Discrete Bayesian Inversion of Satellite Gravity

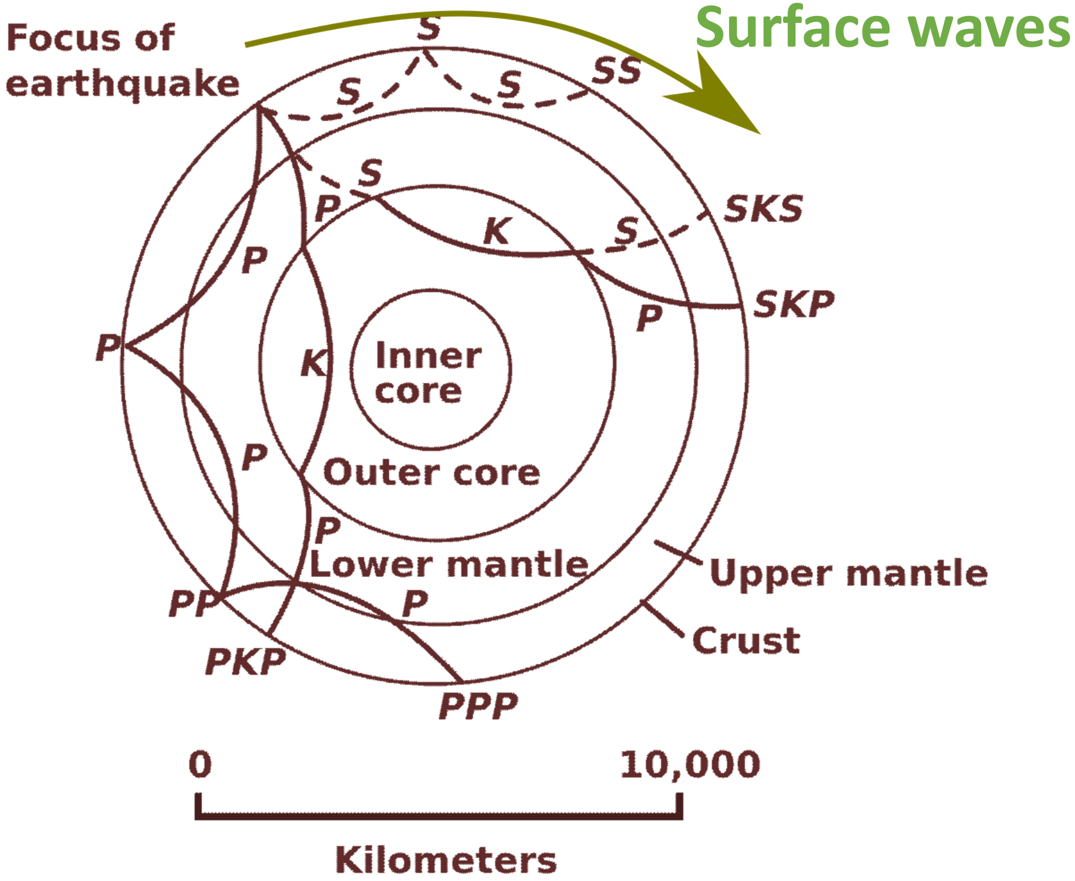
Wolfgang Szwillus,
Kiel University

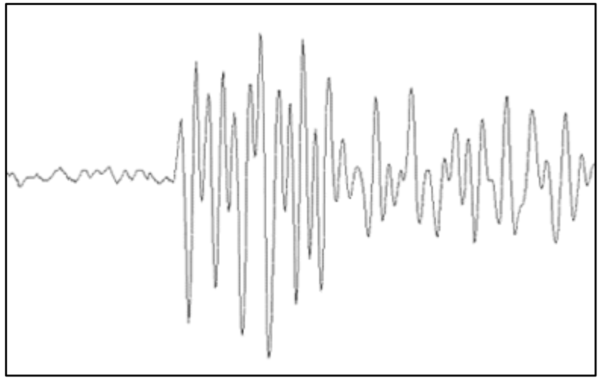
LIVING PLANET FELLOWSHIP

LITHOSPHERE

esa

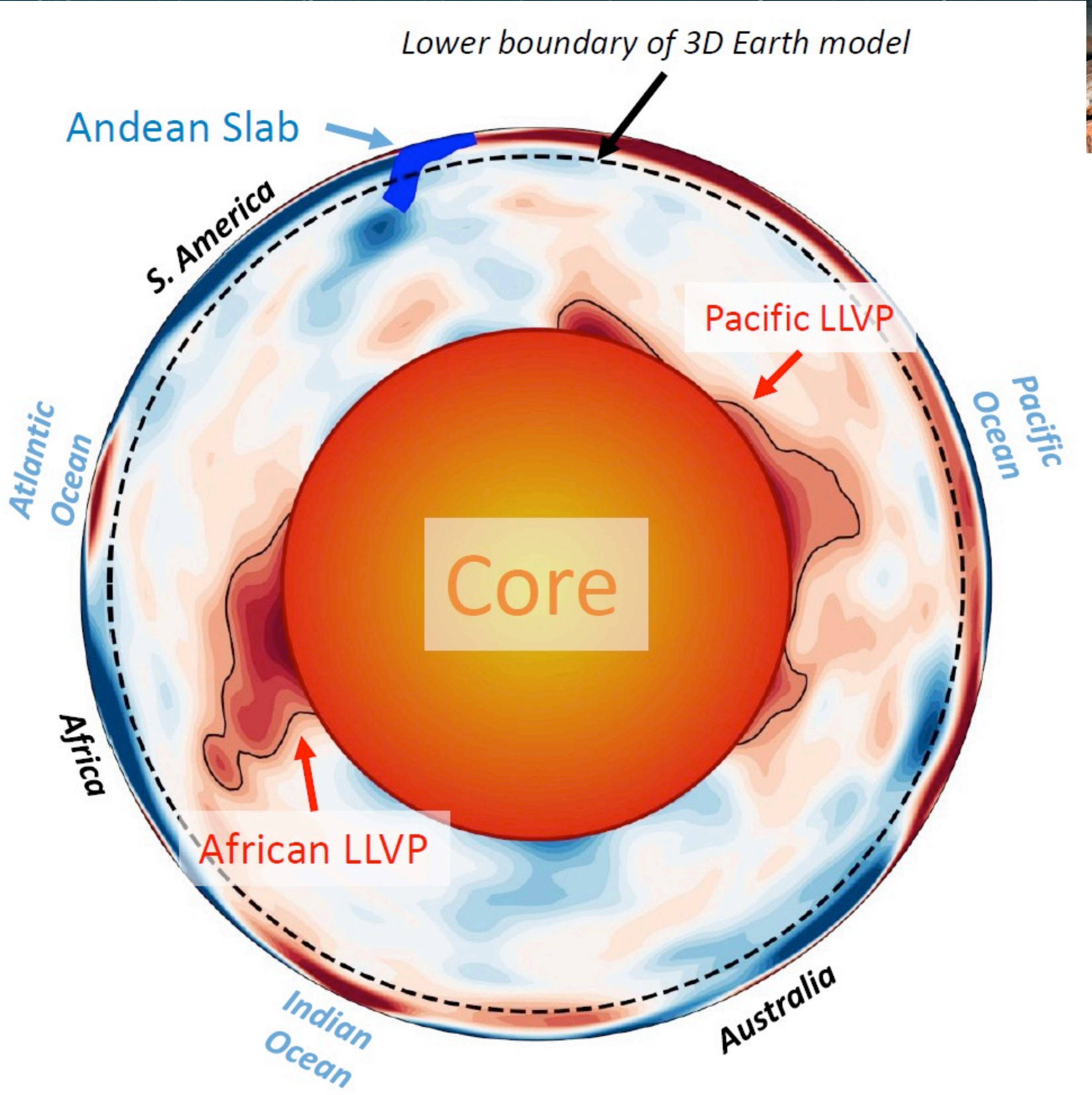
Seismic tomography (Solid Earth's ‚default' tool)





A seismogram

- Using global networks of receivers, you can determine traveltimes of seismic waves
- These traveltimes tell you about velocity structure inside the Earth



Seismic tomography model SMEAN2 (Becker and Boschi, 2012)

Previous work/motivation

- Seismic tomography models show velocity anomalies in the mantle
- **Red = Slow**; **Blue = Fast**
- These anomalies could be due to **temperature** and/or **composition**
- They should also have an associated **density** anomaly
- They should be visible in the satellite **gravity field**

