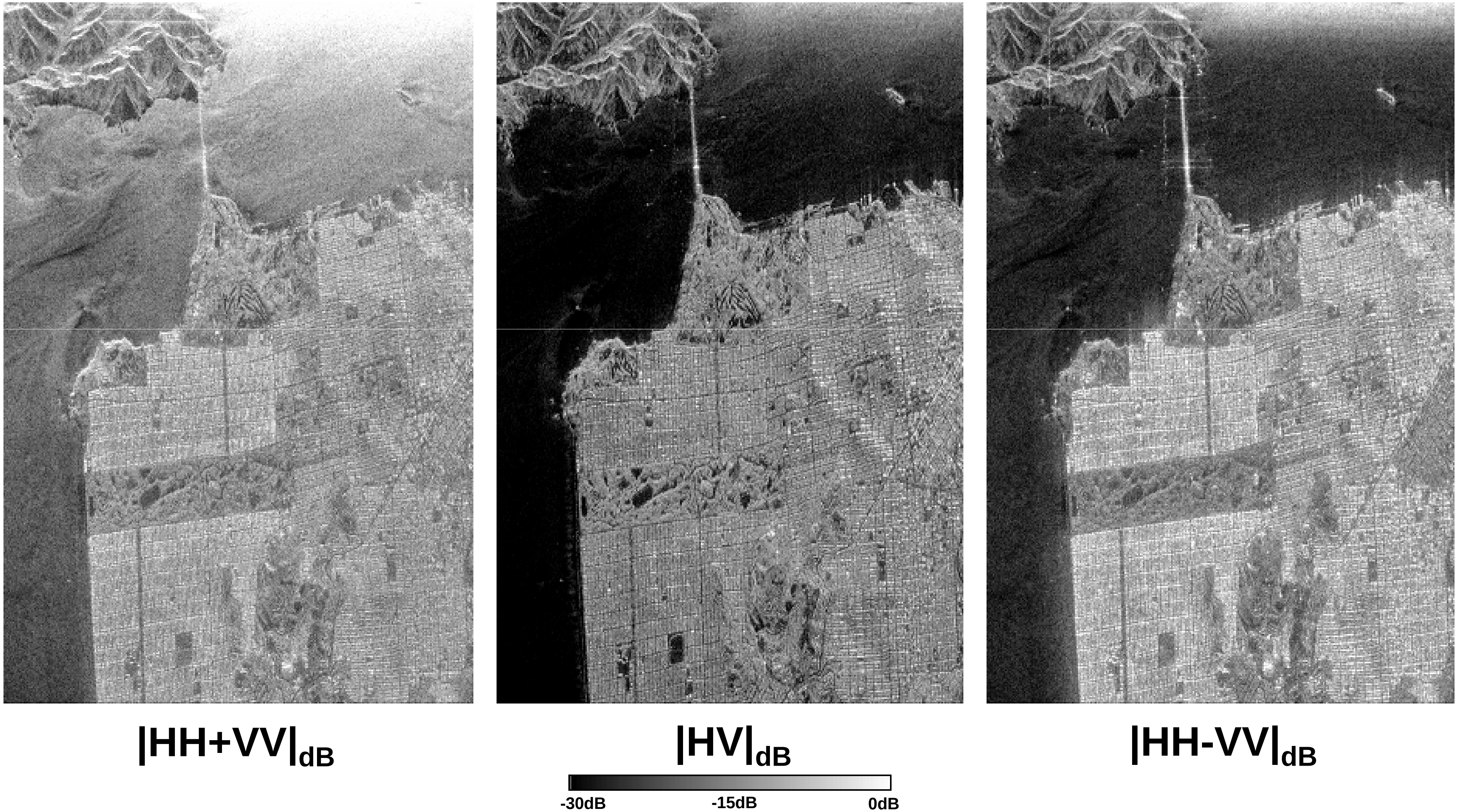
VOLUME SCATTERING



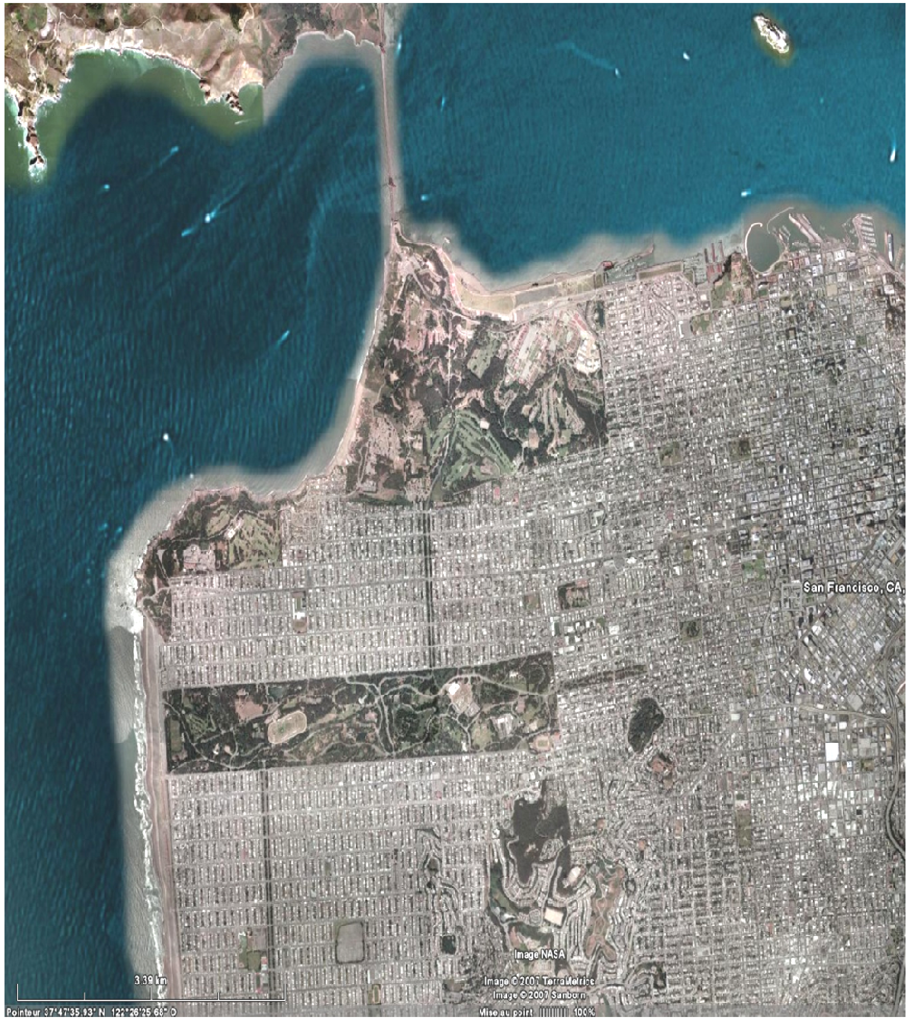
$$T_{11} = 2A_0 = |S_{XX} + S_{YY}|^2$$

$$T_{33} = B_0 - B = 2|S_{XY}|^2$$

$$T_{22} = B_0 + B = |S_{XX} - S_{YY}|^2$$


|HH+VV|dB
|HV|dB
|HH-VV|dB
-30dB
-15dB
0dB

TARGET GENERATORS



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(H,V) POLARISATION BASIS

|HH+VV| |HV | |HH-VV|

ELLIPTICAL BASIS TRANSFORMATION

esa

SPECIAL UNITARY SU(2) GROUP

$$[U_2] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$[U_2(\phi)]$ $[U_2(\tau)]$ $[U_2(\alpha)]$

↓

SPECIAL UNITARY SU(3) GROUP

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\phi) & \sin(2\phi) \\ 0 & -\sin(2\phi) & \cos(2\phi) \end{bmatrix} \begin{bmatrix} \cos(2\tau) & 0 & j\sin(2\tau) \\ 0 & 1 & 0 \\ j\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} \cos(2\alpha) & -j\sin(2\alpha) & 0 \\ -j\sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$[U_3(2\phi)]$ $[U_3(2\tau)]$ $[U_3(2\alpha)]$

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$\underline{E}^{s}_{(A,A_\perp)} = \left[S_{(A,A_\perp)}\right]\underline{E}^{i}_{(A,A_\perp)} \qquad \underline{E}^{s}_{(B,B_\perp)} = \left[S_{(B,B_\perp)}\right]\underline{E}^{i}_{(B,B_\perp)}$$

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^{T}\left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

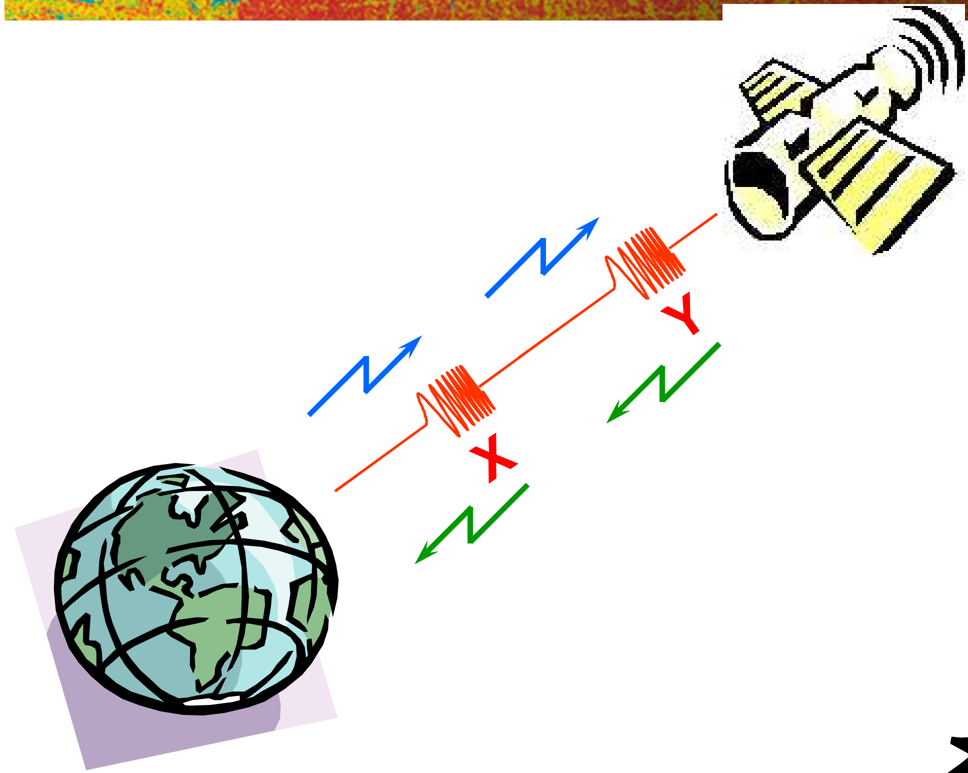
COHERENCY MATRIX

$$\left[T_{(B,B_\perp)}\right] = \left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]\left[T_{(A,A_\perp)}\right]\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]^{-1}$$

SIMILARITY TRANSFORMATION

$$\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]$$ U(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S] SINCLAIR Matrix
k, Ω Target Vectors
[K] KENNAUGH Matrix
[T] Coherency Matrix
[C] Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

BISTATIC CASE

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = [S_{XX} \quad S_{XY} \quad S_{YX} \quad S_{YY}]^T$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX}S_{XX}^* & S_{XX}S_{XY}^* & S_{XX}S_{YX}^* & S_{XX}S_{YY}^* \\ S_{XY}S_{XX}^* & S_{XY}S_{XY}^* & S_{XY}S_{YX}^* & S_{XY}S_{YY}^* \\ S_{YX}S_{XX}^* & S_{YX}S_{XY}^* & S_{YX}S_{YX}^* & S_{YX}S_{YY}^* \\ S_{YY}S_{XX}^* & S_{YY}S_{XY}^* & S_{YY}S_{YX}^* & S_{YY}S_{YY}^* \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

MONOSTATIC CASE

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \left[S_{XX} \quad \sqrt{2} S_{XY} \quad S_{YY} \right]^T$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX} S_{XX}^* & \sqrt{2} S_{XX} S_{XY}^* & S_{XX} S_{YY}^* \\ \sqrt{2} S_{XY} S_{XX}^* & 2 S_{XY} S_{XY}^* & \sqrt{2} S_{XY} S_{YY}^* \\ S_{YY} S_{XX}^* & \sqrt{2} S_{YY} S_{XY}^* & S_{YY} S_{YY}^* \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

ELLIPTICAL BASIS TRANSFORMATION

esa

SINCLAIR MATRIX

$$\underline{E}^{s}_{(A,A_\perp)} = \left[S_{(A,A_\perp)}\right]\underline{E}^{i}_{(A,A_\perp)} \qquad \underline{E}^{s}_{(B,B_\perp)} = \left[S_{(B,B_\perp)}\right]\underline{E}^{i}_{(B,B_\perp)}$$

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^{T}\left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

COVARIANCE MATRIX

$$\left[C_{(B,B_\perp)}\right] = \left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]\left[C_{(A,A_\perp)}\right]\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]^{-1}$$

SIMILARITY TRANSFORMATION

$$\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

U(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

COHERENCY MATRIX

$$[T] = \underline{k} \cdot \underline{k}^{*T}$$

COVARIANCE MATRIX

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T}$$

$$\underline{k} = [D_{3or4}]\underline{\Omega}$$

UNITARY TRANSFORMATION

$$[T] = [D_{3or4}][C][D_{3or4}]^{T*}$$

$[T]$ and $[C]$ HAVE THE SAME EIGENVALUES

Both contain the same information about Polarimetric Scattering Amplitudes, Phase Angles and Correlations

$[T]$ is closer related to Physical and Geometrical Properties of the Scattering Process, and thus allows a better and direct physical interpretation

$[C]$ is directly related to the system measurables

$[T]$ is directly related to the Kennaugh matrix and the Huynen parameters

POLARIMETRIC DESCRIPTORS

esa

SINCLAIR MATRIX

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

KENNAUGH MATRIX

$$[K] = \frac{1}{2}\left([V]^T\left[[S]\otimes[S]^*\right][V]\right)$$

EQUIVALENCE ?

SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

COHERENCY MATRIX $[T]$

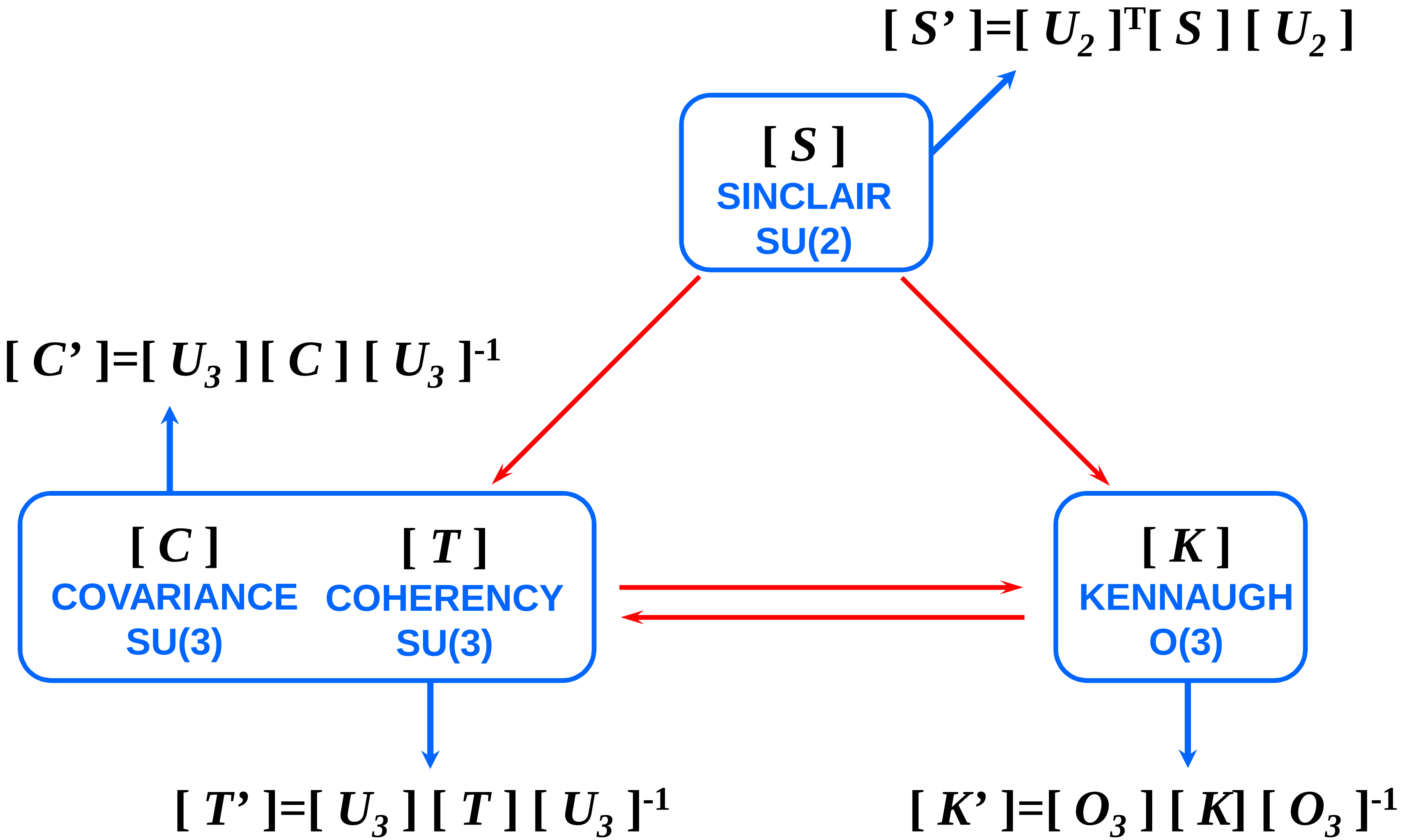
$$[T] = \underline{k} \cdot \underline{k}^{*T}$$

SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \left[S_{XX} \quad \sqrt{2}S_{XY} \quad S_{YY}\right]^T$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega}\,\underline{\Omega}^{T*}$$


$[S'] = [U_2]^T [S] [U_2]$
[S]
SINCLAIR
SU(2)
$[C'] = [U_3] [C] [U_3]^{-1}$
[C]
COVARIANCE
SU(3)
[T]
COHERENCY
SU(3)
[K]
KENNAUGH
O(3)
$[T'] = [U_3] [T] [U_3]^{-1}$
$[K'] = [O_3] [K] [O_3]^{-1}$

ELLIPTICAL BASIS TRANSFORMATION

esa

SPECIAL UNITARY SU(2) GROUP

$$\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

SPECIAL UNITARY SU(3) GROUP (T Matrix)

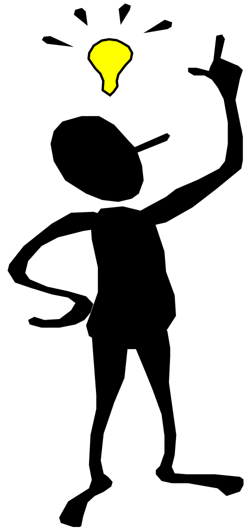
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\phi) & \sin(2\phi) \\ 0 & -\sin(2\phi) & \cos(2\phi) \end{bmatrix} \begin{bmatrix} \cos(2\tau) & 0 & j\sin(2\tau) \\ 0 & 1 & 0 \\ j\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} \cos(2\alpha) & -j\sin(2\alpha) & 0 \\ -j\sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[U_3(2\phi)] \qquad [U_3(2\tau)] \qquad [U_3(2\alpha)]$$

O(3) UNITARY GROUP

$$\begin{bmatrix} \cos 2\phi & -\sin 2\phi & 0 \\ \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 2\tau & 0 & -\sin 2\tau \\ 0 & 1 & 0 \\ \sin 2\tau & 0 & \cos 2\tau \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\alpha & -\sin 2\alpha \\ 0 & \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$[O_3(2\phi)] \qquad [O_3(2\tau)] \qquad [O_3(2\alpha)]$$



POLARIMETRIC GOLDEN NUMBER

POLARIMETRIC TARGET DIMENSION

TARGET EQUATIONS

$$[S]=\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\phi_{XY-XX}, \phi_{YY-XX}$$

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

=

5

KENNAUGH MATRIX $[K]$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(A0, B0, B, C, D, E, F, G, H)

COVARIANCE MATRIX $[C]$

9 REAL PARAMETERS

$|XX|, |XY|, |YY|,$

$\mathrm{Re}(XXXY^*), \mathrm{Im}(XXXY^*)$

$\mathrm{Re}(XXYY^*), \mathrm{Im}(XXYY^*)$

$\mathrm{Re}(XYYY^*), \mathrm{Im}(XYYY^*)$

9 - 5 = 4 TARGET EQUATIONS

PURE TARGET – MONOSTATIC CASE

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

3x3 HERMITIAN MATRIX - RANK 1

↓

9 PRINCIPAL MINORS = 0

$$2A_0(B_0 + B) - C^2 - D^2 = 0 \qquad 2A_0(B_0 - B) - G^2 - H^2 = 0$$

$$-2A_0E + CH - DG = 0 \qquad B_0^2 - B^2 - E^2 - F^2 = 0$$

$$C(B_0 - B) - EH - GF = 0 \qquad -D(B_0 - B) + FH - GE = 0$$

$$2A_0F - CG - DH = 0 \qquad -G(B_0 + B) + FC - ED = 0$$

$$H(B_0 + B) - CE - DF = 0$$

TARGET EQUATIONS

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\phi_{XY-XX}, \phi_{YY-XX}$$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(*A0, B0, B, C, D, E, F, G, H*)

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

=

5

9 - 5 = 4 TARGET EQUATIONS

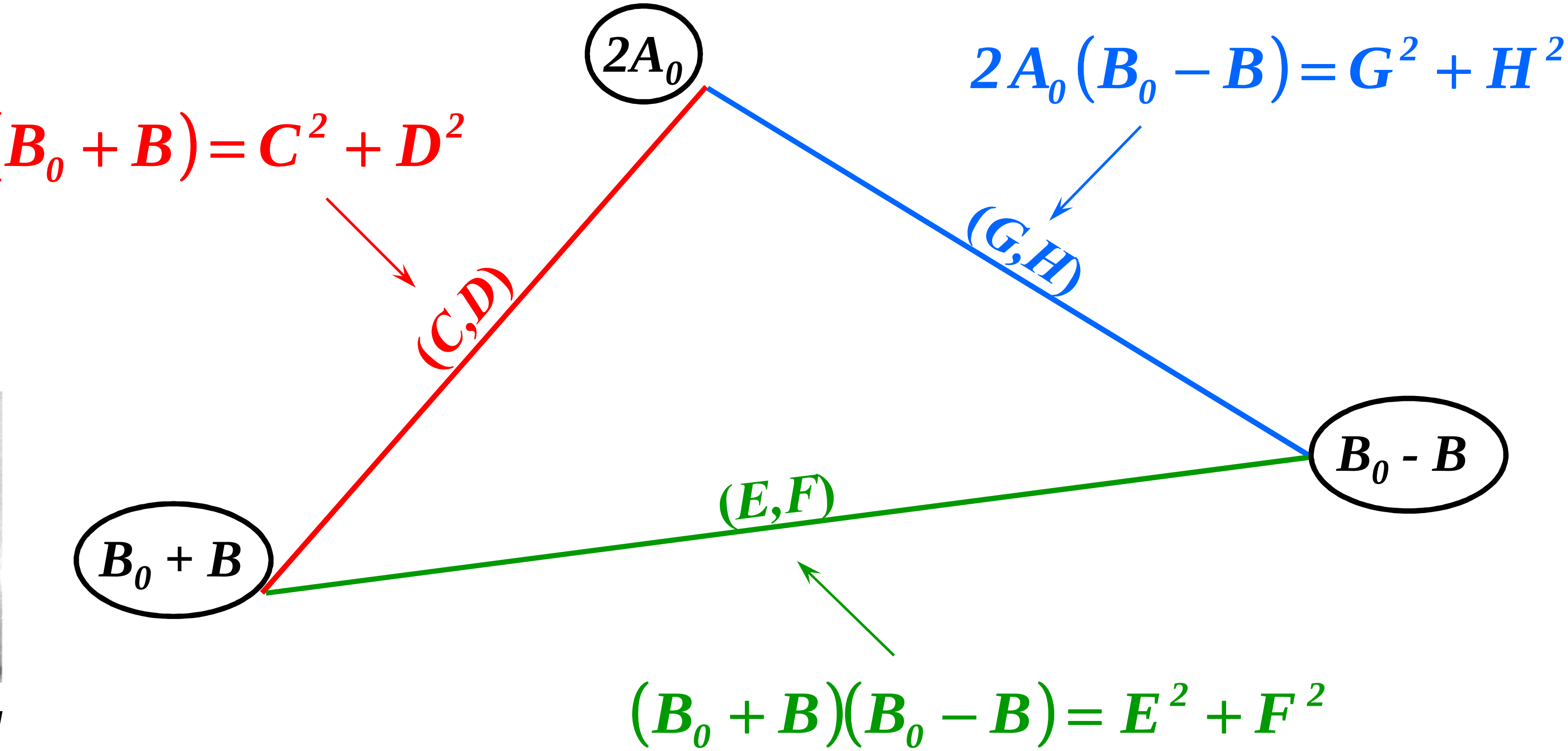
$$2A_0(B_0 + B) = C^2 + D^2$$

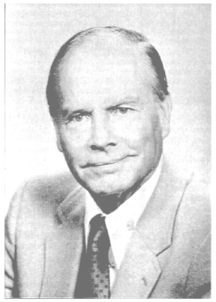
$$2A_0(B_0 - B) = G^2 + H^2$$

$$2A_0 E = CH - DG$$

$$2A_0 F = CG + DH$$

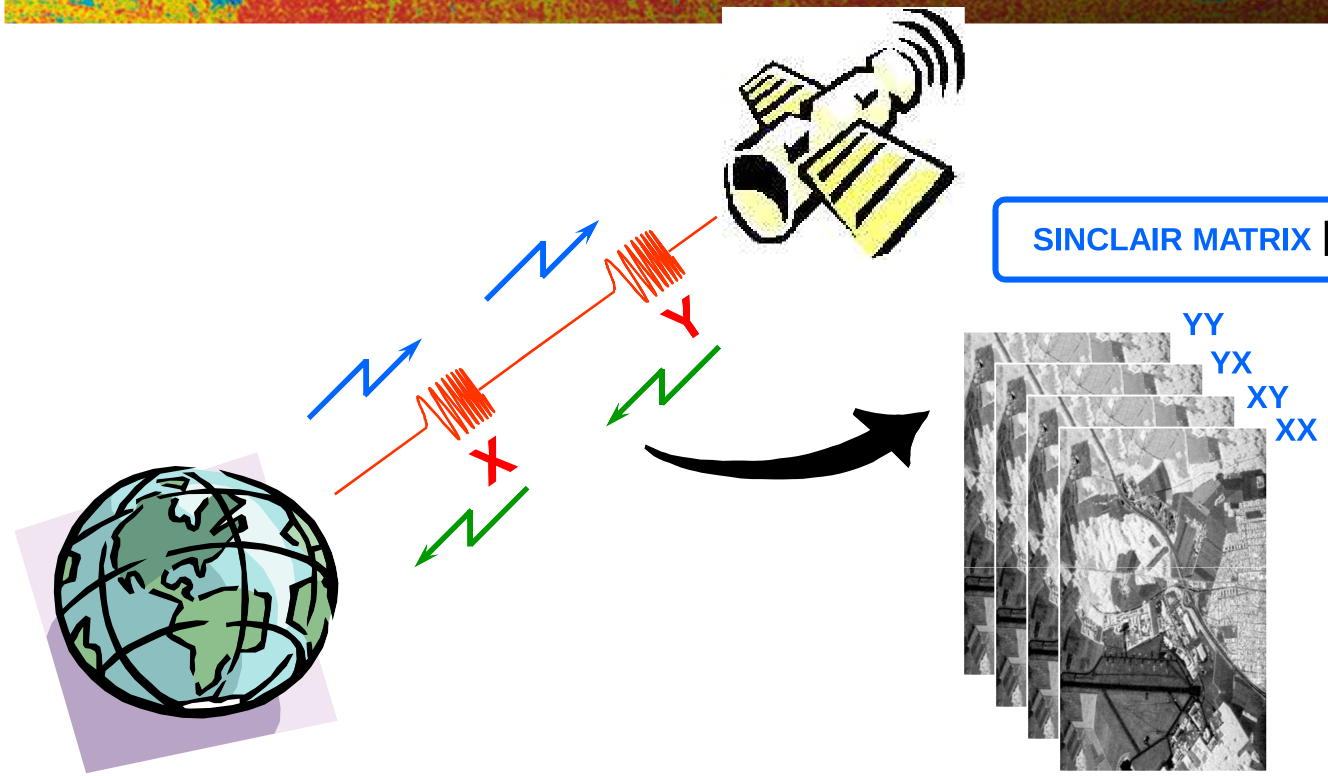
$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$





J.R. HUYNEN
(1920 – 2007)

esa



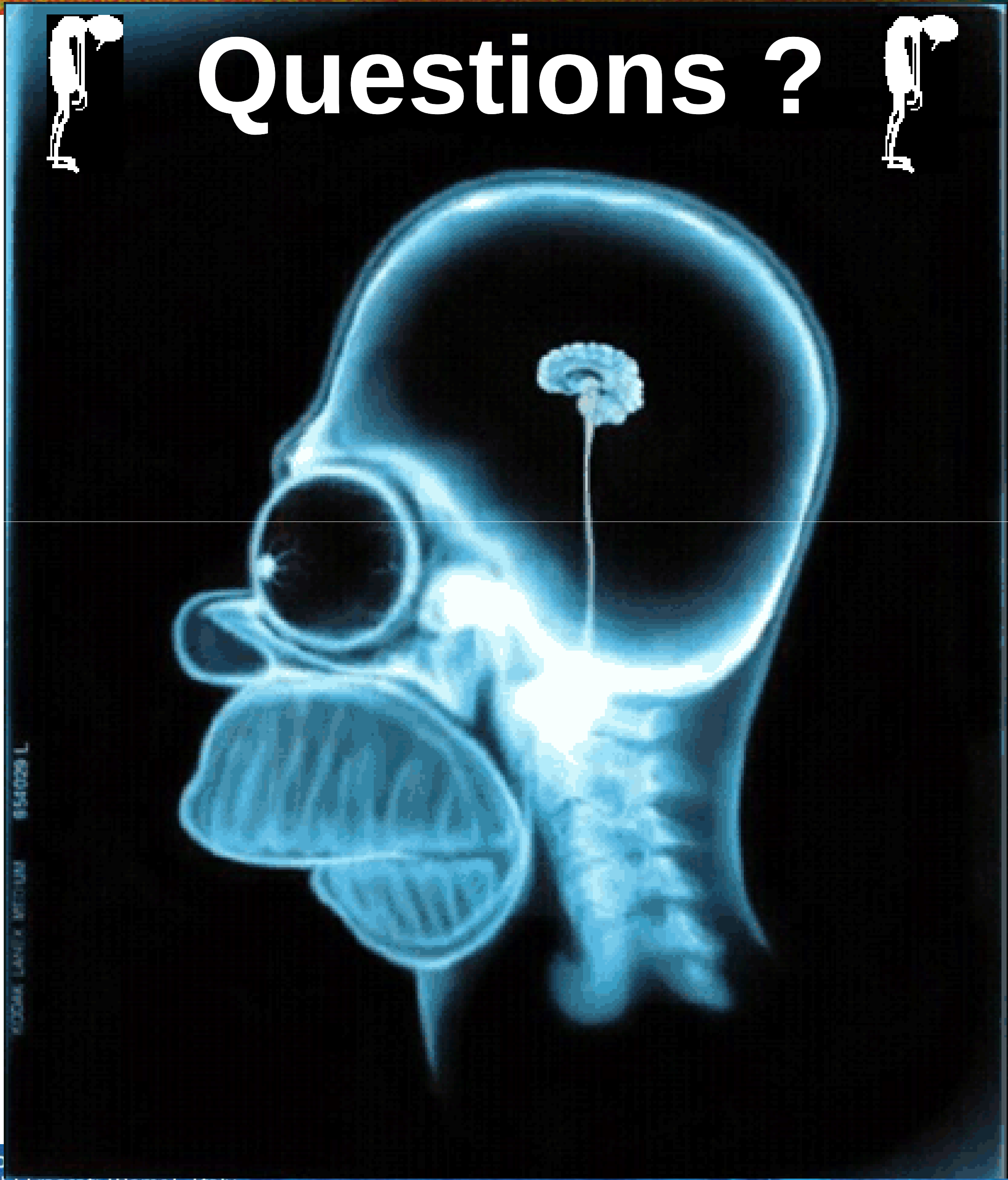
SINCLAIR MATRIX $[S]$

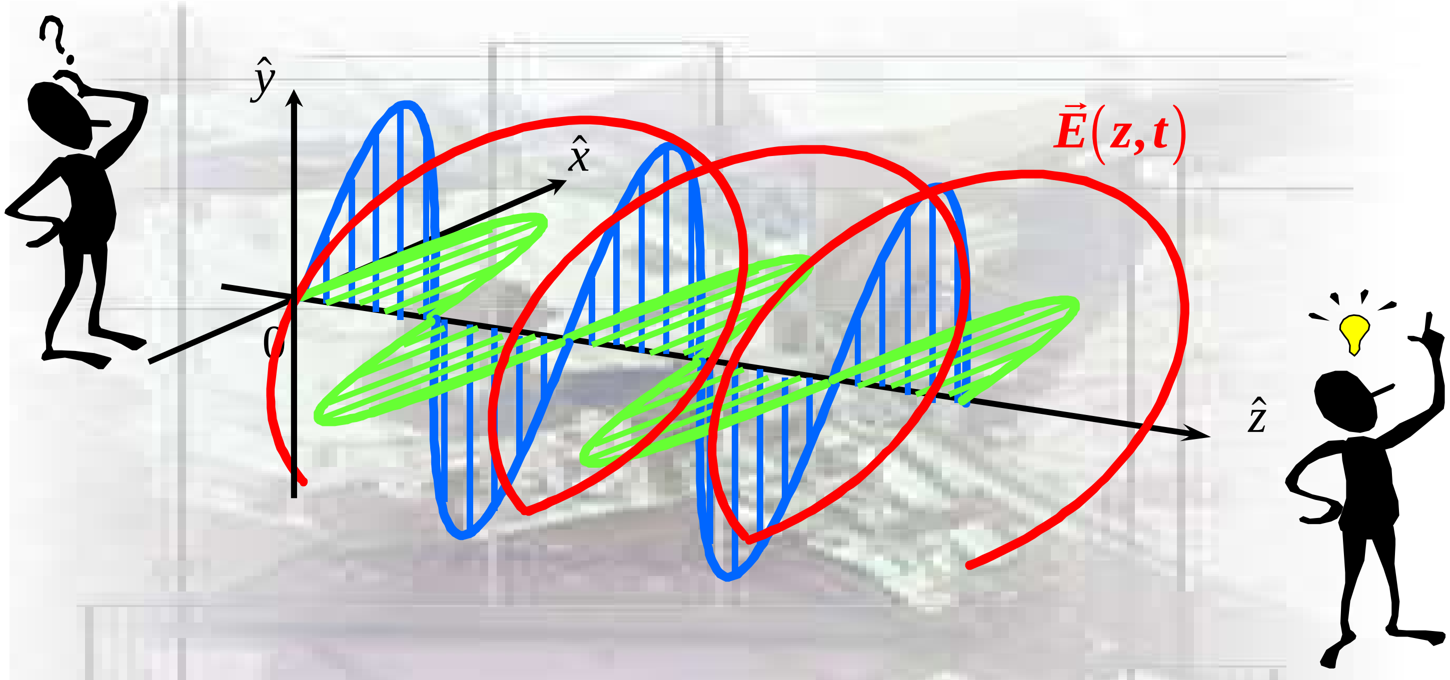
TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

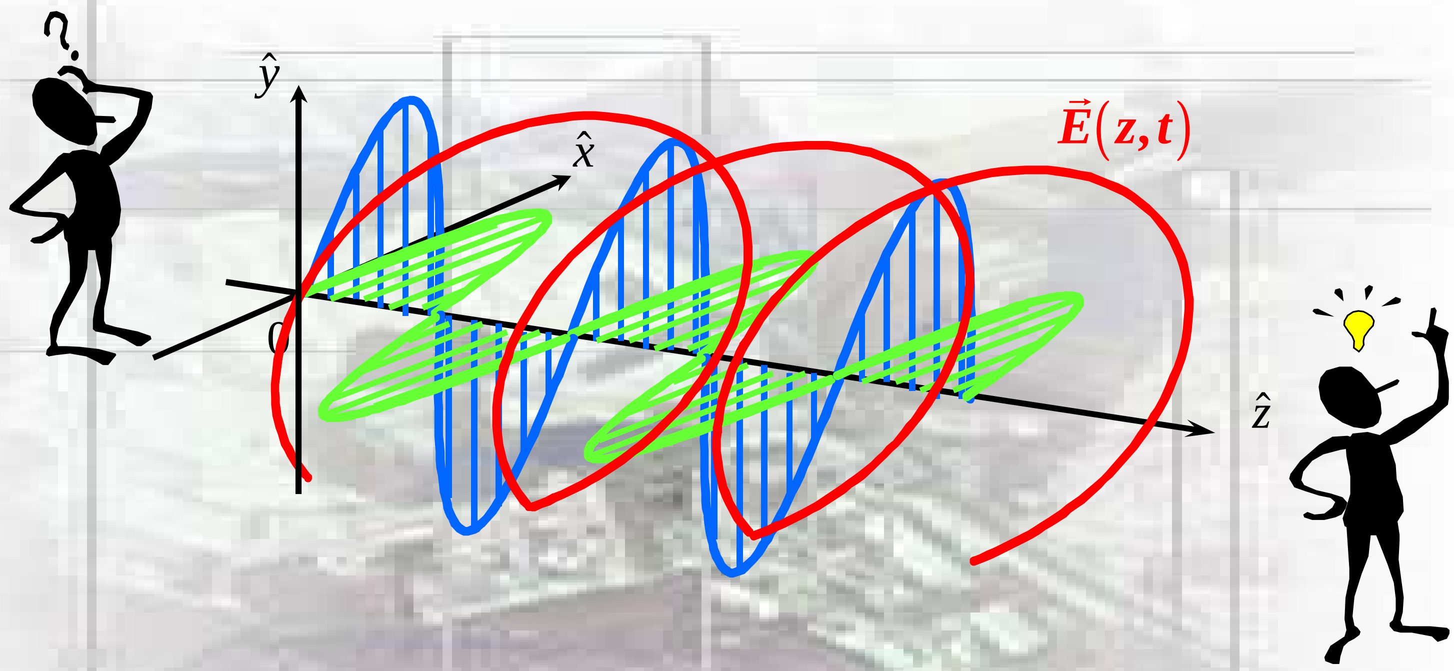
[S]	SINCLAIR Matrix
k, Ω	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

POLARIMETRIC REMOTE SENSING


Questions ?

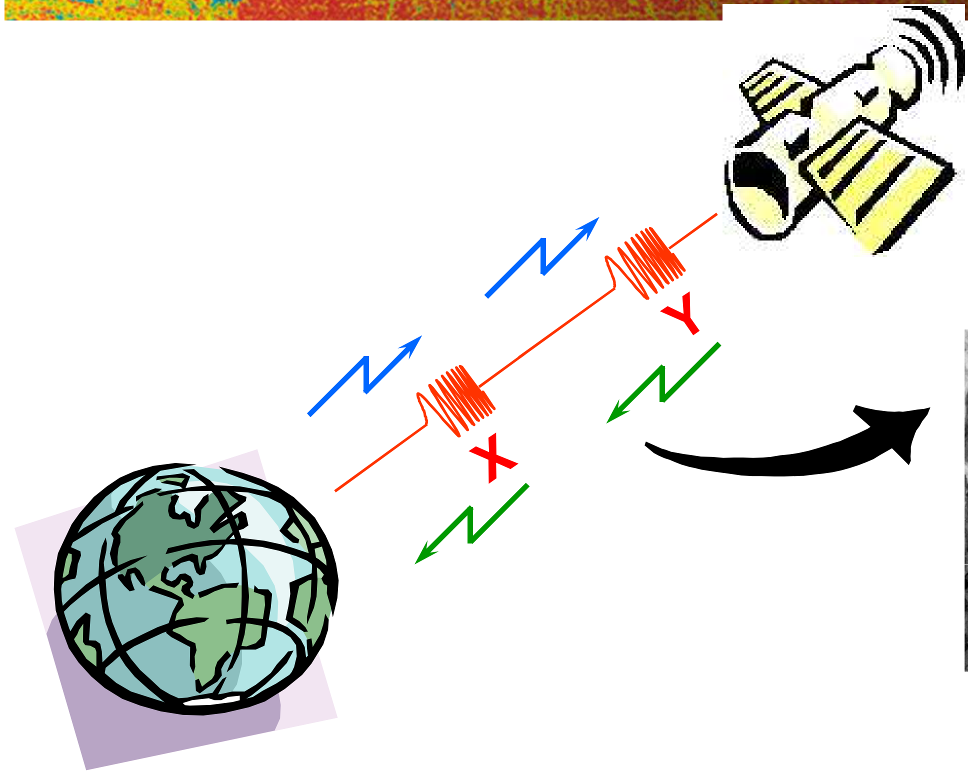


ADVANCED CONCEPTS

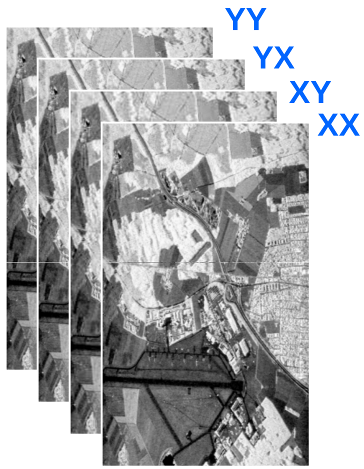


POLARIMETRIC REMOTE SENSING

SCATTERING POLARIMETRY



SINCLAIR MATRIX $[S]$

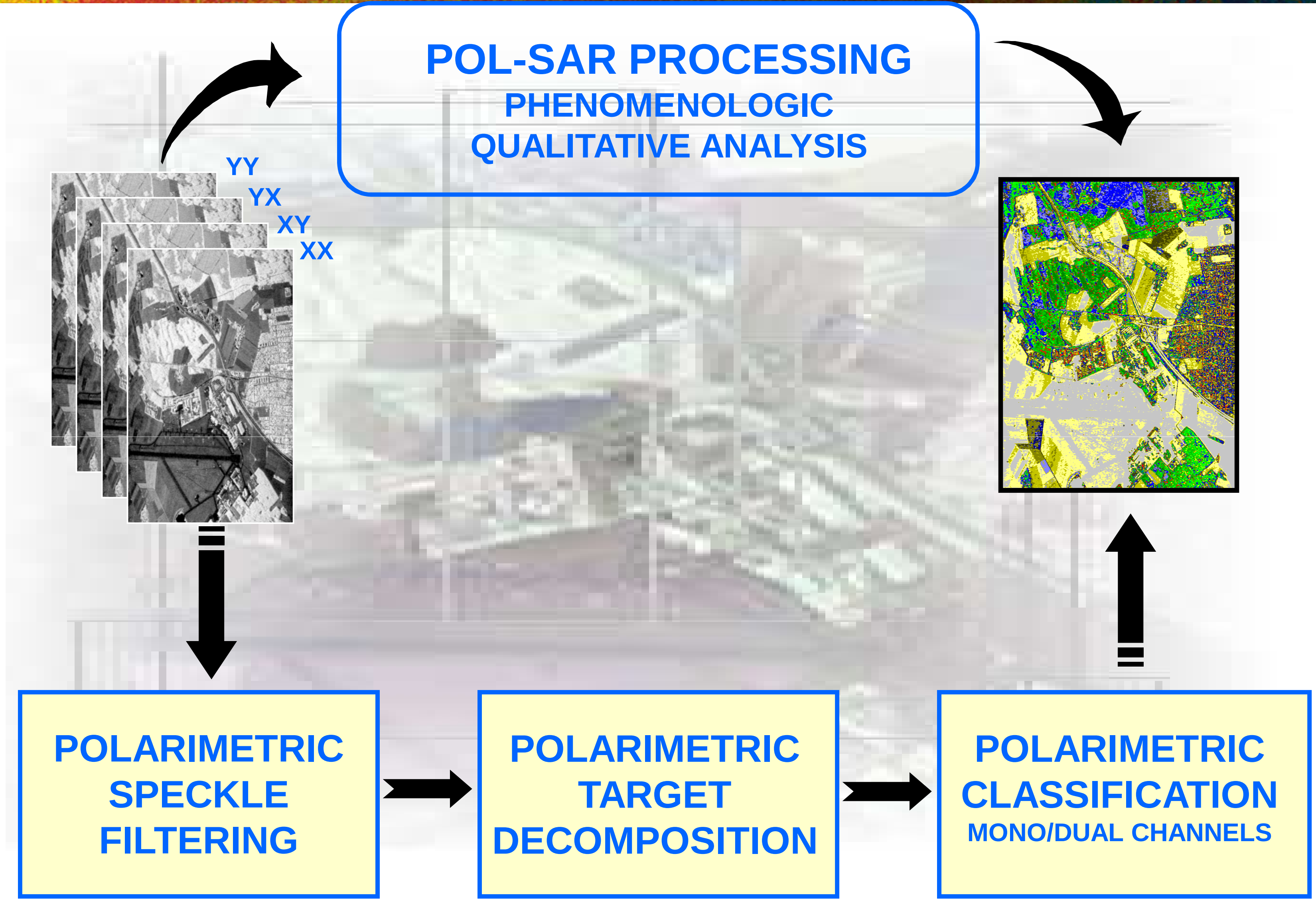


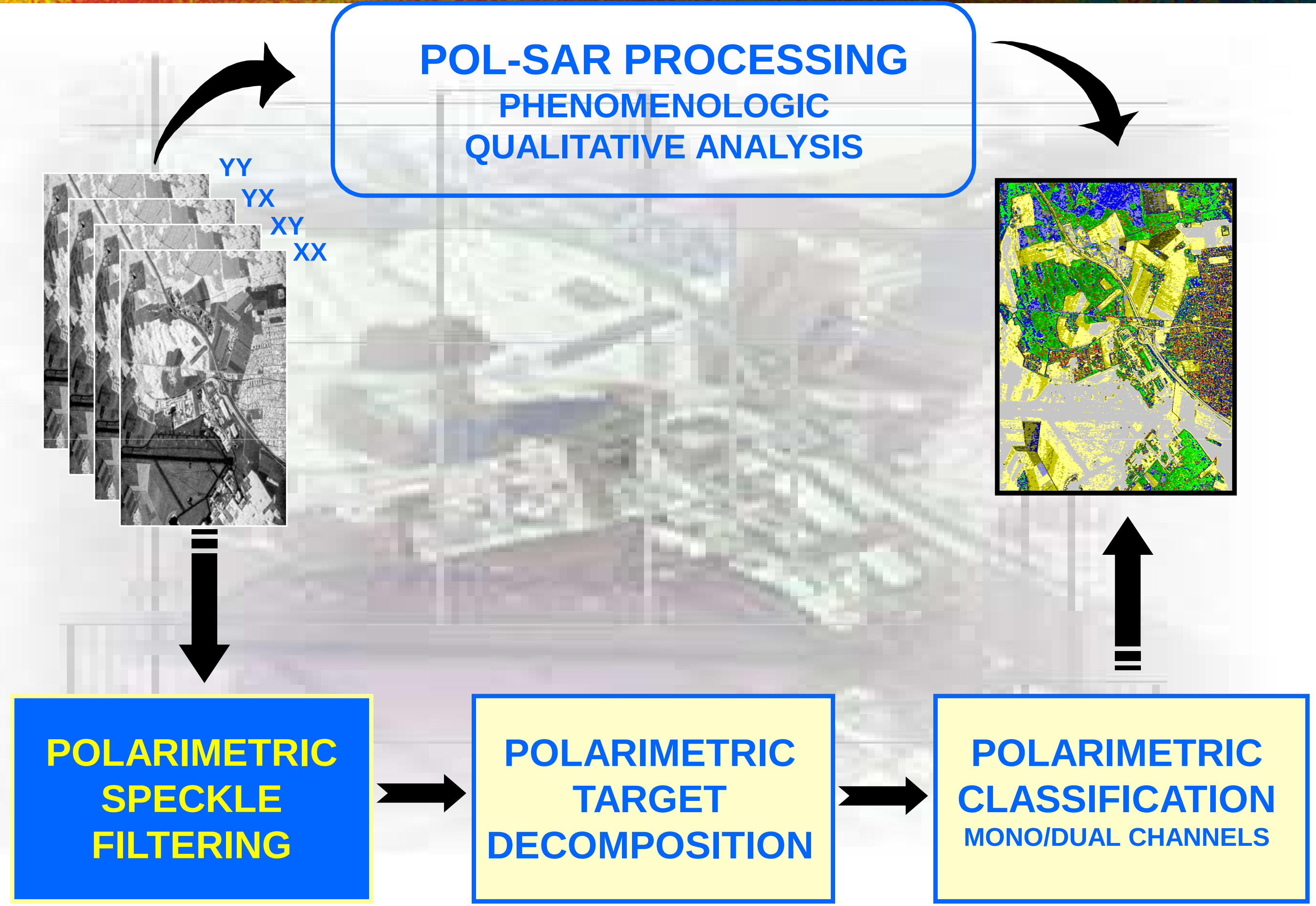
TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
<u>k</u>, <u>Ω</u>	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

POLARIMETRIC REMOTE SENSING


POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS
YY
YX
XY
XX
POLARIMETRIC SPECKLE FILTERING
POLARIMETRIC TARGET DECOMPOSITION
POLARIMETRIC CLASSIFICATION
MONO/DUAL CHANNELS


POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS
YY
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POLARIMETRIC SPECKLE FILTERING
POLARIMETRIC TARGET DECOMPOSITION
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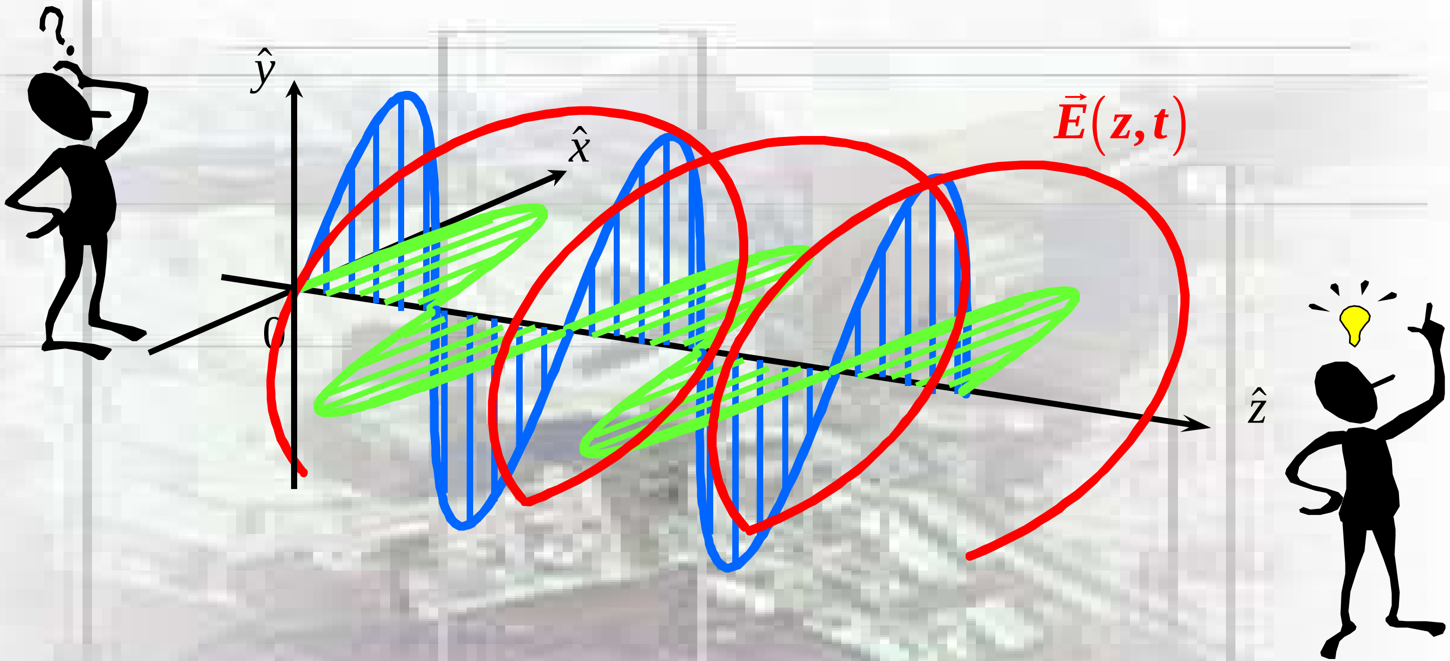
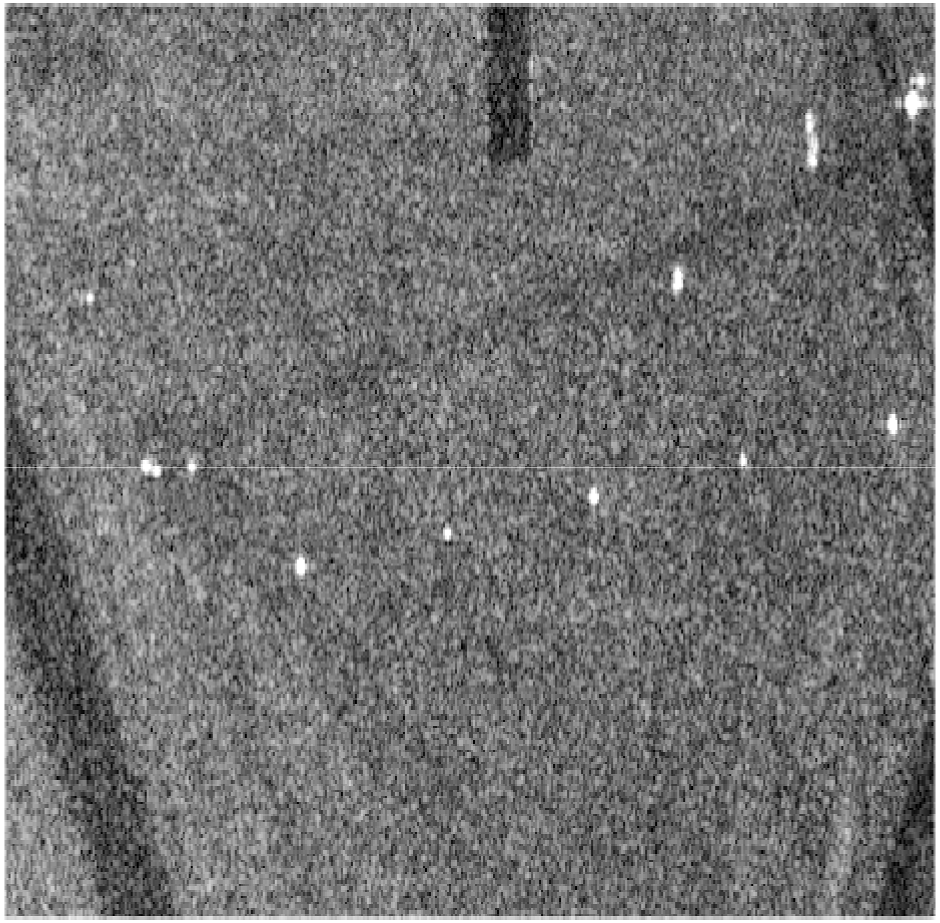
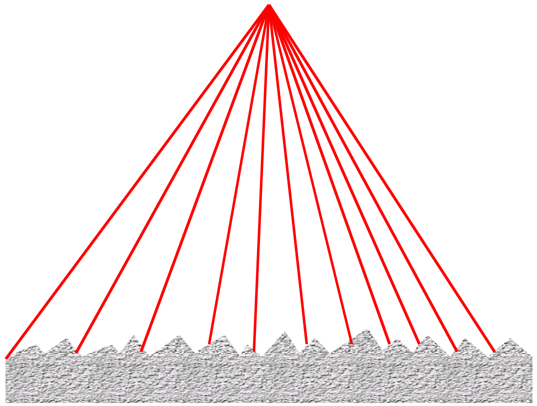


POLARIMETRIC SPECKLE FILTERING

An Introduction

SPECKLE PHENOMENON





SCATTERING FROM DISTRIBUTED SCATTERERS

⬇

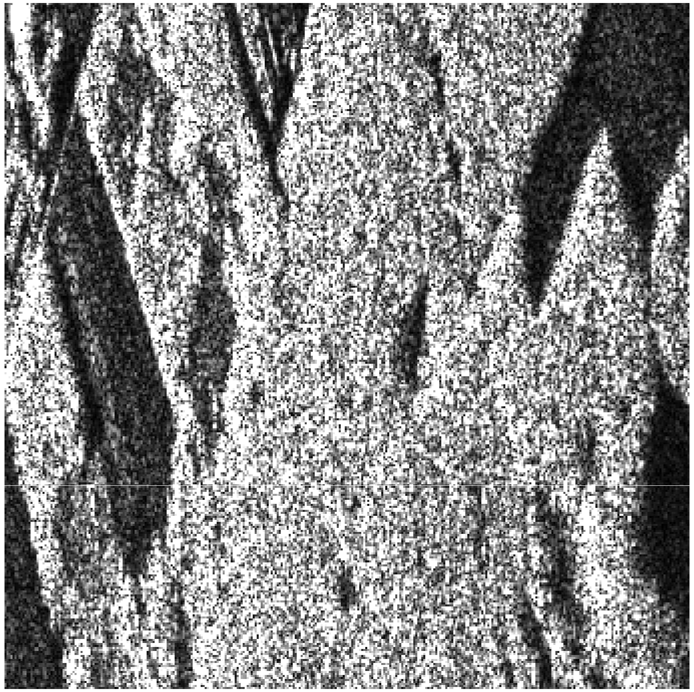
COHERENT INTERFERENCES OF WAVES SCATTERED FROM MANY RANDOMLY DISTRIBUTED ELEMENTARY SCATTERERS INSIDE THE RESOLUTION CELL

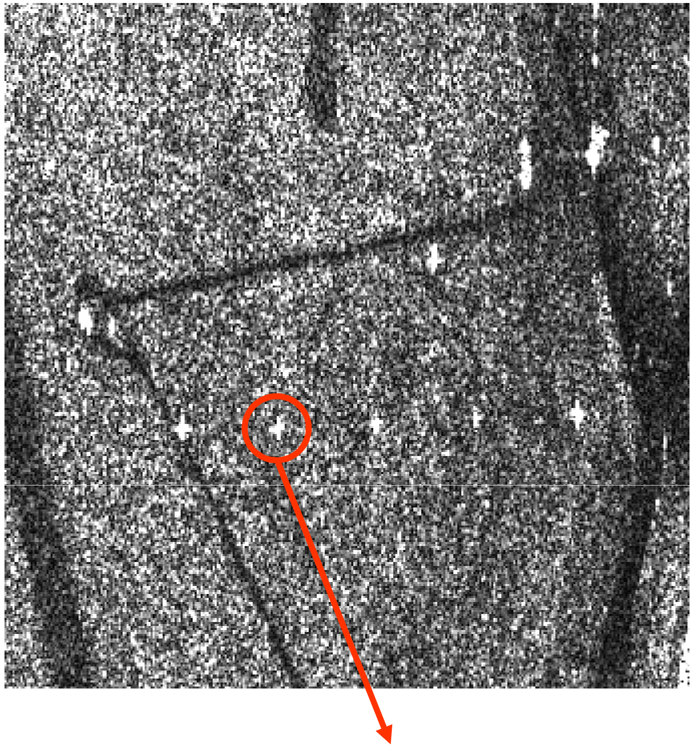
⬇

GRANULAR NOISE

⬇

SPECKLE PHENOMENON





Fully Developed speckle

Bright points: Points where the interference is constructive
Dark points: Points where the interference is destructive

Corner reflector
Dominant scatter
No speckle

DLR

S_{hh} amplitude
E-SAR L-band system

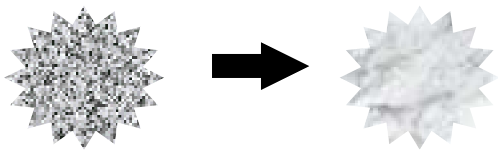
Courtesy of Dr C. Lopez Martinez

SPECKLE PHENOMENON

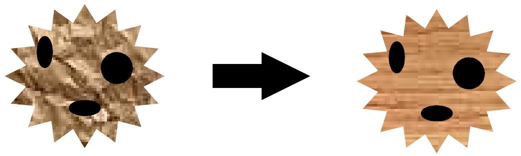
DISTORTION OF THE INTERPRETATION

SPECKLE FILTERING

HOMOGENEOUS AREA



HETEROGENEOUS AREA



SPECKLE REDUCTION (RADIOMETRIC RESOLUTION)

DETAILS PRESERVATION (SPATIAL RESOLUTION)

SPECKLE : MULTIPLICATIVE NOISE MODEL

« SPECKLE is a scattering phenomenon and not a noise. However, from the image SAR processing point of vue, the speckle can be modeled as multiplicative noise for extended target » (Lee, IGARSS-98)

$$\underline{y} = \begin{bmatrix} y_{HH} \\ y_{HV} \\ y_{VV} \end{bmatrix} = \begin{bmatrix} n_{HH} & 0 & 0 \\ 0 & n_{HV} & 0 \\ 0 & 0 & n_{VV} \end{bmatrix} \begin{bmatrix} x_{HH} \\ x_{HV} \\ x_{VV} \end{bmatrix} = \begin{bmatrix} x_{HH} n_{HH} \\ x_{HV} n_{HV} \\ x_{VV} n_{VV} \end{bmatrix}$$

SCATTERING FIELD — NOISE — REFLECTIVITY DENSITY

$$I_{pqpq} = y_{pq} y^*_{pq} = X_{pqpq} v_{pqpq}$$

INTENSITY

$$A_{pqpq} = \sqrt{I_{pqpq}} = \sqrt{y_{pq} y^*_{pq}}$$

AMPLITUDE

LINEAR SPECKLE FILTERS

Intensity / Amplitude – Single / Multi Look – Single Pol Channel

Median Filter
MAP Filter (Kuan)
Gradient Filter
Nagao Filter (Nagao)
Sigma Filter (Lee)
Frost Filter (Frost)
Geometrical Filter (Crimmins)
Morphological Filter (Safa, Flouzat)

.

Local Statistics Filter (Lee 80)
Refined Lee Filter (Lee 81)

J.S. Lee, et al. "Speckle Filtering of SAR images: A Review," Remote Sensing Reviews, Vol. 8, pp. 313-340, 1994.

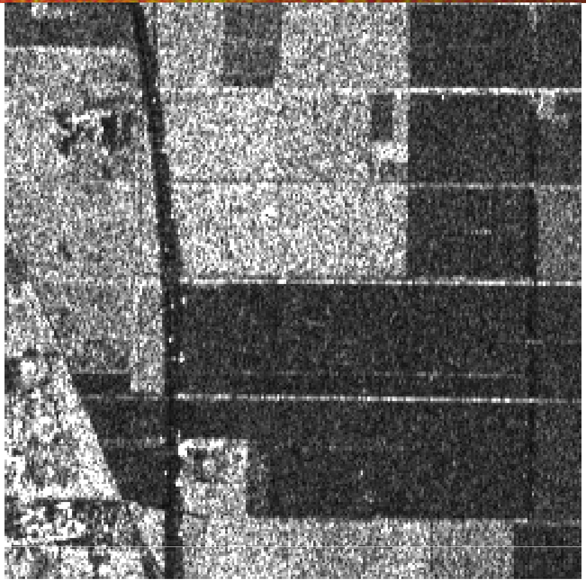
J.S. Lee,"Speckle analysis and smoothing of SAR images," Computer Graphics and Image Processing, Vol. 17, 1981.

J.S. Lee,"Digital image enhancement and noise filtering by use of local statistics," IEEE PAMI, Vol. 2 No. 2, 1980.

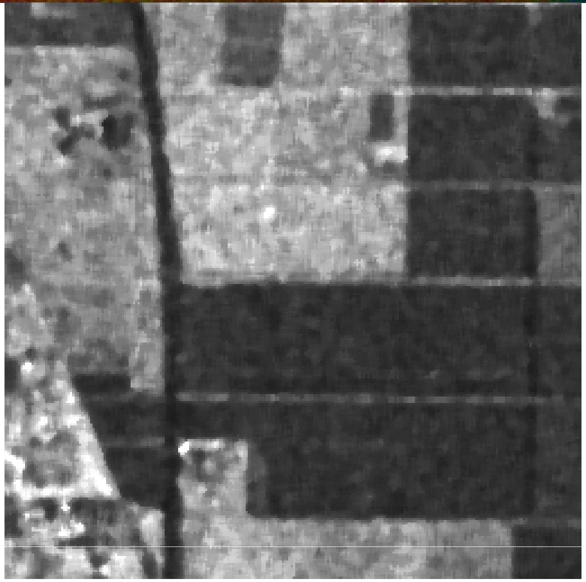
J.S. Lee,"Refined filtering of image noise using local statistics," CVGIP, vol.15, 380-389, 1981.

SPECKLE FILTERING

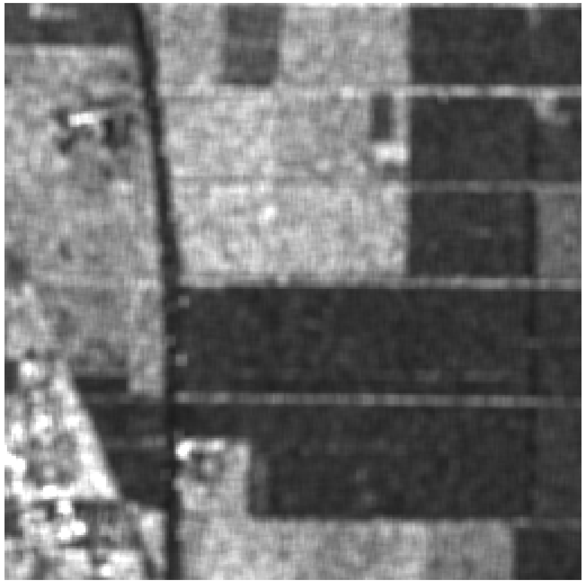
Original
4-look
amplitude



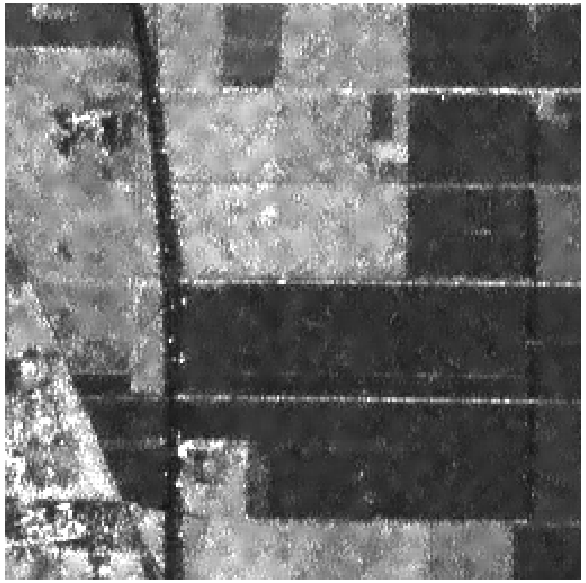
5x5 Median



5x5 Boxcar



Lee Refined
(7x7)



- **Preserving polarimetric properties**
  - **Filter all elements equally like multi-look Processing**
  - **Select pixels with the same scattering property**
- **Introduce no cross-talk**
  - **Filter each element separately but equally**
- **Reduce speckle while preserving image quality**

J.S. Lee, M.R. Grunes and G. De Grandi, "Polarimetric SAR Speckle Filtering and Its Impact on Terrain Classification" *IEEE TGRS*, September 1999

POLARIMETRIC VECTORIAL SPECKLE FILTER

$$\langle [T] \rangle = \frac{1}{N} \sum_{i=1}^{N} \underline{k}_i \underline{k}_i^{*T}$$

$$[\hat{T}] = E([T]) - k\left[E([T]) - [T]\right] \longrightarrow [\hat{T}]$$

SPAN IMAGE

$$S_S = \langle T_{11} \rangle + \langle T_{22} \rangle + \langle T_{33} \rangle$$

LINEAR SCALAR LEE FILTER

$$\hat{S} = E(S_S) - k\left[E(S_S) - S_S\right]$$

$$k = \frac{var(S)}{var(S_S)} = \frac{CV_{S_S}^2 - \sigma_v^2}{CV_{S_S}^2\left[1 + \sigma_v^2\right]}$$

J.S. LEE

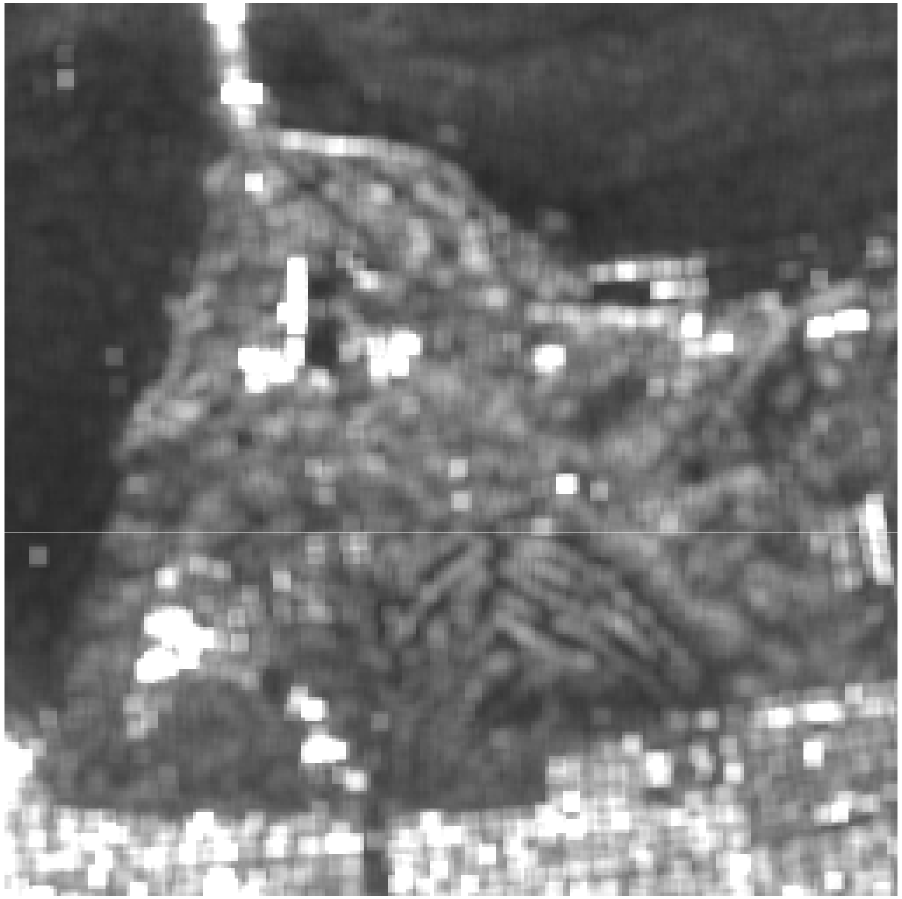
Homogeneous Areas

$$var(S) \approx 0 \quad \Rightarrow \quad k = 0 \quad \Rightarrow \hat{S} = E(S_S)$$

Highly Inhomogeneous Areas

$$var(S) \mapsto var(S_s) \quad \Rightarrow \quad k = 1 \quad \Rightarrow \hat{S} = S_S$$

REFINED FILTER

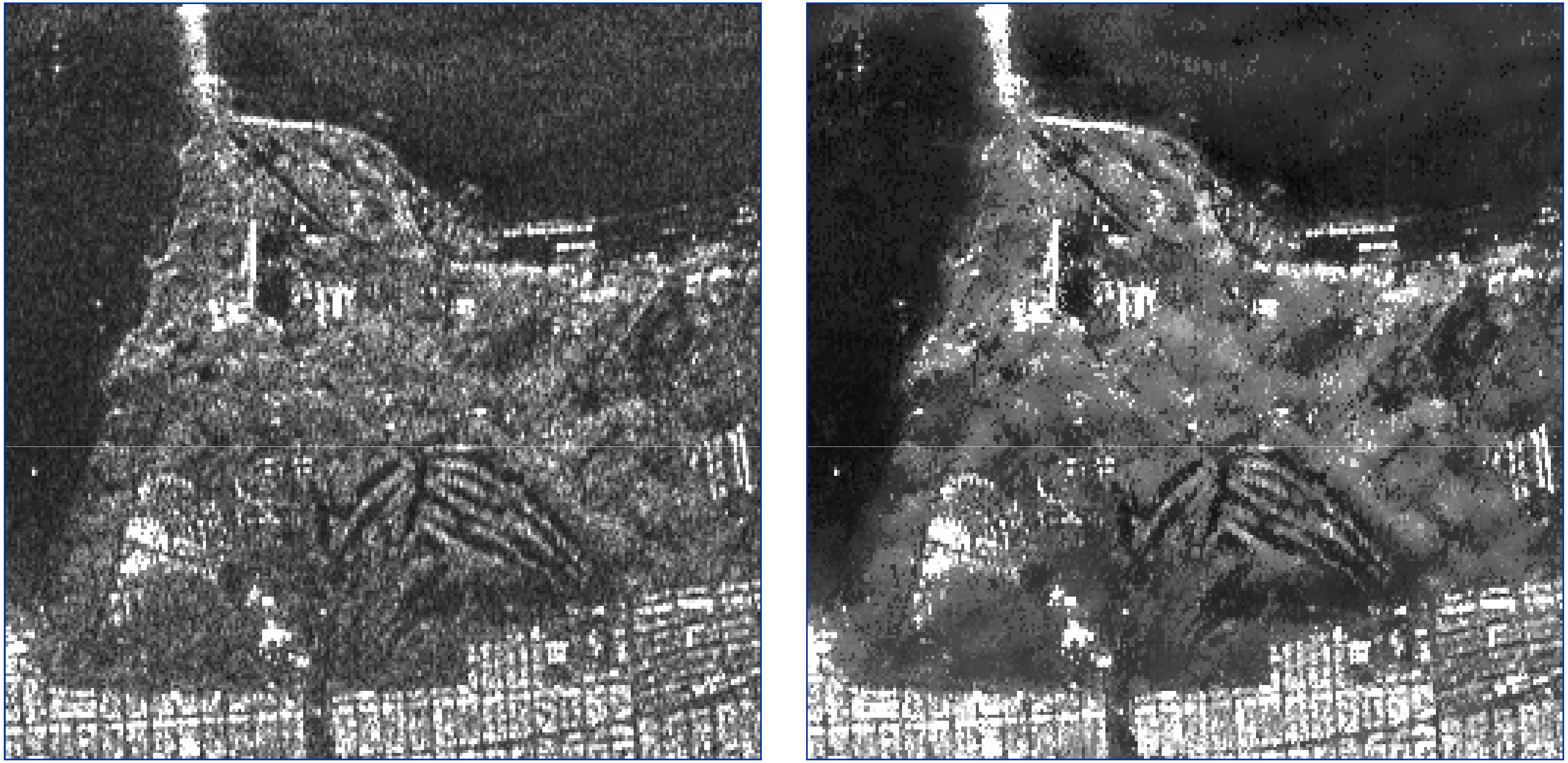


SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

BoxCar Filter

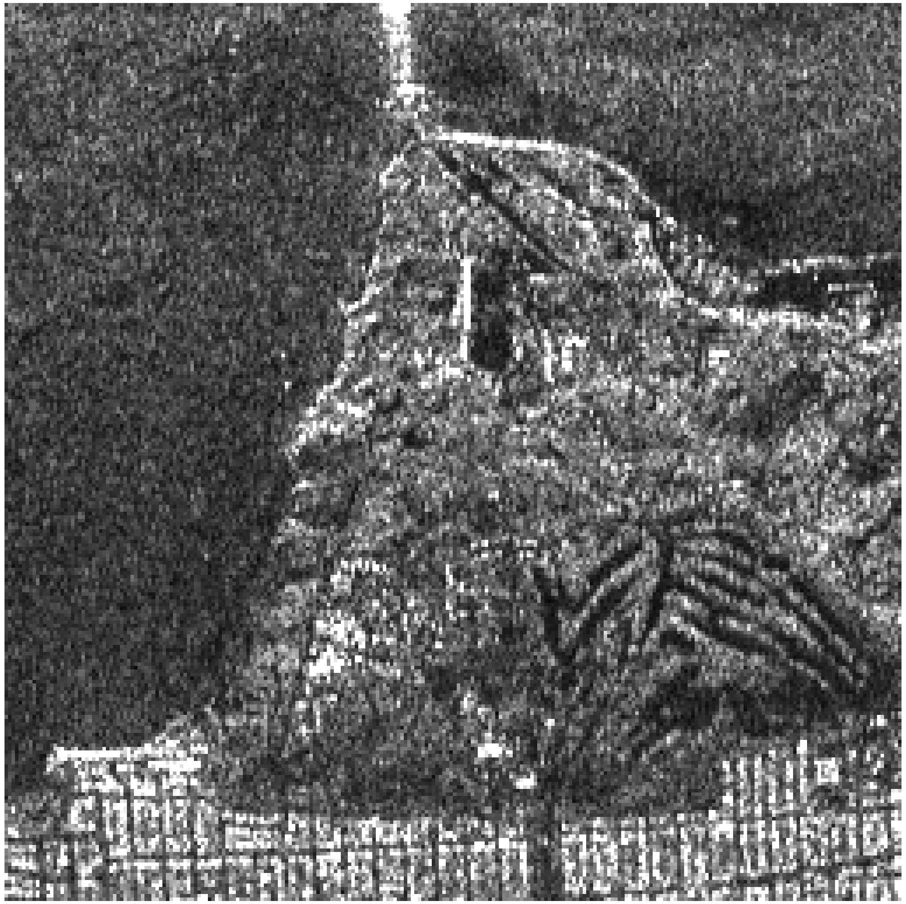
SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

J.S. Lee, M.R. Grunes and G. De Grandi, "Polarimetric SAR Speckle Filtering and Its Impact on Terrain Classification" *IEEE TGRS*, September 1999



SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

J.S. Lee, D.L. Schuler, T.L. Ainsworth, M.R. Grunes, E Pottier, L. Ferro-Famil, "*Scattering Model Based Speckle Filetring of Polarimetric SAR Data*" IEEE – TGRS, vol 1, January 2006



SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

J.S. Lee, J.H Wen, T.L. Ainsworth, K.S. Chen, A.J. Chen, "*Improved Sigma Filter for Speckle Filtering of SAR Imagery*" IEEE – TGRS, vol 1, January 2009

POLARIMETRIC SPECKLE FILTERING IS NOT AN EXACT SCIENCE
SUBJECTIVE, IMAGE DEPENDENT

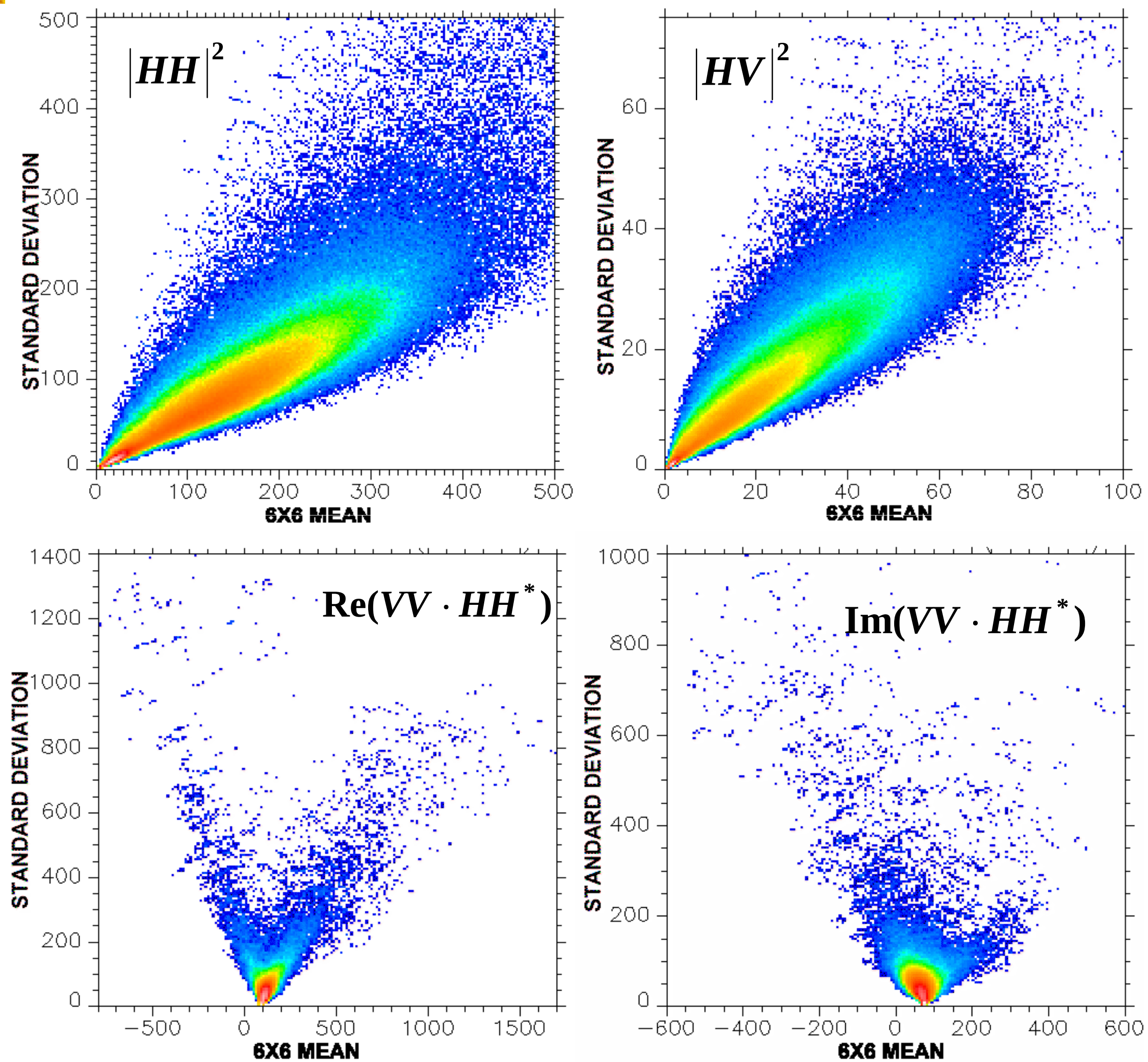
Quantitative Criteria (J.S. Lee - IGARSS 98)

- Speckle Reduction (E.N.L)
- Edge Sharpness Preservation
- Line and Point Target Contrast Preservation
- Retention of Mean Values in Homogeneous Regions
- Retention of Texture Information
- Retention of Polarimetric Information (co, cross-correlations)
- Computational Efficiency
- Implementation Complexity

$$[\hat{T}] = E([T]) - k[E([T]) - [T]]$$

THE POLARIMETRIC SPECKLE LEE FILTER IS TODAY A GOOD COMPROMISE

POLSAR SPECKLE NOISE MODEL



Diagonal Terms

Multiplicative

Off-Diagonal Terms

Additive/Multiplicative

J.S. Lee, M.R. Grunes and G. De Grandi, "Polarimetric SAR Speckle Filtering and Its Impact on Terrain Classification" *IEEE TGRS*, vol. 37, N°5, September 1999

