

Introduction to Polarimetric Synthetic Aperture Radar POLSAR

Armando Marino

The University of Stirling, Scotland, UK

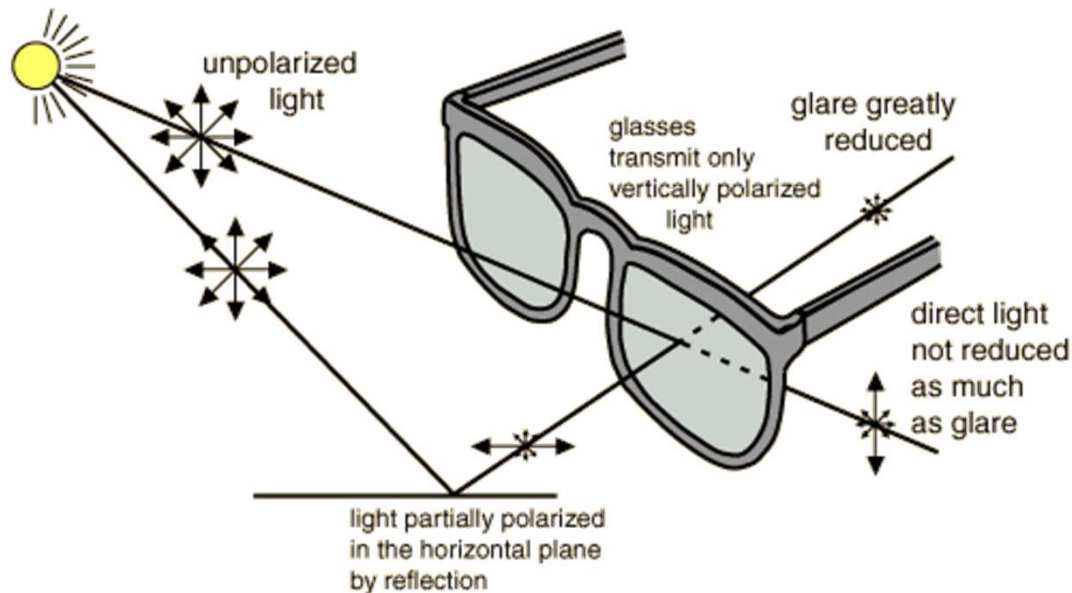
Outline

- ✓ What is polarimetry?
- ✓ Basic concepts in polarimetry:
 - ✓ Wave
 - ✓ Single targets
 - ✓ Partial targets
- ✓ Target decomposition:
 - ✓ Coherent
 - ✓ Incoherent
 - ✓ Non-model based
 - ✓ Model based



What is Polarimetry?

Polaroid Glasses



3D Cinema

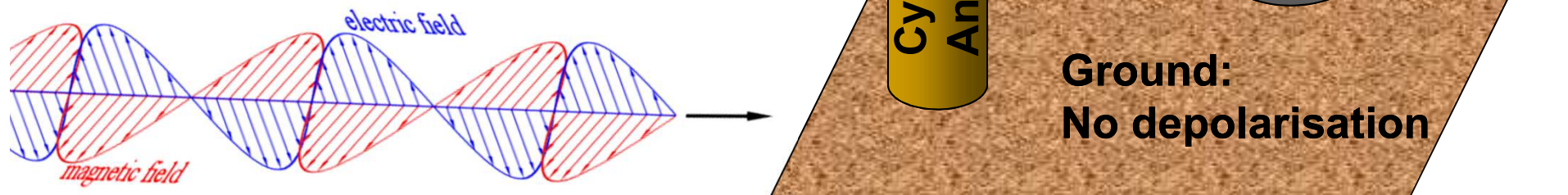


You may know how polarimetry can be exploited in optics:

1. Polaroid glasses
2. Modern 3D Cinema

Why Polarimetry in radar remote sensing?

- ✓ Different targets *generally* interact in a different way when illuminated by differently polarised plane waves
- ✓ We can **use** polarimetry to:
 - ✓ Classify
 - ✓ Detect
 - ✓ Separate returns



Few **definition**... (they will be treated in details later on):

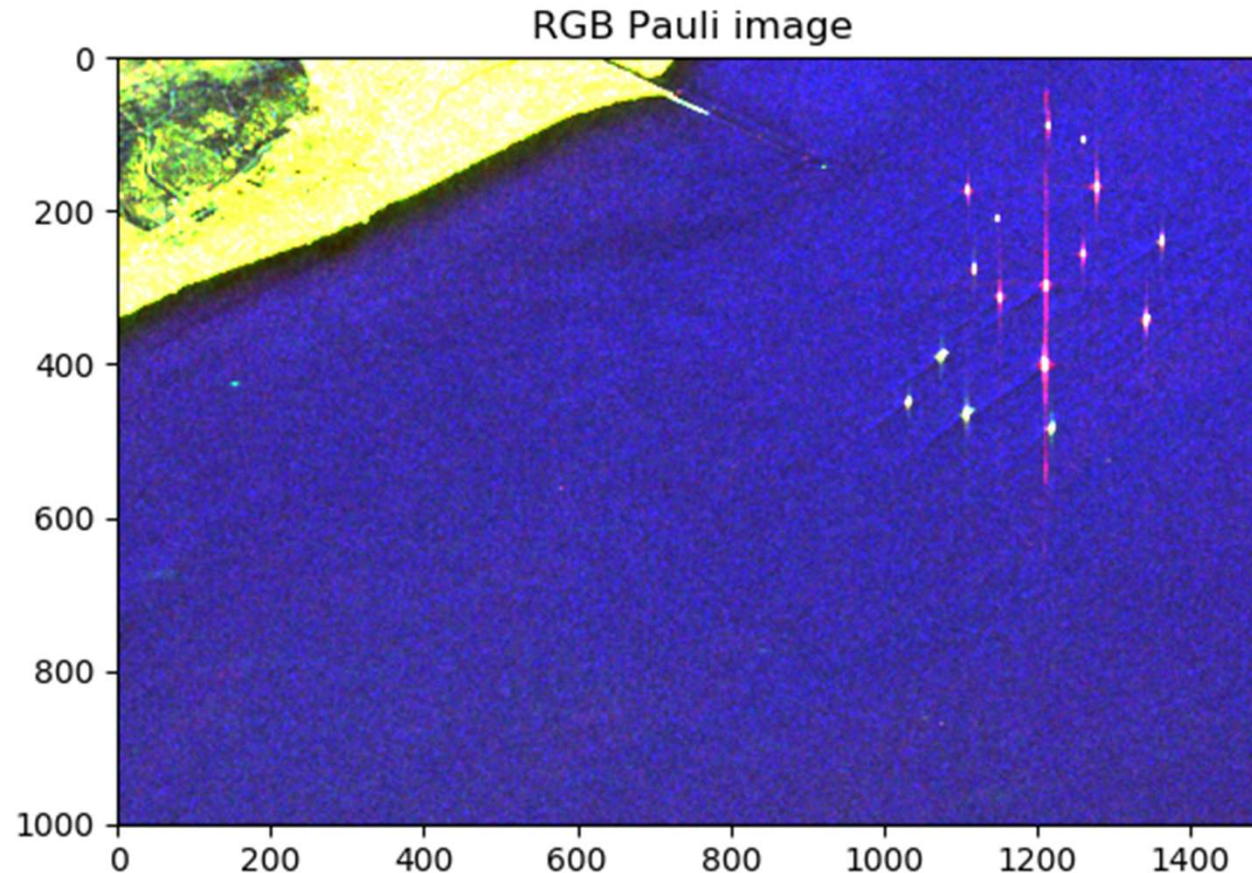
- ✓ **Isotropic**: the target interacts at the same way with any polarisation (the interaction does NOT depend on the direction of the Electric field vector)
- ✓ **Anisotropic**: the scatterer has a different behaviour for different polarisations
- ✓ **Depolarisation**: the tendency of a target to change the polarisation of the incident wave, but in some contexts is only refereed to the lost of polarimetric purity (i.e. the polarisation changes in time/space)

Why Polarimetry in radar remote sensing?



Pauli RGB image of San Francisco Bay (AIRSAR). The polarimetric information is coded in the colours. As you can notice we can use colours to differentiate between targets.
Data courtesy of MDA and Canadian Space Agency.

Why Polarimetry in radar remote sensing?



Pauli RGB image around Buenos Aires (ALOS-1). The polarimetric information is coded in the colours. As you can notice we can use colours to differentiate between targets.

Data courtesy of JAXA.

3713

23 < 33 73 63 68 22 42 < 7A
A 77 77 4A 13 88 07 04 34
33 33 73 63 68 22 42 < 7A
34 34 04 07 88 13 33 4A

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059	2060	2061	2062	2063	2064	2065	2066	2067	2068	2069	2070	2071	2072	2073	2074	2075	2076	2077	2078	2079	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	2098	2099	2100																																																																																																																																																																																															
1990	4.0	3.8	3.6	3.4	3.2	3.0	2.8	2.6	2.4	2.2	2.0	1.8	1.6	1.4	1.2	1.0	0.8	0.6	0.4	0.2	0.0	-0.2	-0.4	-0.6	-0.8	-1.0	-1.2	-1.4	-1.6	-1.8	-2.0	-2.2	-2.4	-2.6	-2.8	-3.0	-3.2	-3.4	-3.6	-3.8	-4.0	-4.2	-4.4	-4.6	-4.8	-5.0	-5.2	-5.4	-5.6	-5.8	-6.0	-6.2	-6.4	-6.6	-6.8	-7.0	-7.2	-7.4	-7.6	-7.8	-8.0	-8.2	-8.4	-8.6	-8.8	-9.0	-9.2	-9.4	-9.6	-9.8	-10.0	-10.2	-10.4	-10.6	-10.8	-11.0	-11.2	-11.4	-11.6	-11.8	-12.0	-12.2	-12.4	-12.6	-12.8	-13.0	-13.2	-13.4	-13.6	-13.8	-14.0	-14.2	-14.4	-14.6	-14.8	-15.0	-15.2	-15.4	-15.6	-15.8	-16.0	-16.2	-16.4	-16.6	-16.8	-17.0	-17.2	-17.4	-17.6	-17.8	-18.0	-18.2	-18.4	-18.6	-18.8	-19.0	-19.2	-19.4	-19.6	-19.8	-20.0	-20.2	-20.4	-20.6	-20.8	-21.0	-21.2	-21.4	-21.6	-21.8	-22.0	-22.2	-22.4	-22.6	-22.8	-23.0	-23.2	-23.4	-23.6	-23.8	-24.0	-24.2	-24.4	-24.6	-24.8	-25.0	-25.2	-25.4	-25.6	-25.8	-26.0	-26.2	-26.4	-26.6	-26.8	-27.0	-27.2	-27.4	-27.6	-27.8	-28.0	-28.2	-28.4	-28.6	-28.8	-29.0	-29.2	-29.4	-29.6	-29.8	-30.0	-30.2	-30.4	-30.6	-30.8	-31.0	-31.2	-31.4	-31.6	-31.8	-32.0	-32.2	-32.4	-32.6	-32.8	-33.0	-33.2	-33.4	-33.6	-33.8	-34.0	-34.2	-34.4	-34.6	-34.8	-35.0	-35.2	-35.4	-35.6	-35.8	-36.0	-36.2	-36.4	-36.6	-36.8	-37.0	-37.2	-37.4	-37.6	-37.8	-38.0	-38.2	-38.4	-38.6	-38.8	-39.0	-39.2	-39.4	-39.6	-39.8	-40.0	-40.2	-40.4	-40.6	-40.8	-41.0	-41.2	-41.4	-41.6	-41.8	-42.0	-42.2	-42.4	-42.6	-42.8	-43.0	-43.2	-43.4	-43.6	-43.8	-44.0	-44.2	-44.4	-44.6	-44.8	-45.0	-45.2	-45.4	-45.6	-45.8	-46.0	-46.2	-46.4	-46.6	-46.8	-47.0	-47.2	-47.4	-47.6	-47.8	-48.0	-48.2	-48.4	-48.6	-48.8	-49.0	-49.2	-49.4	-49.6	-49.8	-50.0	-50.2	-50.4	-50.6	-50.8	-51.0	-51.2	-51.4	-51.6	-51.8	-52.0	-52.2	-52.4	-52.6	-52.8	-53.0	-53.2	-53.4	-53.6	-53.8	-54.0	-54.2	-54.4	-54.6	-54.8	-55.0	-55.2	-55.4	-55.6	-55.8	-56.0	-5