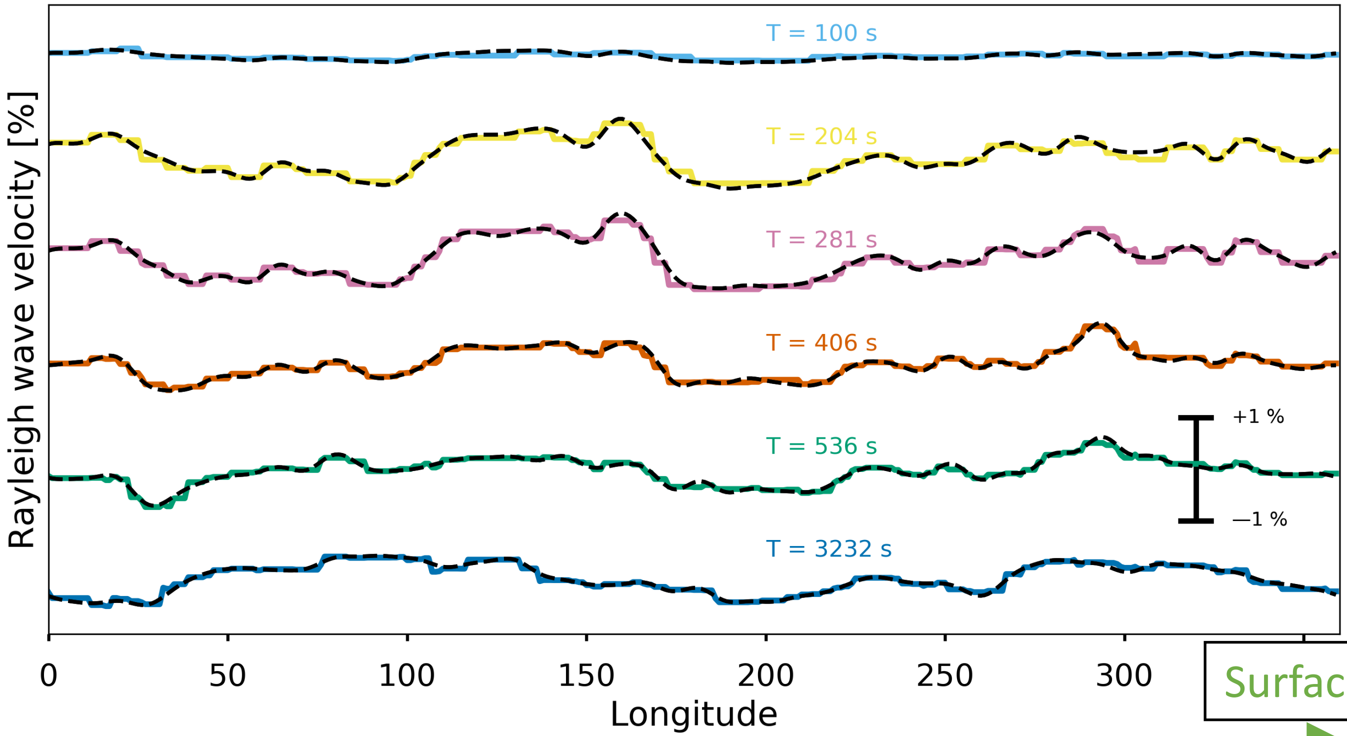
Surface waves

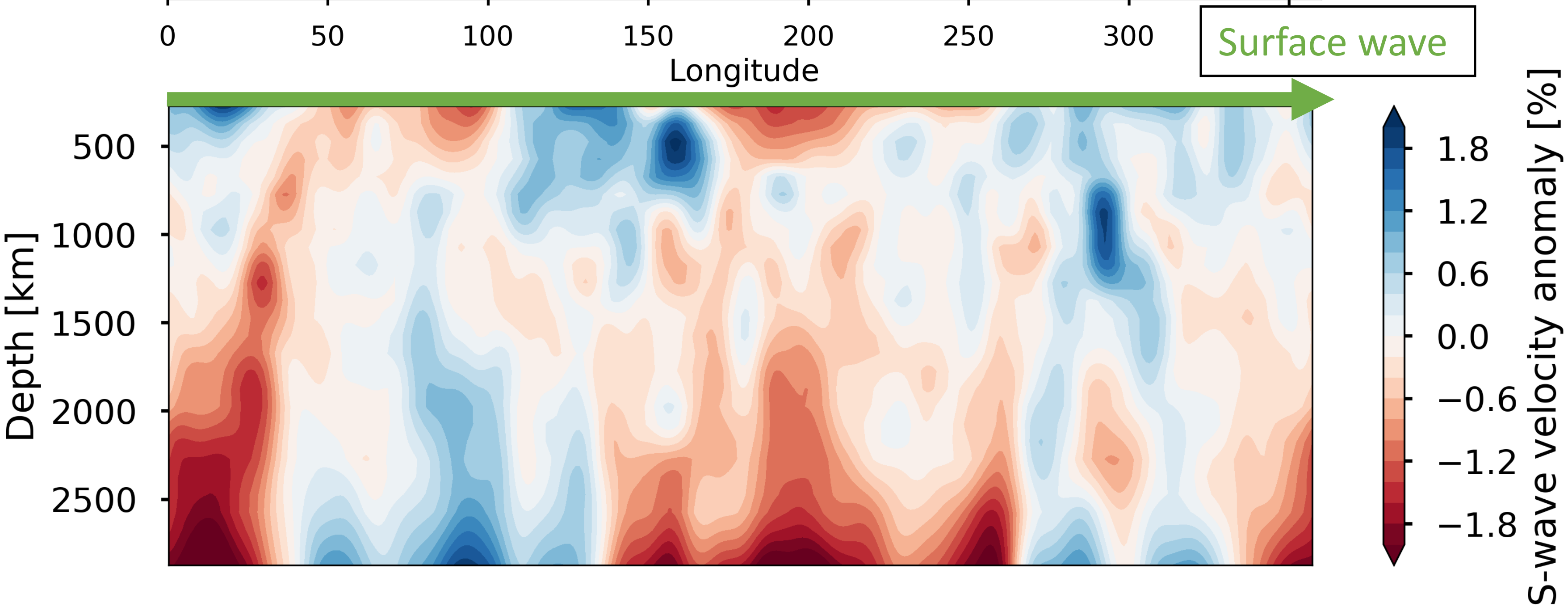
- Rayleigh waves are a type of surface wave
- Their phase speed depends on period (*dispersion*)
- At every location, phase speed only depends on velocity structure underneath

Longer Period

↓