MULTIPLICATIVE-ADDITIVE NOISE MODEL

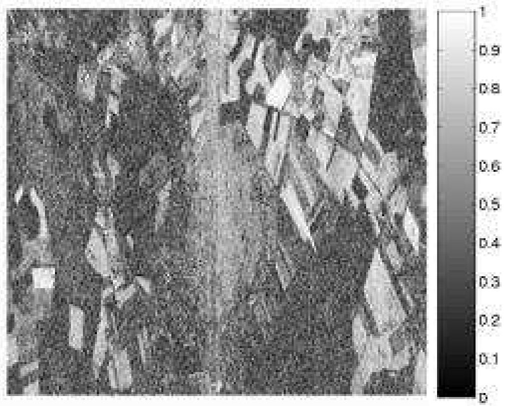
$$S_i S_j^* = \underbrace{\psi \bar{z}_n n_m N_c \exp\left(j\phi_x\right)}_{\text{Multiplicative term}} + \underbrace{\psi\left(|\rho| - N_c \bar{z}_n\right)\exp\left(j\phi_x\right) + \psi\left(n_{ar} + j n_{ai}\right)}_{\text{Additive term}}$$

C. LOPEZ MARTINEZ

Multiplicative speckle noise component: n_m ➡ Important for high coherence areas

Additive speckle noise component: $n_{ar} + jn_{ai}$ ➡ Important for low coherence areas

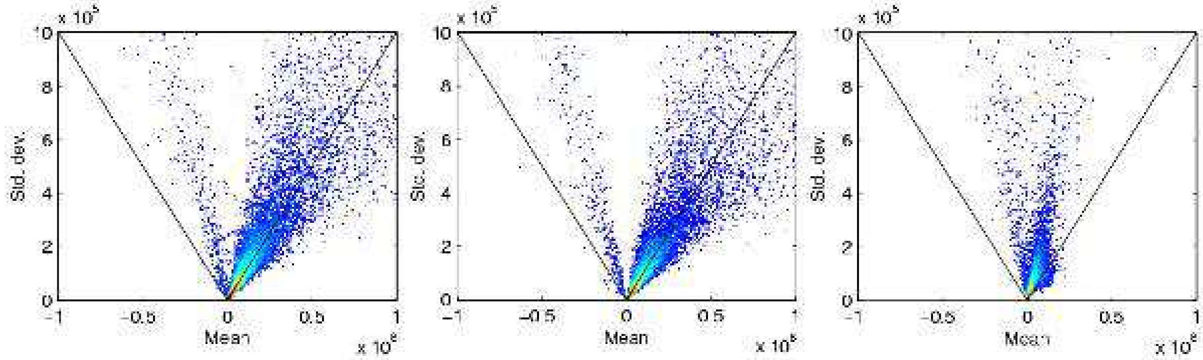
$S_{hh}S_{vv}^*$



$0.8 \leq |\rho| \leq 1$

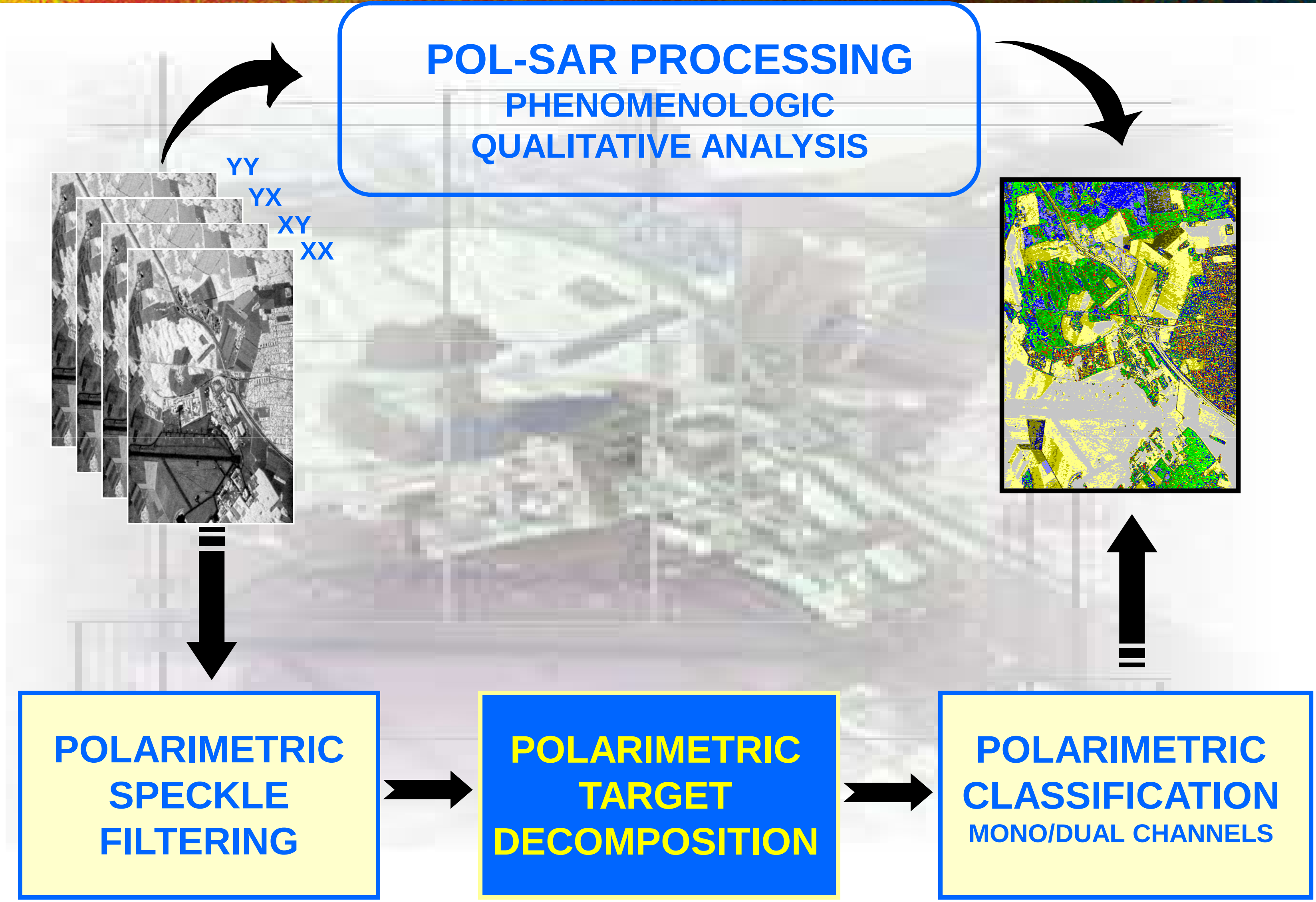
Combination controlled by complex coherence

Total $\Re\{S_{hh}S_{vv}^*\}$ Mult. term $\Re\{S_{hh}S_{vv}^*\}$ Add. term $\Re\{S_{hh}S_{vv}^*\}$



C. López-Martínez and X. Fàbregas, "Polarimetric SAR Speckle Noise Model," IEEE TGRS, vol. 41, no. 10, pp. 2232 – 2242, Oct. 2003

Courtesy of Dr C. Lopez Martinez


POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS
YY
YX
XY
XX
POLARIMETRIC SPECKLE FILTERING
POLARIMETRIC TARGET DECOMPOSITION
POLARIMETRIC CLASSIFICATION
MONO/DUAL CHANNELS

$$[T] = \underline{k}\,\underline{k}^{*T}$$

SPECKLE FILTER

AVERAGING DATA

↓

SECOND ORDER STATISTICS

COHERENCY MATRICES

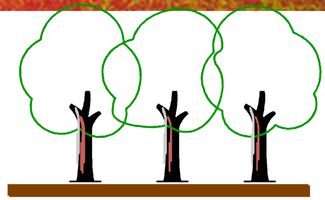
↓

SMOOTHING AVERAGING

↓

CONCEPT OF THE DISTRIBUTED TARGET

$$\langle [T] \rangle = \frac{1}{N}\sum_{i=1}^{N} \underline{k}_i\,\underline{k}_i^{*T}$$



PURE TARGET

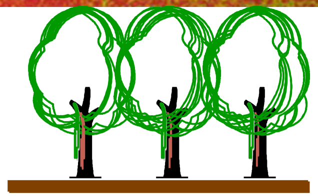
COHERENCY MATRIX [T]

9 REAL DEPENDANT HUYNEN PARAMETERS (A0,B0,B,C,D,E,F,G,H)

POLARIMETRIC DISTRIBUTED TARGET « *DIMENSION* » = 5

9 - 5 = 4 TARGET EQUATIONS

$$2A_0(B_0+B) = C^2+D^2$$
$$2A_0(B_0-B) = G^2+H^2$$
$$2A_0E = CH-DG$$
$$2A_0F = CG+DH$$



DISTRIBUTED TARGET

COHERENCY MATRIX <[*T*]>

9 REAL INDEPENDANT HUYNEN PARAMETERS
(<A0>,<B0>,<B>,<C>,<D>,<E>,<F>,<G>,<H>)

POLARIMETRIC DISTRIBUTED TARGET « *DIMENSION* » = 9

9 TARGET INEQUATIONS

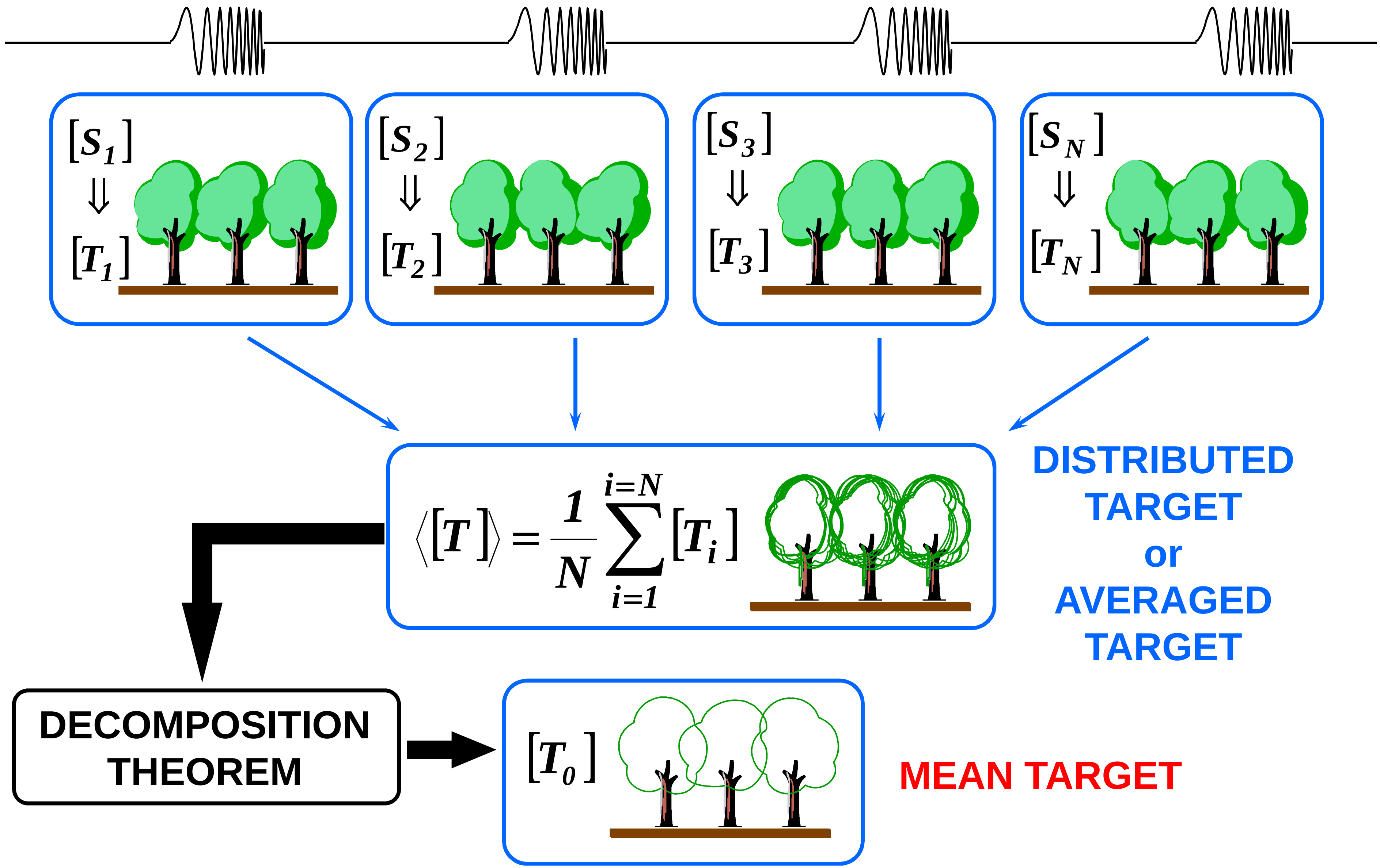
$$2\langle A_0\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle^2+\langle D\rangle^2 \qquad \langle H\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle\langle E\rangle+\langle D\rangle\langle F\rangle$$

$$2\langle A_0\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle G\rangle^2+\langle H\rangle^2 \qquad \langle G\rangle\left(\langle B_0\rangle+\langle B\rangle\right)\geq\langle C\rangle\langle F\rangle-\langle D\rangle\langle E\rangle$$

$$2\langle A_0\rangle\langle E\rangle\geq\langle C\rangle\langle H\rangle-\langle D\rangle\langle G\rangle \qquad \langle C\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle H\rangle\langle E\rangle+\langle F\rangle\langle G\rangle$$

$$2\langle A_0\rangle\langle F\rangle\geq\langle C\rangle\langle G\rangle+\langle D\rangle\langle H\rangle \qquad \langle D\rangle\left(\langle B_0\rangle-\langle B\rangle\right)\geq\langle F\rangle\langle H\rangle-\langle G\rangle\langle E\rangle$$

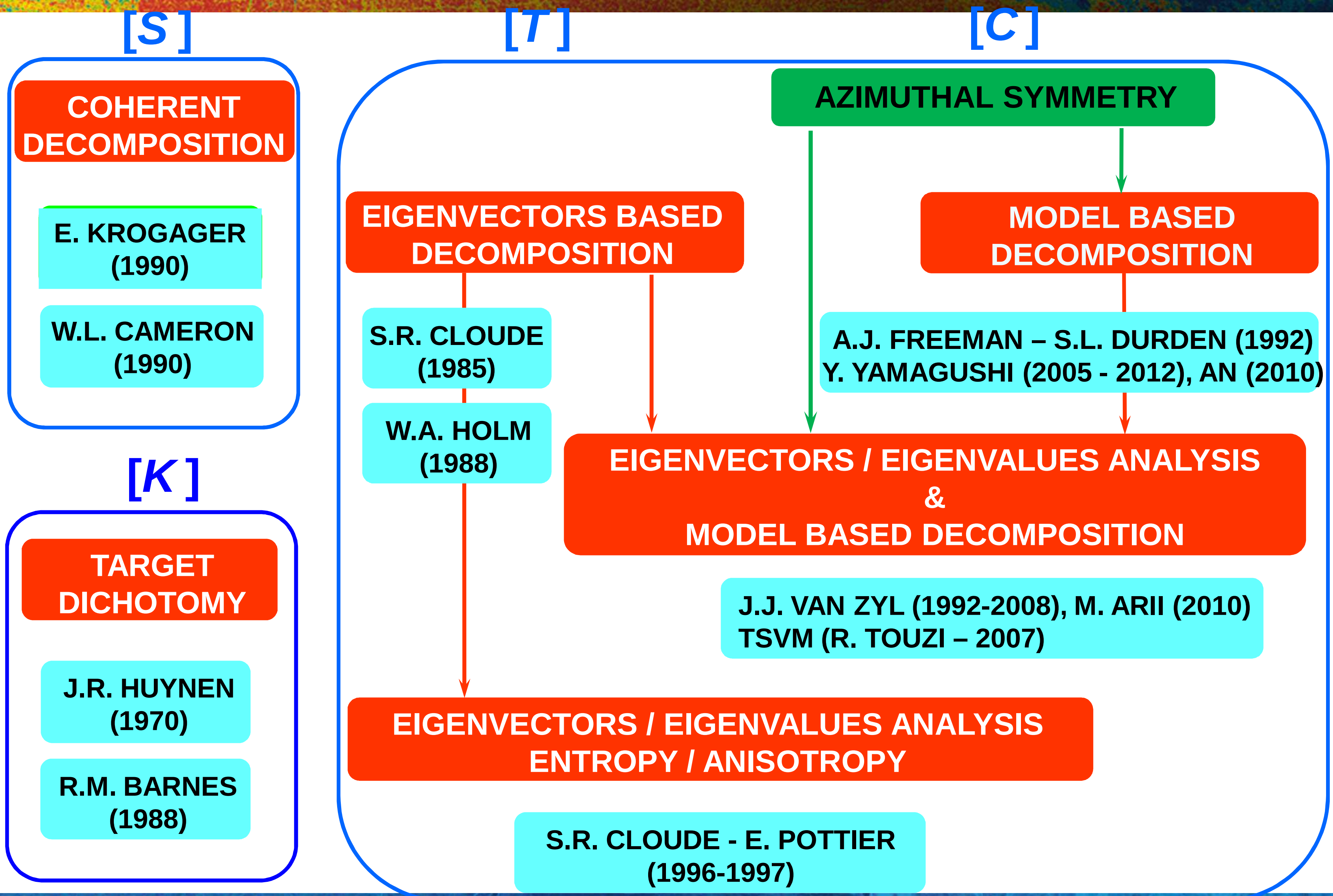
$$\langle B_0\rangle^2\geq\langle B\rangle^2+\langle E\rangle^2+\langle F\rangle^2$$



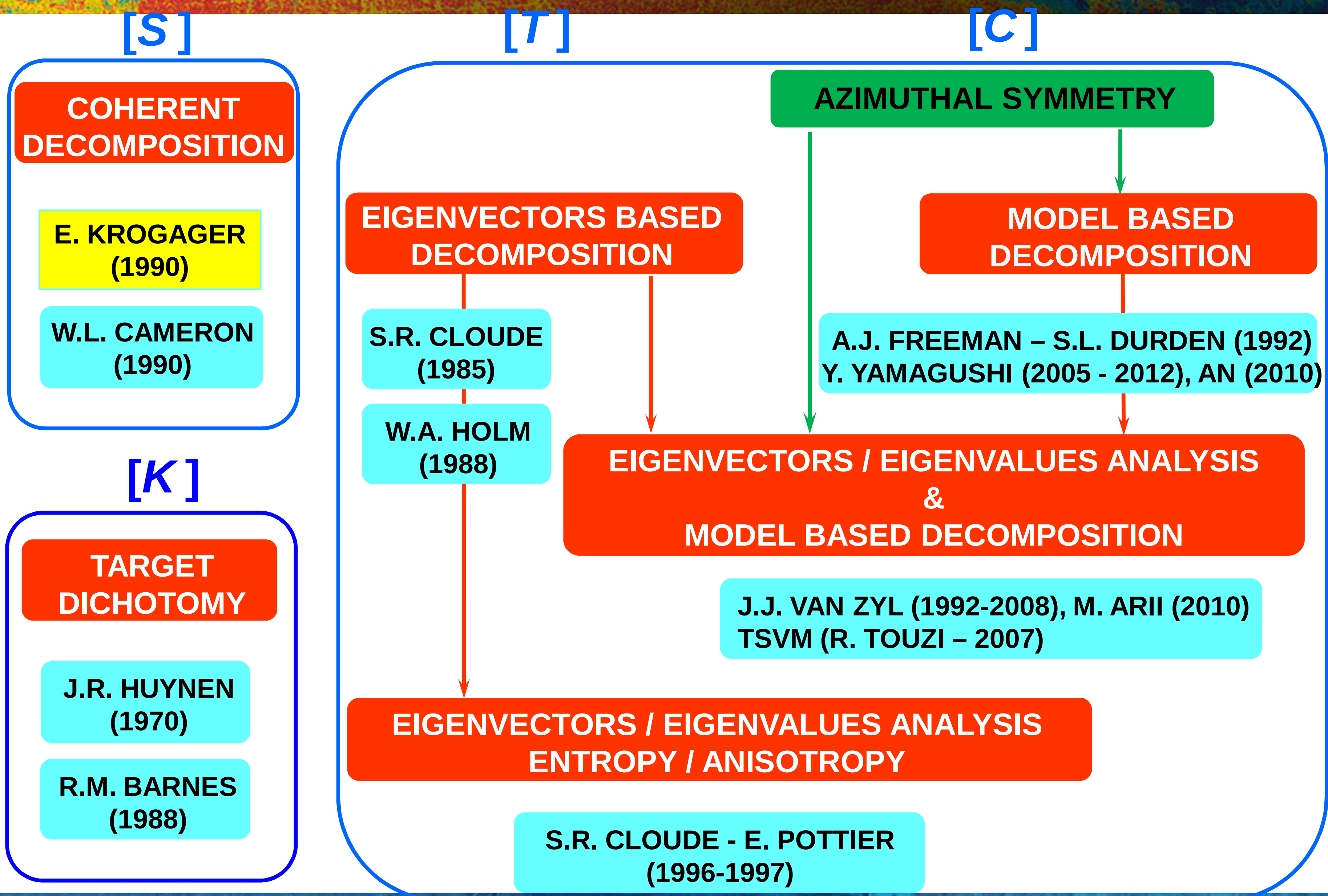
$[S_1] \Downarrow [T_1]$
$[S_2] \Downarrow [T_2]$
$[S_3] \Downarrow [T_3]$
$[S_N] \Downarrow [T_N]$
$\langle [T] \rangle = \frac{1}{N} \sum_{i=1}^{i=N} [T_i]$
DISTRIBUTED TARGET or AVERAGED TARGET
DECOMPOSITION THEOREM
$[T_0]$
MEAN TARGET

TARGET DECOMPOSITIONS

esa



TARGET DECOMPOSITIONS



COHERENT

TARGET DECOMPOSITION

(1990)

ERNST KROGAGER

(1990)

DECOMPOSITION

$[S]$ → **THREE COHERENT COMPONENTS**

$$[S]=\begin{bmatrix} a+b & c \\ c & a-b \end{bmatrix}=e^{j\phi}\left\{k_S[S_S]+e^{j\phi_R}\left(k_D[S_D]+k_H[S_H]\right)\right\}$$

SINGLE BOUNCE SCATTERING | **DOUBLE BOUNCE SCATTERING** | **HELICAL SCATTERING**

COHERENT DECOMPOSITION

$$[S] = e^{j\phi}\left\{k_S\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + e^{j\phi_R}\left(k_D\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} + \frac{k_H}{2}\begin{bmatrix}1 & \pm j\\ \pm j & -1\end{bmatrix}\right)\right\}$$

ROTATION AROUND THE RADAR LINE OF SIGHT

$$[U] = \begin{bmatrix}\cos(\theta) & \sin(\theta)\\ -\sin(\theta) & \cos(\theta)\end{bmatrix}$$

$$\begin{aligned}[S(\theta)] &= [U]^T[S][U]\\ &= e^{j\phi}\left\{k_S\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + e^{j\phi_R}\left(k_D\begin{bmatrix}\cos(2\theta) & \sin(2\theta)\\ \sin(2\theta) & -\cos(2\theta)\end{bmatrix}\cdots\right.\right.\\ &\left.\left.\cdots + \frac{k_H e^{\mp j2\theta}}{2}\begin{bmatrix}1 & \pm j\\ \pm j & -1\end{bmatrix}\right)\right\}\end{aligned}$$

COHERENT DECOMPOSITION

$$[S] = e^{j\phi}\begin{bmatrix} k_S + e^{j\phi_R}\left\{k_D\cos(2\theta) + \frac{\hat{k}_H}{2}\right\} & e^{j\phi_R}\left\{k_D\sin(2\theta) \pm j\frac{\hat{k}_H}{2}\right\} \\ e^{j\phi_R}\left\{k_D\sin(2\theta) \pm j\frac{\hat{k}_H}{2}\right\} & k_S - e^{j\phi_R}\left\{k_D\cos(2\theta) + \frac{\hat{k}_H}{2}\right\} \end{bmatrix}$$

AVEC : $\hat{k}_H = k_H e^{\mp j2\theta}$

$$\underline{k} = \sqrt{2}e^{j\phi}\begin{bmatrix} k_S & e^{j\phi_R}\left\{k_D\cos(2\theta) + \frac{\hat{k}_H}{2}\right\} & e^{j\phi_R}\left\{k_D\sin(2\theta) \pm j\frac{\hat{k}_H}{2}\right\} \end{bmatrix}^T$$

$$\underline{k} = \sqrt{2}k_S e^{j\phi}\begin{bmatrix}1\\0\\0\end{bmatrix} + \hat{k}_H e^{j(\phi+\phi_R)}\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\\pm j\end{bmatrix} + \sqrt{2}k_D e^{j(\phi+\phi_R)}\begin{bmatrix}0\\\cos(2\theta)\\\sin(2\theta)\end{bmatrix}$$

SINGLE SCATTERING CONTRIBUTION

$$k_S = \sqrt{A_0} \qquad [S_S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

DOUBLE SCATTERING CONTRIBUTION

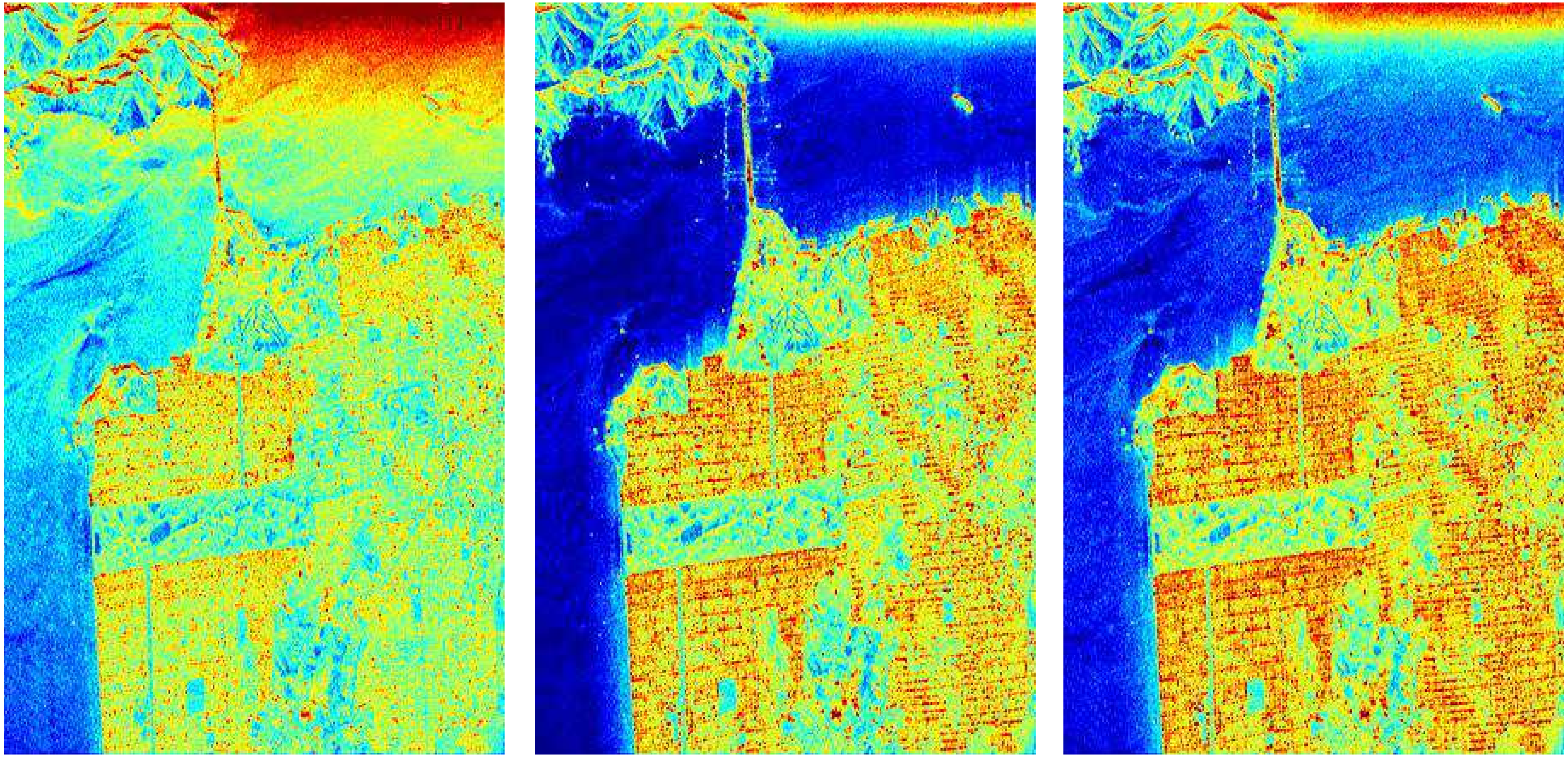
$$k_D = \sqrt{B_0 - |F|} \qquad [S_D] = \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$$

$$\tan(4\theta) = \frac{E}{B}$$

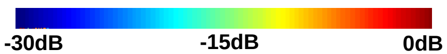
DIPLANE ORIENTATION ANGLE INSIDE THE TARGET

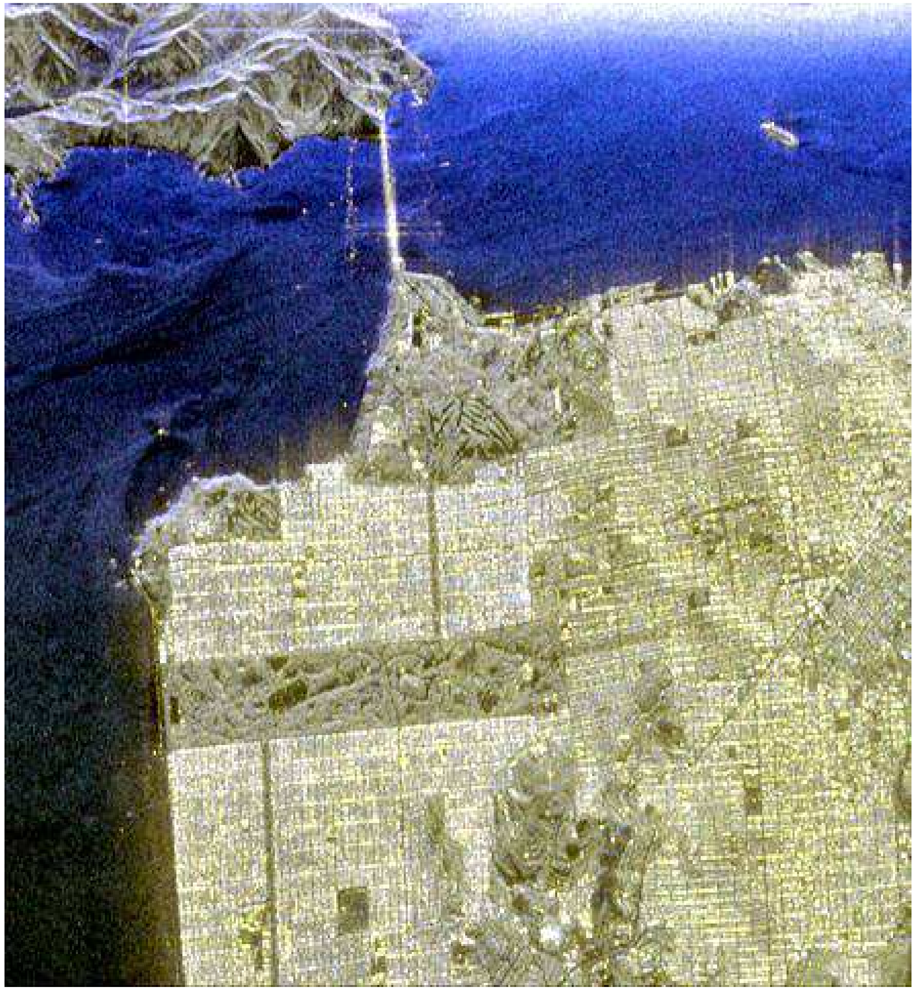
HELICAL SCATTERING CONTRIBUTION

$$k_H = \sqrt{B_0 + |F|} - \sqrt{B_0 - |F|} \qquad [S_H] = \frac{1}{2}\begin{bmatrix} 1 & \pm j \\ \pm j & -1 \end{bmatrix}$$



$(k_S)_{dB}$ $(k_D)_{dB}$ $(k_H)_{dB}$





$2A_0$ $B_0 + B$ $B_0 - B$

k_S k_D k_H

COHERENT DECOMPOSITION

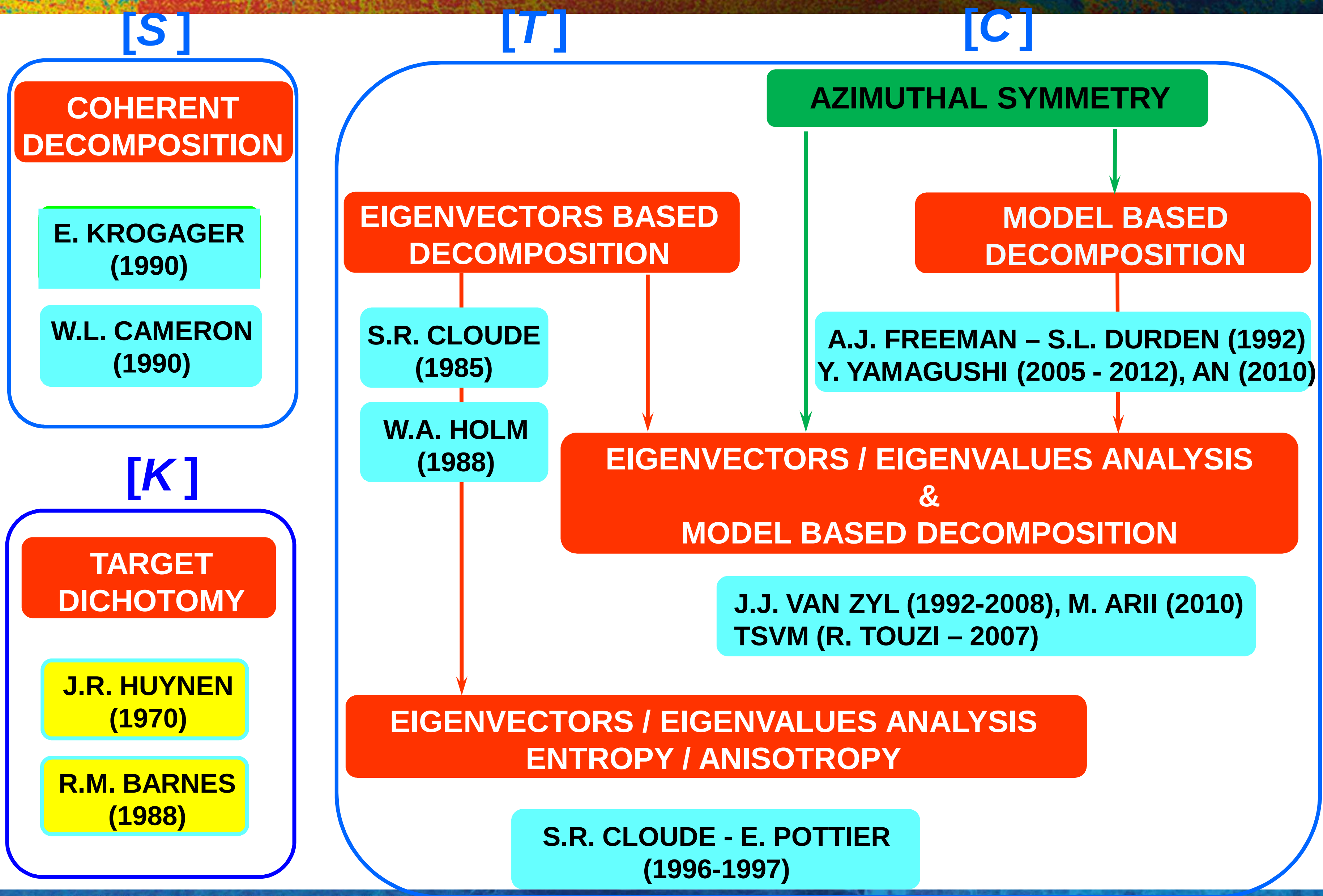
$$\underline{k} = \sqrt{2}k_S e^{j\phi}\begin{bmatrix}1\\0\\0\end{bmatrix} + \hat{k}_H e^{j(\phi+\phi_R)}\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\\varepsilon j\end{bmatrix} + \sqrt{2}k_D e^{j(\phi+\phi_R)}\begin{bmatrix}0\\\cos(2\theta)\\\sin(2\theta)\end{bmatrix}$$

EIGENVECTORS OF $[U_{3R}(\phi)]$ (ROLL INVARIANCE)

➡ NO ORTHOGONALITY OF THE TARGETS COMPONENTS

➡ COHERENT DECOMPOSITION and SPECKLE FILTERING ?

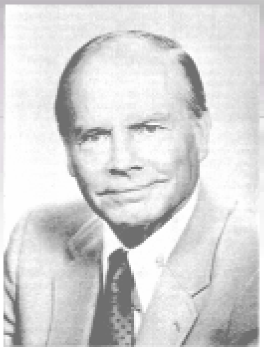
TARGET DECOMPOSITIONS



J.R. HUYNEN

DECOMPOSITION

(1970)



DISTRIBUTED TARGET

$$\langle [T] \rangle = \frac{1}{N} \sum_{i=1}^{i=N} [T_i]$$

DISTRIBUTED TARGET EQUATIONS

$$\begin{aligned}
2\langle A_0 \rangle (\langle B_0 \rangle + \langle B \rangle) &\geq \langle C \rangle^2 + \langle D \rangle^2 \\
2\langle A_0 \rangle (\langle B_0 \rangle - \langle B \rangle) &\geq \langle G \rangle^2 + \langle H \rangle^2 \\
\langle B_0 \rangle^2 &\geq \langle B \rangle^2 + \langle E \rangle^2 + \langle F \rangle^2 \\
2\langle A_0 \rangle \langle E \rangle &\geq \langle C \rangle \langle H \rangle - \langle D \rangle \langle G \rangle \\
2\langle A_0 \rangle \langle F \rangle &\geq \langle C \rangle \langle G \rangle + \langle D \rangle \langle H \rangle
\end{aligned}$$

DECOMPOSITION - TARGET DICHOTOMY

$$\langle B_0 \rangle^2 \geq \langle B \rangle^2 + \langle E \rangle^2 + \langle F \rangle^2$$

$$\begin{bmatrix} \langle B_0 \rangle \\ \langle B \rangle \\ \langle E \rangle \\ \langle F \rangle \end{bmatrix} = \begin{bmatrix} B_{0T} \\ B_T \\ E_T \\ F_T \end{bmatrix} + \begin{bmatrix} B_{0N} \\ B_N \\ E_N \\ F_N \end{bmatrix}$$

J.R. HUYNEN
(1970)

PURE TARGET → NON SYMMETRIC TARGET (N-TARGET)

$$B_{0T}^{\;2} = B_T^{\;2} + E_T^{\;2} + F_T^{\;2}$$

J.R. HUYNEN DECOMPOSITION

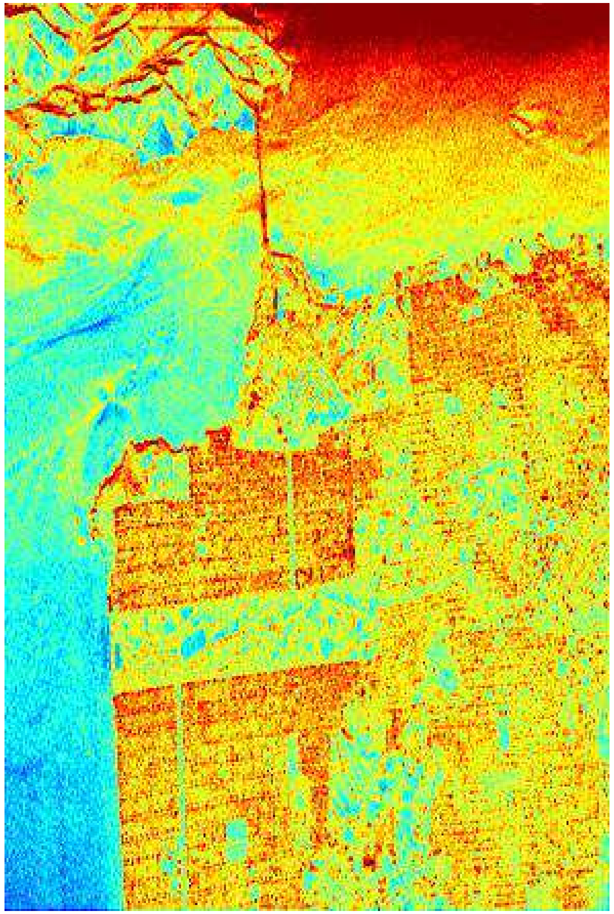
$$\langle [T] \rangle = \begin{bmatrix} \langle 2A_0 \rangle & \langle C \rangle - j\langle D \rangle & \langle H \rangle + j\langle G \rangle \\ \langle C \rangle + j\langle D \rangle & \langle B_0 \rangle + \langle B \rangle & \langle E \rangle + j\langle F \rangle \\ \langle H \rangle - j\langle G \rangle & \langle E \rangle - j\langle F \rangle & \langle B_0 \rangle - \langle B \rangle \end{bmatrix} = [T_0] + [T_N]$$

➡ PURE TARGET

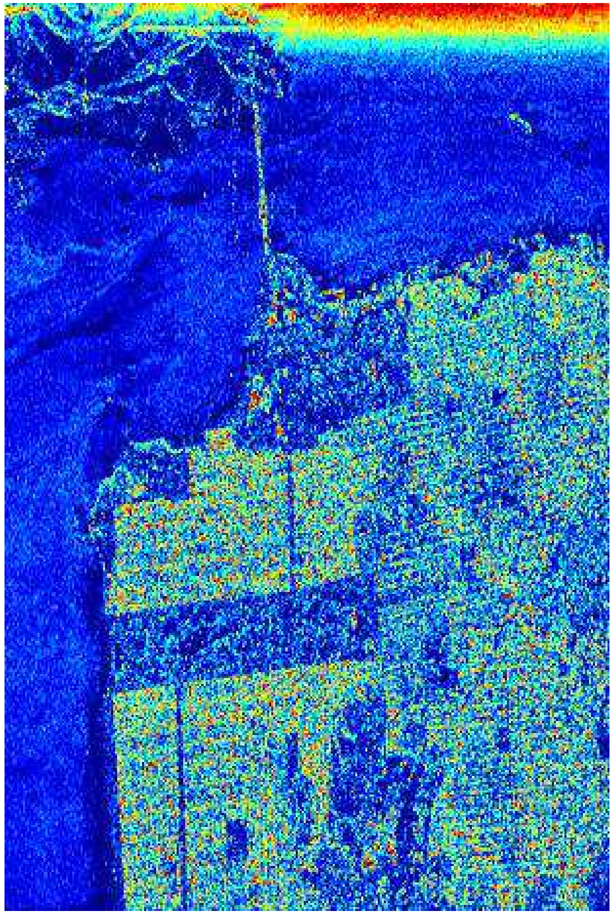
$$[T_0] = \begin{bmatrix} \langle 2A_0 \rangle & \langle C \rangle - j\langle D \rangle & \langle H \rangle + j\langle G \rangle \\ \langle C \rangle - j\langle D \rangle & B_{0T} + B_T & E_T + jF_T \\ \langle H \rangle - j\langle G \rangle & E_T - jF_T & B_{0T} - B_T \end{bmatrix}$$

➡ N-TARGET

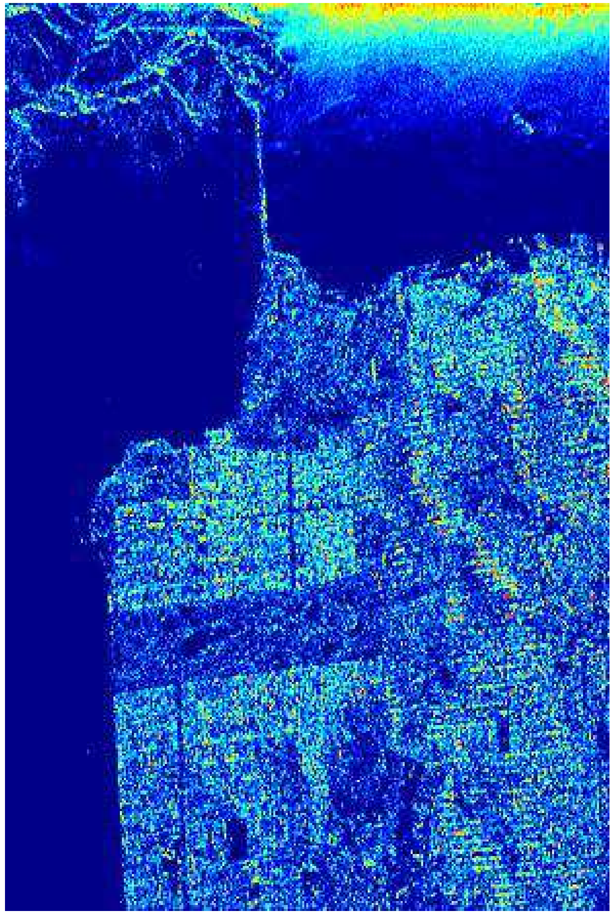
$$[T_N] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_{0N} + B_N & E_N + jF_N \\ 0 & E_N - jF_N & B_{0N} - B_N \end{bmatrix}$$



$$\langle 2A_0 \rangle$$

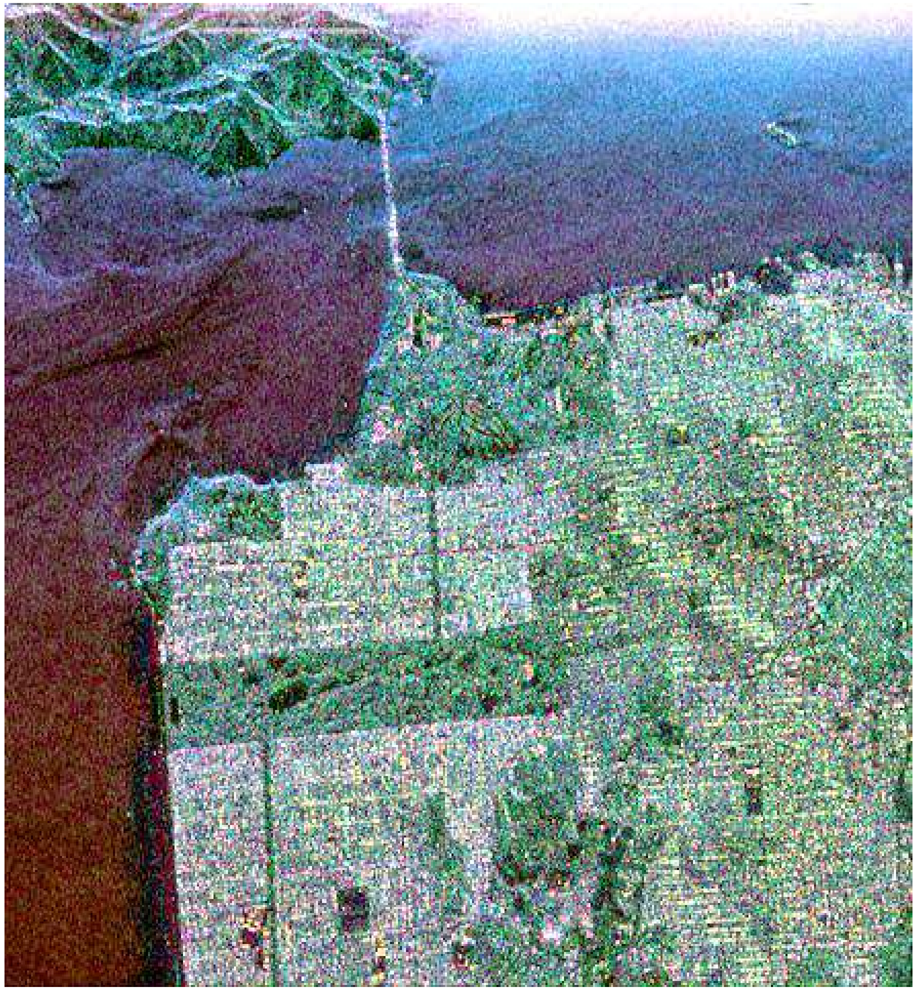


$$\left(\langle C \rangle^2 + \langle D \rangle^2\right) / 2A_0$$



$$\left(\langle H \rangle^2 + \langle G \rangle^2\right) / 2A_0$$

J.R. HUYNEN DECOMPOSITION



$2A_0$ $\quad B_0 + B \quad$ $B_0 - B$

$\langle 2A_0 \rangle \quad \left(\langle C \rangle^2 + \langle D \rangle^2 \right) / 2A_0$

$\left(\langle H \rangle^2 + \langle G \rangle^2 \right) / 2A_0$

BARNES DECOMPOSITION

3 SINGLE TARGET VECTORS

$$\underline{k}_{01} = \frac{\langle [T] \rangle \underline{q}_1}{\sqrt{\underline{q}_1^{T*} \langle [T] \rangle \underline{q}_1}} = \frac{1}{\sqrt{\langle 2A_0 \rangle}} \begin{bmatrix} \langle 2A_0 \rangle \\ \langle C \rangle + j\langle D \rangle \\ \langle H \rangle - j\langle G \rangle \end{bmatrix}$$

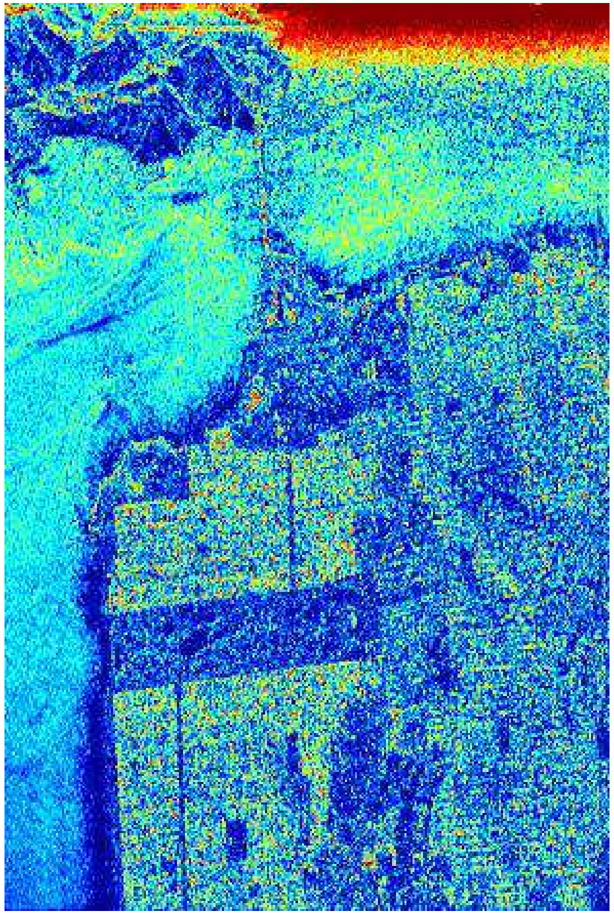
→ HUYNEN DECOMPOSITION

(SYMMETRIC WORLD)

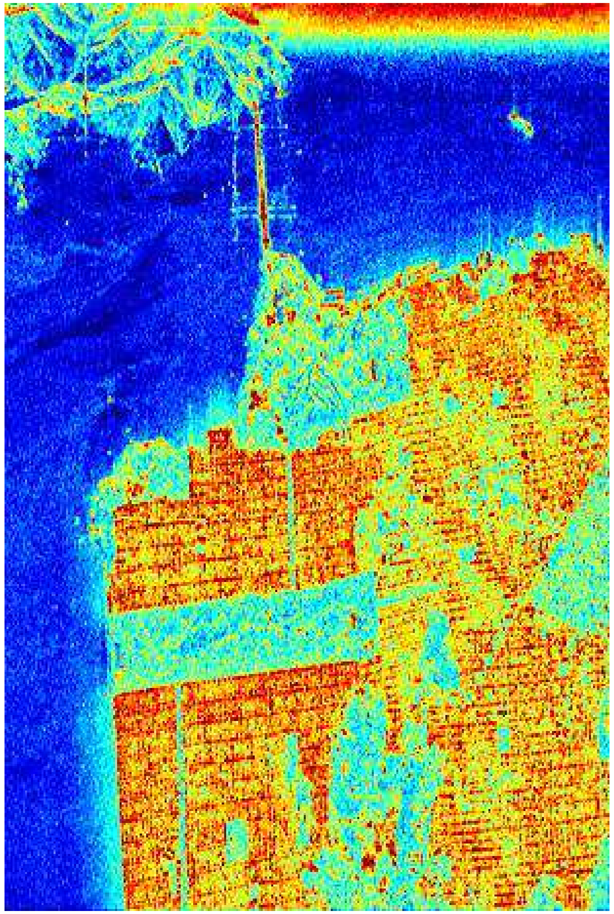
$$\underline{k}_{02} = \frac{\langle [T] \rangle \underline{q}_2}{\sqrt{\underline{q}_2^{T*} \langle [T] \rangle \underline{q}_2}} = \frac{1}{\sqrt{2\left(\langle B_0 \rangle - \langle F \rangle\right)}} \begin{bmatrix} \langle C \rangle - \langle G \rangle + j\langle H \rangle - j\langle D \rangle \\ \langle B_0 \rangle + \langle B \rangle - \langle F \rangle + j\langle E \rangle \\ \langle E \rangle + j\langle B_0 \rangle - j\langle B \rangle - j\langle F \rangle \end{bmatrix}$$

$$\underline{k}_{03} = \frac{\langle [T] \rangle \underline{q}_3}{\sqrt{\underline{q}_3^{T*} \langle [T] \rangle \underline{q}_3}} = \frac{1}{\sqrt{2\left(\langle B_0 \rangle + \langle F \rangle\right)}} \begin{bmatrix} \langle H \rangle + \langle D \rangle + j\langle C \rangle + j\langle G \rangle \\ \langle E \rangle + j\langle B_0 \rangle + j\langle B \rangle + j\langle F \rangle \\ \langle B_0 \rangle - \langle B \rangle + \langle F \rangle + j\langle E \rangle \end{bmatrix}$$

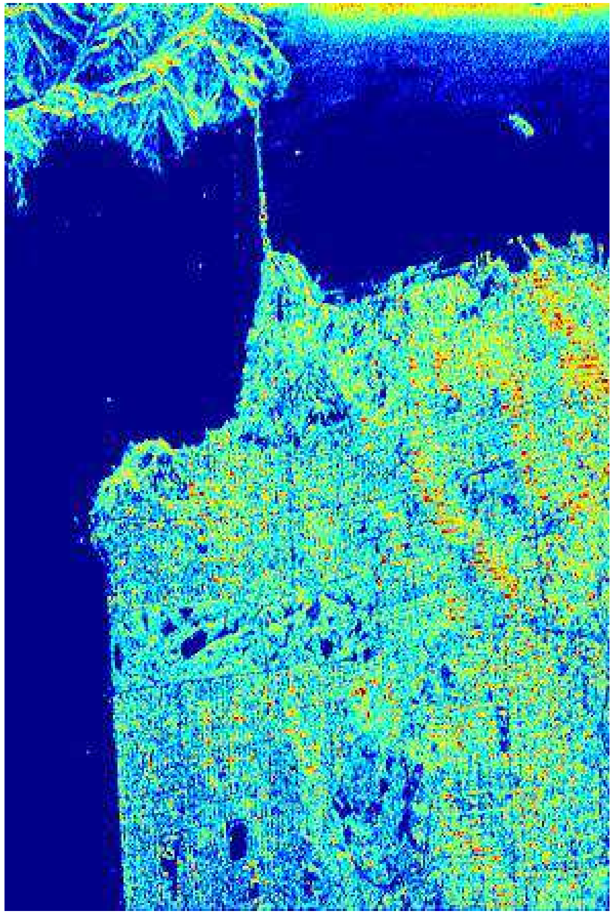
(HELICITY - NON SYMMETRIC COMPONENTS = EXOTIC WORLD)



$$\frac{\sqrt{\left(\langle C\rangle-\langle G\rangle\right)^2+\left(\langle H\rangle-\langle D\rangle\right)^2}}{\sqrt{2\left(\langle B_0\rangle-\langle F\rangle\right)}}$$



$$\frac{\sqrt{\left(\langle B_0\rangle+\langle B\rangle-\langle F\rangle\right)^2+\langle E\rangle^2}}{\sqrt{2\left(\langle B_0\rangle-\langle F\rangle\right)}}$$



$$\frac{\sqrt{\left(\langle B_0\rangle-\langle B\rangle-\langle F\rangle\right)^2+\langle E\rangle^2}}{\sqrt{2\left(\langle B_0\rangle-\langle F\rangle\right)}}$$

BARNES DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

$$\frac{\sqrt{(\langle C\rangle-\langle G\rangle)^2+(\langle H\rangle-\langle D\rangle)^2}}{\sqrt{2(\langle B_0\rangle-\langle F\rangle)}}$$

$$\frac{\sqrt{(\langle B_0\rangle+\langle B\rangle-\langle F\rangle)^2+\langle E\rangle^2}}{\sqrt{2(\langle B_0\rangle-\langle F\rangle)}}$$

$$\frac{\sqrt{(\langle B_0\rangle-\langle B\rangle-\langle F\rangle)^2+\langle E\rangle^2}}{\sqrt{2(\langle B_0\rangle-\langle F\rangle)}}$$

HUYNEN DECOMPOSITION

TARGET DICHOTOMY : PURE TARGET + N TARGET

ROLL INVARIANCE OF THE FORM OF THE N-TARGET

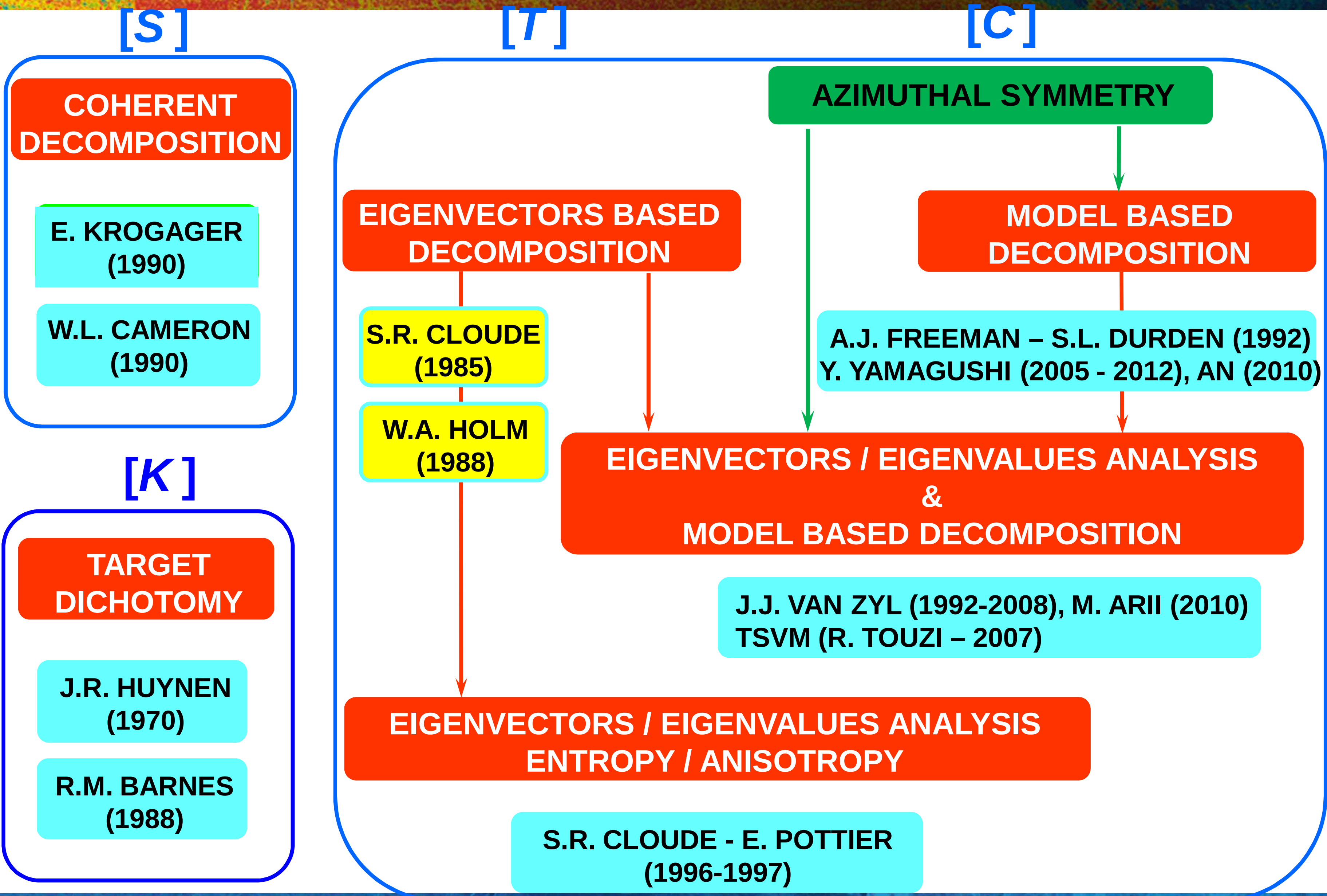
NO UNICITY : 3 DIFFERENT DECOMPOSITIONS

↓

MAN-MADE TARGET DECOMPOSITION
IDENTIFICATION - ANALYSIS

TARGET DECOMPOSITIONS

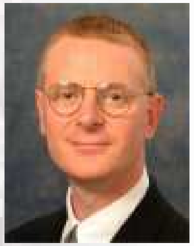
esa



EIGENVECTOR BASED DECOMPOSITION

→ **SHANE R. CLOUDE**

(1985-1992)



→ **WILLIAM A. HOLM**

(1988)

PROPRIETY

EIGENVALUE PROBLEM IS AUTOMATICALLY

BASIS INVARIANT

- **GENERATE A DIAGONAL FORM OF THE COHERENCY MATRIX**
- **PHYSICALLY INTERPRETATION AS STATISTICAL INDEPENDENCE BETWEEN A SET OF VECTORS**
- **GENERAL DECOMPOSITION INTO INDEPENDENT SCATTERING PROCESSES**

EIGENVECTOR-BASED DECOMPOSITION

esa

COHERENCY MATRIX

$$\langle [T] \rangle = \begin{bmatrix} \langle 2A_0 \rangle & \langle C \rangle - j\langle D \rangle & \langle H \rangle + j\langle G \rangle \\ \langle C \rangle + j\langle D \rangle & \langle B_0 \rangle + \langle B \rangle & \langle E \rangle + j\langle F \rangle \\ \langle H \rangle - j\langle G \rangle & \langle E \rangle - j\langle F \rangle & \langle B_0 \rangle - \langle B \rangle \end{bmatrix}$$

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1}$$

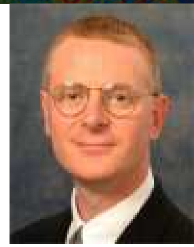
$$[\Sigma] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}_{\lambda_1 \geq \lambda_2 \geq \lambda_3}$$

3x3 DIAGONAL MATRIX OF EIGENVALUES

$$[U_3] = [\underline{u}_1, \underline{u}_2, \underline{u}_3]$$

SU(3) UNITARY MATRIX (EIGENVECTORS)

SHANE R. CLOUDE



(1985-1992)

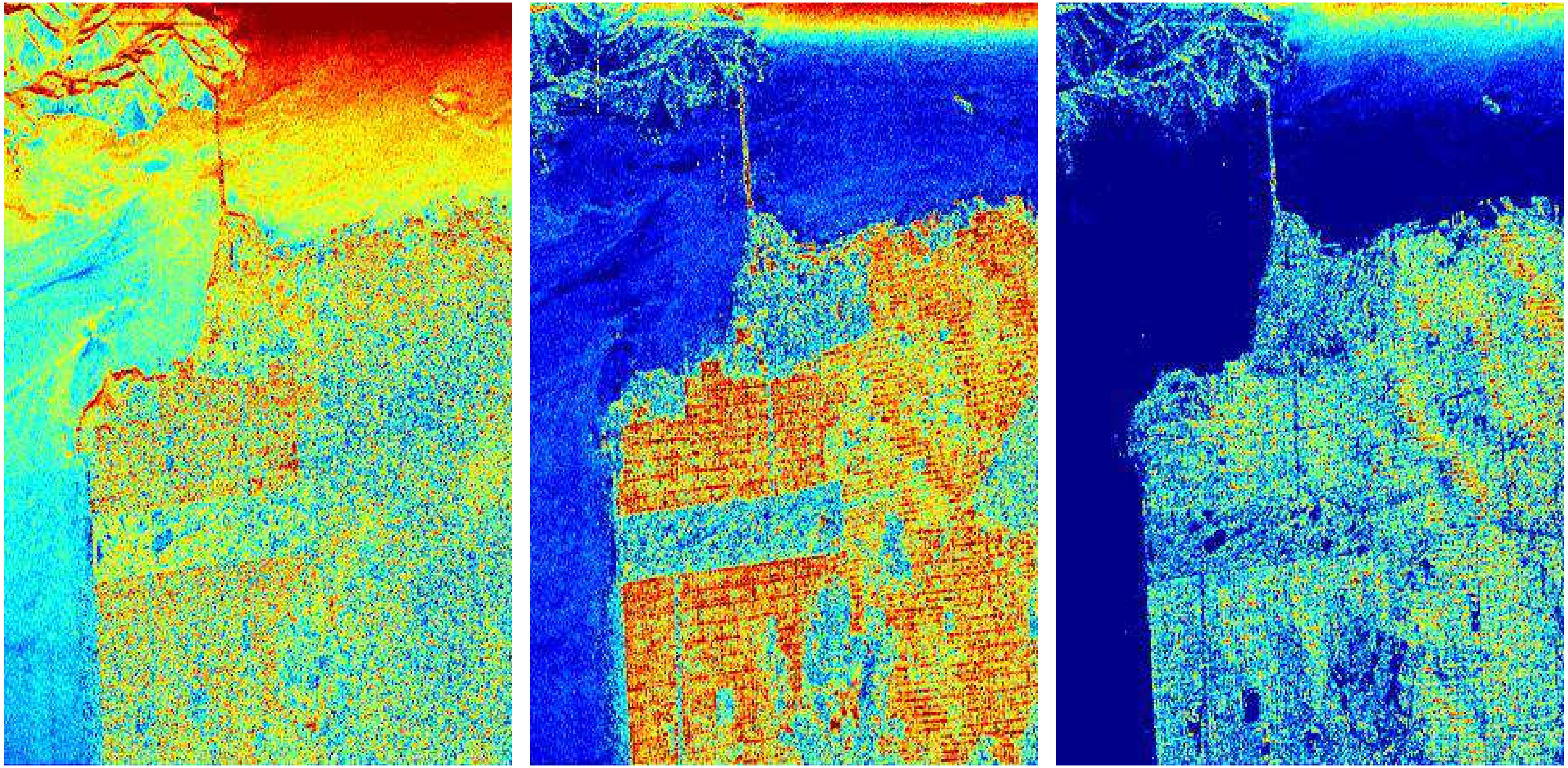
DECOMPOSITION

IDENTIFICATION OF THE DOMINANT SCATTERING MECHANISM

VIA THE

EXTRACTION OF THE LARGEST EIGENVALUE

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} \quad \Rightarrow \quad [T_1] = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} = \underline{k}_1 \underline{k}_1^{T*}$$



$\sqrt{\lambda_1}\,|u_{11}|$ $\sqrt{\lambda_1}\,|u_{12}|$ $\sqrt{\lambda_1}\,|u_{13}|$

$2A_0$ $B_0 + B$ $B_0 - B$

$\sqrt{\lambda_1}\,|u_{11}|$ $\sqrt{\lambda_1}\,|u_{12}|$ $\sqrt{\lambda_1}\,|u_{13}|$

WILLIAM A. HOLM

(1988)

DECOMPOSITION

ALTERNATIVE PHYSICAL INTERPRETATION

OF THE EIGENVALUES SPECTRUM

$$[\Sigma] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$\Downarrow$$

$$[\Sigma] = \begin{bmatrix} \lambda_1 - \lambda_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \lambda_2 - \lambda_3 & 0 & 0 \\ 0 & \lambda_2 - \lambda_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \lambda_3 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$\langle[T]\rangle = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} + \lambda_2 \underline{u}_2 \underline{u}_2^{T*} + \lambda_3 \underline{u}_3 \underline{u}_3^{T*}$$

$$\langle[T]\rangle = (\lambda_1 - \lambda_2)\underline{u}_1 \underline{u}_1^{T*} + (\lambda_2 - \lambda_3)\left(\underline{u}_1 \underline{u}_1^{T*} + \underline{u}_2 \underline{u}_2^{T*}\right) + \lambda_3 [I_{3D}]$$

PURE TARGET (AVERAGE)

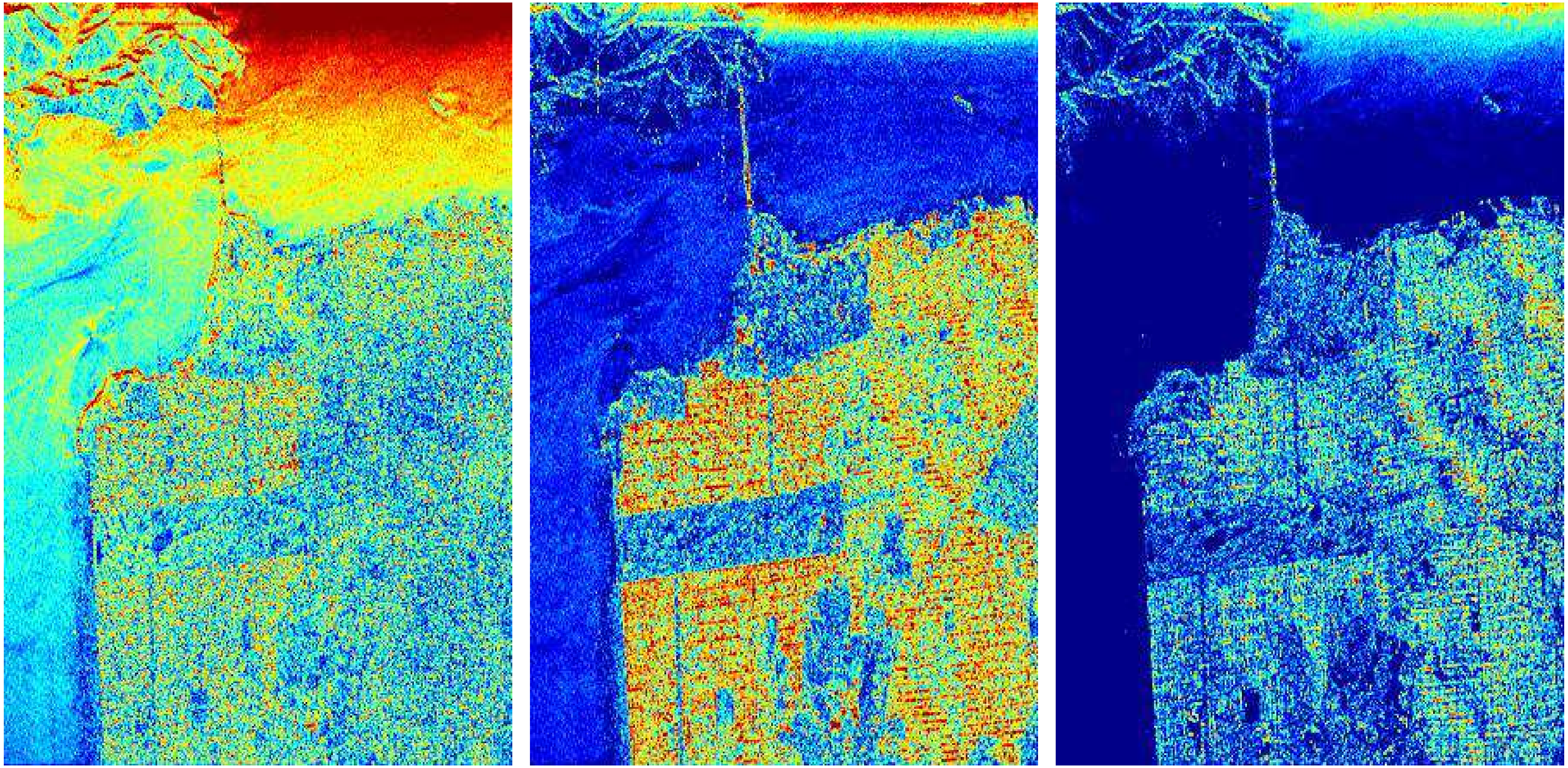
MIXED TARGET (VARIANCE)

NOISE (UNPOLARIZED)

CONCEPT OF : TARGET PLUS NOISE

HYBRID APPROACH OF THE HUYNEN MODEL

W.A HOLM DECOMPOSITION



$\sqrt{\lambda_1 - \lambda_2}\,|u_{11}|$ $\sqrt{\lambda_1 - \lambda_2}\,|u_{12}|$ $\sqrt{\lambda_1 - \lambda_2}\,|u_{13}|$

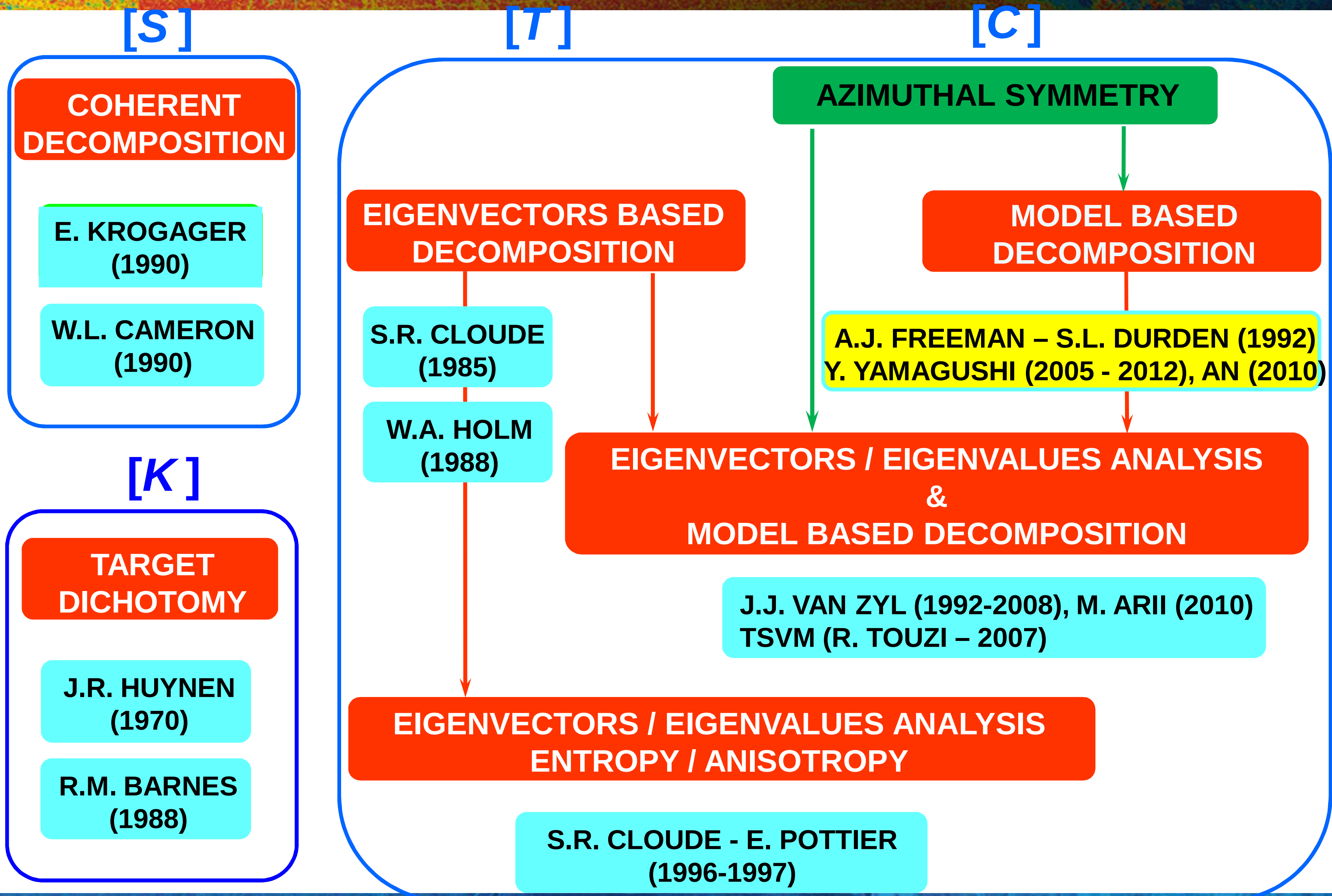
W.A HOLM DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

$\sqrt{\lambda_1 - \lambda_2}\,|u_{11}|$ $\sqrt{\lambda_1 - \lambda_2}\,|u_{12}|$

$\sqrt{\lambda_1 - \lambda_2}\,|u_{13}|$

TARGET DECOMPOSITIONS

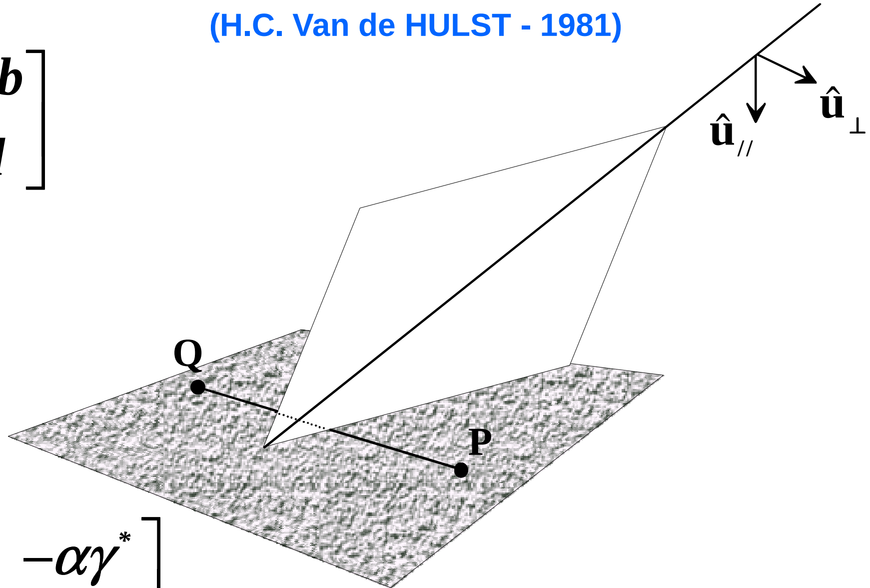


TARGET DECOMPOSITION FOR TARGETS WITH REFLECTION SYMMETRY

MEDIUM WITH REFLECTION SYMMETRY

(H.C. Van de HULST - 1981)

$$[S_Q] = \begin{bmatrix} a & -b \\ -b & d \end{bmatrix} \qquad [S_P] = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$$



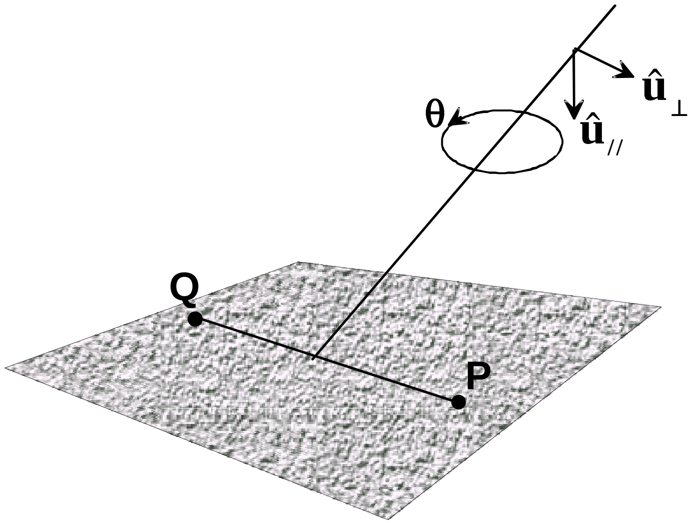
$$\underline{k}_Q = \begin{bmatrix} \alpha \\ \beta \\ -\gamma \end{bmatrix} \qquad \underline{k}_P = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$[T_Q] = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* & -\alpha\gamma^* \\ \beta\alpha^* & |\beta|^2 & -\beta\gamma^* \\ -\gamma\alpha^* & -\gamma\beta^* & |\gamma|^2 \end{bmatrix} \qquad [T_P] = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \beta\alpha^* & |\beta|^2 & \beta\gamma^* \\ \gamma\alpha^* & \gamma\beta^* & |\gamma|^2 \end{bmatrix}$$

$$\langle [T] \rangle = [T_P] + [T_Q] = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* & 0 \\ \beta\alpha^* & |\beta|^2 & 0 \\ 0 & 0 & |\gamma|^2 \end{bmatrix}$$

MEDIUM WITH ROTATION SYMMETRY

(H.C. Van de HULST - 1981)



$$\langle [T(\theta)] \rangle = [R_3(\theta)] \langle [T] \rangle [R_3(\theta)]^{-1}$$

With:

$$[R_3(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\theta) & \sin(2\theta) \\ 0 & -\sin(2\theta) & \cos(2\theta) \end{bmatrix}$$

EIGENVECTORS OF $[U_{3P}^{R}]$

$$[U_{3P}^{R}]\underline{q} = \lambda \underline{q}$$

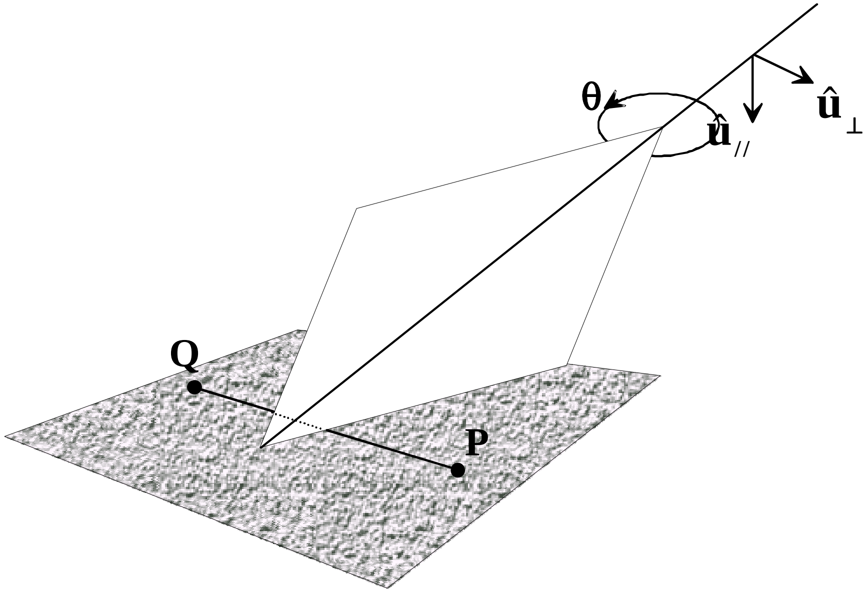
$$\underline{q}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \underline{q}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ j \end{bmatrix} \quad \underline{q}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ j \\ 1 \end{bmatrix}$$

$$\langle [T_R] \rangle = \alpha \underline{q}_1 \underline{q}_1^{T*} + \beta \underline{q}_2 \underline{q}_2^{T*} + \gamma \underline{q}_3 \underline{q}_3^{T*}$$

$$= \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & -j(\beta-\gamma) \\ 0 & j(\beta-\gamma) & \beta+\gamma \end{bmatrix}$$

MEDIUM WITH AZIMUTHAL SYMMETRY

(S.H. NGHIEM et al. - 1992)



$$[T_{PR}] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & -j(\beta-\gamma) \\ 0 & j(\beta-\gamma) & \beta+\gamma \end{bmatrix}$$

$$[T_{QR}] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & j(\beta-\gamma) \\ 0 & -j(\beta-\gamma) & \beta+\gamma \end{bmatrix}$$

AZIMUTHAL SYMMETRY =
REFLECTION + ROTATION SYMMETRIES

$$\langle [T] \rangle = [T_{PR}] + [T_{QR}] = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta+\gamma & 0 \\ 0 & 0 & \beta+\gamma \end{bmatrix}$$

COHERENCY MATRIX

General Case

$$\langle [T] \rangle = \begin{bmatrix} T_1 & T_2 & T_3 \\ T_2^* & T_4 & T_5 \\ T_3^* & T_5^* & T_6 \end{bmatrix}$$

Reflection Symmetry

$$\langle [T] \rangle = \begin{bmatrix} T_1 & T_2 & 0 \\ T_2^* & T_4 & 0 \\ 0 & 0 & T_6 \end{bmatrix}$$

Rotation Symmetry

$$\langle [T] \rangle = \begin{bmatrix} T_1 & 0 & 0 \\ 0 & T_4 & T_5 \\ 0 & T_5^* & T_4 \end{bmatrix}$$

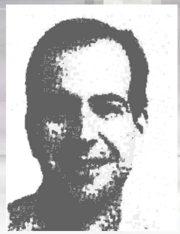
Azimuthal Symmetry

$$\langle [T] \rangle = \begin{bmatrix} T_1 & 0 & 0 \\ 0 & T_4 & 0 \\ 0 & 0 & T_4 \end{bmatrix}$$

TARGET DECOMPOSITION FOR TARGETS WITH REFLECTION SYMMETRY

MODEL BASED DECOMPOSITION

A. FREEMAN – S. DURDEN (1992)



A. Freeman and S.L. Durden, "A Three-Component Scattering Model for Polarimetric SAR Data" IEEE TGRS, vol. 36, no. 3, May 1998

3 COMPONENTS SCATTERING MECHANISM MODEL

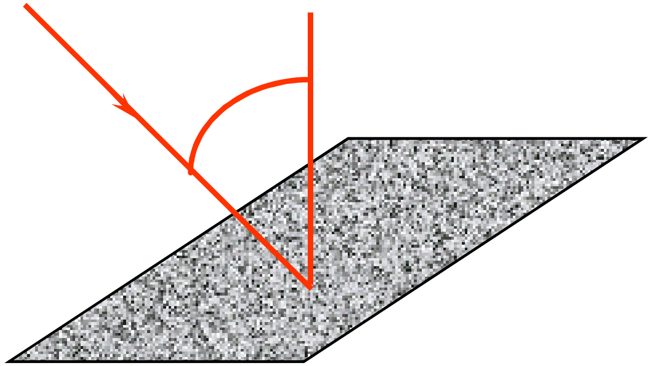
$$\langle [T] \rangle = [T_S] + [T_D] + [T_V]$$

$\downarrow$ SINGLE SCATTERING

$\downarrow$ DOUBLE SCATTERING

$\downarrow$ VOLUME SCATTERING

SINGLE SCATTERING
(ROUGH SURFACE)



MECHANISM

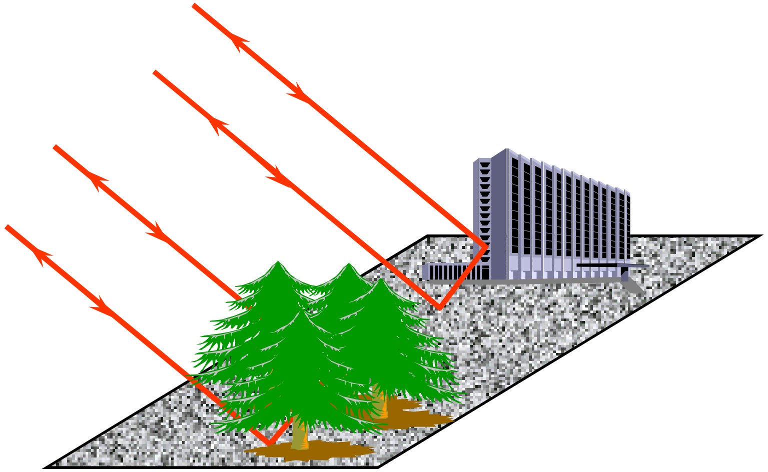
$$[S_S] = \begin{bmatrix} R_H & 0 \\ 0 & R_V \end{bmatrix} \Rightarrow \underline{k}_S = \begin{bmatrix} R_H + R_V \\ R_H - R_V \\ 0 \end{bmatrix}$$

COHERENCY MATRIX

$$[T_S] = f_S \begin{bmatrix} |\beta+1|^2 & (\beta+1)(\beta-1)^* & 0 \\ (\beta+1)^*(\beta-1) & |\beta-1|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$f_S = |R_V|^2 \qquad \beta = \frac{R_H}{R_V}$$

DOUBLE SCATTERING



MECHANISM

$$[S_D] = \begin{bmatrix} R_{GH} R_{TH} & 0 \\ 0 & -R_{GV} R_{TV} \end{bmatrix}$$

$$\Rightarrow \underline{k}_D = \begin{bmatrix} R_{GH} R_{TH} - R_{GV} R_{TV} \\ R_{GH} R_{TH} + R_{GV} R_{TV} \\ 0 \end{bmatrix}$$

COHERENCY MATRIX

$$[T_D] = f_D \begin{bmatrix} |\alpha-1|^2 & (\alpha-1)(\alpha+1)^* & 0 \\ (\alpha-1)^*(\alpha+1) & |\alpha+1|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

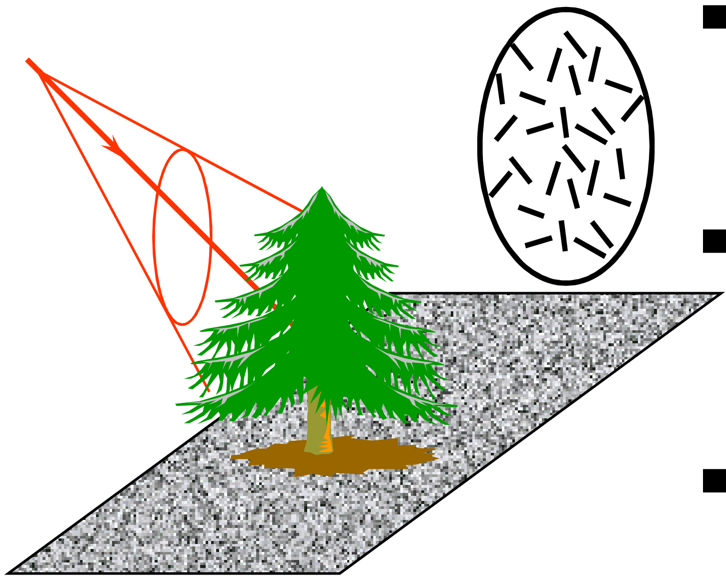
$$f_D = |R_{GV} R_{TV}|^2$$

$$\alpha = \frac{R_{GH} R_{TH}}{R_{GV} R_{TV}}$$

MODEL BASED DECOMPOSITION

VOLUME SCATTERING

(RANDOMLY ORIENTED VERY THIN CYLINDER-LIKE SCATTERERS)



MECHANISM (CYLINDER)

$$[S]=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

ORIENTED MECHANISM

$$[S_\theta]=[U_2(\theta)]^T[S][U_2(\theta)]$$

UNIFORM ORIENTATION

$$P(\theta)=\frac{1}{2\pi}$$

SECOND-ORDER STATISTICS

$$[T_V]=\langle[T_\theta]\rangle=\int_0^{2\pi}[T_V]P(\theta)d\theta$$

COVARIANCE MATRIX

(THIN CYLINDERS)

$a \mapsto 1 \quad b \mapsto 0$

$$[T_V]=\frac{f_V}{3}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

MODEL BASED DECOMPOSITION

esa

3 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_S] + [T_D] + [T_V]$$

SINGLE SCATTERING | DOUBLE SCATTERING | VOLUME SCATTERING

$$T_{11} = f_S |\beta + 1|^2 + f_D |\alpha - 1|^2 + \frac{4 f_V}{3}$$

$$T_{12} = f_S (\beta + 1)(\beta - 1)^* + f_D (\alpha - 1)(\alpha + 1)^*$$

$$T_{22} = f_S |\beta - 1|^2 + f_D |\alpha + 1|^2 + \frac{2 f_V}{3}$$

$$T_{33} = \frac{2 f_V}{3}$$

→ 5 UNKNOWN REAL COEFFICIENTS

→ 4 OBSERVED EQUATIONS

MODEL BASED DECOMPOSITION

$$if\ \Re\left(\langle S_{XX}S_{YY}^{*}\rangle - \frac{f_V}{3}\right) \geq 0 \quad \Rightarrow \quad \alpha = +1$$

$$if\ \Re\left(\langle S_{XX}S_{YY}^{*}\rangle - \frac{f_V}{3}\right) \leq 0 \quad \Rightarrow \quad \beta = +1$$

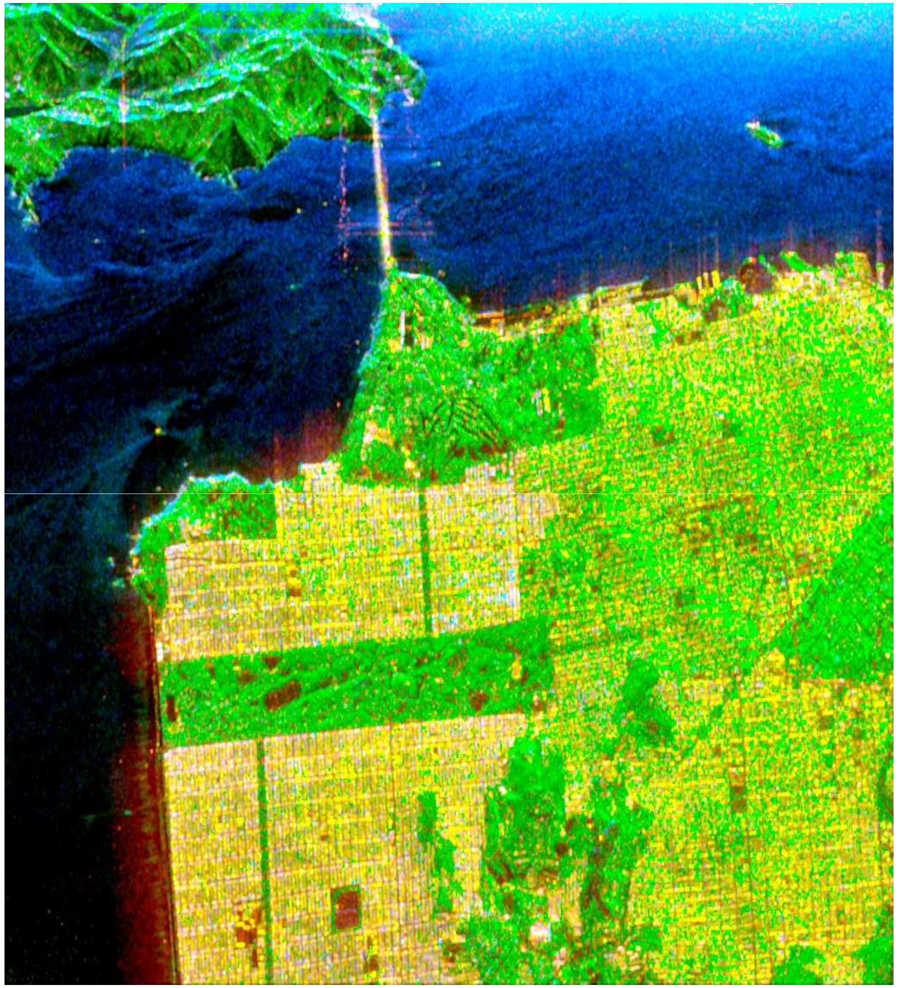
$$\{f_S, |\beta|, f_D, |\alpha|, f_V\}$$

$$span = \langle T_{11}\rangle + \langle T_{22}\rangle + \langle T_{33}\rangle = f_S\left(1+\beta^2\right) + f_D\left(1+|\alpha|^2\right) + \frac{2}{3}f_V$$

SINGLE BOUNCE SCATTERING (ODD)

DOUBLE DOUBLE SCATTERING (DBL)

VOLUME SCATTERING (VOL)



$2A_0$ $B_0 + B$ $B_0 - B$

$$ODD = f_S\left(1+\beta^2\right)$$

$$DBL = f_D\left(1+\alpha^2\right)$$

$$VOL = \frac{2f_V}{3}$$

2 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_G] + [T_V]$$

GROUND SCATTERING

VOLUME SCATTERING

Bragg scatter from a moderately rough surface

Double-bounce scatter from a pair of orthogonal surfaces

Freeman A., "Fitting a Two-Component Scattering Model to Polarimetric SAR Data from Forests", IEEE Trans. Geosci. Remote Sensing, vol. 45, no. 8, pp. 2583–2592, Aug. 2007.