[illegible][illegible][illegible][illegible][illegible][illegible]

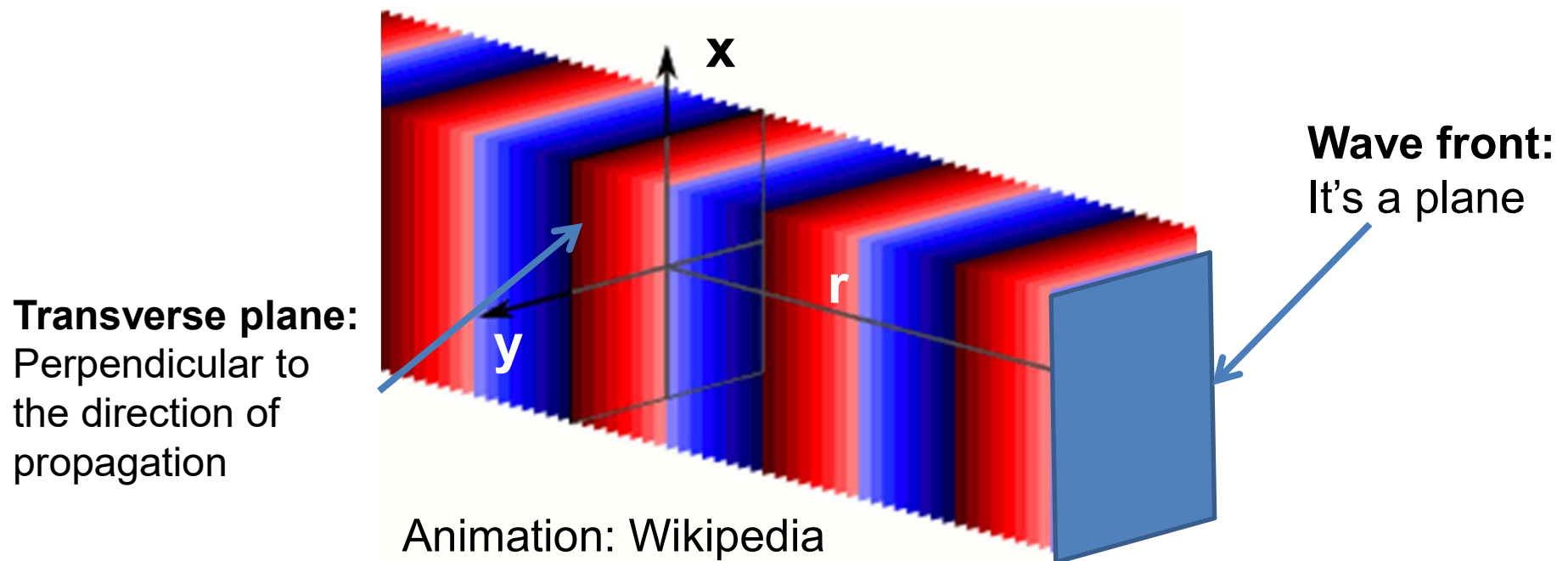
**Three core concepts you should
remember:**

Idea1: Wave polarimetry

Wave Polarimetry: Plane waves

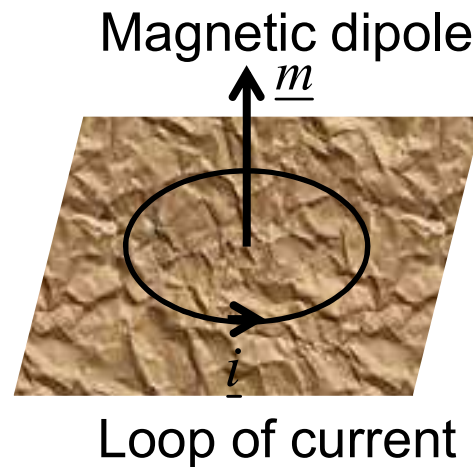
The most **general** way to describe any (macroscopic) electromagnetic phenomenon is by using the legendary **Maxwell equations**

After a series of hypothesis (i.e. monochromatic or narrowband signal, homogeneous, stationary and isotropic medium) we end up with a **Plane Wave** that can be “*easily*” described knowing the currents over the surface of the target



Wave Polarimetry: mathematical expression

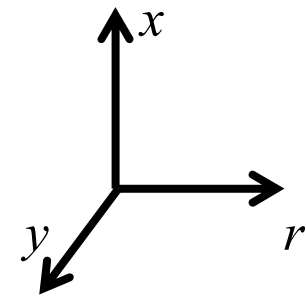
The mathematical expression of the plane wave is the following



$$\underline{E}(\underline{r}) = -\frac{\beta\omega e^{-j\beta R}}{4\pi\mu_0 R}(\underline{r} \times \underline{m})$$

$$\underline{H}(\underline{r}) = \frac{\beta^2 e^{-j\beta R}}{4\pi\mu_0 R} \underline{r} \times (\underline{r} \times \underline{m})$$

$$\beta = \sqrt{\omega^2 \varepsilon_0 \mu_0} \quad \omega = 2\pi f$$



\underline{E} : electric field, it's a complex vector (when \underline{E} is cleaned by the dependences on the distance is sometime refereed as **Jones** vector)

\underline{H} : magnetic field, it's a complex vector and can be derived from \underline{E}

ε_0 : electric permittivity of vacuum

μ_0 : magnetic permeability of vacuum

f : frequency of monocromatic (or narrowband) wave

R : distance from the source (generator) of the wave

Wave Polarimetry: useful abstraction

4 or 2

Parameters needed

- ✓ **Problem:** It is complicated to study the wave polarimetry starting from the **Jones** vectors.
- ✓ **Solution:** we use a geometrical abstraction and wave polarimetry becomes an ellipse. We need:
 - ✓ 2 parameters for the ellipse shape
 - ✓ 1 for the amplitude
 - ✓ 1 for the phase

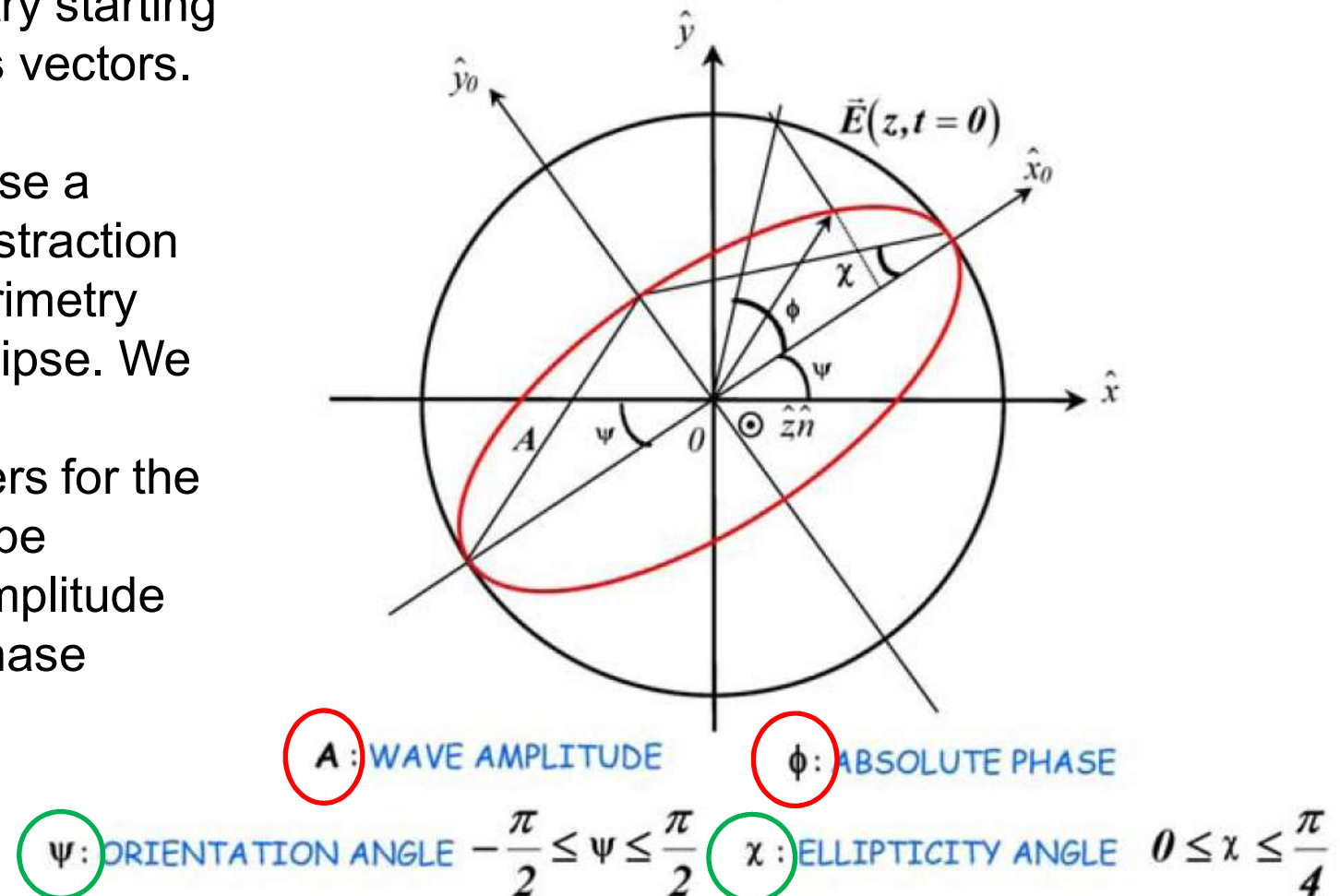


Fig. 2.2 Polarization Ellipse Relations (Courtesy of Prof. E. Pottier)

**Three different concepts you must
remember:**

**Idea2: Scattering polarimetry
Deterministic targets**

Single targets?!?! What is that?!?!?

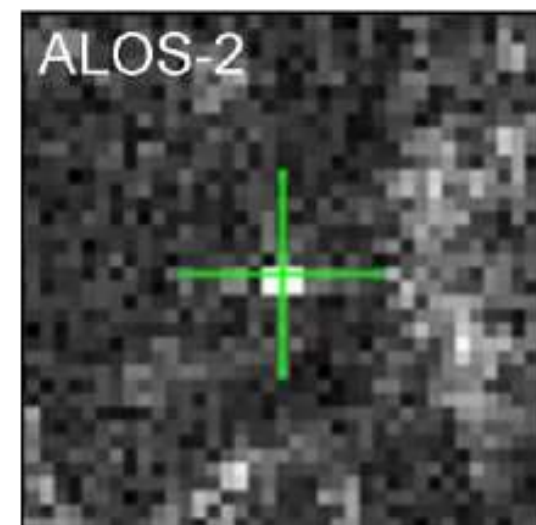
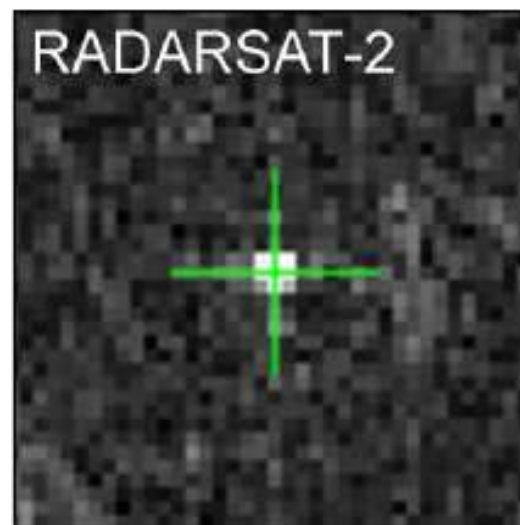
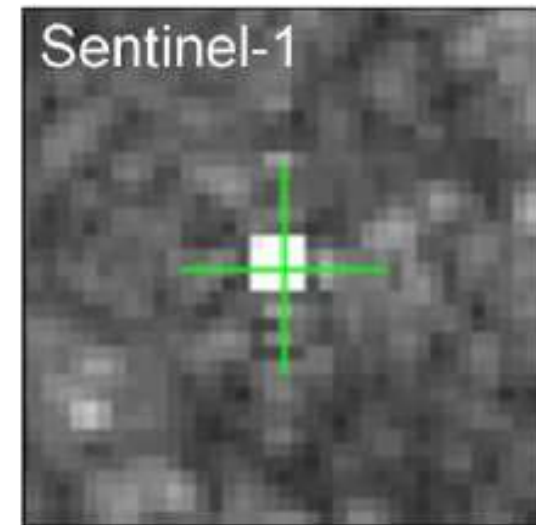
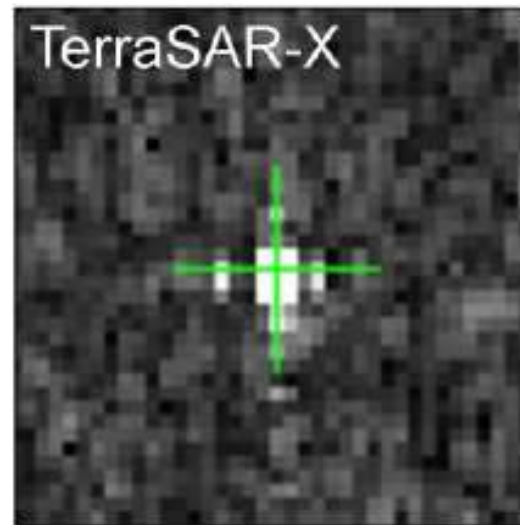
- ✓ A single target is a target that does NOT change its polarimetric signature in time/space: it is a **deterministic** target
- ✓ Examples:
 - ✓ calibration targets: corner reflectors
 - ✓ Some metallic or man-made targets (but not all of them): a car, a wall
 - ✓ Some natural target: a rock



Trihedral
corner reflector

How do they look like?