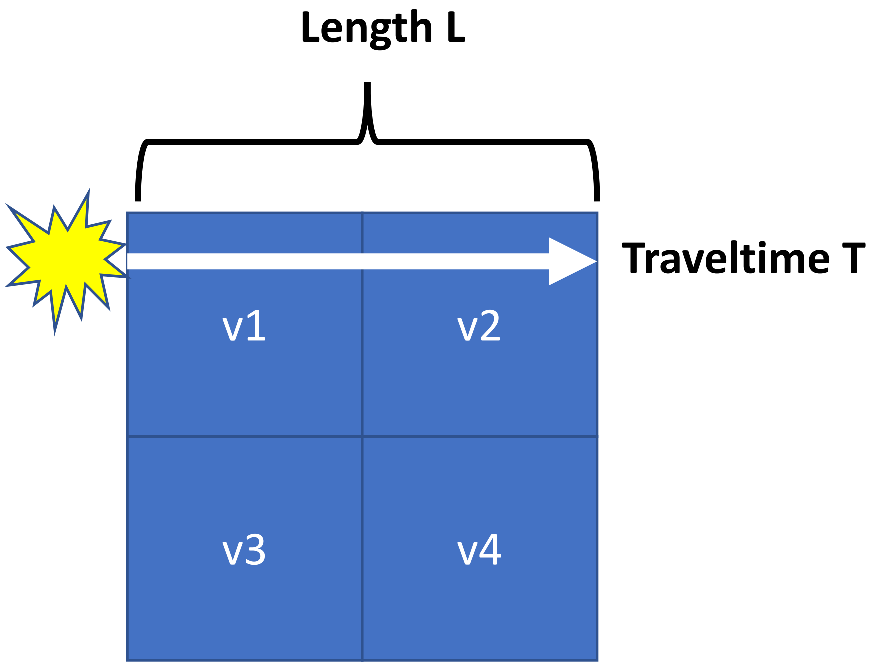
Deeper sensitivity





Spatial parametrization

- Grids are the main form of spatial parametrization in seismology
- The inverse problem is typically ill-posed
- Regularization (smoothing) is required
- Complicates relation to other method (like gravity)