2007

TARGET DECOMPOSITION FOR TARGETS WITHOUT REFLECTION SYMMETRY

MODEL BASED - 4 COMPONENTS DECOMPOSITION

Y. YAMAGUCHI et al. (2005 - 2013)

MODEL BASED DECOMPOSITION

MEDIUM WITHOUT ANY REFLECTION SYMMETRY

4 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [T] \rangle = [T_S] + [T_D] + [T_V] + [T_H]$$

SINGLE SCATTERING | DOUBLE SCATTERING | VOLUME SCATTERING | HELIX SCATTERING

$$[S]_{\pm Helix} = \frac{1}{2}\begin{bmatrix} 1 & \pm j \\ \pm j & -1 \end{bmatrix} \qquad \langle [T] \rangle_{Helix} = \frac{1}{2}\left\langle \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & \pm j \\ 0 & \mp j & 1 \end{bmatrix} \right\rangle$$

Non reflection Symmetric cases

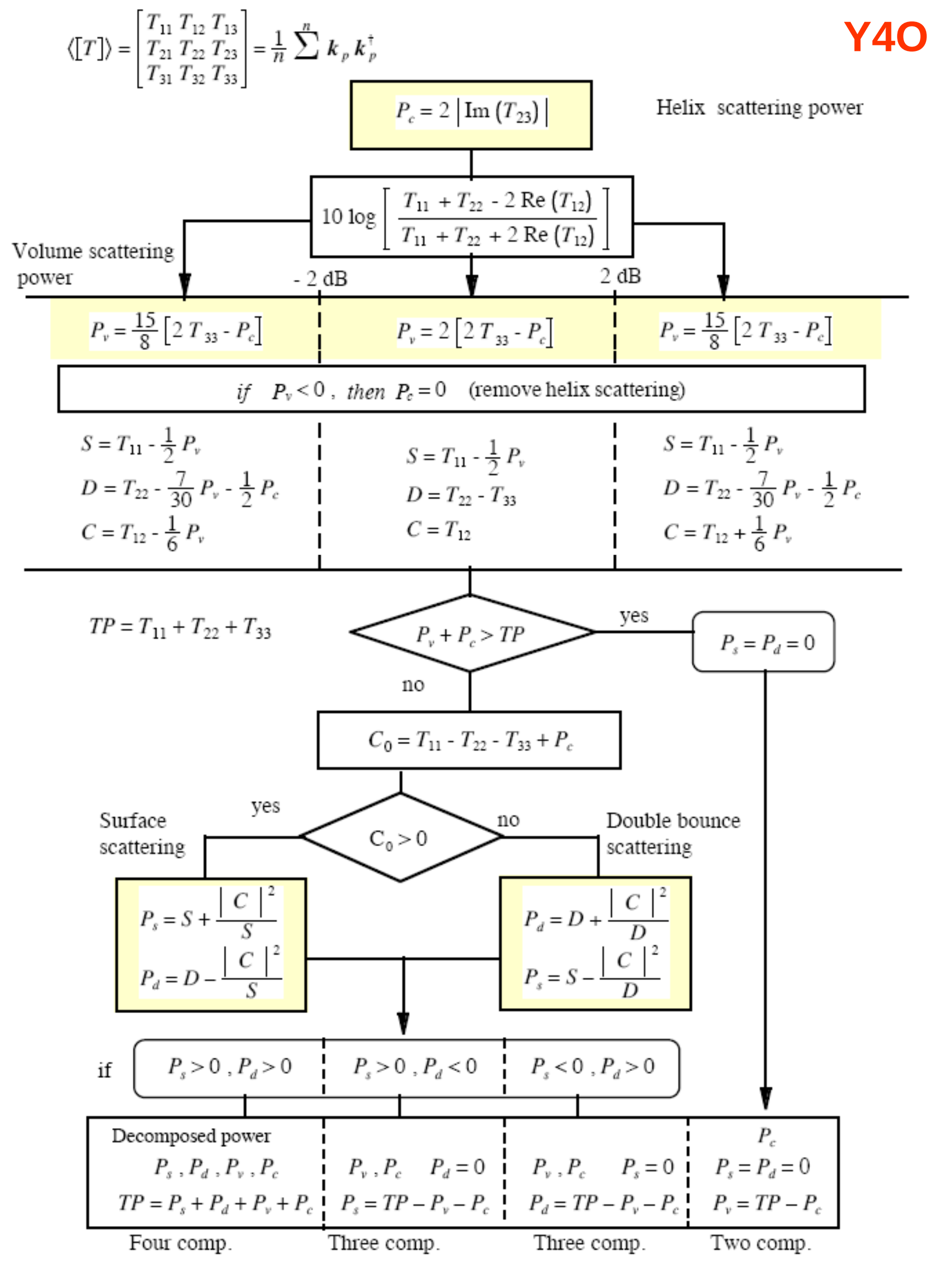
Yamaguchi Y., Moriyama T., Ishido M. and Yamada H., "*Four-Component Scattering Model for Polarimetric SAR Image Decomposition*", IEEE Trans. Geos. Remote Sens., vol. 43, no. 8, August 2005.

Yamaguchi Y., Yajima Y. and Yamada H., "A Four-Component Decomposition of POLSAR Images Based on the Coherency Matrix", IEEE Geos. Rem. Sens. Letters, vol. 3, no. 3, July 2006.

2005 - 2006

MODEL BASED DECOMPOSITION

esa



$2A_0$ $B_0 + B$ $B_0 - B$

$ODD = f_S(1+\beta^2)$

$DBL = f_D(1+\alpha^2)$

$VOL = \frac{2 f_V}{3}$

MODEL BASED DECOMPOSITION

ODD *DBL* *VOL*

Freeman decomposition **Yamaguchi decomposition**

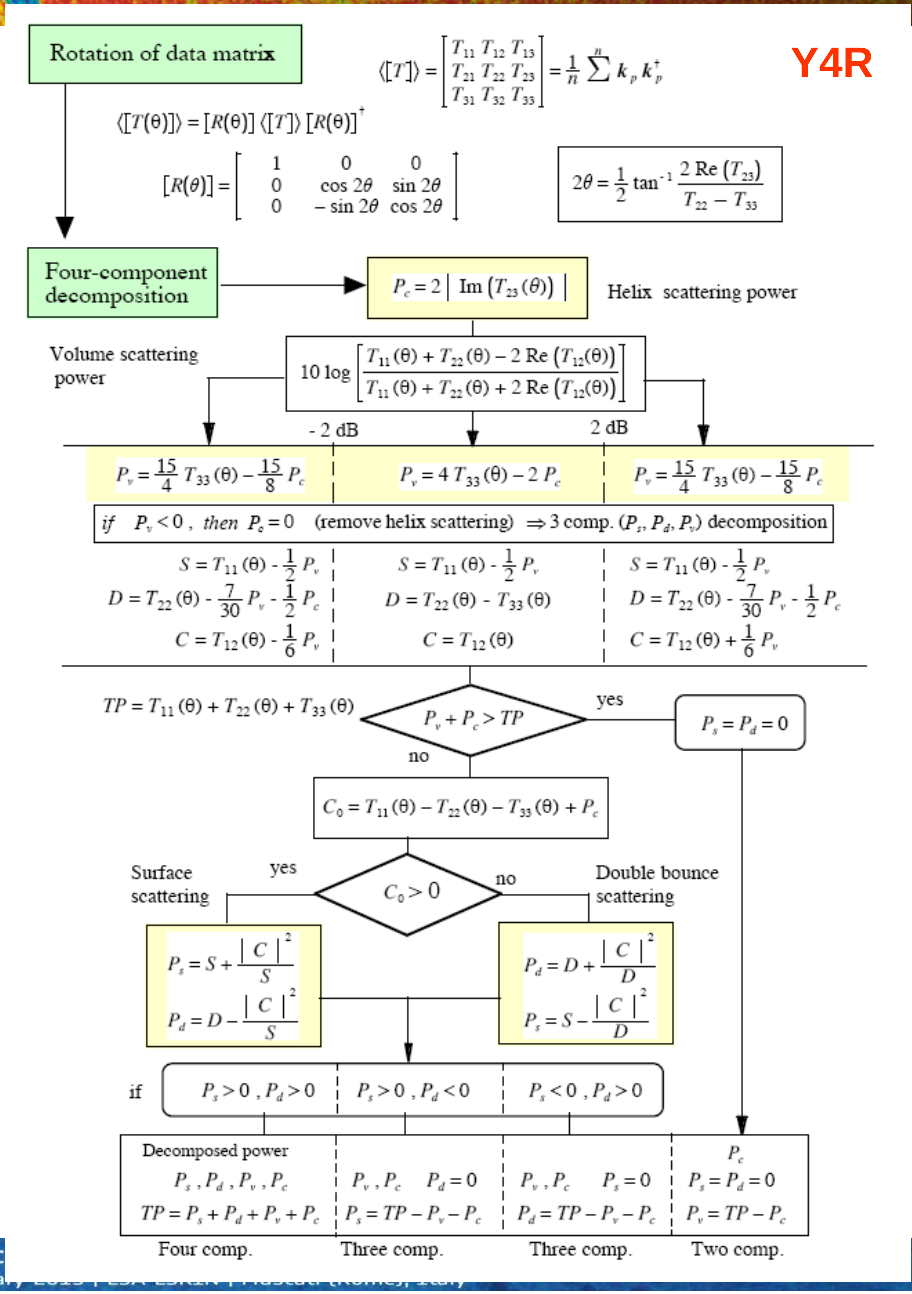
Y. Yamaguchi, A. Sato, W.M. Boerner, R. Sato, H. Yamada, "*4-component scattering power decomposition with rotation of coherency matrix*", IEEE TGRS vol. 49, no. 6, June 2011.

A. Sato, Y. Yamaguchi, G. Singh, and S.-E. Park, "*4-component scattering power decomposition with extended volume scattering model*", IEEE GRS Letters, vol. 9, no. 2, pp. 166–170, Mar. 2012.

G. Singh, Y. Yamaguchi, S.E. Park*, « General Four-Component Scattering Power Decomposition With Unitary Transformation of Coherency Matrix »* IEEE TGRS in press

G. Singh, Y. Yamaguchi, S.E. Park*, Y. Cui, H. Kobayashi, « Hybrid Freeman/Eigenvalue Decomposition Method With Extended Volume Scattering Model »* IEEE GRS Letters, vol. 10, no. 1, Jan. 2013

MODEL BASED DECOMPOSITION



$$2\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\,\mathrm{Re}\{T_{23}\}}{T_{22}-T_{33}}\right)$$

↓

$$[R(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

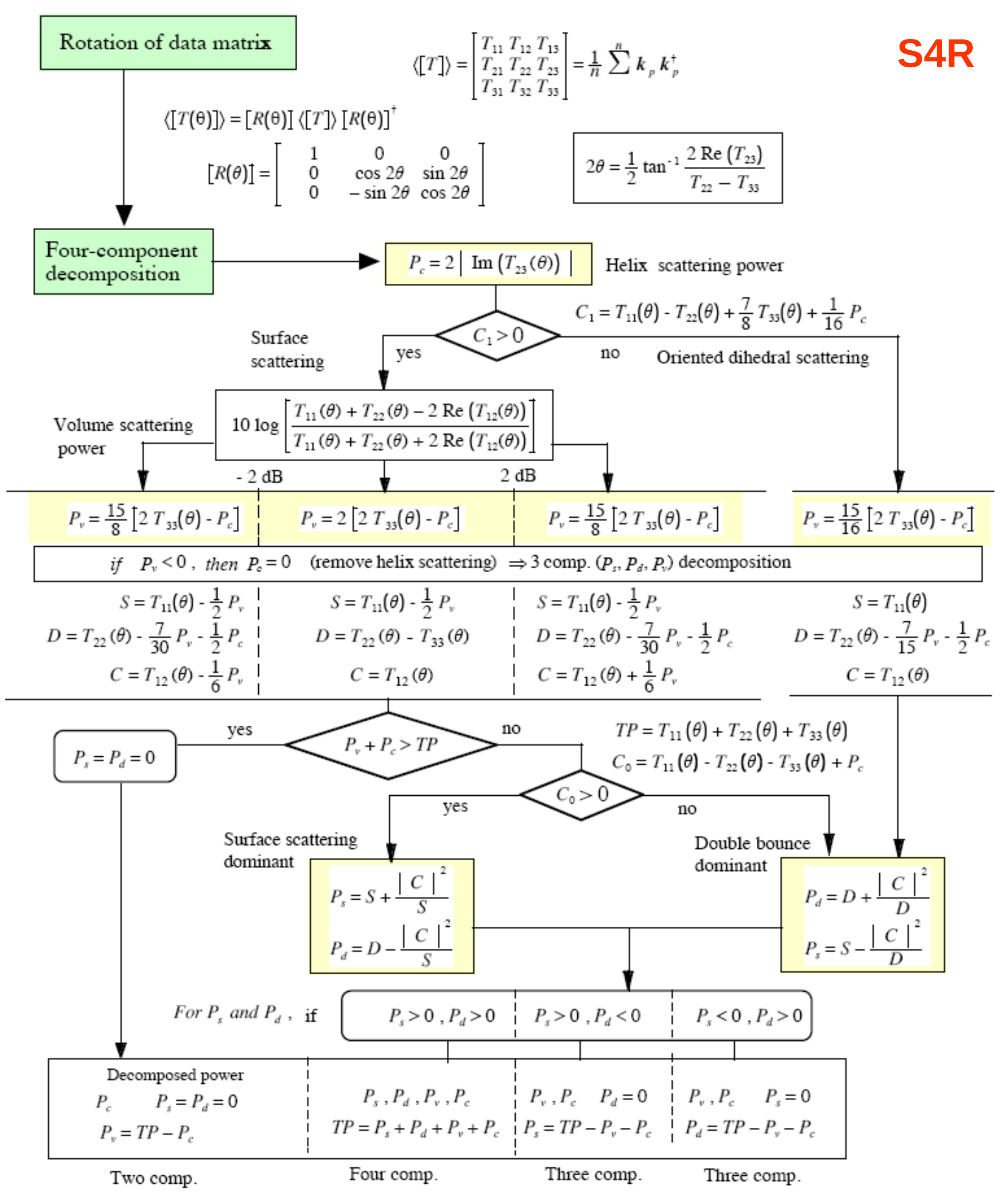
↓

$$\langle [T(\theta)] \rangle = [R(\theta)] \langle [T] \rangle [R(\theta)]^\dagger$$

$2A_0$ $B_0 + B$ $B_0 - B$

ODD *DBL* *VOL*

MODEL BASED DECOMPOSITION



$$\frac{1}{15}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

HV from oriented dihedral components

Extended volume scattering model

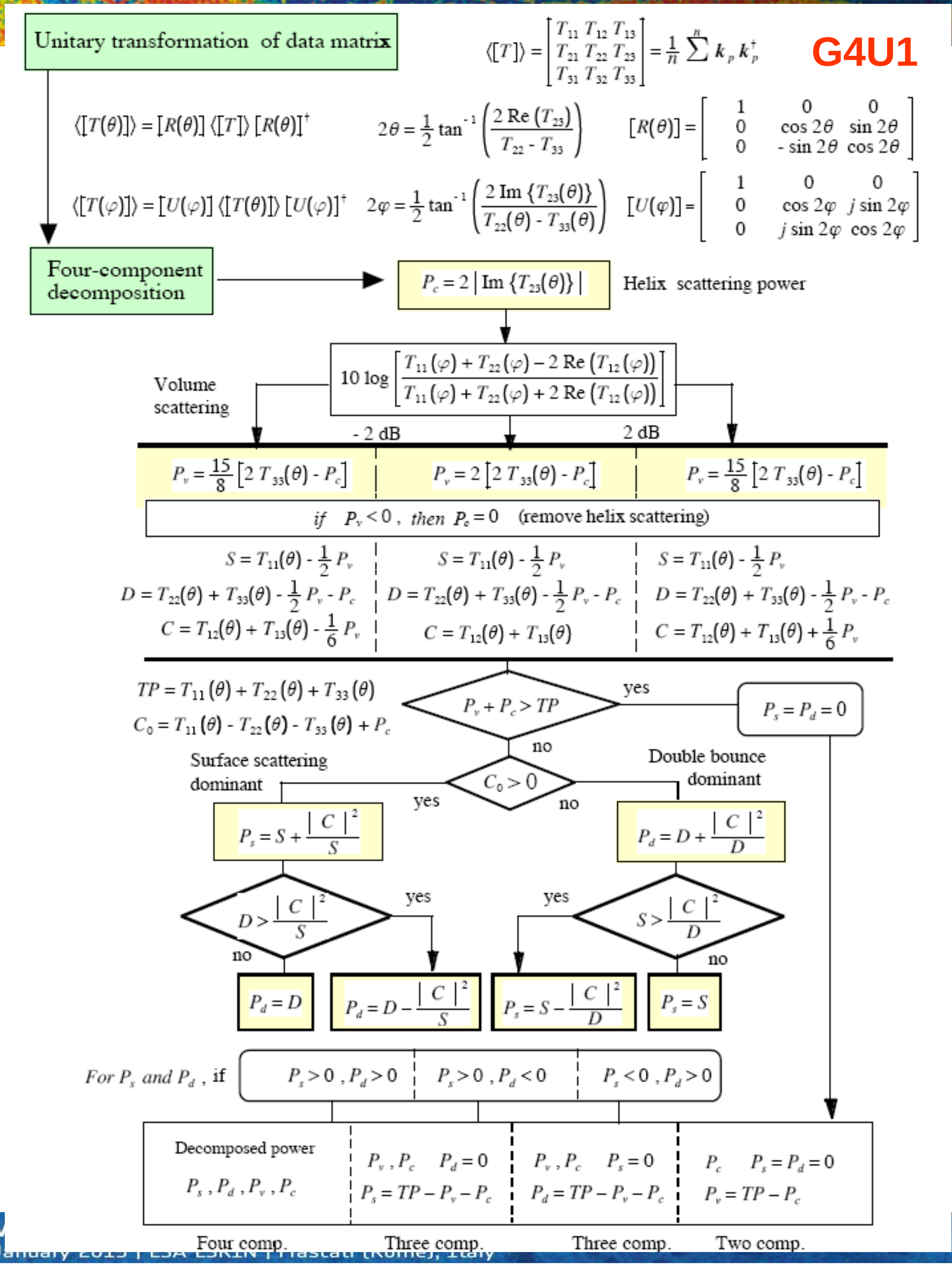
MODEL BASED DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

ODD *DBL* *VOL*

MODEL BASED DECOMPOSITION

esa



$$2\varphi = \frac{1}{2}\tan^{-1}\left(\frac{2\,\mathrm{Im}\{T_{23}(\theta)\}}{T_{22}(\theta) - T_{33}(\theta)}\right)$$

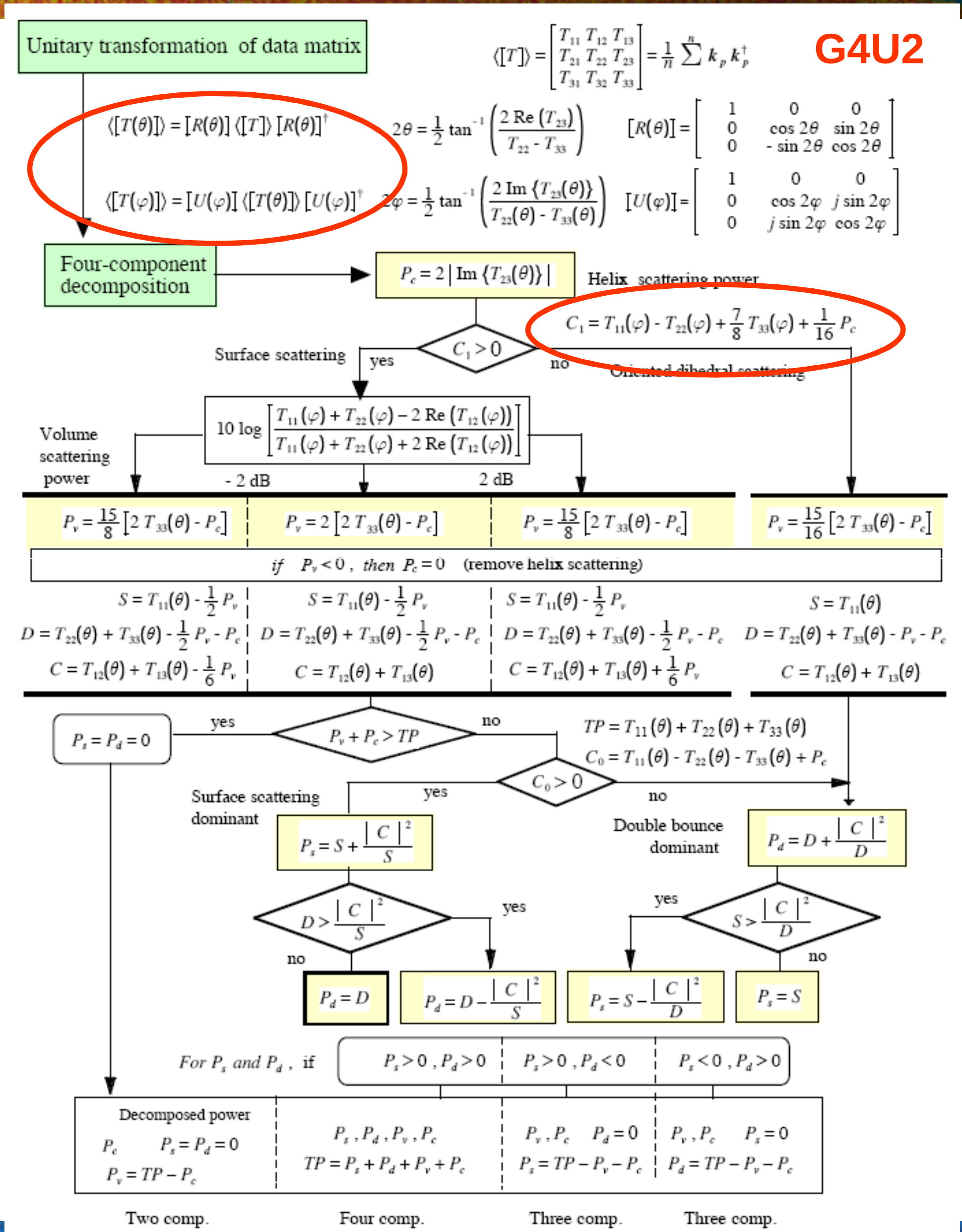
$$[U(\varphi)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\varphi & j\sin 2\varphi \\ 0 & j\sin 2\varphi & \cos 2\varphi \end{bmatrix}$$

$$\langle [T(\varphi)] \rangle = [U(\varphi)]\, \langle [T(\theta)] \rangle\, [U(\varphi)]^{\dagger}$$

$2A_0$ $B_0 + B$ $B_0 - B$

ODD *DBL* *VOL*

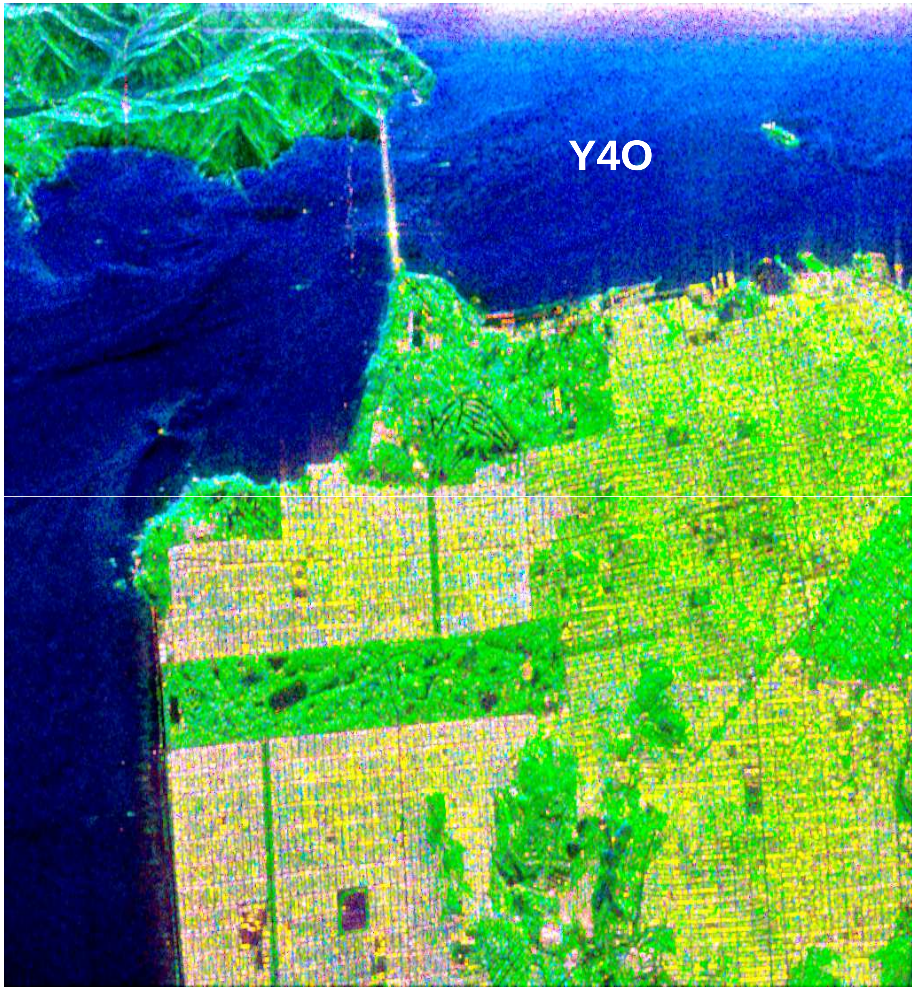
MODEL BASED DECOMPOSITION

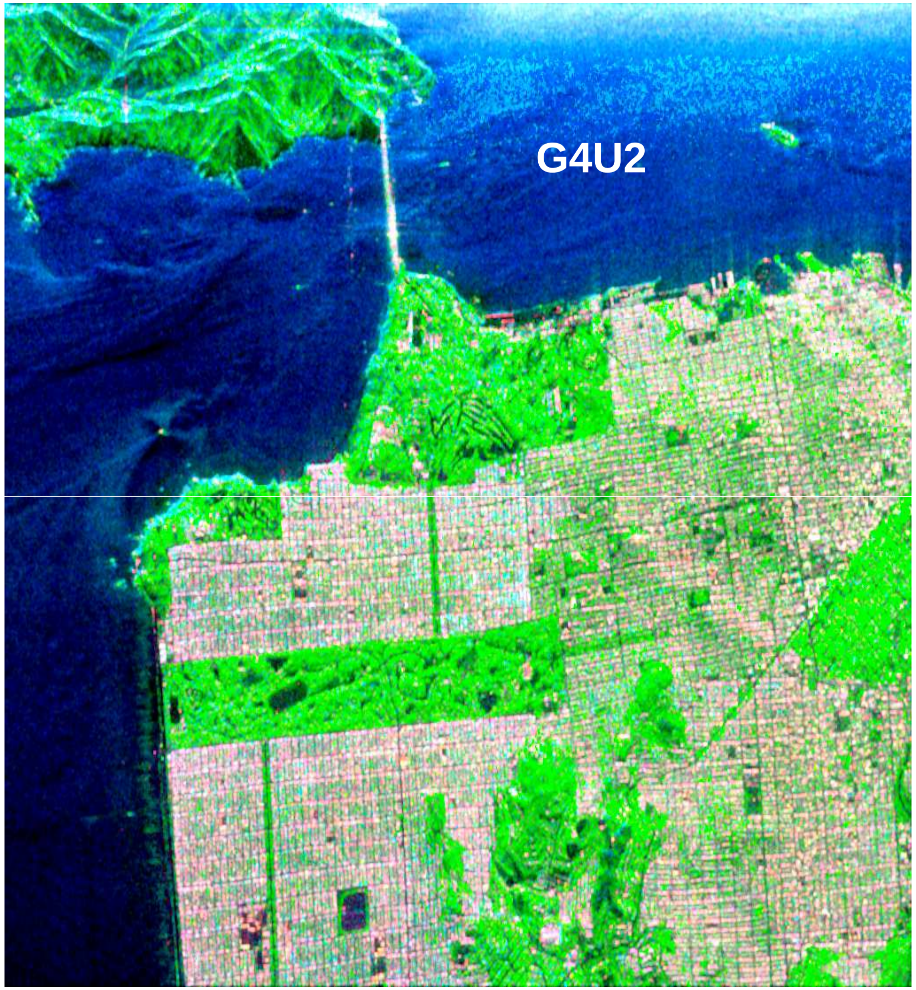


$2A_0$ $B_0 + B$ $B_0 - B$

ODD *DBL* *VOL*

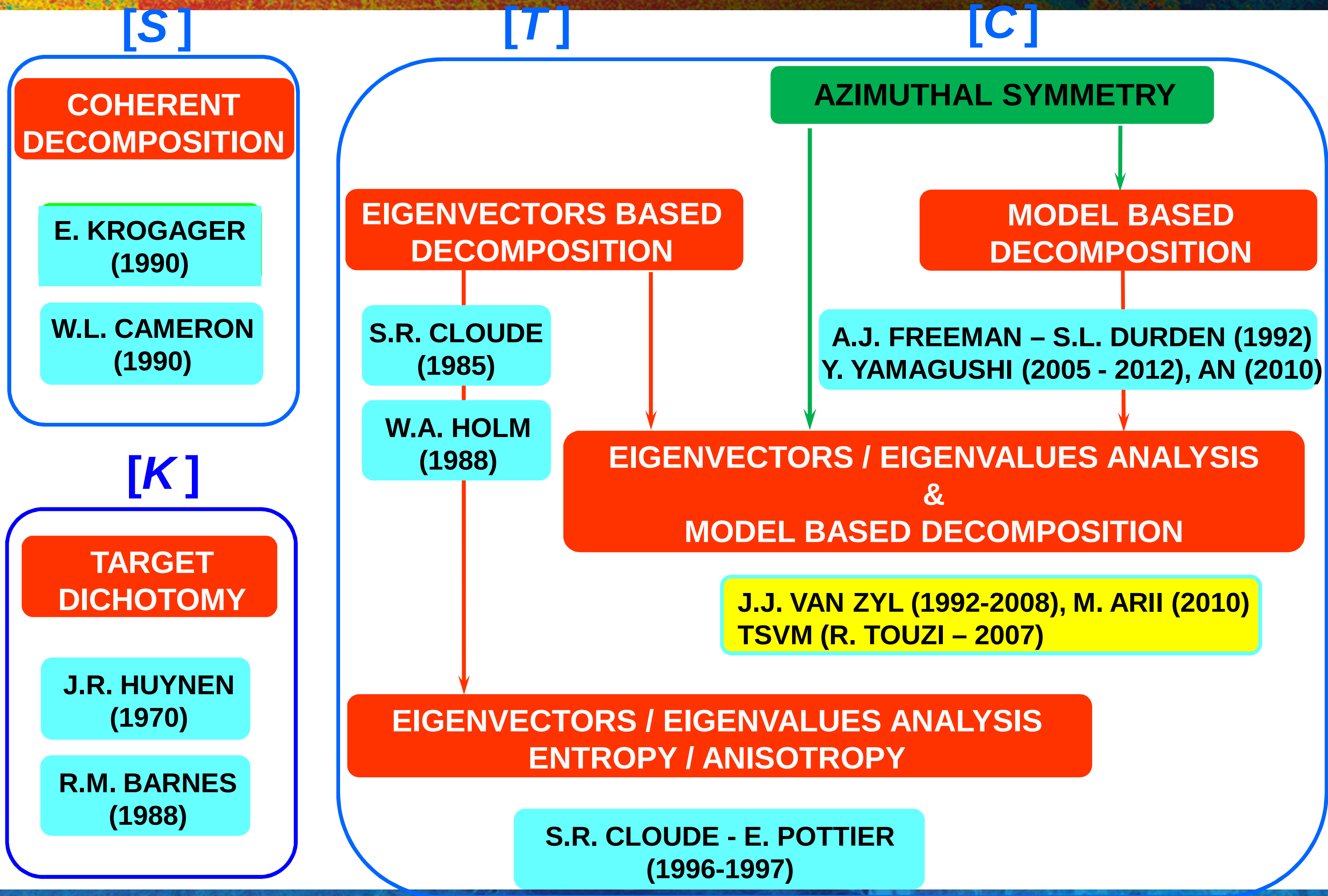
MODEL BASED DECOMPOSITION





ODD DBL VOL

TARGET DECOMPOSITIONS



TARGET DECOMPOSITION FOR TARGETS WITH REFLECTION SYMMETRY

EIGENVECTOR / MODEL BASED DECOMPOSITION

JACOB J. VAN ZYL (1992 - 2008)

van Zyl J. J., "Application of Cloude's target decomposition theorem to polarimetric imaging radar data," ***Radar Polarimetry*****, San Diego, CA, SPIE, vol. 1748, pp. 184–212, July 1992.**

MEDIUM WITH REFLECTION SYMMETRY

$$\langle [C] \rangle = \begin{bmatrix} \langle |S_{HH}|^2 \rangle & 0 & \langle S_{HH} S_{VV}^* \rangle \\ 0 & \langle 2|S_{HV}|^2 \rangle & 0 \\ \langle S_{VV} S_{HH}^* \rangle & 0 & \langle |S_{VV}|^2 \rangle \end{bmatrix} = \alpha \begin{bmatrix} 1 & 0 & \rho \\ 0 & \eta & 0 \\ \rho^* & 0 & \mu \end{bmatrix}$$

With:

$$\alpha = \langle S_{HH} S_{HH}^* \rangle \qquad \rho = \langle S_{HH} S_{VV}^* \rangle / \langle S_{HH} S_{HH}^* \rangle$$

$$\eta = 2 \langle S_{HV} S_{HV}^* \rangle / \langle S_{HH} S_{HH}^* \rangle \qquad \mu = \langle S_{VV} S_{VV}^* \rangle / \langle S_{HH} S_{HH}^* \rangle$$

EIGENVALUES / EIGENVECTORS DECOMPOSITION

$$\lambda_1 = \frac{\alpha}{2}\left\{1 + \mu + \sqrt{(1-\mu)^2 + 4|\rho|^2}\right\}$$

$$\lambda_2 = \frac{\alpha}{2}\left\{1 + \mu - \sqrt{(1-\mu)^2 + 4|\rho|^2}\right\}$$

$$\lambda_3 = \alpha\eta$$

EIGENVALUES / EIGENVECTORS DECOMPOSITION

$$\underline{u}_1 = \sqrt{\frac{\mu - 1 + \sqrt{\Delta}}{\left(\mu - 1 + \sqrt{\Delta}\right)^2 + 4|\rho|^2}} \begin{bmatrix} \dfrac{2\rho}{\mu - 1 + \sqrt{\Delta}} \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{u}_2 = \sqrt{\frac{\mu - 1 - \sqrt{\Delta}}{\left(\mu - 1 - \sqrt{\Delta}\right)^2 + 4|\rho|^2}} \begin{bmatrix} \dfrac{2\rho}{\mu - 1 - \sqrt{\Delta}} \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{u}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \text{with} : \Delta = (1 - \mu)^2 + 4|\rho|^2$$

MEDIUM WITH REFLECTION SYMMETRY

$$\langle [C] \rangle = \sum_{i=1}^{i=3} \lambda_i \underline{u}_i \cdot \underline{u}_i^{*T} = \Lambda_1 \begin{bmatrix} |\alpha|^2 & 0 & \alpha \\ 0 & 0 & 0 \\ \alpha^* & 0 & 1 \end{bmatrix} + \Lambda_2 \begin{bmatrix} |\beta|^2 & 0 & \beta \\ 0 & 0 & 0 \\ \beta^* & 0 & 1 \end{bmatrix} + \Lambda_3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

With:

$$\Lambda_1 = \lambda_1 \left[\frac{\left(\mu - 1 + \sqrt{\Delta}\right)^2}{\left(\mu - 1 + \sqrt{\Delta}\right)^2 + 4|\rho|^2} \right] \qquad \alpha = \frac{2\rho}{\mu - 1 + \sqrt{\Delta}}$$

$$\Lambda_2 = \lambda_2 \left[\frac{\left(\mu - 1 - \sqrt{\Delta}\right)^2}{\left(\mu - 1 - \sqrt{\Delta}\right)^2 + 4|\rho|^2} \right] \qquad \beta = \frac{2\rho}{\mu - 1 - \sqrt{\Delta}}$$

$$\Lambda_3 = \lambda_3$$

MEDIUM WITH REFLECTION SYMMETRY

$$\langle[C]\rangle = \sum_{i=1}^{i=3} \lambda_i \underline{u}_i \cdot \underline{u}_i^{*T} = [C_1] + [C_2] + [C_3]$$

↓ SINGLE SCATTERING

↓ DOUBLE SCATTERING

↓ VOLUME SCATTERING

↓

3 COMPONENTS SCATTERING MECHANISM MODEL

MODEL BASED DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$

ODD DBL VOL

ODD *DBL* *VOL*

Freeman decomposition **Van Zyl decomposition**

TARGET DECOMPOSITION FOR TARGETS WITH / WITHOUT REFLECTION SYMMETRY

REQUIEREMENTS FOR MODEL BASED POLARIMETRIC DECOMPOSITIONS

J.J. VAN ZYL – M. ARII – Y. KIM (2010)



J. J. Van Zyl, M. Arii, Y. Kim, "*Model-Based Decomposition of Polarimetric SAR Covariance Matrices Constrained for Nonnegative Eigenvalues*" IEEE TGRS, vol. 49, n°9, Sept. 2011.

3 COMPONENTS SCATTERING MECHANISM MODEL

$$\langle [C] \rangle = [C_S] + [C_D] + [C_V]$$

The algorithm uses the cross-polarized term to calculate the volume scattering contribution, and subtract that from the observed matrix.

$$[C_{remainder}] = \langle [C] \rangle - [C_V] = [C_S] + [C_D]$$

↓

$[C_{remainder}]$ **COULD BE NOT POSITIVE SEMI-DEFINITE HERMITIAN MATRIX** → $\lambda_i \leq 0$

MODEL BASED DECOMPOSITION

$$\left[C_{remainder}\right] = \langle [C] \rangle - a[C_V]$$

Subtract the volume contribution from a covariance matrix for terrain with reflection symmetry

$$\left[C_{remainder}\right] = \langle [C] \rangle - a[C_V] = \begin{bmatrix} \xi & 0 & \rho \\ 0 & \eta & 0 \\ \rho^* & 0 & \zeta \end{bmatrix} - a \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

The eigenvalues are:

$$\lambda_1 = \frac{1}{2}\left\{\xi + \zeta - 6a + \sqrt{(\xi + \zeta - 6a)^2 - 4(\xi - 3a)(\zeta - 3a) + 4|\rho - a|^2}\right\}$$

$$\lambda_2 = \frac{1}{2}\left\{\xi + \zeta - 6a - \sqrt{(\xi + \zeta - 6a)^2 - 4(\xi - 3a)(\zeta - 3a) + 4|\rho - a|^2}\right\}$$

$$\lambda_3 = \eta - 2a$$

MODEL BASED DECOMPOSITION

• Find those values of ***a*** that will ensure that all three eigenvalues are positive or zero. The solution is

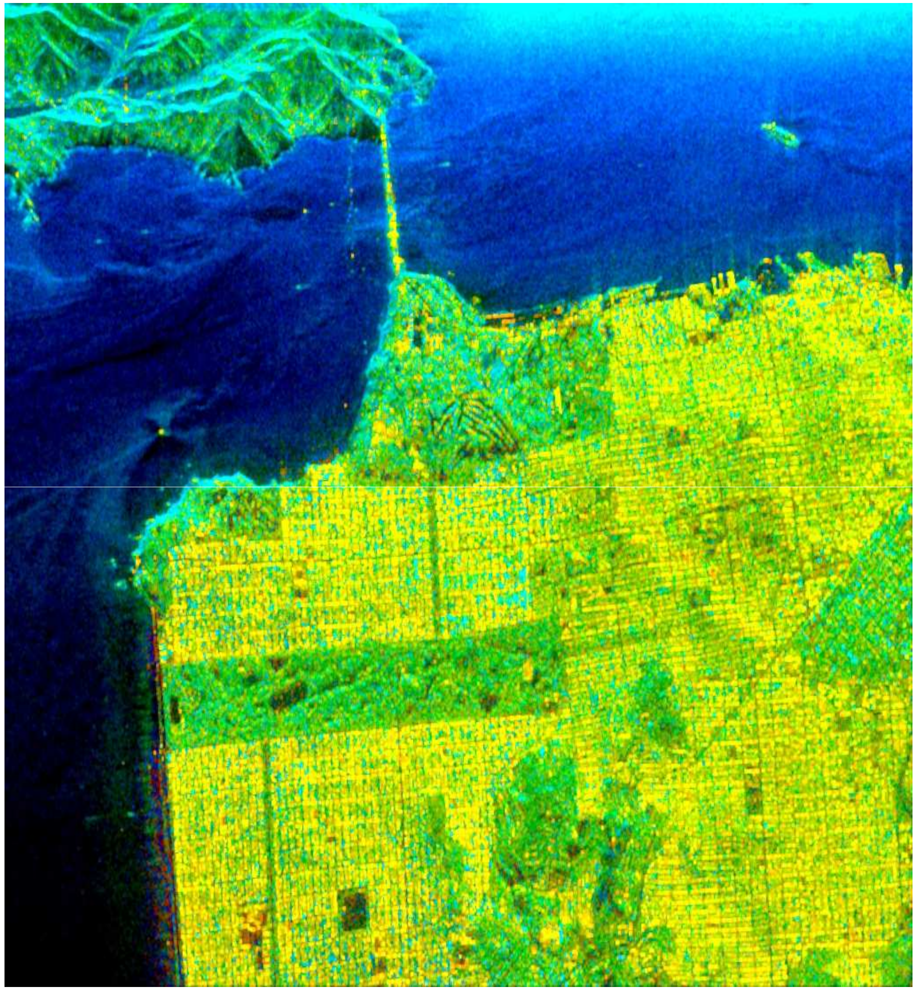
$$a_{\max} = \min \begin{cases} \eta/2 \\ \frac{1}{16}\left\{3(\xi+\zeta)-\rho-\rho^{*}-\sqrt{\left[3(\xi+\zeta)-\rho-\rho^{*}\right]^{2}-32\left(\xi\zeta-|\rho|^{2}\right)}\right\} \end{cases}$$

• Values of ***a*** larger than this, will leave either the second or third eigenvalue negative, resulting in a non-physical solution.

↓

$$[C_{remainder}] = \langle [C] \rangle - a_{\max}[C_V] = [C_S] + [C_D]$$

$2A_0$ $B_0 + B$ $B_0 - B$



ODD DBL VOL

ADAPTATIVE MODEL-BASED DECOMPOSITION

$$[C'_{\text{remainder}}] = \langle [C] \rangle - f_v \langle [C_{\text{vol}}(\theta_0, \sigma)] \rangle$$

$$\langle [C_{\text{vol}}(\theta_0, \sigma)] \rangle = [C_\alpha] + p(\sigma)[C_\beta] + q(\sigma)[C_\gamma].$$

$$[C_\alpha] = \frac{1}{8}\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

$$[C_\beta(2\theta_0)] = \frac{1}{8}\begin{bmatrix} -2\cos 2\theta_0 & \sqrt{2}\sin 2\theta_0 & 0 \\ \sqrt{2}\sin 2\theta_0 & 0 & \sqrt{2}\sin 2\theta_0 \\ 0 & \sqrt{2}\sin 2\theta_0 & 2\cos 2\theta_0 \end{bmatrix}$$

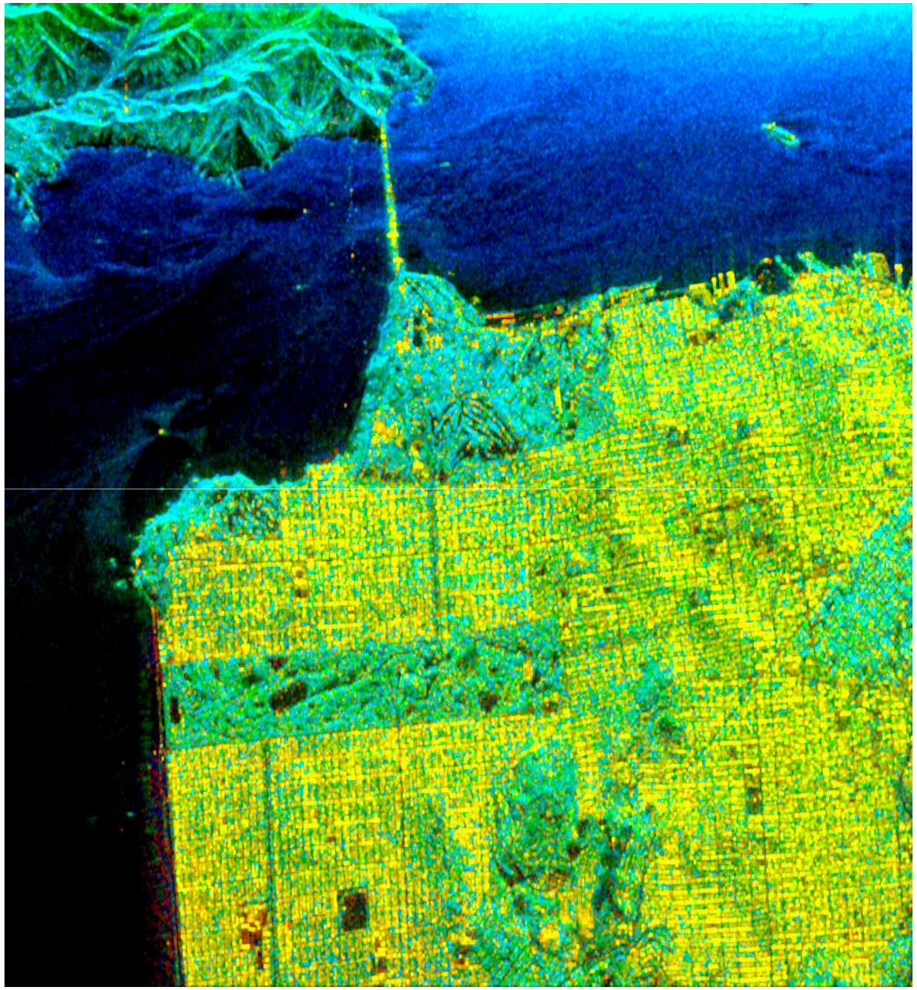
$$[C_\gamma(4\theta_0)] = \frac{1}{8}\begin{bmatrix} \cos 4\theta_0 & -\sqrt{2}\sin 4\theta_0 & -\cos 4\theta_0 \\ -\sqrt{2}\sin 4\theta_0 & -2\cos 4\theta_0 & \sqrt{2}\sin 4\theta_0 \\ -\cos 4\theta_0 & \sqrt{2}\sin 4\theta_0 & \cos 4\theta_0 \end{bmatrix}.$$

$$p(\sigma) = 2.0806\sigma^6 - 6.3350\sigma^5 + 6.3864\sigma^4 - 0.4431\sigma^3 - 3.9638\sigma^2 - 0.0008\sigma + 2.000$$

$$q(\sigma) = 9.0166\sigma^6 - 18.7790\sigma^5 + 4.9590\sigma^4 + 14.5629\sigma^3 - 10.8034\sigma^2 + 0.1902\sigma + 1.000.$$

M. Arii, J. J. Van Zyl, Y. Kim, "Adaptative *Model-Based Decomposition of Polarimetric SAR Covariance Matrices*" IEEE TGRS, vol. 49, n°9, Sept. 2011.

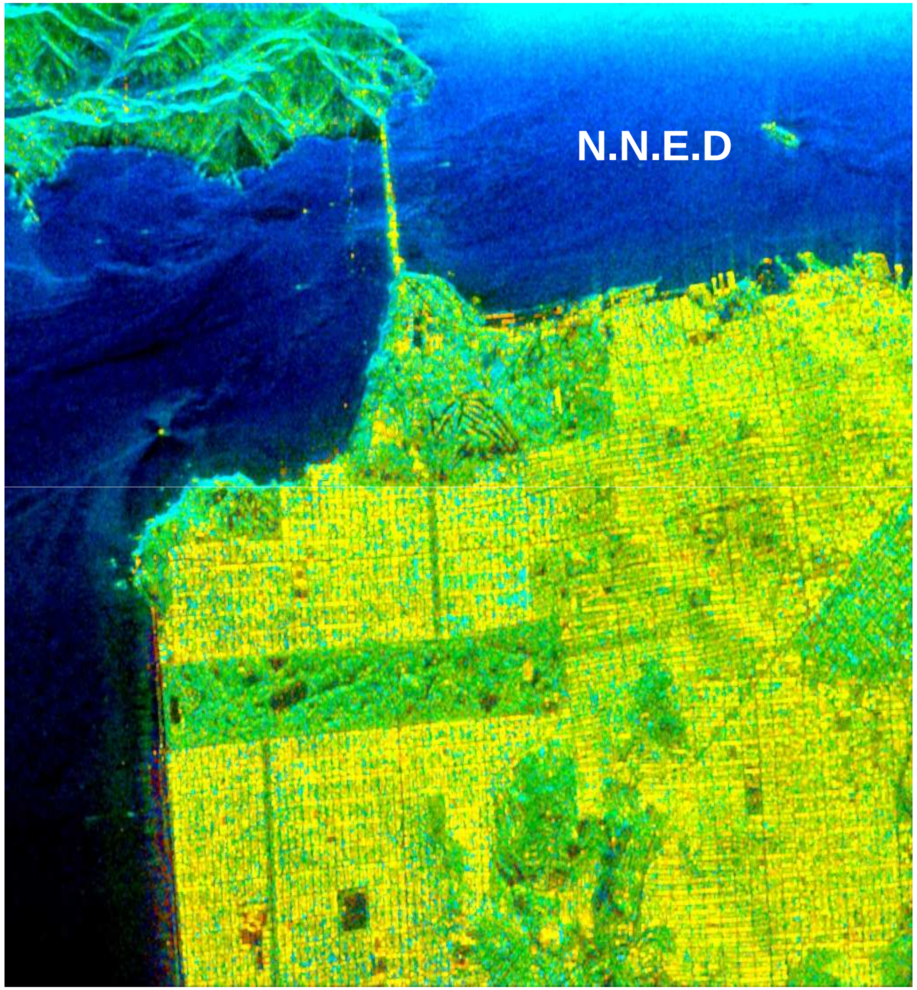
MODEL BASED DECOMPOSITION

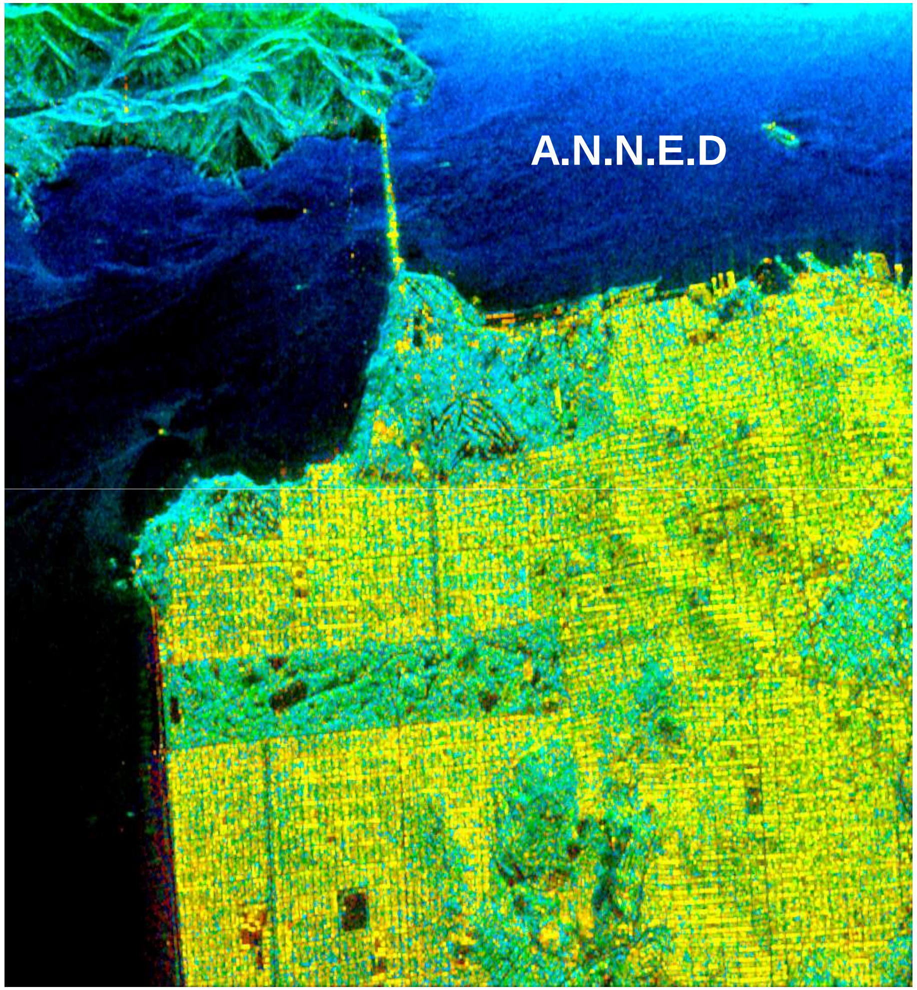


$2A_0$ $B_0 + B$ $B_0 - B$

ODD *DBL* *VOL*

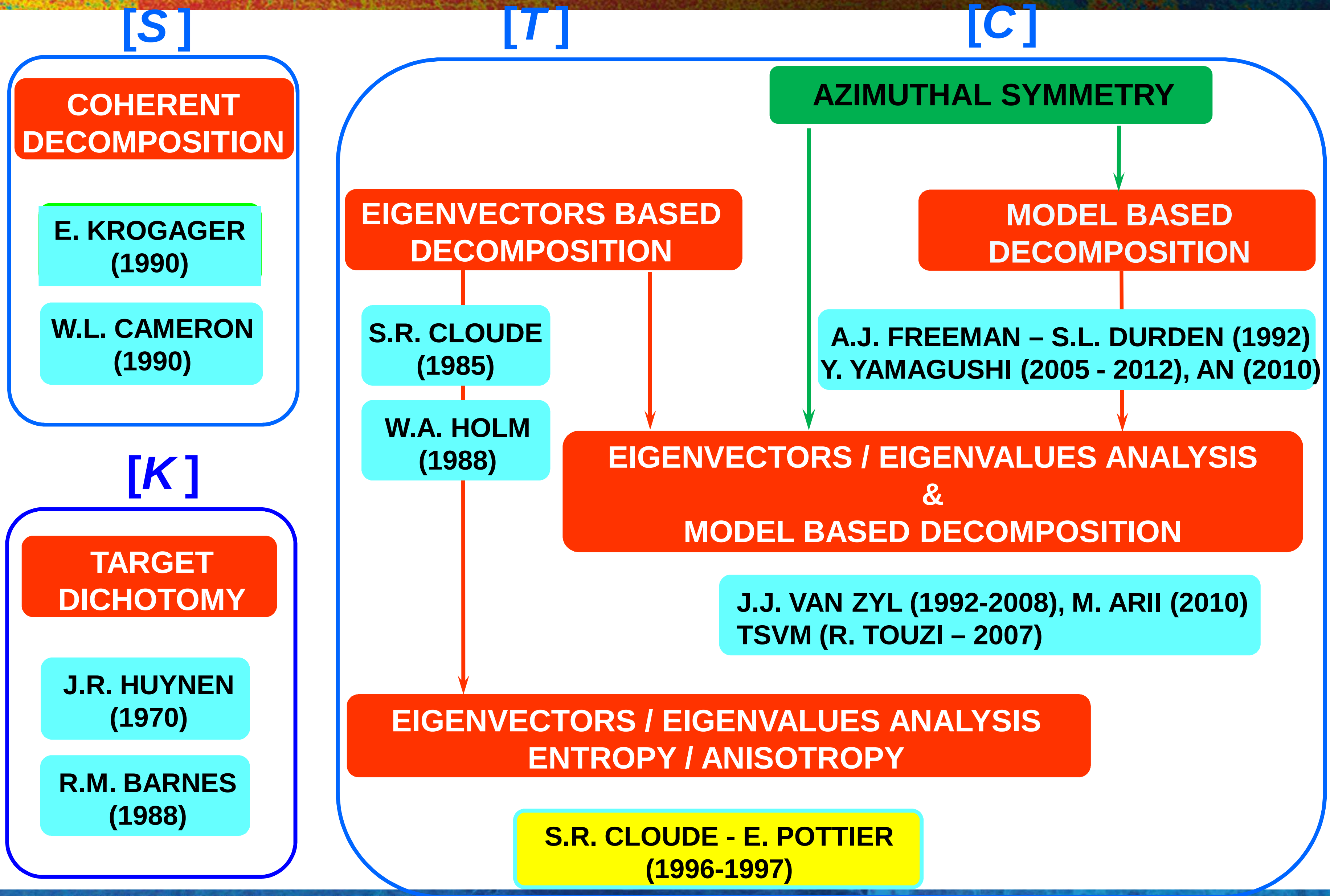
MODEL BASED DECOMPOSITION



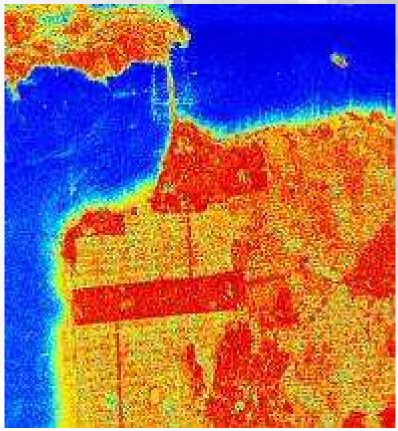


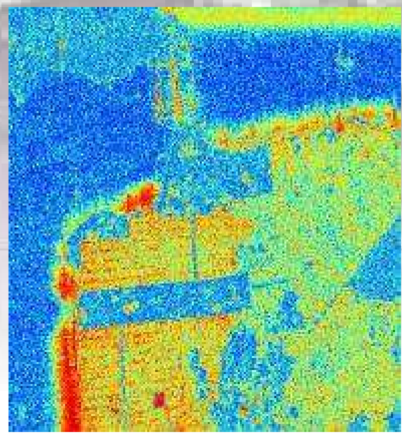
ODD *DBL* *VOL*

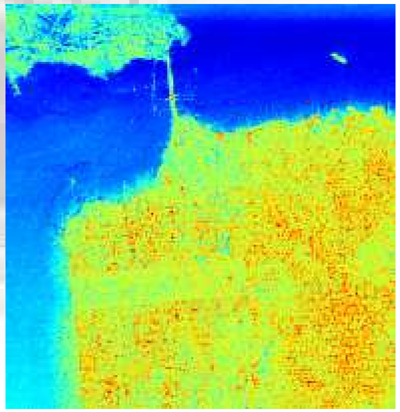
TARGET DECOMPOSITIONS



THE H / A / $\underline{\alpha}$ POLARIMETRIC TARGET DECOMPOSITION THEOREM







S.R. CLOUDE - E. POTTIER (1995 - 1996)

H / A / α DECOMPOSITION

TARGET VECTOR

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

LOCAL ESTIMATE OF THE COHERENCY MATRIX

$$\langle [T] \rangle = \frac{1}{N}\sum_{i=1}^{N} \underline{k}_i \cdot \underline{k}_i^{*T} = \frac{1}{N}\sum_{i=1}^{N} [T_i]$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES $\lambda_1 > \lambda_2 > \lambda_3$

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3} \lambda_k}$$

$$\langle[T]\rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES $\lambda_1 > \lambda_2 > \lambda_3$

PARAMETERISATION OF THE SU(3) UNITARY MATRIX

$$[U_3] = \begin{bmatrix} \cos\alpha_1 e^{j\phi_1} & \cos\alpha_2 e^{j\phi_2} & \cos\alpha_3 e^{j\phi_3} \\ \sin\alpha_1 \cos\beta_1 e^{j\phi_1} e^{j\delta_1} & \sin\alpha_2 \cos\beta_2 e^{j\phi_2} e^{j\delta_2} & \sin\alpha_3 \cos\beta_3 e^{j\phi_3} e^{j\delta_3} \\ \sin\alpha_1 \sin\beta_1 e^{j\phi_1} e^{j\gamma_1} & \sin\alpha_2 \sin\beta_2 e^{j\phi_2} e^{j\gamma_2} & \sin\alpha_3 \sin\beta_3 e^{j\phi_3} e^{j\gamma_3} \end{bmatrix}$$

TARGET 1 | TARGET 2 | TARGET 3

PROBABILITIES

$$P_i = \frac{\lambda_i}{\sum_{k=1}^{3} \lambda_k}$$

AVERAGED PARAMETERS

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3 \qquad \underline{\beta} = P_1\beta_1 + P_2\beta_2 + P_3\beta_3$$

$$\underline{\gamma} = P_1\gamma_1 + P_2\gamma_2 + P_3\gamma_3 \qquad \underline{\delta} = P_1\delta_1 + P_2\delta_2 + P_3\delta_3$$

UNITARY TARGET VECTOR ($\underline{u}_0$) OF THE MEAN DOMINANT MECHANISM

$$\underline{u}_0 = \left[\cos(\underline{\alpha}) \quad \sin(\underline{\alpha})\cos(\underline{\beta})\, e^{j\underline{\delta}} \quad \sin(\underline{\alpha})\sin(\underline{\beta})\, e^{j\underline{\gamma}}\right]^T$$

MEAN SCATTERING MECHANISM

UNITARY VECTOR $\underline{u}_0$

$$\underline{u}_0 = \begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha})\cos(\underline{\beta})e^{j\underline{\delta}} \\ \sin(\underline{\alpha})\sin(\underline{\beta})e^{j\underline{\gamma}} \end{bmatrix}$$

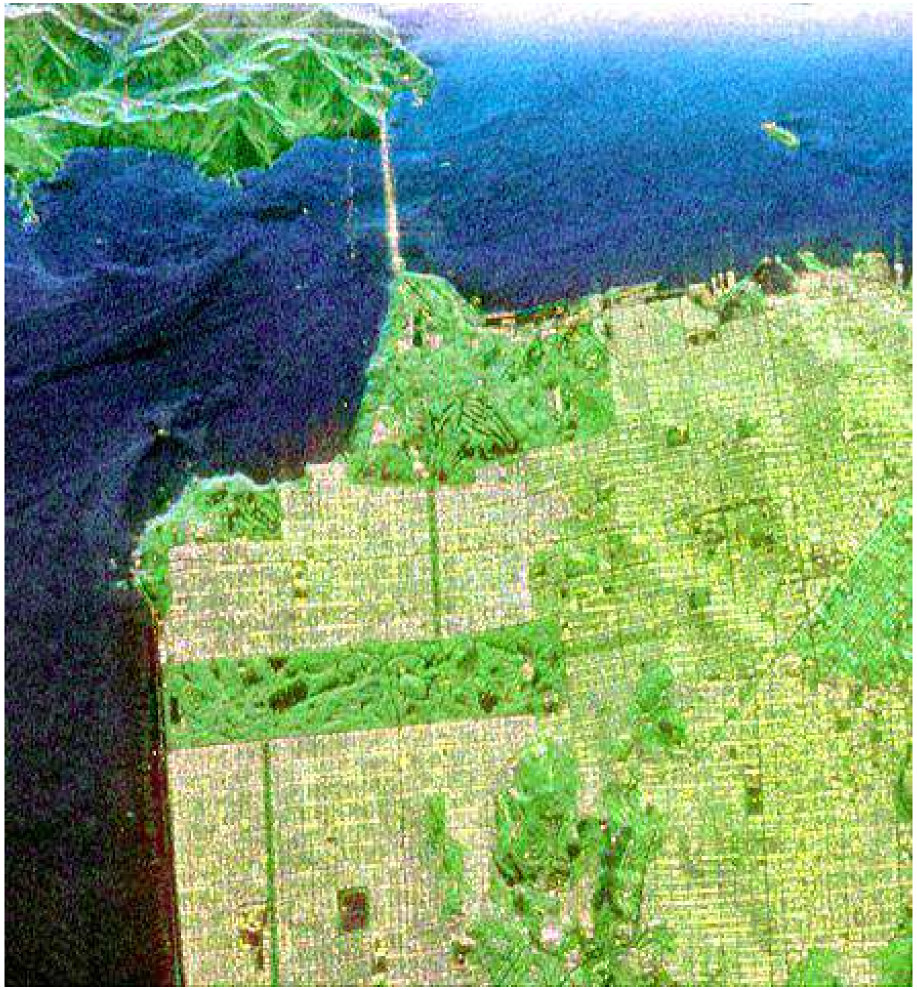
TARGET MAGNITUDE

$$\underline{\lambda} = P_1\lambda_1 + P_2\lambda_2 + P_3\lambda_3 = \frac{\sum_{i=1}^{3}\lambda_i^2}{\sum_{k=1}^{3}\lambda_k}$$

TARGET VECTOR $\underline{k}_0$

$$\underline{k}_0 = \sqrt{\underline{\lambda}}\begin{bmatrix} \cos(\underline{\alpha}) \\ \sin(\underline{\alpha})\cos(\underline{\beta})e^{j\underline{\delta}} \\ \sin(\underline{\alpha})\sin(\underline{\beta})e^{j\underline{\gamma}} \end{bmatrix}$$

$H / A / \underline{\alpha}$ DECOMPOSITION



$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

$\sqrt{\underline{\lambda}}\cos(\underline{\alpha})$ $\quad \sqrt{\underline{\lambda}}\sin(\underline{\alpha})\cos(\underline{\beta})$ $\quad \sqrt{\underline{\lambda}}\sin(\underline{\alpha})\sin(\underline{\beta})$

ROLL INVARIANCE PROPERTY

SAME PHYSICAL PHENOMENOUS **WHATEVER THE ANTENNA** *ORIENTATION ANGLE* **AROUND THE** *RADAR LINE OF SIGHT*

ORIENTED (θ) COHERENCY MATRIX

$$\langle [T(\theta)] \rangle = [U_R(\theta)] \langle [T] \rangle [U_R(\theta)]^{-1}$$

SU(3) UNITARY ROTATION MATRIX (θ)

$$[U_R(\theta)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

EIGENVECTORS / EIGENVALUES ANALYSIS

$$\langle [T(\theta)] \rangle = [U_3(\theta)] [\Sigma] [U_3(\theta)]^{-1}$$

EIGENVALUES $\lambda_1 \ \lambda_2 \ \lambda_3$: ROLL INVARIANT

PROBABILITIES $P_1 \ P_2 \ P_3$: ROLL INVARIANT

EIGENVECTORS UNITARY MATRIX

$$[U_3(\theta)] = [U_R(\theta)][U_3]$$

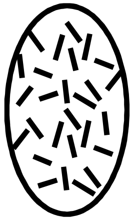
PARAMETERIZATION OF THE UNITARY MATRIX

$$[U_3] = \begin{bmatrix} \cos\alpha_1 e^{j\phi'_1} & \cos\alpha_2 e^{j\phi'_2} & \cos\alpha_3 e^{j\phi'_3} \\ \sin\alpha_1 \cos\beta'_1 e^{j\phi'_1} e^{j\delta'_1} & \sin\alpha_2 \cos\beta'_2 e^{j\phi'_2} e^{j\delta'_2} & \sin\alpha_3 \cos\beta'_3 e^{j\phi'_3} e^{j\delta'_3} \\ \sin\alpha_1 \sin\beta'_1 e^{j\phi'_1} e^{j\gamma'_1} & \sin\alpha_2 \sin\beta'_2 e^{j\phi'_2} e^{j\gamma'_2} & \sin\alpha_3 \sin\beta'_3 e^{j\phi'_3} e^{j\gamma'_3} \end{bmatrix}$$

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3$$: ROLL INVARIANT

PHYSICAL INTERPRETATION

ANISOTROPIC PARTICLES CLOUD



$$[S]=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \Rightarrow [T]=\begin{bmatrix} \varepsilon & \mu & 0 \\ \mu^* & \nu & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\langle [T(\theta)] \rangle_\theta = \frac{1}{2}\begin{bmatrix} 2\varepsilon & 0 & 0 \\ 0 & \nu & 0 \\ 0 & 0 & \nu \end{bmatrix}$$

AVERAGING OVER ALL ORIENTATION ANGLES

WITH : $P(\theta)=\frac{1}{2\pi}$

$$\lambda_1=\varepsilon \Rightarrow P_1=\frac{\varepsilon}{(\varepsilon+\nu)}$$

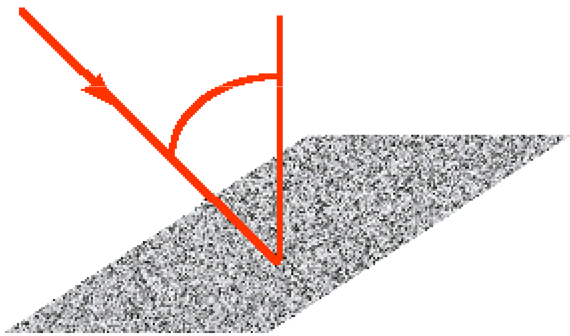
$$\lambda_2=\lambda_3=\frac{\nu}{2} \Rightarrow P_2=P_3=\frac{\nu}{2(\varepsilon+\nu)}$$

$$[U_3]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{matrix} \alpha_1=0 \\ \alpha_2=\alpha_3=\frac{\pi}{2} \end{matrix}$$

$$\underline{\alpha} = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 = \frac{\pi}{2}(P_2+P_3)$$

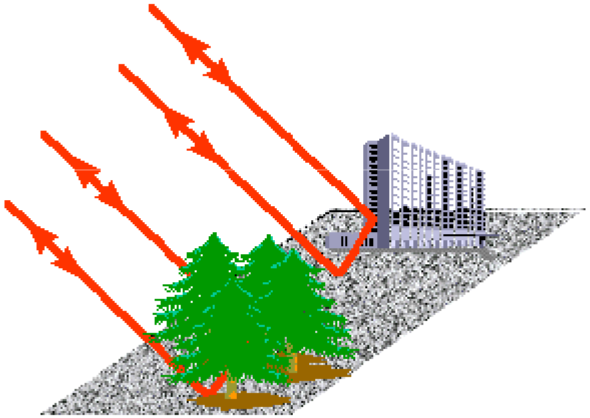
$\underline{\alpha}$ PHYSICAL INTERPRETATION

SINGLE BOUNCE SCATTERING
(ROUGH SURFACE)



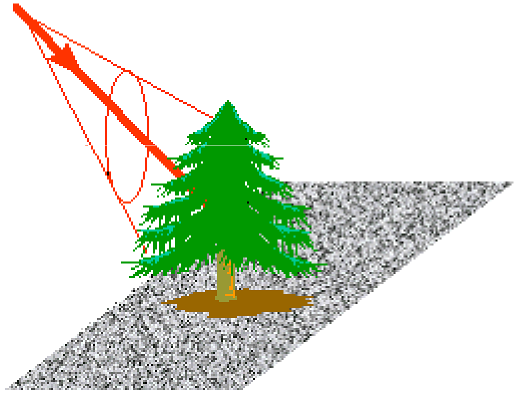
$$a \mapsto b \Rightarrow \nu \mapsto 0$$
$$\Downarrow$$
$$\underline{\alpha} \mapsto 0$$

DOUBLE BOUNCE SCATTERING



$$a \mapsto -b \Rightarrow \varepsilon \mapsto 0$$
$$\Downarrow$$
$$\underline{\alpha} \mapsto \frac{\pi}{2}$$

VOLUME SCATTERING



$$a >> b \Rightarrow \varepsilon \approx \nu$$
$$\Downarrow$$
$$\underline{\alpha} \mapsto \frac{\pi}{4}$$