Corner reflectors in X-,
C- and L-band SAR
imagery.



Corner Reflectors as the tie between
InSAR and GNSS measurements: Case
Study of Resource Extraction in Australia
March 2015

DOI: 10.5270/Fringe2015.pp60

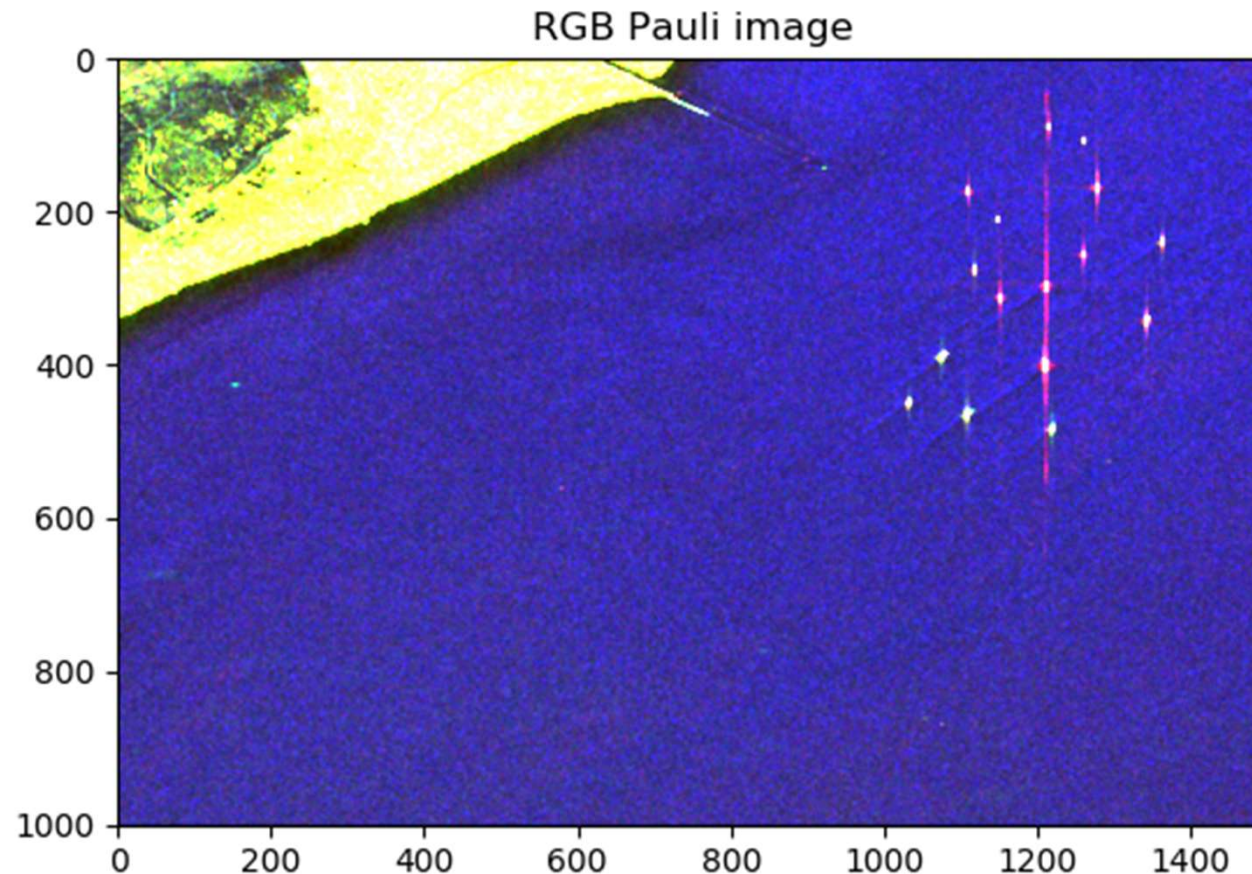
Armando Marino

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Are ships single targets?



It depends on the size of the ship, but generally they are a collection of several single targets.

Pauli RGB image around Buenos Aires (ALOS-1). The polarimetric information is coded in the colours. As you can notice we can use colours to differentiate between targets.

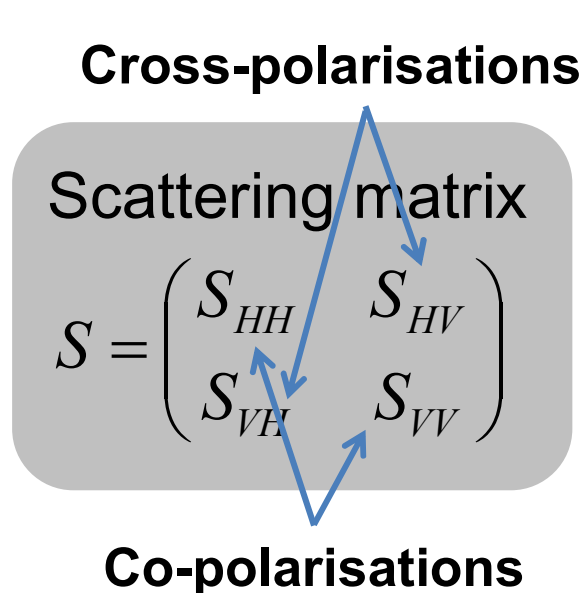
Data courtesy of JAXA.

Single targets: how to study polarimetric targets

- ✓ We want to use polarimetry to detect single targets and we can transmit and receive polarised waves:
 - ✓ **How many polarimetric acquisitions would we need?**
- ✓ To characterise **any polarised wave we need 2 polarisations**, since the plane wave is 2 dimensional (2-D).
- ✓ If we send a polarised wave (e.g. linear horizontal), this will generate currents on our target and these currents will scatter a wave with some polarisation. Therefore **we need to collect 2 polarisations to characterise such scattered wave**.
- ✓ But what happen if we **change the polarisation of the transmitted wave** (the wave that we send from the satellite)?...
 - ✓ Well, we will have different currents on the surface.
- ✓ In order to cover each possible transmitter waves, **we need to send two polarimetrically orthogonal waves**.
- ✓ Summarising, **we transmit 2 orthogonal waves and collect 2 orthogonal waves**:
2x2=4 channels/acquisitions needed

Single targets: same as before, but with math

- ✓ We can arrange the 4 acquisitions discussed before in a matrix: the **Scattering** or **Sinclair matrix**
 - ✓ **H**: horizontal linear
 - ✓ **V**: vertical linear
- ✓ The matrix will represent a **transformation from transmitted polarised waves to received waves**: i.e. it describes the polarimetric behaviour of the target



Scattered
(received)
wave

Incident
(transmitted)
wave

$$\underline{E}_r = C(r) [S] \underline{E}_i$$

Complex scalar depending on distance and medium where the wave propagates (e.g. air)

Single Look Complex

Data are stored in complex form, that is real plus imaginary part

Spyder (Python 3.6)

File Edit Search Source Run Debug Consoles Projects Tools View Help

Editor - C:\MyC\Talks\2018\CONAE\programs\Tutorial_CONAE_180927_solutions.py

Variable explorer

Name	Type	Size	Value
HHFull	complex64	(1248, 18432)	array([[1.70937300e-01-0.07887045j, 2.42148161e-01+0.09545995j, ...
HVFull	complex64	(1248, 18432)	array([[0.04068684+5.5295121e-02j, -0.07140828+2.2488926e-02j, ...
VHFull	complex64	(1248, 18432)	array([[0.03976065+0.05898558j, -0.03031691+0.03130209j, ...
VVFull	complex64	(1248, 18432)	array([[0.23054181+0.04004149j, 0.2626125 +0.18533355j, ...
envi	module	1	module object of builtins module
filei_HH	str	1	i_HH
filei_HV	str	1	i_HV

HHFull - NumPy array

	6	7	8	9	10	11	12
208	(0.07107788+0.0001118965j)	(-0.2165731-0.11895645j)	(-0.037696168+0.027938405j)	(0.16672957-0.25383106j)	(0.09106964-0.16733629j)	(-0.2292527+0.13909335j)	(-0.11490008+0.11884244j)
209	(-0.28057334-0.042206895j)	(0.38021272+0.029810535j)	(0.43548566+0.26694945j)	(-0.138532+0.40491346j)	(-0.5769423+0.1847391j)	(-0.015036544-0.05962692j)	(0.26831898-0.1743832j)
210	(0.030044155+0.34601098j)	(-0.5461137-0.025732089j)	(-0.29259968-0.14295235j)	(0.20367235-0.5841537j)	(0.37775946-0.262635j)	(-0.21820979+0.16362272j)	(-0.17708418-0.09521227j)
211	(-0.41205642+0.051513102j)	(0.7546207+0.32724288j)	(1.0687426+0.5270206j)	(0.054809444+0.82367045j)	(-0.3507389+0.1318827j)	(0.21761155-0.41316128j)	(0.08479531-0.22683923j)
212	(-1.1572694-0.38786665j)	(1.9590037+0.3208726j)	(1.9236386+1.241693j)	(-1.1034379+2.4863272j)	(-2.1077378+0.6184698j)	(0.90856504-0.7555746j)	(1.2048659-0.31549478j)
213	(0.03483691+0.24929173j)	(-0.06724173-0.19649513j)	(-0.052806515+0.14856093j)	(-0.33817375+0.041899033j)	(-0.325384-0.35427806j)	(0.15580273+0.25812522j)	(0.3568068+0.18579605j)
214	(0.05983149+0.14247231j)	(0.09123571-0.019802434j)	(-0.0056780856-0.08619735j)	(0.0276807+0.026372923j)	(0.004342114+0.07160738j)	(-0.16508183-0.13053997j)	(-0.10213288-0.1436391j)
215	(-0.23911689+0.13675046j)	(0.2510476+0.20256251j)	(0.1657779+0.29408714j)	(-0.14119186+0.3051973j)	(-0.14424676+0.11273051j)	(0.019122131-0.149513j)	(0.011736556-0.17397477j)
216	(0.14865826+0.14563574j)	(-0.14296752+0.18294339j)	(-0.08458038+0.04554746j)	(0.11610123-0.26472777j)	(0.27141842-0.2700538j)	(0.014428847+0.092346266j)	(-0.16020864+0.009083515j)
217	(0.14413048+0.050818104j)	(0.1473479+0.08817709j)	(0.023391057-0.08530626j)	(0.035040505+0.107569896j)	(-0.057849325+0.24947123j)	(0.12625532-0.06831674j)	(0.2286333-0.123649314j)
218	(0.035153415+0.10457542j)	(-0.011003431+0.120111205j)	(0.03158148+0.0710468j)	(0.17607468-0.20794402j)	(0.22993661-0.093825236j)	(-0.052824214+0.10840995j)	(-0.13797311+0.015566013j)
219	(0.03727213-0.21260321j)	(0.1176478-0.031192722j)	(0.03230082-0.16893798j)	(-0.08027681-0.031375308j)	(-0.05084166+0.05267703j)	(-0.06616261-0.09878135j)	(-0.100596376-0.028747175j)
220	(0.26260537-0.008604212j)	(0.15181771-0.14325972j)	(-0.21277122-0.3136897j)	(-0.06775208-0.13144417j)	(0.060304996+0.14034642j)	(-0.038857333+0.015634881j)	(0.023326596+0.09556501j)
221	(-0.042971827+0.23937722j)	(0.13073908+0.1270226j)	(0.048672948-0.06346875j)	(0.021206522-0.088813424j)	(0.1901794-0.2169216j)	(-0.029284136+0.024858948j)	(-0.29060984+0.03422946j)
222	(-0.037735436+0.0066418382j)	(-0.099647336-0.12131322j)	(-0.05169738-0.36016986j)	(-0.14253348+0.10819008j)	(-0.116834626+0.09651822j)	(0.056414068-0.17803384j)	(0.1667285-0.24515238j)
223	(0.070434734-0.07451398j)	(-0.1756553-0.12745978j)	(-0.08949843-0.15957025j)	(0.03456987-0.07367534j)	(-0.032520145+0.09075477j)	(0.0029854525+0.062434208j)	(0.08751949-0.08023951j)