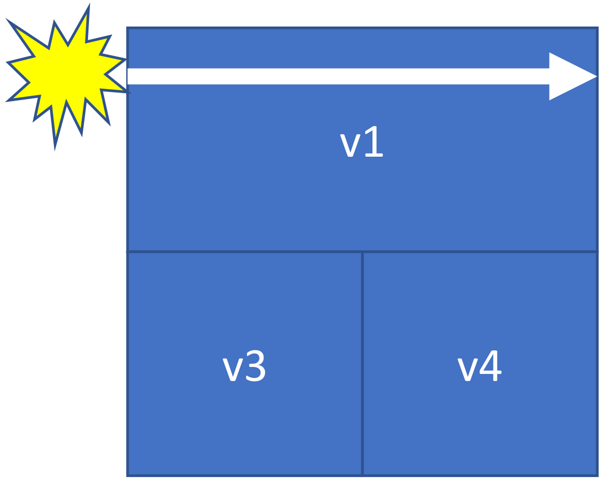


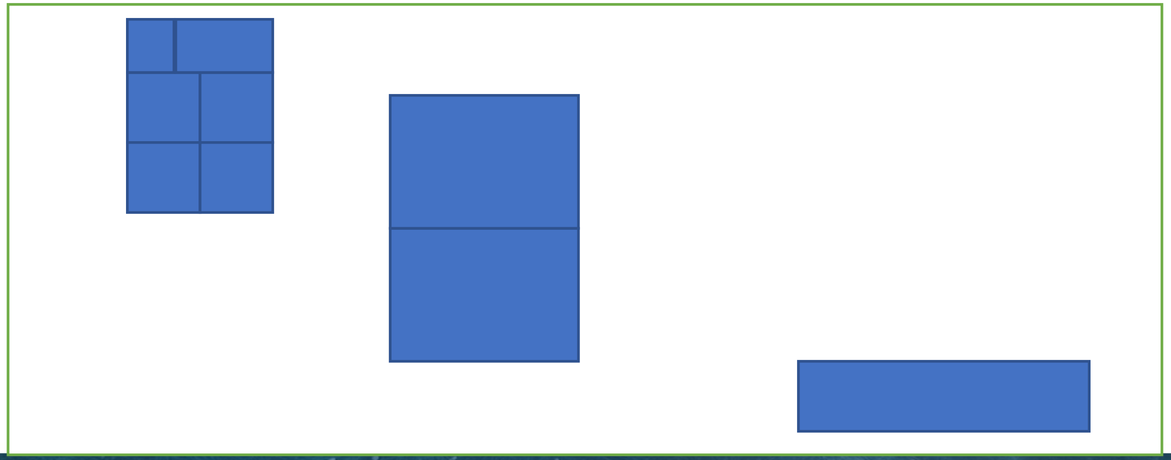
$$T = L * \left(\frac{1}{v_1} + \frac{1}{v_2}\right)$$

No independent information about the two velocities!

Discrete Parametrization

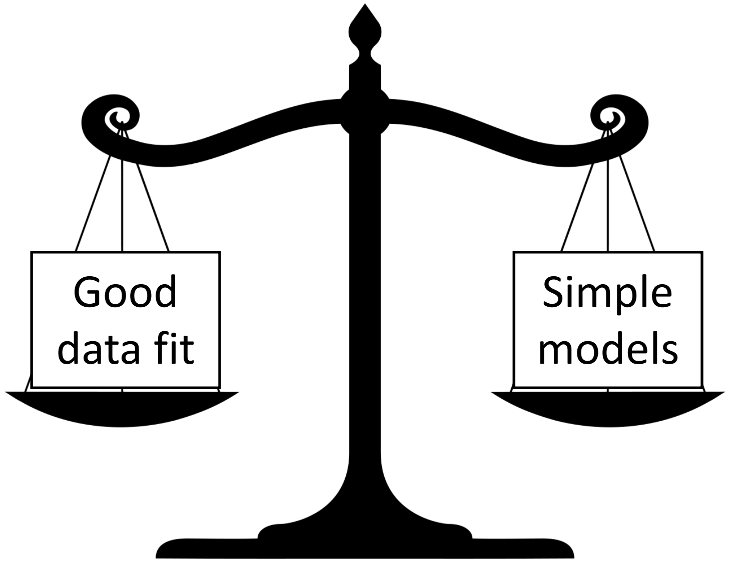
- Describe spatial structure as a collection of discrete geometric shapes (**Sambridge et al. 2006**)
- Here, I use rectangles
  - Center (x,y)
  - Width (x,y)
  - Values (e.g. velocity, density,...)
- Goal: **Use as many (or few) rectangles as required by the data**





Bayesian perspective

- Goal: „Judge“ a given model m_k with k rectangle using Bayes‘ Rule:



$$Posterior\ (m_k) = Likelihood(\Delta y, \sigma^2) * Prior(m_k) * C$$

Better models have **higher** posterior

How well does the model fit the data?