H / A / α DECOMPOSITION

EIGENVALUES λ_1 λ_2 λ_3 : ROLL INVARIANT

PROBABILITIES P_1 P_2 P_3 : ROLL INVARIANT

↓

ENTROPY

(DEGREE OF RANDOMNESS
STATISTICAL DISORDER)

$$H = -\sum_{i=1}^{3} P_i \log_3(P_i)$$

↙ ↘

PURE TARGET

$\lambda_1 = SPAN \quad \lambda_2 = 0 \quad \lambda_3 = 0$

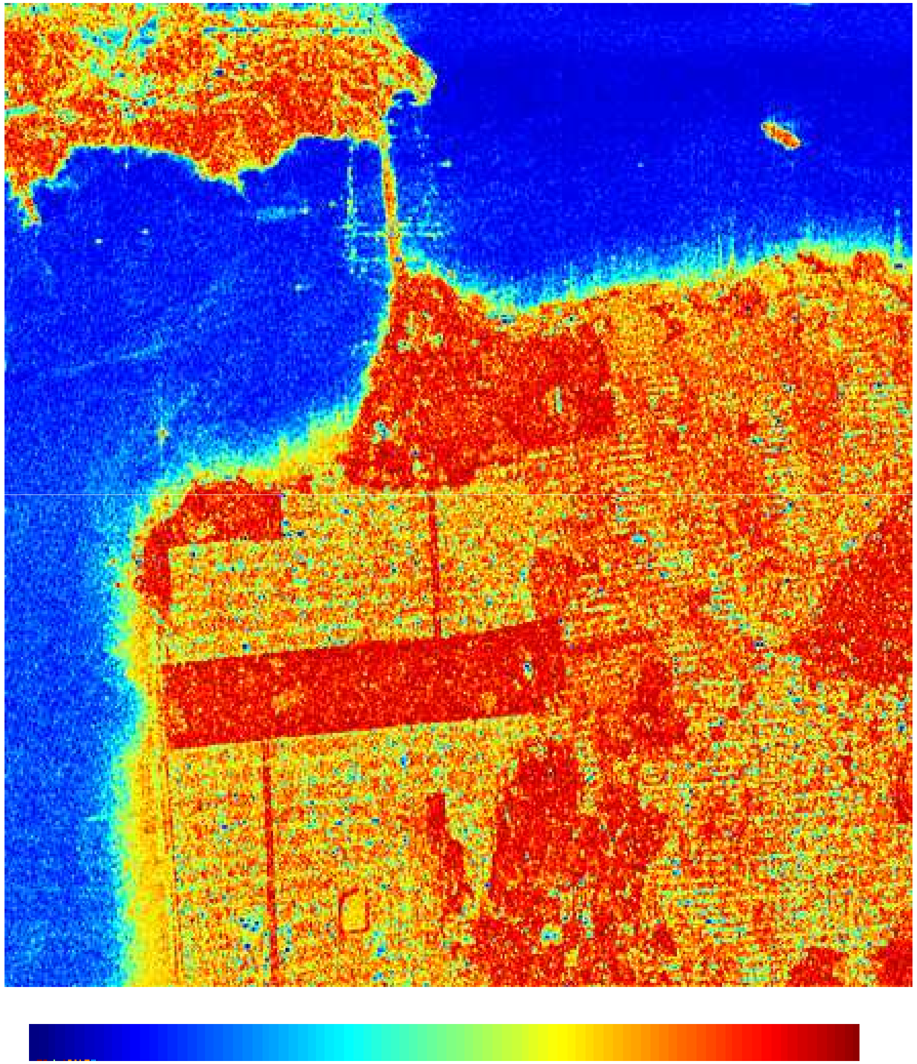
$H = 0$

DISTRIBUTED TARGET

$\lambda_1 = \lambda_2 = \lambda_3 = SPAN / 3$

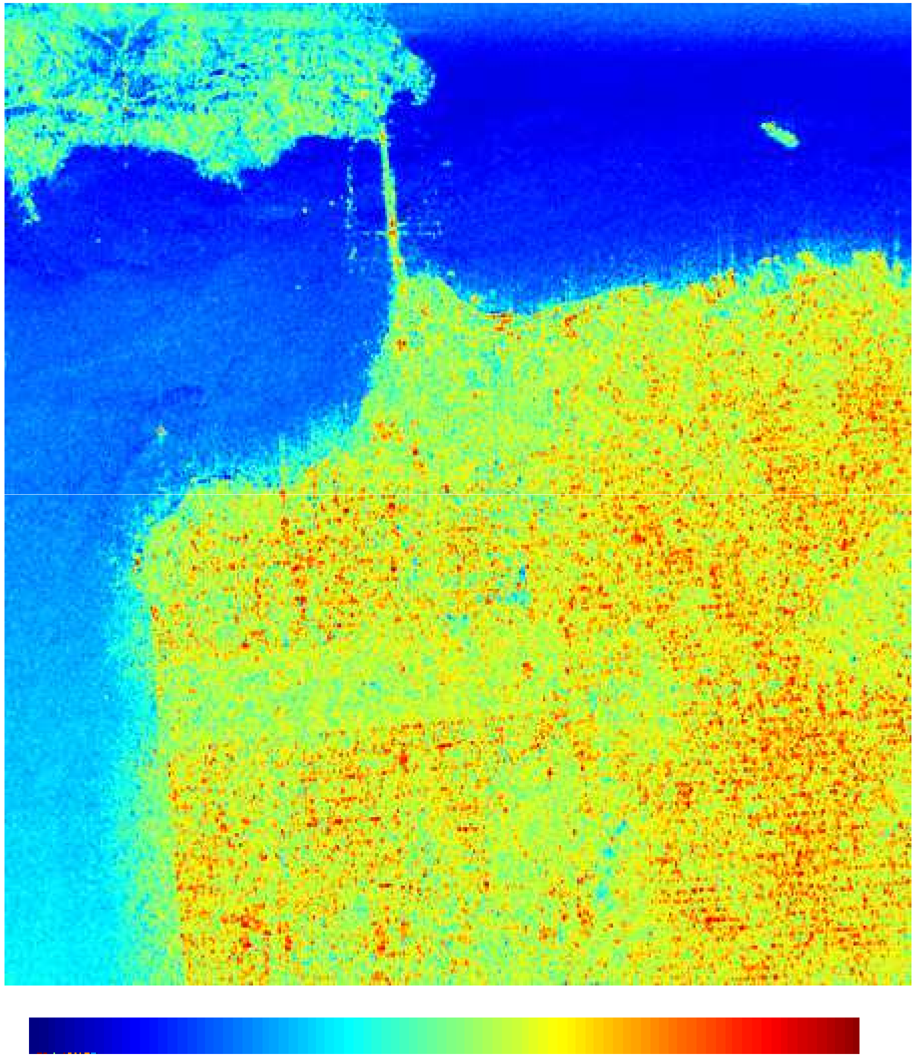
$H = 1$

H / A / $\underline{\alpha}$ DECOMPOSITION



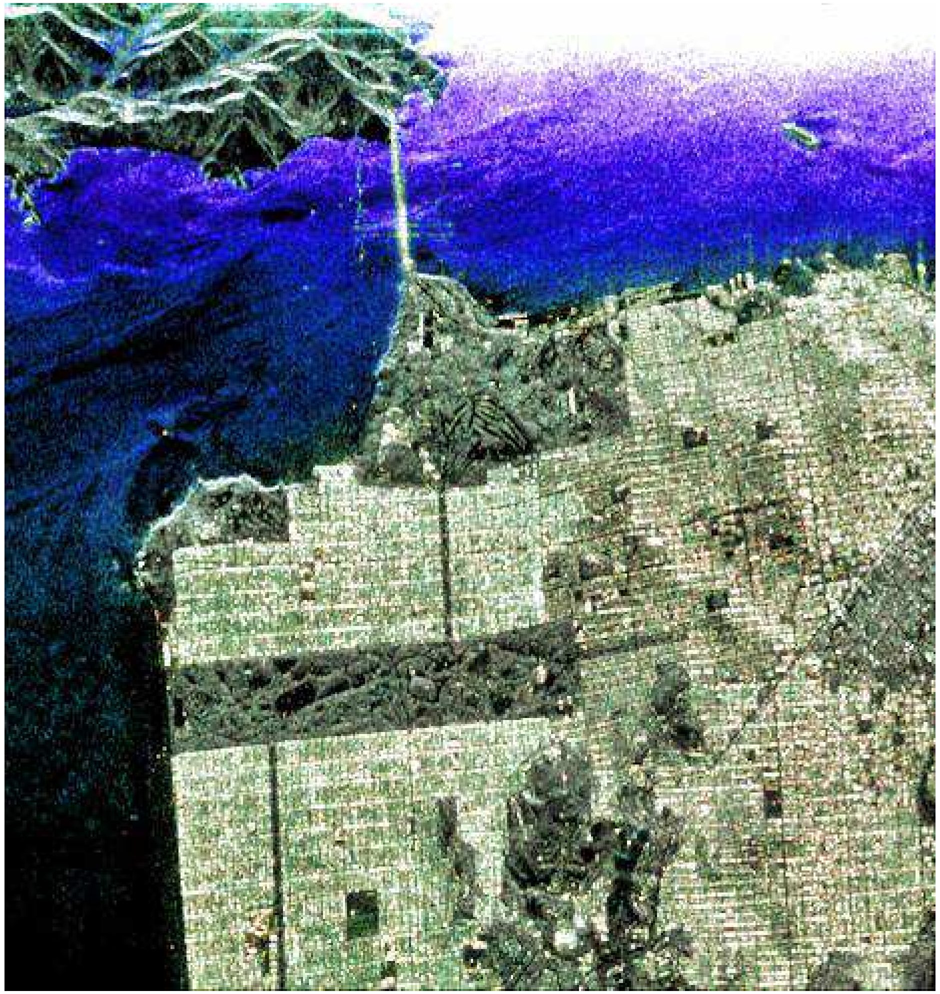
H / A / $\underline{\alpha}$ DECOMPOSITION

$2A_0$ $B_0 + B$ $B_0 - B$



α PARAMETER

HSL Representation



$2A_0$ $B_0 + B$ $B_0 - B$ $\underline{\lambda}$ **Intensity, (1-H) Saturation,** $\underline{\alpha}$ **Intensity**

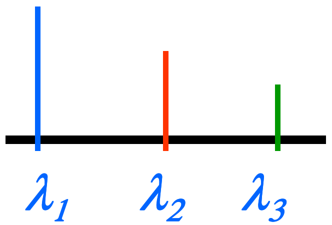
H / A / $\underline{\alpha}$ DECOMPOSITION

DIFFICULT MECHANISM DISCRIMINATION WHEN : H > 0.7

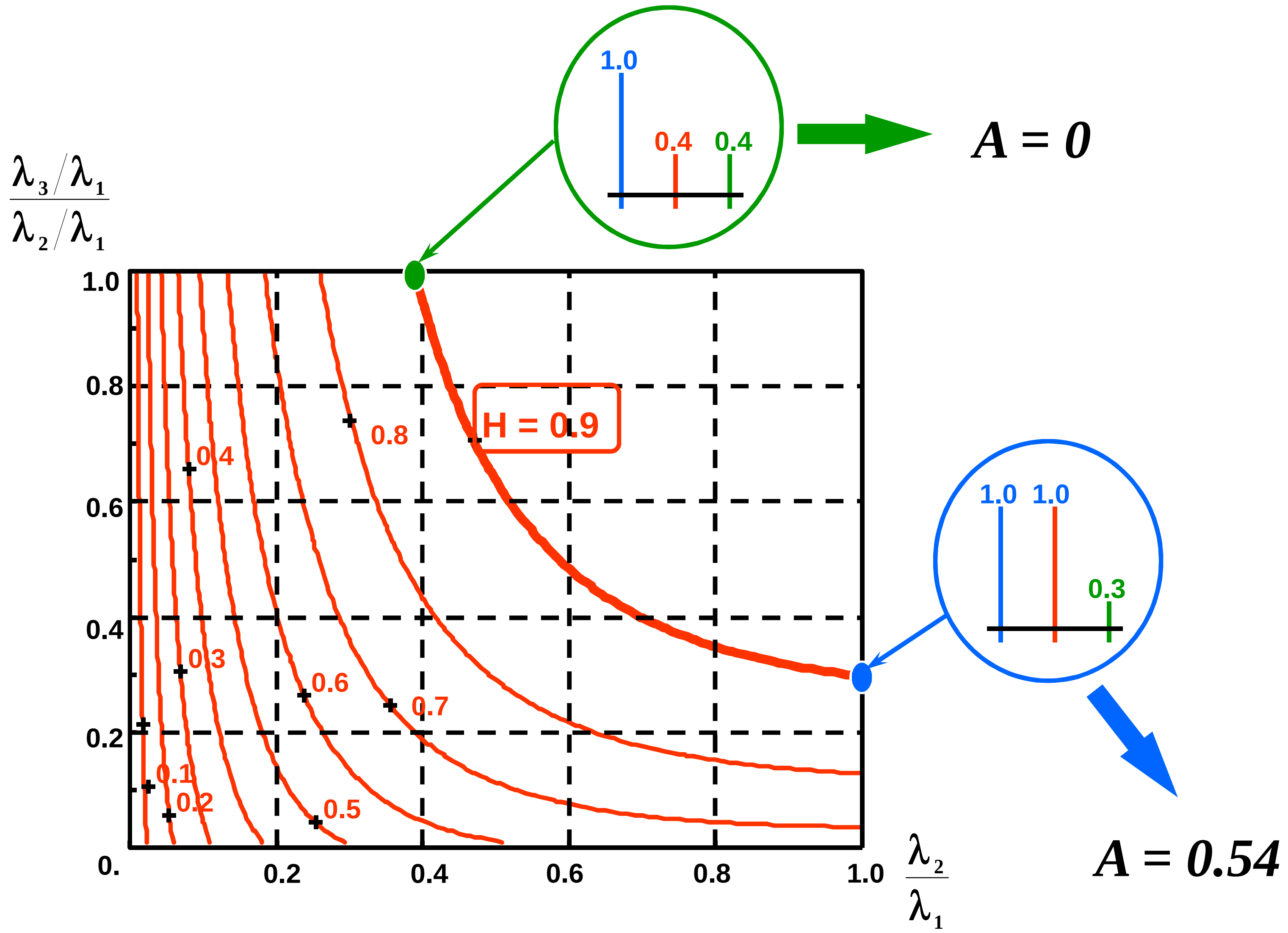
ANISOTROPY

(EIGENVALUES SPECTRUM)

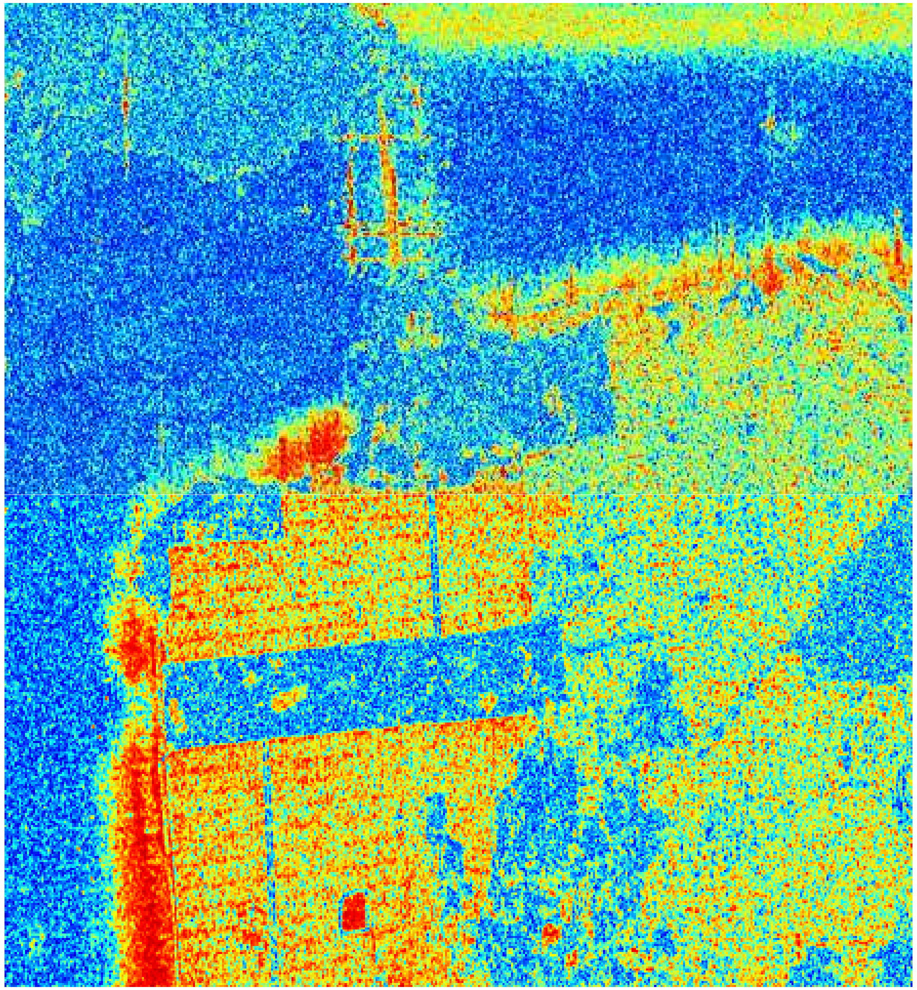
$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$



- COMPLEMENTARY TO ENTROPY
- DISCRIMINATION WHEN H > 0.7
- ROLL INVARIANT

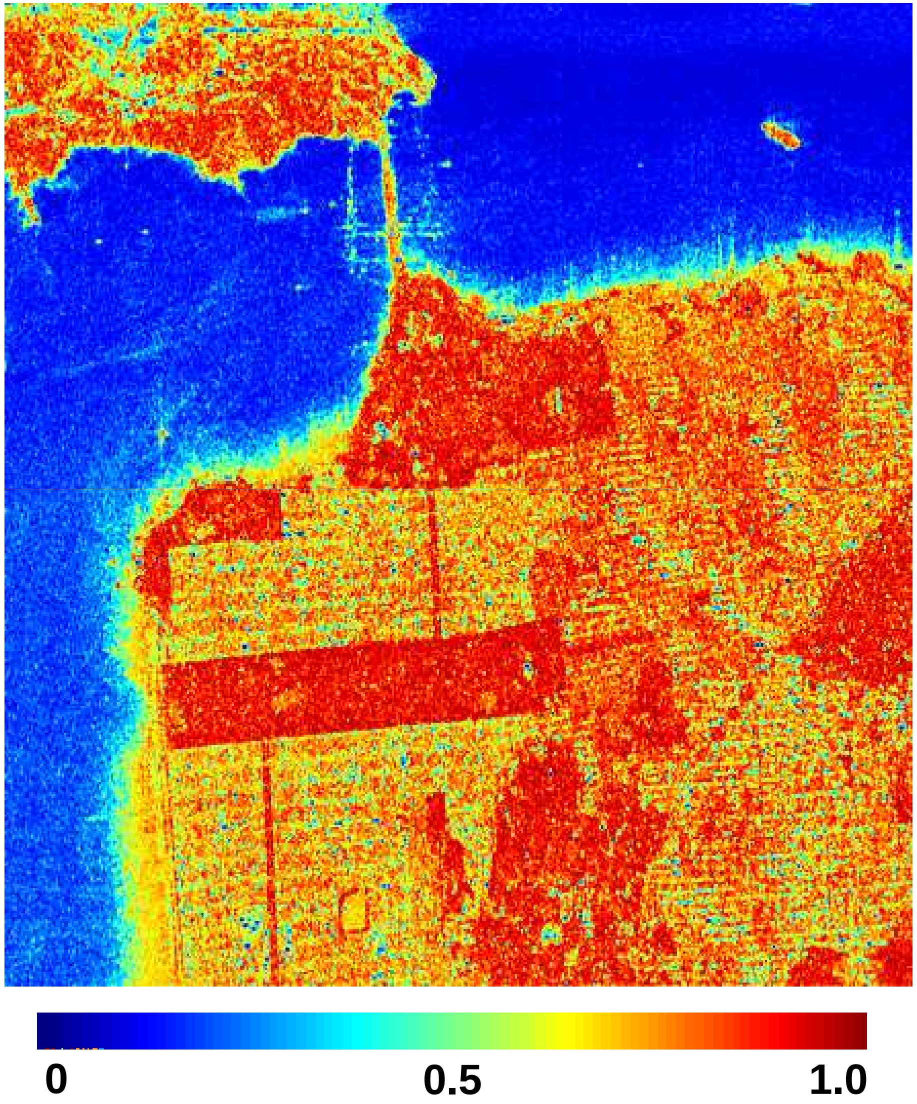


$\frac{\lambda_3/\lambda_1}{\lambda_2/\lambda_1}$
1.0
0.8
0.6
0.4
0.2
0.
0.2
0.4
0.6
0.8
1.0
$\frac{\lambda_2}{\lambda_1}$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
H = 0.9
1.0
0.4
0.4
A = 0
1.0
1.0
0.3
A = 0.54

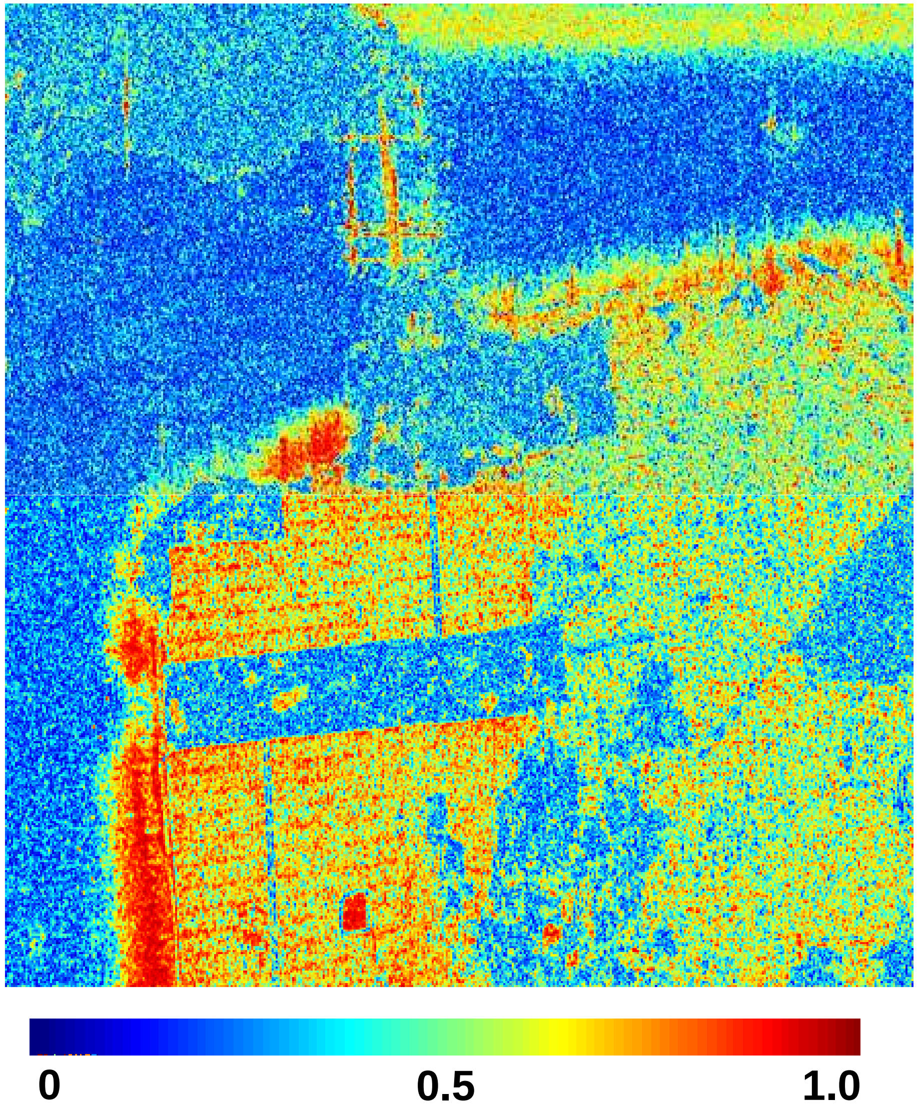


$2A_0$ $B_0 + B$ $B_0 - B$

H / A / α DECOMPOSITION



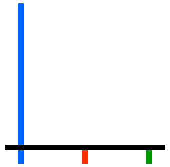
ENTROPY (H)



ANISOTROPY (A)

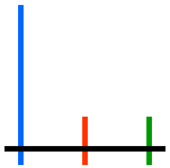
H / A / α DECOMPOSITION

(1-H)(1-A)

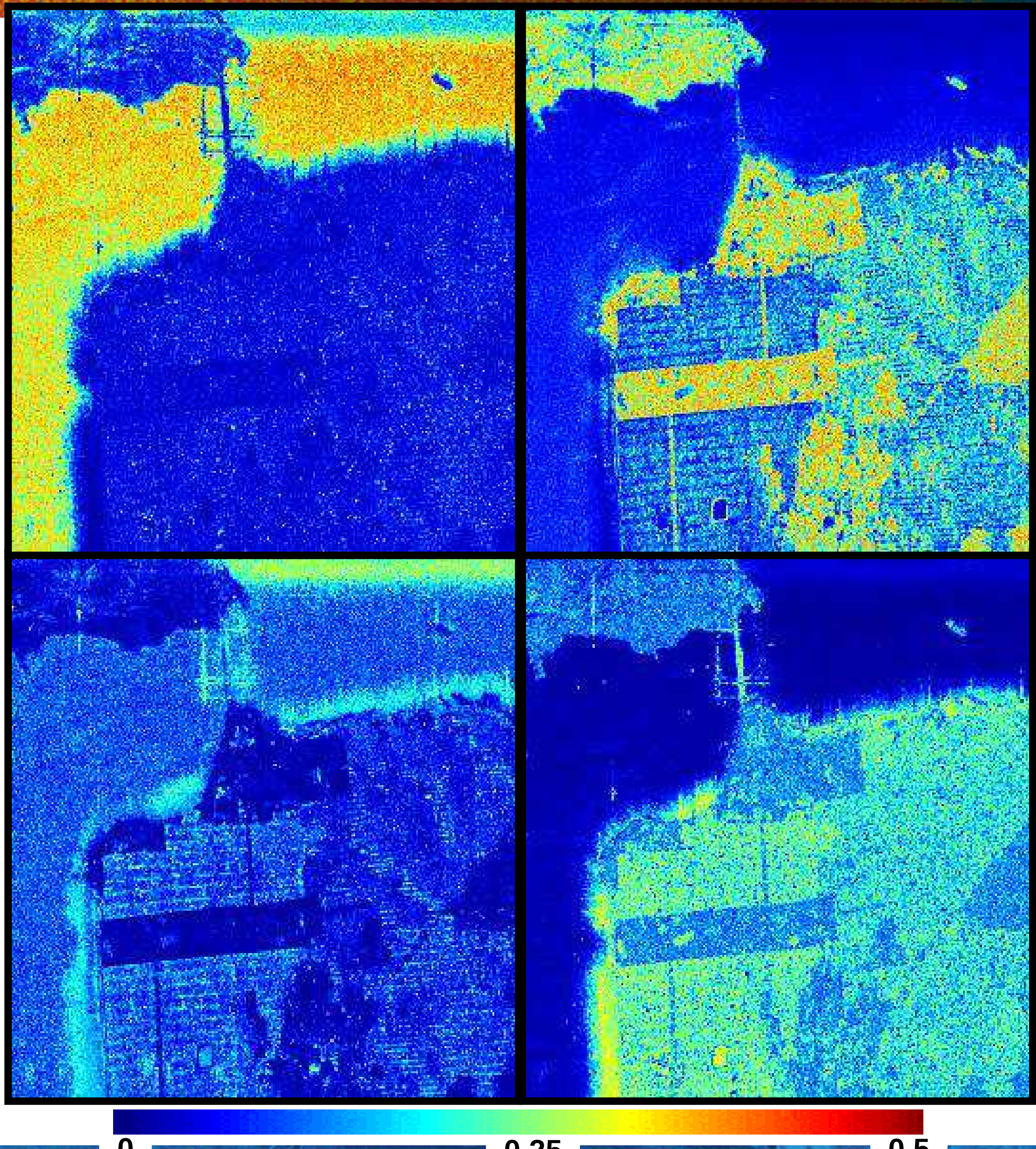


1 MECHANISM

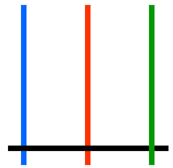
A(1-H)



2 MECHANISMS

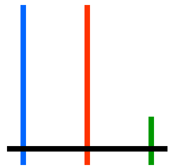


H(1-A)



3 MECHANISMS

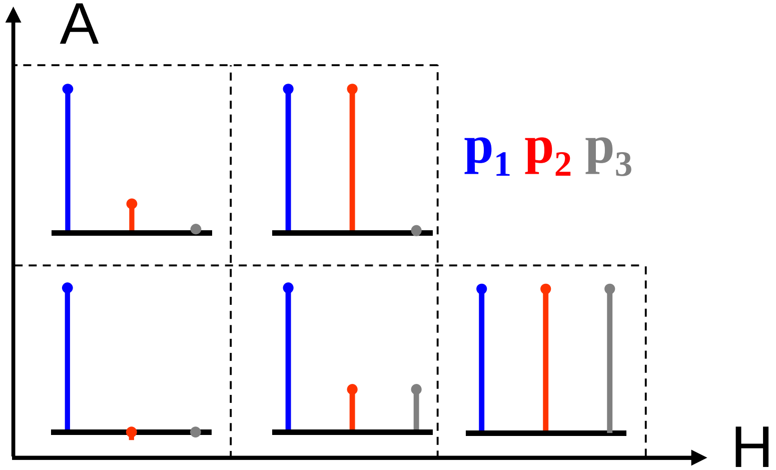
HA

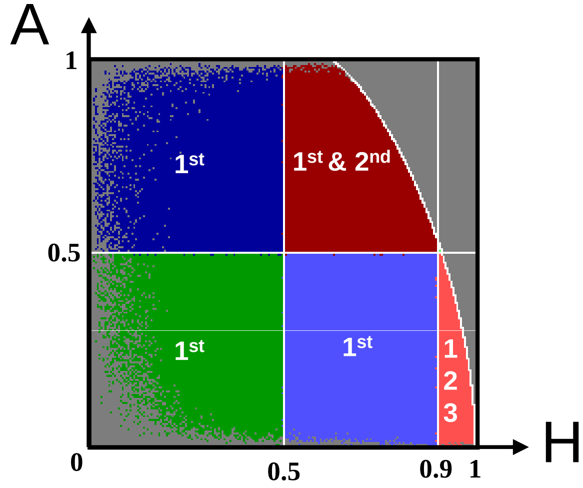


2 MECHANISMS

Basic scattering mechanism identification

Selection of polarimetric contributions





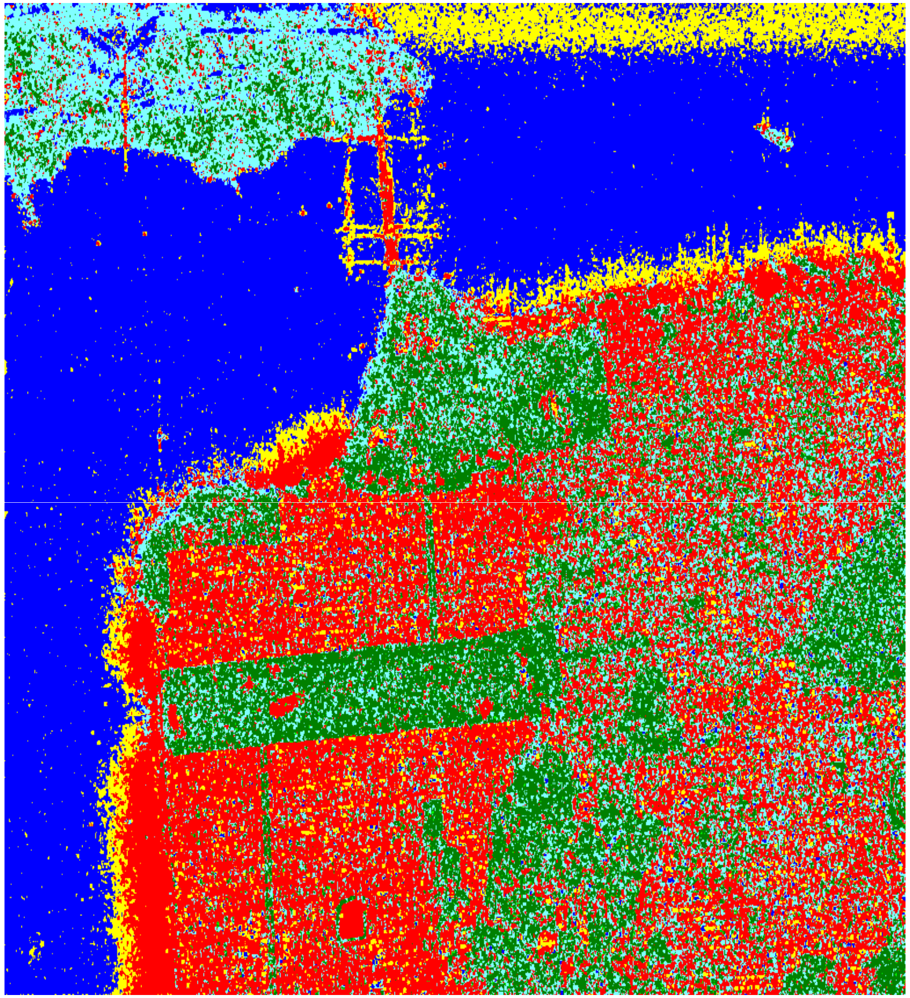
Scattering mechanism identification

H > 0.9 ⇨ **Volume Diffusion** (random scattering)

1 scattering mechanism ⇨ $\alpha_1 \gtrless \frac{\pi}{4} \rightarrow$ SR,DR

2 scattering mechanisms ⇨ $\mathbf{M} = p_1 \mathbf{v}_1 \mathbf{v}_1^\dagger + p_2 \mathbf{v}_2 \mathbf{v}_2^\dagger$; Huynen generators $\rightarrow$ SR,DR

H / A / $\underline{\alpha}$ DECOMPOSITION



$2A_0$ $B_0 + B$ $B_0 - B$

Basic scattering mechanism identification

S. Allain

S.E.R.D and D.E.R.D PARAMETERS
(*Single- and Double-bounce Eigenvalue Relative Difference*)

Reflection Symmetry

$$\langle [T] \rangle = \begin{bmatrix} T_1 & T_2 & 0 \\ T_2^* & T_4 & 0 \\ 0 & 0 & T_6 \end{bmatrix}$$

$$\lambda_{1_{NOS}} = \frac{1}{2}\left\{ \langle |S_{HH}|^2 \rangle + \langle |S_{VV}|^2 \rangle + \sqrt{\left(\langle |S_{HH}|^2 \rangle - \langle |S_{VV}|^2 \rangle \right)^2 + 4 \langle |S_{HH} S_{VV}^*|^2 \rangle} \right\}$$

$$\lambda_{2_{NOS}} = \frac{1}{2}\left\{ \langle |S_{HH}|^2 \rangle + \langle |S_{VV}|^2 \rangle - \sqrt{\left(\langle |S_{HH}|^2 \rangle - \langle |S_{VV}|^2 \rangle \right)^2 + 4 \langle |S_{HH} S_{VV}^*|^2 \rangle} \right\}$$

$$\lambda_{3_{NOS}} = 2 \langle |S_{HV}|^2 \rangle$$

Non-Ordered in Size Eigenvalues (*NOS*)

S. Allain

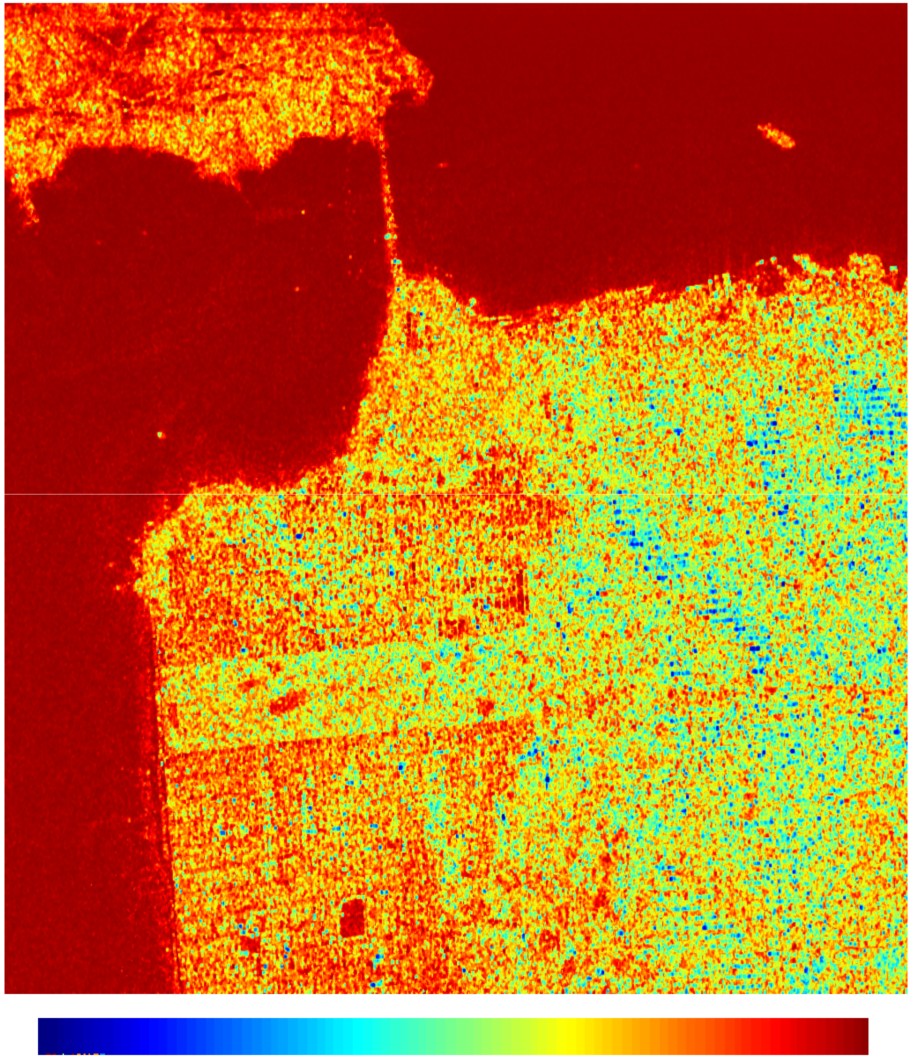
S.E.R.D and D.E.R.D PARAMETERS
(*Single- and Double-bounce Eigenvalue Relative Difference*)

$$\text{if } \alpha_1 \leq \frac{\pi}{4} \text{ or } \alpha_2 \geq \frac{\pi}{4} \Rightarrow \begin{cases} \lambda_S = \lambda_{1_{NOS}} \\ \lambda_D = \lambda_{2_{NOS}} \end{cases}$$

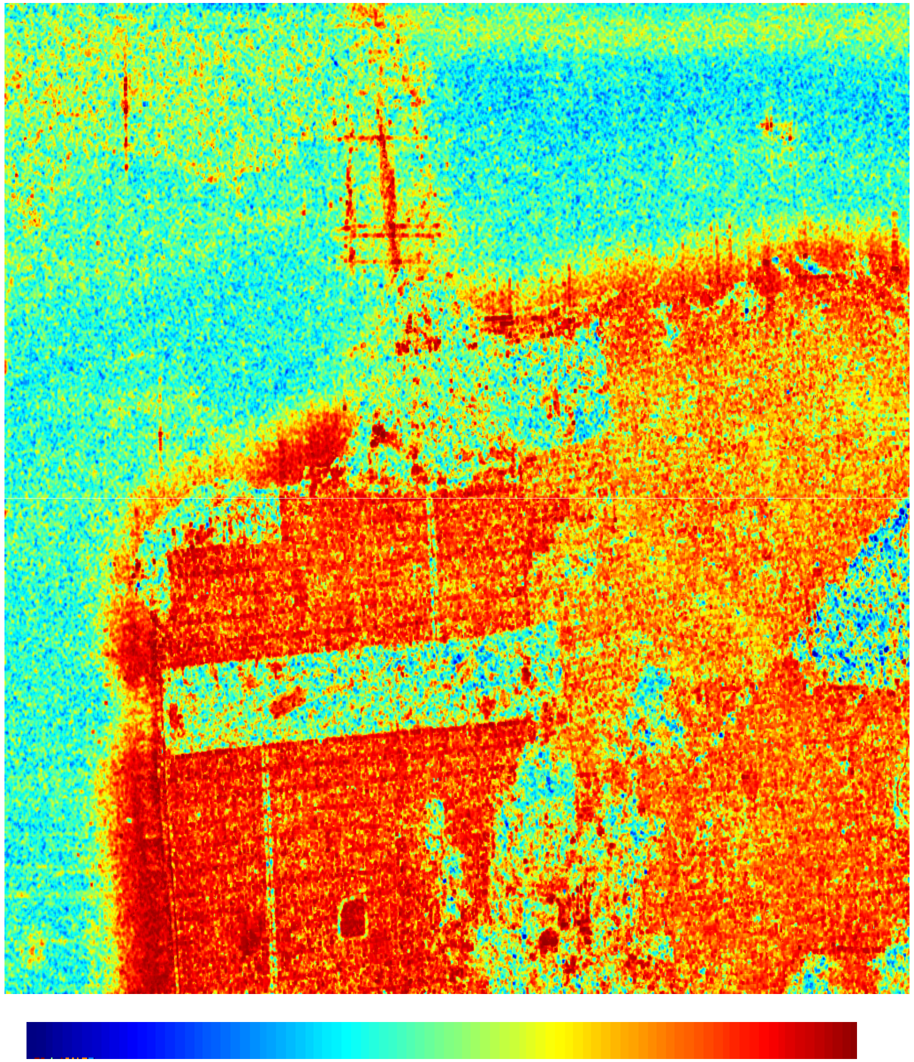
$$\text{if } \alpha_1 \geq \frac{\pi}{4} \text{ or } \alpha_2 \leq \frac{\pi}{4} \Rightarrow \begin{cases} \lambda_S = \lambda_{2_{NOS}} \\ \lambda_D = \lambda_{1_{NOS}} \end{cases}$$

$$SERD = \frac{\lambda_S - \lambda_{3_{NOS}}}{\lambda_S + \lambda_{3_{NOS}}} \qquad DERD = \frac{\lambda_D - \lambda_{3_{NOS}}}{\lambda_D + \lambda_{3_{NOS}}}$$

EIGENVALUE-BASED PARAMETERS



S.E.R.D



D.E.R.D

SHANNON ENTROPY
(*J. Morio – P. Refrégier*)

3D Circular Gaussian Process

$$P_{T_3}(\underline{\mathbf{k}}) = \frac{1}{\pi^3 |\mathbf{T}_3|} exp\left(\underline{\mathbf{k}}^{T*} \mathbf{T}_3^{-1} \underline{\mathbf{k}}\right)$$

⬇

$$I_T = \mathrm{Tr}(\mathbf{T}_3) \qquad p_T = \sqrt{1 - 27 \frac{|\mathbf{T}_3|}{\mathrm{Tr}(\mathbf{T}_3)^3}}$$

INTENSITY — DEGREE OF POLARIZATION

SHANNON ENTROPY
(J. Morio – P. Refrégier)

Shannon Entropy (SE)

$$S\left[P_{T_\cdot}(\underline{\mathbf{k}})\right] = \int P_{T_\cdot}(\underline{\mathbf{k}}) \log\left[P_{T_\cdot}(\underline{\mathbf{k}})\right] d\underline{\mathbf{k}}$$

$$SE = \log\left(\pi^3 e^3 \left|\mathbf{T}_3\right|\right) = SE_I + SE_P$$

$$SE_I = 3\log\left(\frac{\pi e I_T}{3}\right) = 3\log\left(\frac{\pi e \,\mathrm{Tr}(\mathbf{T}_3)}{3}\right)$$

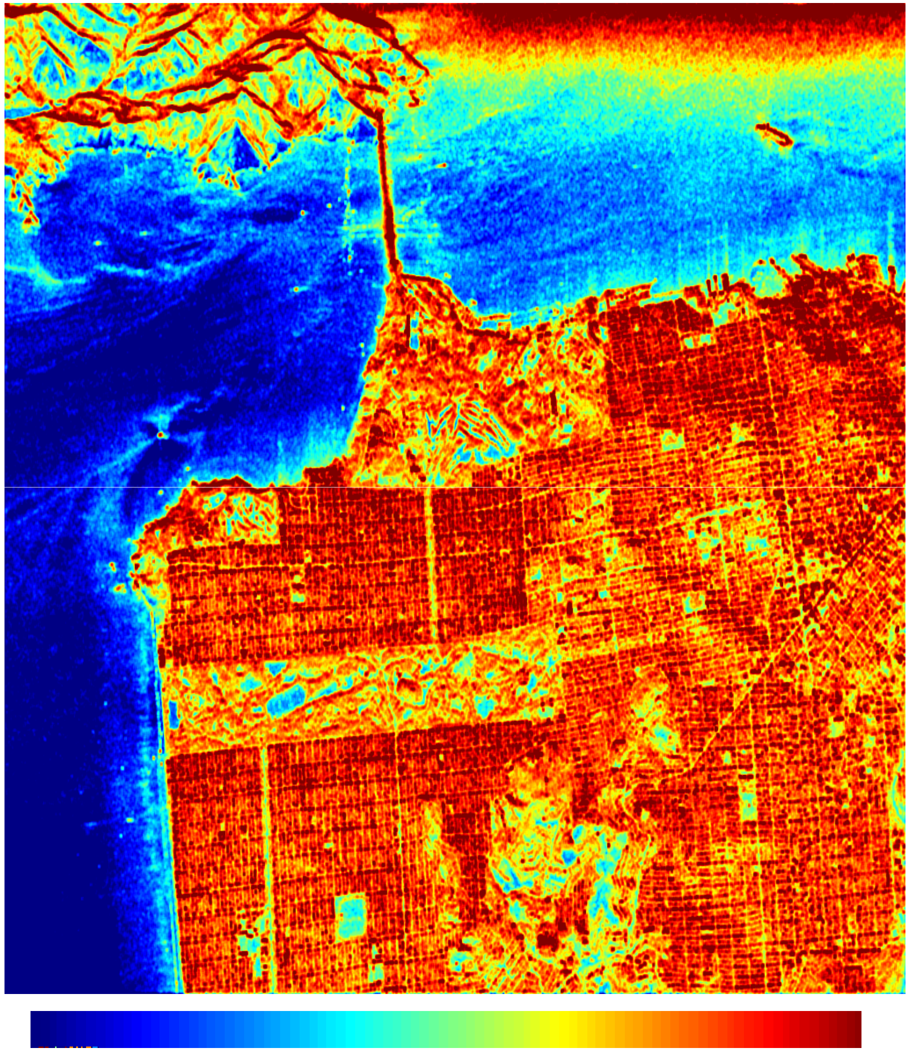
INTENSITY

$$SE_P = \log\left(1 - p_T^2\right) = \log\left(27\frac{\left|\mathbf{T}_3\right|}{\mathrm{Tr}(\mathbf{T}_3)^3}\right)$$

DEGREE OF POLARIZATION

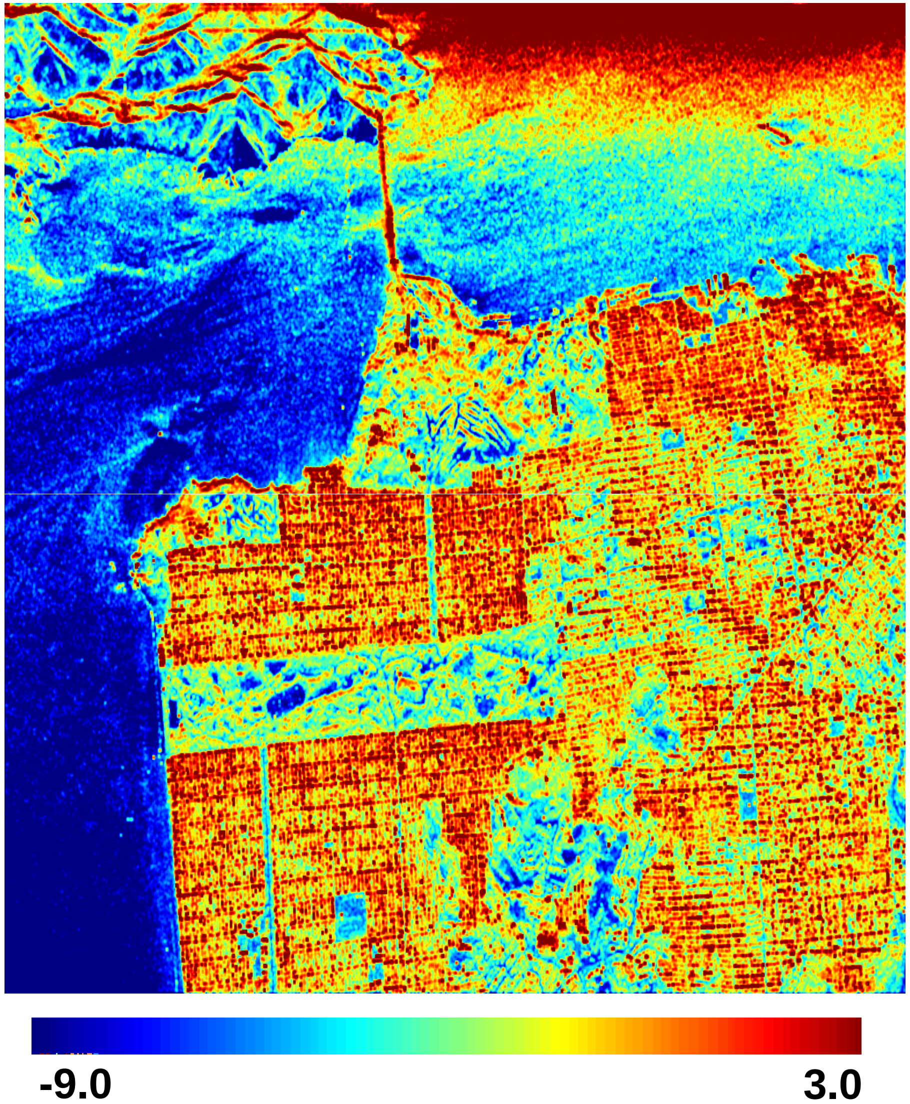
EIGENVALUE-BASED PARAMETERS

$2A_0$ $B_0 + B$ $B_0 - B$

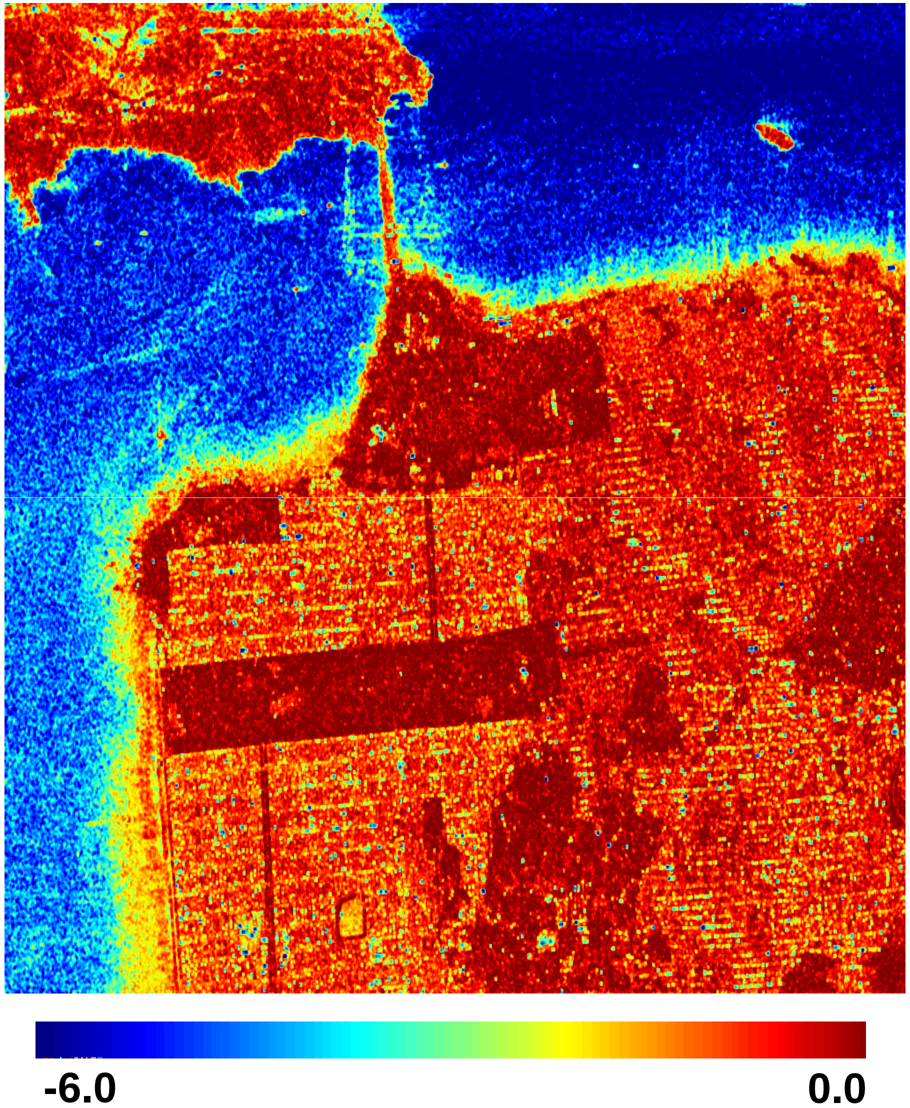


SHANNON ENTROPY (SE-norm)

EIGENVALUE-BASED PARAMETERS



SHANNON ENTROPY (SE-I)



SHANNON ENTROPY (SE-P)

POLARIZATION ASYMMETRY – POLARIZATION FRACTION

T. Ainsworth

$$\mathbf{T}_3 = \mathbf{U}_3 \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \mathbf{U}_3^{-1}$$

$$= \mathbf{U}_3 \begin{bmatrix} \lambda_1 - \lambda_3 & 0 & 0 \\ 0 & \lambda_2 - \lambda_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{U}_3^{-1} + \mathbf{U}_3 \begin{bmatrix} \lambda_3 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \mathbf{U}_3^{-1}$$

Power that remains completely unpolarized

$$PF = 1 - \frac{3\lambda_3}{Span} = 1 - \frac{3\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$0 \le PF \le 1$$

POLARIZATION FRACTION

EIGENVALUE-BASED PARAMETERS

POLARIZATION ASYMMETRY – POLARIZATION FRACTION

T. Ainsworth

$$\mathbf{T}_3 = \mathbf{U}_3 \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \mathbf{U}_3^{-1}$$

$$= \mathbf{U}_3 \begin{bmatrix} \lambda_1 - \lambda_3 & 0 & 0 \\ 0 & \lambda_2 - \lambda_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{U}_3^{-1} + \mathbf{U}_3 \begin{bmatrix} \lambda_3 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \mathbf{U}_3^{-1}$$

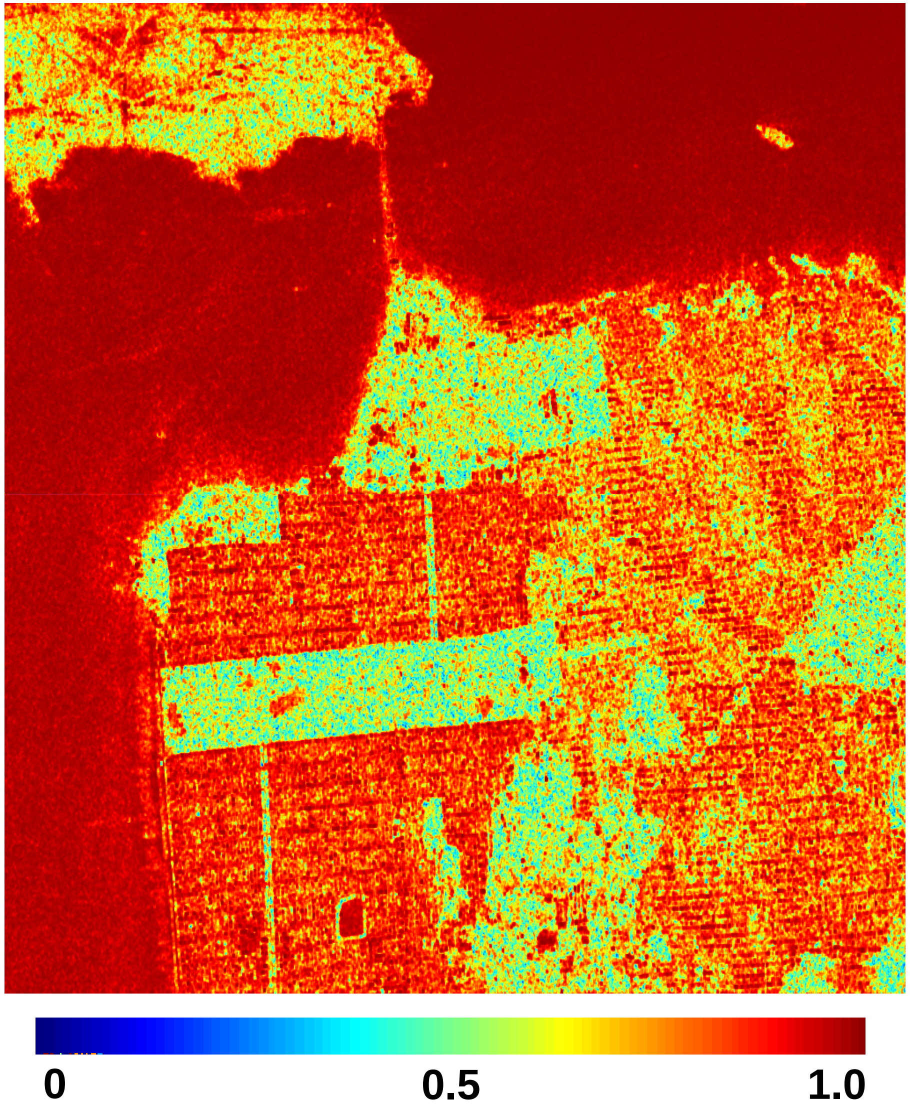
Power that remains completely unpolarized

$$PA = \frac{(\lambda_1 - \lambda_3) - (\lambda_2 - \lambda_3)}{(\lambda_1 - \lambda_3) + (\lambda_2 - \lambda_3)} = \frac{\lambda_1 - \lambda_2}{Span - 3\lambda_3}$$

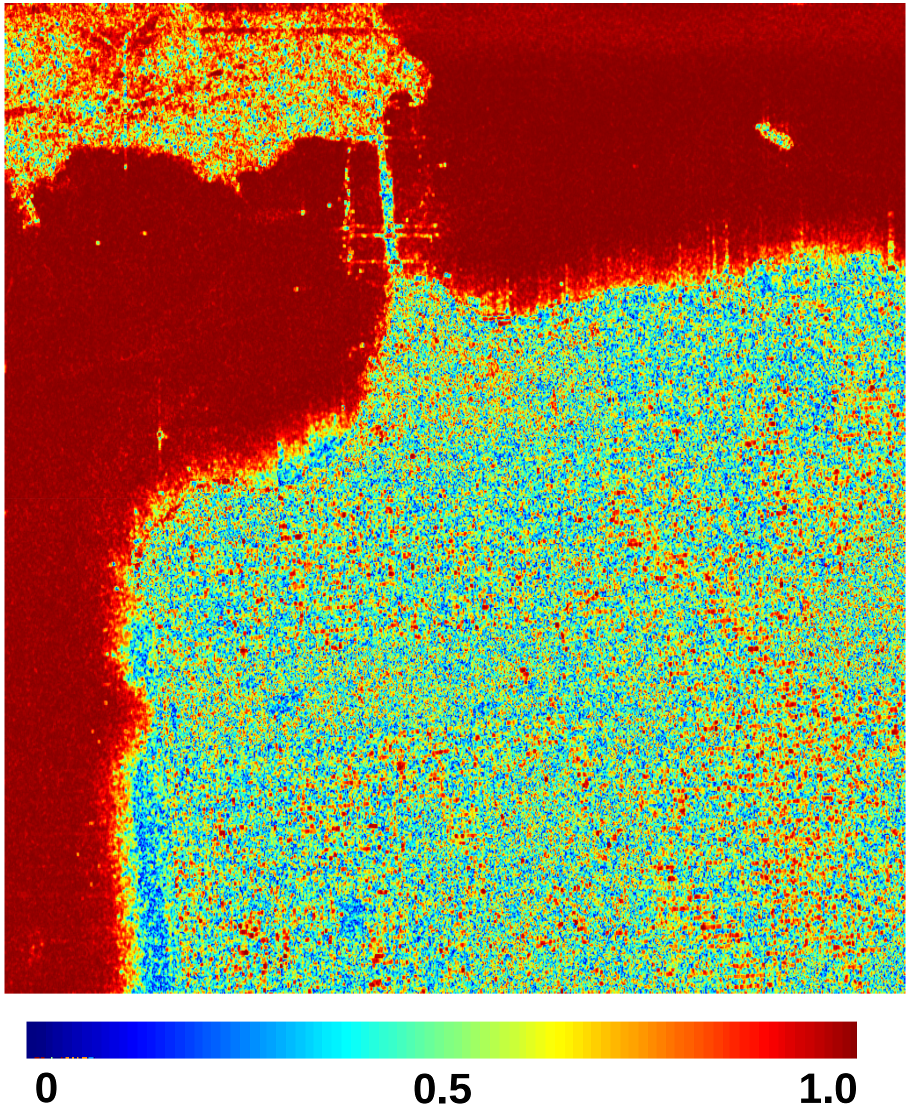
$$0 \leq PA \leq 1$$

POLARIZATION ASYMMETRY

EIGENVALUE-BASED PARAMETERS



POLARIZATION FRACTION



POLARIZATION ASYMMETRY

EIGENVALUE-BASED PARAMETERS

J. Van Zyl

RADAR VEGETATION INDEX

$$RVI = \frac{4\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \qquad 0 \leq RVI \leq \frac{4}{3}$$

S.L. Durden

PEDESTAL HEIGHT

$$PH = \frac{min(\lambda_1, \lambda_2, \lambda_3)}{max(\lambda_1, \lambda_2, \lambda_3)} = \frac{\lambda_3}{\lambda_1} \qquad 0 \leq PH \leq 1$$

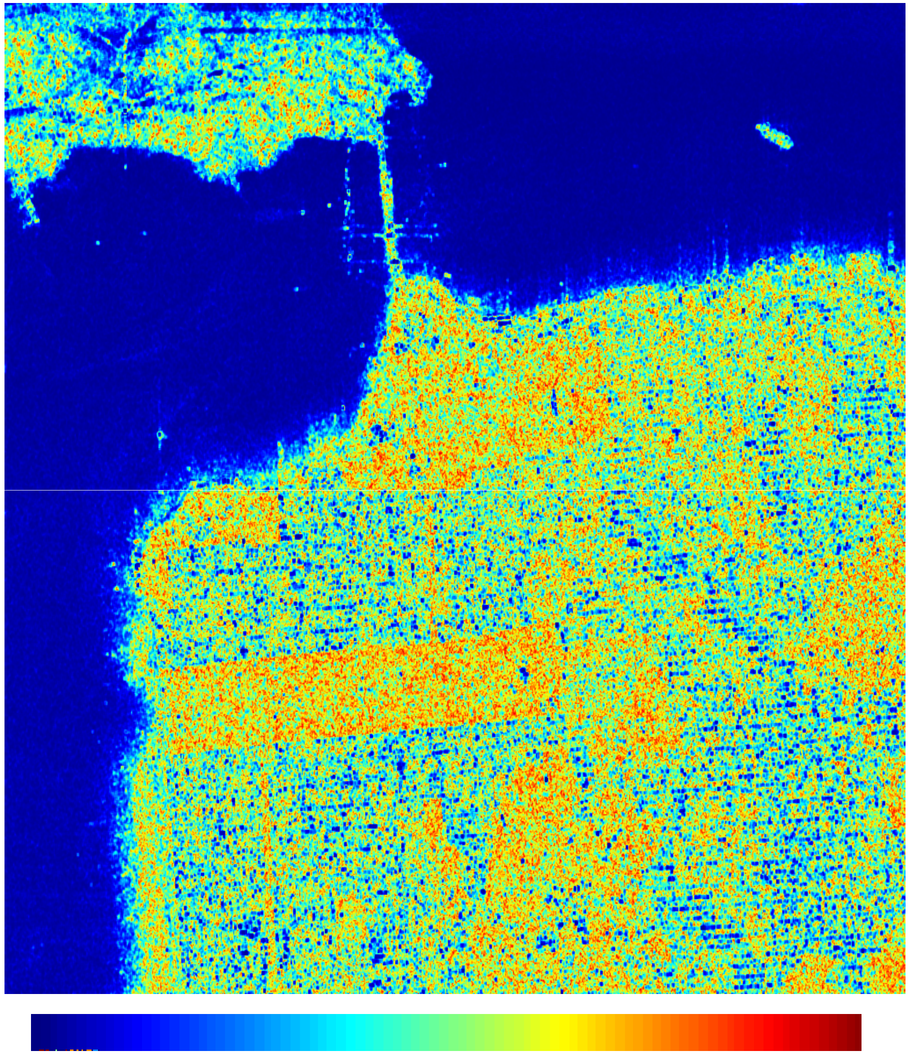
E. Luneuburg

TARGET RANDOMNESS

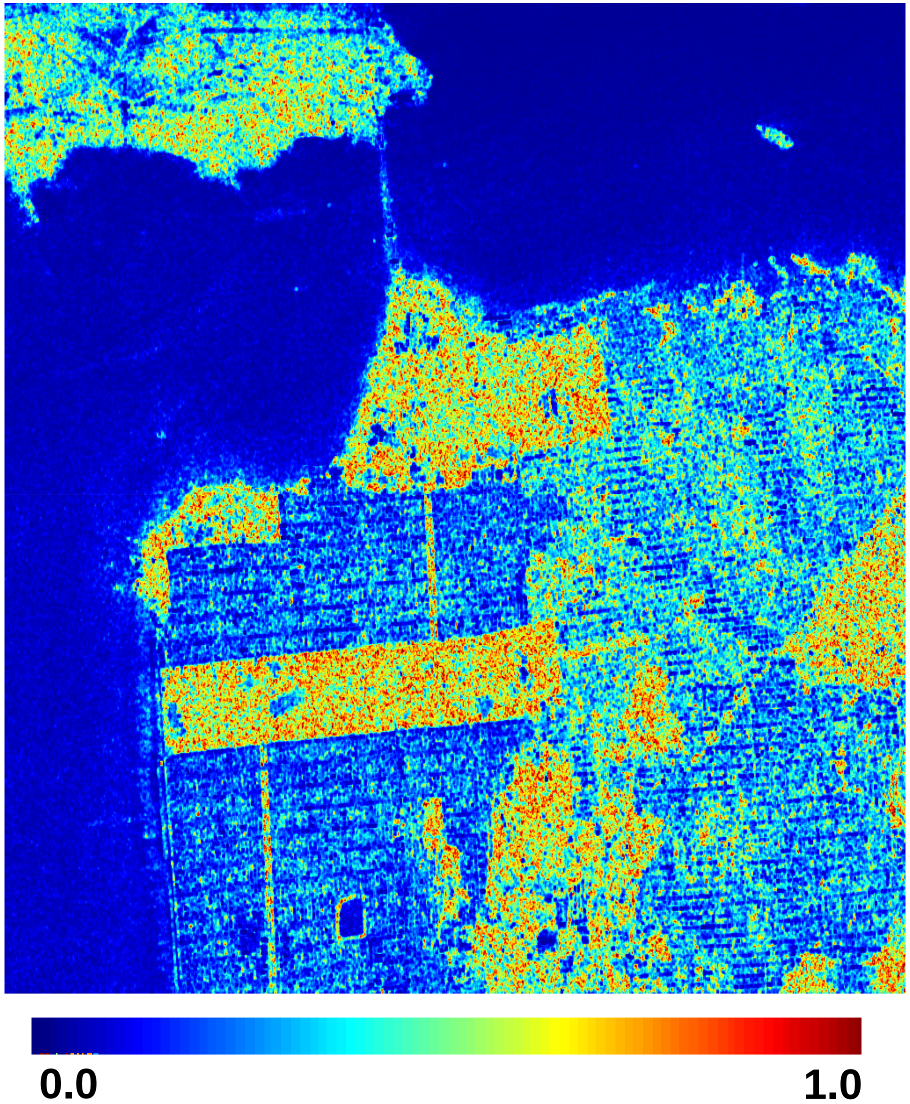
$$p_R = \sqrt{\frac{3}{2}} \sqrt{\frac{\lambda_2^2 + \lambda_3^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}} \qquad 0 \leq p_R \leq 1$$

EIGENVALUE-BASED PARAMETERS

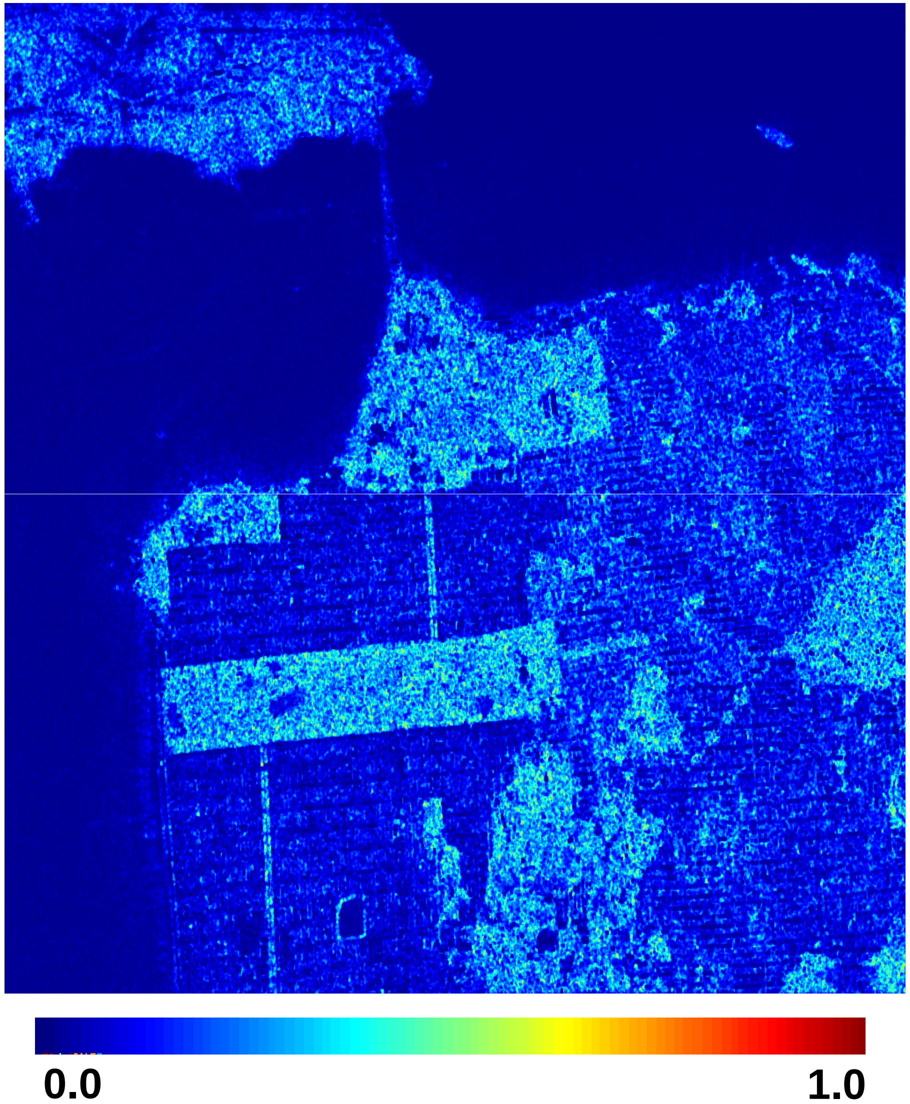
$2A_0$ $B_0 + B$ $B_0 - B$



TARGET RANDOMNESS



RADAR VEGETATION INDEX



PEDESTAL HEIGHT

ALTERNATIVE ENTROPY AND ALPHA PARAMETERS DERIVATION

J. Praks

E. Colin

Normalized Coherency Matrix

$$\mathbf{N}_3 = \left\langle \underline{\mathbf{k}}^{T*} \cdot \underline{\mathbf{k}} \right\rangle^{-1} \left\langle \underline{\mathbf{k}} \cdot \underline{\mathbf{k}}^{T*} \right\rangle = \frac{\mathbf{T}_3}{\mathbf{Tr}(\mathbf{T}_3)}$$

$$H \approx 2.52 + 0.78\, log_3\left(\left| \mathbf{N}_3 + 0.16\mathbf{I}_{D3} \right| \right)$$

ENTROPY

With:

$$\begin{aligned}\left| \mathbf{N}_3 + 0.16\mathbf{I}_{D3} \right| = \; & \left(\langle N_{11}\rangle + 0.16\right)\left(\langle N_{22}\rangle + 0.16\right)\left(\langle N_{33}\rangle + 0.16\right) \\ & - \left(\langle N_{11}\rangle + 0.16\right)\left|\langle N_{23}\rangle\right|^2 - \left(\langle N_{22}\rangle + 0.16\right)\left|\langle N_{13}\rangle\right|^2 \\ & - \left(\langle N_{33}\rangle + 0.16\right)\left|\langle N_{12}\rangle\right|^2 + \langle N_{12}^*\rangle\langle N_{13}\rangle\langle N_{23}^*\rangle \\ & + \langle N_{12}\rangle\langle N_{13}^*\rangle\langle N_{23}\rangle\end{aligned}$$

J. Praks

E. Colin

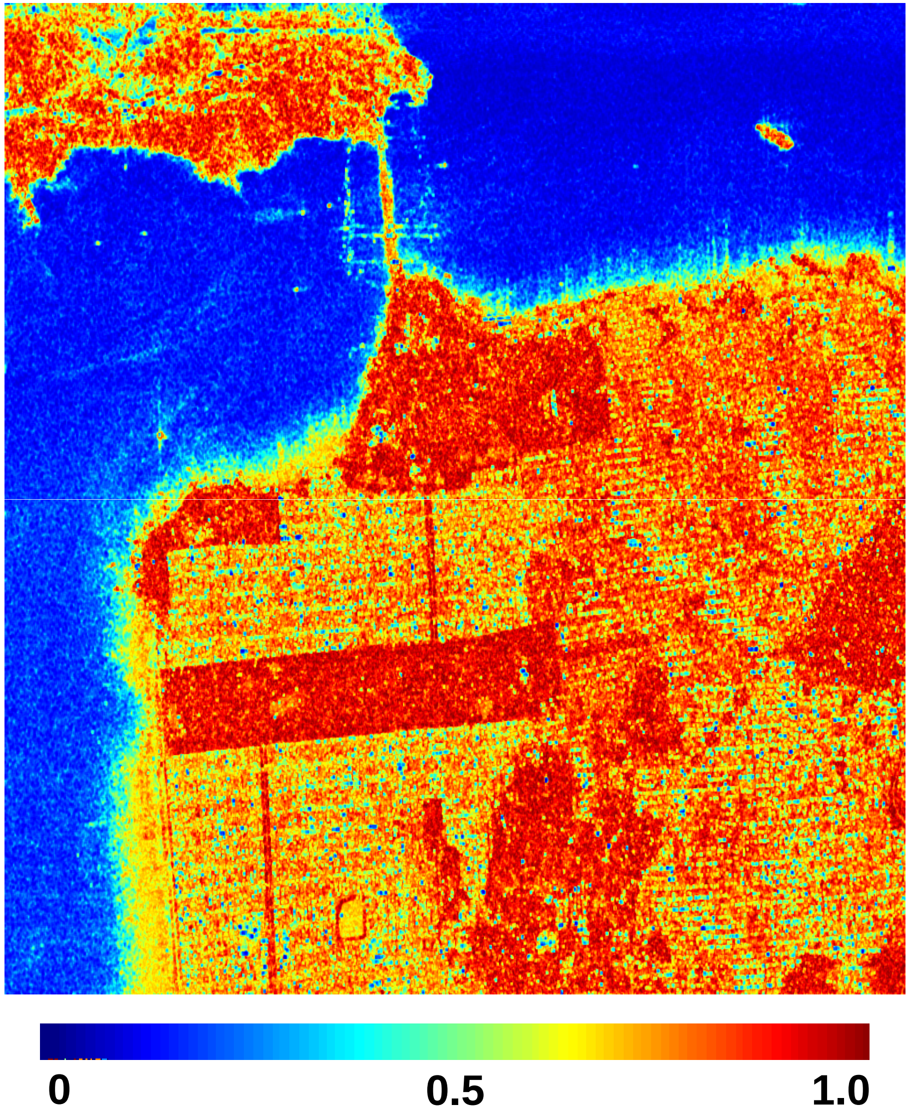
ALTERNATIVE ENTROPY AND ALPHA PARAMETERS DERIVATION

Normalized Coherency Matrix

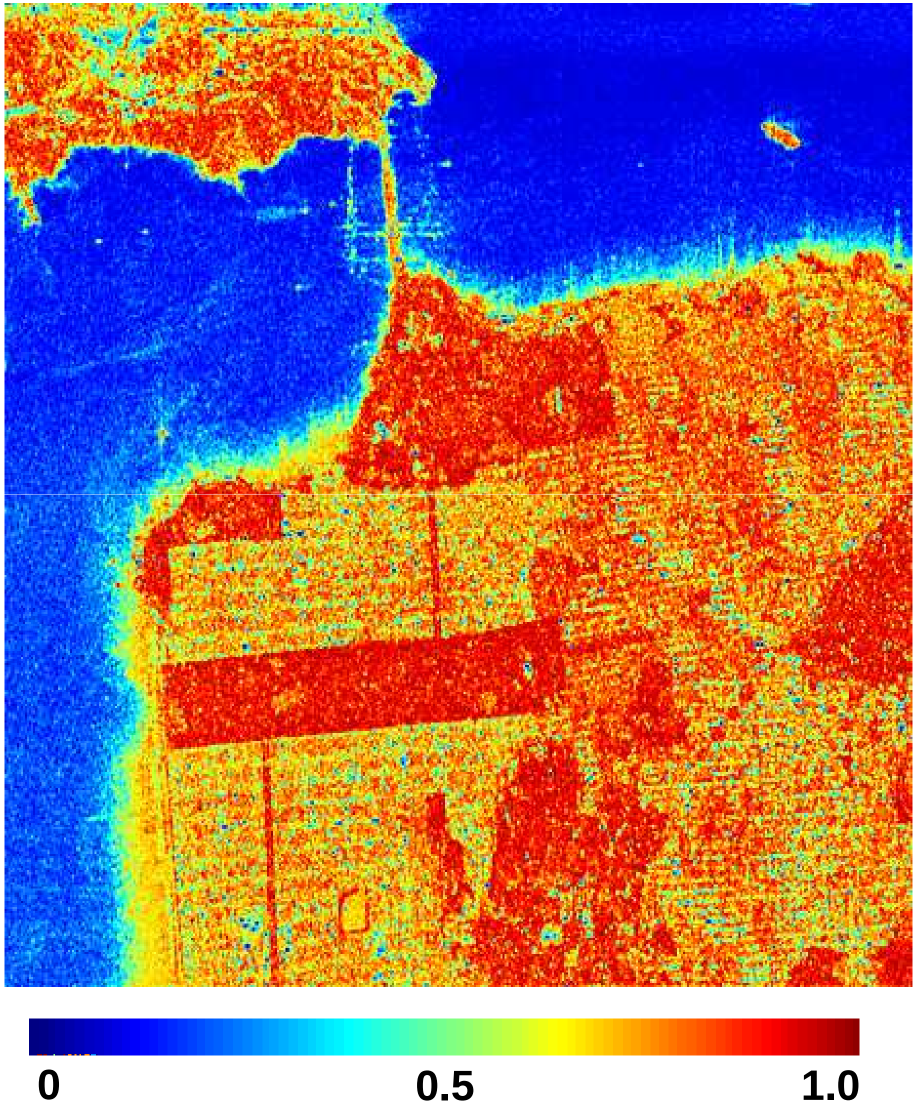
$$\mathbf{N}_3 = \left\langle \underline{\mathbf{k}}^{T*} \cdot \underline{\mathbf{k}} \right\rangle^{-1} \left\langle \underline{\mathbf{k}} \cdot \underline{\mathbf{k}}^{T*} \right\rangle = \frac{\mathbf{T}_3}{\mathbf{Tr}(\mathbf{T}_3)}$$

$$\overline{\alpha}_{Low\ H} = \cos^{-1}\left(\sqrt{\langle N_{11} \rangle}\right) \qquad \overline{\alpha}_{High\ H} = \left(1 - \langle N_{11} \rangle\right)\frac{\pi}{2}$$

ALPHA

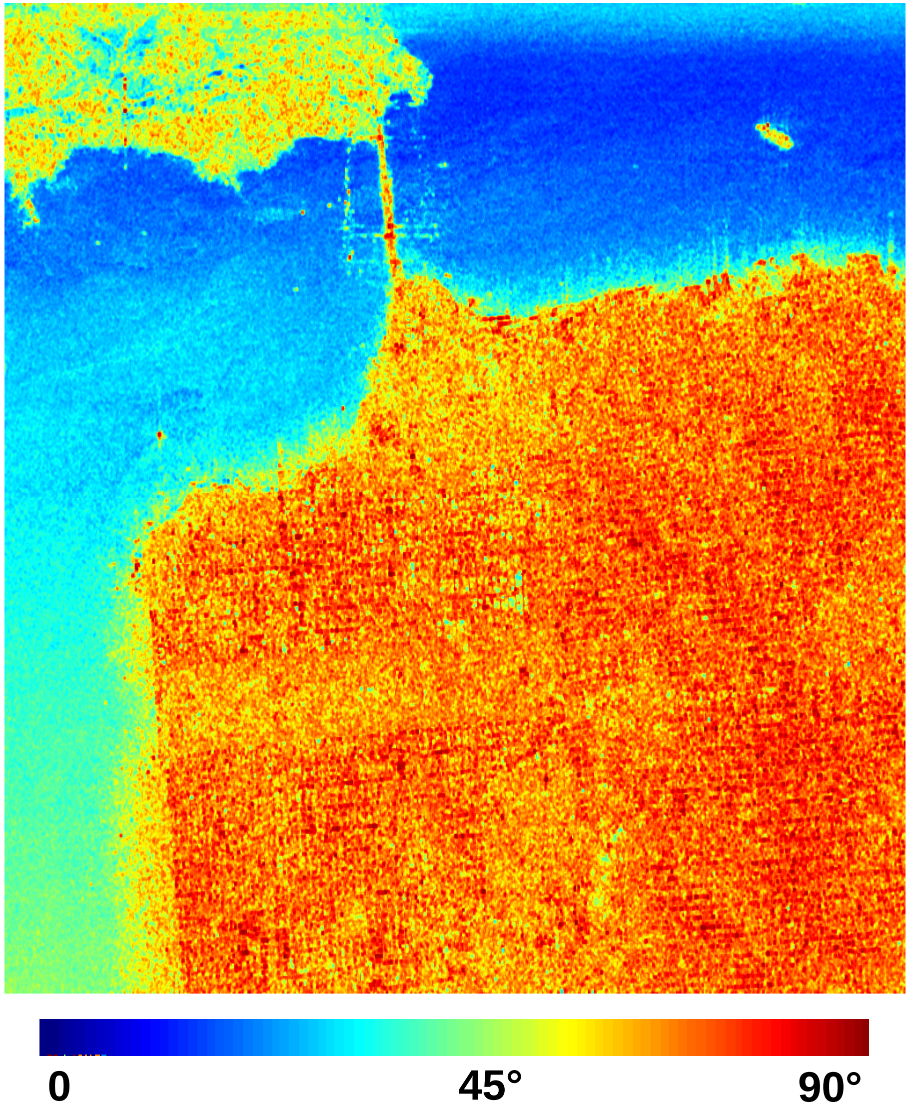


ENTROPY – Praks Colin

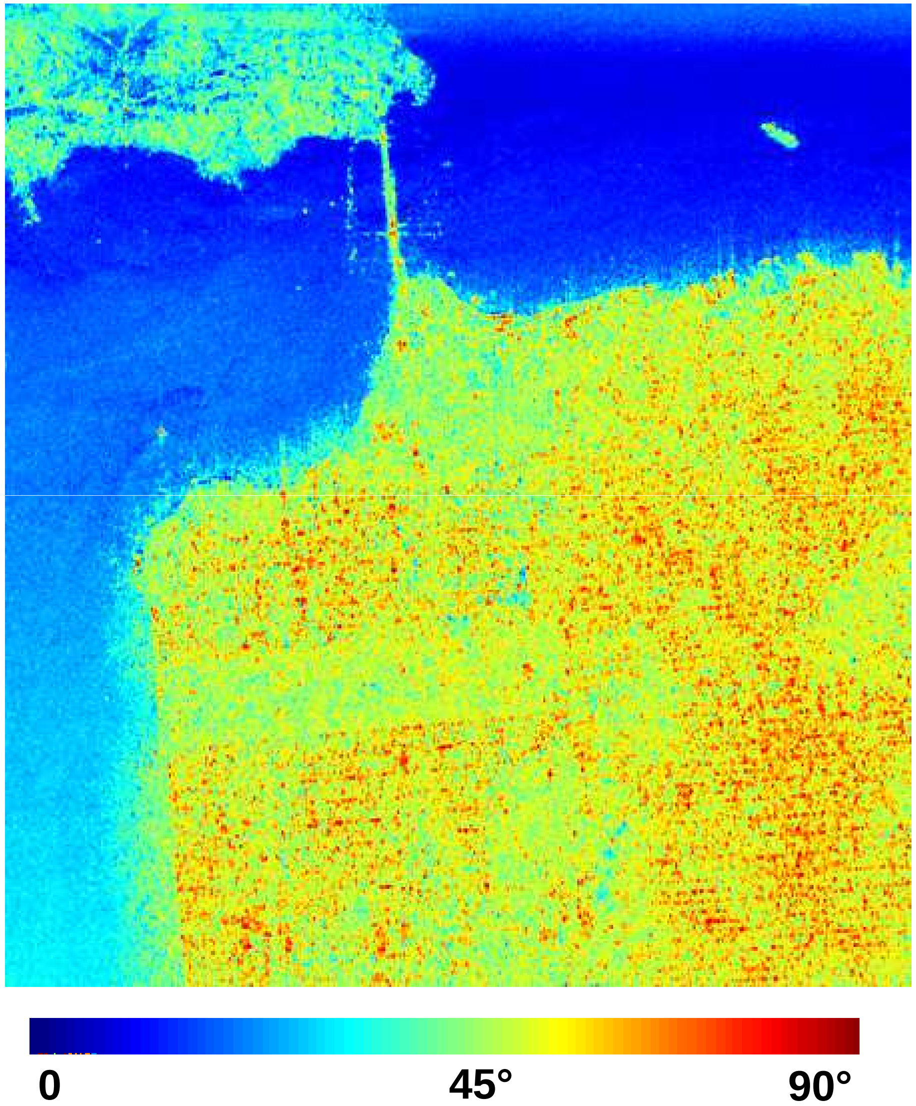


ENTROPY (H)