Format: Resize Background color

Permissions: RW End-of-lines: CRLF Encoding: UTF-8 Line: 69 Column: 1 Memory: 22 %



Single targets: same as before, but with vectors

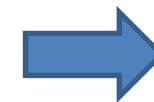
8 or 6

Parameters needed

- ✓ An easier way to see the polarimetric information is **vectorising** the scattering matrix
- ✓ Also, we are often interested in polarimetry alone and not how much the target scatter: so we **normalise** the scattering vector and obtain the **scattering mechanism** (this is sometime called projection vector).
- ✓ How many parameters we need:
 - ✓ In general 8 since we have 4 complex numbers
 - ✓ But for scattering mechanism we remove the overall power (length of vector)
 - ✓ It can also be showed that the “absolute phase” (a phase term that we can put as overall factor) does not keep information.
 - ✓ We end up with 6 parameters for a scattering vector

Scattering vector

$$\underline{k} = \frac{1}{2} \text{Trace}([S]\Psi) = [k_1, k_2, k_3, k_4]^T$$



Scattering Mechanism

$$\underline{\omega} = \underline{k} / |\underline{k}|$$



Single targets: same as before, but with a simplification

6 or 4

Parameters needed

- ✓ In case the system is **monostatic** (one antenna as transmitter-receiver), and the medium observed is **reciprocal** (it behaves the same way independently by the direction of propagation of the wave) the two cross-polarisations are the same.
- ✓ We need less parameters to characterise targets in such situation.
- ✓ *Please note*, HV and VH are exactly the same except for **thermal noise**.
- ✓ *Please note*, at low frequencies (P and sometime L band) the **ionosphere** is not reciprocal introducing non-reciprocity (i.e. Faraday rotation). This is a problem only for satellites.

$$\underline{k} \in \mathbb{C}^3$$

$$S_{HV} = S_{VH}$$



Scattering vector

$$\underline{k} = \frac{1}{2} \text{Trace}([S]\Psi) = [k_1, k_2, k_3]^T$$

Single targets: some physical interpretation

- ✓ The idea of using polarimetry is based on the physical concept that different targets will be excited with different currents. Therefore, *it has a narrow relation with some physical interpretation.*
- ✓ The scattering vector (as any vector) has to be expressed in some **basis**. The one that list the elements of the S matrix is called **Lexicographic basis**.
- ✓ There are also other basis that helps having some physical interpretation of the target. An example is the **Pauli basis**.
 - ✓ Each of the components is sensitive to a specific target (you will learn more on this when specking about *Decompositions*).

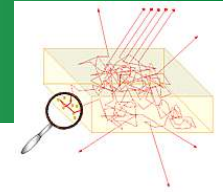
Pauli bases:
$$\vec{k}_P = \frac{1}{\sqrt{2}} [S_{HH} + S_{VV}, S_{HH} - S_{VV}, S_{HV} + S_{VH}]^T$$

Lexicographic bases:
$$\underline{k}_l = [S_{HH}, S_{HV}, S_{VV},]^T$$

**Three different concepts you must
remember:**

**Idea3: scattering polarimetry
Distributed/partial targets**

Problem: statistic or distributed targets



- ✓ This is a concept narrowly related with what we said in the lecture about **Speckle**
- ✓ When the target *changes* spatially it can NOT be represented by a *unique* Scattering matrix. It is a random process.
- ✓ In this image, you can imagine the squares as the **resolution cells**...
- ✓ The target under analysis is the same (the same forest)... but the objects things inside the squares are different (as you can see):
 - ✓ So, how do we deal with this variation? The scattering matrix will change pixel by pixel due to the speckle (because the targets in each resolution cell is slightly different).



Solution: second order statistics

- ✓ In order to extract information the **second order statistics** of the target can be extracted
- ✓ In case of Gaussian complex pixels, these contain all the information about the random process
- ✓ In *Lexicographic basis*, we often talk about **COVARIANCE** matrix
- ✓ In *Pauli basis*, we often talk about **COHERENCY** matrix (the difference is only the basis and we can transform one into the other)

General formulation (any basis)

Second order statistics as outer product.

$$[C_3] = \langle \underline{k} \cdot \underline{k}^{*T} \rangle$$



$$[C_3] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

Average

Pauli basis

$$[T] = \begin{bmatrix} \langle |S_{HH} + S_{VV}|^2 \rangle & \langle (S_{HH} + S_{VV})(S_{HH} - S_{VV})^* \rangle & 2\langle (S_{HH} + S_{VV})S_{HV}^* \rangle \\ \langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* \rangle & \langle |S_{HH} - S_{VV}|^2 \rangle & 2\langle (S_{HH} - S_{VV})S_{HV}^* \rangle \\ 2\langle S_{HV}(S_{HH} + S_{VV})^* \rangle & 2\langle S_{HV}(S_{HH} - S_{VV})^* \rangle & 4\langle |S_{HV}|^2 \rangle \end{bmatrix}$$

Properties of Target Coherency Matrix 9 or 8

Parameters needed

It is $[C] = [C]^{*T}$
Hermittian:
Semi-positive
Definite $I = \underline{\omega}^{*T} [C] \underline{\omega} \geq 0$

Rank 1 [C] matrices has a unique representation as [S] matrices (unless one absolute phase)... i.e. they are built with one single scattering vector

Real positive Complex

$$[C] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

Lower and upper triangular parts are complex conjugate

- ✓ How many **parameters** we need?
 - ✓ 3 for the diagonal real positive terms
 - ✓ 6 for the off diagonal complex terms
 - ✓ If we neglect the overall amplitude (trace of the matrix) we reduce one parameter

How do we practically use this?

- ✓ An easy way to use this is by creating one image each element of the covariance matrix
- ✓ Pay attention that cross-diagonal elements are Complex numbers.

The screenshot shows the Spyder Python IDE interface. The editor window displays a Python script with the following code:

```
175 HV = HVFull[dr1:dr2, da1:da2]
176 VH = VHFull[dr1:dr2, da1:da2]
177 VV = VVFull[dr1:dr2, da1:da2]
178 del HHFull, HVFull, VHFull, VVFull
179
180 # Check the Variable Explorer and see how t
181
182 #%%
183 #####
184 #
185 # BUILDING THE COVARIANCE M
```

The Variable explorer shows the following variables:

Name	Type	Size	Value
C11Full	float32	(1000, 6000)	array([[0.02843286, 0.11281456, 0.19216399, ..., 0.11451569, 0.0627168 ...
C12Full	complex64	(1000, 6000)	array([[1.45457825e-02+0.01128196j, 7.70821190e-03+0.00376824j, ...
C13Full	complex64	(1000, 6000)	array([[0.01435393-0.01395321j, 0.11613512+0.00352166j, ...
C22Full	float32	(1000, 6000)	array([[1.19179888e-02, 6.52541290e-04, 4.98843566e-03, ..., ...
C23Full	complex64	(1000, 6000)	array([[1.80669781e-03-0.01283378j, 8.05272441e-03-0.00363853j, ...
C33Full	float32	(1000, 6000)	array([[0.01409382, 0.11966334, 0.06784319, ..., 0.10910278, 0.0680876 ...

Two NumPy array viewers are shown below the variable explorer:

C11Full - NumPy array

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	0.0284329	0.112815	0.192164	0.0696964	0.00977801	0.091945	0.0765197	0.115472	0.0576391	0.0892011	0.27376	0.0235134	0.00875837	0.2234	0.0880832	0.019835	0.0285191	0.00658374
1	0.15936	0.0309374	0.241725	0.100041	0.0452113	0.314009	0.0665284	0.0419675	0.123675	0.0274823	0.0421274	0.0443142	0.240537	0.00357138	0.138227	0.0619979	0.0970914	0.0458256
2	0.0438354	0.214579	0.058059	0.092448	0.258334	0.0500391	0.199267	0.101509	0.0680541	0.128033	0.00781235	0.00219264	0.0293388	0.0250444	0.12638	0.131971	0.0275621	0.016508
3	0.207855	0.119369	0.000719917	0.148365	0.0301519	0.0247315	0.00830485	0.07295	0.209398	0.330004	0.0114797	0.0991345	0.235359	0.134207	0.011273	0.0753751	0.00321607	0.0527594
4	0.0570053	0.132377	0.0792781	0.103252	0.247283	0.130179	0.117291	0.353488	0.505952	0.0482432	0.0584333	0.0743887	0.0500249	0.0111193	0.0628825	0.098449	0.288889	0.162527
5	0.624295	0.26706	0.0635061	0.00456994	0.00365802	0.0246228	0.00758548	0.0763015	0.0308939	0.2255	0.0952573	0.023491	0.21724	0.0472077	0.0946453	0.0102976	0.0233078	0.0614216
6	0.107482	0.749921	0.521945	0.0526716	0.128643	0.202174	0.0588344	0.236714	0.0425943	0.165698	0.00416545	0.0774513	0.0740566	0.337991	0.284731	0.0110099	0.0759539	0.131773

C12Full - NumPy array

	0	1	2	3	4	5	6	7
0	(0.0145457825+0.0112819625j)	(0.007708212+0.003768239j)	(-0.030748777+0.0036208266j)	(-0.012237186+0.009193638j)	(0.0021310826-0.00048958056j)	(-0.0055696126+0.013920422j)	(-0.023065198+0.0055310074j)	(0.019580366-0.00019580366j)
1	(0.0005545728+0.030781973j)	(-0.009317523+0.012127403j)	(0.007266458+0.034893423j)	(-0.011122643+0.01017262j)	(-0.0018902698-0.000510095j)	(-0.0021518082-0.00040983662j)	(0.0045812223+0.021286389j)	(-0.006360942-0.00019580366j)
2	(-0.01802452-0.0024115592j)	(-0.01955137+0.0012981007j)	(-0.034278706-0.0059847683j)	(-0.007957747+0.059697345j)	(-0.06374178+0.03488311j)	(0.0106113115-0.021453666j)	(-0.043256726-0.0047023185j)	(0.015702989-0.00019580366j)
3	(0.032029703+0.0025845626j)	(-0.0054205414-0.024452088j)	(0.0005135682+0.00049541594j)	(0.009434683+0.0063087144j)	(0.0035462547-0.027169514j)	(-0.017103825-0.008943477j)	(0.001655386-0.006919672j)	(0.008182942-0.00019580366j)
4	(-0.006390075+0.016714128j)	(0.039637692+0.0011262847j)	(0.05489466+0.008085463j)	(0.0129286945+0.01801482j)	(0.0052719237-0.0061142254j)	(0.023920152+0.01425597j)	(0.019047242-0.053946618j)	(-0.022478918-0.00019580366j)
5	(0.024336044-0.021858297j)	(-0.043478243+0.04129299j)	(0.02379073+0.0077017555j)	(0.0022853983+0.003018279j)	(0.0052460665-0.0066166753j)	(-0.0096098855+0.00792744j)	(-0.01565314+0.00020128489j)	(0.031087045+0.00019580366j)
6	(0.0024066973-0.0026878137j)	(-0.046150185-0.011261776j)	(0.058944046-0.046654627j)	(-0.00066997623+0.01203581j)	(0.0021754978-0.008668138j)	(0.021249032+0.031643406j)	(0.0031516273+0.012148932j)	(0.009013542+0.00019580366j)
7								

Summary of basic concepts

Reminder: single and partial target representation

Scattering matrix:

$$[S] = \begin{pmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{pmatrix}$$