Depends on requested data accuracy $\boldsymbol{\sigma^2}$

How complicated is the model?

Basically $\propto D^{-k}$

Hierarchical Bayesian

- Problem: How to pick the requested accuracy σ^2?
- Solution: Include it as an unknown! **(Bodin et al. 2012)**

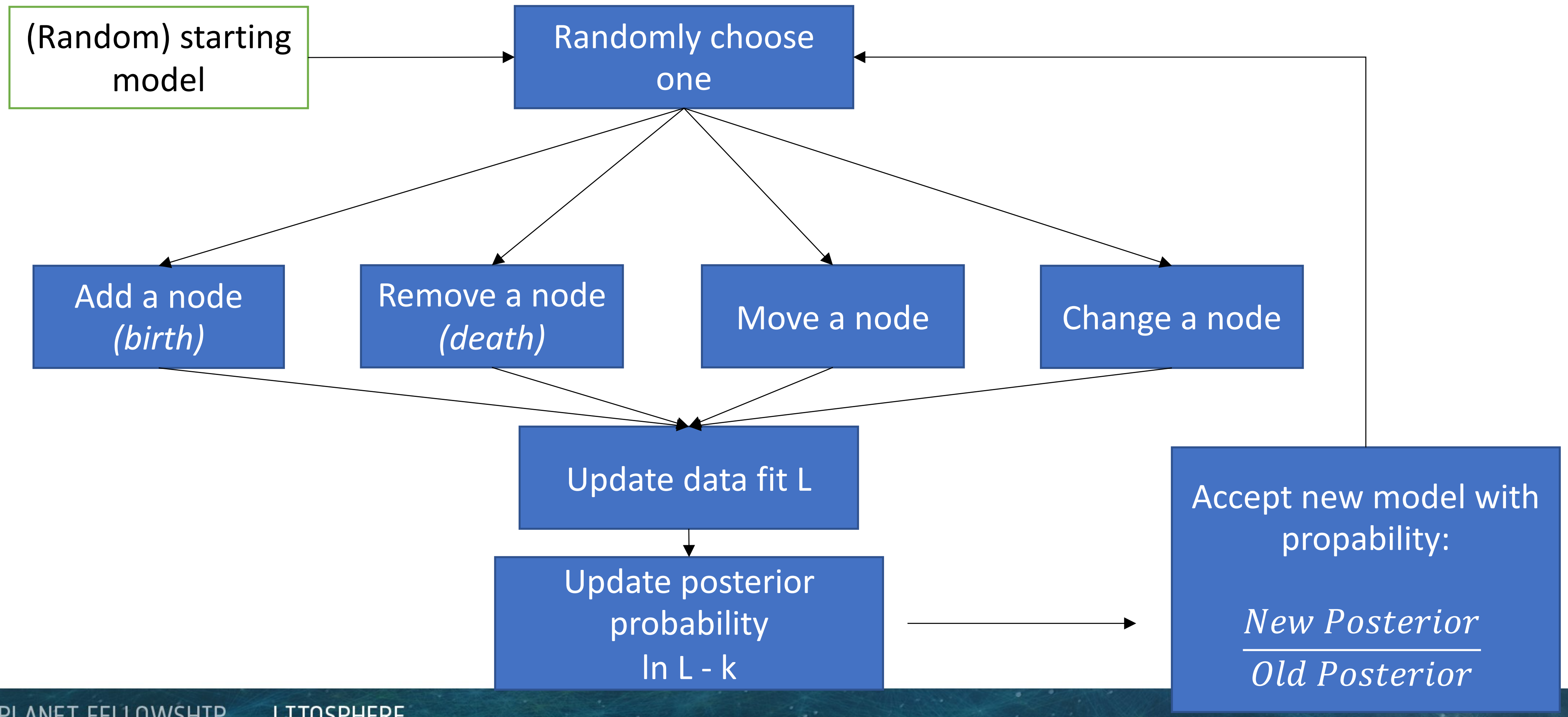
$$Posterior\ (m_k) = Likelihood(\Delta y, \sigma^2) * Prior(m_k) * C$$