EIGENVALUE-BASED PARAMETERS



α PARAMETER – Praks Colin



α PARAMETER

TARGET SCATTERING VECTOR

MODEL DECOMPOSITION

(2007)

T.S.V.M DECOMPOSITION

$$\langle [T] \rangle = [U_3][\Sigma][U_3]^{-1} = \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \underline{u}_1 & \underline{u}_2 & \underline{u}_3 \end{bmatrix}^{*T}$$

ORTHOGONAL EIGENVECTORS

REAL EIGENVALUES $\lambda_1 > \lambda_2 > \lambda_3$

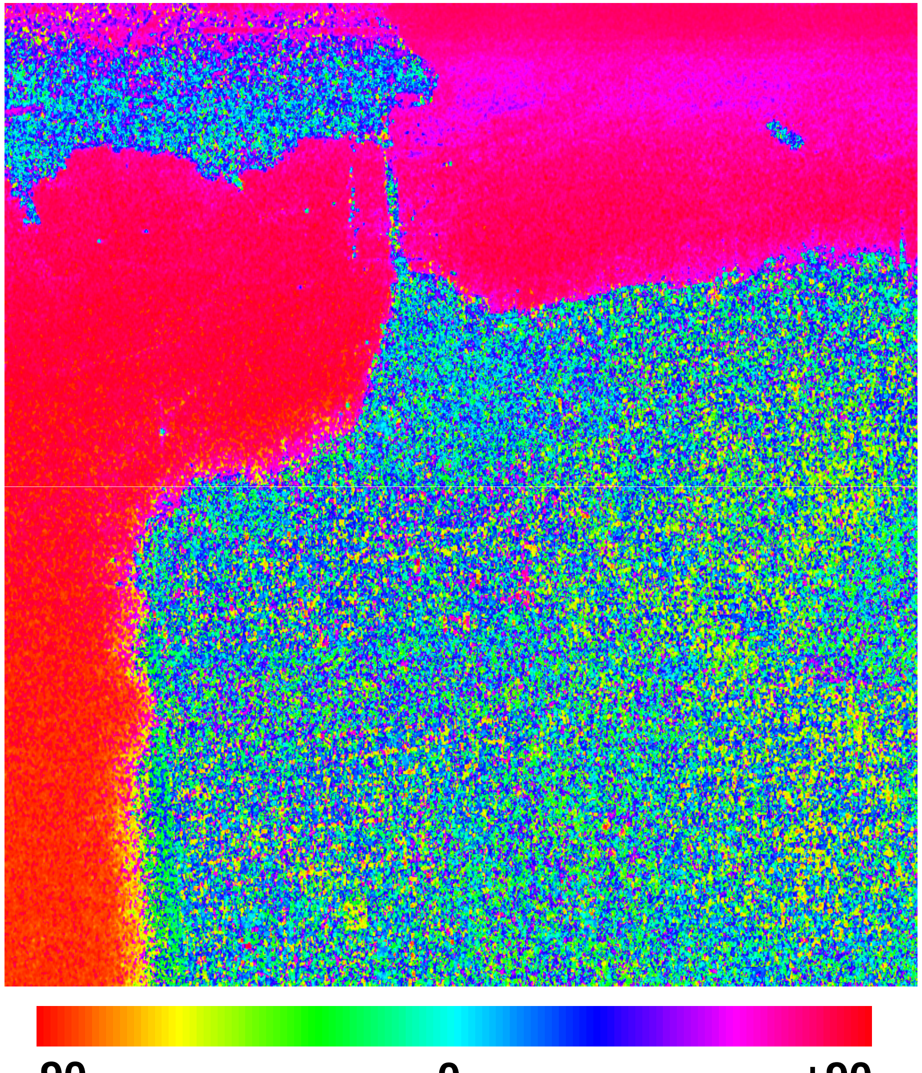
PARAMETERISATION OF THE EIGENVECTOR

$$\begin{bmatrix} \cos\alpha\, e^{j\phi} \\ \sin\alpha\cos\beta\, e^{j\phi} e^{j\delta} \\ \sin\alpha\sin\beta\, e^{j\phi} e^{j\gamma} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\psi & -\sin 2\psi \\ 0 & \sin 2\psi & \cos 2\psi \end{bmatrix} \begin{bmatrix} \cos\alpha_s \cos 2\tau_m \\ \sin\alpha_s\, e^{j\phi_{\alpha_s}} \\ -j\cos\alpha_s \sin 2\tau_m \end{bmatrix}$$

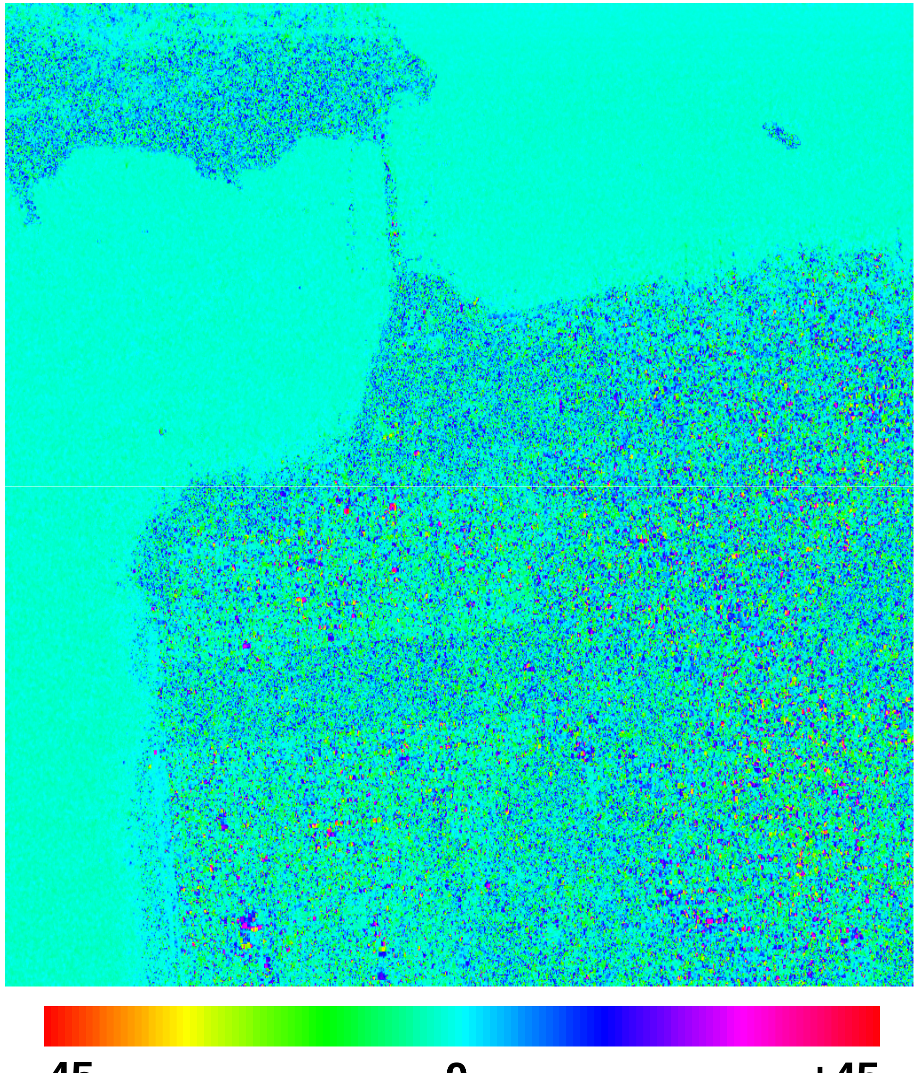
ψ : Target Orientation τ_m : Target Helicity

α_s , ϕ_{α_s} : Symmetric scattering type vector parameters

T.S.V.M DECOMPOSITION

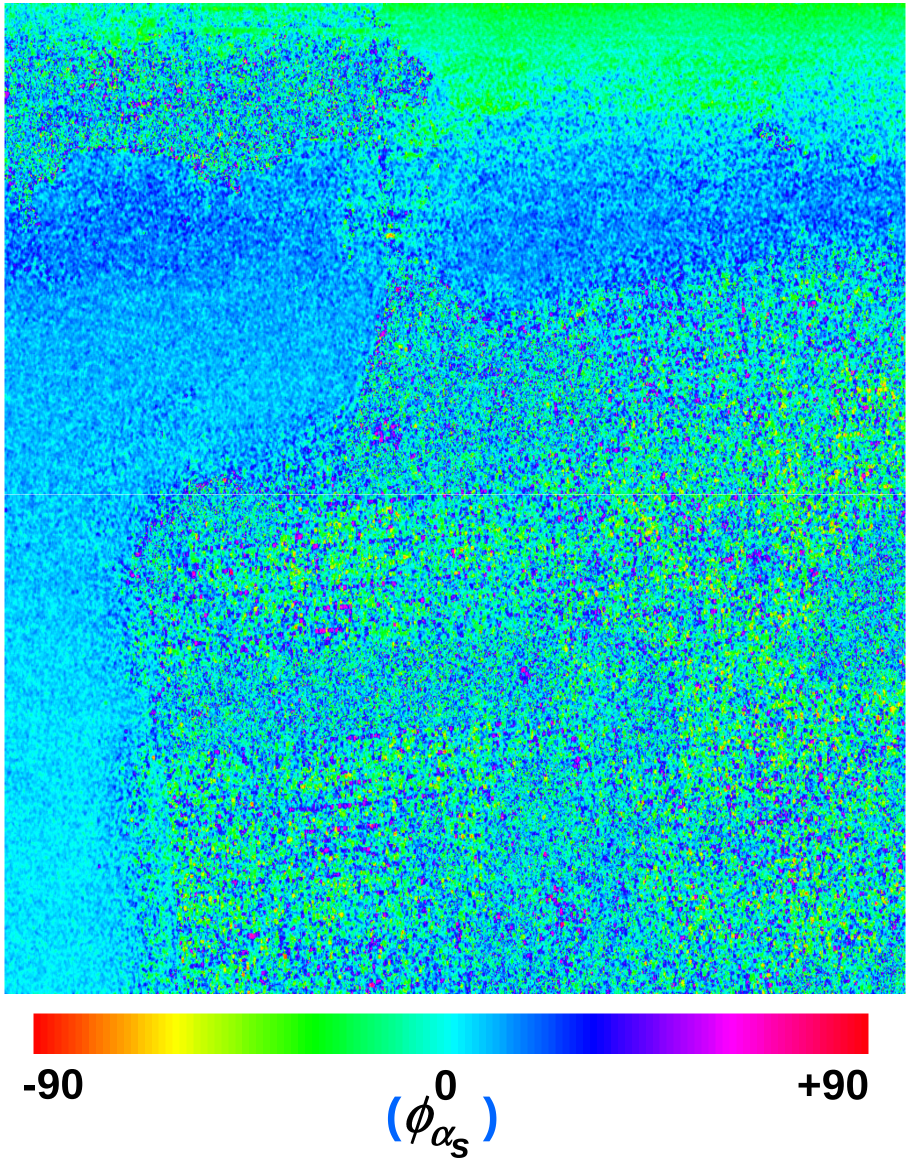


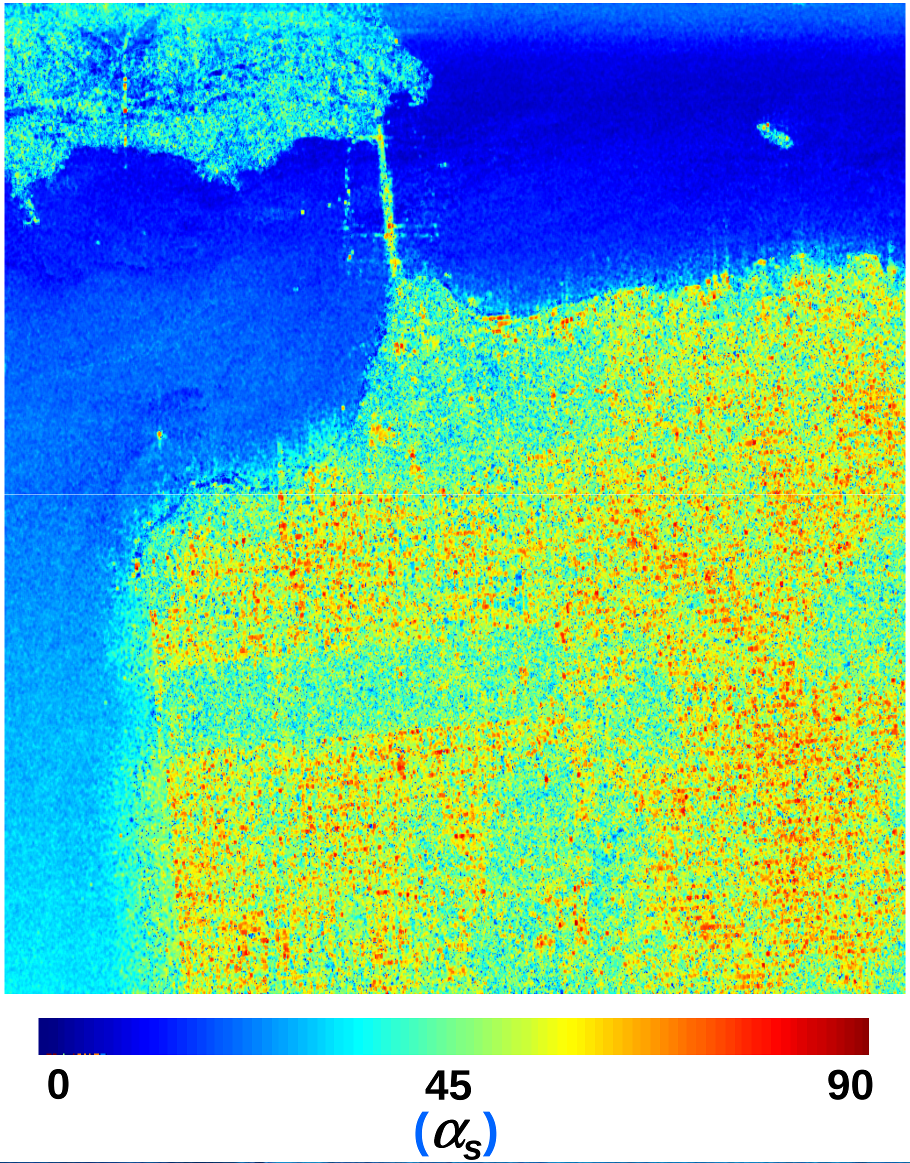
Target Orientation (ψ)



Target Helicity (τ_m)

T.S.V.M DECOMPOSITION





H / A / $\underline{\alpha}$ DECOMPOSITION

ENTROPY

$\underline{\alpha}$ PARAMETER

ANISOTROPY

$$H = -\sum_{i=1}^{3} P_i \log_3(P_i)$$

$$\underline{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3$$

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$

3 ROLL INVARIANT PARAMETERS

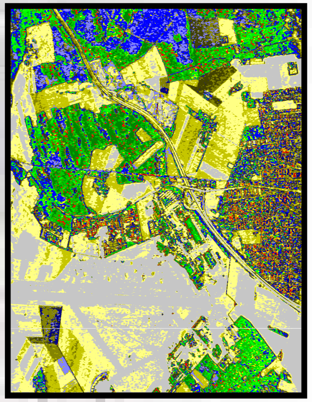
$$\underline{I} = \begin{bmatrix} \underline{\alpha} \\ H\,A \\ H(1-A) \\ (1-H)A \\ (1-H)(1-A) \end{bmatrix}$$

→ PHYSICAL SCATTERING MECHANISM

→ TYPE OF SCATTERING PROCESS

SEGMENTATION / CLASSIFICATION

POL-SAR PROCESSING
PHENOMENOLOGIC
QUALITATIVE ANALYSIS



POLARIMETRIC SPECKLE FILTERING → **POLARIMETRIC TARGET DECOMPOSITION** → **POLARIMETRIC CLASSIFICATION**
MONO/DUAL CHANNELS

PolSAR TERRAIN and LAND-USE CLASSIFICATION

J.S. Lee, M.R. Grunes, E. Pottier, L. Ferro-Famil, "Unsupervised terrain classification preserving scattering characteristics," IEEE Transactions on Geoscience and Remote Sensing,vol. 42, no.4, pp. 722-731, April, 2004.

J.S. Lee, M. R. Grunes and E. Pottier, "Quantitative Comparison of Classification Capability: Fully polarimetric versus Dual- and Single polarization SAR," IEEE TGRS, November 2002

E. Pottier and J.S. Lee, "Application of the « H / A / $\underline{\alpha}$ » polarimetric decomposition theorem for unsupervised classification of fully polarimetric SAR data based on the Wishart distribution" Proceedings of EUSAR2000

J.S. Lee, M.R. Grunes, T.L. Ainsworth, L. Du, D.L. Schuler, and S.R. Cloude, " Unsupervised Classification of Polarimetric SAR Imagery Based on Target Decomposition and Wishart Distribution," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 37, no. 5, 2249-2258, September 1999.

J.S. Lee, M. R. Grunes and R. Kwok," Classification of Polarimetric SAR Images Based on the Complex Wishart Distribution," ***Int. J. Remote Sensing, vol.32, No. 5, Sept. 1994.***

J.S. Lee, E. Pottier, *Polarimetric Radar Imaging: From Basics to Applications,* Taylor & Francis/CRC, 2009

WISHART CLASSIFIER

Target Vector

$$\underline{X} = \begin{bmatrix} S_{HH} & \sqrt{2}S_{HV} & S_{VV} \end{bmatrix}^T \qquad P(\underline{X}) = \frac{1}{\pi^3 |[C]|} e^{-\underline{X}^{*T}[C]^{-1}\underline{X}}$$

$$\underline{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH}+S_{VV} & S_{HH}-S_{VV} & 2S_{HV} \end{bmatrix}^T \qquad P(\underline{k}) = \frac{1}{\pi^3 |[T]|} e^{-\underline{k}^{*T}[T]^{-1}\underline{k}}$$

Coherency Matrix

$$\langle [T] \rangle = \frac{1}{N}\sum_{i=1}^{N} \underline{k}_i \cdot \underline{k}_i^{*T} = \frac{1}{N}\sum_{i=1}^{N} [T_i]$$

$$P(\langle [T] \rangle / [T_m]) = \frac{L^{Lp} \left| \langle [T] \rangle \right|^{L-p} e^{-LTr\left([T_m]^{-1}\langle [T] \rangle\right)}}{\pi^{\frac{p(p-1)}{2}} \Gamma(L) \ldots \Gamma(L-p+1) \left| [T_m] \right|^{L}}$$

COMPLEX WISHART DISTRIBUTION

L: Number of Look p: Polarimetric Dimension

WISHART CLASSIFIER

$$P\left(\langle[T]\rangle/[T_m]\right)=\frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p}e^{-LTr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)\left|[T_m]\right|^{L}}$$

$$\Downarrow$$

BAYES MAXIMUM LIKELIHOOD CLASSIFICATION PROCEDURE

$$\langle[T]\rangle\in[T_m]\quad if\quad P\left([T_m]/\langle[T]\rangle\right)\geq P\left([T_j]/\langle[T]\rangle\right)\quad\forall\ j\neq m$$

Applying Bayes rule $$P\left([T_m]/\langle[T]\rangle\right)=\frac{P\left(\langle[T]\rangle/[T_m]\right)}{P\left(\langle[T]\rangle\right)}P\left([T_m]\right)$$

It follows

$$\langle[T]\rangle\in[T_m]\quad if\quad P\left(\langle[T]\rangle/[T_m]\right)P\left([T_m]\right)\geq P\left(\langle[T]\rangle/[T_j]\right)P\left([T_j]\right)\quad\forall\ j\neq m$$

$[T_m]$: Cluster Center of the class *m*

WISHART CLASSIFIER

$$P\left(\langle[T]\rangle/[T_m]\right) = \frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p} e^{-LTr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)\left|[T_m]\right|^{L}}$$

BAYES MAXIMUM LIKELIHOOD CLASSIFICATION PROCEDURE

$$\langle[T]\rangle \in [T_m] \quad if \quad d_m\left(\langle[T]\rangle\right) < d_j\left(\langle[T]\rangle\right) \quad \forall\, j \neq m$$

with

$$d_m\left(\langle[T]\rangle\right) = LTr\left([T_m]^{-1}\langle[T]\rangle\right) + L\ln\left(\left|[T_m]\right|\right) - \ln\left(P\left([T_m]\right)\right) + K$$

$[T_m]$: Cluster Center of the class *m*

ROBUSTENESS OF WISHART CLASSIFIER

$$d_m(\langle[T]\rangle) = L\,Tr\left([T_m]^{-1}\langle[T]\rangle\right) + L\ln\left(|[T_m]|\right) - \ln\left(P([T_m])\right) + K$$

INDEPENDENT OF # OF LOOKS

INDEPENDENT OF POLARIZATION BASIS

[T] or [C] IDENTICAL CLASSIFICATION RESULTS

INDEPENDENT OF WEIGHTING

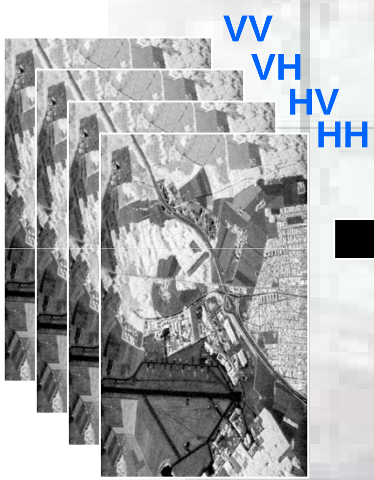
$$u = \begin{bmatrix} S_{hh} \\ \sqrt{2}S_{hv} \\ S_{vv} \end{bmatrix} \qquad u_1 = \begin{bmatrix} S_{hh} \\ S_{hv} \\ S_{vv} \end{bmatrix}$$

For Dual-Pol ($p=2$), PolSAR ($p=3$), Pol-InSAR ($p=6$)

J.S. Lee, E. Pottier, *Polarimetric Radar Imaging: From Basics to Applications,* Taylor & Francis/CRC, 2009

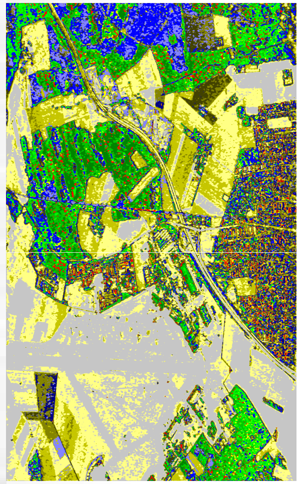
WISHART PDF

$$P\left(\langle[T]\rangle/[T_m]\right)=\frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p}e^{-L\,Tr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)\left|[T_m]\right|^{L}}$$



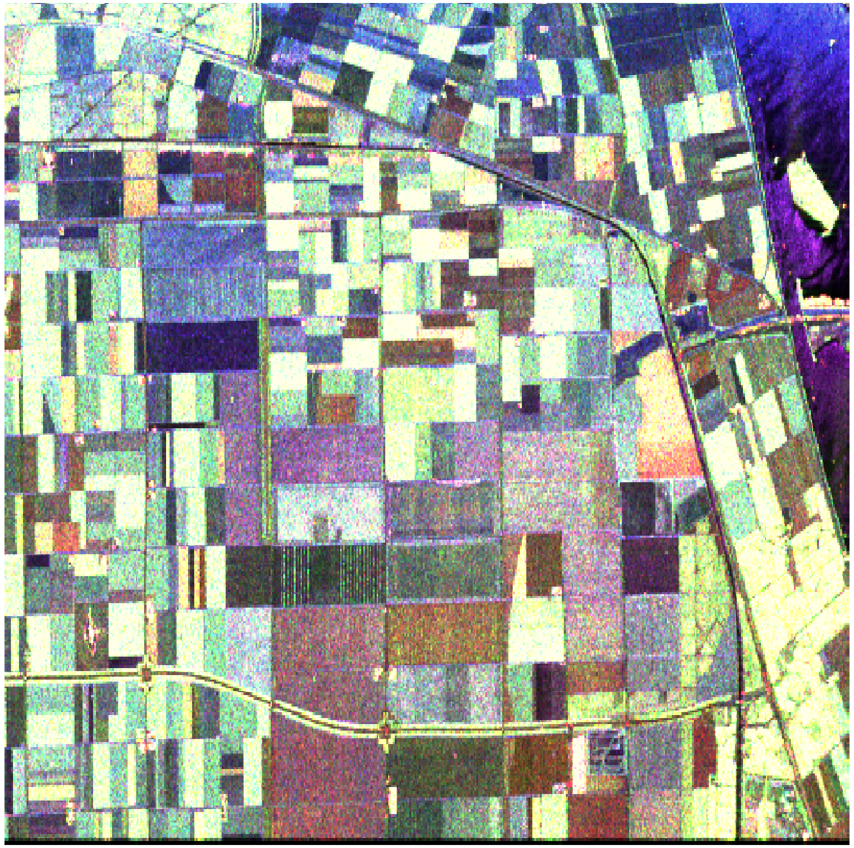
SUPERVISED POLSAR CLASSIFICATION

J.S LEE, M.R GRUNES, E.POTTIER (2002)



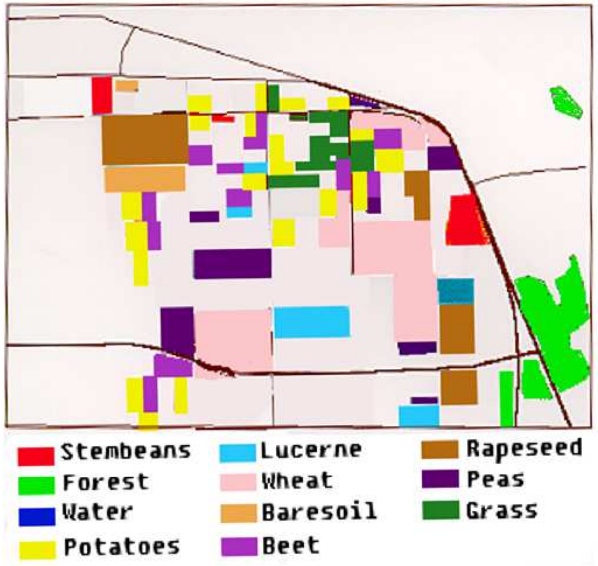
WISHART CLASSIFIER

Courtesy of Dr J.S Lee



$2A_0$ $B_0 + B$ $B_0 - B$

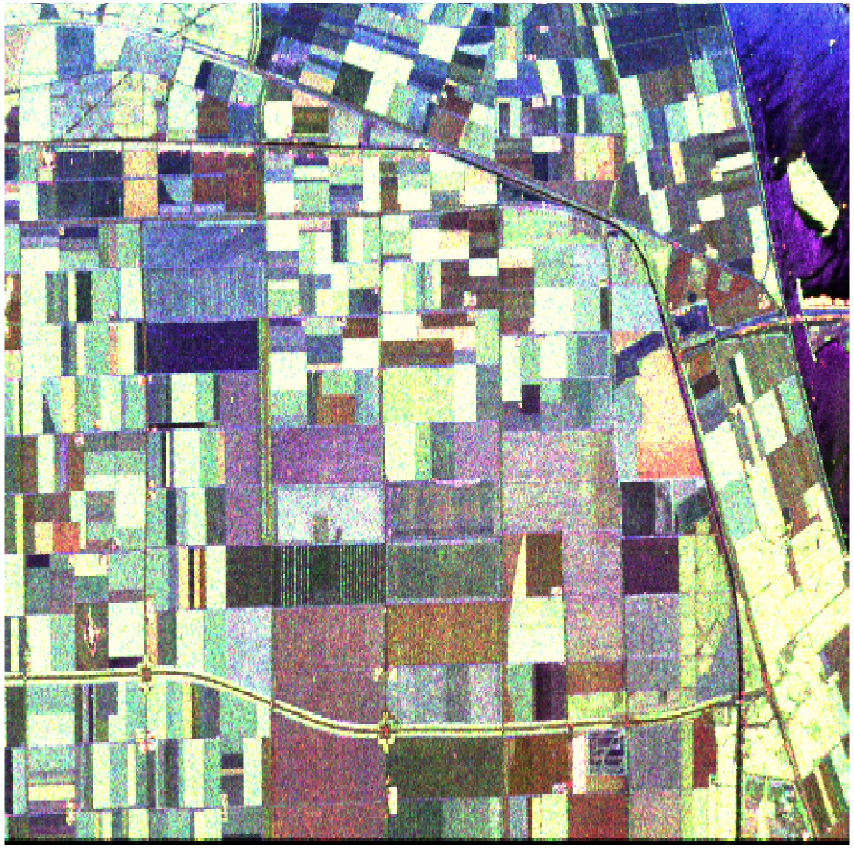
JPL AIRSAR
P-L-C Band Flevoland Data



Original Ground- Truth

Training Sets / Reference map

Courtesy of Dr J.S Lee



$2A_0$ $B_0 + B$ $B_0 - B$

JPL AIRSAR
L-Band Flevoland Data

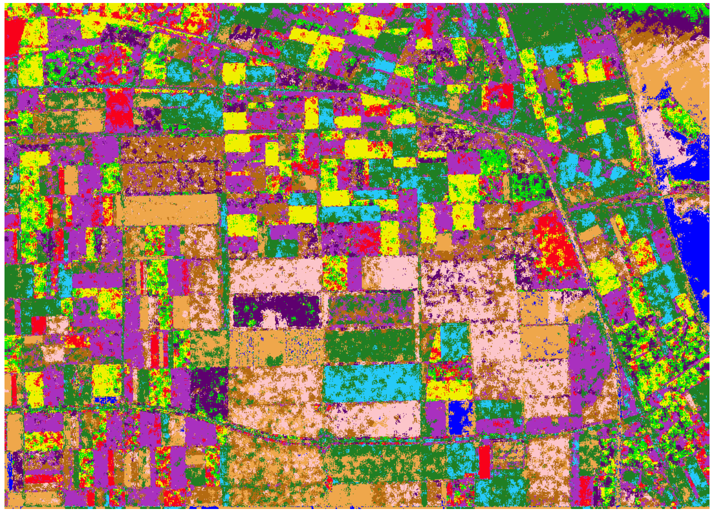
L-band (81.63%)

SUPERVISED CLASSIFIER

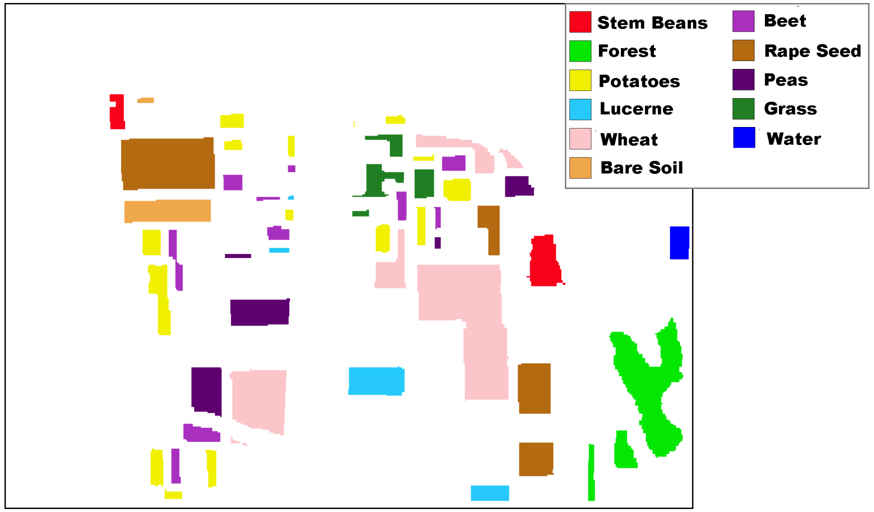
Courtesy of Dr J.S Lee

L-band Fully Pol. (81.63%)

L-band complex HH and VV (80.91%)



L-band HH and VV Intensities (56.35%)



Reference map for comparison

SUPERVISED CLASSIFIER

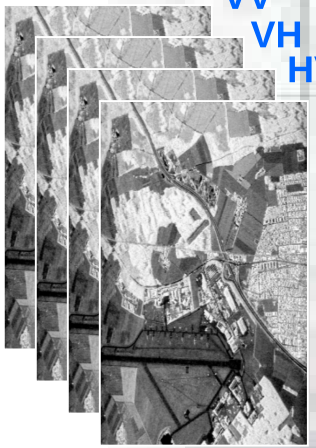
Courtesy of Dr J.S Lee

	Fully Polarimetric	Complex HH, HV	Intensity HH, HV	Complex HH, VV	Intensity HH, VV	Complex VV, HV	Intensity VV, HV
Stem Bean	95.32	51.16	63.27	90.64	61.73	35.97	31.29
Forest	81.07	66.73	68.39	75.75	33.83	60.05	60.91
Potatoes	82.89	67.53	66.36	81.52	49.35	54.40	59.15
Lucerne	97.91	39.29	38.23	99.26	65.15	67.49	65.30
Wheat	64.80	49.77	44.27	68.02	53.72	49.43	41.65
Bare Soil	99.36	90.04	82.86	98.42	93.15	90.93	63.74
Beet	89.26	68.80	66.36	86.22	81.98	75.94	74.77
Rape Seed	89.05	55.01	53.23	87.18	49.85	82.31	77.12
Peas	86.47	50.77	39.25	84.59	65.21	81.82	79.59
Grass	91.05	66.44	65.06	90.13	71.08	75.36	75.19
Water	100	90.39	87.33	100	99.86	96.30	70.53
TOTAL	81.63	59.16	55.38	80.91	56.35	64.72	60.12

L-Band Crop Classification Results

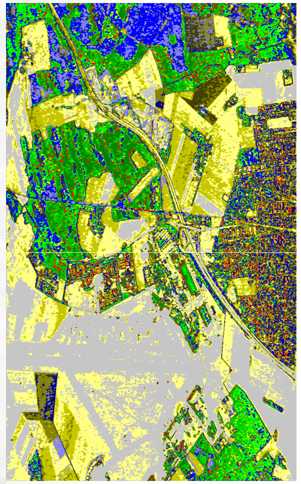
WISHART PDF

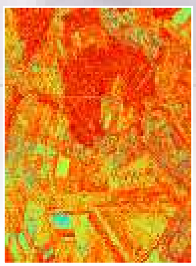
$$P(\langle[T]\rangle/[T_m]) = \frac{L^{Lp}\,|\langle[T]\rangle|^{L-p}\,e^{-L\,Tr([T_m]^{-1}\langle[T]\rangle)}}{\pi^{\frac{p(p-1)}{2}}\,\Gamma(L)\ldots\Gamma(L-p+1)\,|[T_m]|^{L}}$$

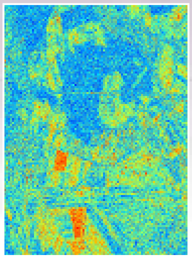


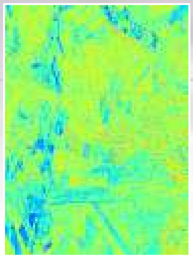
UNSUPERVISED POLSAR CLASSIFICATION

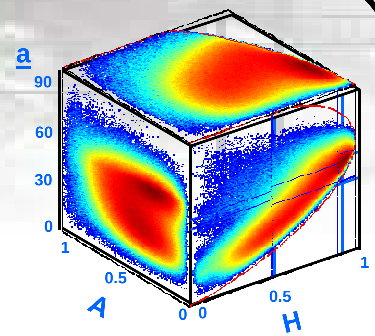
E.POTTIER, J.S LEE (2000)




H

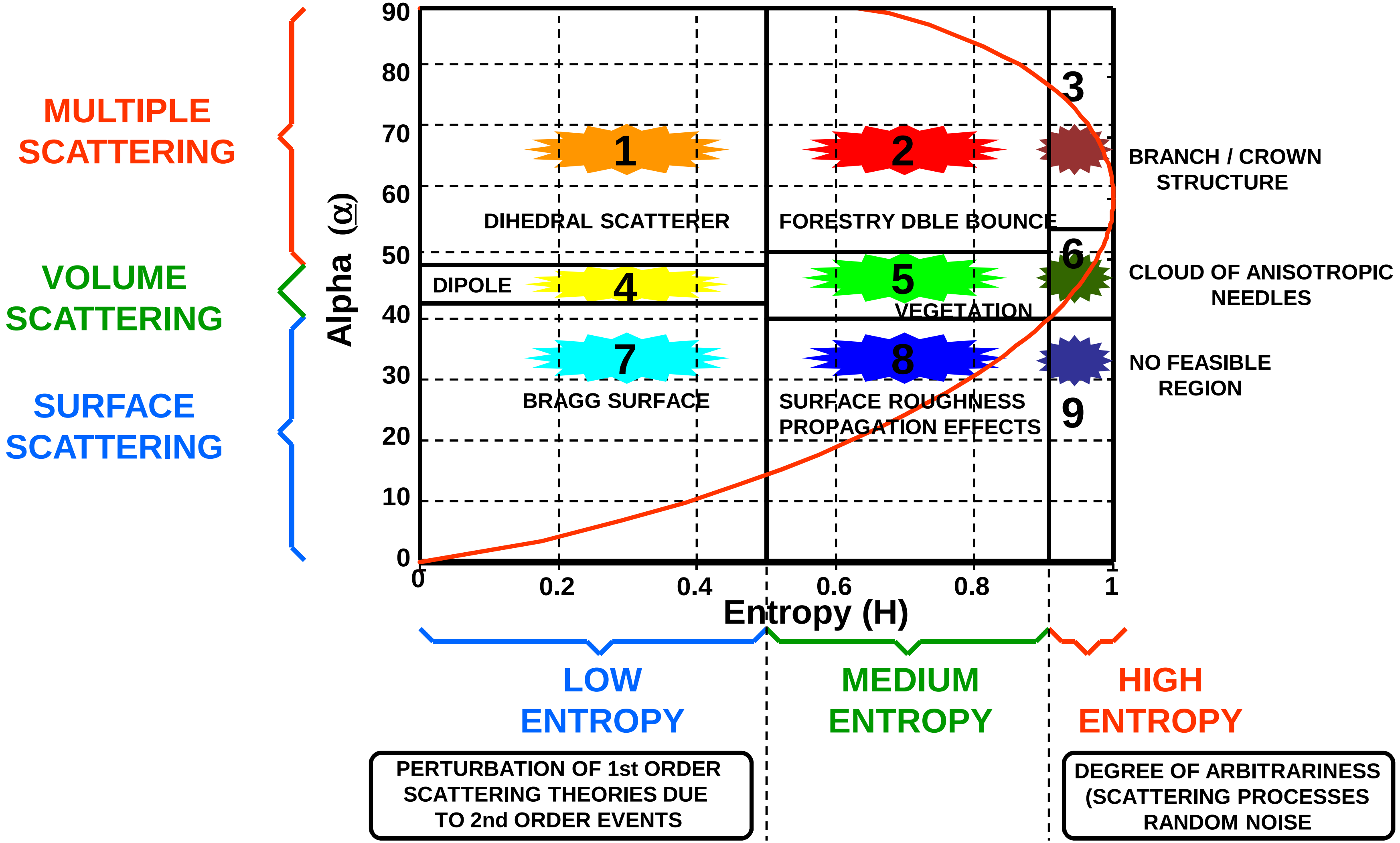

A


α

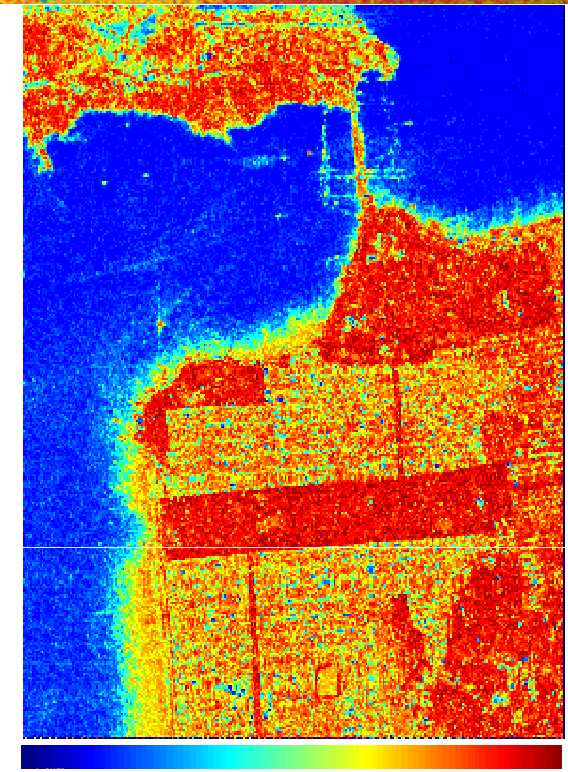


H / A / a DECOMPOSITION THEOREM

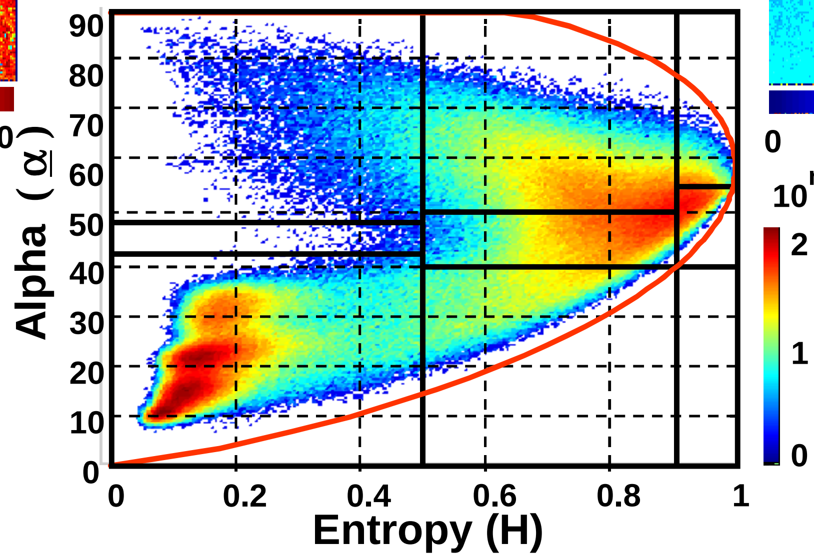
SEGMENTATION OF THE H / α SPACE

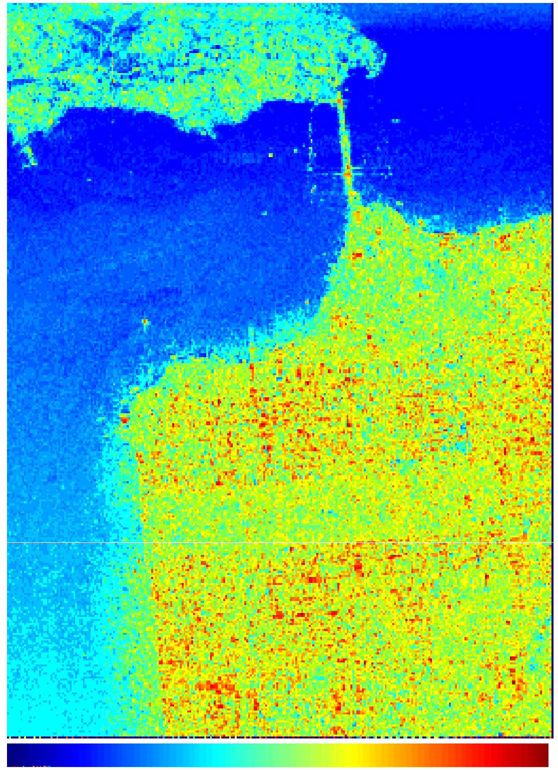


H / α CLASSIFICATION



POLSAR DATA DISTRIBUTION IN THE H / $\underline{\alpha}$ PLANE

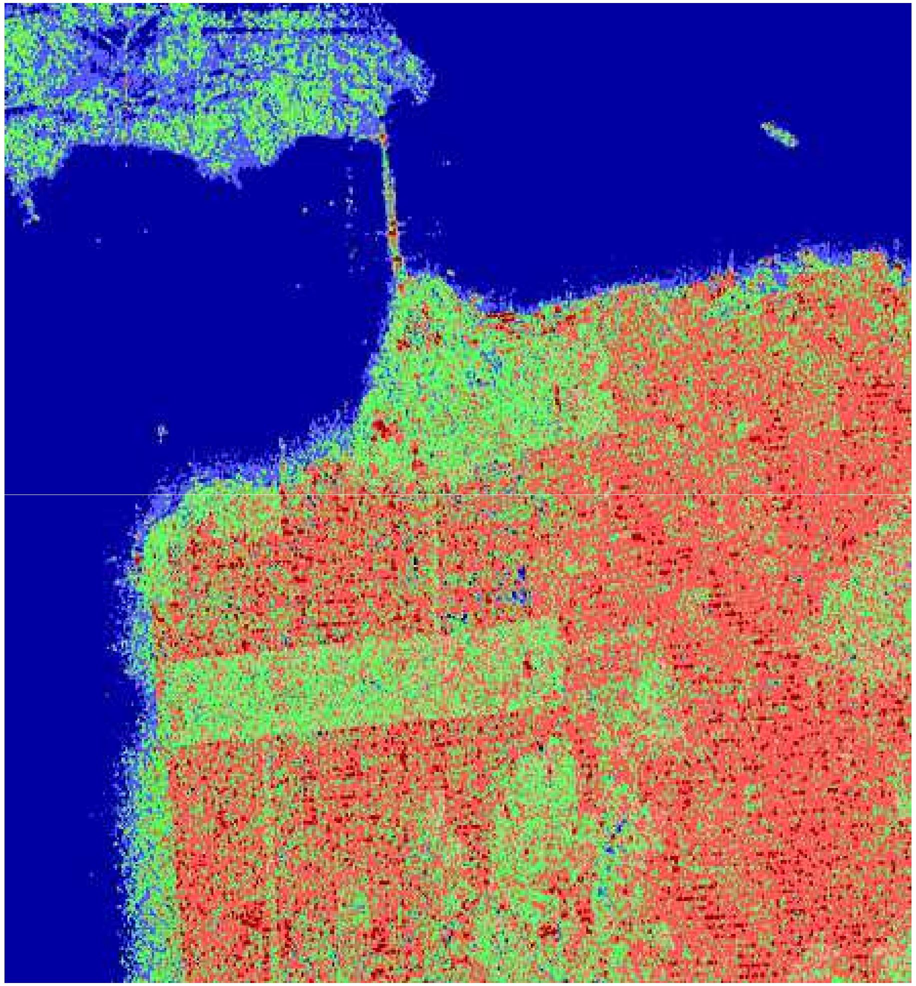


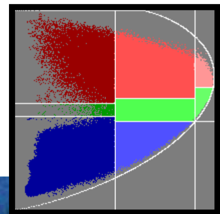


H / α CLASSIFICATION

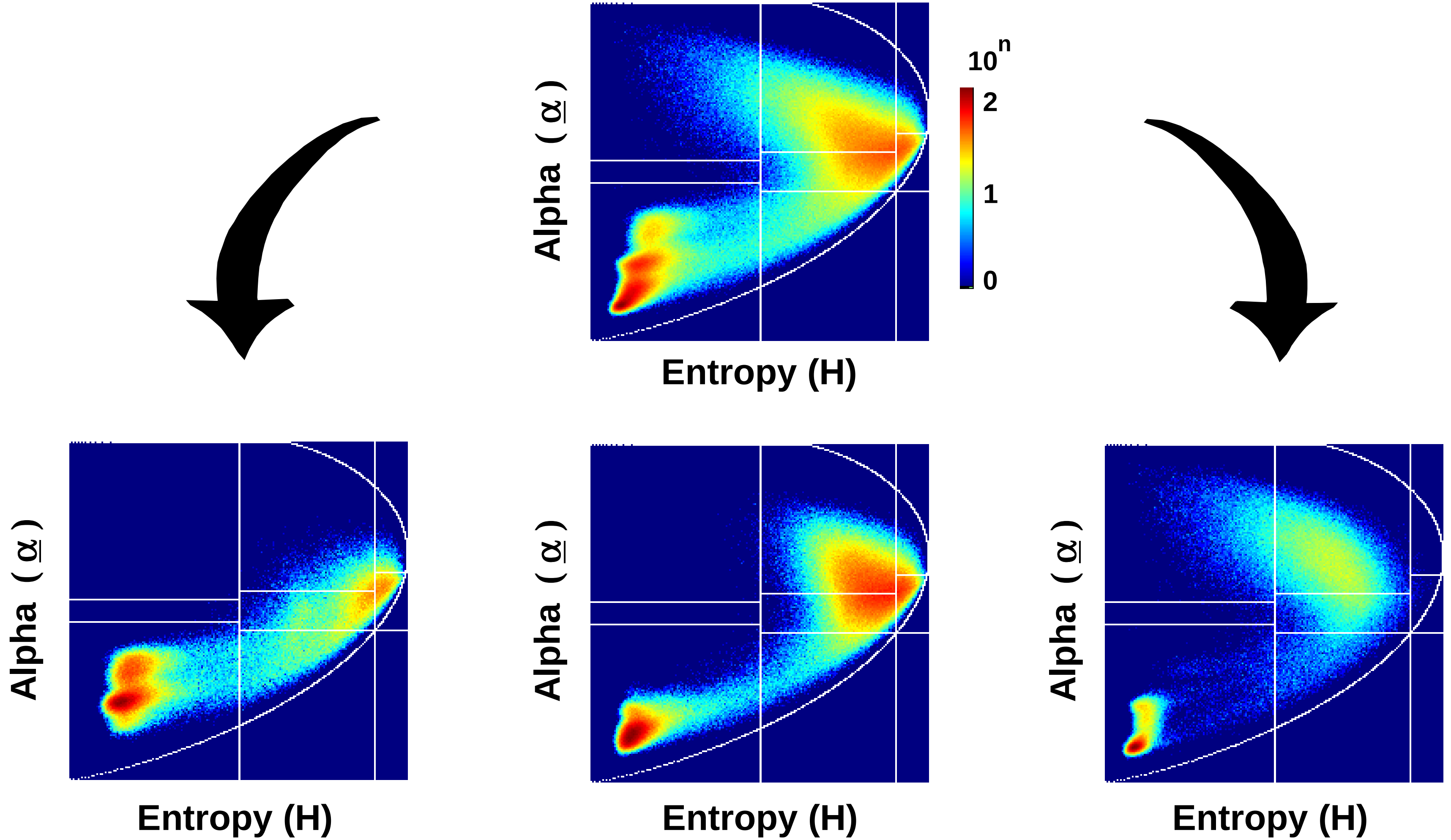
$2A_0$ $B_0 + B$ $B_0 - B$

H - α classification





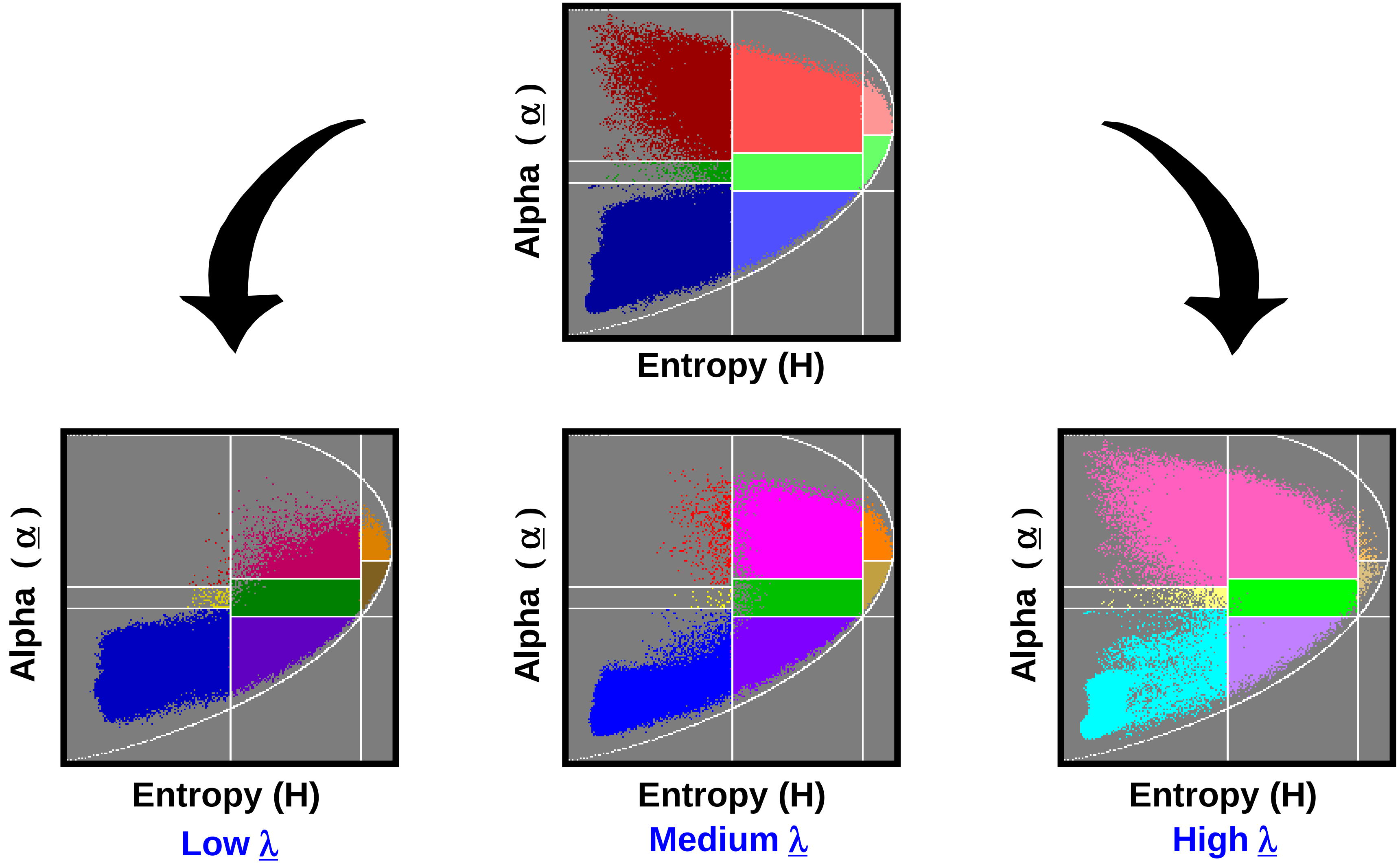
POLSAR DATA DISTRIBUTION IN THE H / α PLANE



Low λ   Medium λ   High λ

Cao Fang, Hong Wen *A New Classification Method Based on Cloude-Pottier Eigenvalue / Eigenvector Decomposition*, IGARSS 05, Seoul, Korea

POLSAR DATA DISTRIBUTION IN THE H / α PLANE

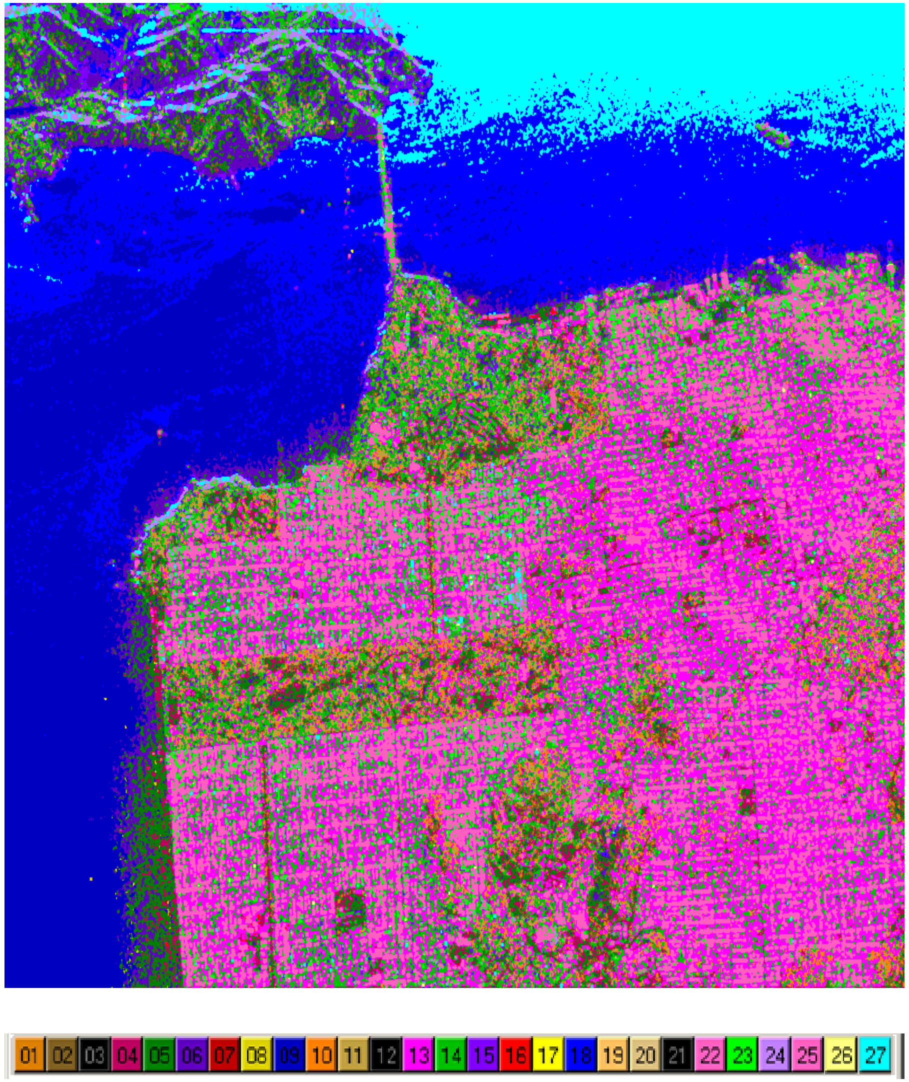


Cao Fang, Hong Wen *A New Classification Method Based on Cloude-Pottier Eigenvalue / Eigenvector Decomposition,* IGARSS 05, Seoul, Korea

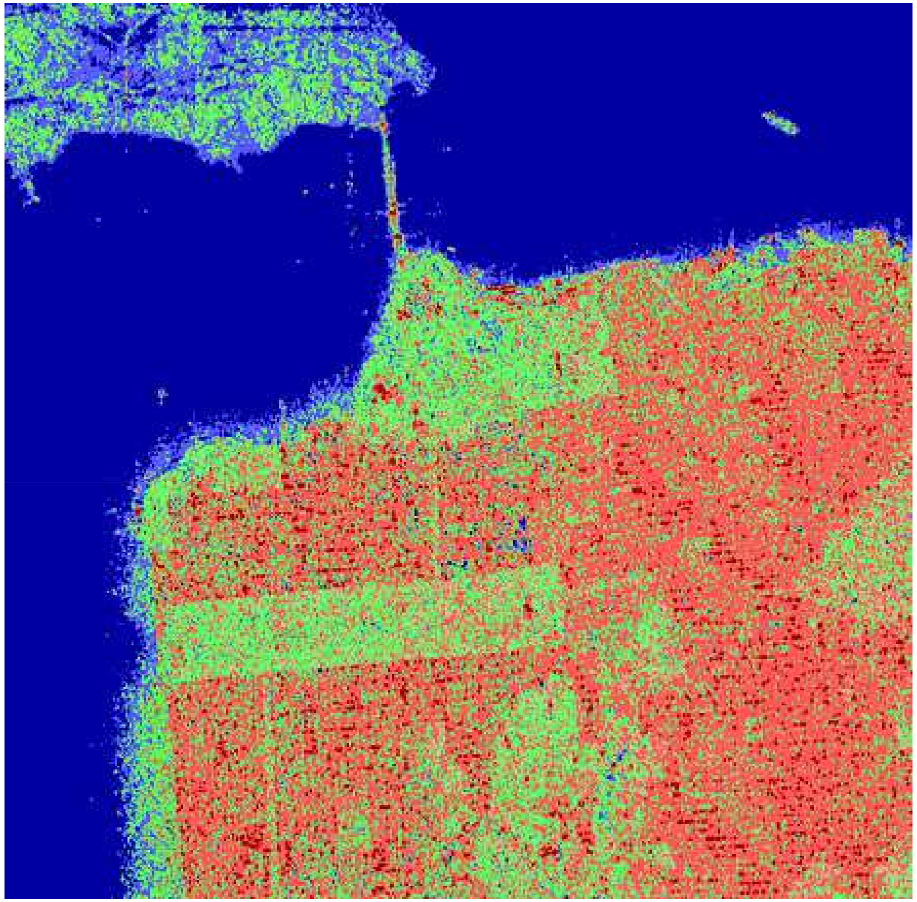
H / $\underline{\alpha}$ / span CLASSIFICATION

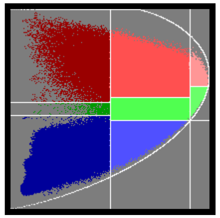
H – $\underline{\alpha}$ ($\underline{\lambda}$) classification

$2A_0$ $B_0 + B$ $B_0 - B$



H-α classification





H / α Classification Space
Sub-divised into 9 basic zones

Location of the boundaries is arbitrary and generically

Degree of arbitrariness on the setting of these boundaries

Segmentation is offered merely to illustrate the unsupervised classification strategy and to emphasize the geometrical segmentation of physical scattering processes

Dr J.S. LEE
N.R. L US -NAVY

1994 ***LEE et al.*** **PROPOSED A SUPERVISED ALGORITHM BASED ON THE COMPLEX WISHART DISTRIBUTION FOR THE COMPLEX COVARIANCE / COHERENCY MATRIX.**

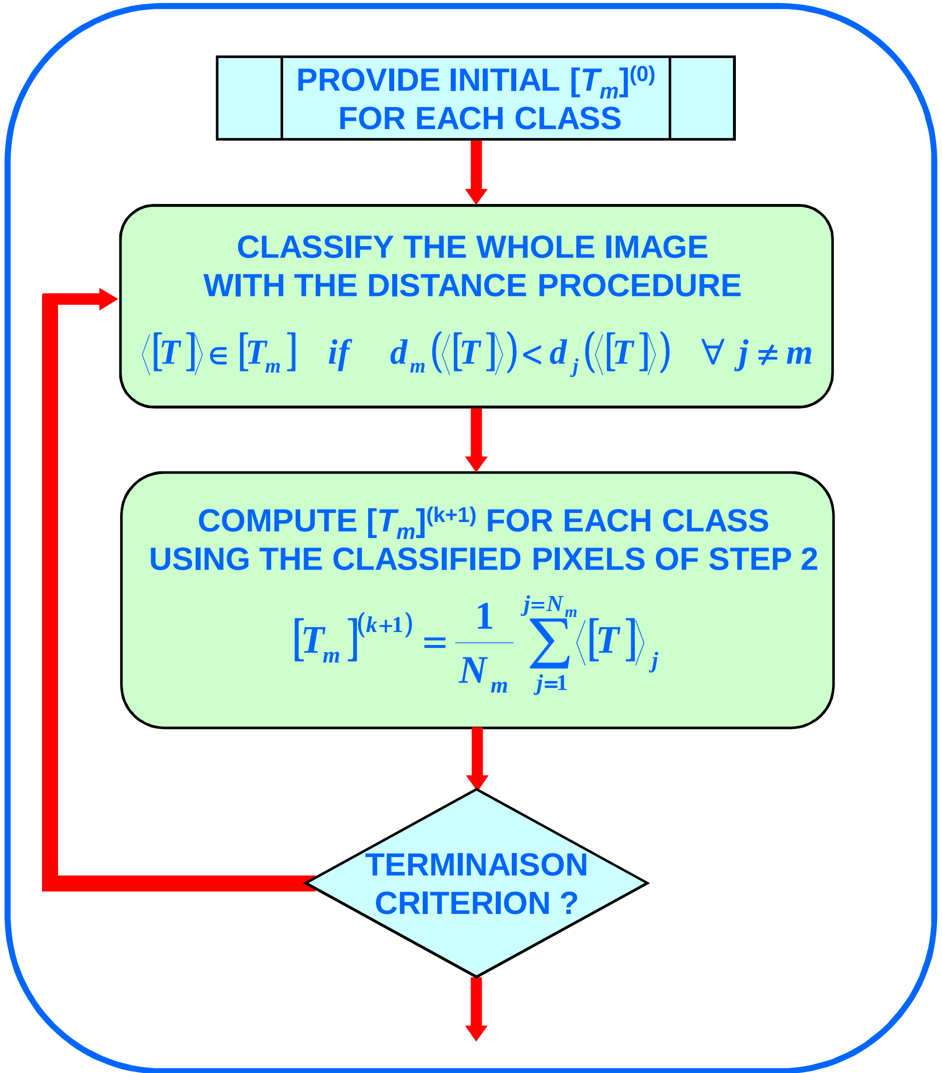
1998 ***LEE et al.*** **DEVELOPED A COMBINED UNSUPERVISED CLASSIFICATION METHOD THAT USES THE H / $\underline{a}$ PLANE WHICH INITIALLY CLASSIFIES THE POLSAR IMAGE. THIS SEGMENTED IMAGE IS THEN USED AS TRAINING SETS FOR THE INITIALIZATION OF THE SUPERVISED WISHART CLASSIFIER.**

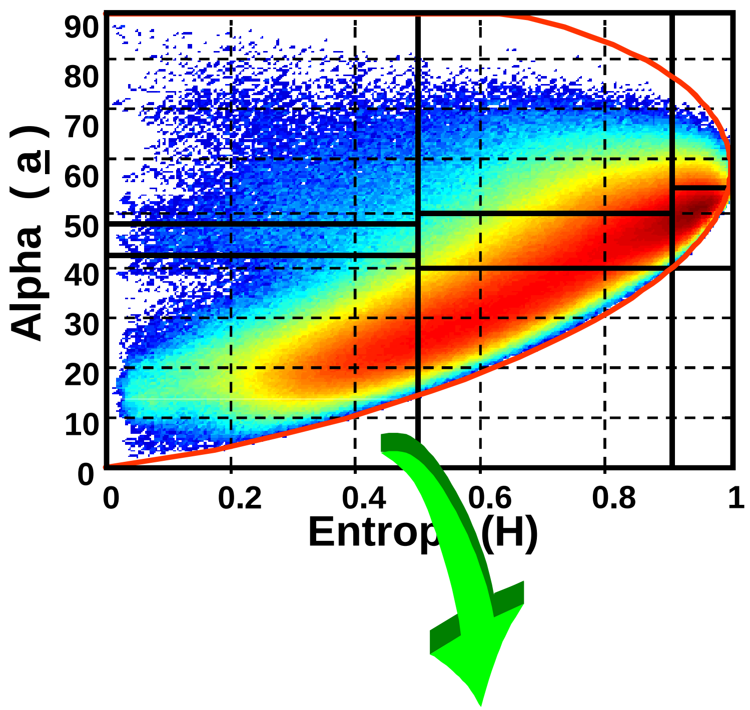
1999 **INTRODUCTION OF THE ANISOTROPY (*E. POTTIER - J.S.LEE*)**

IMPROVEMENT OF THE CAPABILITY TO DISTINGUISH BETWEEN DIFFERENT CLASSES WHOS CENTERS END IN THE SAME ENTROPY (H) AND ALPHA ($\underline{\alpha}$) ZONE.

H / α - WISHART CLASSIFIER

k - mean CLASSIFICATION PROCEDURE





$$[T_m]^{(0)} = \frac{1}{N_m} \sum_{k=1}^{k=N_m} \langle [T] \rangle_k$$

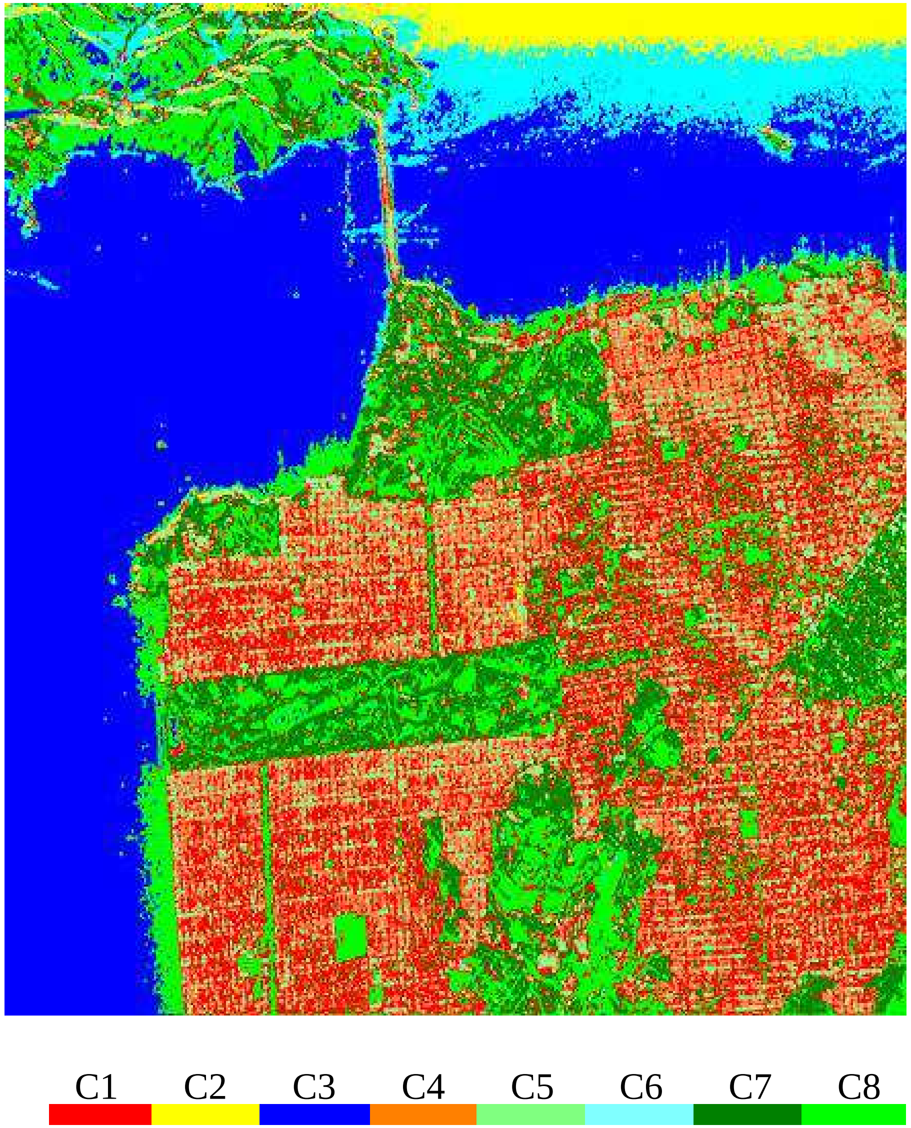
Cluster Center of the class *m*
(Lee 1998)

H / α - WISHART CLASSIFIER

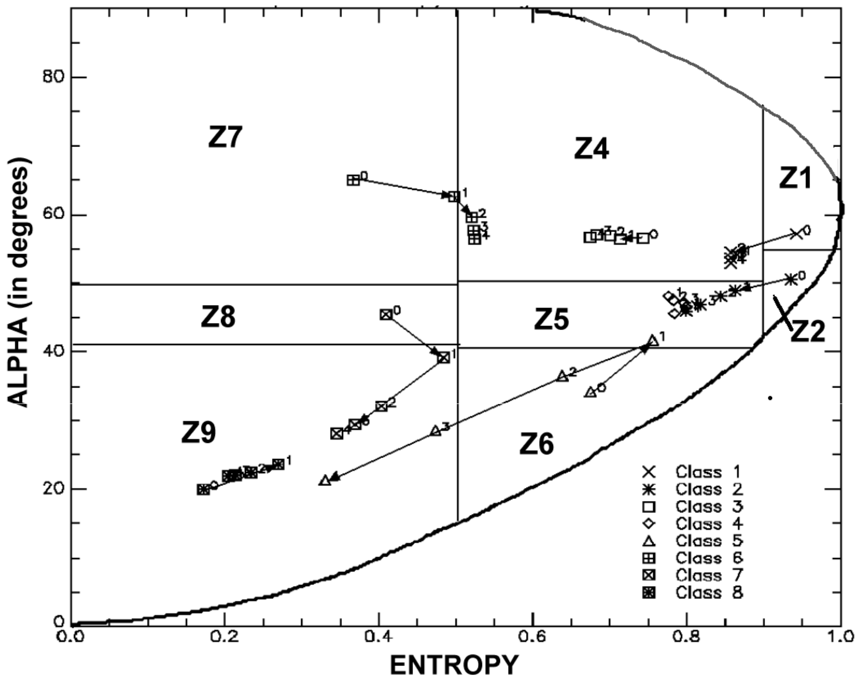
SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

$2A_0$ $B_0 + B$ $B_0 - B$

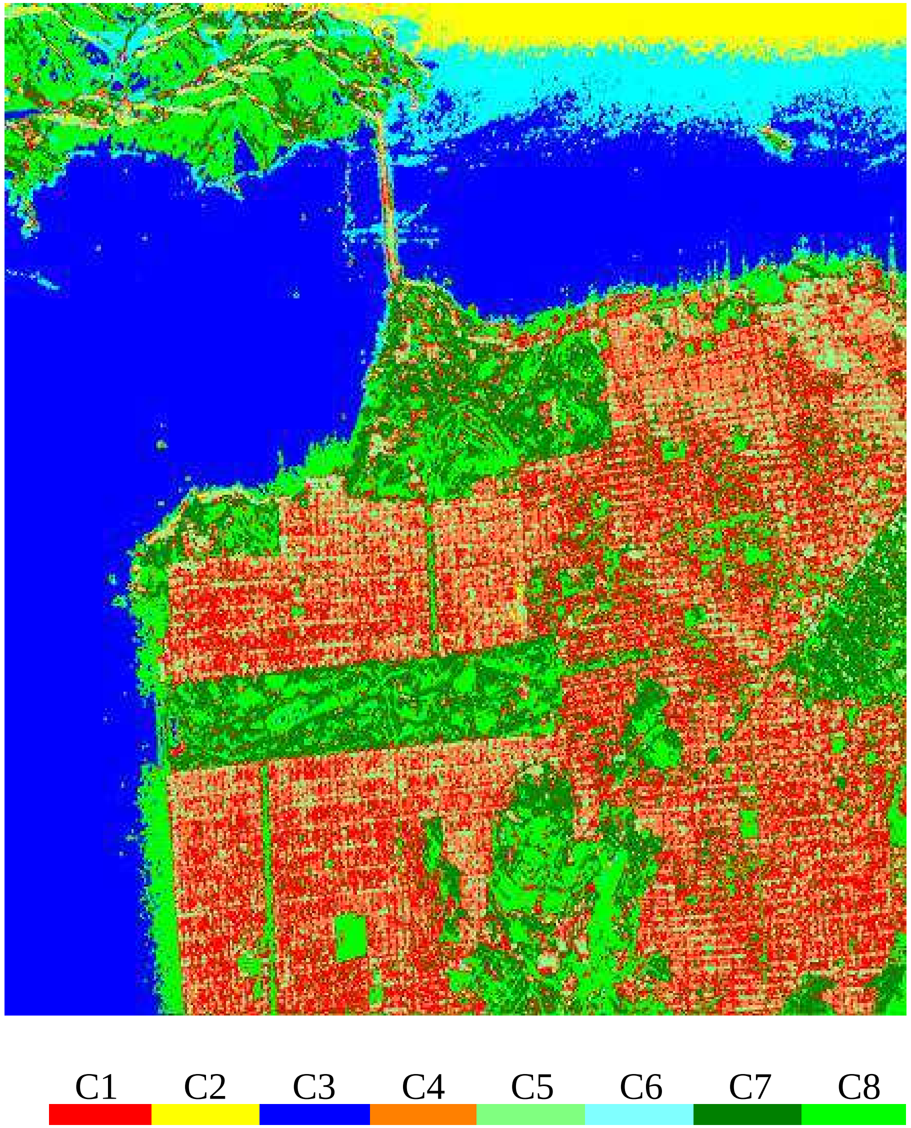
4th ITERATION



4th ITERATION



Cluster centers shifting after each iteration



H / $\underline{\alpha}$ - WISHART CLASSIFIER

During the classification, the cluster centers can move out their zones or several clusters may end in the same zone

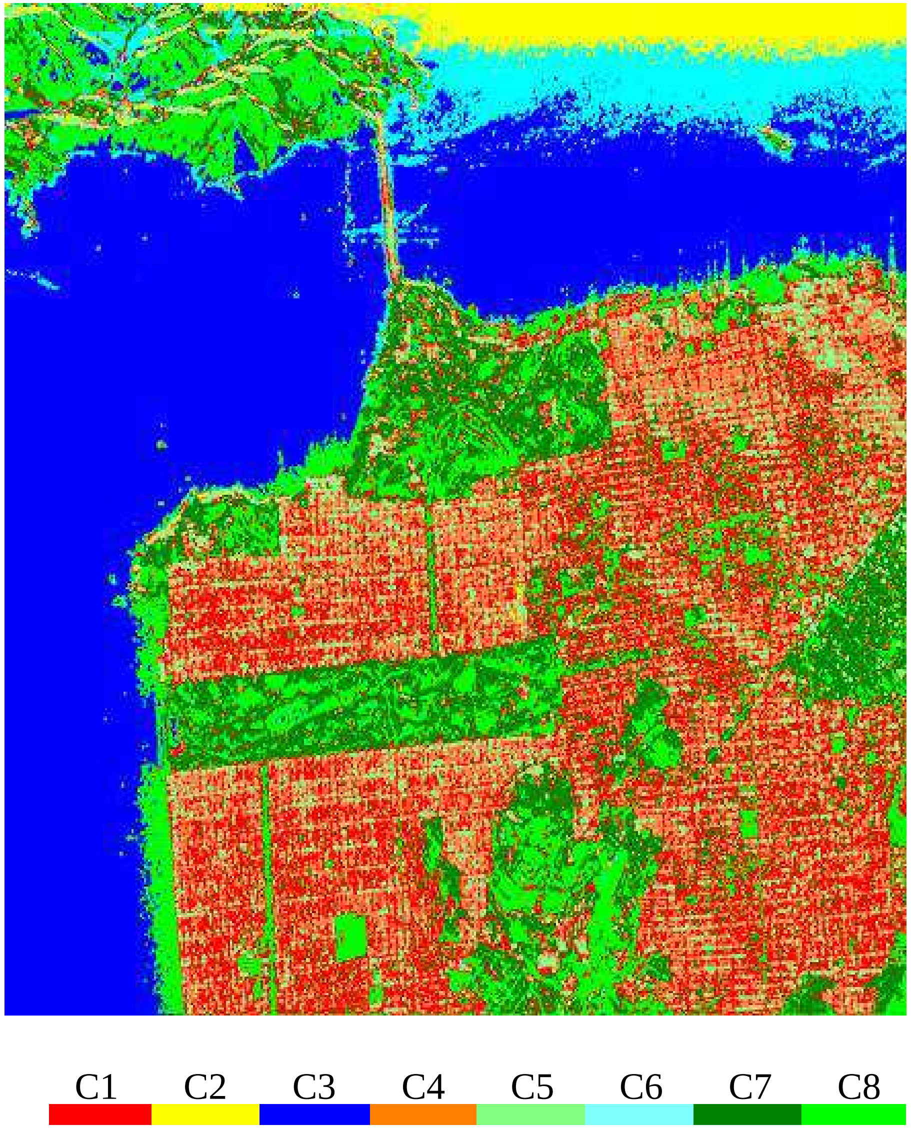
↓

Identification of the terrain type may cause some confusion due to the color scheme