Scattering mechanism:

$$\underline{\omega} = \underline{k}/|\underline{k}|$$

Scattering vector:

$$\underline{k} = \frac{1}{2} \text{Trace}([S]\Psi) = [k_1, k_2, k_3, k_4]^T$$

Backscattering & reciprocity

$$\underline{k} = \frac{1}{2} \text{Trace}([S]\Psi) = [k_1, k_2, k_3]^T$$

The second order statistics are necessary.

$$[C_3] = \langle \underline{k} \cdot \underline{k}^+ \rangle$$

Covariance matrix:

$$[C_3] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

Scattering matrix

Data are stored in complex form, that is real plus imaginary part

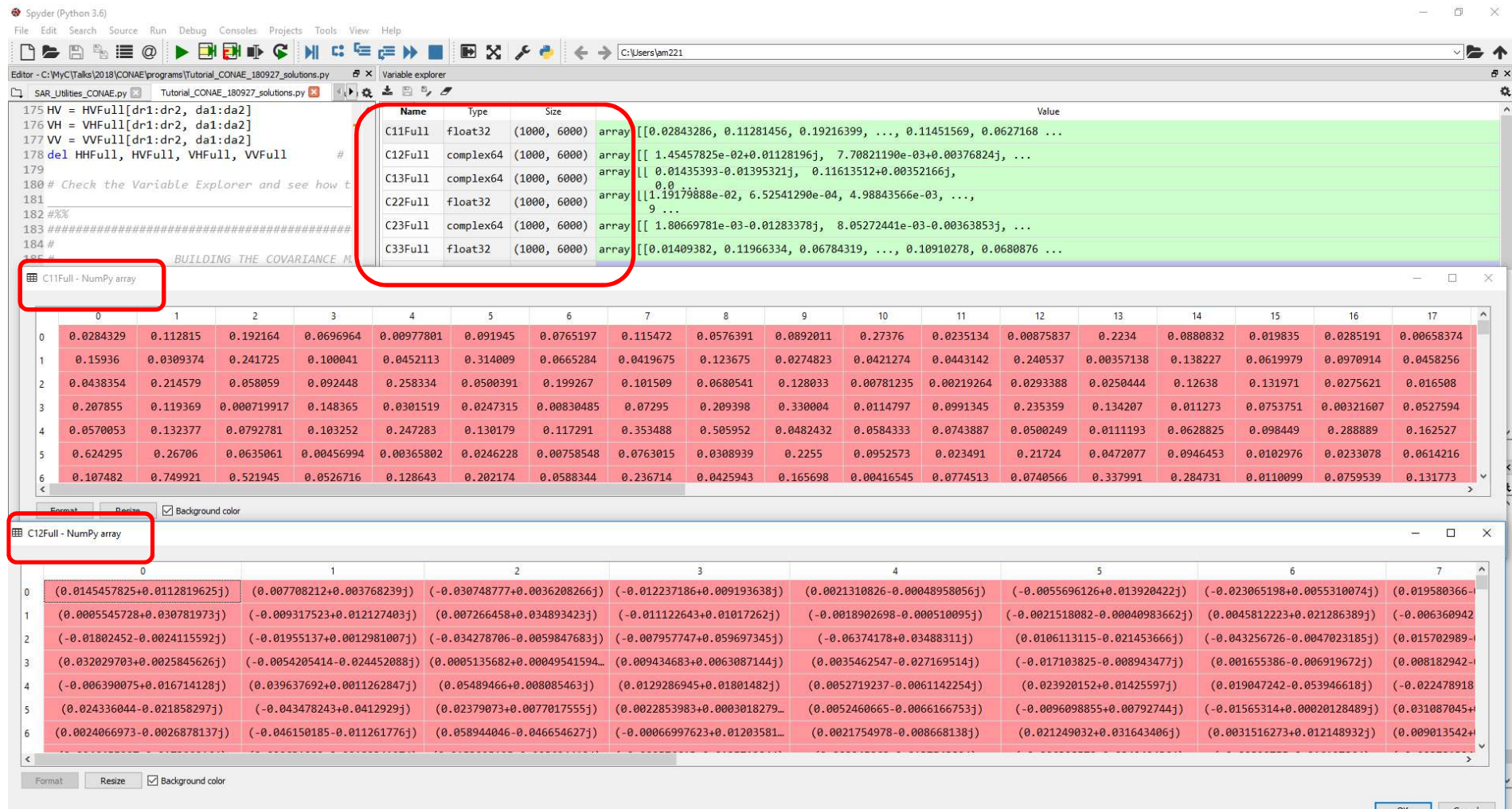
The screenshot shows the Spyder Python IDE interface. The Variable explorer on the right lists variables: HHFull (complex64 array), HVFull (complex64 array), VHFull (complex64 array), VVFull (complex64 array), env1 (module), filei_HH (str), and filei_HV (str). The HHFull - NumPy array window is open, displaying a 2D array of complex numbers. The array has 12 columns and 23 rows. The first column is labeled '6' and the second column is labeled '7'. The array contains complex numbers in the form (real + imaginary*j).

	6	7	8	9	10	11	12
208	(0.07107788+0.0001118965j)	(-0.2165731-0.11895645j)	(-0.037696168+0.027938405j)	(0.16672957-0.25383106j)	(0.09106964-0.16733629j)	(-0.2292527+0.13909335j)	(-0.11490008+0.11884244j)
209	(-0.28057334-0.042206895j)	(0.38021272+0.029810535j)	(0.43548566+0.26694945j)	(-0.138532+0.40491346j)	(-0.5769423+0.1847391j)	(-0.015036544-0.05962692j)	(0.26831898-0.1743832j)
210	(0.030044155+0.34601098j)	(-0.5461137-0.025732089j)	(-0.29259968-0.14295235j)	(0.20367235-0.5841537j)	(0.37775946-0.262635j)	(-0.21820979+0.16362272j)	(-0.17708418-0.09521227j)
211	(-0.41205642+0.051513102j)	(0.7546207+0.32724288j)	(1.0687426+0.5270206j)	(0.054809444+0.82367045j)	(-0.3507389+0.1318827j)	(0.21761155-0.41316128j)	(0.08479531-0.22683923j)
212	(-1.1572694-0.38786665j)	(1.9590037+0.3208726j)	(1.9236386+1.241693j)	(-1.1034379+2.4863272j)	(-2.1077378+0.6184698j)	(0.90856504-0.7555746j)	(1.2048659-0.31549478j)
213	(0.03483691+0.24929173j)	(-0.06724173-0.19649513j)	(-0.052806515+0.14856093j)	(-0.33817375+0.041899033j)	(-0.325384-0.35427806j)	(0.15580273+0.25812522j)	(0.3568068+0.18579605j)
214	(0.05983149+0.14247231j)	(0.09123571-0.019802434j)	(-0.0056780856-0.08619735j)	(0.0276807+0.026372923j)	(0.004342114+0.07160738j)	(-0.16508183-0.13053997j)	(-0.10213288-0.1436391j)
215	(-0.23911689+0.13675046j)	(0.2510476+0.20256251j)	(0.1657779+0.29408714j)	(-0.14119186+0.3051973j)	(-0.14424676+0.11273051j)	(0.019122131-0.149513j)	(0.011736556-0.17397477j)
216	(0.14865826+0.14563574j)	(-0.14296752+0.18294339j)	(-0.08458038+0.04554746j)	(0.11610123-0.26472777j)	(0.27141842-0.2700538j)	(0.014428847+0.092346266j)	(-0.16020864+0.009083515j)
217	(0.14413048+0.050818104j)	(0.1473479+0.08817709j)	(0.023391057-0.08530626j)	(0.035040505+0.107569896j)	(-0.057849325+0.24947123j)	(0.12625532-0.06831674j)	(0.2286333-0.123649314j)
218	(0.035153415+0.10457542j)	(-0.011003431+0.120111205j)	(0.03158148+0.0710468j)	(0.17607468-0.20794402j)	(0.22993661-0.093825236j)	(-0.052824214+0.10840995j)	(-0.13797311+0.015566013j)
219	(0.03727213-0.21260321j)	(0.1176478-0.0311192722j)	(0.03230082-0.16893798j)	(-0.08027681-0.031375308j)	(-0.05084166+0.05267703j)	(-0.06616261-0.09878135j)	(-0.100596376-0.028747175j)
220	(0.26260537-0.008604212j)	(0.15181771-0.14325972j)	(-0.21277122-0.3136897j)	(-0.06775208-0.13144417j)	(0.060304996+0.14034642j)	(-0.038857333+0.015634881j)	(0.023326596+0.09556501j)
221	(-0.042971827+0.23937722j)	(0.13073908+0.1270226j)	(0.048672948-0.06346875j)	(0.021206522-0.088813424j)	(0.1901794-0.2169216j)	(-0.029284136+0.024858948j)	(-0.29060984+0.03422946j)
222	(-0.037735436+0.0066418382j)	(-0.099647336-0.12131322j)	(-0.05169738-0.36016986j)	(-0.14253348+0.10819008j)	(-0.116834626+0.09651822j)	(0.056414068-0.17803384j)	(0.1667285-0.24515238j)
223	(0.070434734-0.07451398j)	(-0.1756553-0.12745978j)	(-0.08949843-0.15957025j)	(0.03456987-0.07367534j)	(-0.032520145+0.09075477j)	(0.0029854525+0.062434208j)	(0.08751949-0.08023951j)



Covariance matrix

- ✓ An easy way to use this is by creating one image each element of the covariance matrix
- ✓ Pay attention that cross-diagonal elements are Complex numbers.



Target decomposition

What is a decomposition?

Wikipedia definition: **Decomposition (or rotting)** is the process by which organic substances are broken down into simpler forms of matter.

Collins definition:

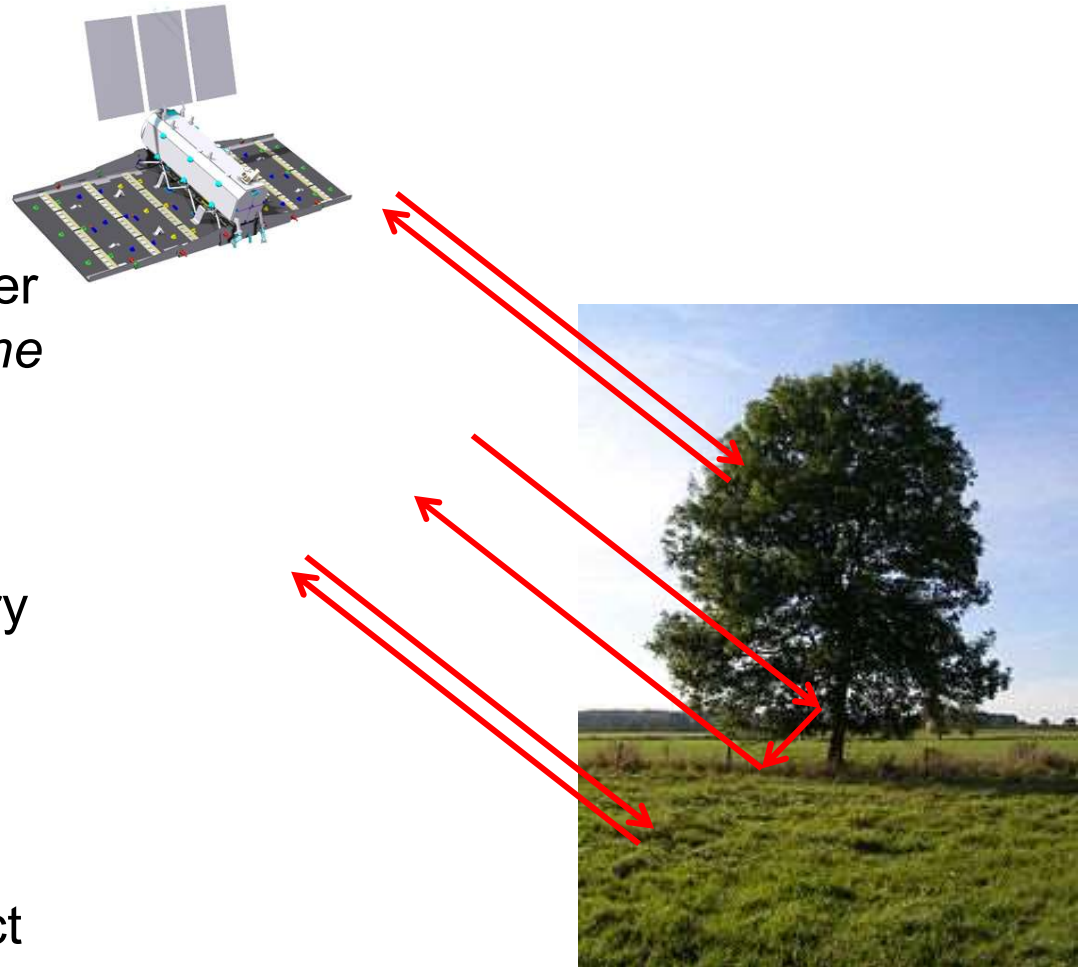
decompose (,di:kəm'pəʊz)

- 1) to break down (organic matter) or (of organic matter) to be broken down physically and chemically by bacterial or fungal action;
- 2) chem to break down or cause to break down into simpler chemical compounds
- 3) to break up or separate into constituent parts
- 4) (tr) maths to express in terms of a number of independent simpler components, as a set as a canonical union of disjoint subsets, or a vector into orthogonal components



What is a decomposition?