↓

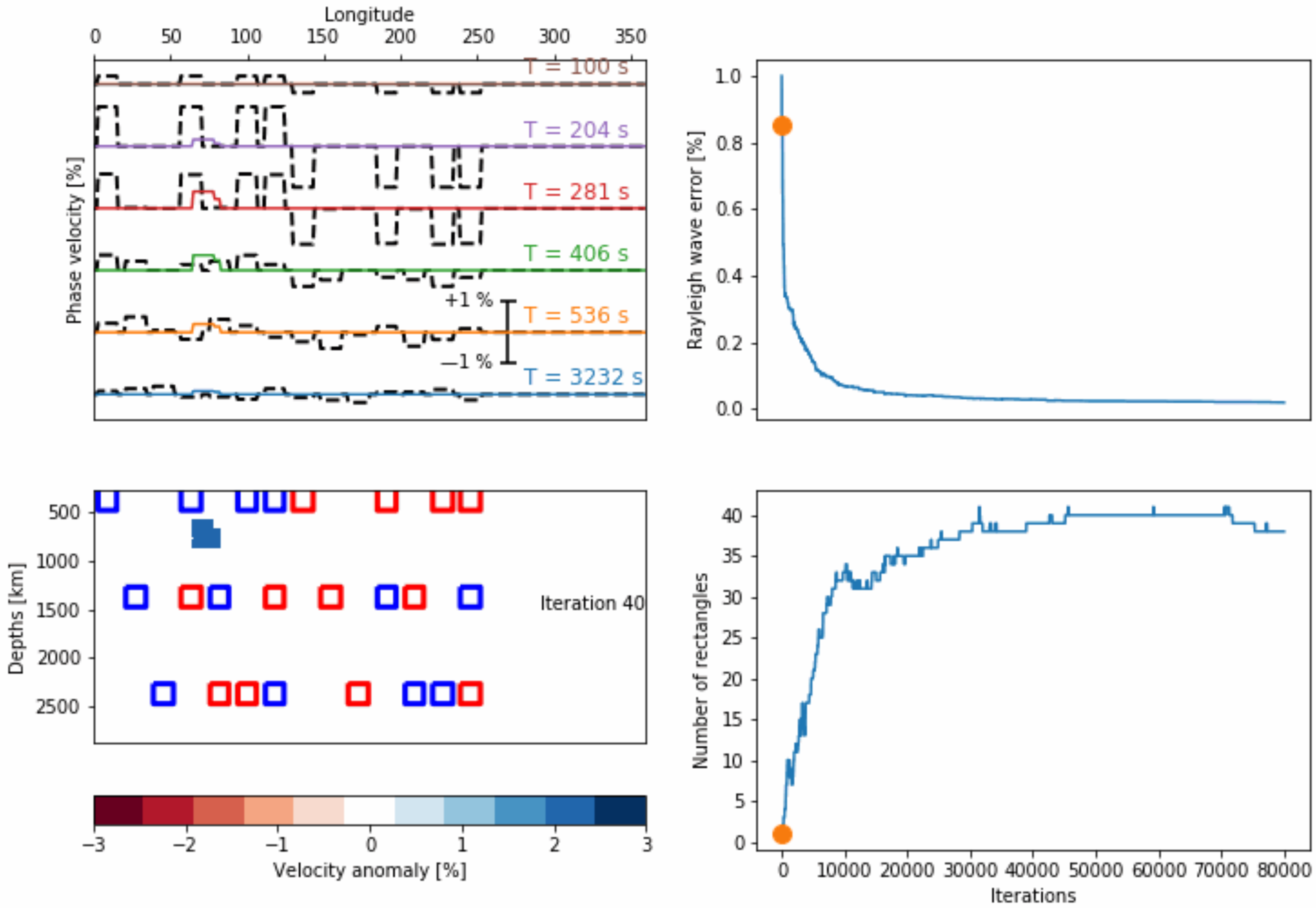
$$Posterior\ (m_k, \sigma^2) = Likelihood(\Delta y, \sigma^2) * Prior(m_k) * Prior(\sigma^2)$$

Inversion (Birth-Death MCMC*)