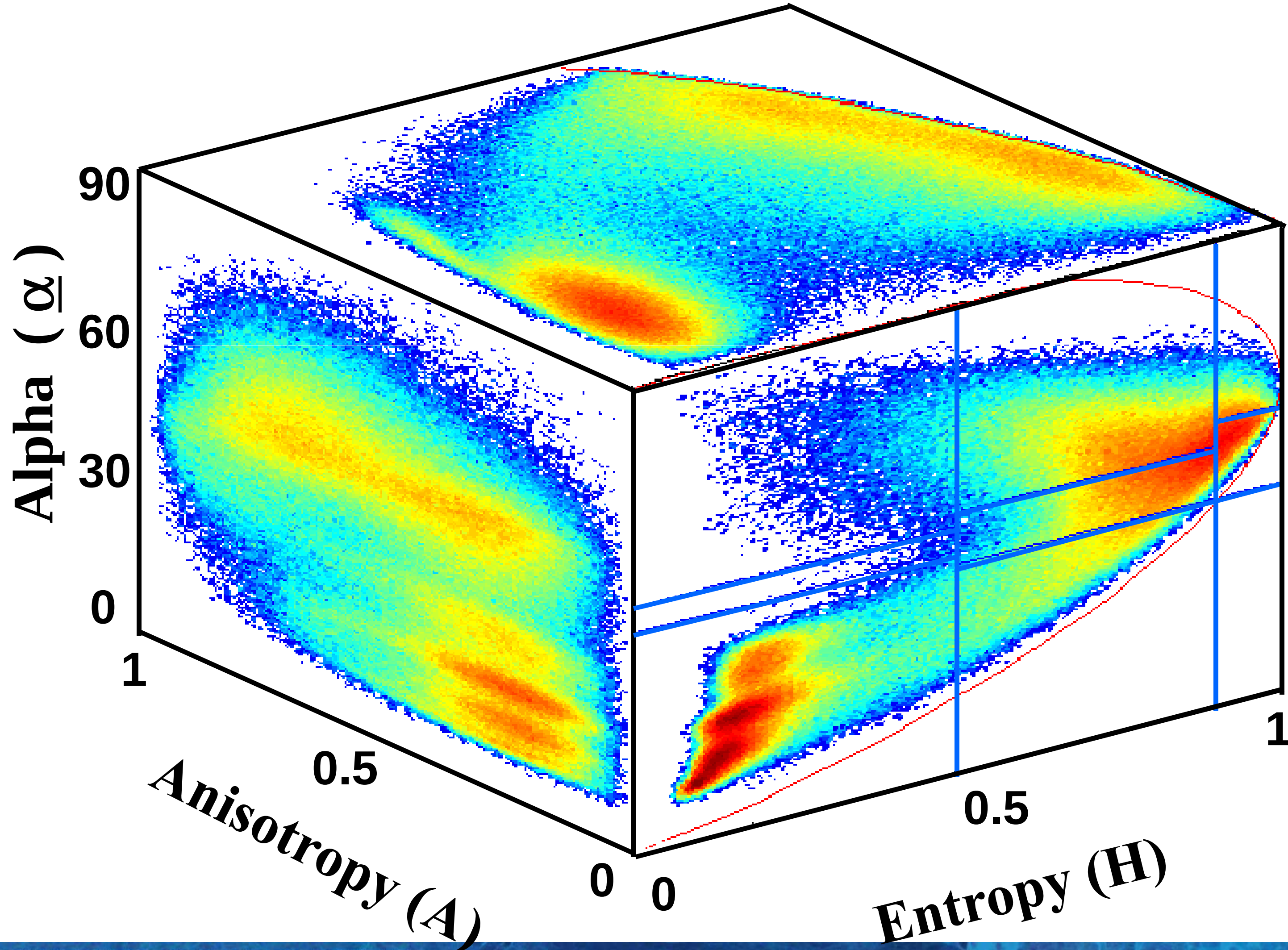
↓

The combined Wishart classifier is extended and complemented with the introduction of the Anisotropy (A)

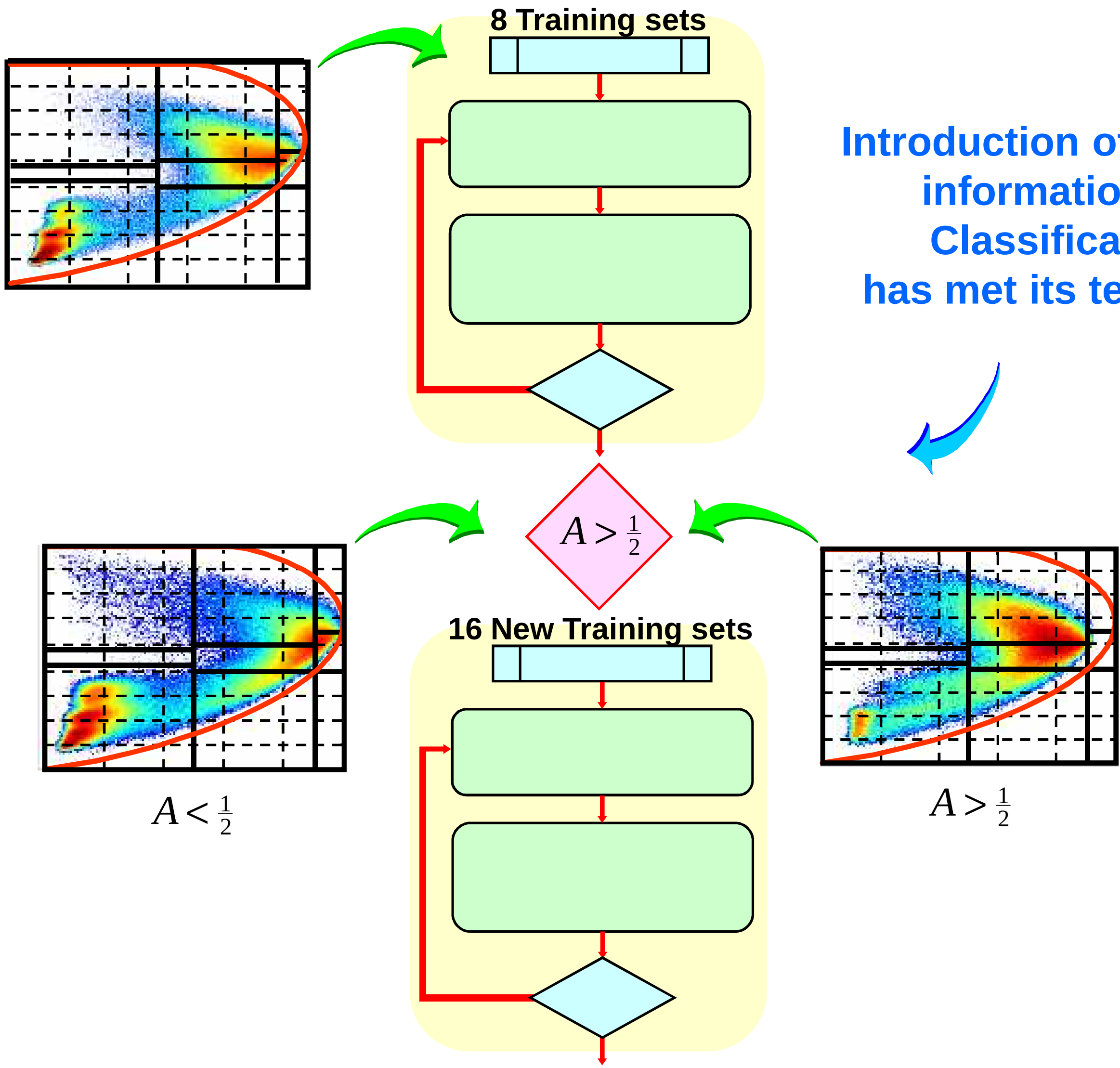
4th ITERATION



POLSAR DATA DISTRIBUTION IN THE H / A / $\underline{\alpha}$ SPACE



2 Successive k - mean Classification procedures



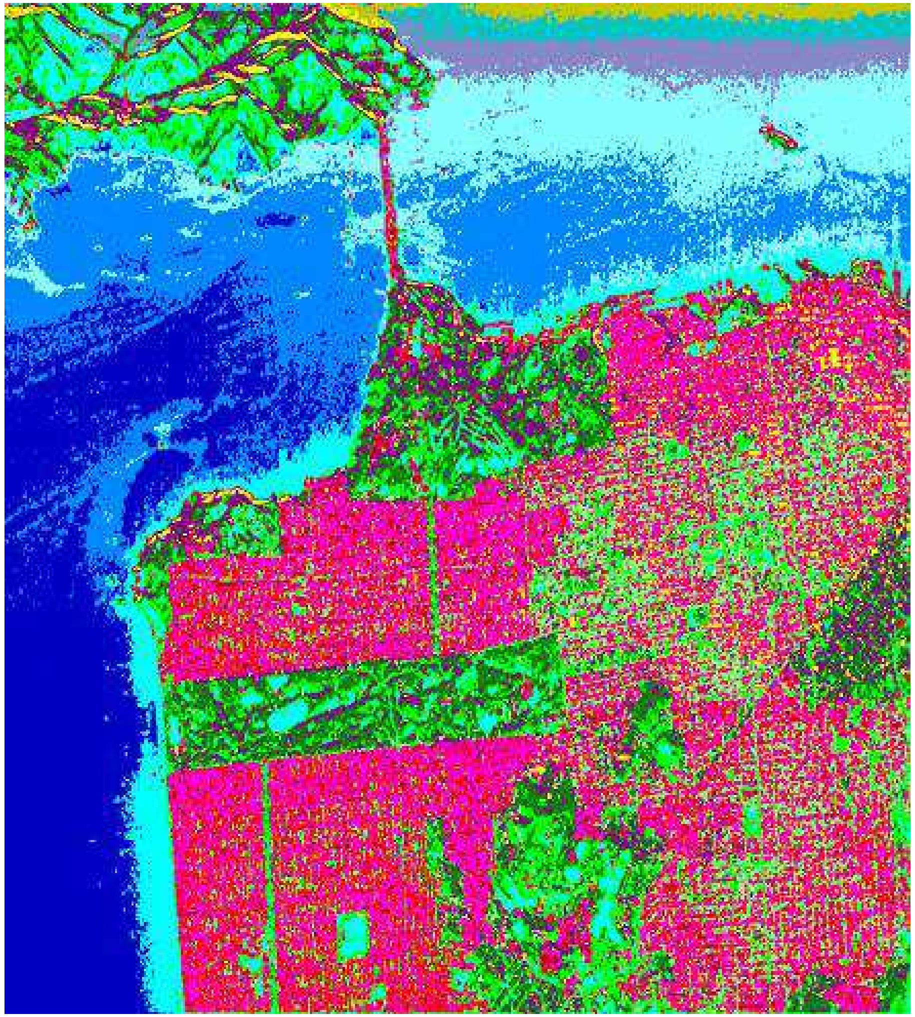
Introduction of the Anisotropy (A) information once the first Classification procedure has met its termination criterion

H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

SAN FRANCISCO BAY JPL - AIRSAR L-band 1988

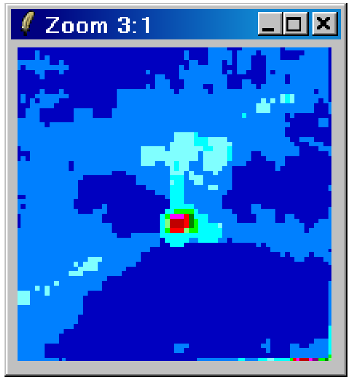
$2A_0$ $B_0 + B$ $B_0 - B$

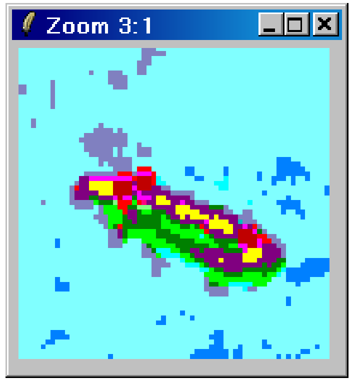
4th ITERATION



C1	C2	C3	C4	C5	C6	C7	C8
C9	C10	C11	C12	C13	C14	C15	C16

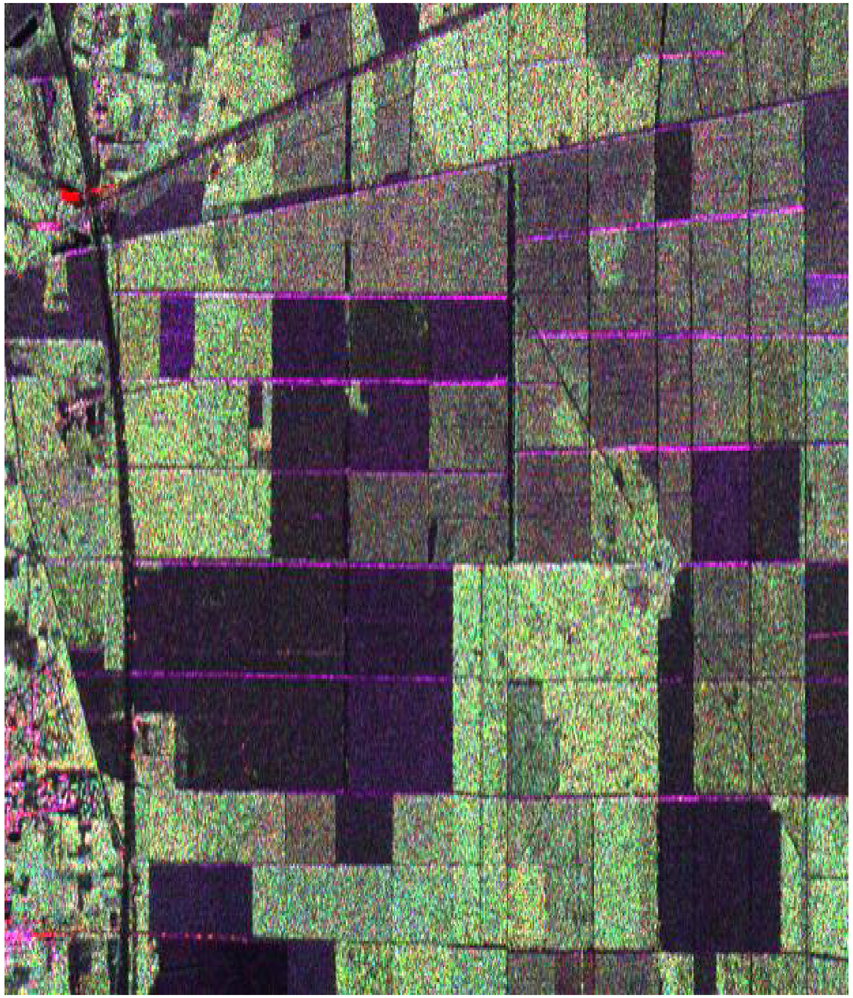
SAN FRANCISCO BAY JPL - AIRSAR L-band 1988



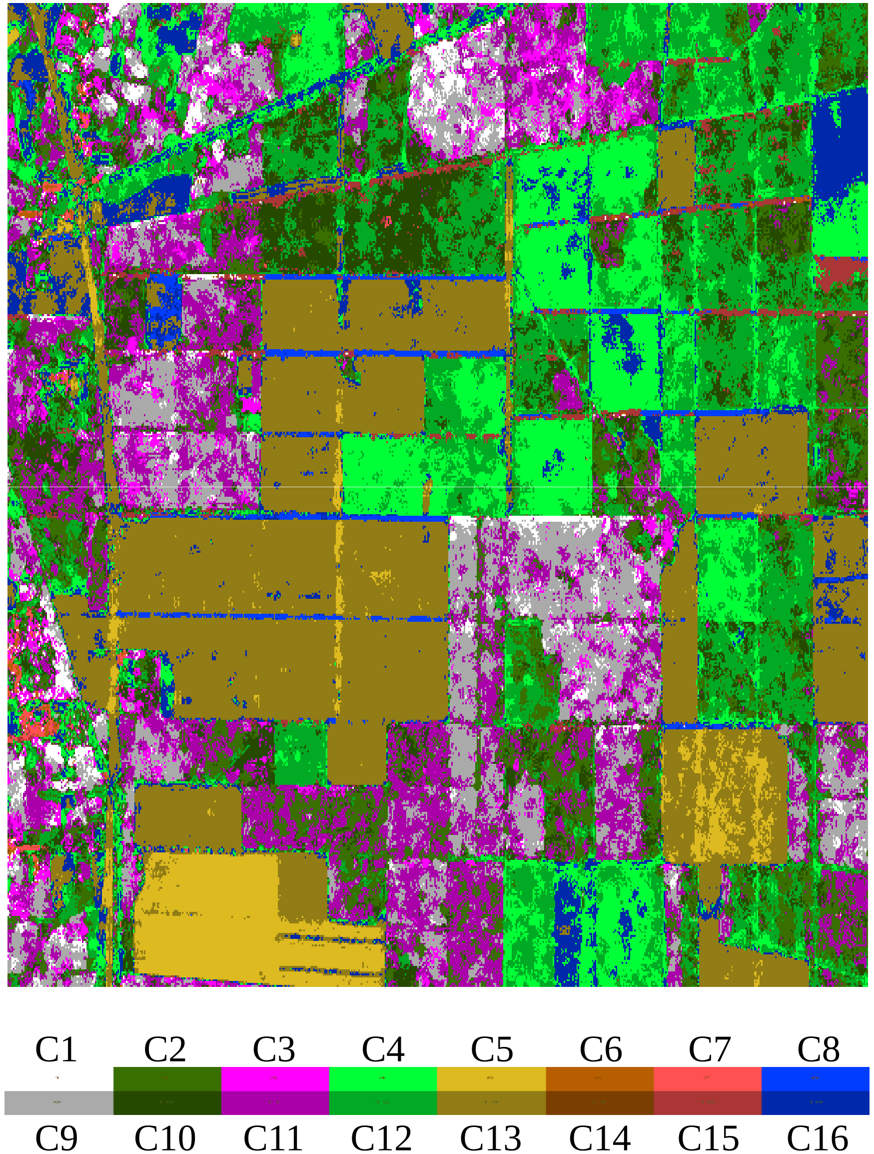


$2A_0$ $B_0 + B$ $B_0 - B$

NEZER FOREST JPL - AIRSAR L-band

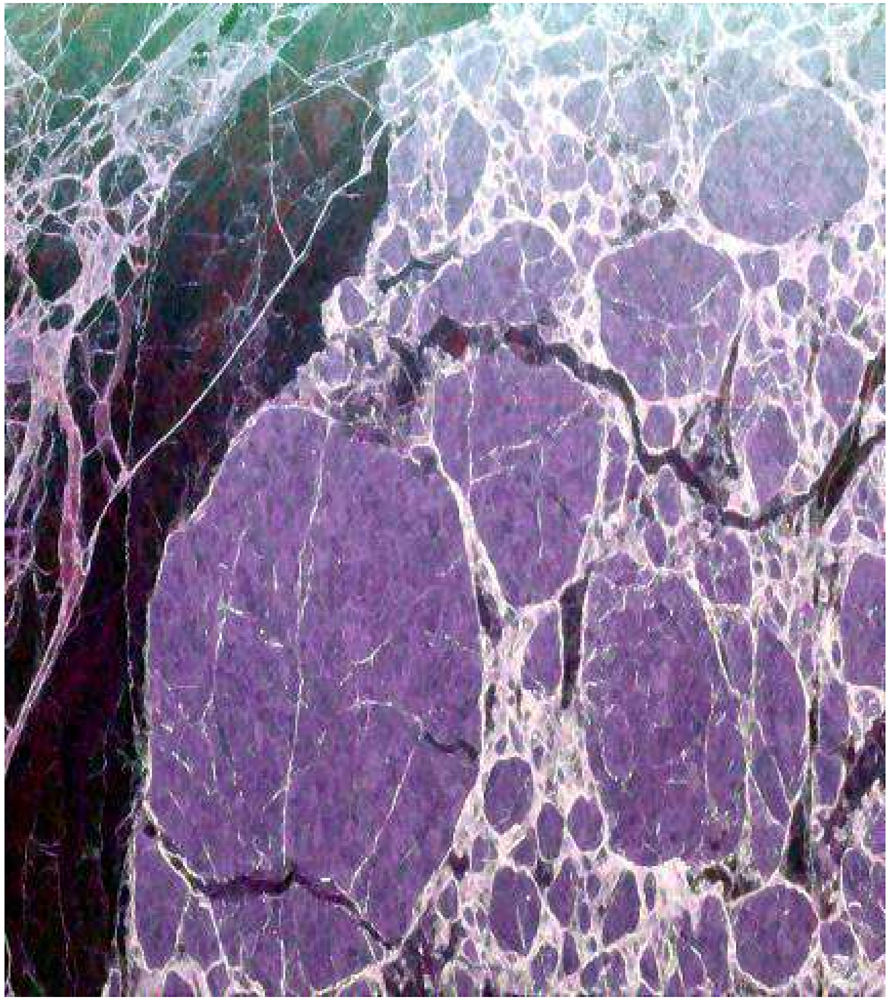


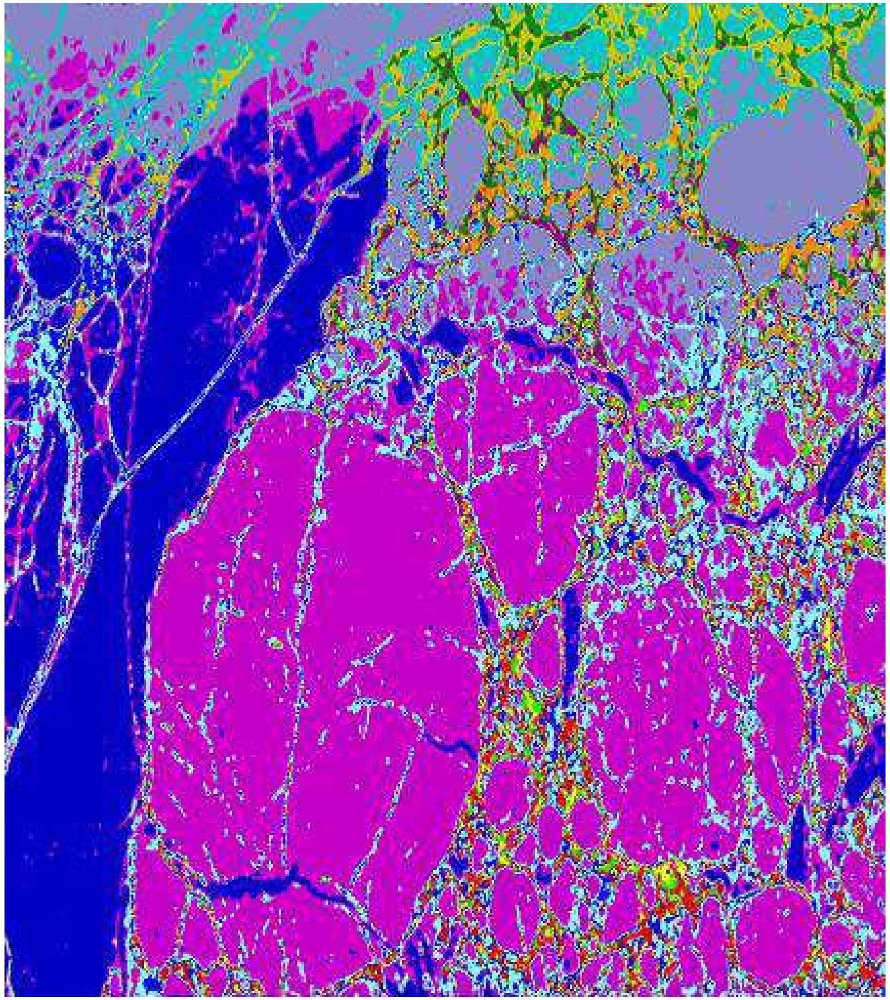
$2A_0$ $B_0 + B$ $B_0 - B$



H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

ICE AREA JPL - AIRSAR L-band





$2A_0$ $B_0 + B$ $B_0 - B$

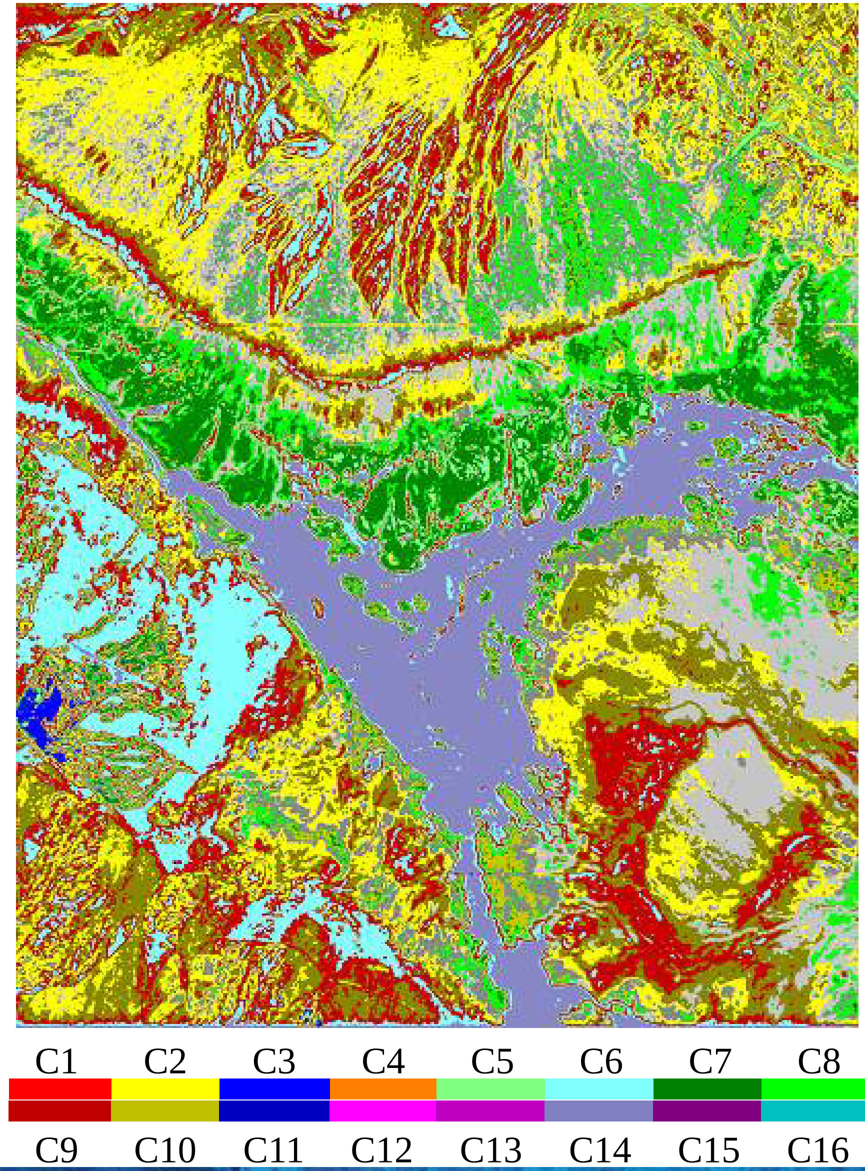
C1	C2	C3	C4	C5	C6	C7	C8
C9	C10	C11	C12	C13	C14	C15	C16

H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

esa

DEATH VALLEY JPL - AIRSAR L-band

$2A_0$ $\quad B_0 + B$ $\quad B_0 - B$

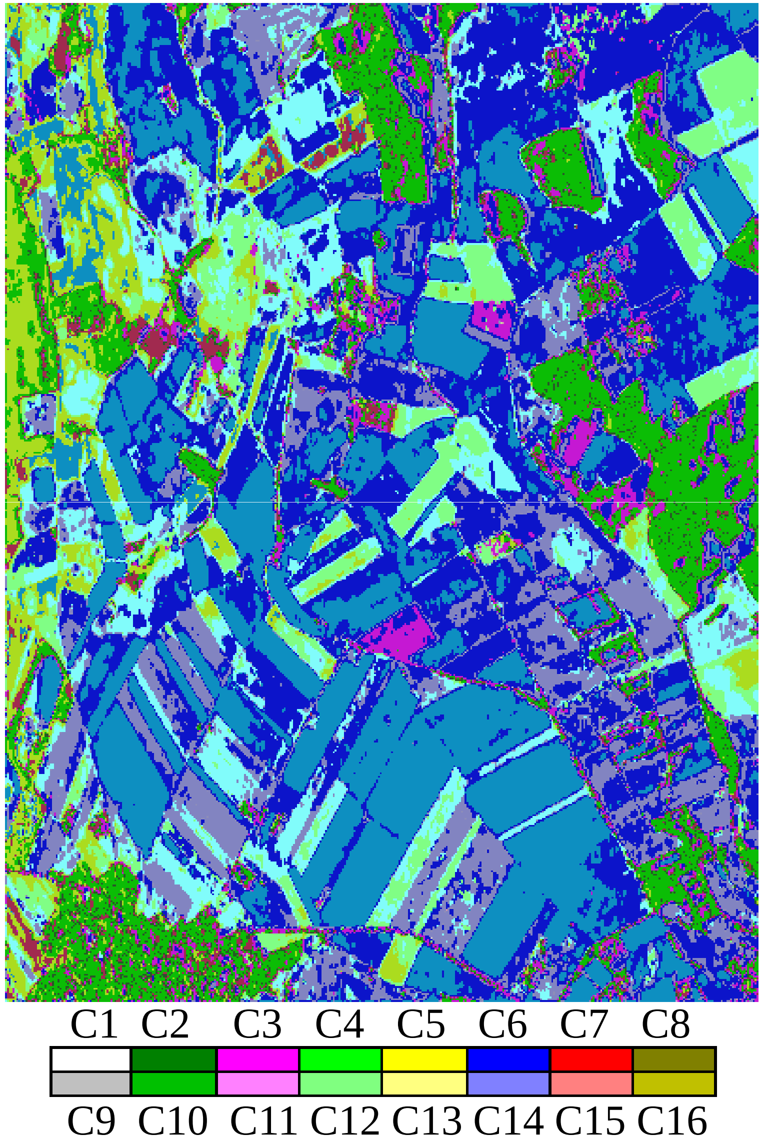


H / A / α - WISHART CLASSIFIER

ALLING - ESAR L-band

$2A_0$ $B_0 + B$ $B_0 - B$

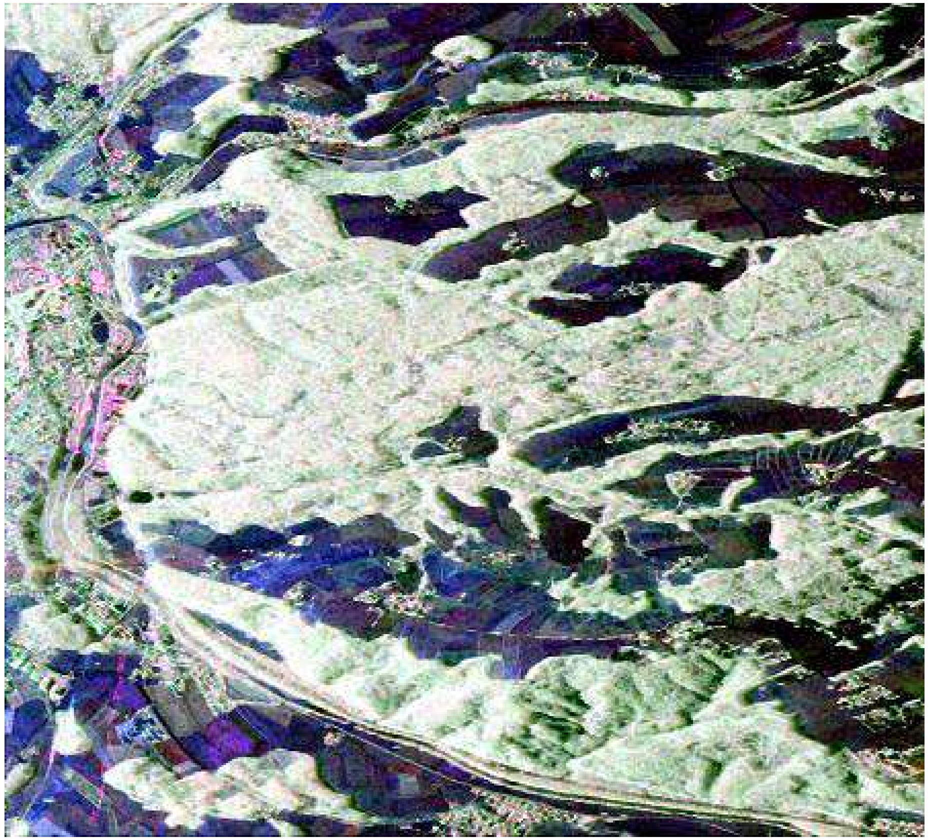
H / A / α and WISHART CLASSIFIER



H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

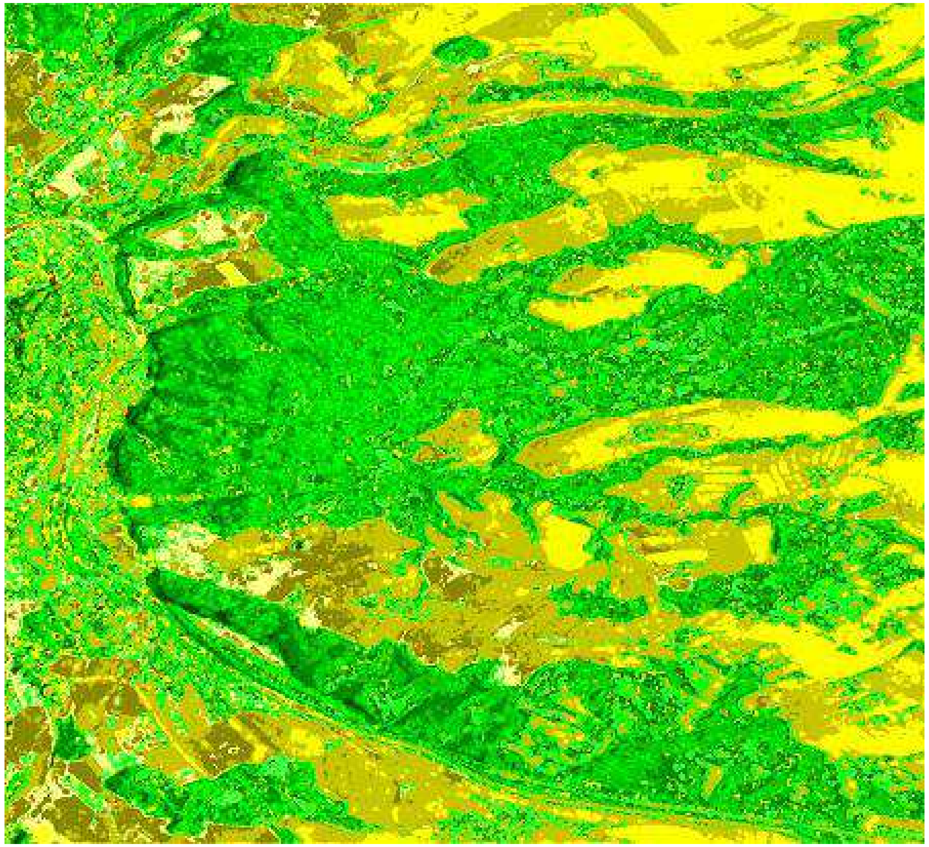
POLinSAR Project

TRAUNSTEIN - ESAR L-band



$2A_0$ $\quad B_0 + B \quad B_0 - B$

H / A / $\underline{\alpha}$ and WISHART CLASSIFIER

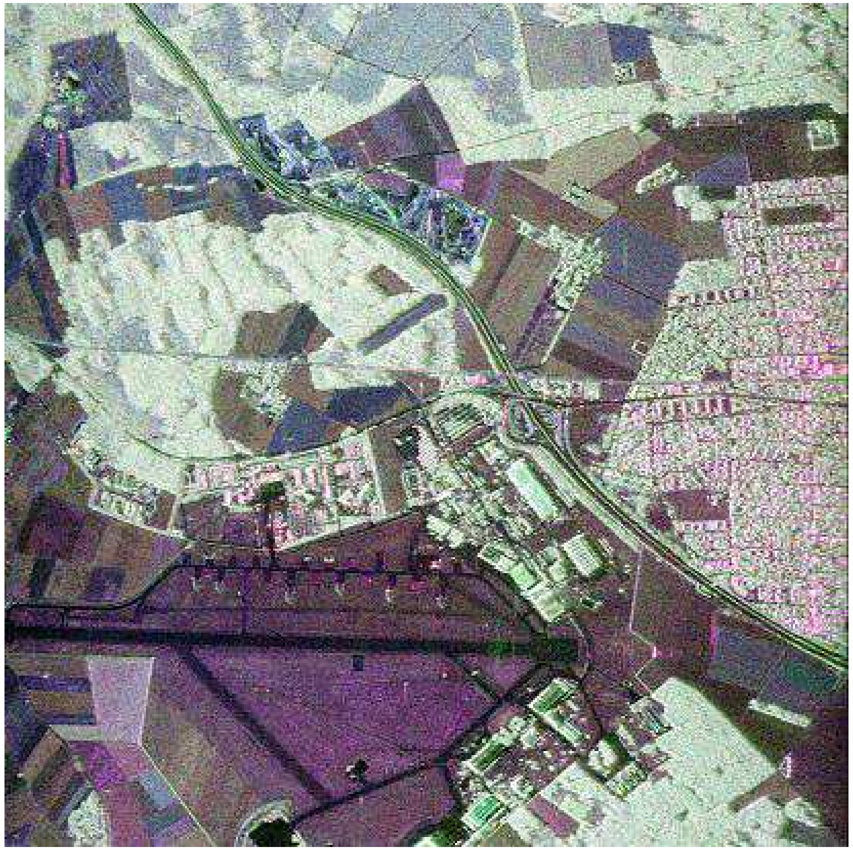


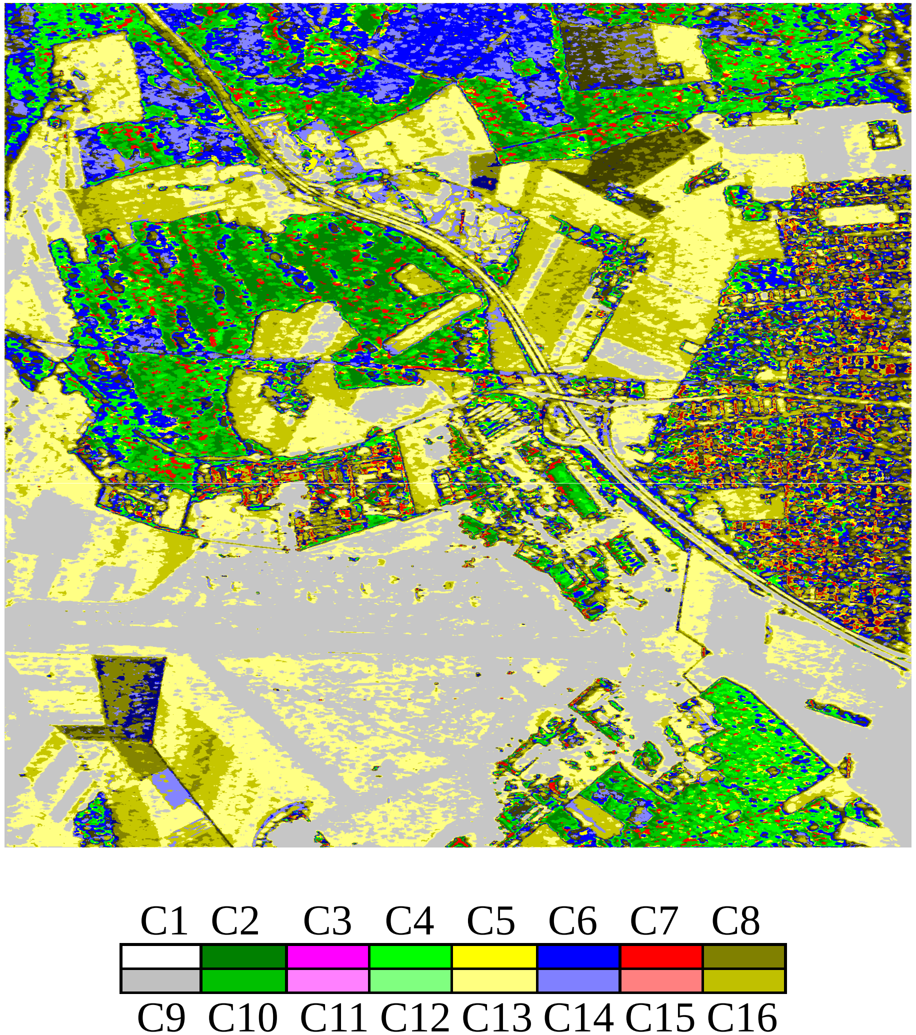
C1	C2	C3	C4	C5	C6	C7	C8
C9	C10	C11	C12	C13	C14	C15	C16

DLR

OBERPFAFFENHOFEN - ESAR L-band

H / A / $\underline{\alpha}$ and WISHART CLASSIFIER



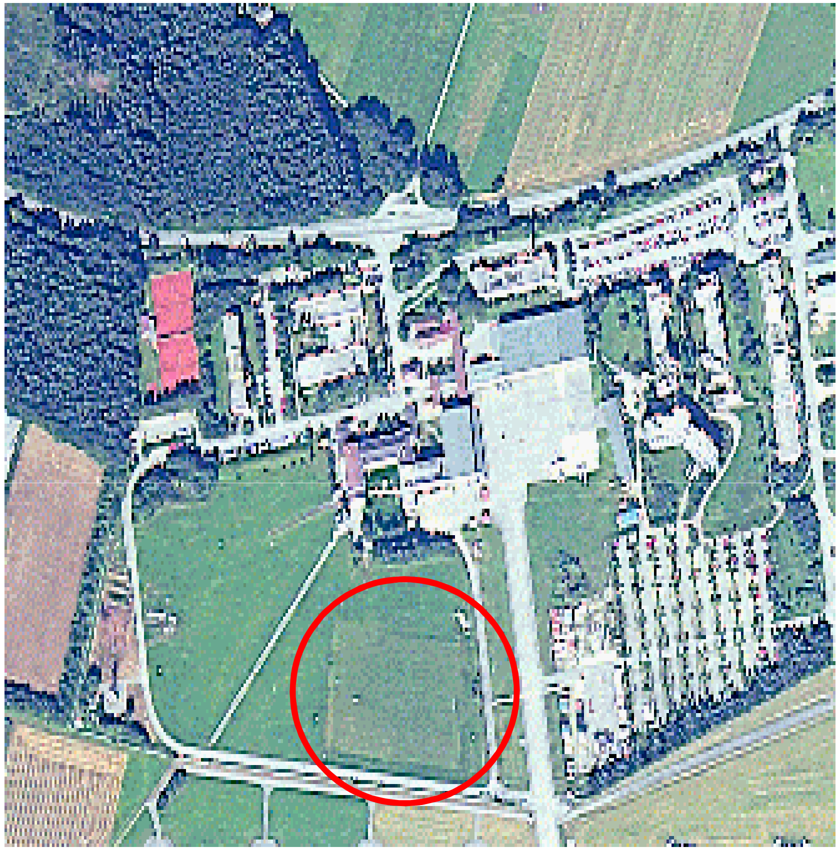


$2A_0$ $B_0 + B$ $B_0 - B$

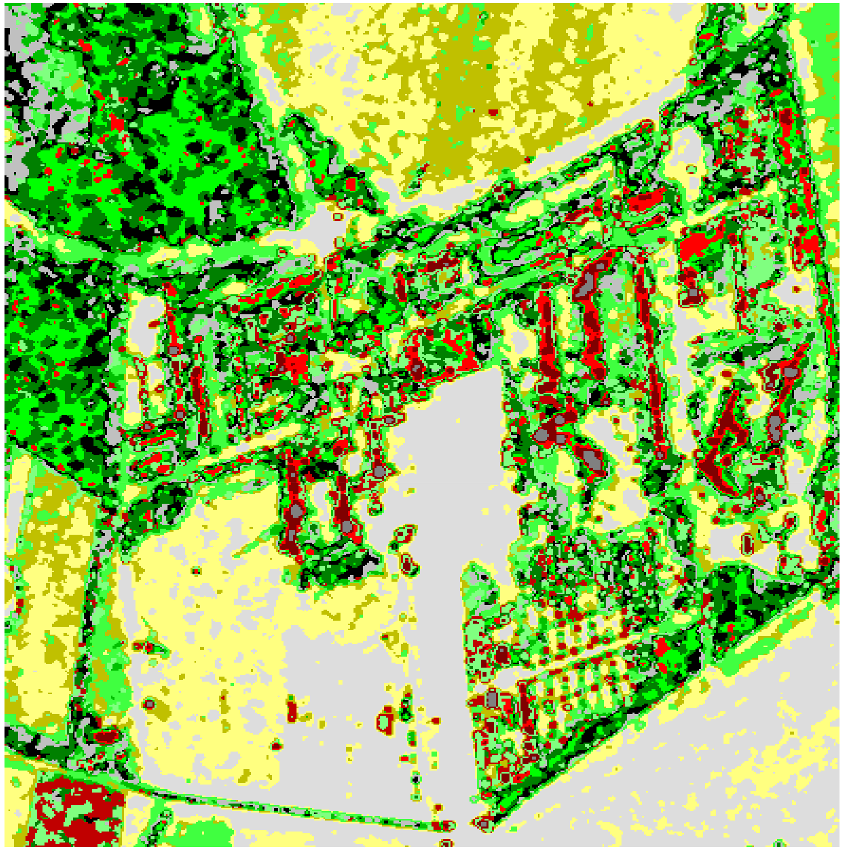
DLR

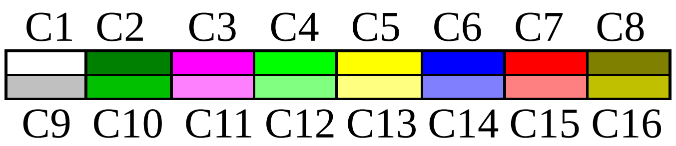
H / A / $\underline{\alpha}$ - WISHART CLASSIFIER

OBERPFAFFENHOFEN - ESAR L-band



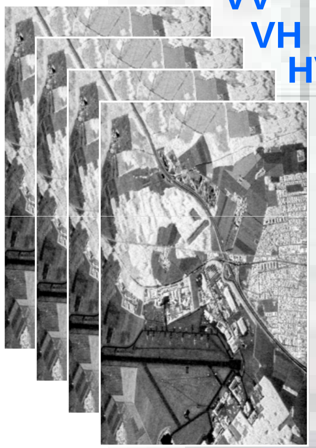
H / A / $\underline{\alpha}$ and WISHART CLASSIFIER





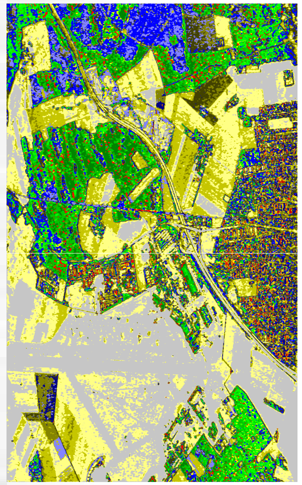
WISHART PDF

$$P\left(\langle[T]\rangle/[T_m]\right)=\frac{L^{Lp}\left|\langle[T]\rangle\right|^{L-p}e^{-LTr\left([T_m]^{-1}\langle[T]\rangle\right)}}{\pi^{\frac{p(p-1)}{2}}\Gamma(L)\ldots\Gamma(L-p+1)\left|[T_m]\right|^{L}}$$



UNSUPERVISED POLSAR CLASSIFICATION

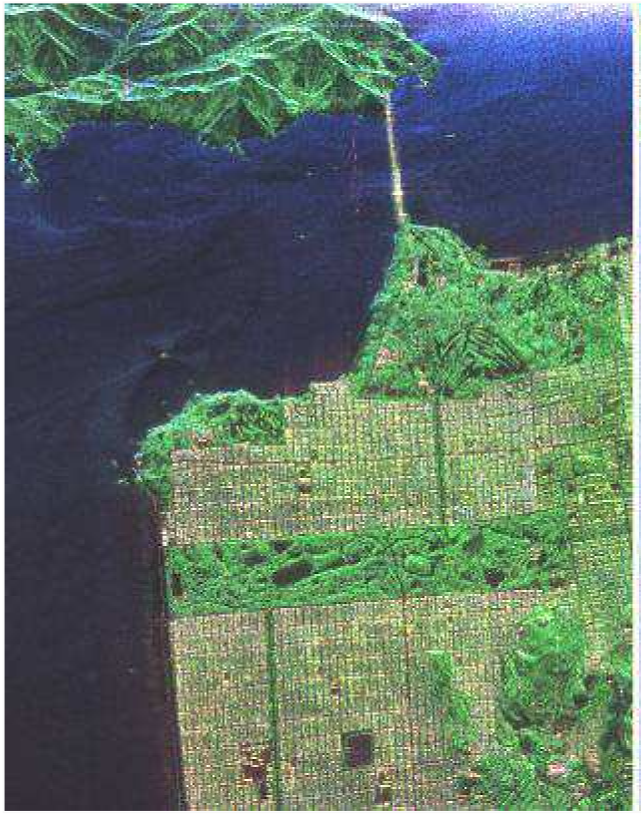
E.POTTIER, J.S LEE (2000)



Unsupervised Classification Preserving Scattering Mechanisms

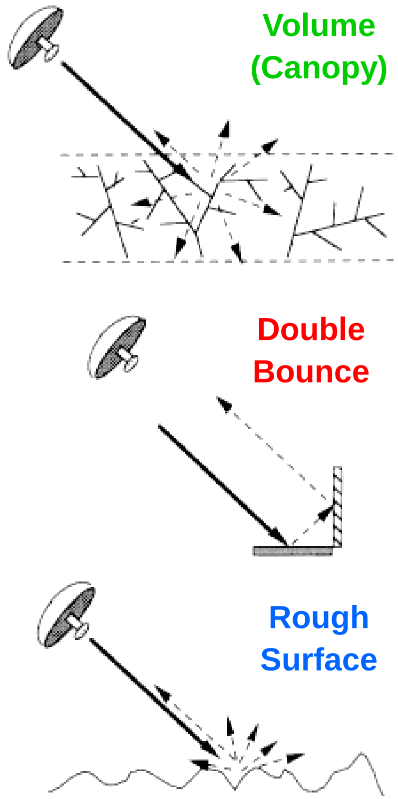
J.S. Lee, M.R. Grunes, E. Pottier and L. Ferro-Famil,"Segmentation of polarimetric SAR images that preserves scattering mechanisms" Proceedings of EUSAR2002

Courtesy of Dr J.S Lee



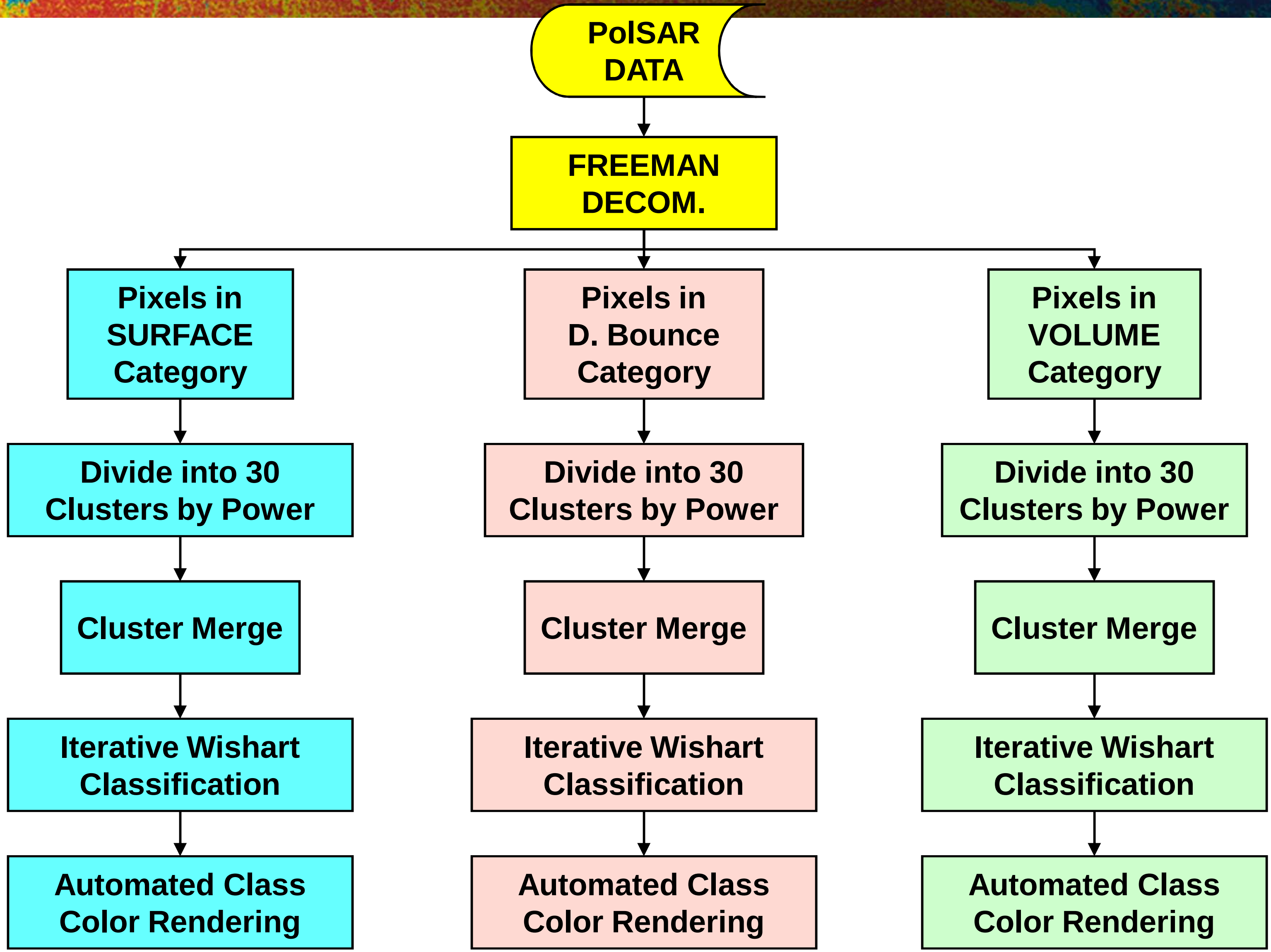
|HH-VV|, |HV|, |HH+VV|

Freeman and Durden



A. Freeman and S.L. Durden, "A Three-Component Scattering Model for Polarimetric SAR Data" IEEE TGRS, vol. 36, no. 3, May 1998

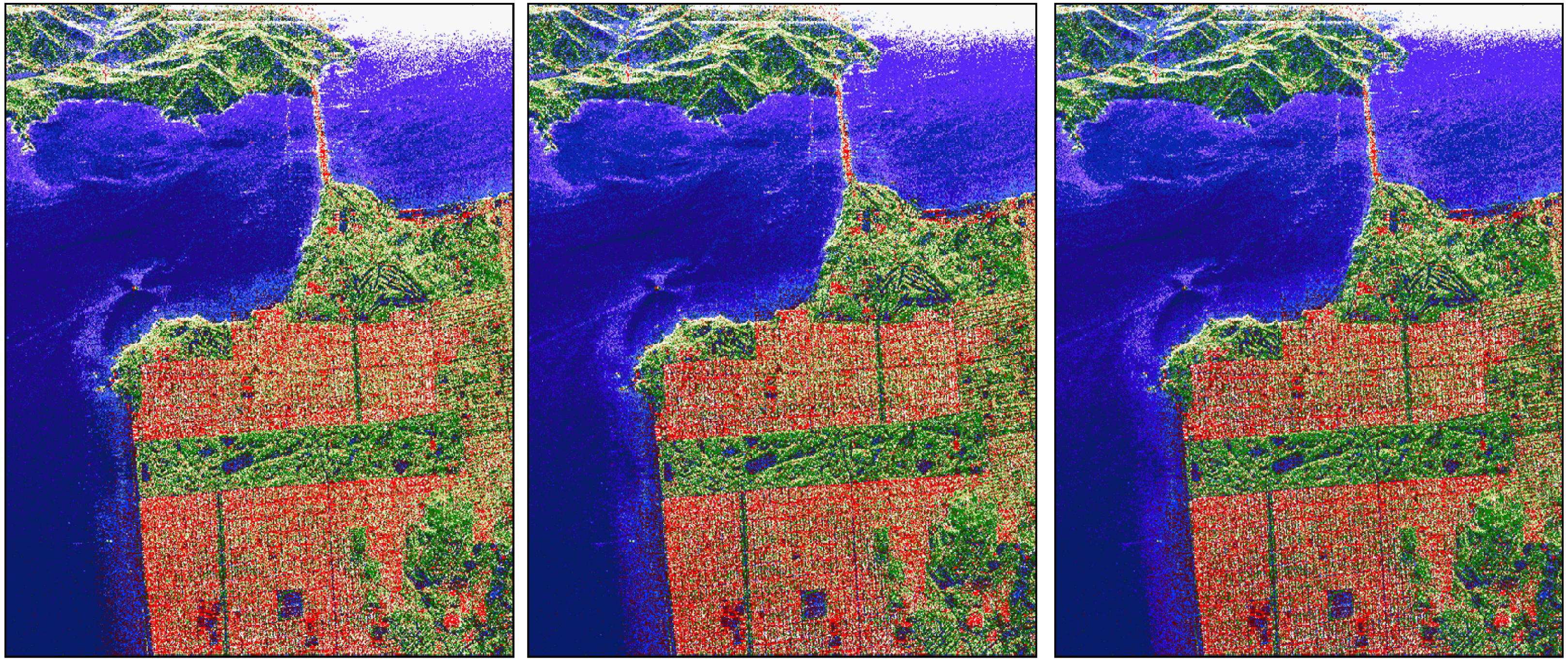
PROCEDURE – FLOW CHART



Cluster Merging $D_{ij} = \frac{1}{2}\{ln(|V_i|) + ln(|V_j|) + Tr(V_i^{-1}V_j + V_j^{-1}V_i)\}$

Wishart Iteration – After Class Merge

Classification Maps



First Iteration **Second Iteration** **Third Iteration**

Note: Stability insures good convergence

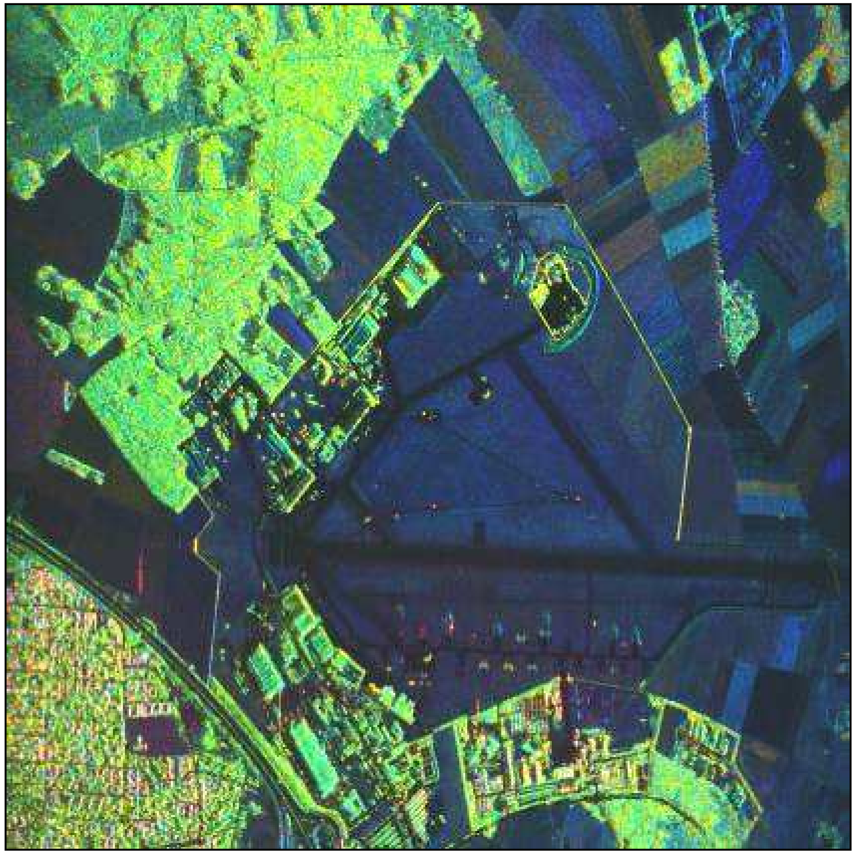
FREEMAN - WISHART CLASSIFIER

Courtesy of Dr J.S Lee

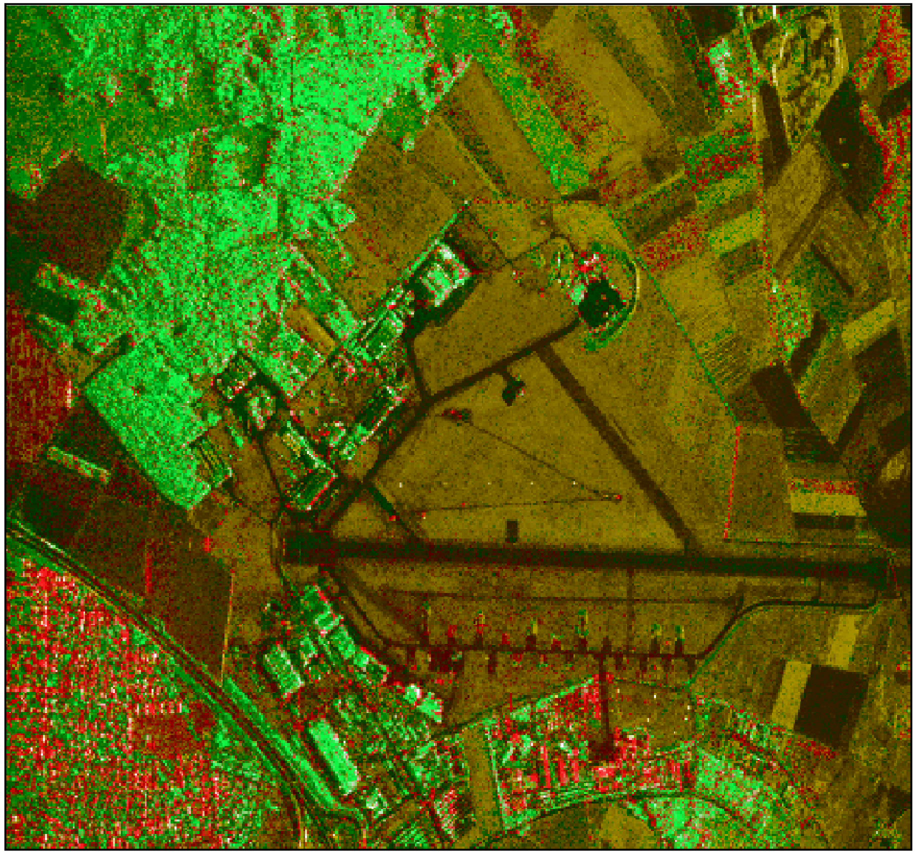
|HH-VV|, |HV|, |HH+VV|

4th Iteration (15 classes)

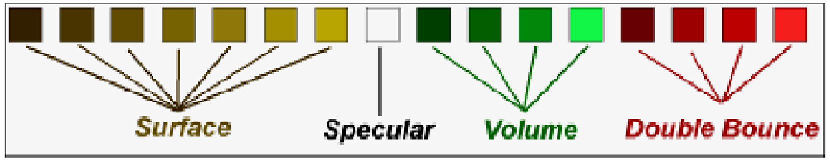
Courtesy of Dr J.S Lee



$2A_0$ $B_0 + B$ $B_0 - B$



4th Iteration (15 classes)

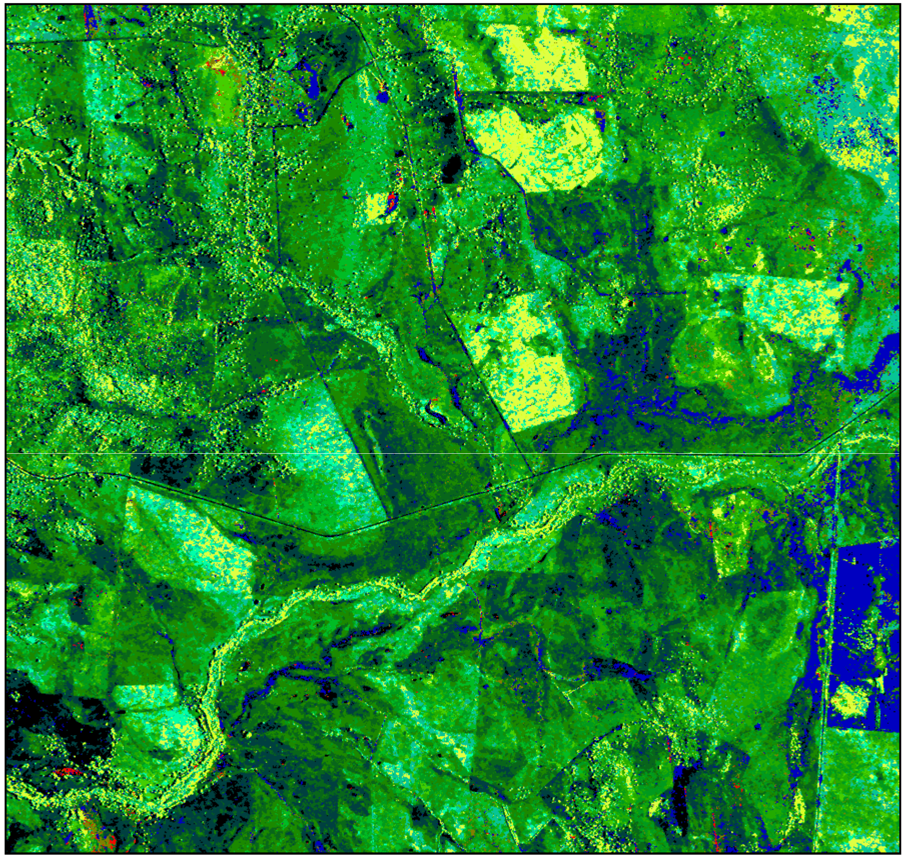


FREEMAN - WISHART CLASSIFIER

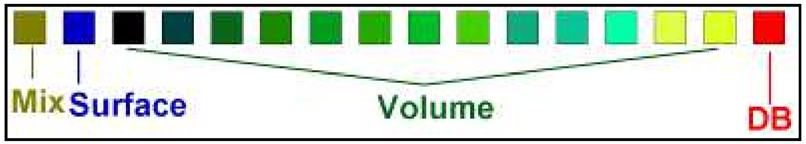
Courtesy of Dr J.S Lee

$2A_0$ $B_0 + B$ $B_0 - B$

Australian Pasture



4th Iteration (15 classes)

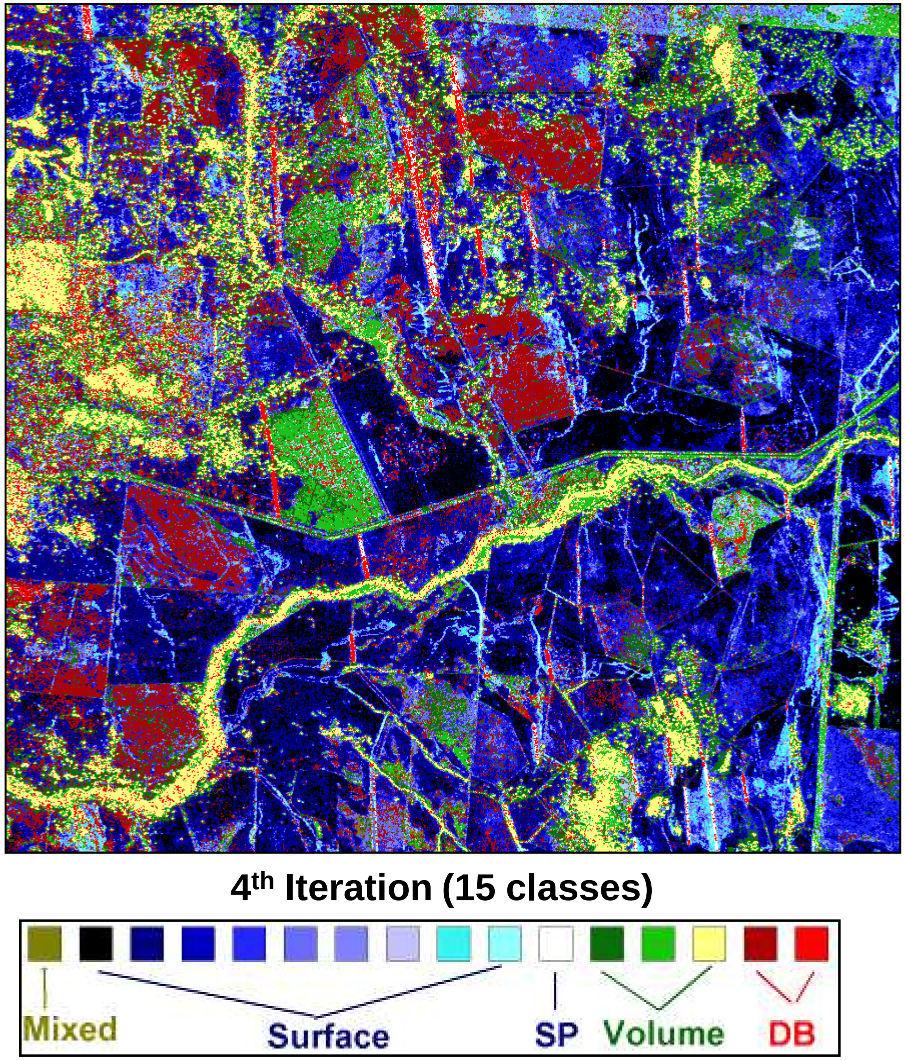


C-Band Volume Dominated

Courtesy of Dr J.S Lee

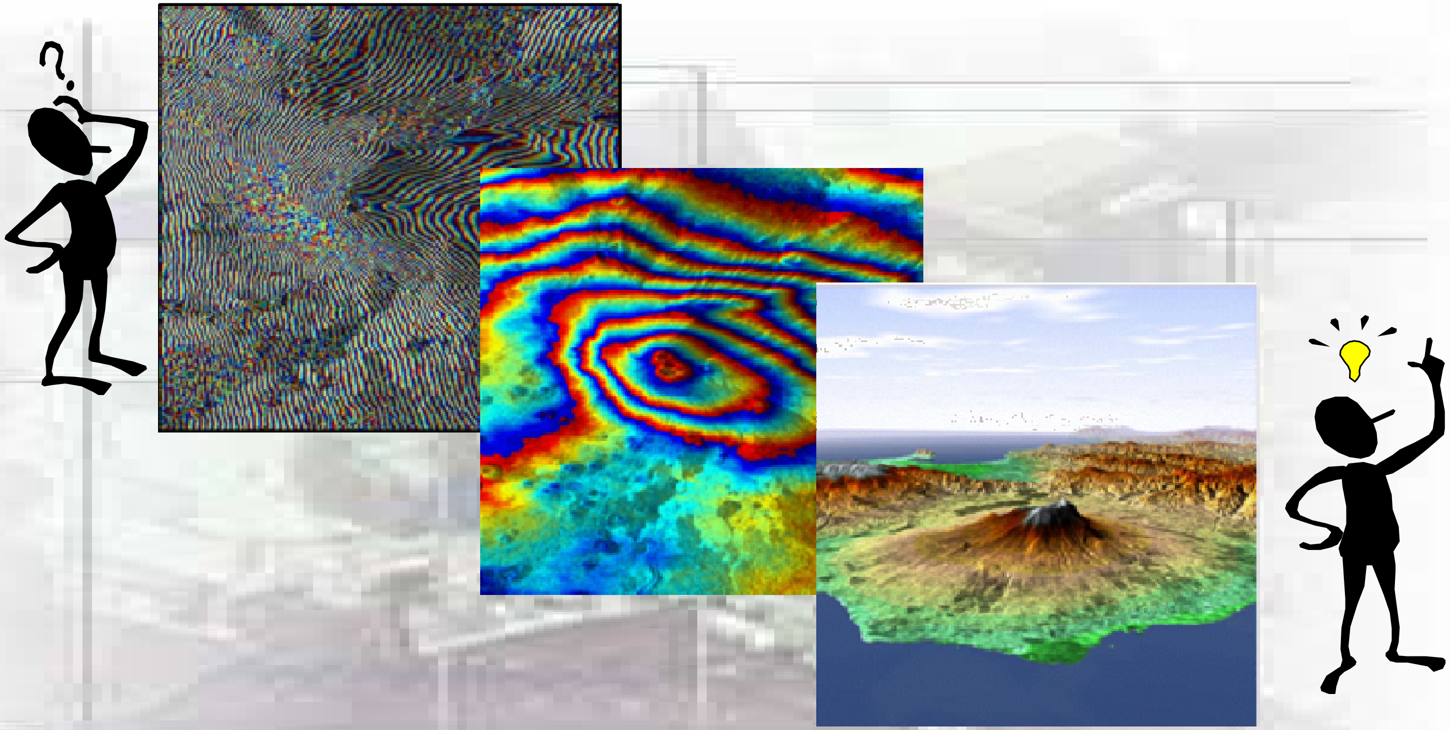
$2A_0$ $B_0 + B$ $B_0 - B$

Australian Pasture



4th Iteration (15 classes)

L-Band Volume Dominated



POLARIMETRIC INTERFEROMETRIC SAR POL-InSAR

ESAR

DO 228

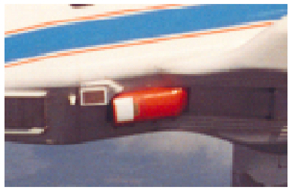
P, L, S-Band (Quad)

C, X-Band (Sngl)

P-Band

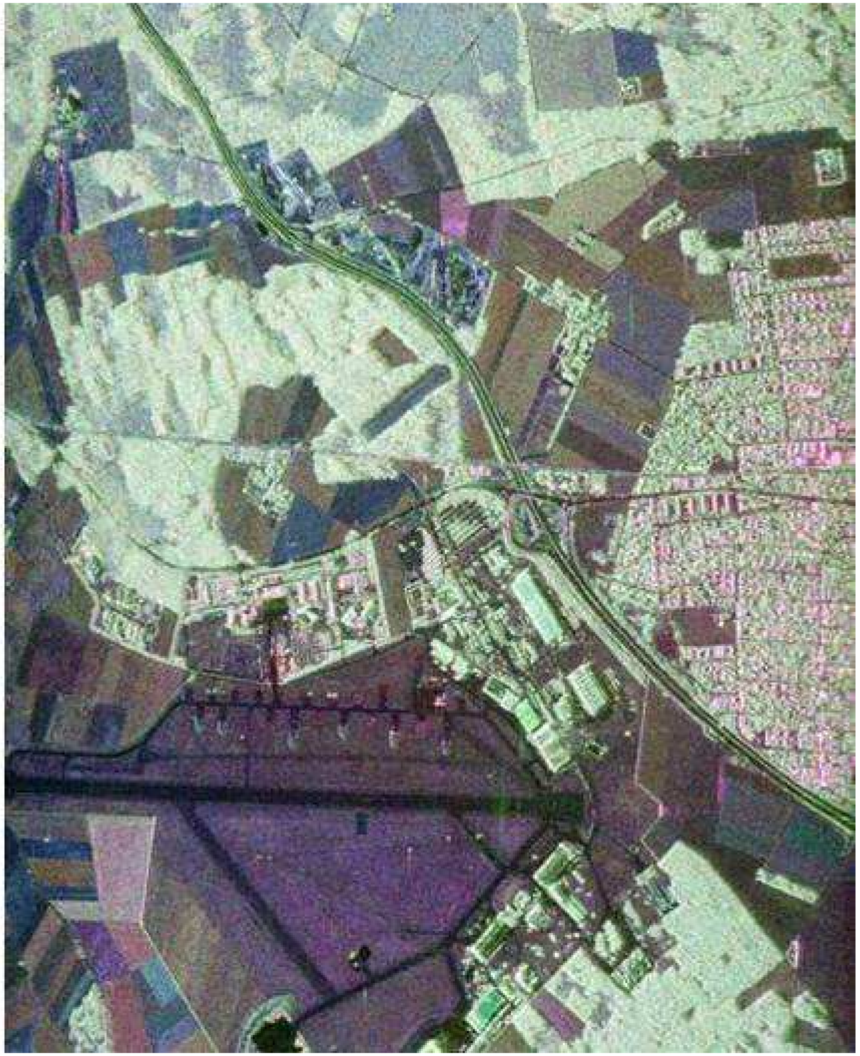
X-Band

L-Band



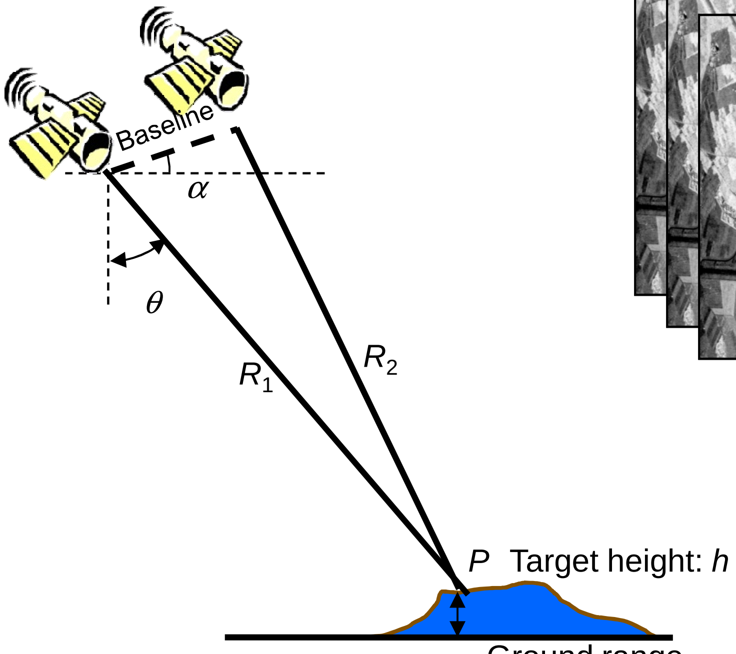
C-Band

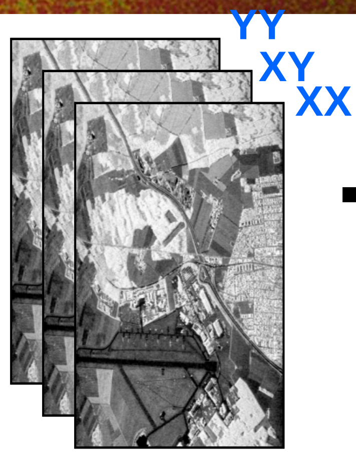
Courtesy of DLR



|HH+VV| T11=2A0 |HV| T33=B0-B |HH-VV| T22=B0+B

POL-InSAR

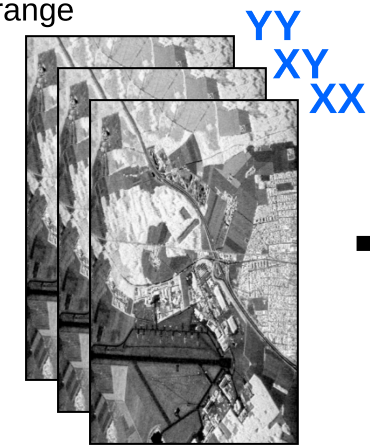




$$\underline{k}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX_1} + S_{YY_1} \\ S_{XX_1} - S_{YY_1} \\ 2S_{XY_1} \end{bmatrix}$$

$$\underline{\underline{k}} = \begin{bmatrix} \underline{k}_1 \\ \underline{k}_2 \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC TARGET VECTOR



$$\underline{k}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX_2} + S_{YY_2} \\ S_{XX_2} - S_{YY_2} \\ 2S_{XY_2} \end{bmatrix}$$

$$\underline{\underline{k}} = \begin{bmatrix} \underline{\underline{k}}_1 \\ \underline{\underline{k}}_2 \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC TARGET VECTOR

$$\langle [T_6] \rangle = \langle \underline{\underline{k}} \cdot \underline{\underline{k}}^{T*} \rangle = \begin{bmatrix} \langle \underline{\underline{k}}_1 \cdot \underline{\underline{k}}_1^{T*} \rangle & \langle \underline{\underline{k}}_1 \cdot \underline{\underline{k}}_2^{T*} \rangle \\ \langle \underline{\underline{k}}_2 \cdot \underline{\underline{k}}_1^{T*} \rangle & \langle \underline{\underline{k}}_2 \cdot \underline{\underline{k}}_2^{T*} \rangle \end{bmatrix} = \begin{bmatrix} \langle [T_1] \rangle & \langle [\Omega_{12}] \rangle \\ \langle [\Omega_{12}]^{T*} \rangle & \langle [T_2] \rangle \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC COHERENCY MATRIX (6x6)

$\langle [T_1] \rangle$ HERMITIAN POLARIMETRIC COHERENCY MATRIX (3x3)

$\langle [T_2] \rangle$ HERMITIAN POLARIMETRIC COHERENCY MATRIX (3x3)

$\langle [\Omega_{12}] \rangle$ NON HERMITIAN POLARIMETRIC INTER-COHERENCY MATRIX (3x3)

DUAL CHANNELS POLINSAR UNSUPERVISED SEGMENTATION

$$\langle [T_6] \rangle = \langle \underline{\underline{k}} \cdot \underline{\underline{k}}^{T*} \rangle = \begin{bmatrix} \langle \underline{k}_1 \cdot \underline{k}_1^{T*} \rangle & \langle \underline{k}_1 \cdot \underline{k}_2^{T*} \rangle \\ \langle \underline{k}_2 \cdot \underline{k}_1^{T*} \rangle & \langle \underline{k}_2 \cdot \underline{k}_2^{T*} \rangle \end{bmatrix} = \begin{bmatrix} \langle [T_1] \rangle & \langle [\Omega_{12}] \rangle \\ \langle [\Omega_{12}]^{T*} \rangle & \langle [T_2] \rangle \end{bmatrix}$$

POLARIMETRIC INTERFEROMETRIC COHERENCY MATRIX (6x6)

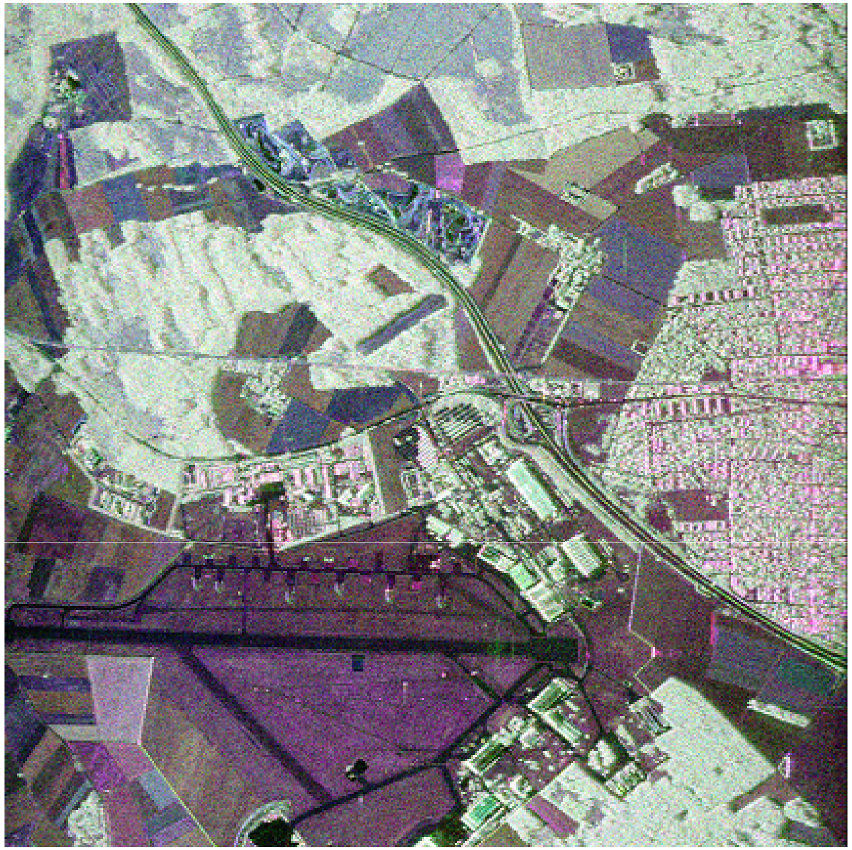
$\langle [T_6] \rangle$ FOLLOWS A WISHART DISTRIBUTION