- ✓ On the scene, several targets are **combined/mixed** each other inside the *resolution cell* AND the *averaging window*.
- ✓ It makes image **interpretation** and **retrieval** of parameters very complex
- ✓ We want to use polarimetry to **separate** (or decompose) the different contributors and extract some physical interpretation.



What shall we decompose?

As for the basic **concepts** of polarimetry, we should separate **deterministic** and **statistical** targets

Scattering matrix

$$S = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

1) Coherent Decompositions

2) Incoherent decompositions

Covariance matrix

$$[T] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

Coherent decompositions: Scattering matrix

Coherent decompositions

$$[S] = c_1 [S_1] + c_2 [S_2] + c_3 [S_3]$$

$$c_1, c_2, c_3 \in \mathbb{C}$$

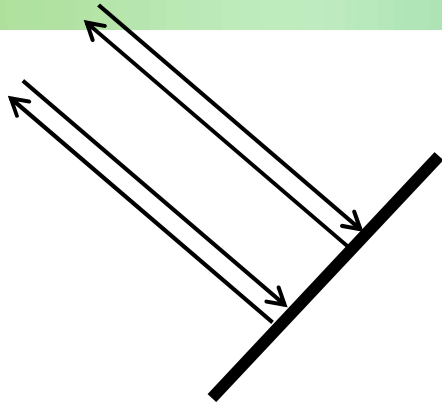
$$S = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

$$S = e^{j\varphi_{HH}} \begin{bmatrix} A_{HH} & A_{HV} e^{j(\varphi_{HV} - \varphi_{HH})} \\ A_{VH} e^{j(\varphi_{VH} - \varphi_{HH})} & A_{VV} e^{j(\varphi_{VV} - \varphi_{HH})} \end{bmatrix}$$

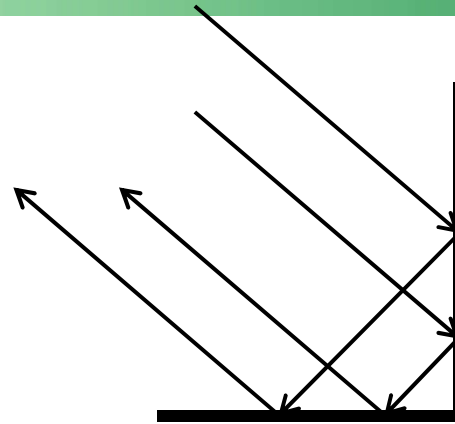
Absolute phase

- ✓ Definition: they are called **COHERENT** because they separate the contributions at the sub-pixel level starting from the **scattering matrix** and the contributors sum “coherently” (i.e. with the phase)

Pauli coherent decomposition



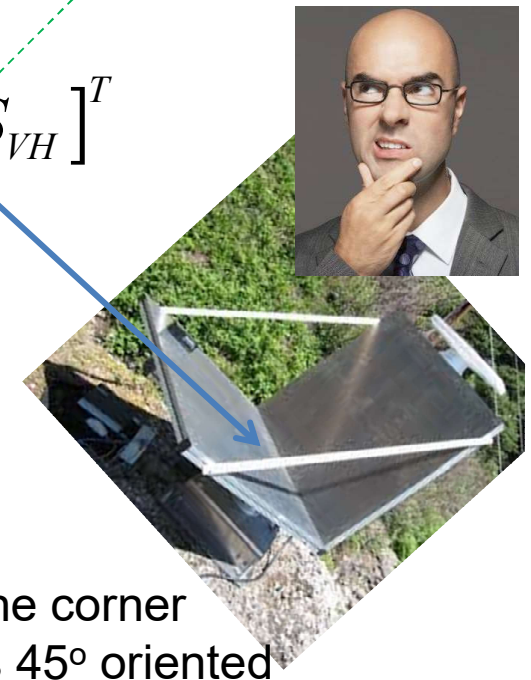
Odd-bounce



Even-bounce

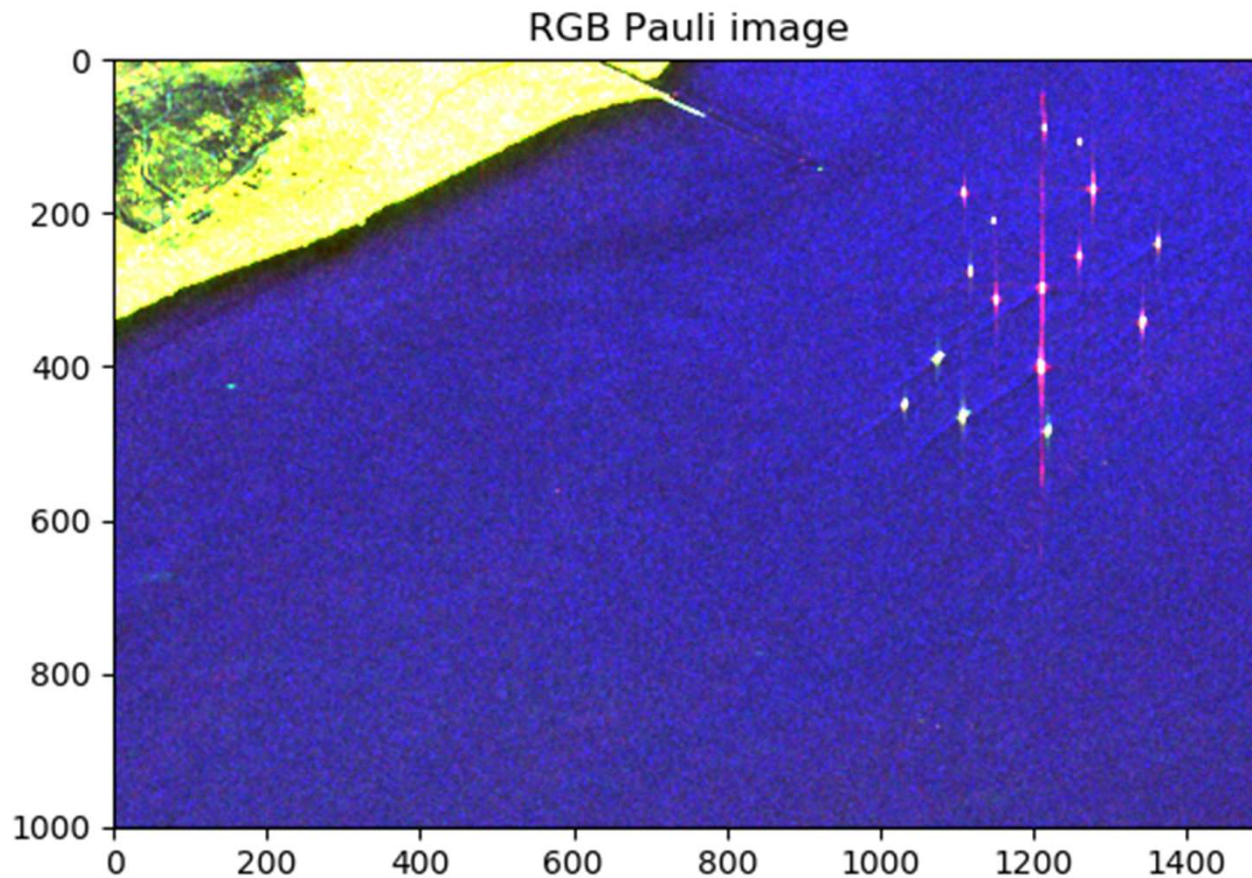
Even-bounce
45° oriented

$$\underline{k}_p = \frac{1}{\sqrt{2}} [S_{HH} + S_{VV}, S_{HH} - S_{VV}, S_{HV} + S_{VH}]^T$$



The corner
is 45° oriented

Pauli decomposition



Pauli RGB image around Buenos Aires (ALOS-1). The polarimetric information is coded in the colours. As you can notice we can use colours to differentiate between targets.
Data courtesy of JAXA.

Incoherent decompositions: Covariance matrix

Incoherent decompositions

$$[T] = c_1 [T_1] + c_2 [T_2] + c_3 [T_3]$$

$$c_1, c_2, c_3 \in R$$

$$[T] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

- ✓ Definition: they are defined incoherent because they separate the contribution starting from the **coherency matrix**, therefore the components sum each other **WITHOUT the phase**
- ✓ This is based on the assumption that the components/contributors are **independent** of each other and therefore they sum incoherently (without phase).

**Incoherent decompositions:
Non-model based**

Diagonalising the coherency matrix: Cloude-Pottier

It is based on the **diagonalisation** of the coherency matrix which is *Hermittian positive semi-definite*

$$I = \underline{\omega}^{*T} [T] \underline{\omega} \geq 0$$

$$[T] = [U][\Sigma][U]^{*T} = \sum_{i=1}^3 \lambda_i \underline{u}_i \underline{u}_i^{*T} = \lambda_1 \underline{u}_1 \underline{u}_1^{*T} + \lambda_2 \underline{u}_2 \underline{u}_2^{*T} + \lambda_3 \underline{u}_3 \underline{u}_3^{*T}$$

$$[U]^{*T} [U] = [I] \Rightarrow [U]^{*T} = [U]^{-1}$$

Unitary matrix

Eigenvalues

Eigenvectors

$$[T] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

- ✓ Each component represents a **deterministic** target (it could be expressed by a single scattering matrix): i.e. each component is a rank one matrix.

Cloude-Pottier: interpreting eigenvalues

Nice math, but what all this eigenvalues tells us?

We can define a **probability** of each eigenvalue $P_i = \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}$

We can calculate the **Entropy**:
of the scattering process

$$H = \sum_{i=1}^3 (-P_i \log_3 P_i)$$

We can also calculate the **Anisotropy**:

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}$$

An interesting property is that the parameters on this slide are **basis invariant**: i.e. the same results are obtained independently on the basis used to represent the scattering vector. This is a property of diagonalisations... and we like it, since it makes the result more general.

Cloude-Pottier: interpreting eigenvalues

- ✓ The entropy tells us the **confusion** of the scattering process. If there is a component (i.e. one eigenvector) that is much stronger than the other, then the entropy is **LOW** (close to 0) and we know there is only one dominant target in the scene (i.e. this is a more deterministic problem that could be treated with a single scattering matrix). An example is a man-made target.
- ✓ If the entropy is **HIGH** (close to 1) there are three or more equally strong scattering processes in the scene that they confuse a lot the polarisation of the pixels. An example is a forested area.
- ✓ The anisotropy tells about the **imbalance** of second and third scattering mechanisms (eigenvalues). It is used to complement the entropy... you will learn more next lecture.