*MCMC = Monte Carlo Markov Chain

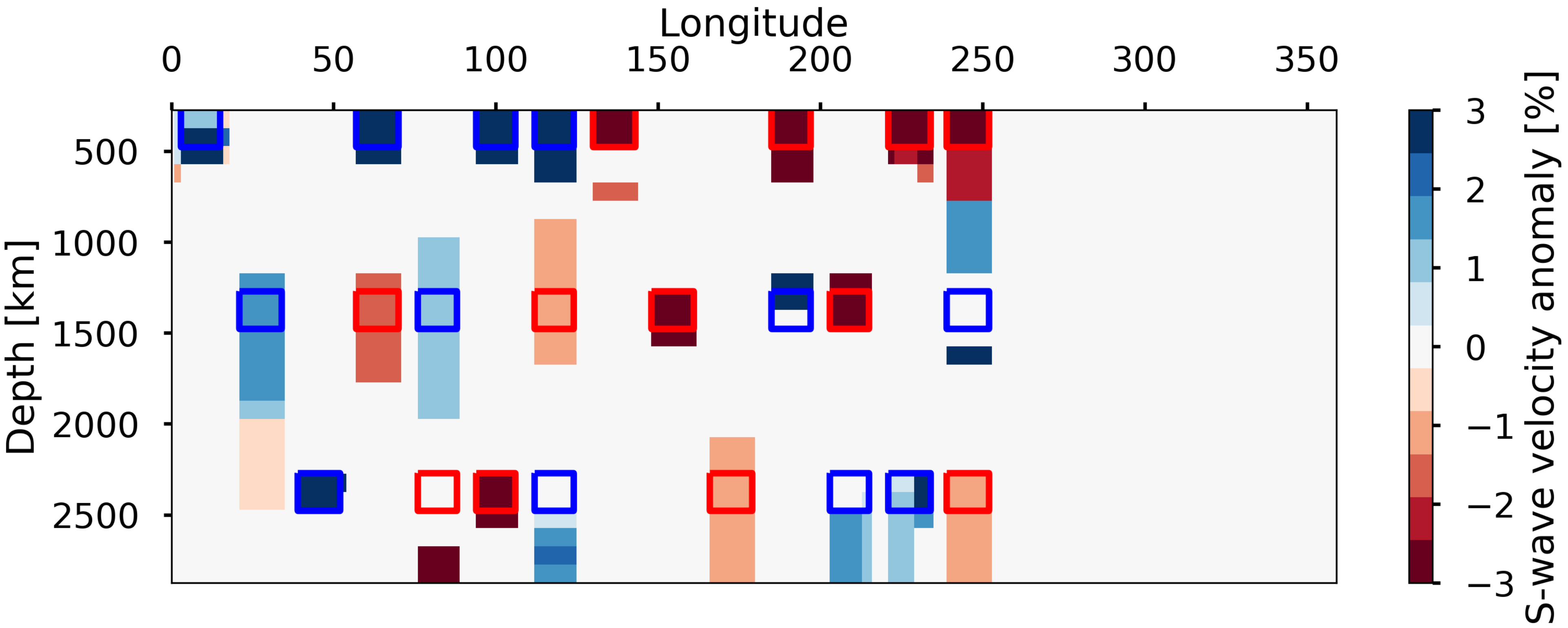


Synthetic example



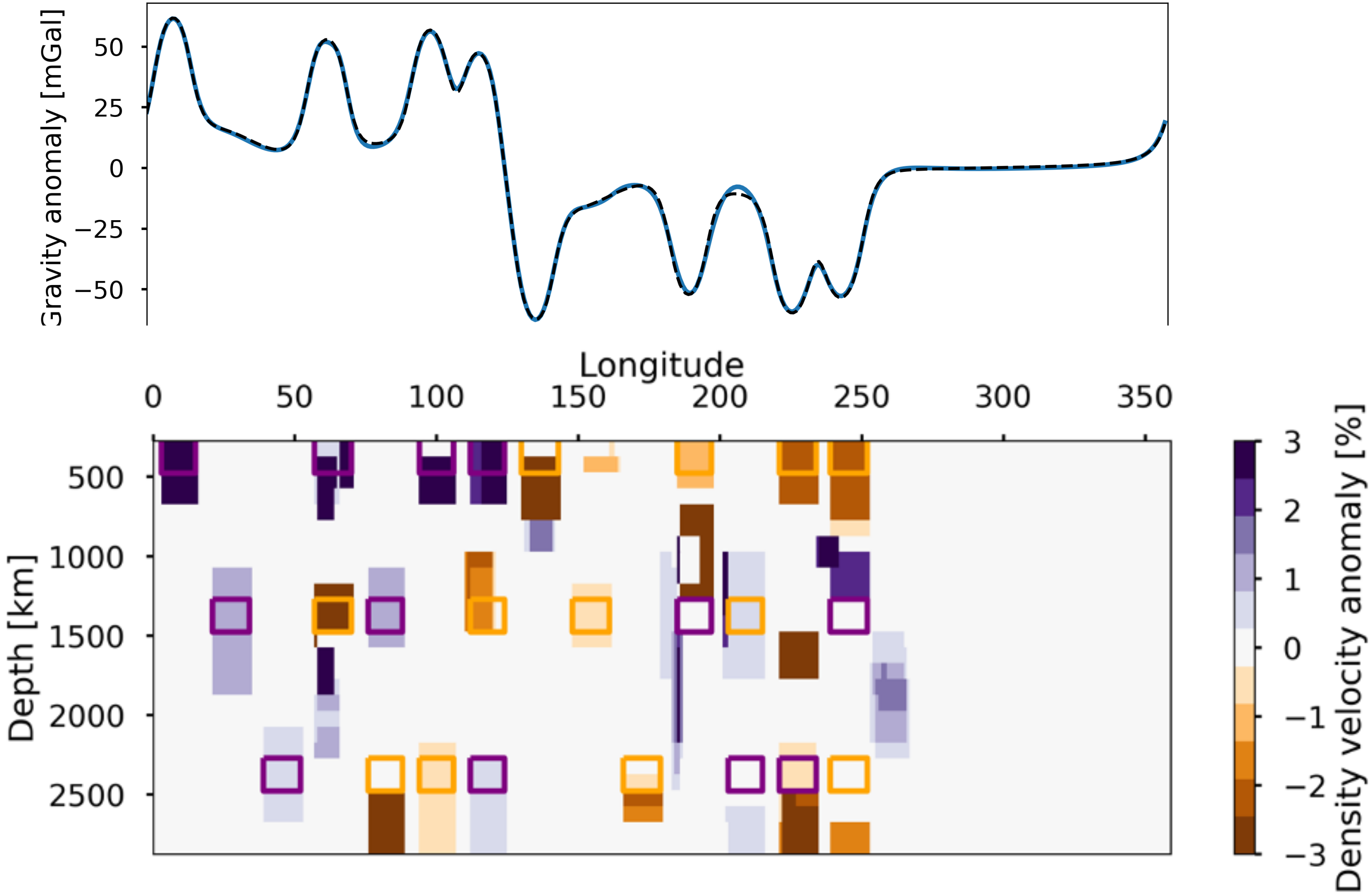
Synthetic example (final model)