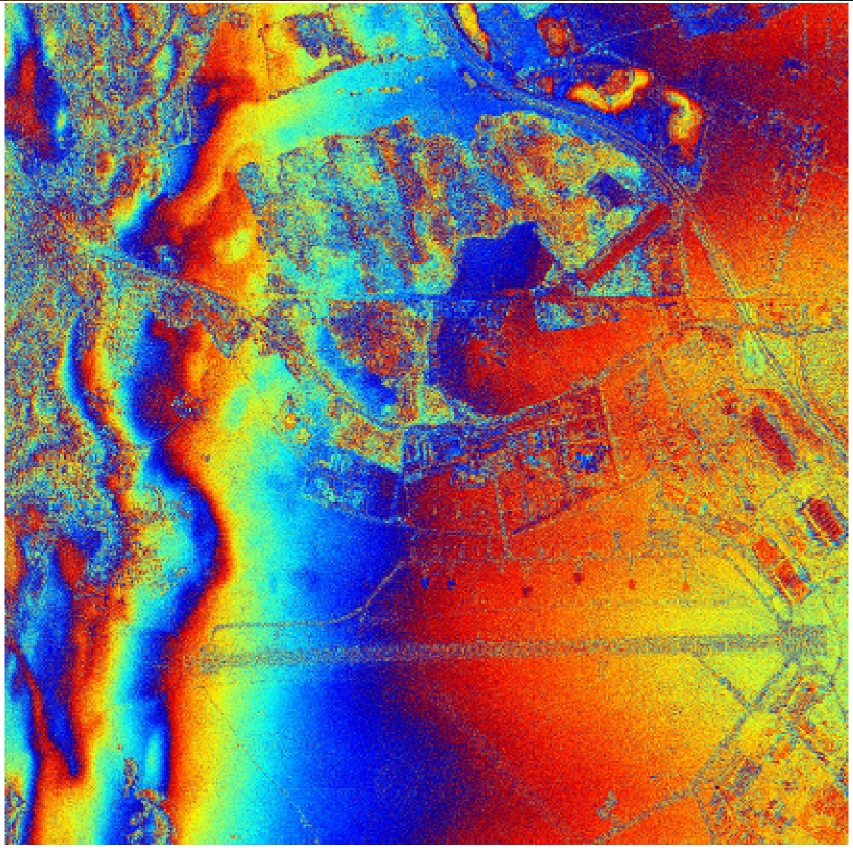
$$P(\langle [T_6] \rangle / [\Sigma_m]) = \frac{|\langle [T_6] \rangle|^{L-p} \exp\left(-tr\left([\Sigma_m]^{-1} \langle [T_6] \rangle\right)\right)}{K(L,p)\,|[\Sigma_m]|^{L}} = W_C(L,[\Sigma_m])$$

L: Number of Look
p: Polarimetric Dimension

With: $$K(L,p) = \frac{\pi^{\frac{p(p-1)}{2}}}{L^{Lp}} \Gamma(L) \ldots \Gamma(L-p+1)$$

$[\Sigma m]$: Cluster Center of the class *m*

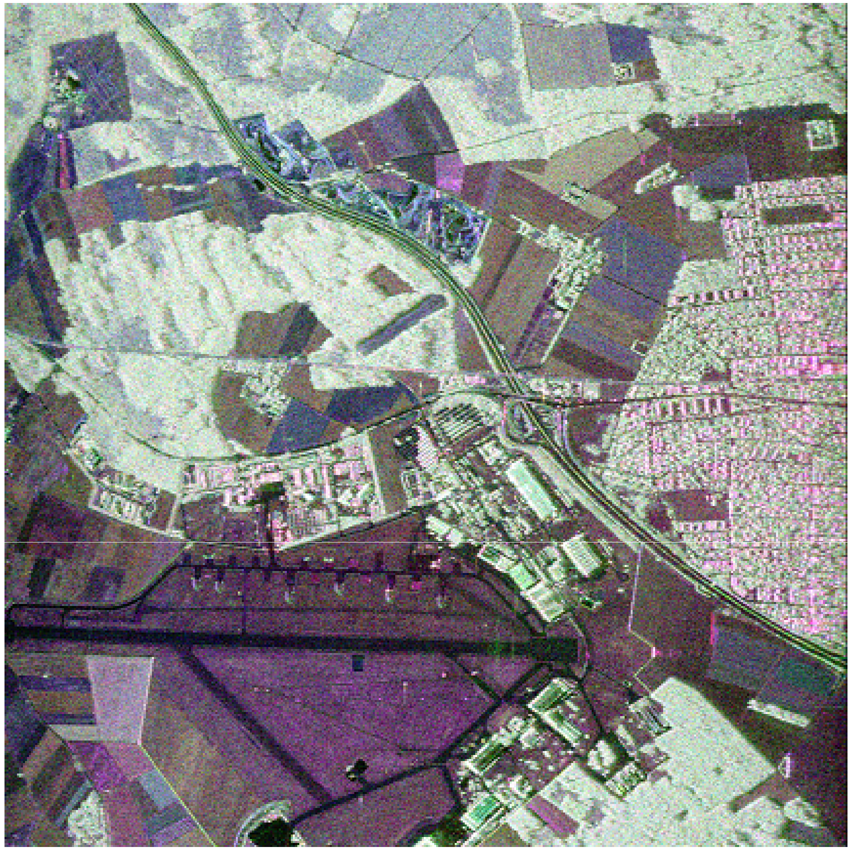




DLR E-SAR L Band
Pol-In SAR (1.5m x 3m) – Baseline 15m

POL-SAR INFORMATION

IN-SAR INFORMATION **Arg(γ)**

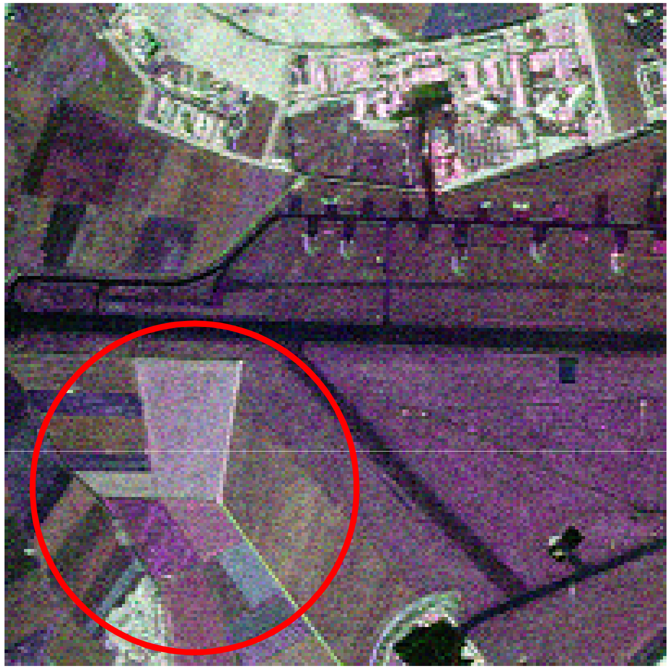


DLR E-SAR L Band
Pol-In SAR (1.5m x 3m) – Baseline 5m

POL-SAR INFORMATION

IN-SAR INFORMATION $|\gamma|$

COMPLEMENTARY INFORMATION

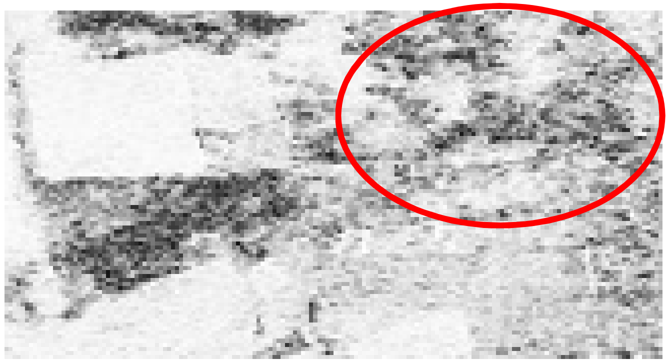


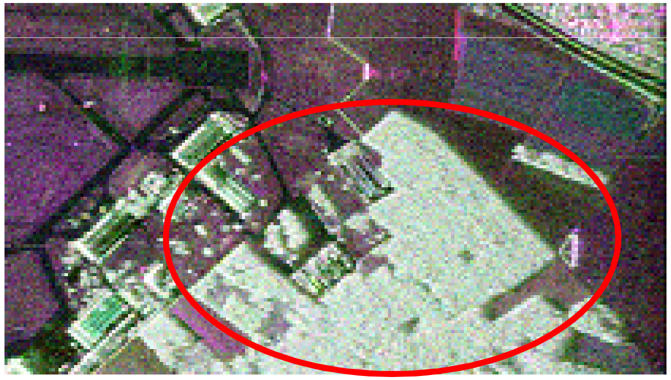
HETEROGENEOUS AREA

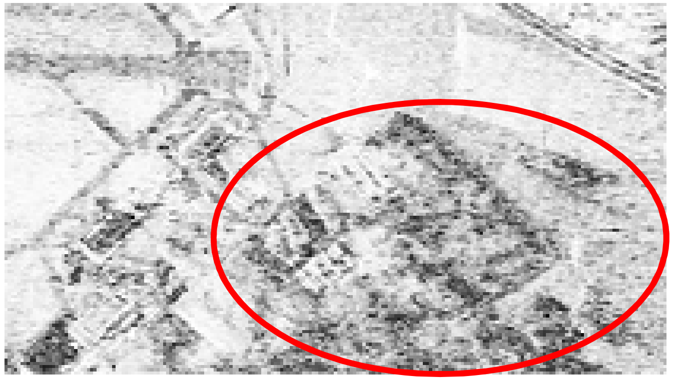
HOMOGENEOUS AREA

DIFFERENT POLARIMETRIC SCATTERING MECHANISMS

CONSTANT INTERFEROMETRIC COHERENCE





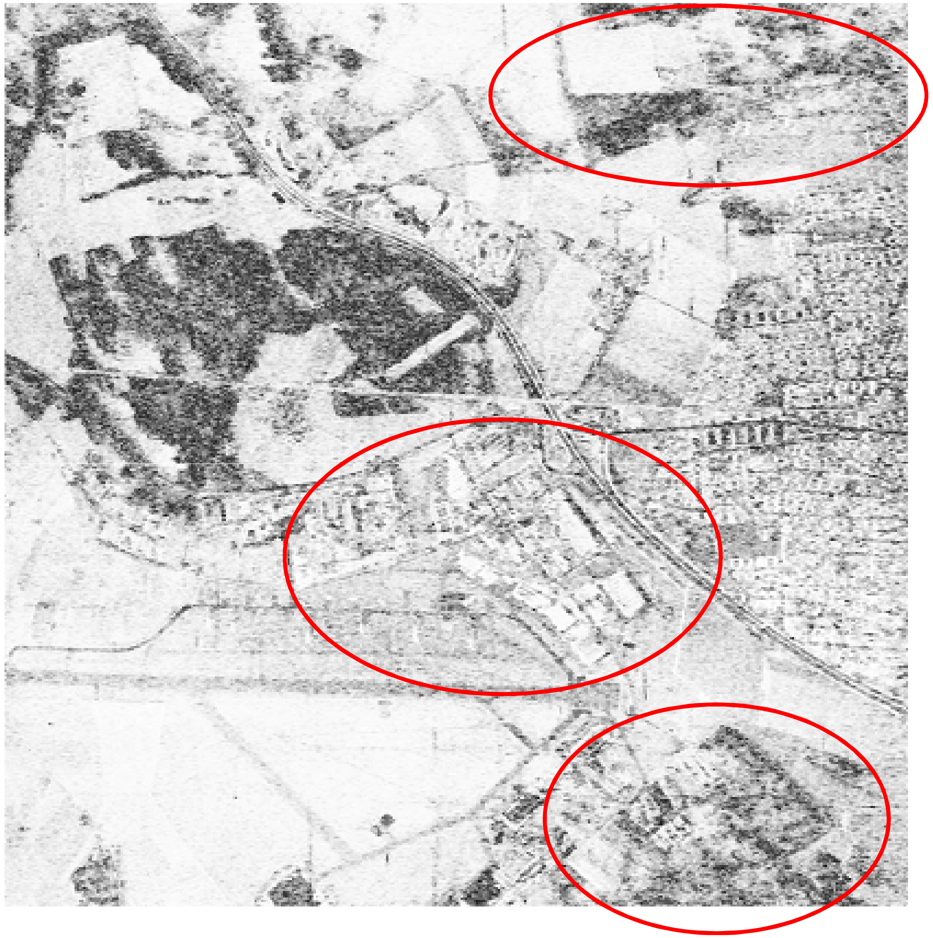


HOMOGENEOUS AREA

HETEROGENEOUS AREA

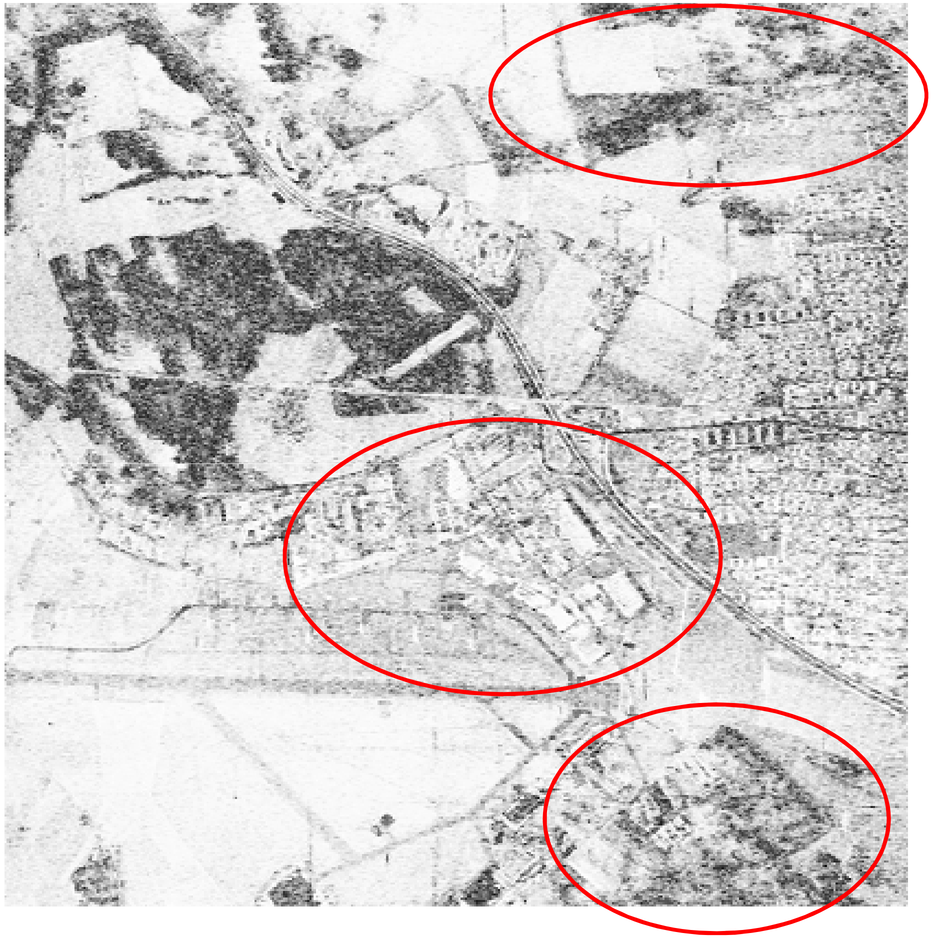
SAME POLARIMETRIC SCATTERING MECHANISMS

DIFFERENT INTERFEROMETRIC COHERENCE



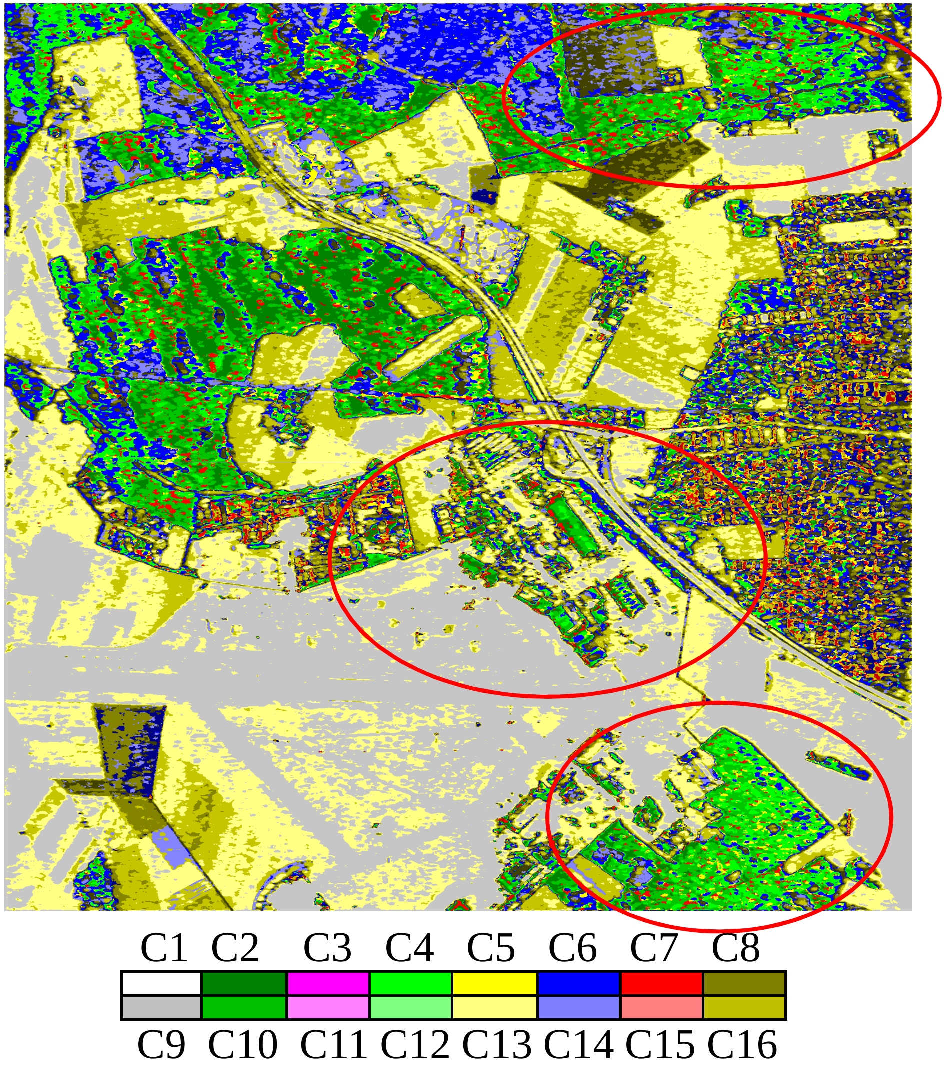
INTERFEROMETRIC COHERENCE γ

$2A_0$ $B_0 + B$ $B_0 - B$



INTERFEROMETRIC COHERENCE γ

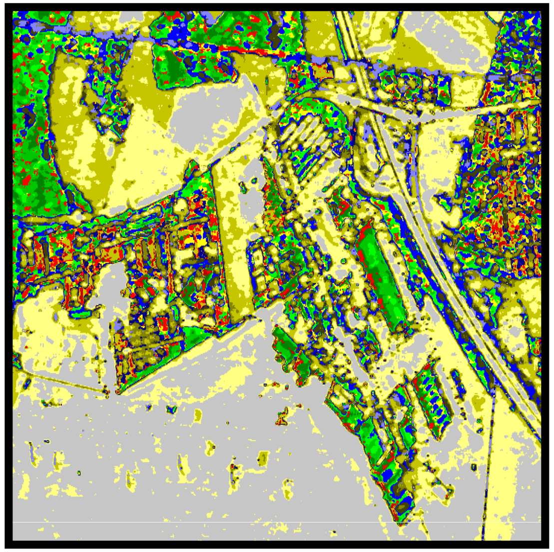
Wishart H-A-$\underline{\alpha}$ segmentation



Optical Image

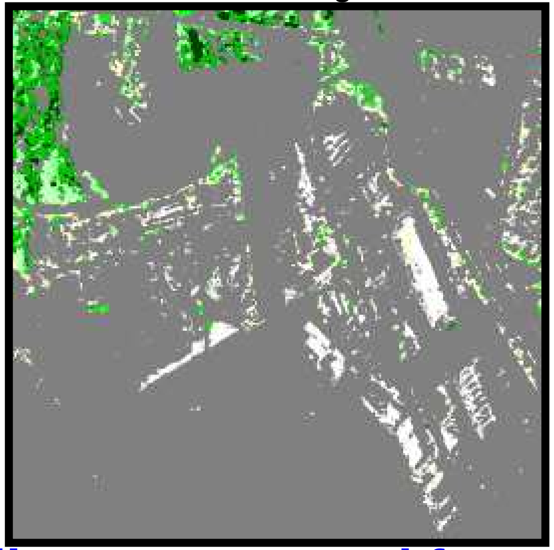
POLSAR Image

POLSAR Segmentation

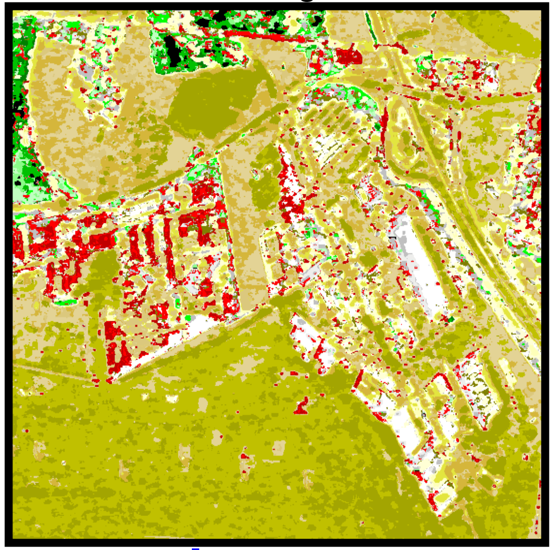


INSAR Image

VOL POLINSAR Segmentation



POLINSAR Segmentation



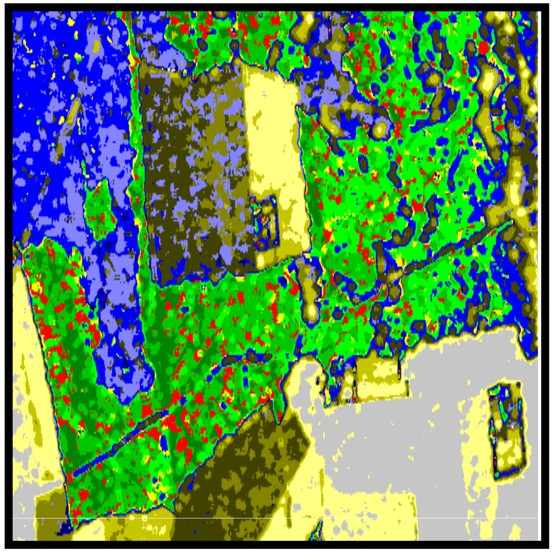
Oriented buildings segmented from vegetated areas

POL-InSAR

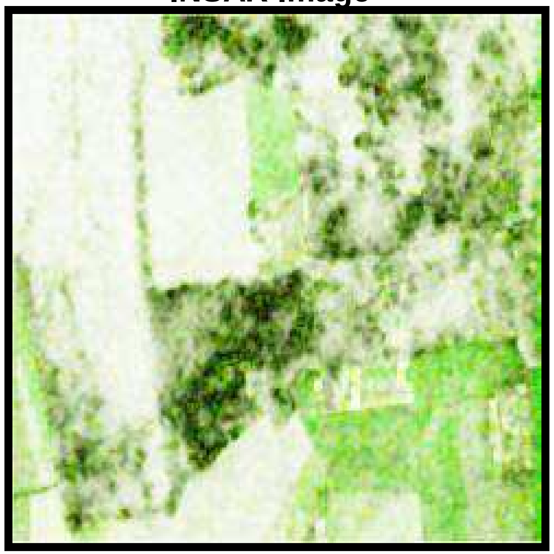
Optical Image

POLSAR Image

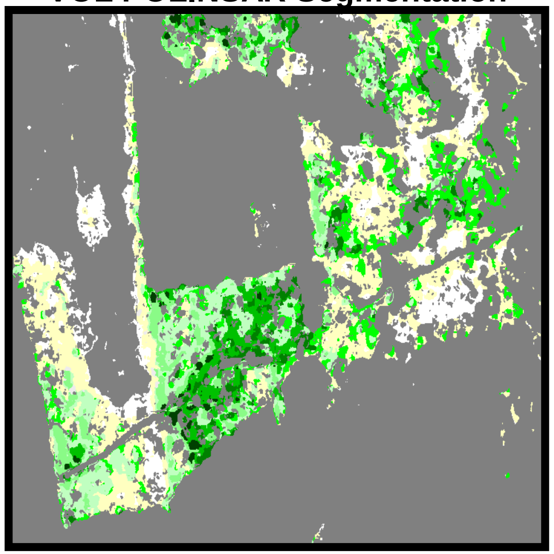
POLSAR Segmentation



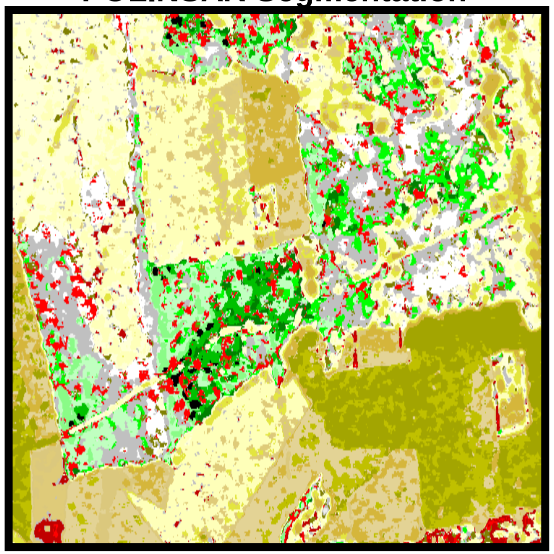
INSAR Image



VOL POLINSAR Segmentation

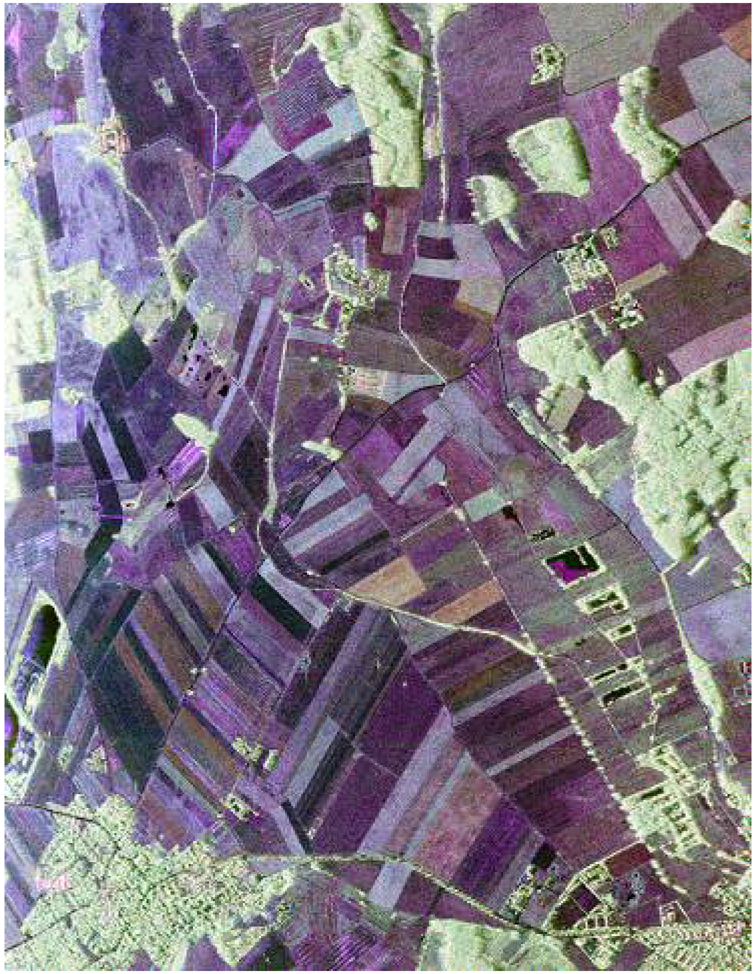


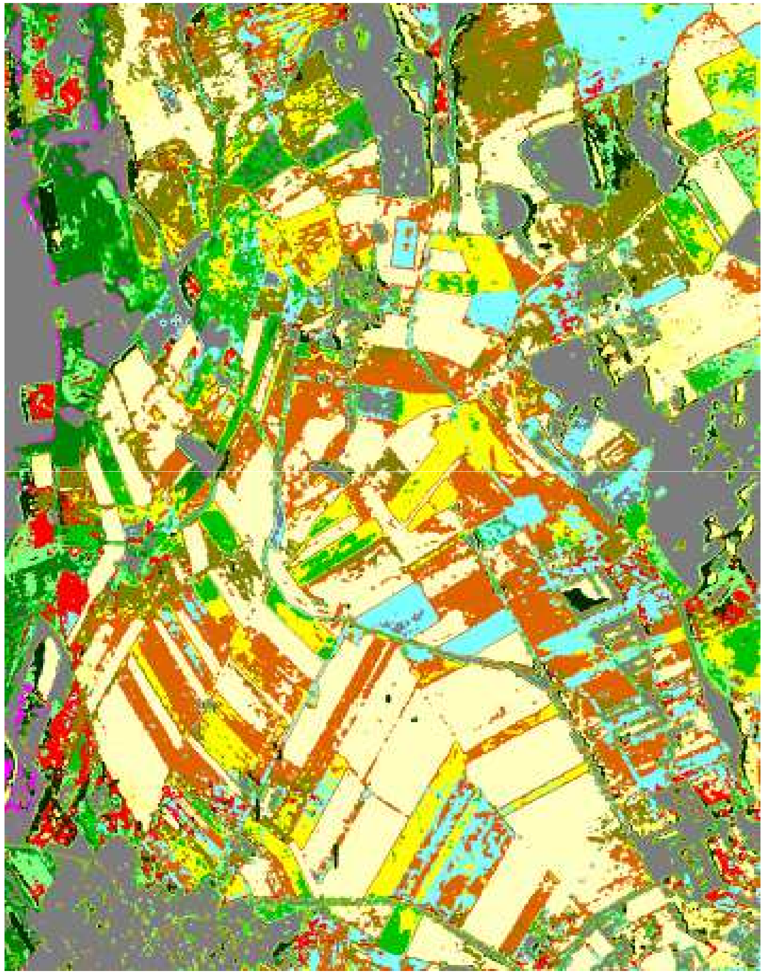
POLINSAR Segmentation



Low density forested areas segmented from dense forest

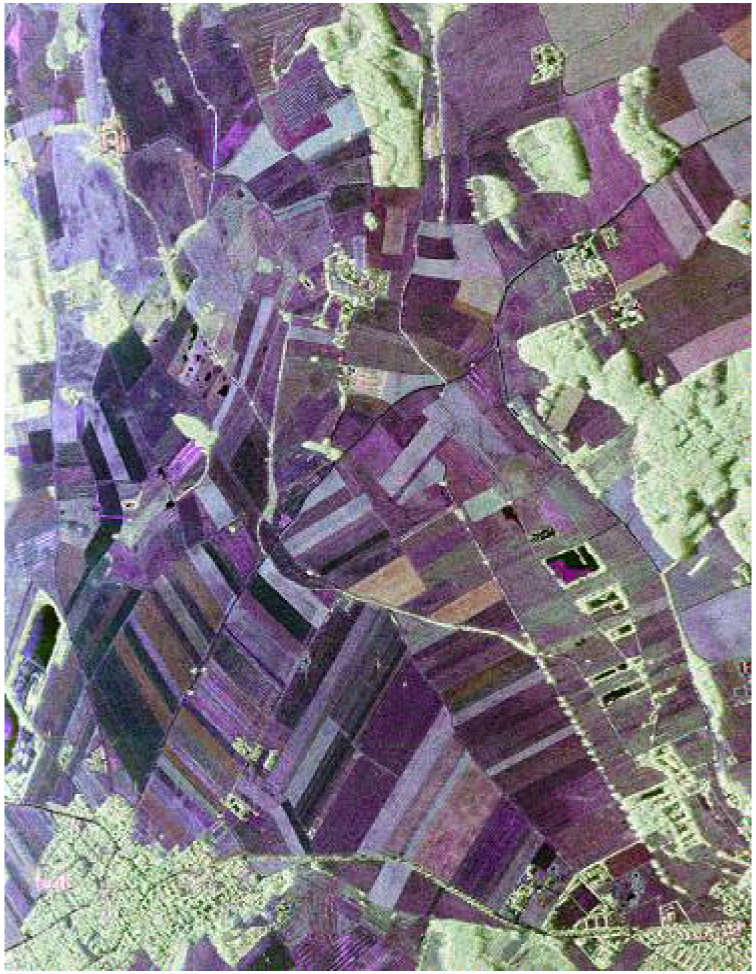
ALLING - ESAR L-band





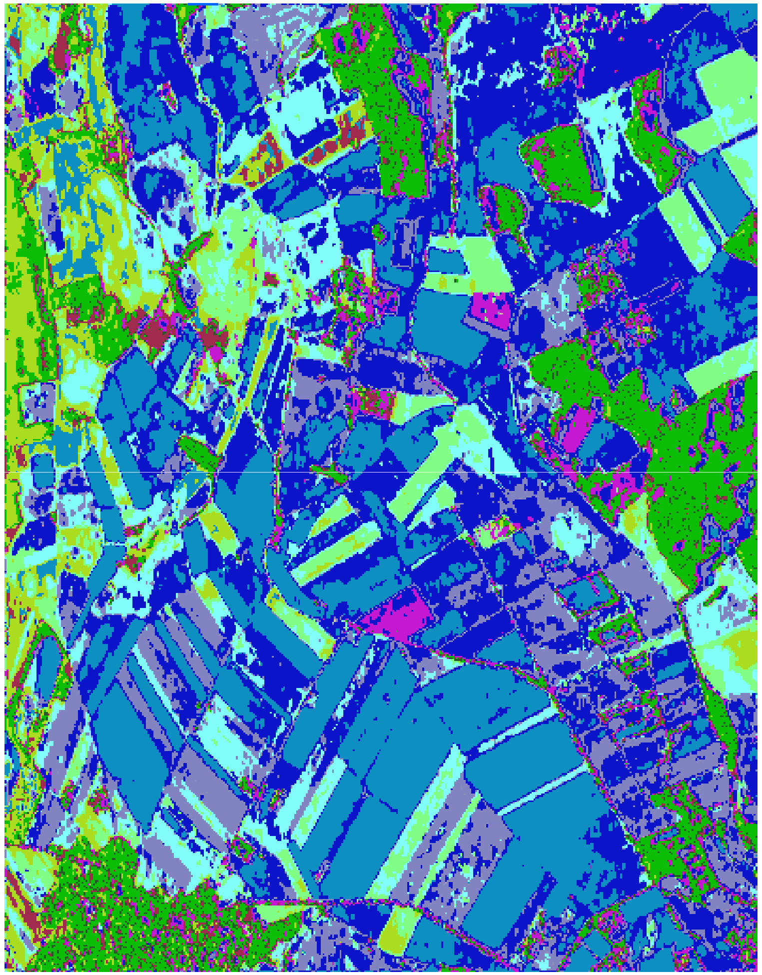
$2A_0$ $B_0 + B$ $B_0 - B$

ALLING - ESAR L-band



$2A_0$ $B_0 + B$ $B_0 - B$

H / A / α and WISHART CLASSIFIER



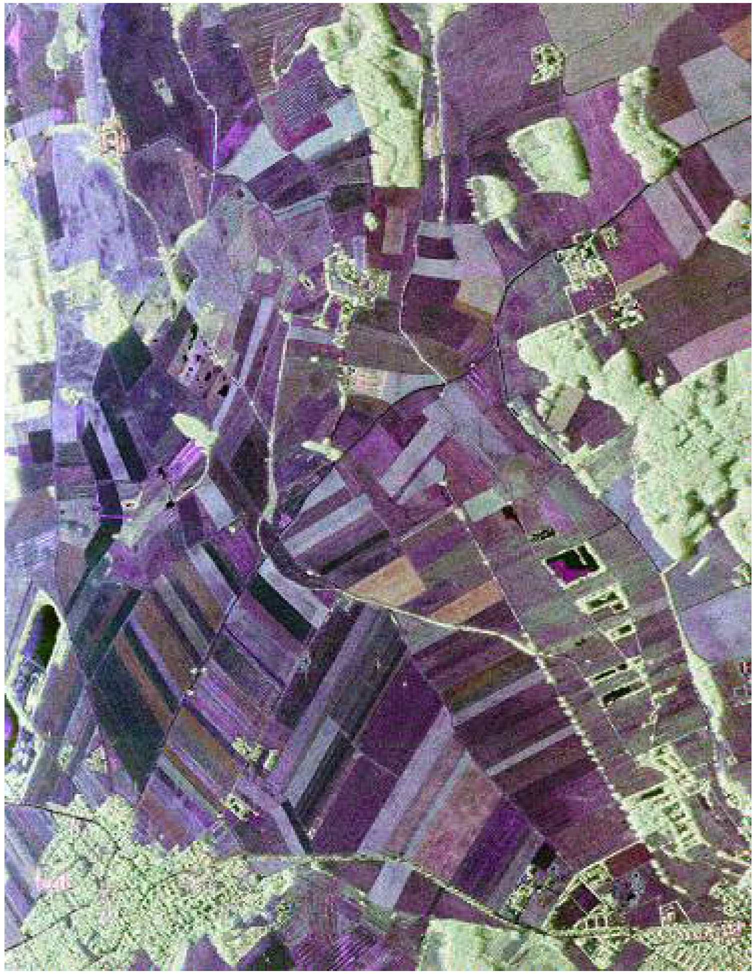
C1 C2 C3 C4 C5 C6 C7 C8

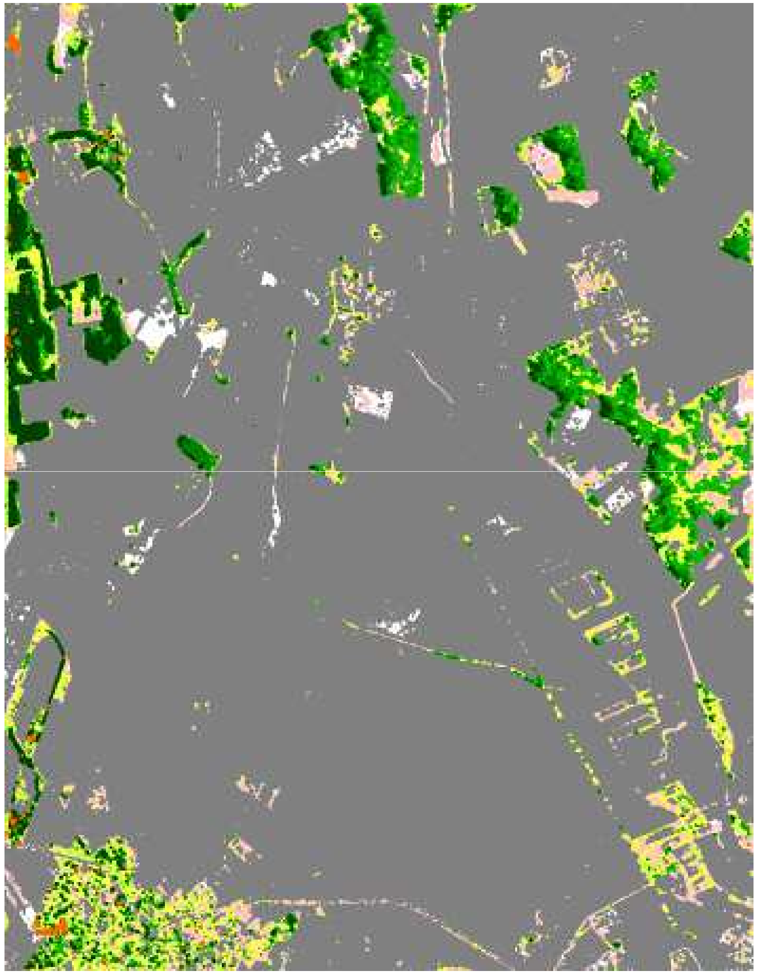
C9 C10 C11 C12 C13 C14 C15 C16

DLR E-SAR L Band – Pol-InSAR (1.5m x 3m) – Baseline 5m

POL-SAR INFORMATION

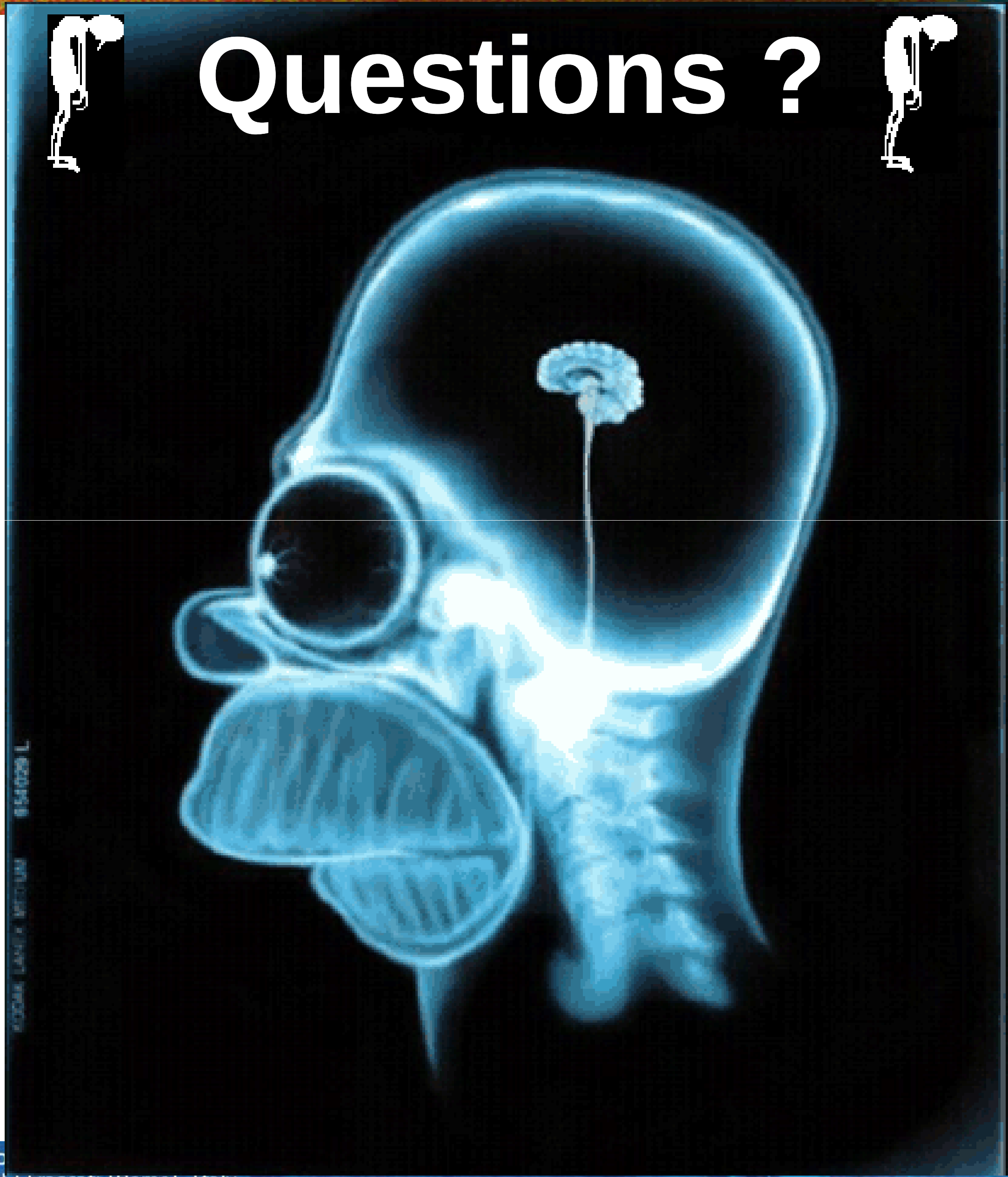
IN-SAR INFORMATION $|\gamma|$

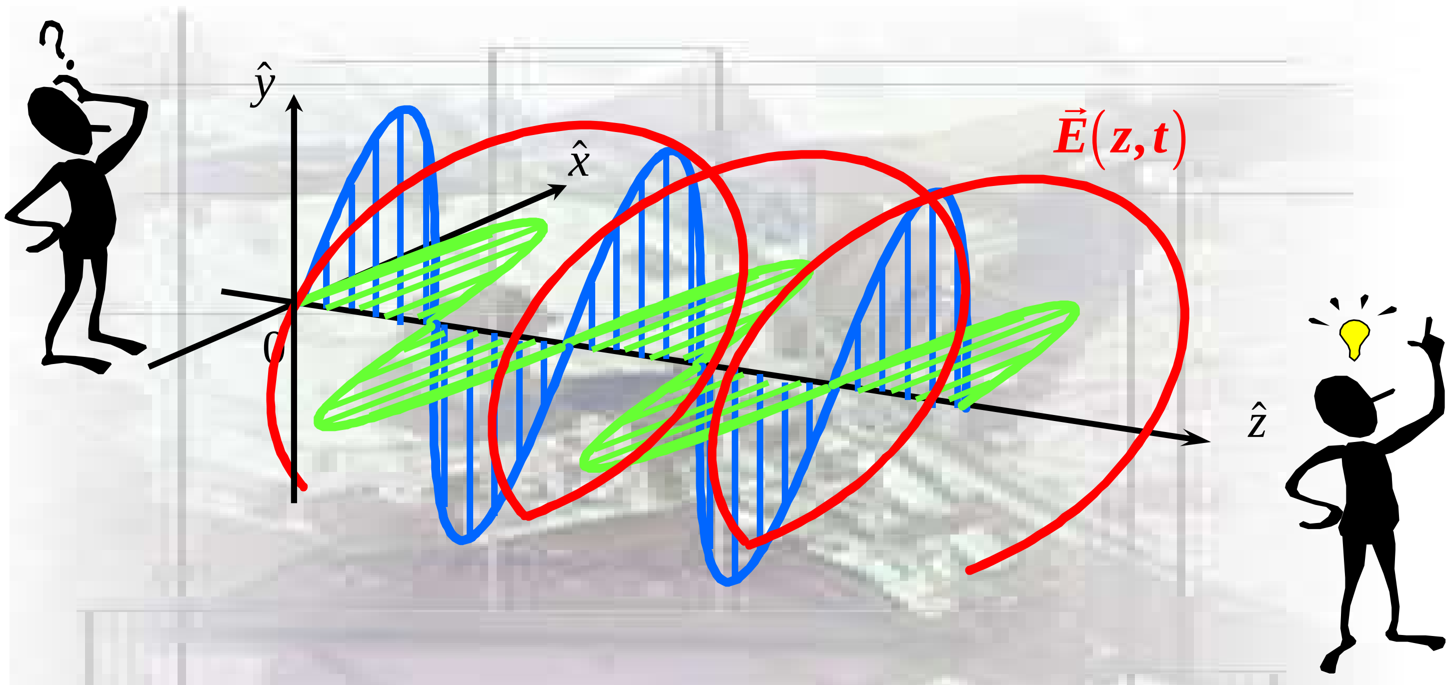




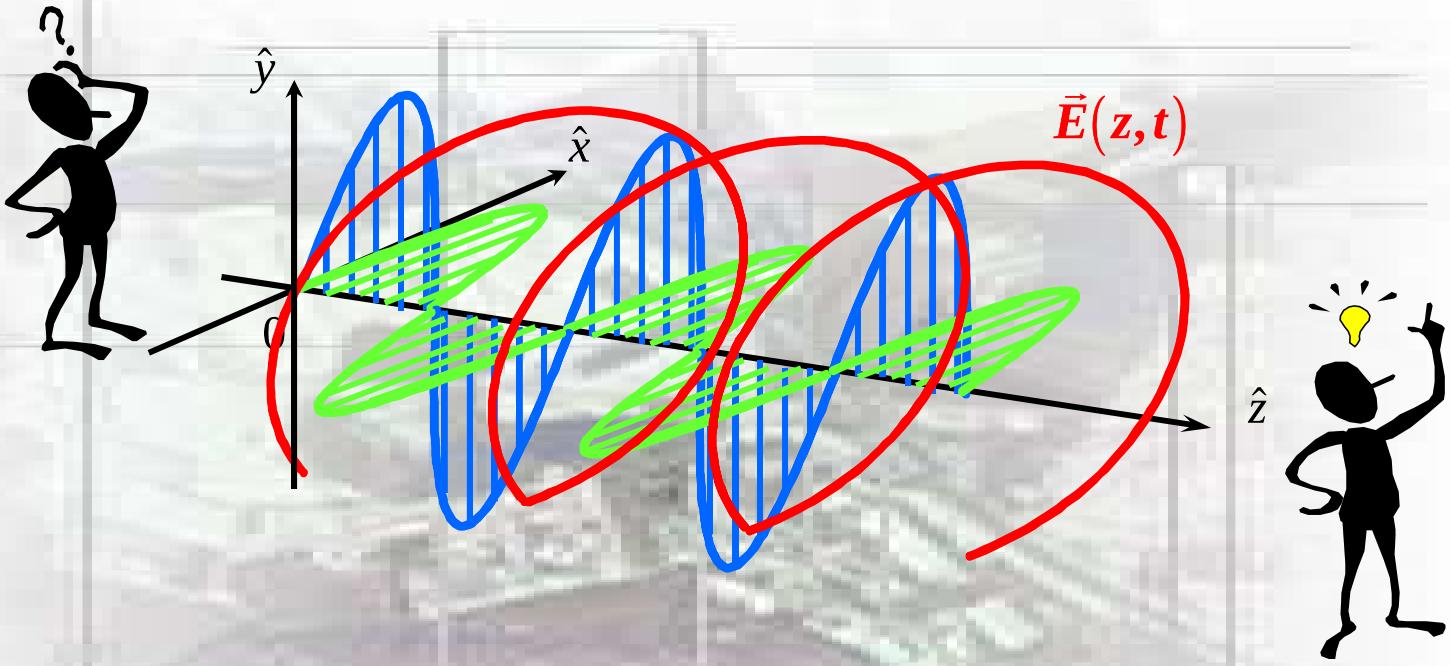
Oriented Targets segmented from Vegetated Areas

Questions ?



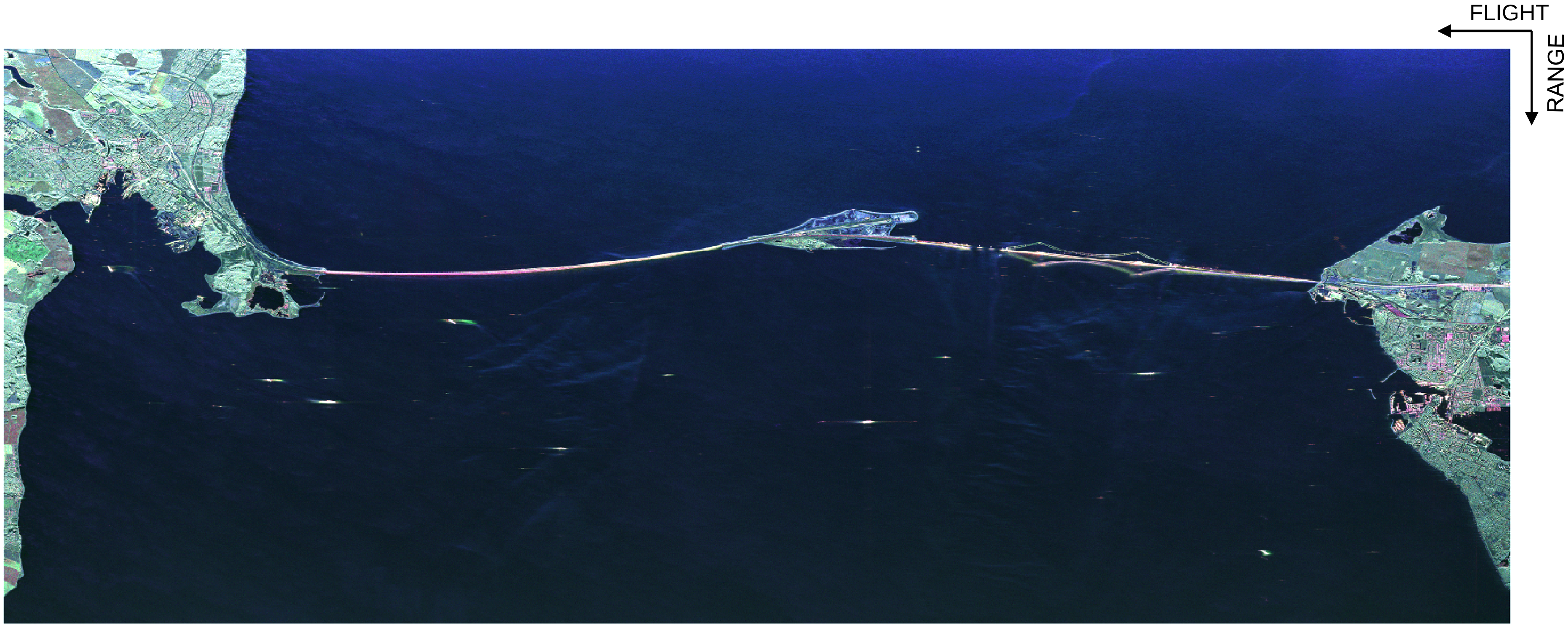


APPLICATIONS



POLARIMETRIC TARGET SIGNATURES: An Example

A Man-Made Structure - An Example

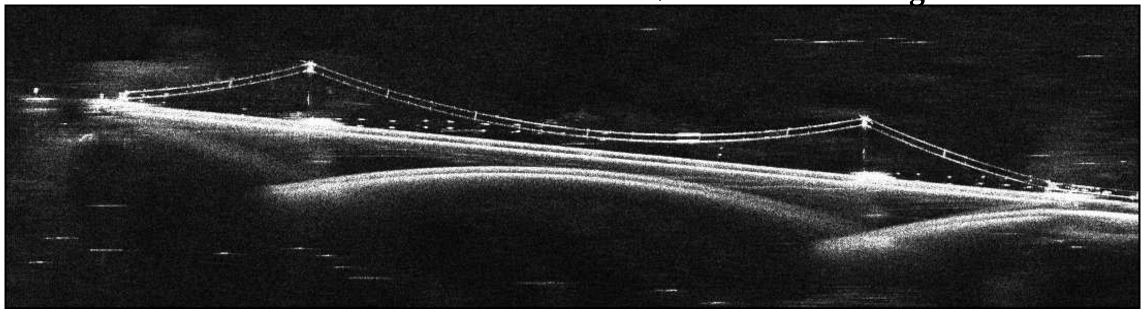


|HH-VV|, |HV|, |HH+VV|

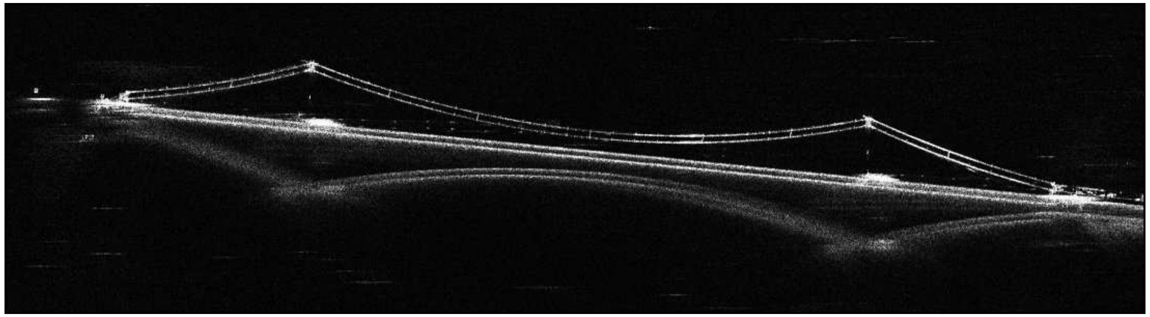
EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

J.S. Lee, E. Krogager, T.L. Ainsworth, and W.-M. Boerner, "Polarimetric analysis of radar signature of a manmade structure," *IEEE Remote Sensing Letters*, vol. 3 no. 4, 555-559, October 2006.

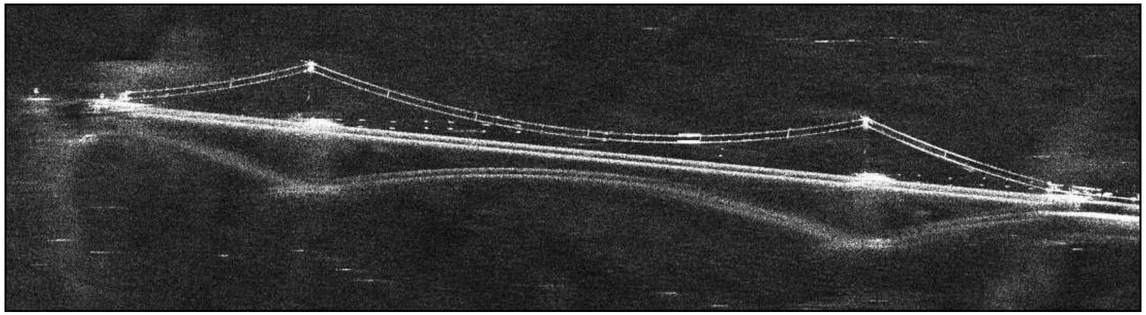
← Flight Direction



|HH|



|HV|



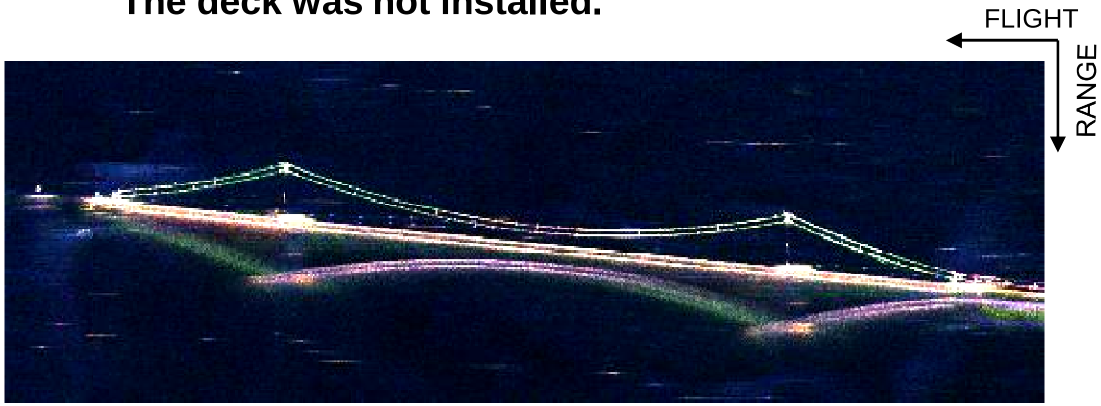
|VV|

EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

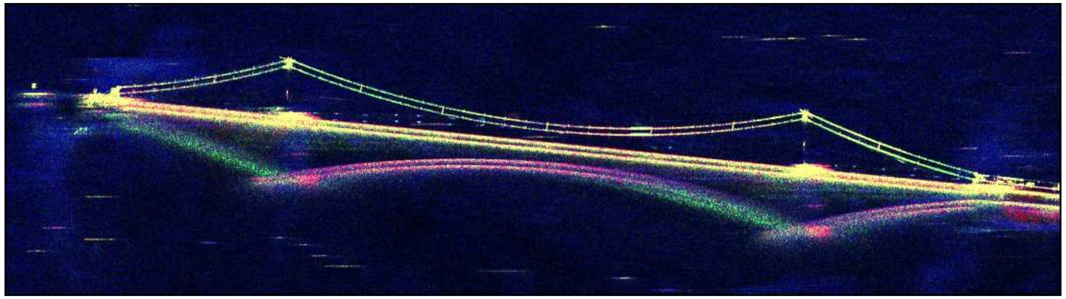
POLARIMETRIC DECOMPOSITION

High-resolution POLSAR signature of a suspension bridge under construction
The deck was not installed.

Aerial Photo



Pauli Decomposition, |HH-VV|, |HV|, |HH+VV|



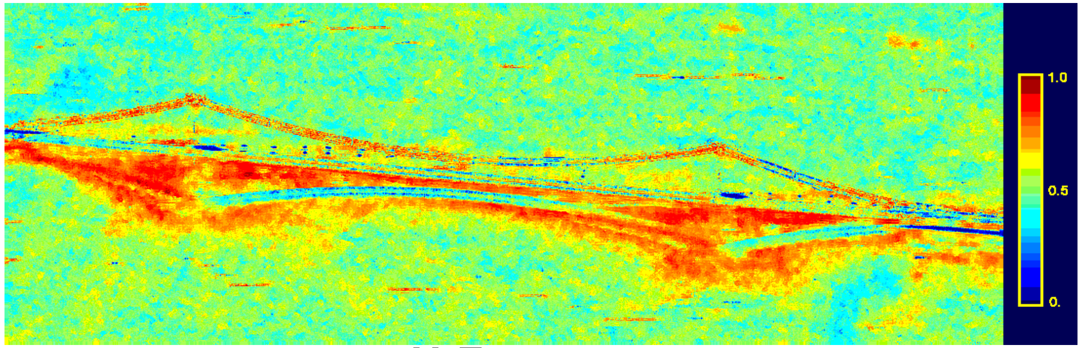
Krogager SDH Decomposition

- Blue= Sphere Green = Helix Red = Diplane.

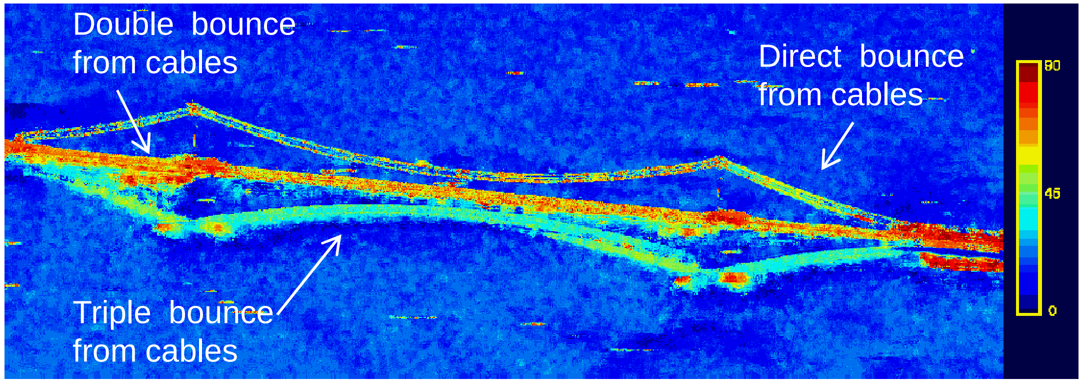
EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

High-resolution POLSAR signature of a suspension bridge under construction
The deck was not installed.

Aerial Photo



H *Entropy*

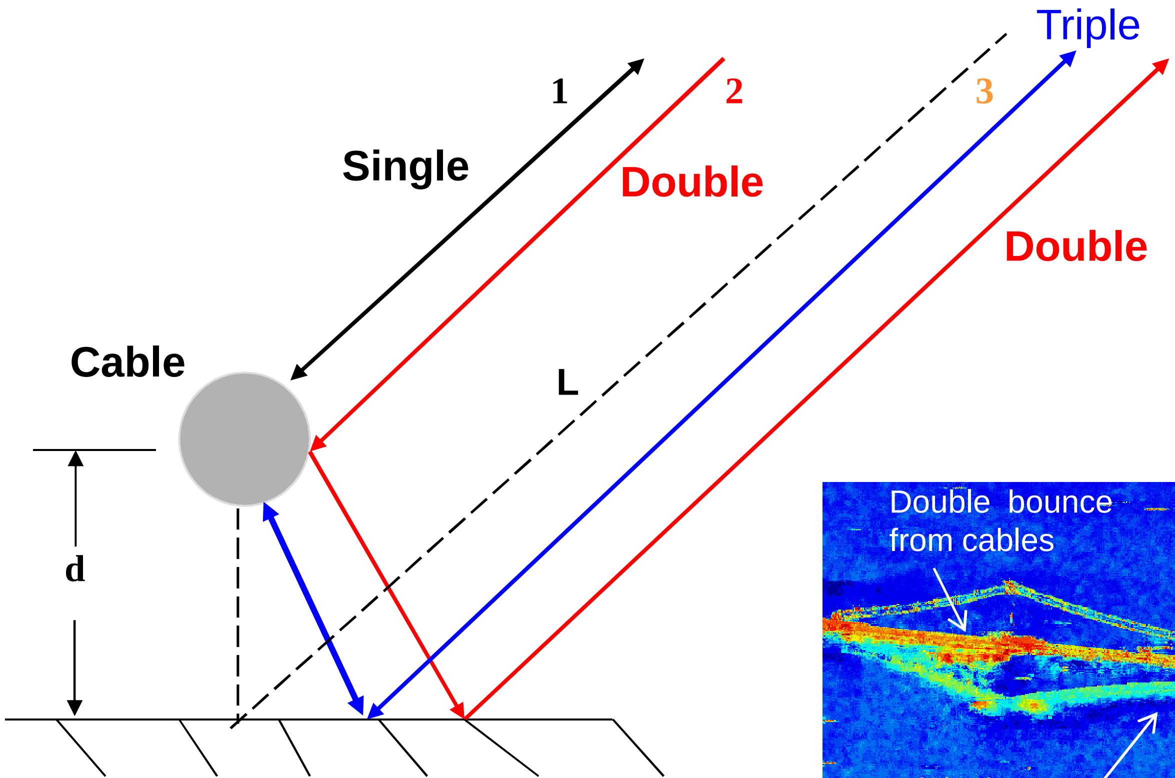


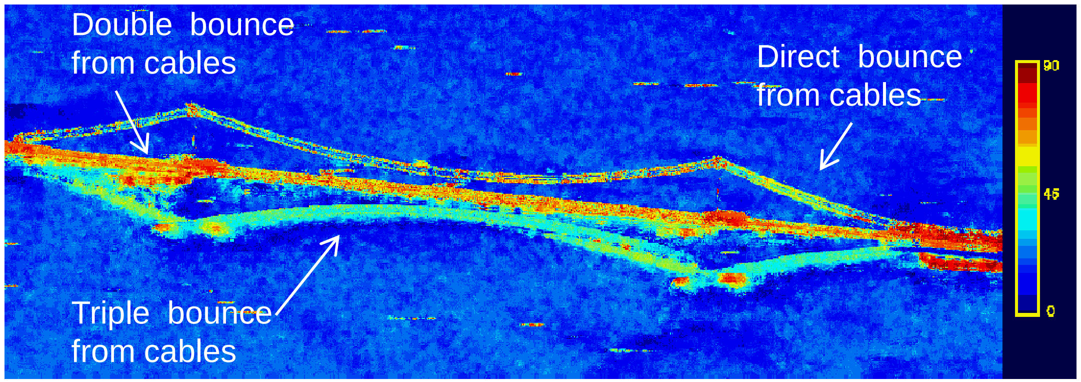
α *Angle*

Dominant scattering mechanisms are extracted by applying target decomposition:
Blue= Surface Green = Dipole Red = Double Bounce.

EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

Multi – Bounce Scattering mechanisms





α *Angle*

Roundtrip distances:

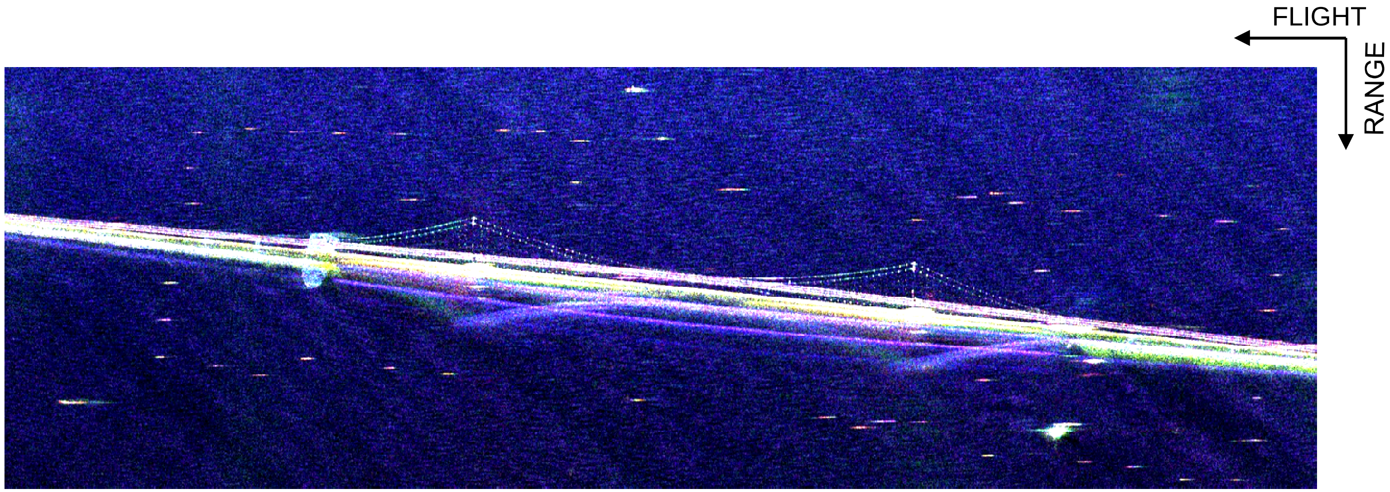
Single bounce return: $2(L - d\cos\theta)$

Double bounce return: $2L$ ⇐ ***Independent of height***

Triple bounce return: $2(L + d\cos\theta)$

θ is the local incidence angle.

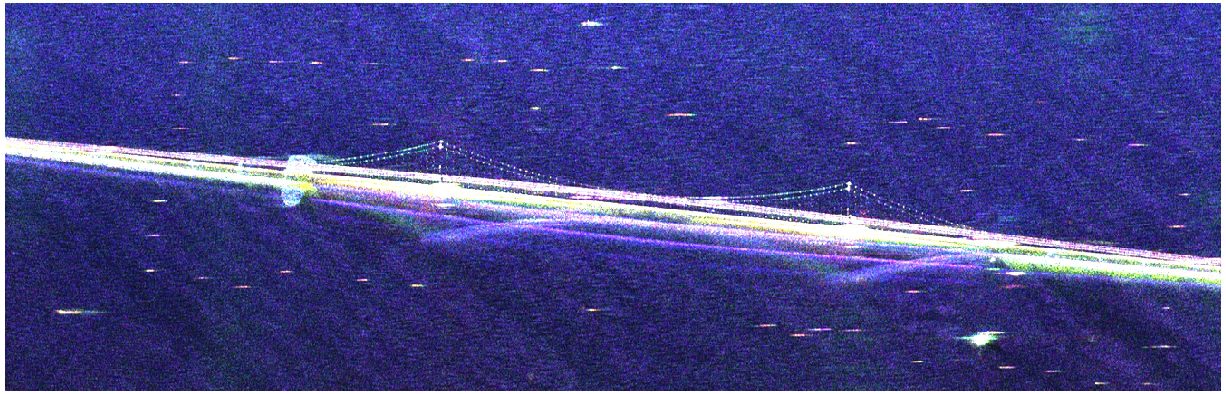
**High-resolution POLSAR signature of a suspension bridge after completion.
The deck is installed.**

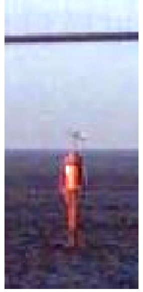


|HH-VV|, |HV|, |HH+VV|

EMISAR C-Band Polarimetric SAR Image of StoreBelt Bridge

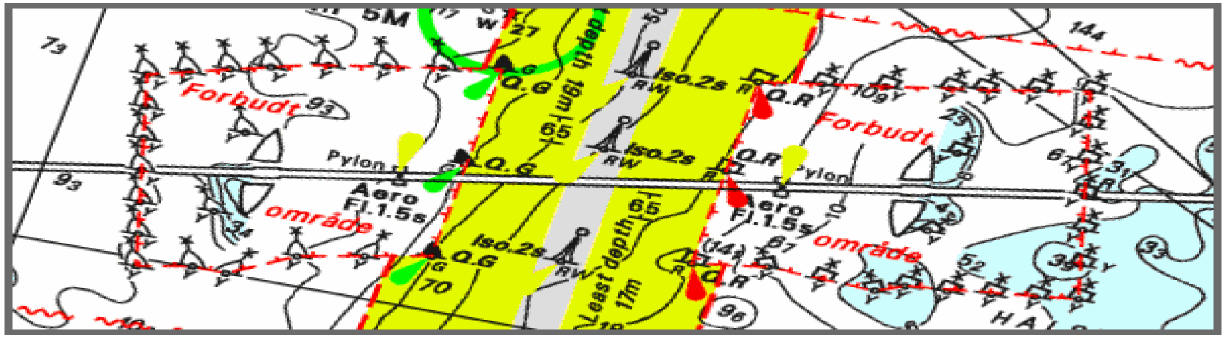
Buoy



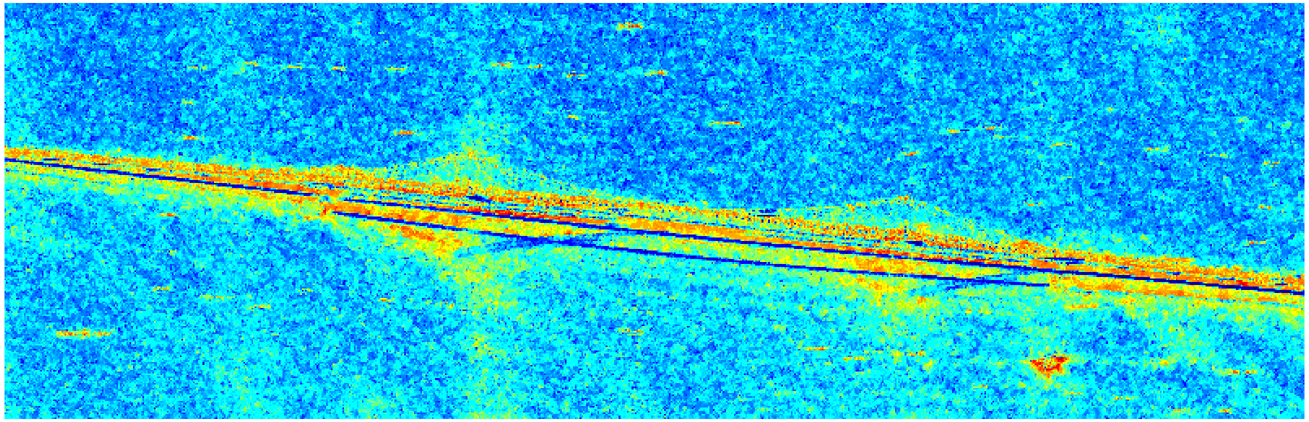


Buoy

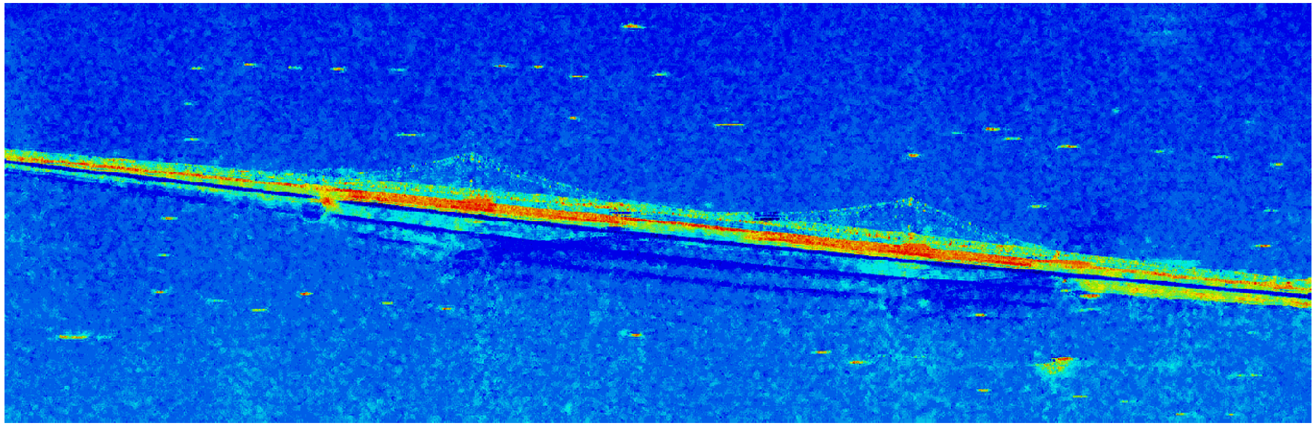
|HH-VV|, |HV|, |HH+VV|



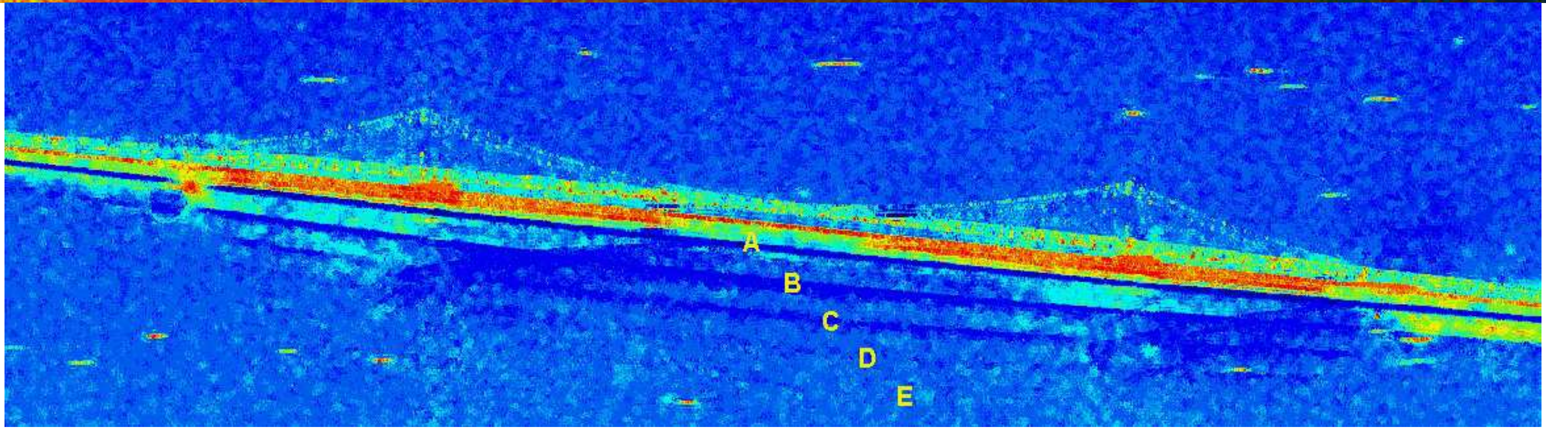
Navigation Map of Storebelt, Denmark



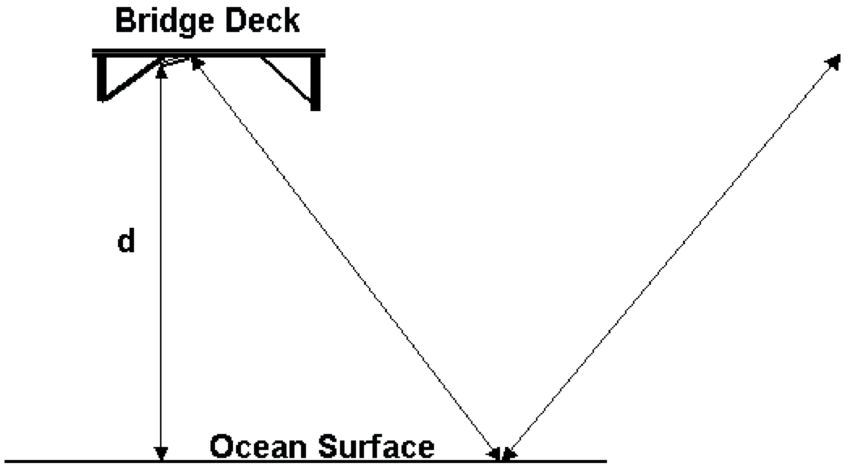
H *Entropy*



α *Angle*



α *Angle*



Roundtrip distances:

A: Triple bounces, $2(L + d\cos\theta)$

B: 5 (or 7) bounces, $2(L + d\cos\theta) + 2d$

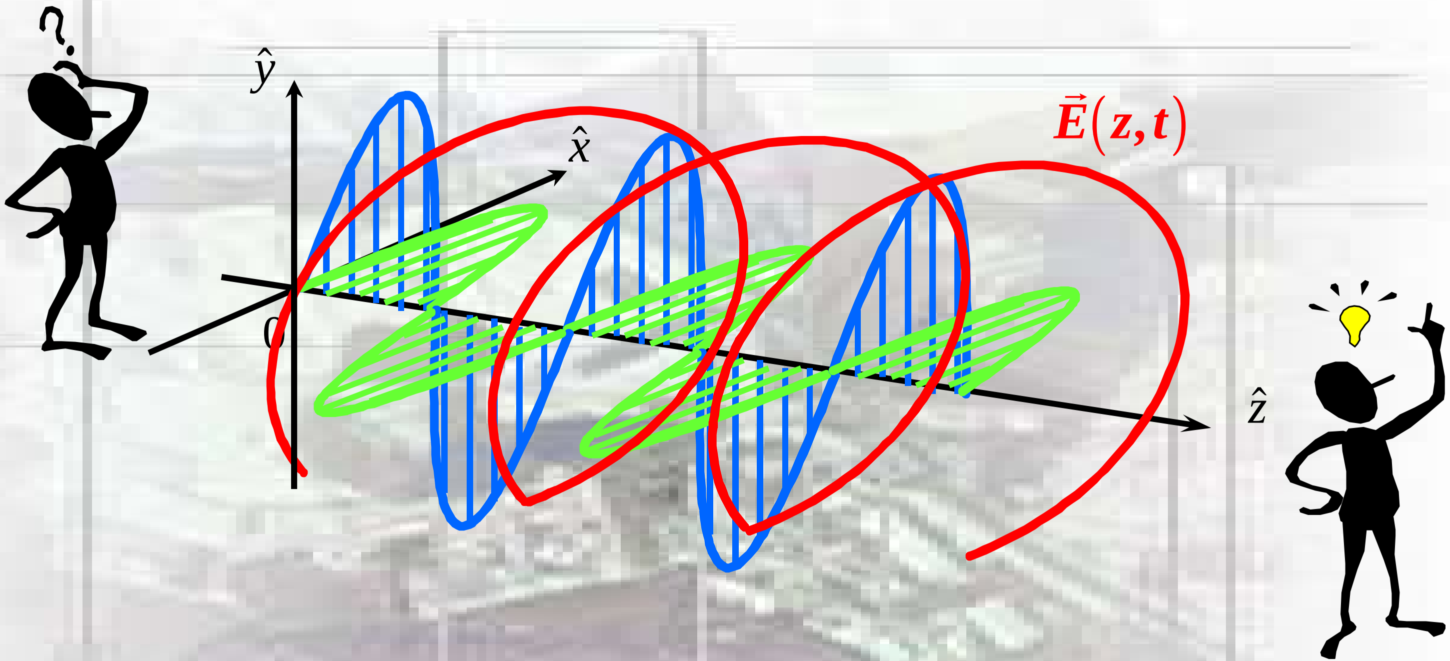
C: 7 (or 9) bounces, $2(L + d\cos\theta) + 4d$

D: 9 (or 11) bounces, $2(L + d\cos\theta) + 6d$

(…) indicates additional two bounces from the bottom of the deck.

RESONANT CAVITY

Dominant scattering mechanisms are extracted by applying target decomposition:
Blue= Surface Green = Dipole Red = Double Bounce.



POLARIZATION ORIENTATION ANGLE ESTIMATION AND APPLICATIONS

POLARIZATION ORIENTATION ANGLE ESTIMATION AND APPLICATIONS

D.L. Schuler, J.S. Lee and G. De Grandi, "Measurement of Topography Using Polarimetric SAR Images," *IEEE Trans. on Geoscience and Remote Sensing*, vol. 34, no. 5, 1266-1277, September, 1996.