Cloude-Pottier: interpreting eigenvectors

- ✓ What about the eigenvectors? They are **3 scattering mechanisms orthogonal each other**
- ✓ Their **representation** (i.e. the numbers in the vector components) is not basis invariant and we need to select a basis to visualise them (since they are vectors)
- ✓ The Cloude-Pottier decomposition consider using the **Pauli** basis and perform a parameterisation based on spherical coordinates (with unitary radius)

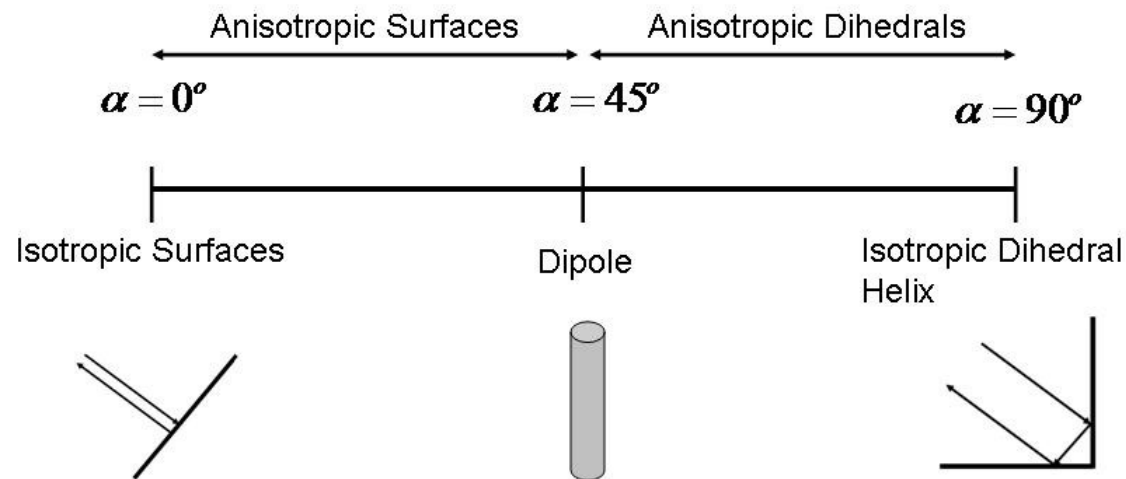
Scattering vector in Pauli basis with spherical coordinates

$$\underline{u}_i = \left[\cos \alpha_i, \sin \alpha_i \cos \beta_i \cdot e^{j\varepsilon_i}, \sin \alpha_i \sin \beta_i \cdot e^{j\eta_i} \right]^T, \quad i=1,2,3$$

Each one of the eigenvectors can be represented this way

Cloude-Pottier: interpreting eigenvectors

1) The parameter α is related to the type of scattering mechanism (it can be easily proved substituting the values of alpha in the previous parameterisation)



2) The parameter β is related to the orientation of the scattering mechanism (also can be easily proved substituting the values in the previous parameterisation)

3) The parameters ϵ and η are phases with complicated physical interpretation (but they stay the same once decided the target to represent)

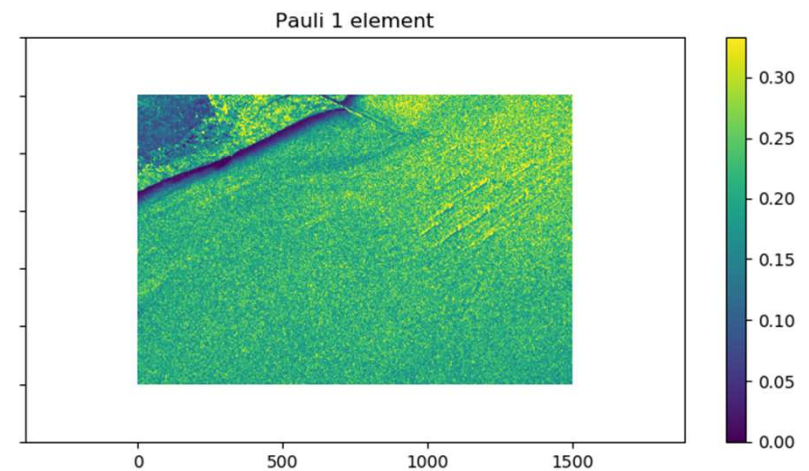
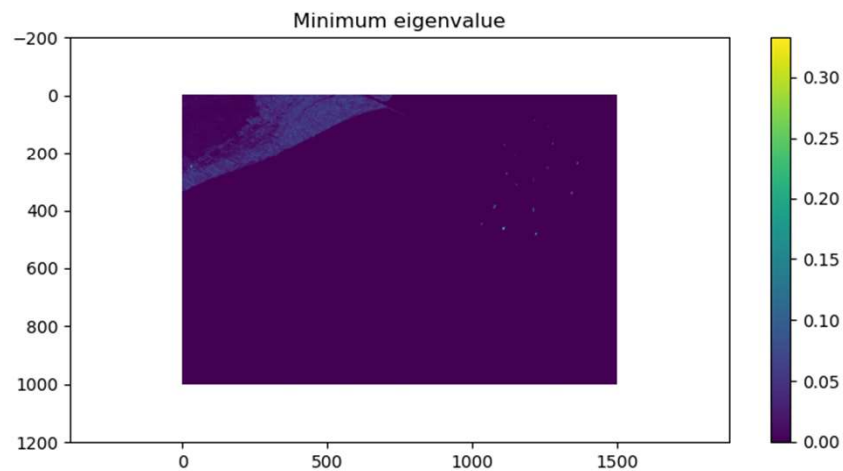
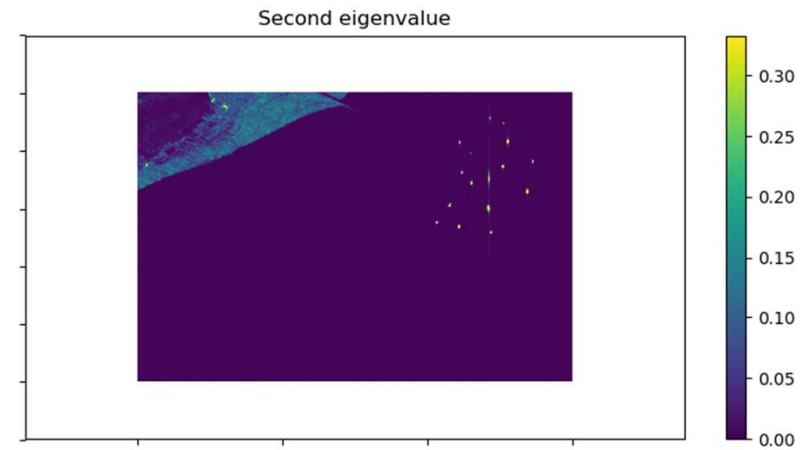
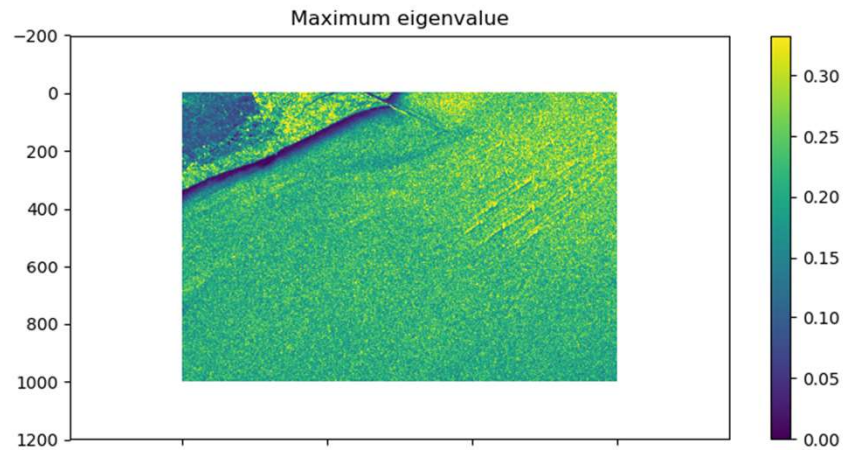
Cloude-Pottier: interpreting eigenvectors

- ✓ We have three α angles in the decomposition (one from each scattering mechanism). Which one shall we use?
 - ✓ If the entropy is **low** (one dominant target) we can use the dominant α
 - ✓ If the entropy is **high**, the process is very confused and it is better to use an **averaged** value for α .
 - ✓ We consider a **Bernulli** process to average the α (i.e. we do a weighted average where the weights are the probability of the eigenvalues).
- ✓ The same is for β , we can consider dominant or averaged values

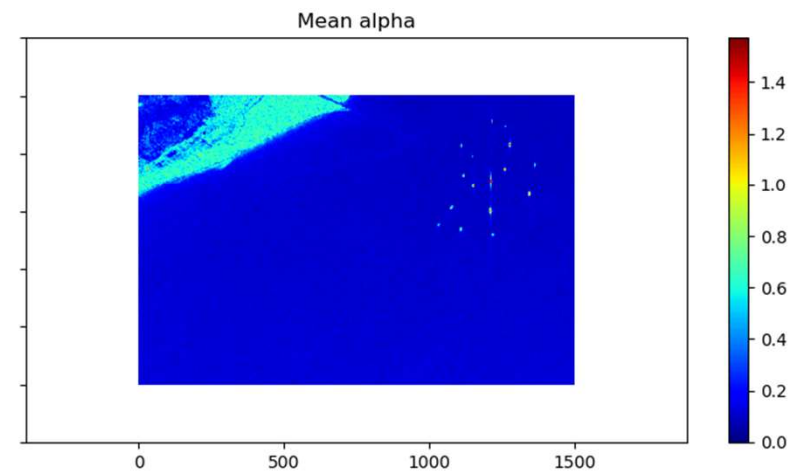
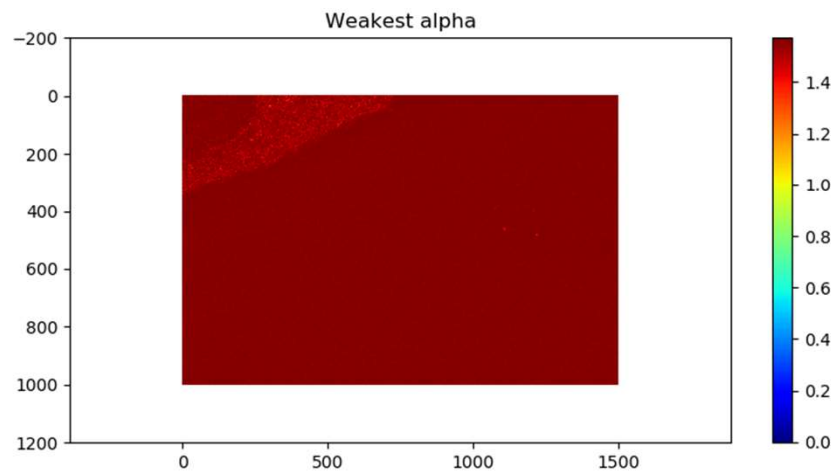
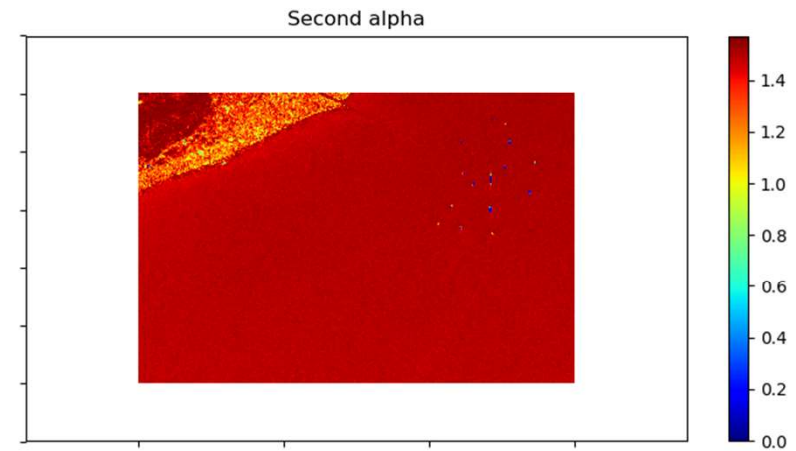
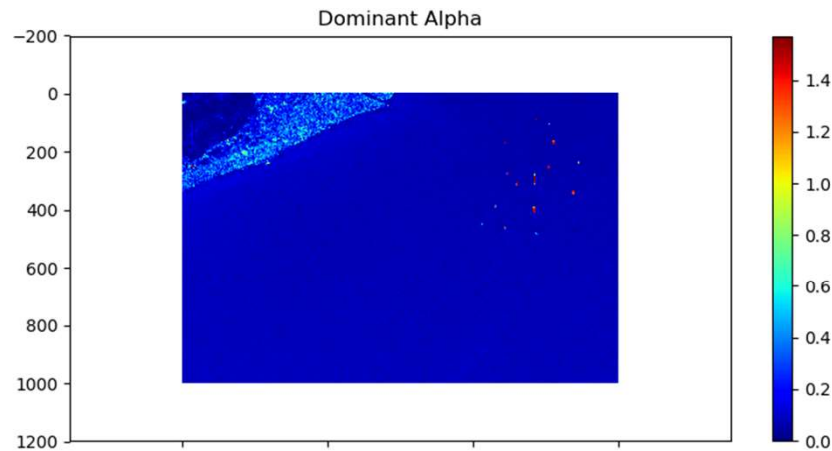
It is useful to calculate an average α ,
obtained as the result of a Bernulli process