- Mean of last 20,000 models



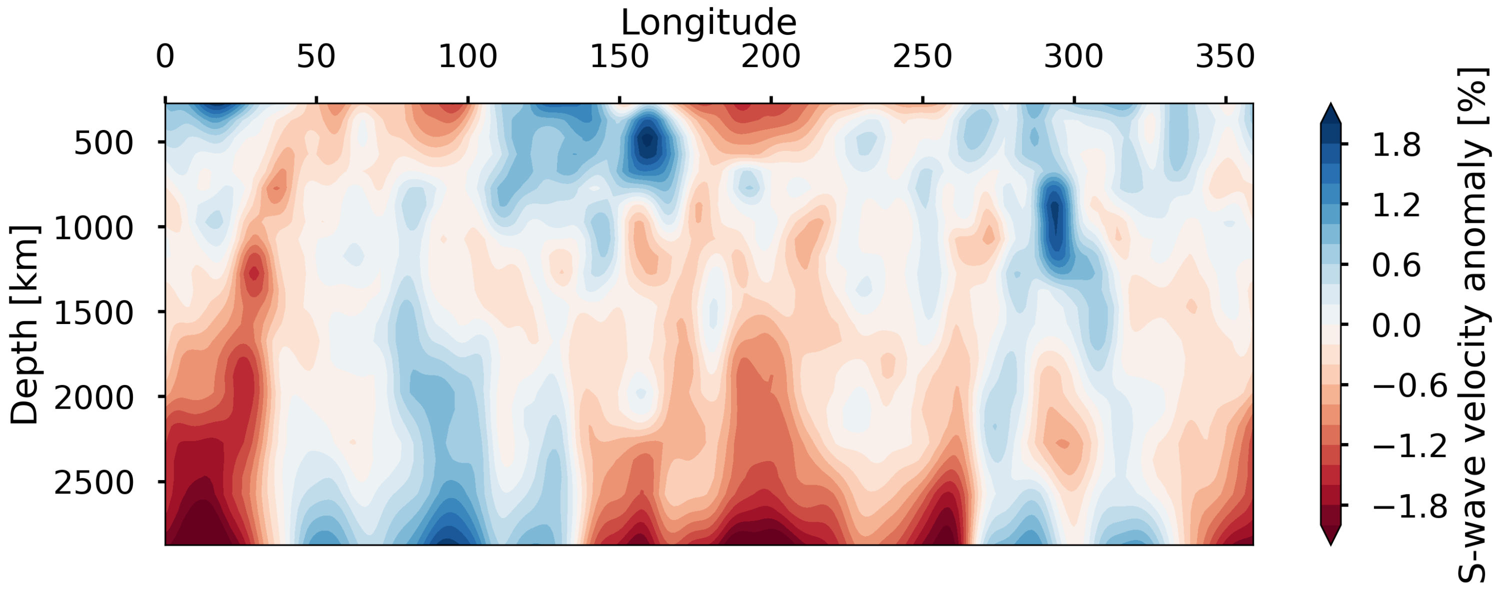
Adding **Satellite gravity**

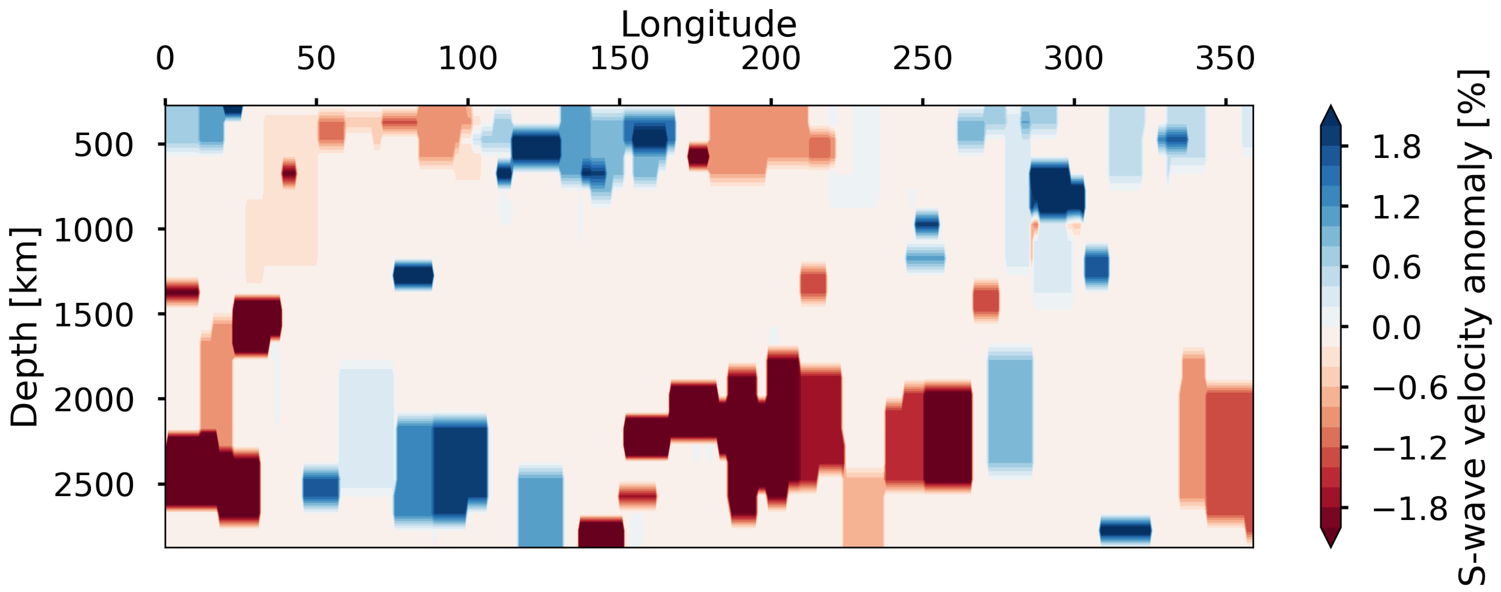
- Joint inversion of Surface waves and satellite gravity data



SMEAN2

1. Use seismic tomography as input
2. Calculate dispersion curves
3. Invert with MCMC





Conclusions

- Proof-of-concept for discrete trans-dimensional inversion
- The discrete parametrization is useful for combining data with vastly different spatial sensitivities, like surface waves and satellite gravity.
- Insights into the mantle density structure important for interpretation of seismic results

Thank you for your attention!