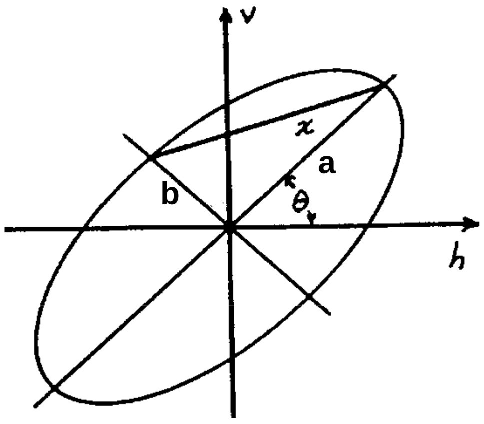
J.S. Lee, D.L. Schuler and T.L. Ainsworth, “Polariemtric SAR data compensation for terrain azimuth slope variations,” IEEE TGRS (September, 2000)

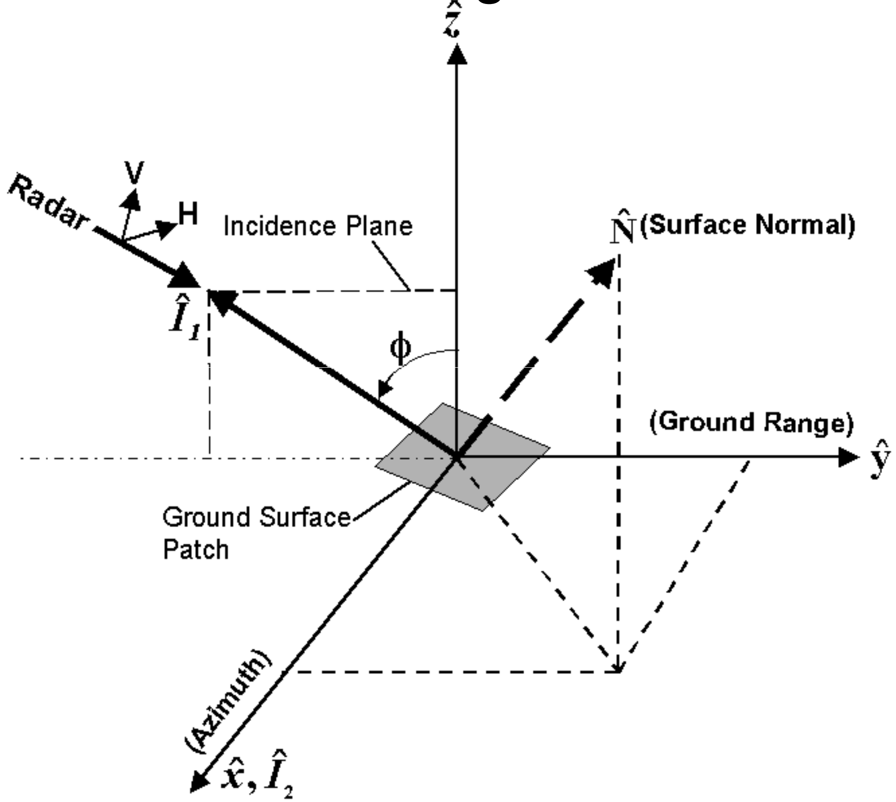
J.S. Lee, D.L. Schuler, T.L. Ainsworth, and W. M. Boerner “A Review of Polarization orientation angle estimation and Applications,” Proceedings of EUSAR 2006, E. Lueneburg Memorial Session, 2006

F. Xu, and Y.-Q. Jin, “Deorientation theory of polarimetric scattering targets and application to terrain surface classification,” *IEEE Trans. on Geoscience and Remote Sensing,* vol.43, no.10, pp. 2351-2364, 2005.

Polarization Orientation Shifts

Orientation angles = rotation about the line of sight





J.S. Lee, D.L. Schuler and T.L. Ainsworth, “Polarimetric SAR data compensation for terrain azimuth slope variations,” IEEE TGRS (September, 2000)

J.S. Lee, D.L. Schuler, T.L. Ainsworth, and W. M. Boerner “A Review of Polarization orientation angle estimation and Applications,” Proceedings of EUSAR 2006, E. Lueneburg Memorial Session, 2006

Polarization Orientation Shifts

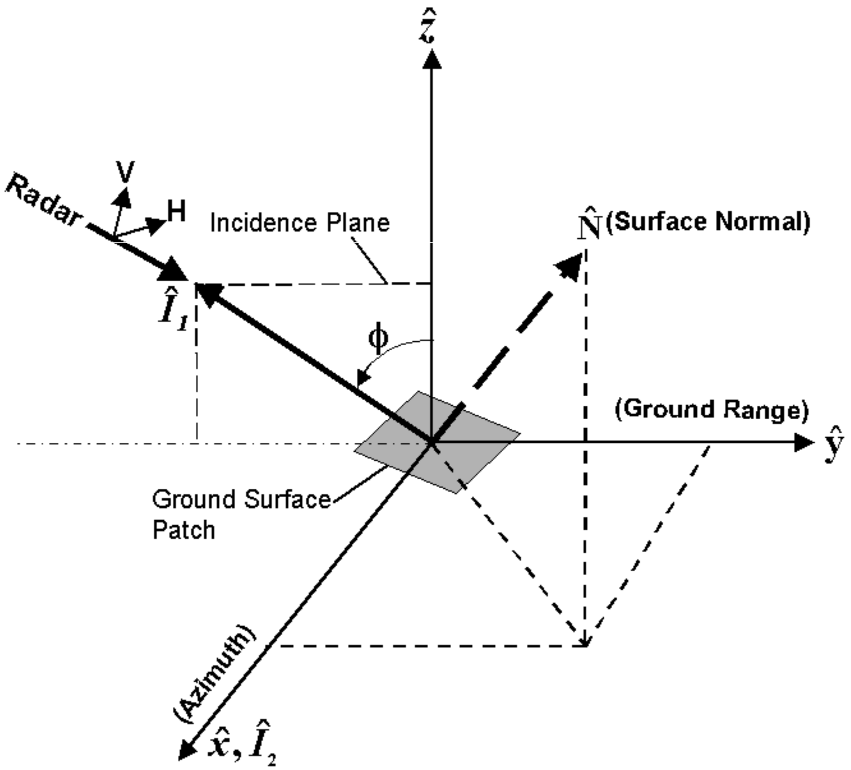
Orientation angle shifts induced by azimuthal slopes
Orientation information imbedded in Pol-SAR data

$$\tan\theta = \frac{\tan\omega}{-\tan\gamma\cos\phi + \sin\phi}$$

ϕ = Radar look angle

$\tan\omega$ = Azimuth slope

$\tan\gamma$ = Ground range slope



Orientation Estimation

Scattering Matrix

$$S^{(new)} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Coherency Matrix

$$T^{(new)} = UTU^{T}$$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

Orientation Rotation

Circular Polarizations (only phase is affected)

$$S_{LL} = (S_{hh} - S_{vv} + 2jS_{hv})/2$$
$$S_{L_\perp L_\perp} = (-S_{hh} + S_{vv} + 2jS_{hv})/2$$
$$S_{LL_\perp} = j(S_{hh} + S_{vv})/2$$

ROTATION →

$$\tilde{S}_{LL} = S_{LL}\, e^{-i2\theta}$$
$$\tilde{S}_{L_\perp L_\perp} = S_{L_\perp L_\perp}\, e^{i2\theta}$$
$$\tilde{S}_{LL_\perp} = S_{LL_\perp}$$

Circular Covariance Matrix

$$\tilde{C} = \begin{bmatrix} \left\langle \left|S_{LL}\right|^2 \right\rangle & \sqrt{2}\left\langle (S_{LL} S^*_{LL_\perp}) e^{-i2\theta} \right\rangle & \left\langle (S_{L_\perp L_\perp} S^*_{LL}) e^{-i4\theta} \right\rangle \\ \sqrt{2}\left\langle (S_{LL_\perp} S^*_{LL}) e^{i2\theta} \right\rangle & 2\left\langle \left| S_{LL_\perp} \right|^2 \right\rangle & \sqrt{2}\left\langle (S_{LL_\perp} S^*_{L_\perp L_\perp}) e^{-i2\theta} \right\rangle \\ \left\langle (S_{LL} S^*_{L_\perp L_\perp}) e^{i4\theta} \right\rangle & \sqrt{2}\left\langle (S_{L_\perp L_\perp} S^*_{LL_\perp}) e^{i2\theta} \right\rangle & \left\langle \left|S_{L_\perp L_\perp}\right|^2 \right\rangle \end{bmatrix}$$

Estimation Methods

Circular Polarization Estimators

$$\tilde{S}_{LL} = S_{LL}\, e^{-i2\theta}$$

$$\Rightarrow \left\langle \tilde{S}_{LL}\tilde{S}^*_{L_\perp L_\perp} \right\rangle = \left\langle (S_{LL} S^*_{L_\perp L_\perp}) e^{-i4\theta} \right\rangle \approx \left\langle (S_{LL} S^*_{L_\perp L_\perp}) \right\rangle e^{-i4\theta}$$

$$\left\langle \tilde{S}_{LL}\tilde{S}^*_{LL_\perp} \right\rangle = \left\langle (S_{LL} S^*_{LL_\perp}) e^{-i2\theta} \right\rangle \approx \left\langle (S_{LL} S^*_{LL_\perp}) \right\rangle e^{-i2\theta}$$

Which estimator is the good one ?

The Circular Co-Pol Algorithm

For Multi-look or single-look complex data

Circular co-pol method (Lee et al. 1999)

$$\left\langle \tilde{S}_{LL}\tilde{S}^*_{L_\perp L_\perp}\right\rangle \approx \left\langle (S_{LL}S^*_{L_\perp L_\perp})\right\rangle e^{-i4\theta}$$

For a reflection symmetrical medium, $<S_{RR}S^*_{LL}>$ **should be real.**

$$<S_{LL}S^*_{L_\perp L_\perp}> = (-<|S_{HH}-S_{VV}|^2> + 4<|S_{HV}|^2>)/4$$

From.

$$<\tilde{S}_{LL}\tilde{S}^*_{L_\perp L_\perp}> = \frac{1}{4}\{<-|\tilde{S}_{HH}-\tilde{S}_{VV}|^2+4|\tilde{S}_{HV}|^2> - i\,4\,\mathrm{Re}(<(\tilde{S}_{HH}-\tilde{S}_{VV})\tilde{S}^*_{HV}>)\}$$

$$-4\theta = Arg(<\tilde{S}_{LL}\tilde{S}^*_{L_\perp L_\perp}>) = \tan^{-1}\left(\frac{-4\,\mathrm{Re}(<(\tilde{S}_{HH}-\tilde{S}_{VV})\tilde{S}^*_{HV}>)}{-<|\tilde{S}_{HH}-\tilde{S}_{VV}|^2>+4<|\tilde{S}_{HV}|^2>}\right)$$

The Circular Co-Pol Algorithm

From :

$$-4\theta = Arg(<\tilde{S}_{LL}\tilde{S}^{*}_{L_\perp L_\perp}>) = \tan^{-1}\left(\frac{-4\,\mathrm{Re}(<(\tilde{S}_{HH}-\tilde{S}_{VV})\tilde{S}^{*}_{HV}>)}{-<|\tilde{S}_{HH}-\tilde{S}_{VV}|^2>+4<|\tilde{S}_{HV}|^2>}\right)$$

A bias of $\pm\pi$ is introduced, since

$$\{-<|\tilde{S}_{HH}-\tilde{S}_{VV}|^2>+4<\left|\tilde{S}_{HV}\right|^2>\}<0$$

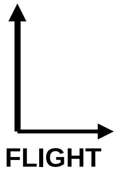
The algorithm is

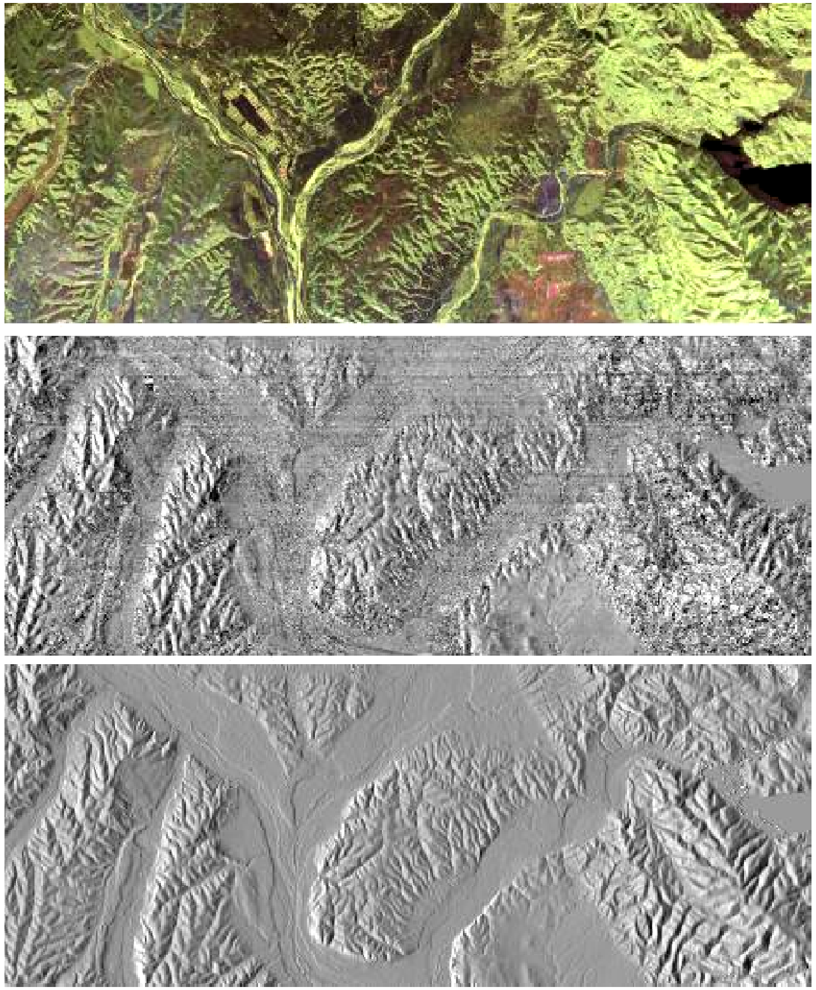
$$\theta = \begin{cases} \eta, & if \quad \eta \le \pi/4 \\ \eta - \pi/2, & if \quad \eta > \pi/4 \end{cases}$$

$$\eta = \frac{1}{4}\left[\tan^{-1}\left(\frac{-4\,\mathrm{Re}(<(\tilde{S}_{HH}-\tilde{S}_{VV})\tilde{S}^{*}_{HV}>)}{-<|\tilde{S}_{HH}-\tilde{S}_{VV}|^2>+4<|\tilde{S}_{HV}|^2>}\right)+\pi\right]$$

Measurement range: $-\pi/4 \le \theta \le \pi/4$

Terrain Vegetation and Topography Representative of Camp Roberts, CA





JPL AIRSAR <u>L-Band</u>
(Camp Roberts)
|HH-VV| |HV|+|VH|
|HH+VV|

From POLSAR data

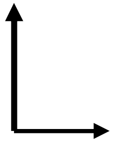
$$-45^{o} \leq \theta \leq 45^{o}$$

From C-band Interferometry

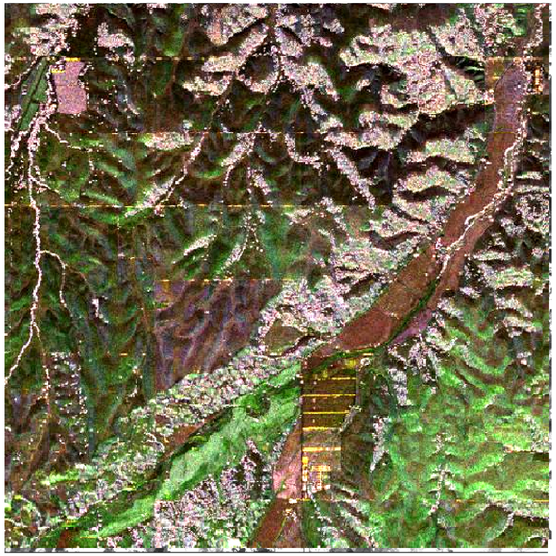
$$\tan\theta = \frac{\tan\omega}{-\tan\gamma\cos\phi + \sin\varphi}$$

J.S. Lee, D.L. Schuler and T.L. Ainsworth, "Polarimetric SAR data compensation for terrain azimuth slope variations," IEEE TGRS (September, 2000)

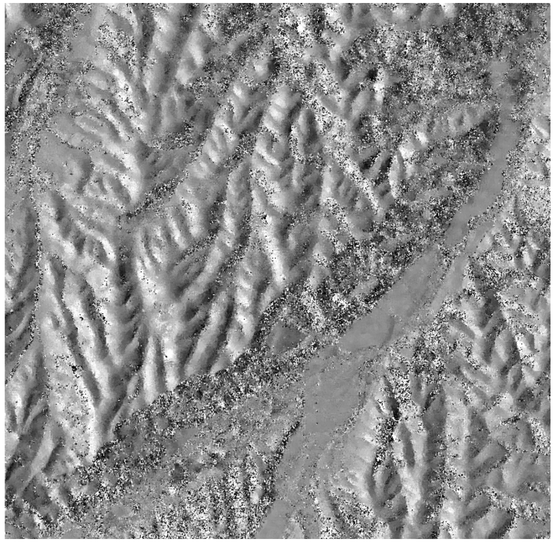
P.O.A ESTIMATION



FLIGHT



|HH-VV|, |HV|+|VH|, |HH+VV|

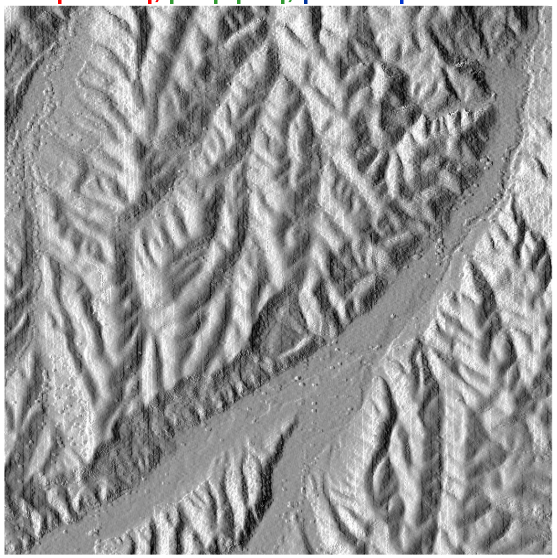


L-Band PolSAR derived PO angle

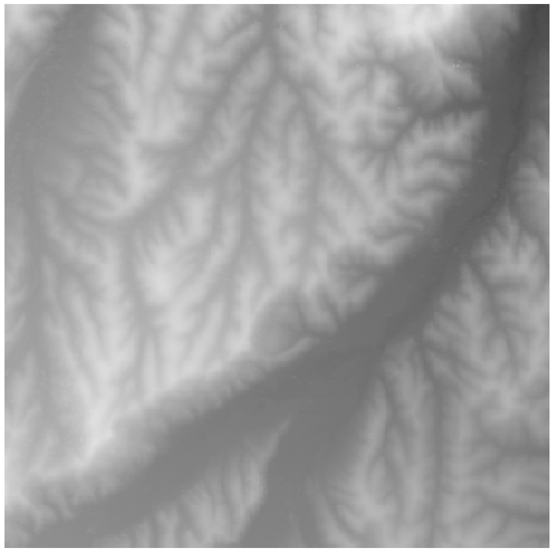
PO angles derived By L-Band PolSAR

PO angles derived from DEM of C-Band interferometry

$$\tan\theta = \frac{\tan\omega}{-\tan\gamma\cos\phi + \sin\varphi}$$



PO angles from C-band DEM



C-Band DEM

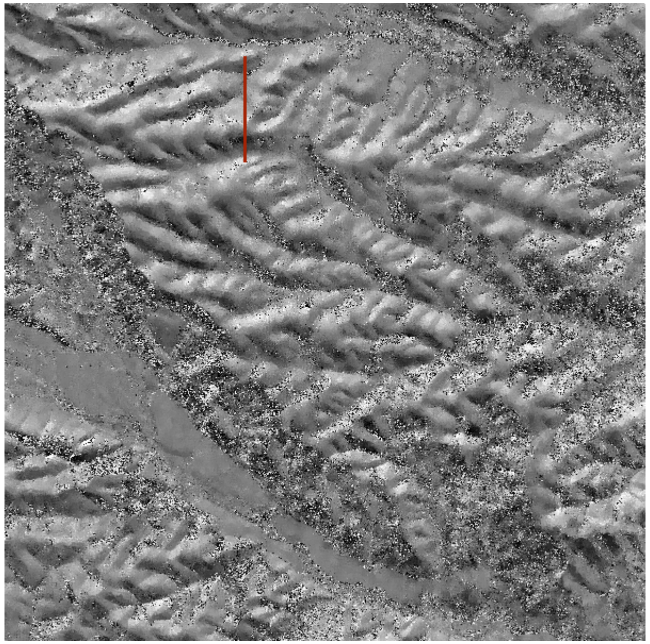
The effect of POA Compensation

The Polarization Orientation (PO) angle effect :

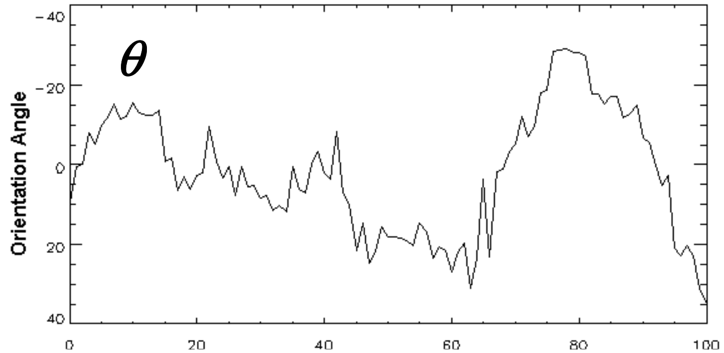
- **Azimuth slopes and buildings induce PO angle shift**
- **Model based decompositions based on uncompensated data may mis-interpret scattering mechanisms**
    - **High relief terrain = Forest (volume scattering)**
    - **Buildings = Forest**

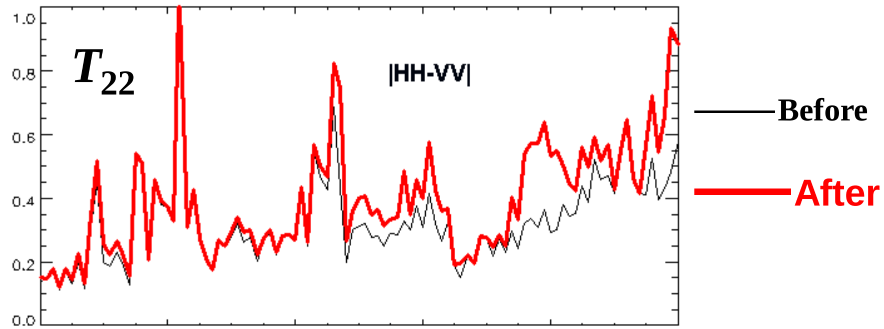
P.O.A COMPENSATION

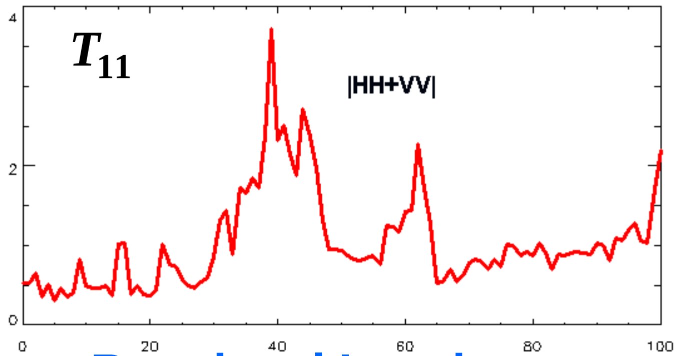
The effect on Coherency Matrix



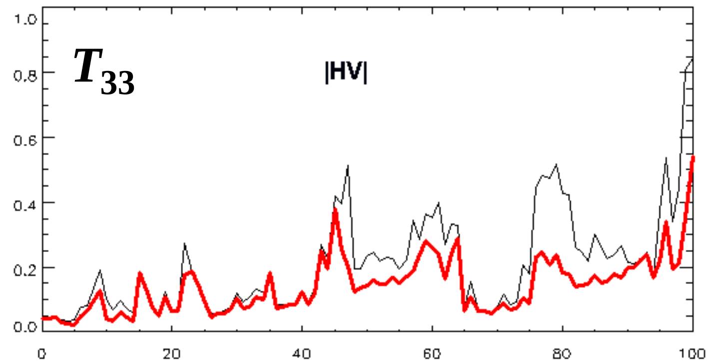
Orientation angle map $(-45^o \le \theta \le 45^o)$



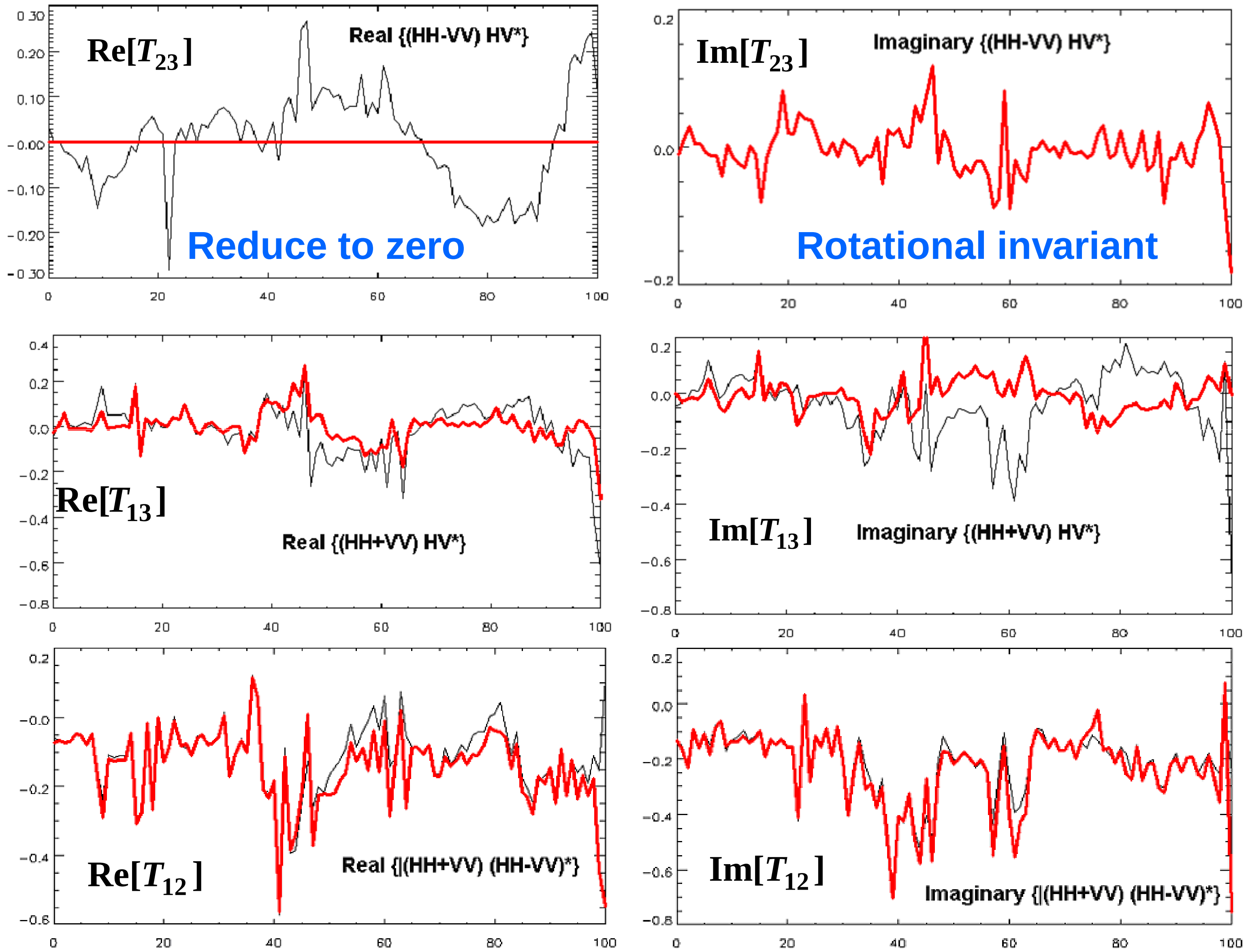


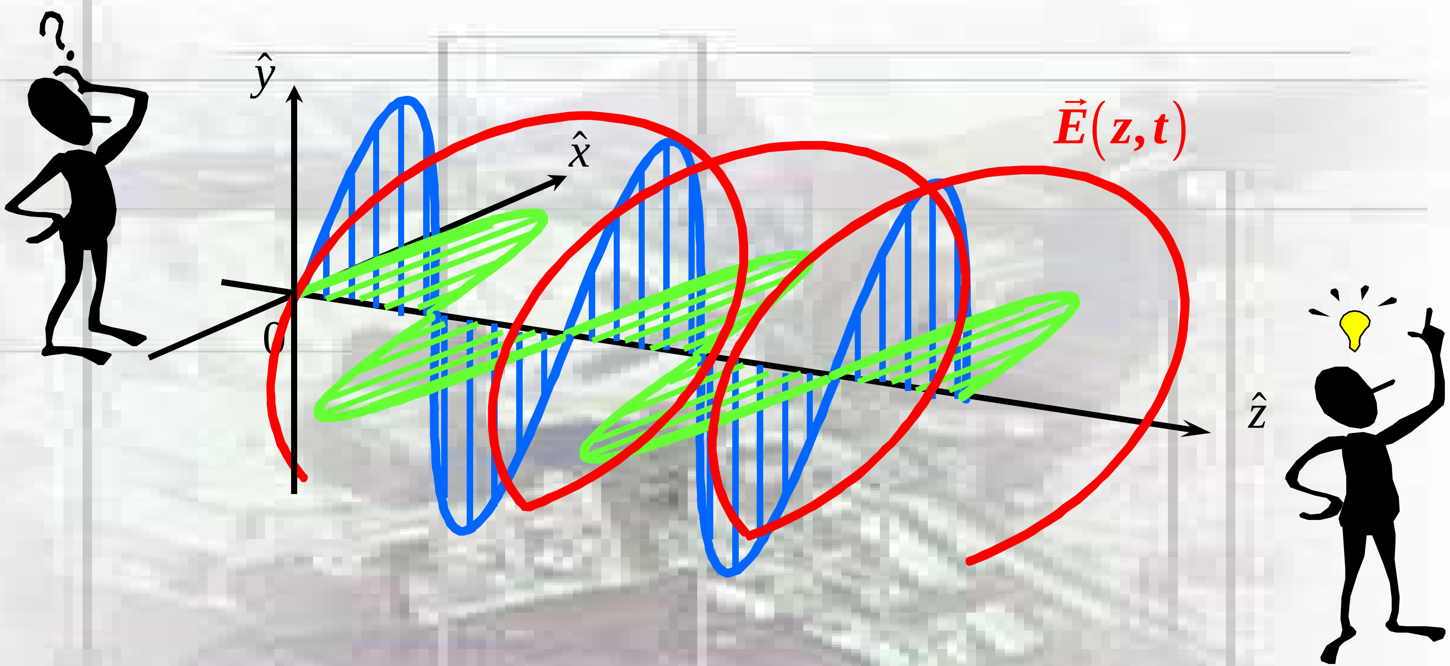


Rotational Invariant



P.O.A COMPENSATION



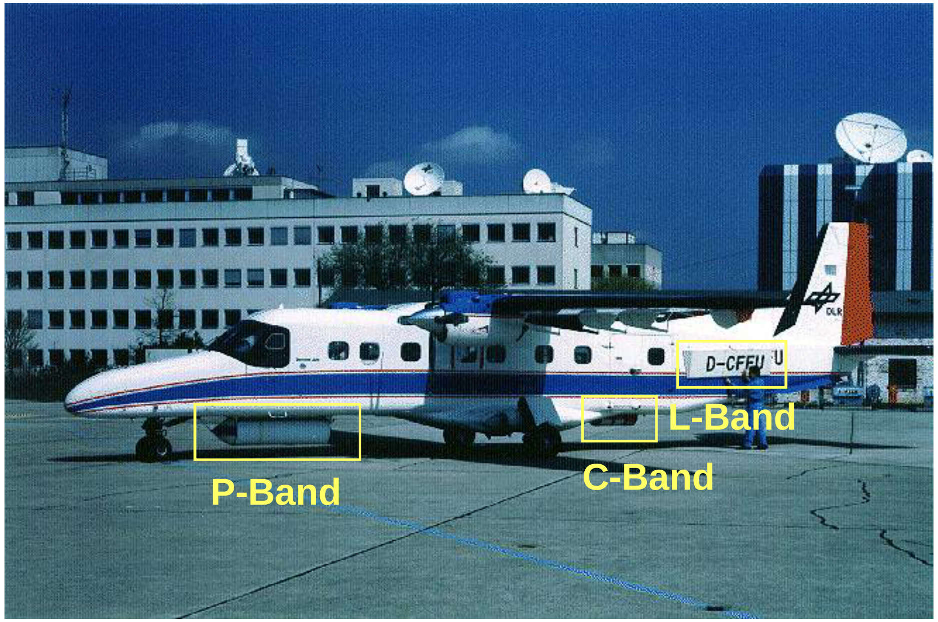


SOIL MOISTURE ESTIMATION

An Example

ALLING

DLR / E-SAR   L-band



DLR

Deutsches Zentrum für
Luft- und Raumfahrt
Institut für Hochfrequenztechnik
und Radarsysteme

$$\theta \in [25° ... 55°]$$

$$res = (1.5m \times 3m)$$

LAND – AGRICULTURE APPLICATIONS

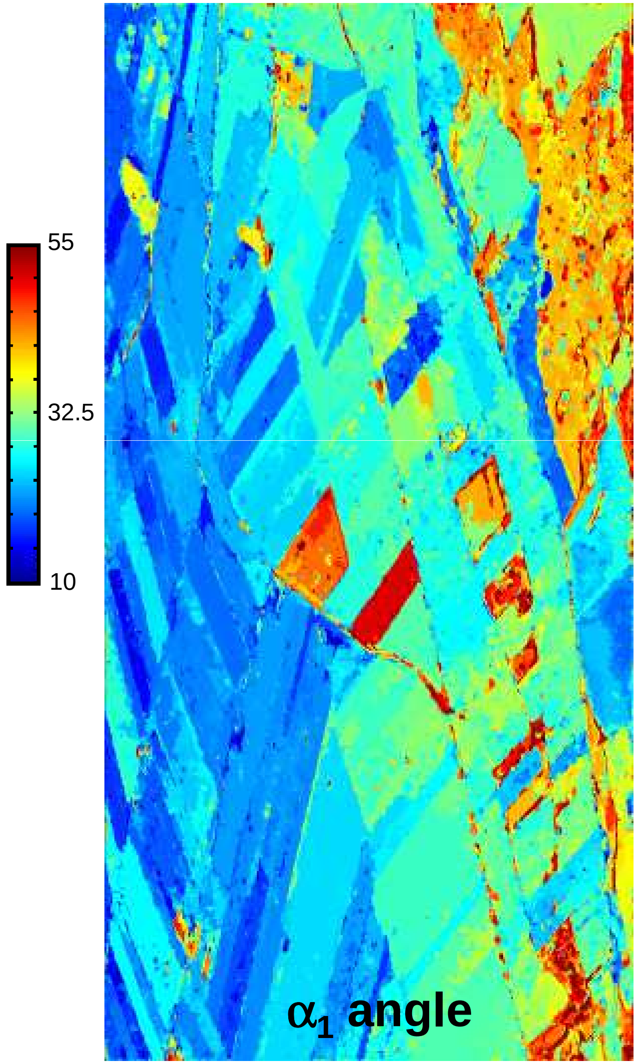
ALLING Site
DLR – ESAR
L-band

LAND – AGRICULTURE APPLICATIONS

Soil Moisture & Biomass Estimation using Polarimetric Scattering Theory

- **Oh**
- **Shi**
- **Dubois**
- **Francesco Mattia**
- **X-Bragg (I. Hajnsek - 2000)**
- **E.R.D (Eigenvalue Relative Difference) (S. Allain – 2003)**
- **S.E.R.D and D.E.R.D (S. Allain – 2005)**

SOIL MOISTURE CHARACTERIZATION

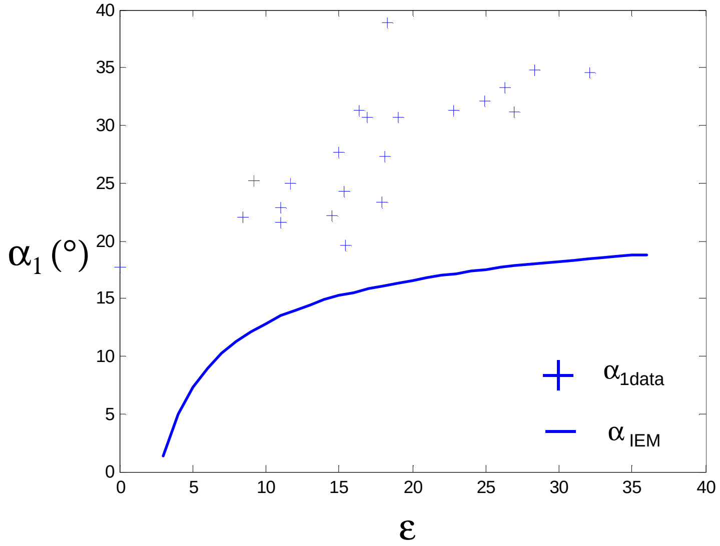


SOIL MOISTURE CHARACTERIZATION

Observations

α_{1_IEM} increases with soil moisture
Quasi roughness independent
α_1 angle sensitive to soil moisture

$8 < \varepsilon < 33$
Gaussian spectrum shape
σ = 2cm
Lc = 10 cm
θ = 50°



Curves have the same behavior but different amplitude level !!!

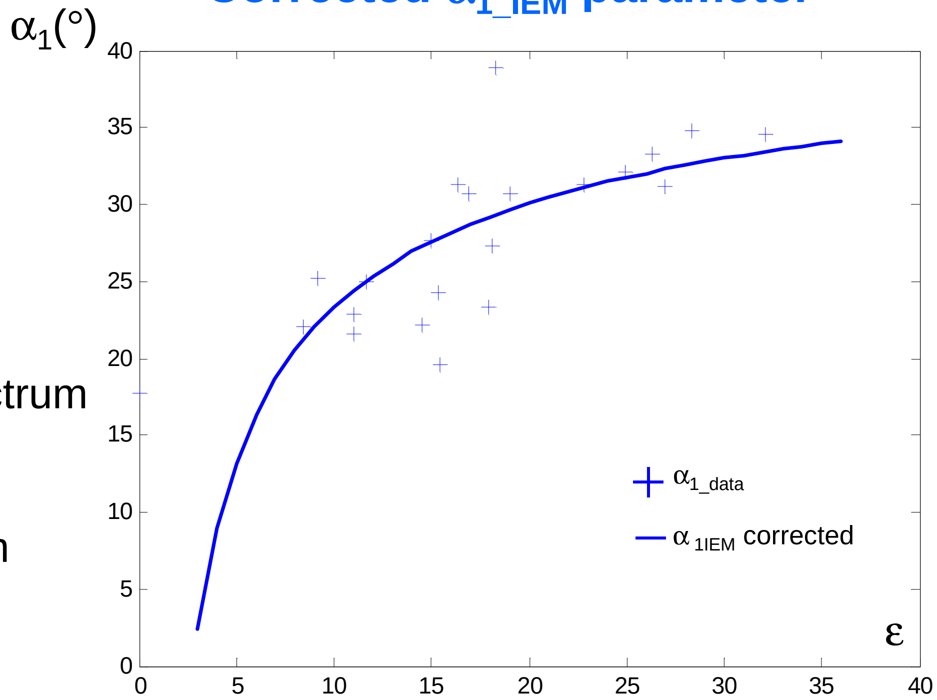
Corrected α_{1_IEM} parameter →

$$\alpha_{1\_IEM\_Corrected} = \alpha_{1\_IEM} \frac{\langle \alpha_{1\_data} \rangle}{\langle \alpha_{1\_IEM} \rangle}$$

SOIL MOISTURE CHARACTERIZATION

$8 < \varepsilon < 33$
Gaussian spectrum shape
σ = 2cm
Lc = 10 cm
θ = 50°

Corrected α_{1_IEM} parameter



α_1 angle sensitive to soil moisture

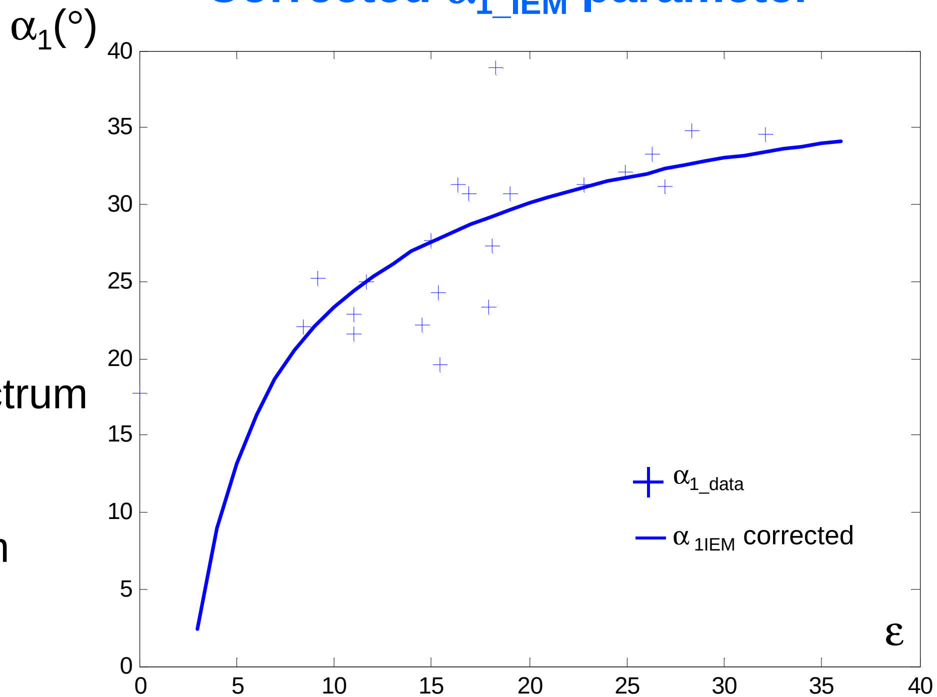
$$\alpha_{1\_IEM\_Corrected} = \alpha_{1\_IEM} \frac{\langle \alpha_{1\_data} \rangle}{\langle \alpha_{1\_IEM} \rangle}$$

Correction coefficient derived for each dataset independently

SOIL MOISTURE CHARACTERIZATION

$8 < \varepsilon < 33$
Gaussian spectrum shape
$\sigma = 2cm$
Lc = 10 cm
$\theta = 50°$

Corrected α_{1_IEM} parameter



SOIL MOISTURE INVERSION SCHEME

$\alpha_{1\_data/L\text{-}band}$ → ε retrieved ← $\alpha_{1\_IEM/\ L\text{-}band}$ corrected

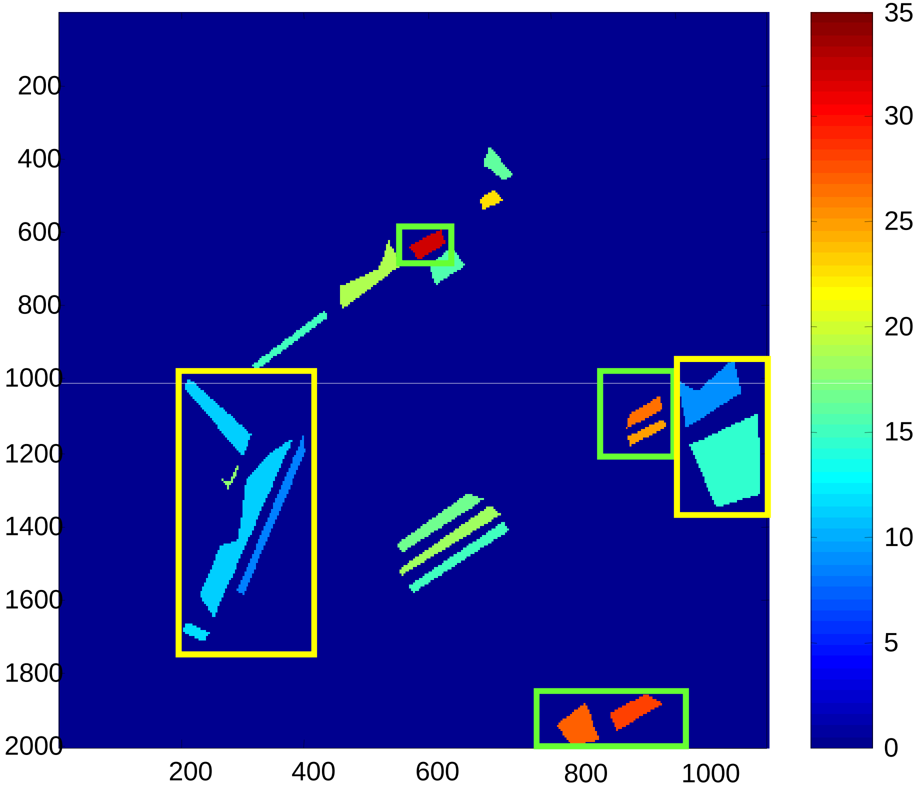
Dobson, Topp model

Retrieved soil moisture : mv

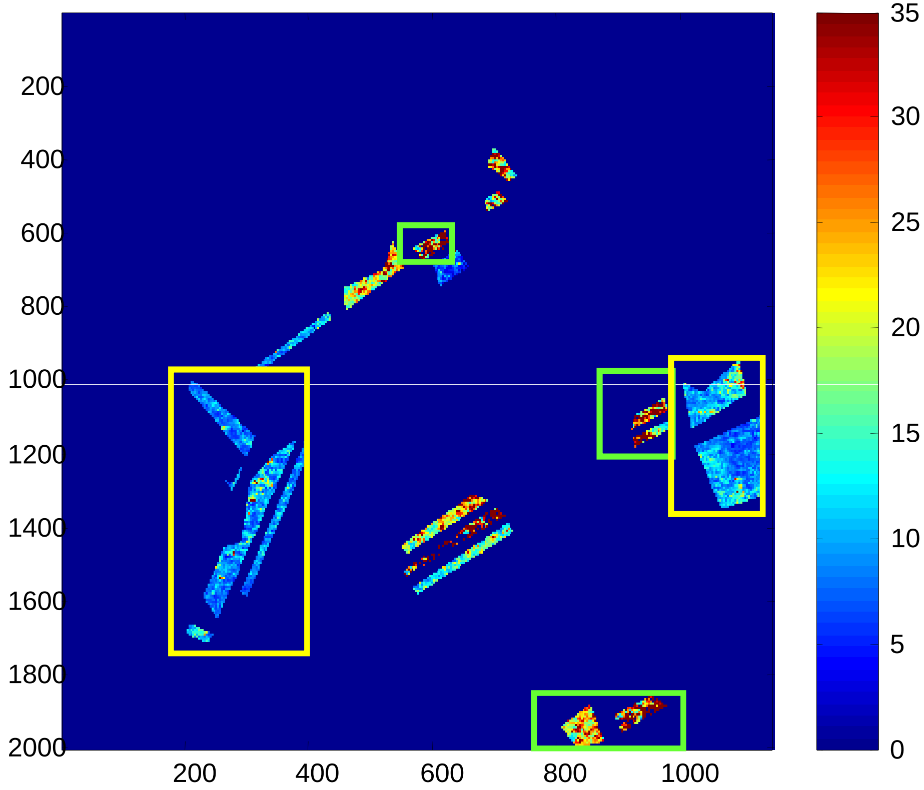
QUANTITATIVE ANALYSIS

SOIL MOISTURE CHARACTERIZATION

Ground truth measurements



Retrieved dielectric constant

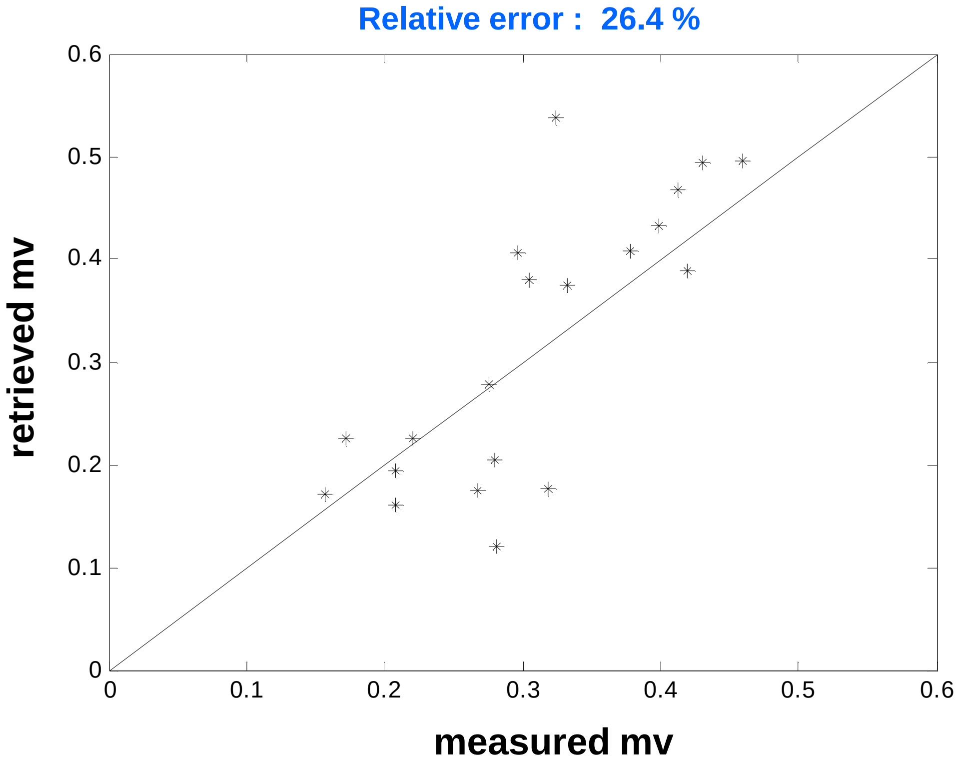


High ε values

Low ε values

Good agreement between ground truth and estimated moisture

SOIL MOISTURE CHARACTERIZATION



$$relative\ error = \left| \frac{retrieved\ mv - measured\ mv}{retrieved\ mv} \right| .100$$

SOIL MOISTURE CHARACTERIZATION

1 : Seedbed winter cereals

2 : Seedbed winter cereals (80%,15 cm)

3 : Harrowed

4 : Harrowed

5 : Pasture, grassland (95%,10 cm)

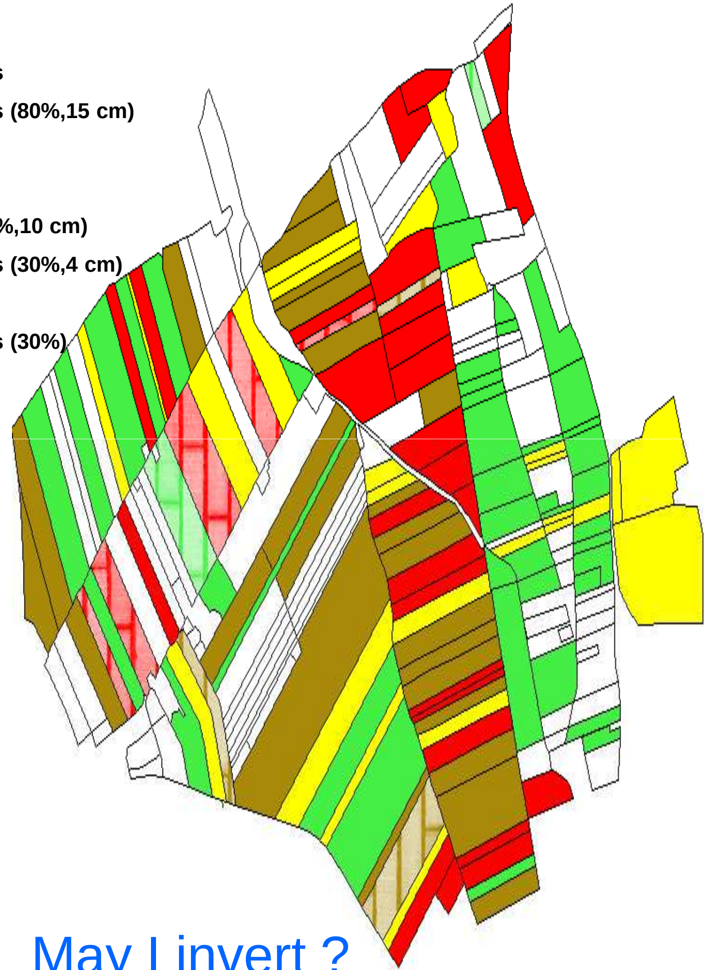
6 : Seedbed winter cereals (30%,4 cm)

7 : Seedbed

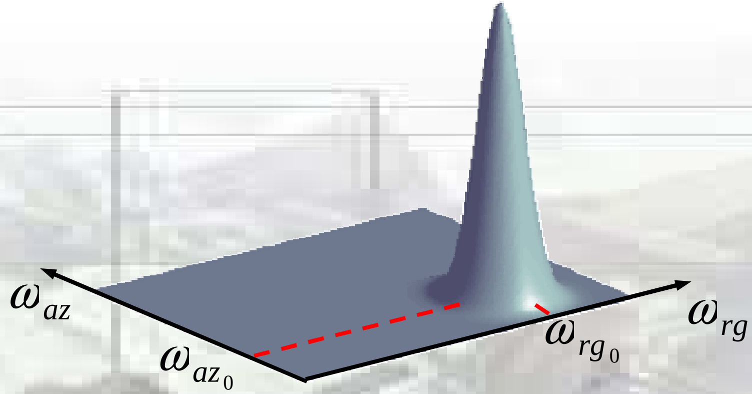
8 : Seedbed winter cereals (30%)

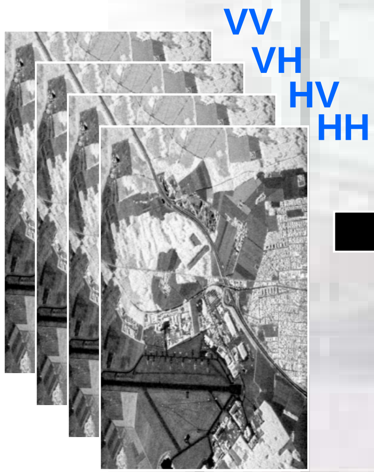
9 : Harrowed

10 : Pasture, grassland



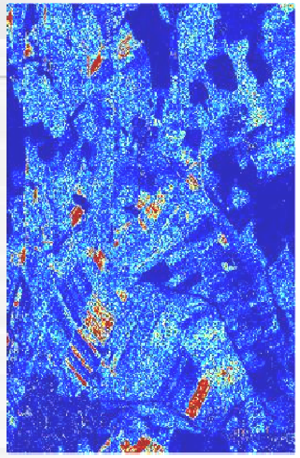
May I invert ?
Where can I invert ?

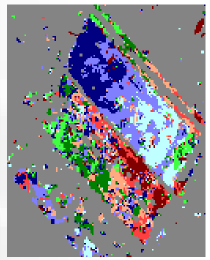




NON STATIONARY SCATTERING ANALYSIS

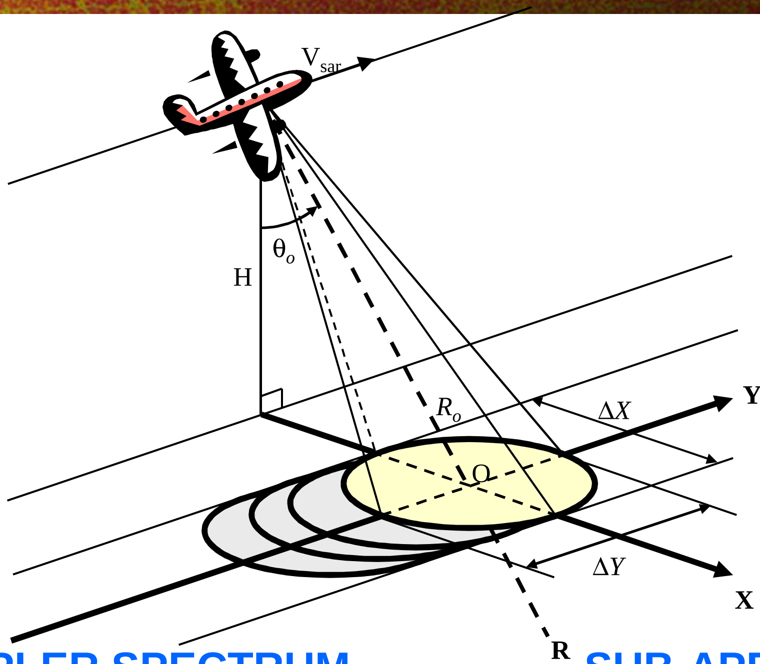
L. FERRO-FAMIL, A. REIGBER (2003)



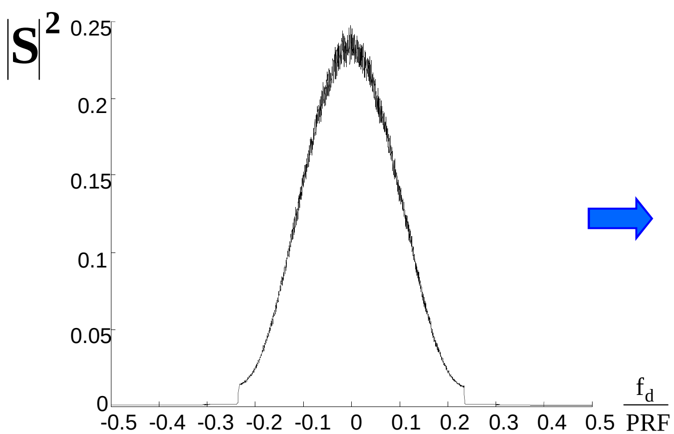


L. Ferro-Famil, A. Reigber, E. Pottier, W.M. Boerner"Scene Characterization Using Subaperture Polarimetric SAR Data" IEEE Transactions on Geoscience and Remote Sensing, Vol 41, n° 10, October 2003.

SUB-IMAGE ANALYSIS



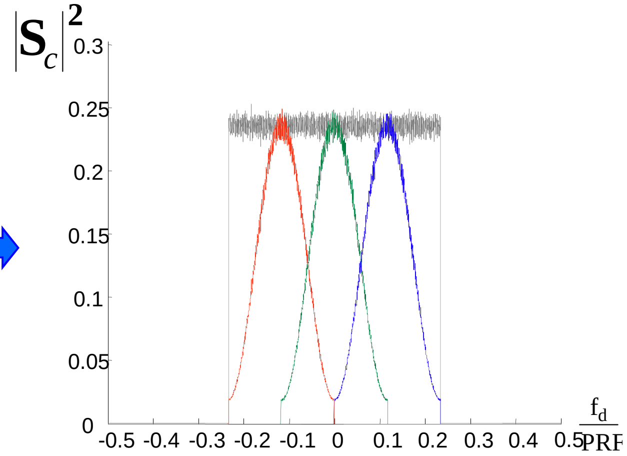
AZIMUT AVERAGE DOPPLER SPECTRUM



Side-lobes weighting / Antenna pattern

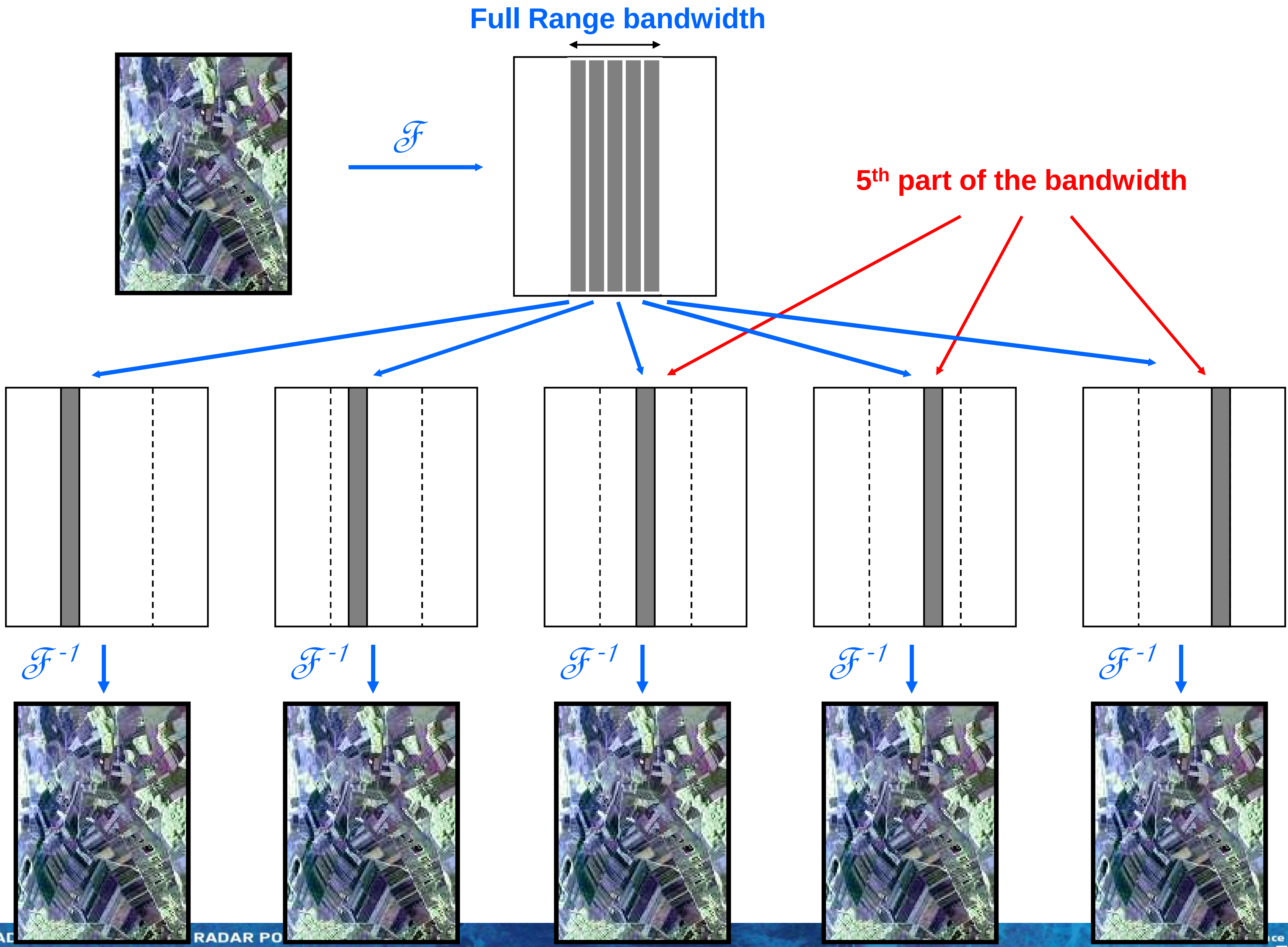
Amplitude correction

SUB-APERTURE DECOMPOSITION

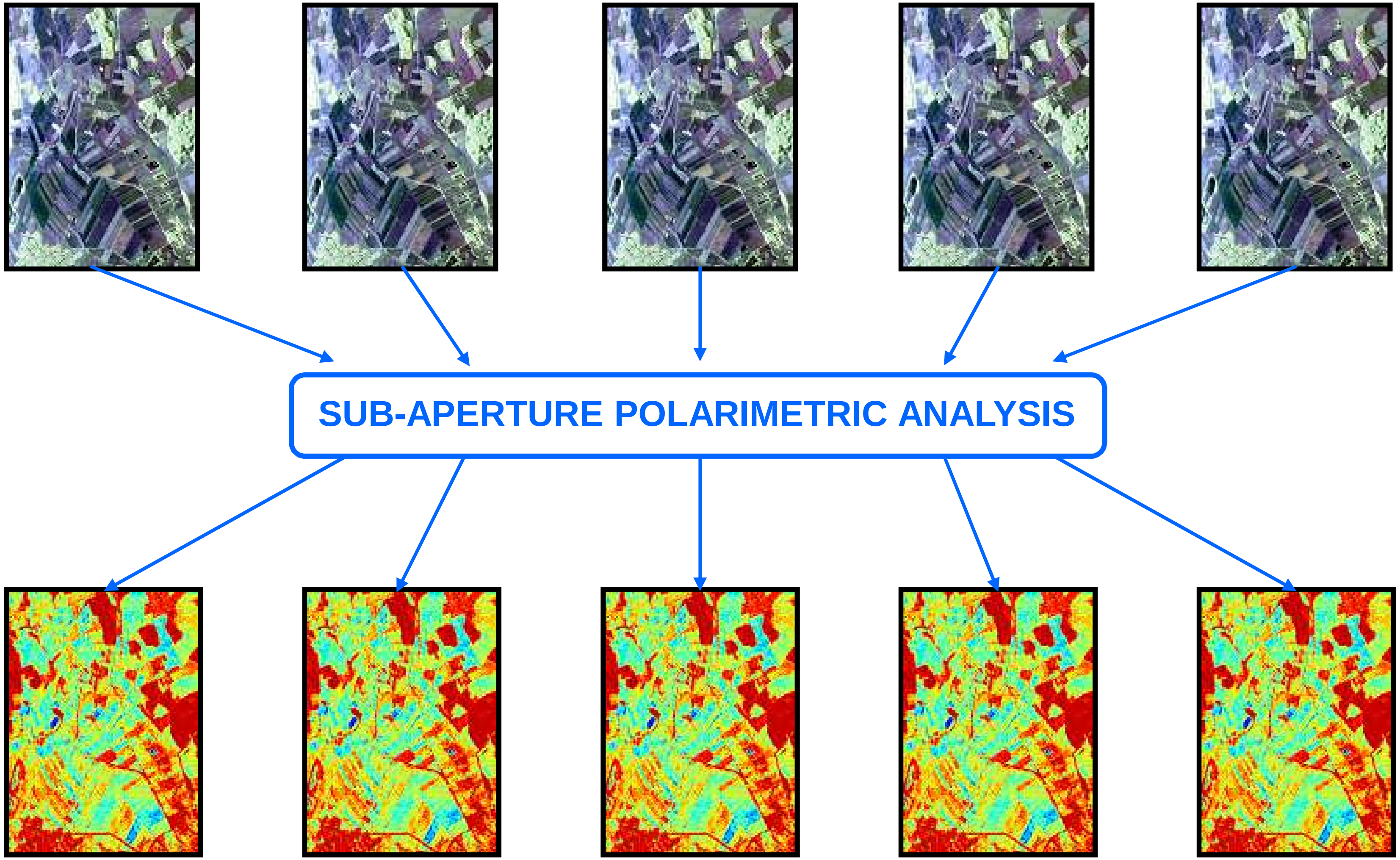


- **Reduced azimuth resolution**
- **Varying center look angle ϕ_d**

SUB-IMAGE ANALYSIS

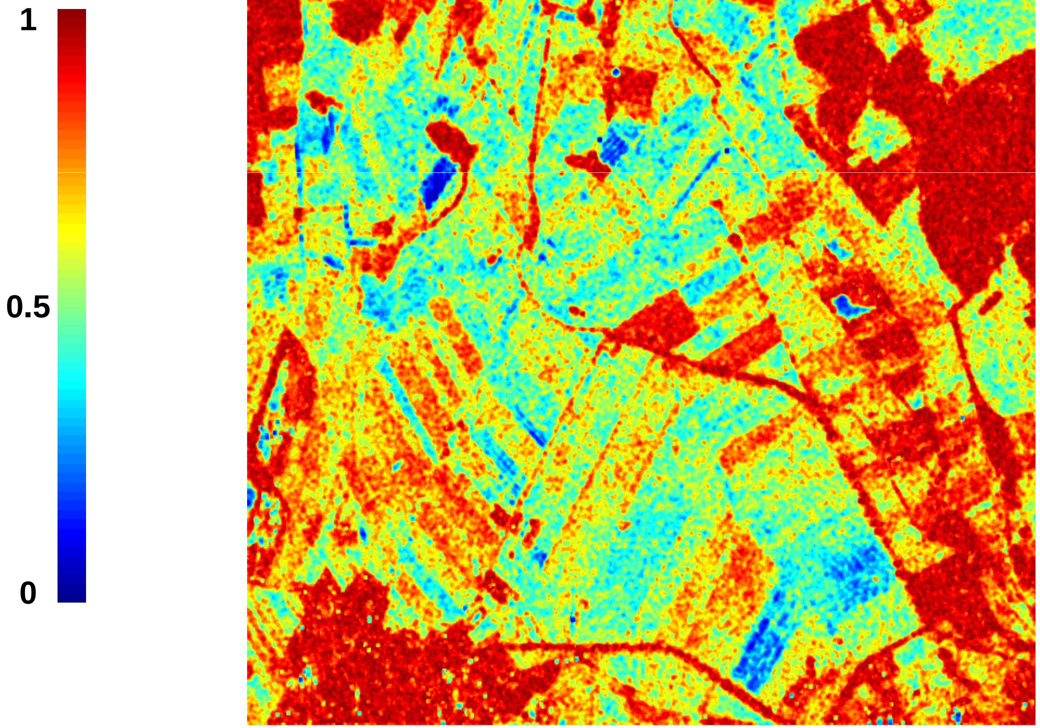


SUB-IMAGE ANALYSIS



SUB-IMAGE ANALYSIS

T_{11} T_{22} T_{33}



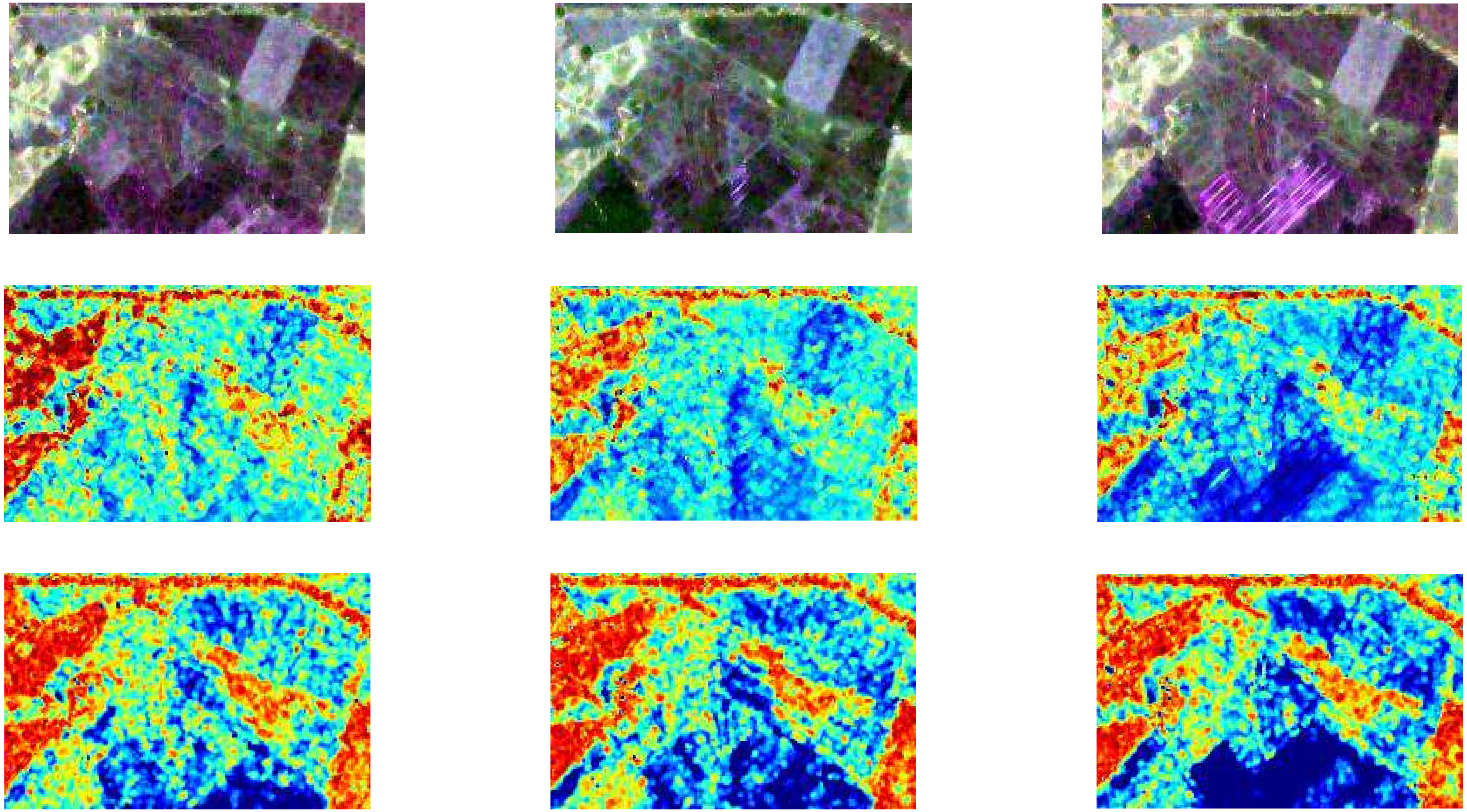
DLR
ALLING DATA
DLR / E-SAR L-band

Visualization of polarimetric variations

SUB-IMAGE ANALYSIS

α

H



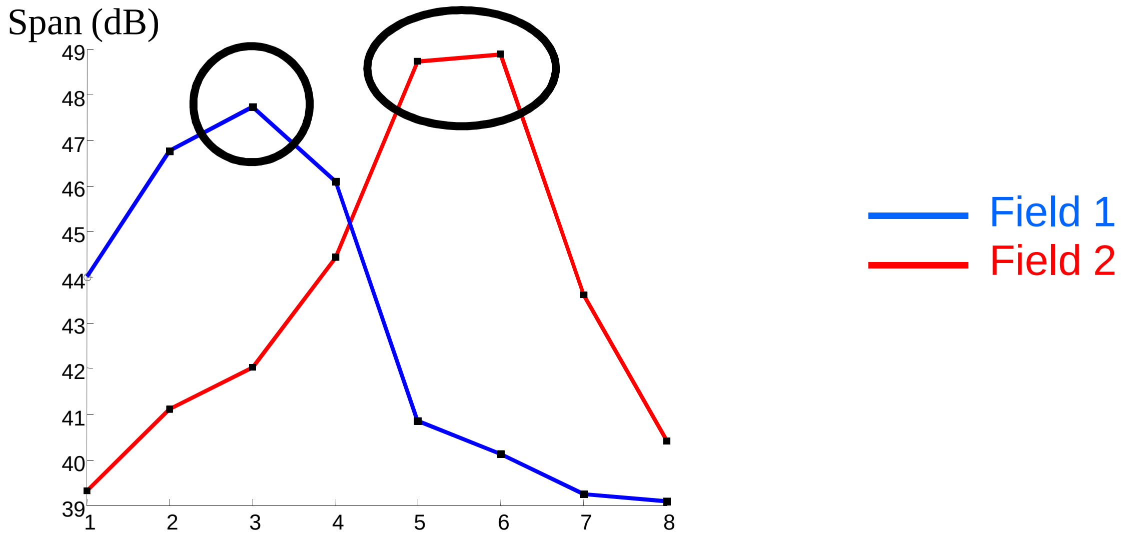
$$\phi_d = \phi_{\min}$$

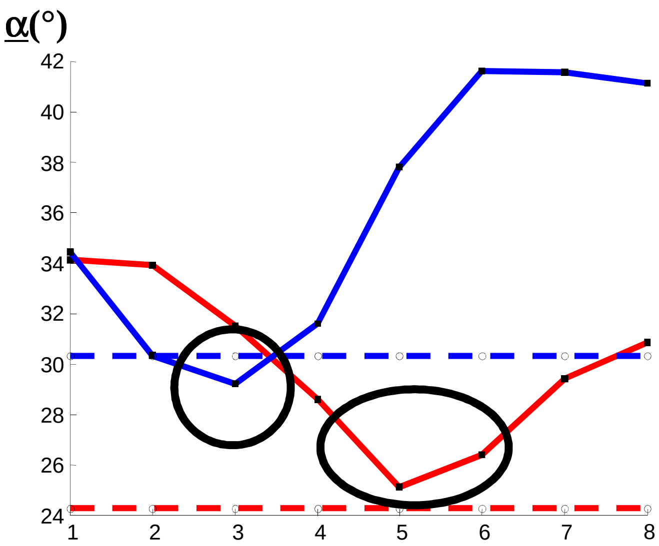
$$\phi_d = \frac{\phi_{\min} + \phi_{\max}}{2}$$

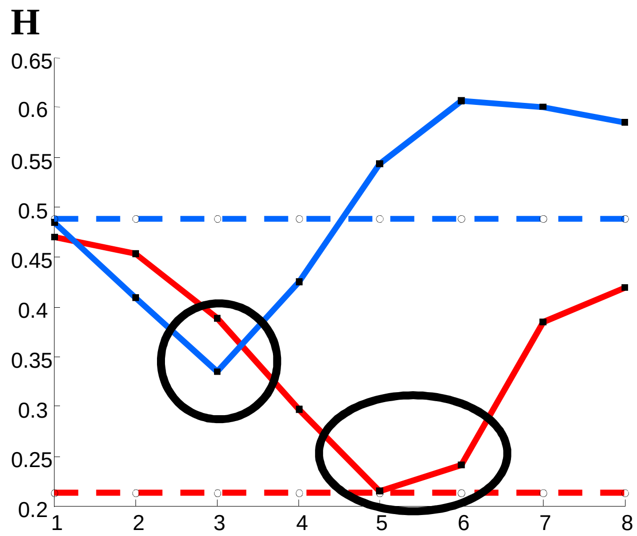
$$\phi_d = \phi_{\max}$$

Variations of Polarimetric Indicators

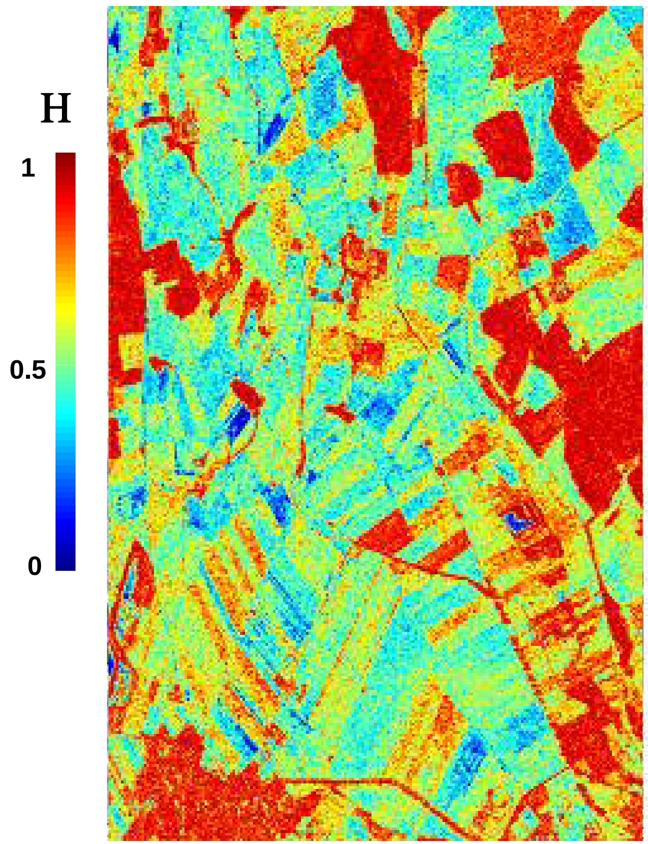
Resonant surface backscattering

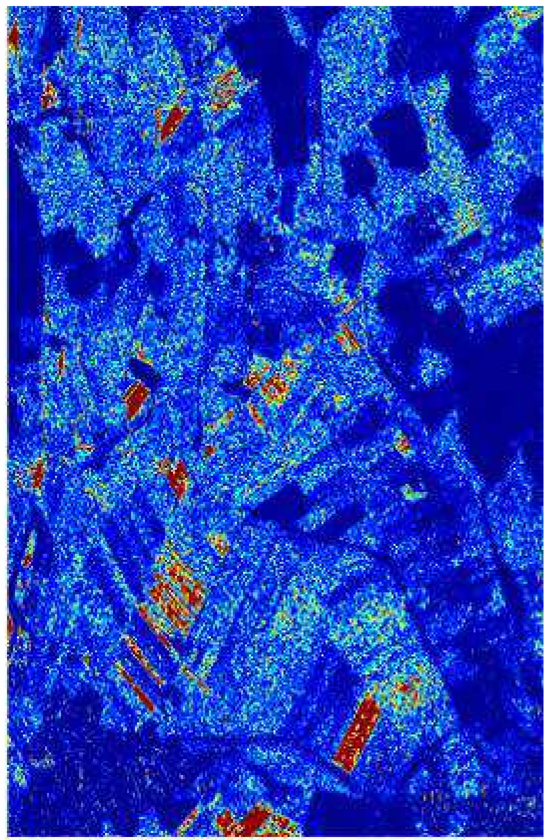






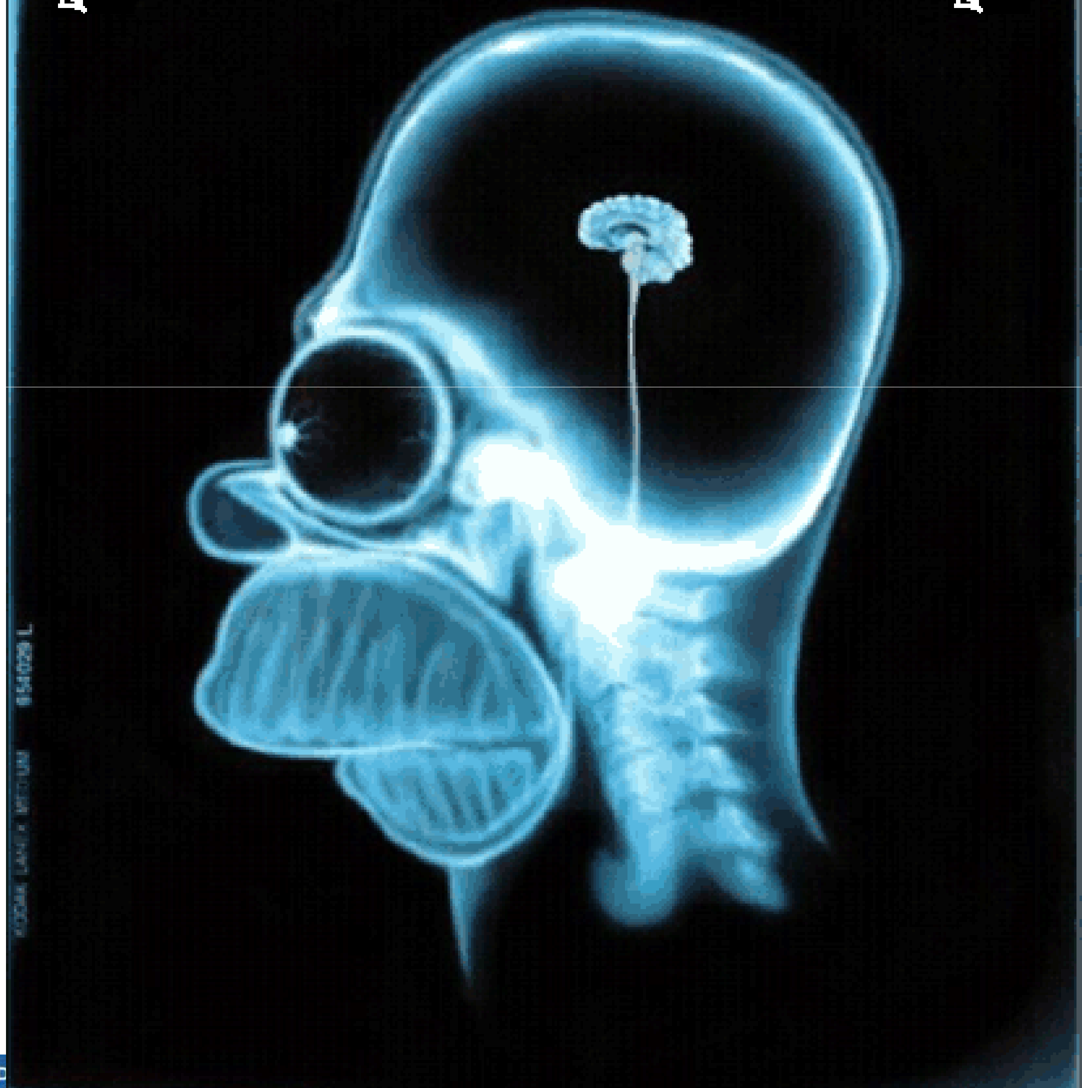
Non-stationary media discrimination





CV(H)

Questions ?



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