$$\hat{\alpha} = \sum_{i=1}^3 (P_i \alpha_i)$$

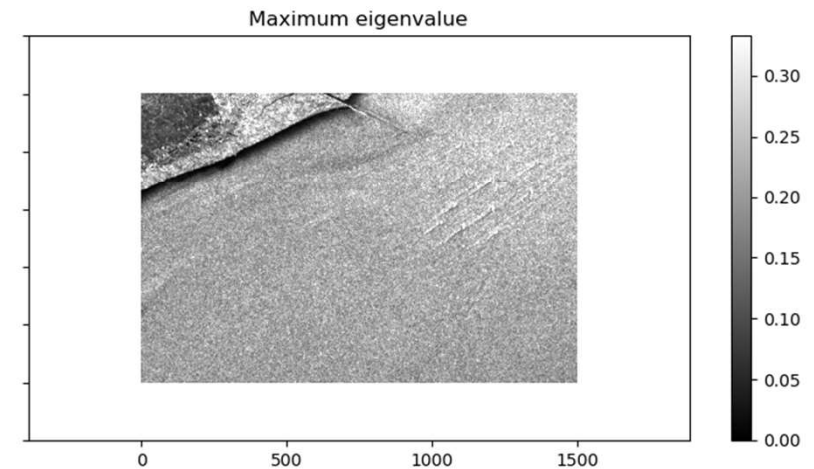
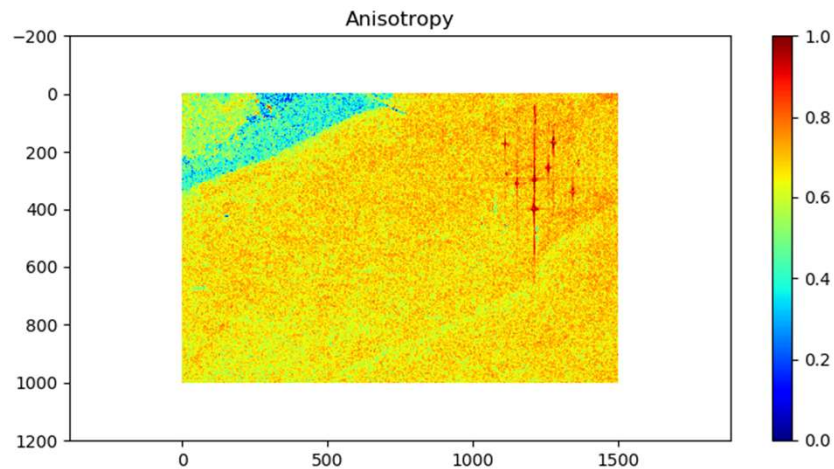
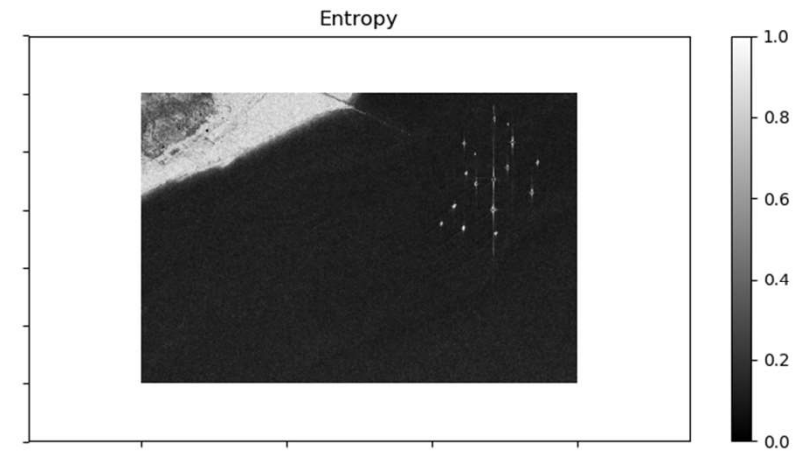
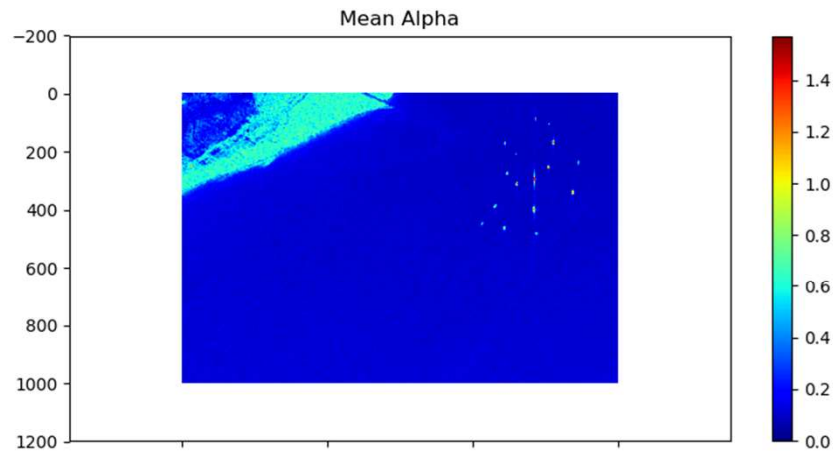
Cloude-Pottier: Buenos Aires (ALOS-1)



Cloude-Pottier: Buenos Aires (ALOS-1)



Cloude-Pottier: Buenos Aires (ALOS-1)



Incoherent decompositions: Model based

Yamaguchi decomposition

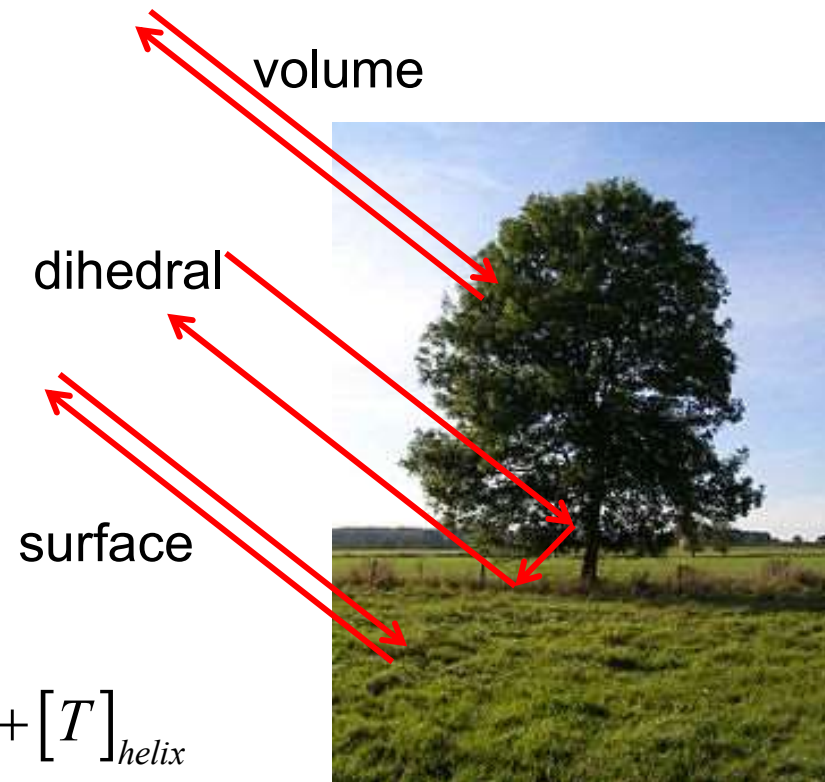
It is based on a model for the backscattering of **forested areas**.

The total return is decomposed in
Surface, **Dihedral**, **Volume** and Helix scattering.

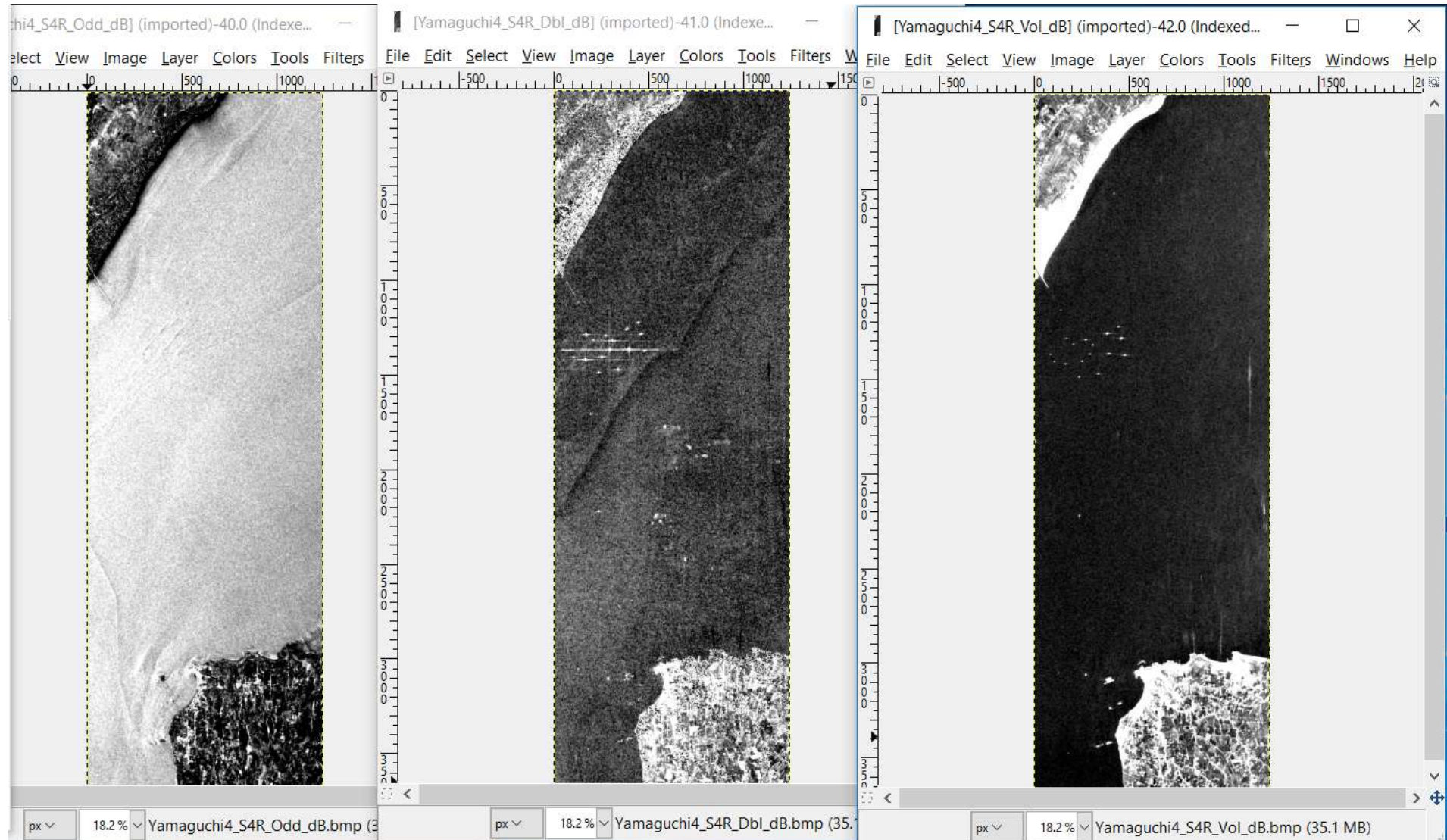
In order to solve the problem with the **orientation** of the dihedrals, it perform a **correction** for the orientation angle

It rotates the partial target in order to give it an *overall horizontal orientation*

$$[T] = [T]_{surface} + [T]_{dihedral} + [T]_{volume} + [T]_{helix}$$



Yamaguchi decomposition: Buenos Aires (ALOS-1)



Practical

PolSARPro

- ✓ Today you will use the POLSARpro software to investigate some of the polarimetric theory you have studied.
 - ✓ Pauli decomposition
 - ✓ Covariance and Coherency matrix
 - ✓ Claude Pottier decomposition
 - ✓ Yamaguchi decomposition
 - ✓ Compact pol (Raney decomposition)
 - ✓ Ship detection

Python

- ✓ Tomorrow you will use Python to process polarimetric data.
 - ✓ Pauli decomposition
 - ✓ Covariance and Coherency matrix
 - ✓ Claude Pottier decomposition
 - ✓ Ship detection
- ✓ You will be give the code with missing parts to complete.